1^{st} ICTP-Trieste/ICTP-SAIFR School on Particle Physics: Dark Matter and Particle Physics – Some exercises

Patrick J. Fox*

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Below are a list of questions, some simple, some involved, that will help illustrate some of the points raised in lectures. The questions approximately follow the order of topics in the lectures, but you should feel free to approach them in any order you wish.

1 Lecture III

Question 1. *Co-annihilation a.* If co-annihilation modes are available during freeze out they can dramatically alter the final DM abundance. Including the additional channels leads to an "effective" annihilation cross section,

$$\langle \sigma_{eff} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}} , \tag{1}$$

with $n = \sum_i n_i$. Consider the toy example of two states in the dark sector $\chi_{1,2}$ where $m_1 < m_2$. χ_1 has only weak interactions whereas χ_2 has strong interactions. We thus take the various annihilation cross sections to be in the ratios $\langle \sigma_{22} v \rangle = \frac{\alpha_s}{\alpha} \langle \sigma_{21} v \rangle = \left(\frac{\alpha_s}{\alpha}\right)^2 \langle \sigma_{11} v \rangle$. Show that

$$\langle \sigma_{eff} v \rangle = \langle \sigma_{11} v \rangle \left(\frac{1 + \frac{\alpha_s \omega}{\alpha}}{1 + \omega} \right)^2 \tag{2}$$

with $\omega = (1 + \Delta)^{3/2} e^{-\Delta x} g_2 / g_1$, $\Delta = (m_2 - m_1)/m_1$. How close do the masses of χ_1 and χ_2 have to be for the presence of χ_2 to have an appreciable effect on the relic abundance?

Question 2. *Co-annihilation b.* Consider another interesting limit where there is little actual co-annihilation but the second state still impacts the final abundance, namely $\sigma_{12} \ll \sigma_{11}$, σ_{22}. Show that this can lead to a smaller effective cross section, therefore larger relic abundance. Explain what is going on physically.

*pjfox@fnal.gov