Problems for 2018 ICTP SAIFR Lectures on Higgs and Flavor Physics

Benjamín Grinstein

June 2018

Contents

Lecture 0 – Review

0.1 Spontaneous Symmetry breaking, Nambu-Goldstone, Higgs Mechanism

---

Exercises

Exercise 1-1: For a single complex scalar field, $|\phi(x)| = v$ is not sufficient to characterize the minimum. Why? (Hint: Compute $E[e^{i\alpha(x)}v]$).

Exercise 1-2: If $\chi(x)$ is a real scalar field, invent an action integral with SSB and explain which symmetry(ies) is(are) broken.

Exercise 1-3: In lecture we considered a model of two complex scalars, ϕ and χ with

$$\mathcal{L} = |\partial_\mu \phi|^2 + |\partial_\mu \chi|^2 - V_1(|\phi|) - m^2|\chi|^2 + \tfrac{1}{2}\lambda[(\phi^*\chi)^2 + (\chi^*\phi)^2],$$

and assumed that V_1 is minimized at $|\phi| = v$. We found that due to SSB the complex field χ breaks into two real fields, A and B, of mass-squared $m_A^2 = m^2 + \lambda v^2$ and $m_B^2 = m^2 - \lambda v^2$. Discuss the case $m^2 < \lambda v^2$.

Exercise 1-4: Show that the spectrum of the model of a single complex scalar $\phi(x)$ symmetric under global $U(1)$ transformations (that is, under $\phi(x) \to e^{i\alpha}\phi(x)$) consists of two degenerate states when the symmetry is not spontaneously broken.

Exercise 1-5: Determine the Energy functional for a U(1)-gauge invariant single complex field with Lagrangian density $\mathcal{L} = D_\mu\phi^* D^\mu\phi$.

Exercise 1-6: Consider scalar-QED: a theory of a single complex scalar with a gauged U(1) symmetry:

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(|\phi|)$$

Determine the equations of motion of the gauge fields A_0 and $\vec{A}$. What's peculiar about the equation of motion for A_0?

0.2 Group theory

Exercises

Exercise 2-1: Give explicitly a 4-dimensional irreducible representation of SU(2) (more specifically, the three matrices that form a basis for the Lie Algebra of SU(2)).
Follow-up: Find explicitly the 1-dimensional ("trivial") representation.

Exercise 2-2: Find all inequivalent 4-dimensional representations of the Lie algebra of SU(2)

Exercise 2-3: Suppose the Lie algebra is spanned by n hermitian, traceless matrices, T^a, $a = 1, \ldots, n$. Show that $(t^a)^{bc} = \kappa f^{abc}$, where f^{abc} are the structure constants, constitute an n-dimensional representation, for some constant κ, and determine the value of this constant.

Exercise 2-4: For $SU(N)$ show that the adjoint representation Adj (defined in the previous problem) had $\dim(Adj) = N^2 - 1$.

Exercise 2-5: For $SU(3)$, why are triplets necessarily complex but octets can be real?

0.3 Non-abelian (yang-Mills) gauge theories

Exercises

Exercise 3-1: Under $A_\mu \to A'_\mu = U(A_\mu + \frac{1}{ig}\partial_\mu)U^{-1}$, show that $F_{\mu\nu} \to F'_{\mu\nu} = UF_{\mu\nu}U^{-1}$.

Exercise 3-2: Show that $\mathrm{Tr}(F_{\mu\nu}) = 0$.

Exercise 3-3: Write $-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ in terms of the component fields $A^a_\mu(x)$ (where $A_\mu(x) = A^a_\mu(x)T^a$).

Exercise 3-4: (i) Let $\phi(x)$ be an N-component complex scalar, transforming as a fundamental of $SU(N)$. Assume ϕ has a vacuum expectation value (VEV) that breaks this symmetry spontaneously,

$$\langle\phi(x)\rangle = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ v \end{pmatrix}$$

(ii) Display explicitly the $2N-1$ broken generators of $SU(N)$.

(iii) The mass-square matrix, $(\mathcal{M}^2)^{ab}$ for the vector bosons of gauged $SU(N)$ generated by this VEV is

$$\tfrac{1}{2}g^2v^2(T^aT^b)_{NN}$$

Show that all non-zero eigenvalues of $(\mathcal{M}^2)^{ab}$ are common (ie, all equal).

(iv) Write out the Feynman rules for this model.

Chapter 1

Lecture 1

1.1 The Standard Model

Exercises

Exercise 1.1-1: In the Standard Model (SM) take the fields to have quantum numbers as in lecture. In particular H, a doublet, has $Y = \frac{1}{2}$. Assume

$$\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Describe what must be changed with respect to the calculations done in class, where we took instead

$$\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Exercise 1.1-2: Write explicitly the interaction terms of quarks and leptons with the weak vector bosons and the photons, that is, the gauge fields of $SU(2) \times U(1)$, in the SM.

Exercise 1.1-3: This is a longer exercise, more like a small **project**. The aim is to compute some decay properties of the Z boson. We could have just as well done this for the $W^\pm$, but the Z is a good example and there is some interesting physics that goes along with this.

(i) Compute, at tree level, the partial decay width of the Z boson into the following final states: e^+e^-, $\nu\bar{\nu}$, $u\bar{u}$, $d\bar{d}$.

(ii) The μ and τ leptons have the same quantum numbers as the electron. Look up their masses in the PDG (not in google), and the mass of the Z boson, and argue that to good approximation the partial decay widths are equal, $\Gamma(Z \to \tau^+\tau^-) \approx \Gamma(Z \to \mu^+\mu^-) \approx \Gamma(Z \to e^+e^-)$. How good is this approximation?

(iii) Similarly, find out the masses of the quarks and argue that for decays to the down, strange and bottom quarks $\Gamma(Z \to d\bar{d}) \approx \Gamma(Z \to s\bar{s})\Gamma(Z \to b\bar{b})$ and that for decays to the up and charm quarks $\Gamma(Z \to u\bar{u}) \approx \Gamma(Z \to c\bar{c})$.

(iv) However, for decays to the top quark show that $\Gamma(Z \to t\bar{t}) = 0$.

(v) Argue that at lowest order in perturbation theory the total decay width (that is, the inverse lifetime) of the Z boson is given by

$$\Gamma(Z) = 3\Gamma(Z \to e^+e^-) + 3\Gamma(Z \to \nu\bar{\nu}) + 3\Gamma(Z \to d\bar{d}) + 2\Gamma(Z \to u\bar{u})$$

(vi) Compute the branching fractions for all these decays. The branching fraction for the decay of Z into a final state f is defined by

$$\mathrm{Br}(Z \to f) = \frac{\Gamma(Z \to f)}{\Gamma(Z)}$$

(vii) Evaluate your answers numerically. Check against the PDG.

(viii) The PDG gives that the branching fraction into four leptons ("$\ell^+\ell^-\ell^+\ell^-$") is four orders of magnitude than the one into a lepton pair ("$\ell^+\ell^-$"). How does the SM account for this?

The PDG also gives a lot of interesting information. Notably, $\mathrm{Br}(Z- \to e^\pm\mu^\mp <$ 7.5×10^{-7}. As we will see, Z interactions are "diagonal in flavor space", that is Z couples to, eg, $\bar{e}e$ and to $\bar{\mu}\mu$ but not to $\bar{e}\mu$ or $\bar{\mu}e$.

Chapter 2

Lecture 2

2.1 Charged & neutral currents and the ρ parameter

Exercise 2.1-1: Using Fermi theory, that is, an effective interaction hamiltonian given by

$$\mathcal{H}' = \frac{G_F}{\sqrt{2}} J_\lambda^\dagger J^\lambda$$

where the charged current is given by

$$J^\lambda(x) = \bar{\nu}_e(x)\gamma^\lambda(1-\gamma_5)e(x) + \bar{\nu}_\mu(x)\gamma^\lambda(1-\gamma_5)\mu(x),$$

determine the partial width for $\mu \to e\nu_\mu\bar{\nu}_e$ decay. Neglect the masses of electron an neutrinos. You should obtain

$$\Gamma(\mu \to e\nu_\mu\bar{\nu}_e) = \frac{1}{192\pi^3} G_F^2 m_\mu^5$$

The lifetime is the inverse total width. First argue that this is the dominant decay mode of the muon (question: what else can it decay into?), and then compute the lifetime using the PDG values of G_F and m_μ, and compare with the PDG value of the lifetime.
Suggestion: Save a little time by using the PDG expressions for 3-body phase space integrals.

Exercise 2.1-2: (i) Compute $\Delta\rho/\rho$ for a model with two doublet fields, that is, H_1 and H_2 both in the $2_{\frac{1}{2}}$ representation of $SU(2)\times U(1)$, both with non-zero VEVs, $\langle H_i\rangle \neq 0$ and both preserving charge, $Q\langle H_i\rangle = 0$ (where $Q = T^3 + Y$).

(ii) Generalize the result to the case of an arbitrary number n of higgs doublets, H_i.

2.2 Custodial symmetry

Exercise 2.2-1: In lecture we introduced the 2×2 matrix field

$$\phi(x) = \left(\begin{pmatrix} \tilde{H}(x) \end{pmatrix} \begin{pmatrix} H(x) \end{pmatrix} \right)$$

and explained how, in the absence of hypercharge interactions, the Lagrangian $\mathcal{L} = \frac{1}{2}\mathrm{Tr}(D_\mu \phi^\dagger D^\mu \phi)$ is invariant under an $SU(2)_L \times SU(2)_R$ group of transformations, with action

$$\phi \to L\phi R^\dagger, \qquad \text{with} \quad L \in SU(2)_L \quad \text{and} \quad R \in SU(2)_R$$

(i) Show that hypercharge is a $U(1)$ subgroup of $SU(2)_R$, with $R \in U(1)$ generated by $\frac{1}{2}\sigma^3$, that is, $R = \exp(i\omega\sigma^3/2)$.

(ii) Write an expression of the covariant derivative $D_\mu \phi$ in matrix notation, that includes both the W and B fields.

(iii) Give the generators of the custodial symmetry, $SU(2)_C$, in terms of the generators T_L^a and T_R^a of $SU(2)_L$ and $SU(2)_R$, respectively.

(iv) Show that hypercharge breaks custodial symmetry. Argue that this contributes to $\Delta\rho$ only at 1-loop order.

Exercise 2.2-2: If $y^U = y^D \equiv y^Q$ combine the right handed fields u_R and d_R into a doublet q_R of $SU(2)_R$ (see previous exercise). Rewrite the Yukawa couplings of quarks of the SM in terms of the matrix ϕ and the doublets q_L and q_R in a manner that makes explicit that these terms are symmetric under $SU(2)_L \times SU(2)_R$ and that they are symmetric under the $SU(2)_C$ custodial symmetry. Argue that $\Delta\rho \propto m_d - m_u$

Exercise 2.2-3: **Harder** Let the self-energy of the $W^\pm$ and W^3bosons be $\Pi_W^{\mu\nu}(p)$ and $\Pi_3^{\mu\nu}(p)$, respectively. They give a contribution to $\Delta\rho$ by shifting the denominators in propagators of vector bosons at small momentum, so that

$$\frac{\Delta\rho}{\rho} = \frac{\Pi_W(0) - \Pi_3(0)}{M_W^2},$$

where $\Pi(0)$ stands for the coefficient of $\eta^{\mu\nu}$ in $\Pi^{\mu\nu}$.
Calculate the 1-loop contribution of the u and d quarks to $\Delta\rho/\rho$.
Note: The diagrams that need to be computed and the result of the computation are given in the lecture notes, at the bottom of page 9, in "Lecture 1-2".

Chapter 3

Lecture 3 – Flavor Theory

3.1 Introduction: What/Why/How?

3.2 Flavor in the Standard Model

3.3 The CKM matrix and the KM model of CP-violation

Exercise 3.3-1: In lecture we saud: Now, choose to make a redefinition by matrices that diagonalize the mass terms,

$$V_{u_L}^\dagger y^U V_{u_R} = y^{U\prime}, \quad V_{d_L}^\dagger y^D V_{d_R} = y^{D\prime}\ . \tag{3.1}$$

Here the matrices with a prime, $y^{U\prime}$ and $y^{D\prime}$, are diagonal, real and positive. Show that this can always be done. That is, that an arbitrary matrix M can be transformed into a real, positive diagonal matrix $M' = P^\dagger M Q$ by a pair of unitary matrices, P and Q.

Exercise 3.3-2: Warning: This problem is difficult, deceptively so; do not spend too much timem on it.

In QED, charge conjugation is $\bar{e}\gamma^\mu e \to -\bar{e}\gamma^\mu e$ and $A^\mu \to -A^\mu$. So $\bar{e}\not{A}e$ is invariant under C.

So what about QCD? Under charge conjugation $\bar{q}T^a\gamma^\mu q \to \bar{q}(-T^a)^T\gamma^\mu q$, but $(-T^a)^T = (-T^a)^*$ does not equal $-T^a$ (nor T^a). So what does charge conjugation mean in QCD? How does the gluon field, A^a_μ, transform?

Exercise 3.3-3: If two entries in m_U (or in m_D) are equal show that V can be brought into a real matrix and hence is an orthogonal transformation (an element of $O(3)$).

Exercise 3.3-4: Show that

(i) $\beta = \arg\left(-\dfrac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$, $\alpha = \arg\left(-\dfrac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$ and $\gamma = \arg\left(-\dfrac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$.

(ii) These are invariant under phase redefinitions of quark fields (that is, under the remaining arbitrariness, often called "re-phasing of quark fields"). Hence these are candidates for observable quantities.

(iii) The area of the triangle is $-\frac{1}{2}\,\mathrm{Im}\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = -\frac{1}{2}\,\frac{1}{|V_{cd}V_{cb}^*|^2}\,\mathrm{Im}\,(V_{ud}V_{cd}^*V_{cb}V_{ub}^*)$.

(iv) The product $J = \mathrm{Im}\,(V_{ud}V_{cd}^*V_{cb}V_{ub}^*)$ (a "Jarlskog invariant") is invariant under re-phasing of quark fields.

Note that $\mathrm{Im}\left(V_{ij}V_{kl}V_{il}^*V_{kj}^*\right) = J(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})$ is the common area of all the un-normalized triangles. The area of a normalized triangle is J divided by the square of the magnitude of the side that is normalized to unity.

Exercise 3.3-5: (i) Show that

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*},$$

hence $\bar{\rho}$ and $\bar{\eta}$ are indeed the coordinates of the apex of the unitarity triangle and are invariant under quark phase redefinitions.

(ii) Expand in $\lambda \ll 1$ to show

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Chapter 4

Lecture 4

4.1 Flavor Symmetry and MFV

Exercise 4.1-1: Had we considered an operator like $O_1 = G^a_{\mu\nu} H \bar{u}_R T^a \sigma^{\mu\nu} y^U q_L$, but with $\widetilde{H} \bar{d}_R$ instead of $H\bar{u}_R$ the flavor off-diagonal terms would have been governed by $y^{D,diag} V^\dagger$. Show this is generally true, that is, that flavor change in any operator is governed by V and powers of y^{diag}.

Exercise 4.1-2: Exhibit examples of operators of dimension 6 that produce flavor change without involving $y^{U,D}$. Can these be such that only quarks of charge $+2/3$ are involved? (These would correspond to Flavor Changing Neutral Currents; see Sec. **??** below).

4.2 2HDM

Exercise 4.2-1: Write explicitly the most general potential, $V(H_1, H_2)$, for the 2HDM, up to quartic terms (so that it is renormalizable). Which terms depend on the alignment of $\langle H_2 \rangle$ relative to that of $\langle H_1 \rangle$? How would you arrange coefficients so that $\langle H_1 \rangle$ and $\langle H_2 \rangle$ are aligned?

Exercise 4.2-2: In the 2HDM, show that the cubic couplings of h^0 and H^0 to vector bosons are given by

$$2\frac{M_W^2}{v} \sin(\alpha + \beta) h^0 W^+_\mu W^-_\mu + 2\frac{M_Z^2}{v} \sin(\alpha + \beta) h^0 Z^0_\mu Z^0_\mu$$

and

$$2\frac{M_W^2}{v} \cos(\alpha + \beta) H^0 W^+_\mu W^-_\mu + 2\frac{M_Z^2}{v} \cos(\alpha + \beta) H^0 Z^0_\mu Z^0_\mu$$

Exercise 4.2-3: In the 2HDM the cubic couplings $h^\pm W^\mp Z^0$, and $A^0 W^+ W^-$ and $A^0 Z^0 Z^0$ vanish.

Exercise 4.2-4: In the 2HDM compute the $H^0 A^0 Z^0_\mu$ coupling.

Exercise 4.2-5: How do the 5 scalars of the 2HDM transform under custodial $S(2)_C$ symmetry?

4.3 Georgi-Machacek model

Exercise 4.3-1: If

$$\langle H\rangle = \frac{v_H}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}, \quad \langle\chi\rangle = v_\chi\begin{pmatrix}0\\0\\1\end{pmatrix}, \quad \text{and} \quad \langle\xi\rangle = v_\chi\begin{pmatrix}0\\1\\0\end{pmatrix},$$

show that $M_W^2 = \frac{1}{4}g_2^2 v^2$ where $v^2 = v_H^2 + 8v_\chi^2$

Exercise 4.3-2: **Project** This model has 13 real scalars – 3 eaten scalars = 10 physical scalars.

(i) Work it out: find the physical fields in terms of the components of

$$H = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}, \quad \chi = \begin{pmatrix}\chi^{++}\\ \chi^+\\ \chi^0\end{pmatrix}, \quad \text{and} \quad \xi = \begin{pmatrix}\xi^+\\ \xi^0\\ \xi^-\end{pmatrix}.$$

(ii) Find their couplings to two vector bosons.

(iii) Can a singly charged scalar decay to WZ? Can a doubly charged scalar decay to W^+W^+? If so compute the decay rates.

(iv) Can a doubly charged higgs decay to a W^+ and a singly charged higgs, and if so what is the decay rate?

(v) Neglecting the decay of a doubly charged higgs to a pair of singly charged higgsses, and the decays into more than two bodies, compute the branching fractions for decays of doubly and singly charged higgses as a function of their masses.

(vi) Come up with one more question about the properties of higgses in the Georgi-Machacek model and solve it. Particularly intersting questions highlight differences between this model and the 2HDM or the SM.

Exercise 4.3-3: Show that under $SU(2)_C$ the 10 states transform as $5\oplus3\oplus1\oplus1$ (and as always the eaten fields transform as 3). It is slightly more difficult (and more work) to give explicitly the components of these representations in terms of the components of H, χ and ξ.

Chapter 5

Lecture 5

5.1 Flavor in the 2HDM

Exercise 5.1-1: If $V^\mu(x) = \bar{u}(x)\gamma^\mu d(x)$ (a vector current), show that parity (P) invariance of the strong and electromagnetic interactions implies $\langle 0|V^\mu(0)|\pi^-(\vec{p})\rangle = 0$.

Exercise 5.1-2: If $A^\mu(x) = \bar{u}(x)\gamma^\mu\gamma_5 d(x)$ (a vector current), show that $\langle 0|A^\mu(x)|\pi^-(\vec{p})\rangle = e^{-ip\cdot x} f_\pi p^\mu$.

Exercise 5.1-3: By taking the divergence (a derivative) of the axial current, show that

$$\langle 0|\bar{u}(0)\gamma_5 d(0)|\pi^-(\vec{p})\rangle = \frac{f_\pi m_\pi^2}{m_u + m_d}$$

Exercise 5.1-4: Add to the SM a charged scalar $h^\pm$ of mass M with $\mathcal{L}_{Yuk} = \kappa_q h^- \bar{d}\gamma_5 u + \kappa_\ell h^- \bar{e}(1-\gamma_5)\nu + \text{h.c.}$. Compute the contribution, $\Delta\Gamma$, to the partial width for $\pi^- \to e^-\bar{\nu}$. Compare with SM. If $\kappa_q \sim \kappa_\ell \sim 1$ give a rough lower bound on M so that $\Delta\Gamma$ is not in conflict with $\Gamma^{exp}(\pi^- \to e^-\bar{\nu})$

Exercise 5.1-5: Compute charged higgs couplings to quarks. Express your answer in terms of V (CKM matrix), the diagonal mass matrices, v (the SM VEV) and $\beta = v_2/v$.

5.2 More flavor

Exercise 5.2-1: When we presented in class the equation for the SUSY-SM,

$$\Delta\mathcal{L}_{\text{SUSY-bkg}} = \phi_q^*\mathcal{M}_q^2\phi_q + \phi_u^*\mathcal{M}_u^2\phi_u + \phi_d^*\mathcal{M}_d^2\phi_d + (\phi_{h_1}\phi_u g_U \phi_q + \phi_{h_2}\phi_d g_D \phi_q + \text{h.c.}) \quad (5.1)$$

we said, "This breaks the flavor symmetry unless $\mathcal{M}^2_{q,u,d} \propto \mathbf{1}$ and $g_{U,D} \propto y_{U,D}$." This is not strictly correct (or, more bluntly, it is a lie). While not correct it is the simplest choice. Why? Exhibit alternatives, that is, other forms for $\mathcal{M}^2_{q,u,d}$ and $g_{U,D}$ that respect the symmetry.

Exercise 5.2-2: Classify all possible dim-4 interactions of Yukawa form made out of SM fermions and possibly new scalar fields. To this end list all possible Lorentz scalar combinations you can form out of pairs of SM quark fields. Then give explicitly the transformation properties of the scalar field, under the gauge and flavor symmetry groups, required to make the Yukawa interaction invariant. Do this first without including the SM Yukawa couplings as spurions and then including also one power of the SM Yukawa couplings.

5.3 FCNC

Exercise 5.3-1: Compute the amplitude for $Z \to b\bar{s}$ in the SM to lowest order in perturbation theory (in the strong and electroweak couplings). Don't bother to compute integrals explicitly, just make sure they are finite (so you could evaluate them numerically if need be). Of course, if you can express the result in closed analytic form, you should. See *Flavor Changing Decays of the* Z^0, by M. Clements, *et al*, in Phys. Rev. D **27**, 570 (1983).

5.4 GIM-mechanism: more suppression of FCNC in SM

5.4.1 Old GIM

5.4.2 Modern GIM

Exercise 5.4.2-1: Consider $s \to d\gamma$. Show that the above type of analysis suggests that virtual top quark exchange no longer dominates, but that in fact the charm and top contributions are roughly equally important. *Note: For this you need to know the mass of charm relative to* M_W. *If you don't, look it up!*

5.4.3 Bounds on New Physics, GIM and MFV

Exercise 5.4.3-1: Determine how much each of the bounds in Fig. **??** is weakened if you assume MFV. You may not be able to complete this problem if you do not have some idea of what the symbols ΔM_K, ϵ_K, etc, mean or what type of operators contribute to each process; in that case you should postpone this exercise until that material has been covered later in these lectures.

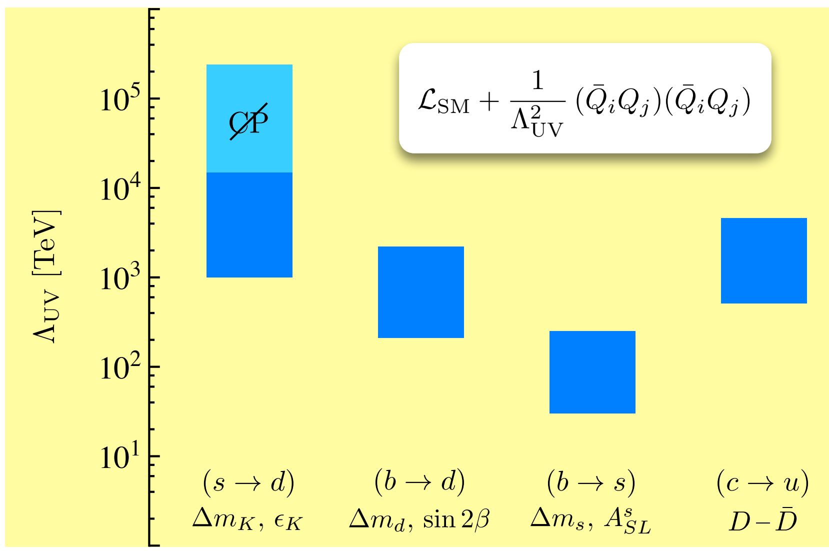

Figure 5.1: Bounds on the scale of NP scale from various processes. The NP is modeled as dimension 6 operators. No accidental suppression of the coefficient (as in MFV) is included. The $b \to s$ case is consistent with the explicit $b \to s\gamma$ example worked out in these notes. The figure is taken from M. Neubert's talk at EPS 2011.

5.5 Determination of CKM Elements

Exercise 5.5-1: Show that $q \cdot (V - A) \sim m_\ell$ for the leptonic charged current. Be more precise than "$\sim$."