Practical QCD at colliders

Giulia Zanderighi (CERN & University Oxford)

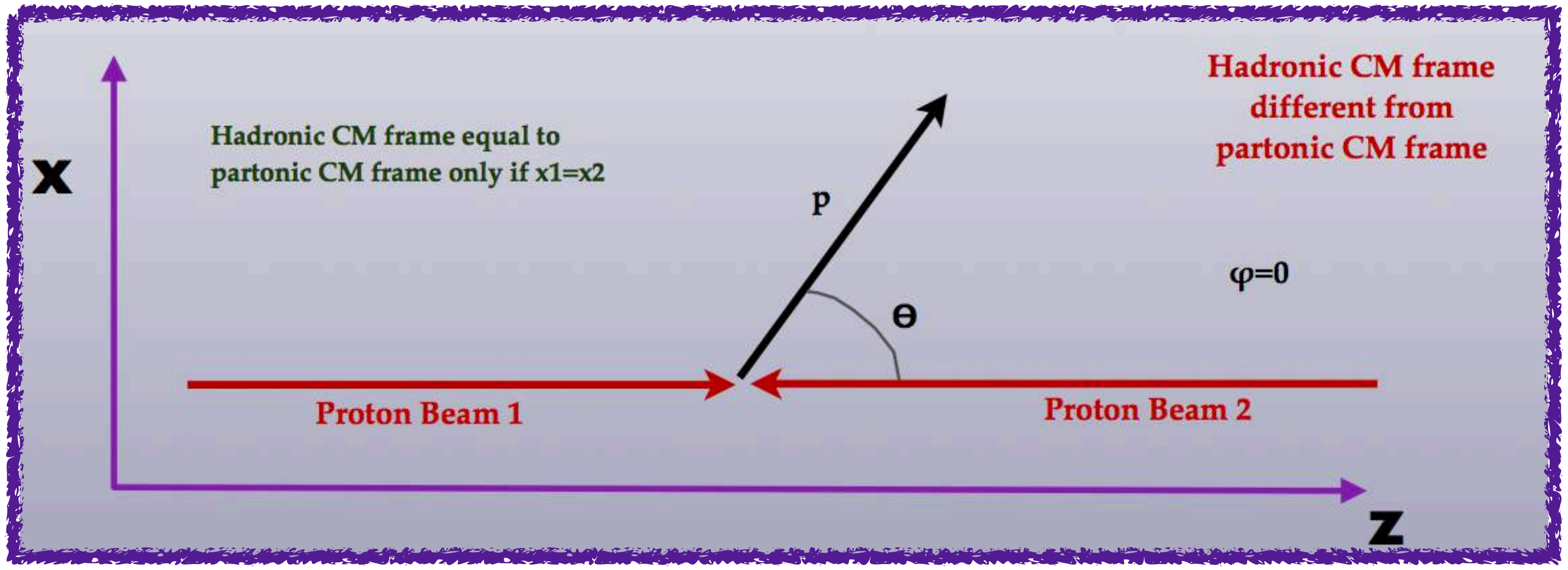
4^{th} Lecture

Joint ICTP-SAIFR school on Particle Physics — June 2018

LHC kinematics

In this lecture we want to review the application of perturbative QCD in high-energy LHC collisions

Before discussing calculations, it is important to understand the kinematics in proton-proton collisions



$$(E, p_x, p_y, p_z) = (\sqrt{\vec{p}^2 + m^2}, |\vec{p}| \sin\theta \cos\phi, |\vec{p}| \sin\theta \sin\phi, |\vec{p}| \cos\theta)$$

The total longitudinal momentum of the colliding system is unknown (one can measure missing transverse momentum, but not missing longitudinal one)

LHC kinematics

A more common parametrisation relies on rapidity and transverse mass

$$(E, p_x, p_y, p_z) = (m_T \cosh y, |p_T| \cos\phi, |p_T| \sin\phi, m_Y \sinh y)$$

With

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \qquad p_T = \sqrt{p_x^2 + p_y^2} \qquad m_T = \sqrt{p_T^2 + m^2}$$

Exercise: check that the two parametrisations are equivalent

Exercise: check that the rapidity transform linearly under a longitudinal boost

Exercise: given two particles, can you easily construct a boost-invariant quantity?

LHC kinematics

For particles with negligible mass the rapidity coincides with the pseudo-rapidity

$$y = \eta \equiv \frac{1}{2} \log \frac{1 + \cos\theta}{1 - \cos\theta} = - \log \tan \frac{\theta}{2}$$

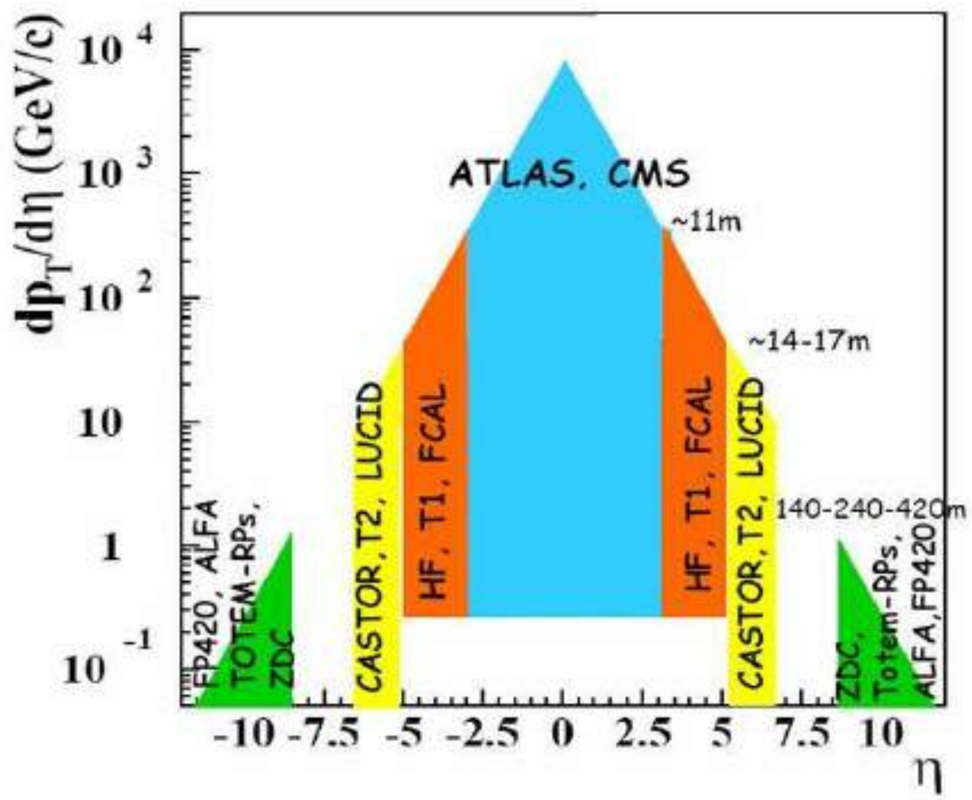
The pseudo-rapidity can then be easily translated to the detector geometric acceptance as used in experimental measurements

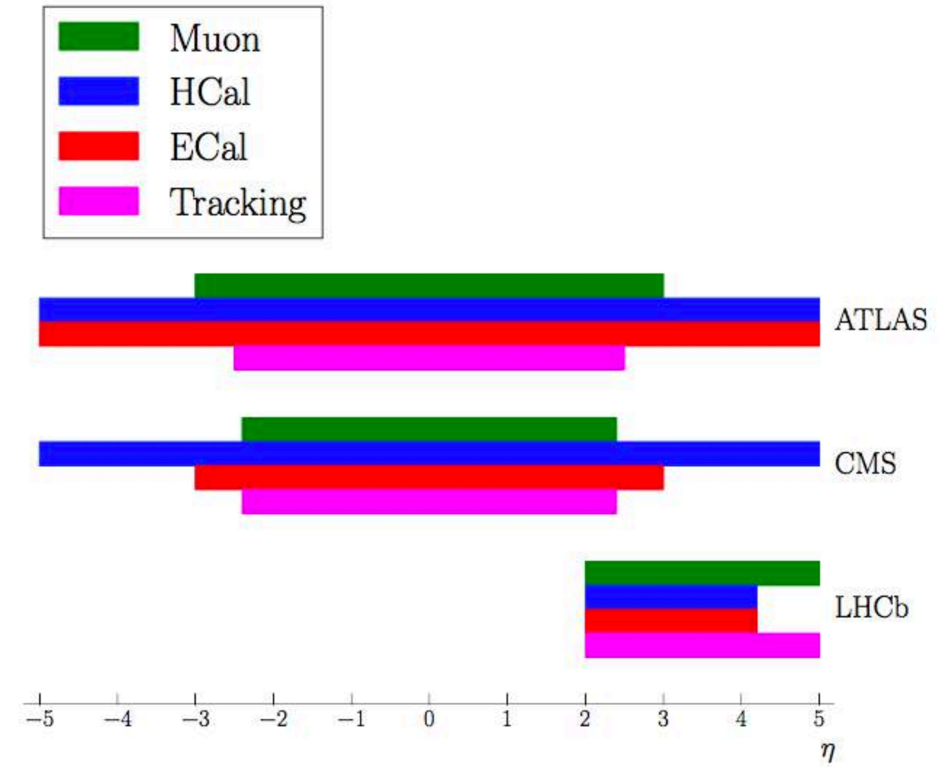
Θ	10^{-4}	10^{-3}	10^{-2}	0.1	0.5	1	$\pi/2$
Y	9.9	7.6	5.3	3	1.36	0.6	0

(The other hemisphere has same but negative numbers)

Rapidity coverage of LHC detectors

The achieved maximum rapidity coverage is important in LHC detectors





- For ATLAS and CMS: muons can be detected only in the central regions, while for jets and hadrons, hadronic calorimetry extends up to 4.5-5 (essential for processes like vector boson fusion Higgs production)
- LHCb covers better the forward region, but only forward one
- Studies are ongoing to determine the required/possible rapidity coverage of future detectors

LHC kinematics

Rapidity is also interesting from a theoretical point of view, as the single particle phase space is uniform in rapidity

$$\frac{d^3p}{2E(2\pi)^3} = \frac{1}{2(2\pi)^3} d^2p_T dy$$

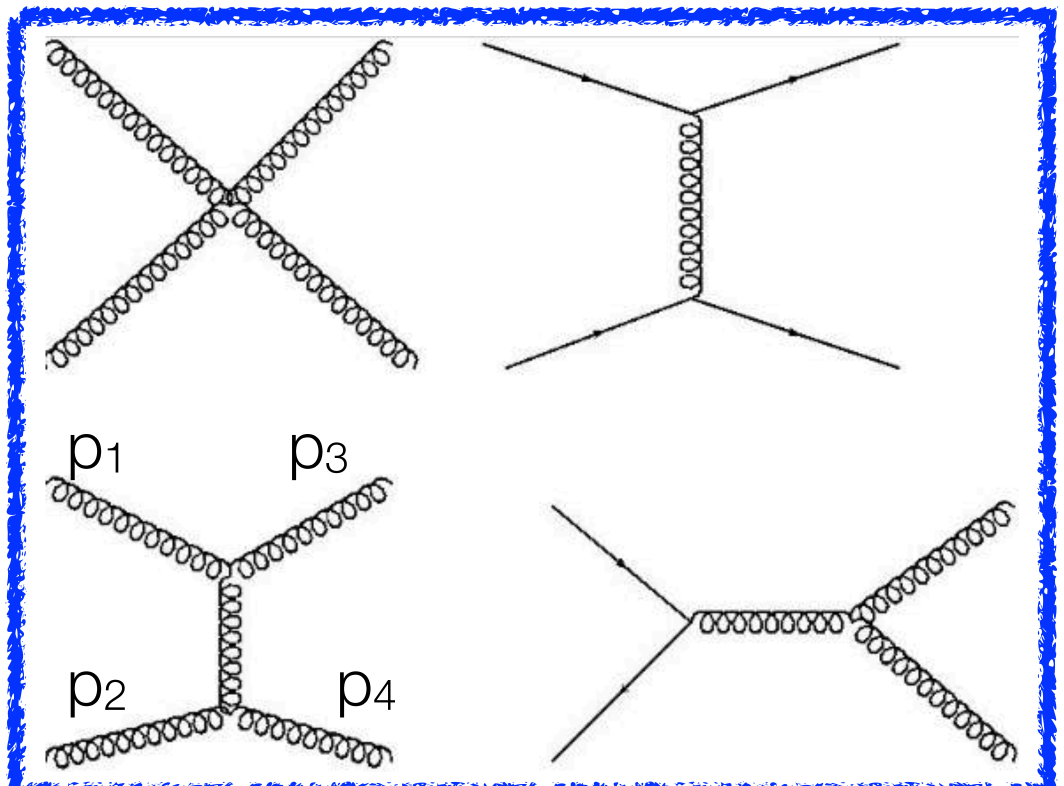
Exercise: derive the above expression (change variables and include the Jacobian of the transformation)

The above relation has already deep implications: for instance incoherent radiation (e.g. soft underlying event) is to a large extend uniform in rapidity

Dijet production

Before discussing higher-order corrections, let's discuss go through the leading order calculation of one of the main LHC process: di-jet production

Sample diagrams (all must be included)



Many partonic subprocesses contribute

Process	$\frac{d\hat{\sigma}}{d\Phi_2}$
$qq' \to qq'$	$\frac{1}{2\hat{s}}\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$
$qq \to qq$	$\frac{1}{2}\frac{1}{2\hat{s}}\left[\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)-\frac{8}{27}\frac{\hat{s}^2}{\hat{u}\hat{t}}\right]$
$q\bar{q} \to q'\bar{q}'$	$\frac{1}{2\hat{s}}\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$q\bar{q} \to q\bar{q}$	$\frac{1}{2\hat{s}}\left[\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)-\frac{8}{27}\frac{\hat{u}^2}{\hat{s}\hat{t}}\right]$
$q\bar{q} \to gg$	$\frac{1}{2}\frac{1}{2\hat{s}}\left[\frac{32}{27}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{8}{3}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$
$gg \to q\bar{q}$	$\frac{1}{2\hat{s}}\left[\frac{1}{6}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{3}{8}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$
$gq \to gq$	$\frac{1}{2\hat{s}}\left[-\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}+\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}\right]$
$gg \to gg$	$\frac{1}{2}\frac{1}{2\hat{s}}\frac{9}{2}\left(3-\frac{\hat{t}\hat{u}}{\hat{s}^2}-\frac{\hat{s}\hat{u}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$

Mandelstam variables: $\hat{s} = (p_1 + p_2)^2 \quad \hat{t} = (p_1 + p_3)^2 \quad \hat{u} = (p_1 + p_4)^2$

Dijet production

The hadronic cross-section — **PDFs for initial state partons** — **Phase space**

$$d\sigma^{\text{dijet}} = \sum_{ijkl} \frac{1}{1+\delta_{kl}} \int dx_1 dx_2 f_i(x_1,\mu^2) f_j(x_2,\mu^2) \frac{d\hat{\sigma}_{ij\to kl}}{d\Phi_2} d\Phi_2 \Theta_{\text{cuts}}$$

Symmetry factor — **Matrix elements** — **Measurement function**

We have seen that in the LAB frame

$$p_3 = (p_T \cosh y_3, p_T \cos\phi, p_T \sin\phi, p_T \sinh y_3)$$

$$p_4 = (p_T \cosh y_4, -p_T \cos\phi, -p_T \sin\phi, p_T \sinh y_4)$$

Exercise: show that the rapidities are related to the Bjorken-x variables by

$$x_1 = \frac{p_T}{\sqrt{s}}\left(e^{y_3} + e^{y_4}\right) \qquad x_2 = \frac{p_T}{\sqrt{s}}\left(e^{-y_3} + e^{-y_4}\right)$$

Dijet production

Exercise: show that the rapidities in the partonic centre-of-mass frame are given by

$$\hat{y}_3 = \frac{1}{2}\left(y_3 - y_4\right) = -\hat{y}_4$$

Exercise: show that the scattering angle in the partonic frame is given by

$$\cos\hat{\theta} = \tanh\hat{y}_3 = \tanh\left(\frac{y_3 - y_4}{2}\right)$$

this relation shows that the difference in rapidities between the jets gives direct access to the dynamics in the partonic frame

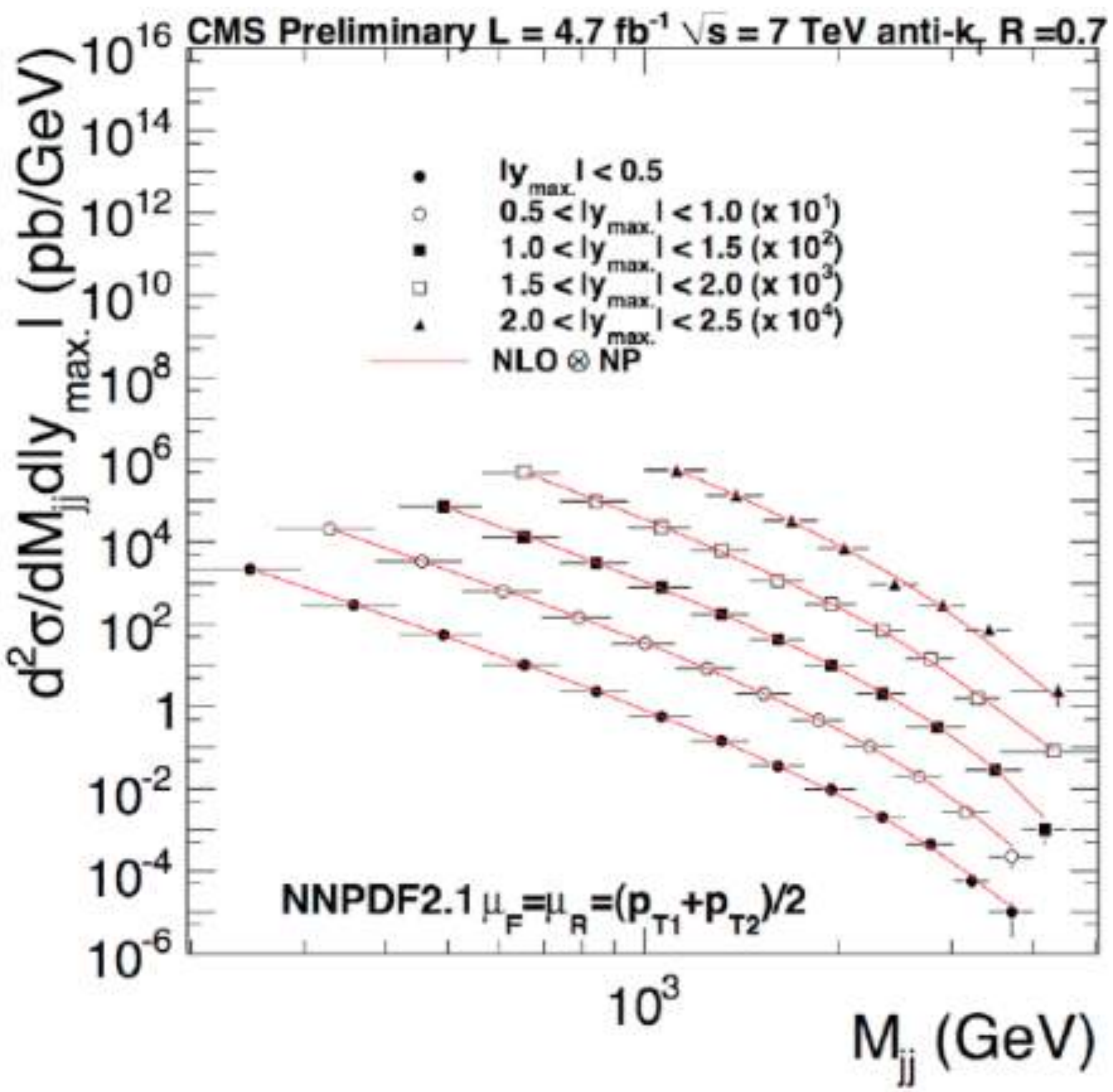
Exercise: show that in terms of rapidities the cross-section becomes

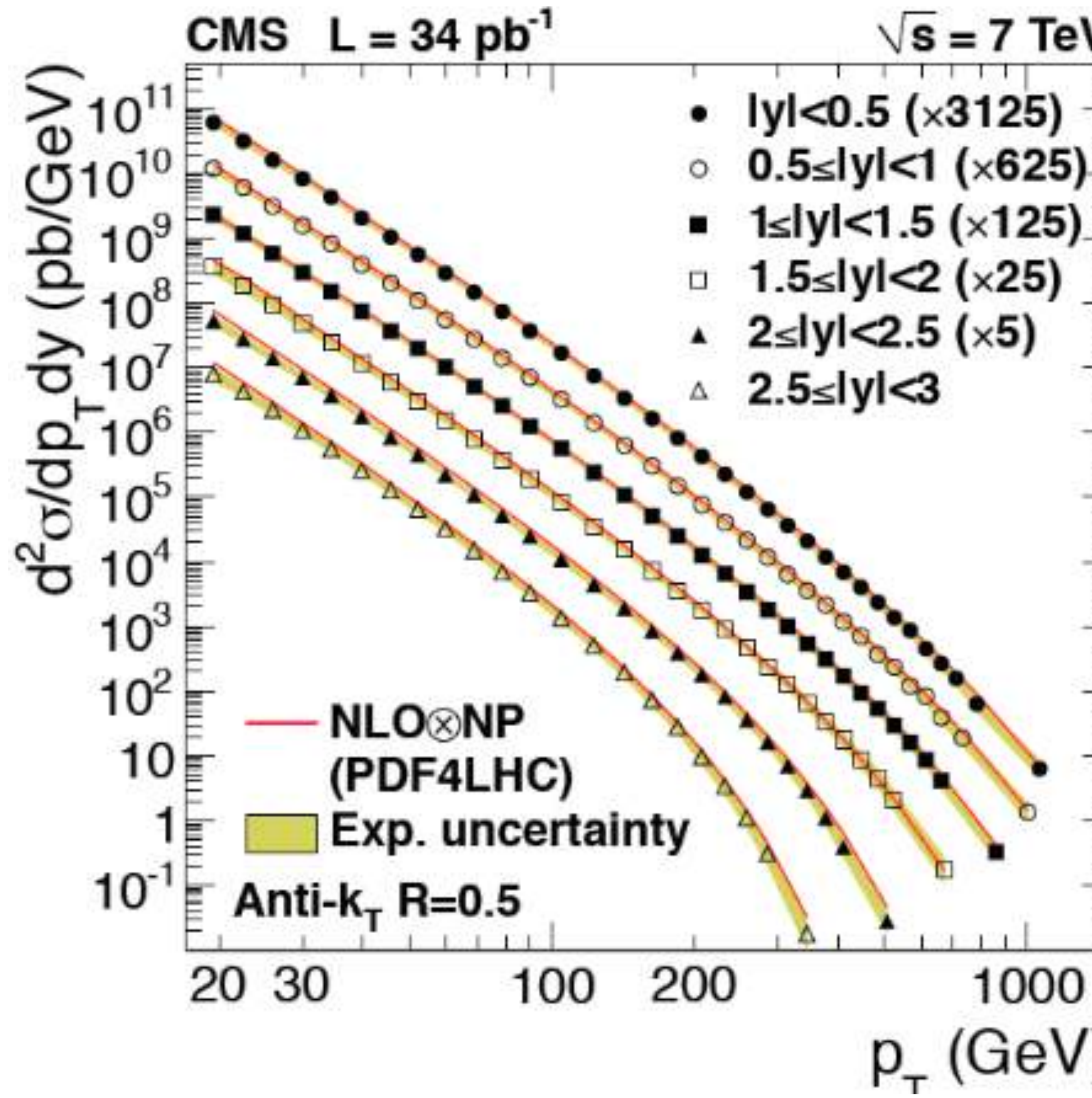
$$\frac{d^3\sigma^{\text{dijet}}}{dy_1 dy_2 dp_T^2} = \frac{1}{16\pi s}\sum_{ijkl}\frac{1}{1+\delta_{kl}}\int\frac{dx_1}{x_1}\frac{dx_2}{x_2} f_i(x_1,\mu^2) f_j(x_2,\mu^2)\frac{d\hat{\sigma}_{ij\to kl}}{d\Phi_2}$$

The above expression can be integrated numerically and provides a leading order estimate of the cross section

Dijet production

Inclusive and dijet production are extensively studies at the LHC, both for SM measurements and in searches for New physics



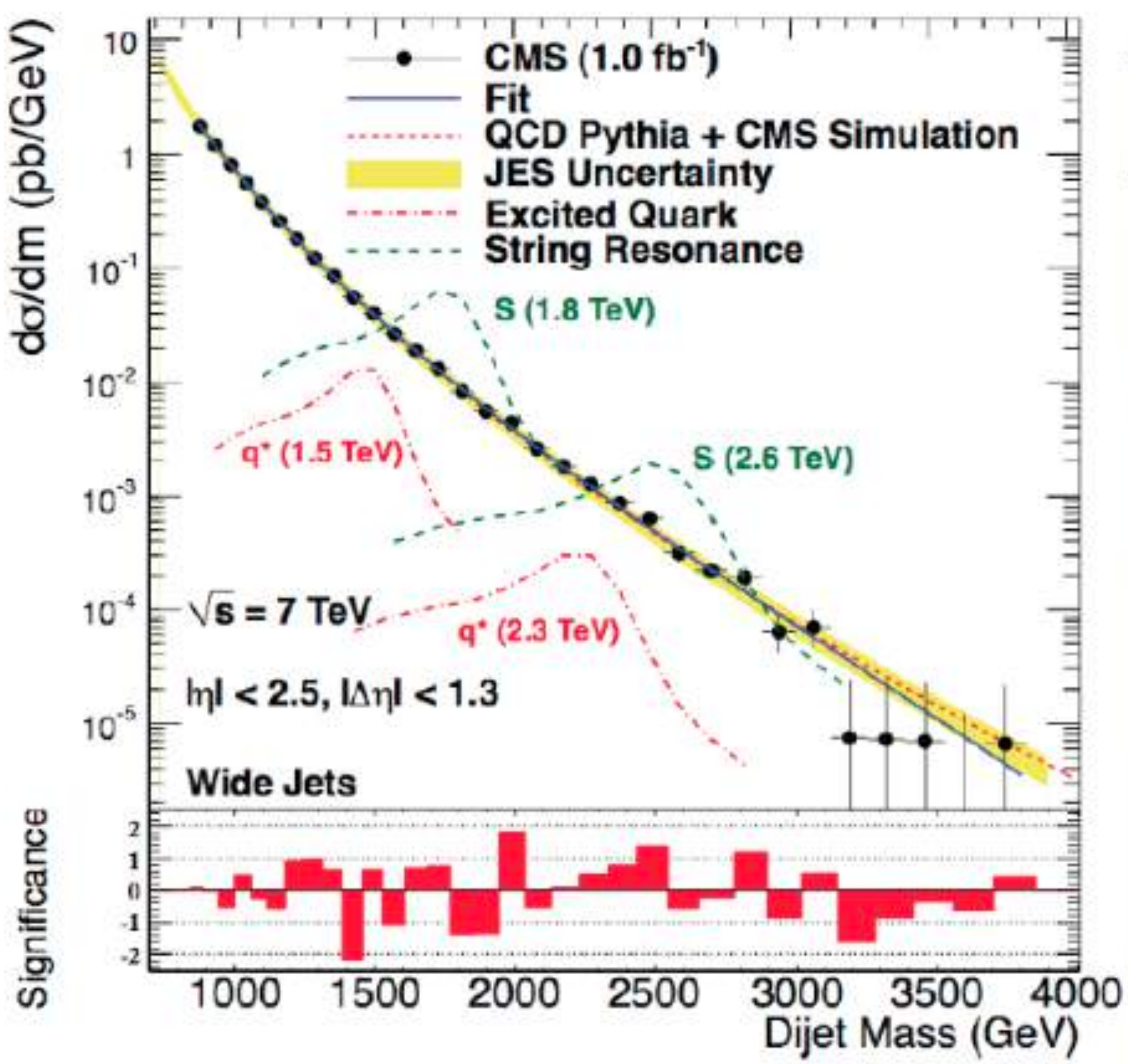


Direct determination of gluon PDF, constraints of other PDFS, measurement of α_s, probe of QCD running at TeV scales …

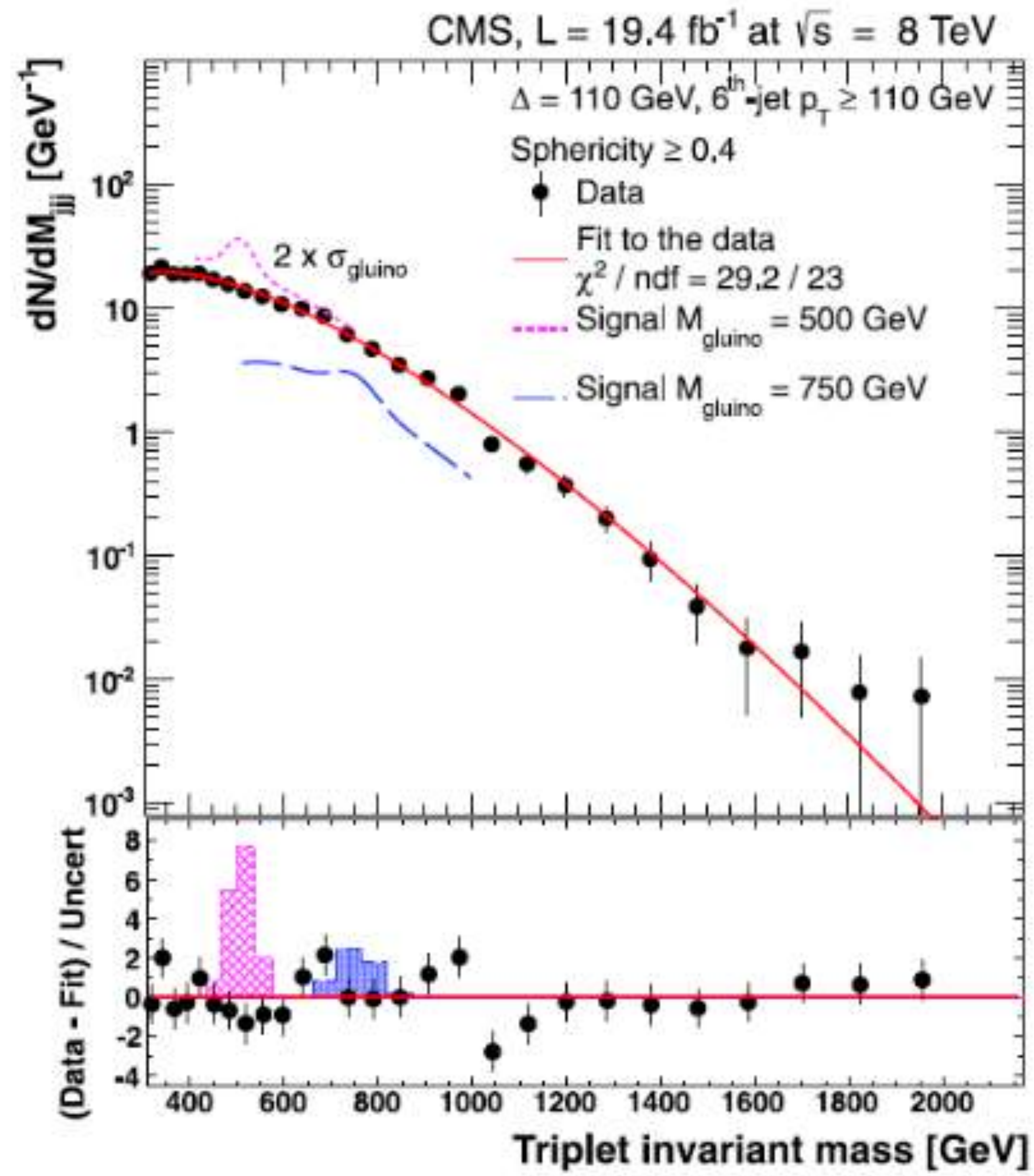
Dijet production

Inclusive and dijet production are extensively studies at the LHC, both for SM measurements and in searches for New physics

Search for excited quarks



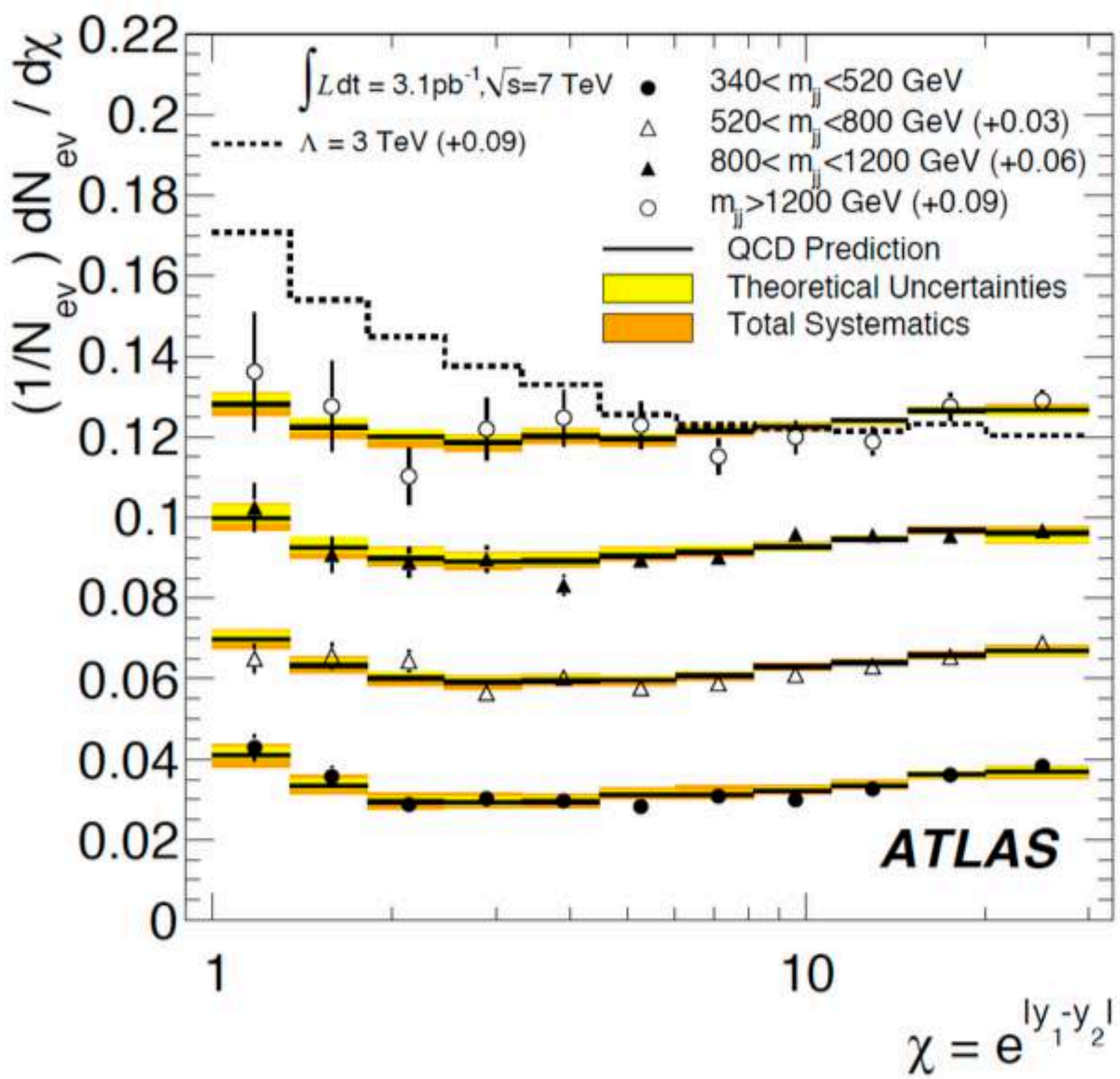
Search for gluions



Dijet production

Inclusive and dijet production are extensively studies at the LHC, both for SM measurements and in searches for New physics

Explore substructure of quarks



It is clear that the smaller the uncertainties, the more one can exclude exotic scenarios.
Above we sketched a leading order calculation, in the following we'll discuss higher-order corrections in a more generic case

Perturbative calculations

Perturbative calculations rely on the idea of an order-by-order expansion in the small coupling

$$\sigma \sim A + B\alpha_s + C\alpha_s^2 + D\alpha_s^3 + \ldots$$

LO NLO NNLO NNNLO

- Perturbative calculations are possible because the coupling is small at high energy
- In QCD (or in a generic QFT) the coupling depends on the energy (renormalization scale)
- So changing scale the result changes. By how much? What does this dependence mean?
- In the following will discuss these issues through examples

Hard cross section

Born level cross section straightforward in principle

$$\sigma_{LO} = \int_m d\Phi_m |\mathcal{M}^{(0)}(\{p_i\})|^2 S(\{p_i\})$$

m-particle phase space
(e.g. Vegas)

Matrix element

measurement function
(constraint on phase space)

Leading order with Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

Leading order with Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

Bottlenecks

a) number of Feynman diagrams diverges factorially

b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO

Diagrams for gluon amplitudes

Number of diagrams for gg → n gluons

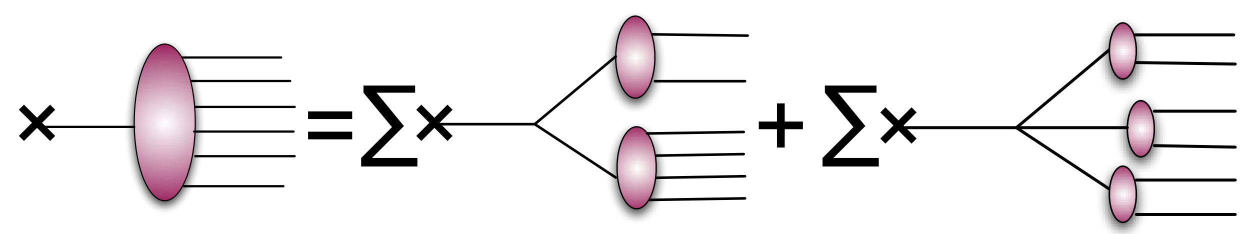
n	2	3	4	5	6	7	8
diag.	4	25	220	2485	34300	559405	10525900

- number of diagrams grows very fast
- complexity of each diagrams grows with n

Alternative methods?

Techniques beyond Feynman diagrams

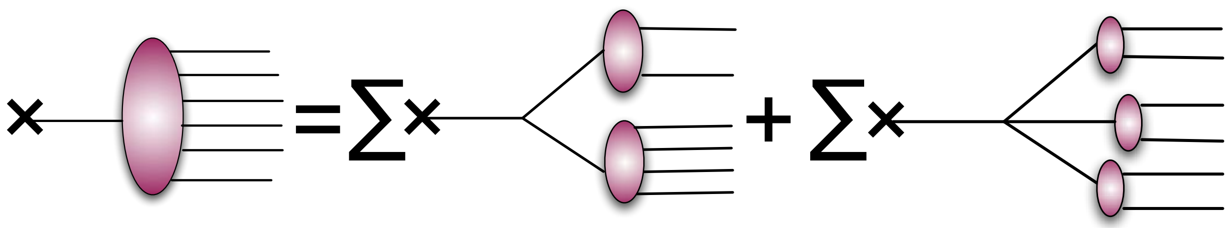
✓ Berends-Giele relations: compute helicity amplitudes **recursively** using off-shell currents



Berends, Giele '88

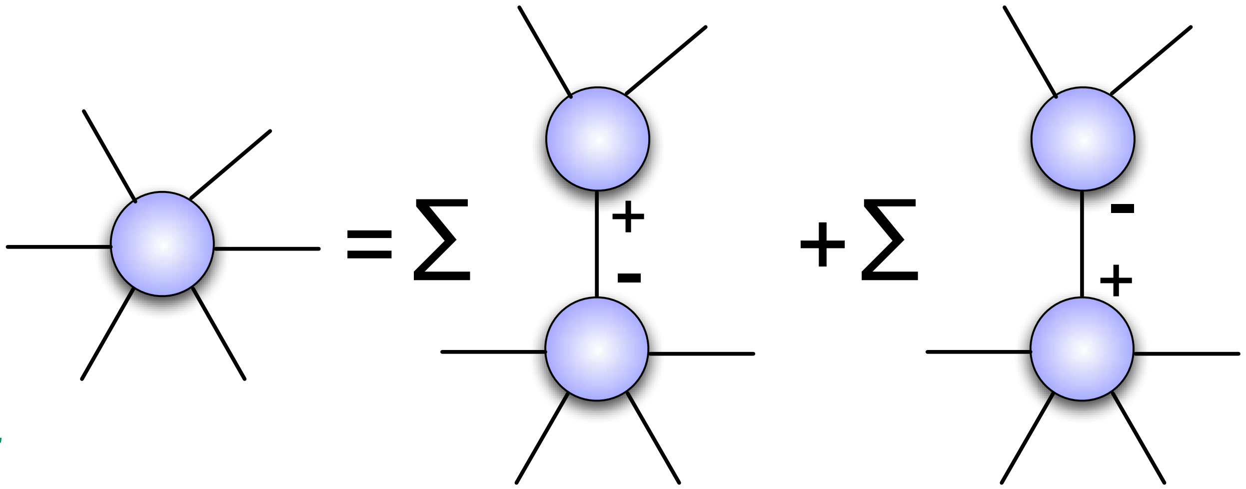
Techniques beyond Feynman diagrams

✓ Berends-Giele relations: compute helicity amplitudes recursively using off-shell currents



Berends, Giele '88

✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)

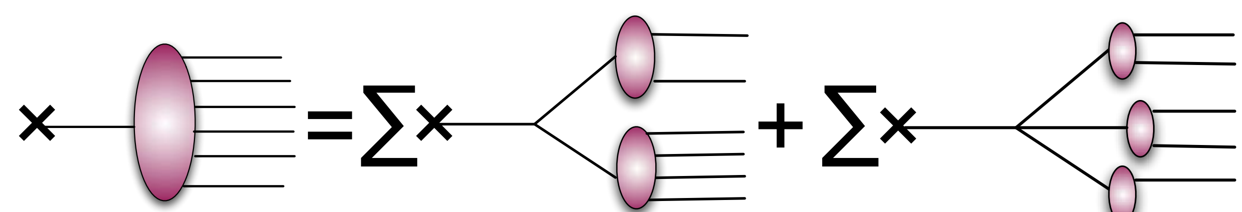


Britto, Cachazo, Feng '04

Techniques beyond Feynman diagrams

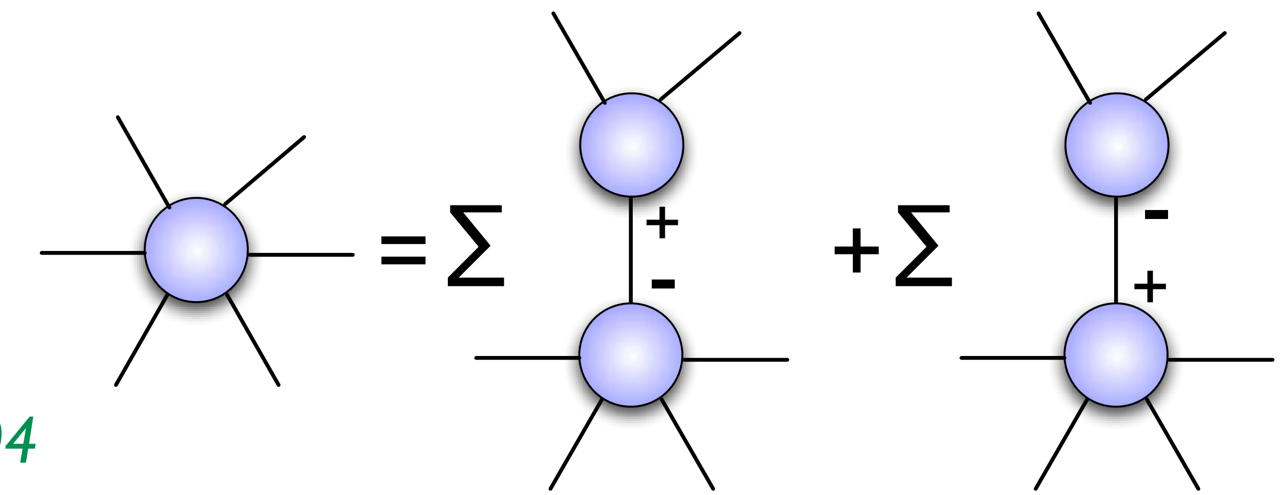
✓ Berends-Giele relations: compute helicity amplitudes recursively using off-shell currents

Berends, Giele '88



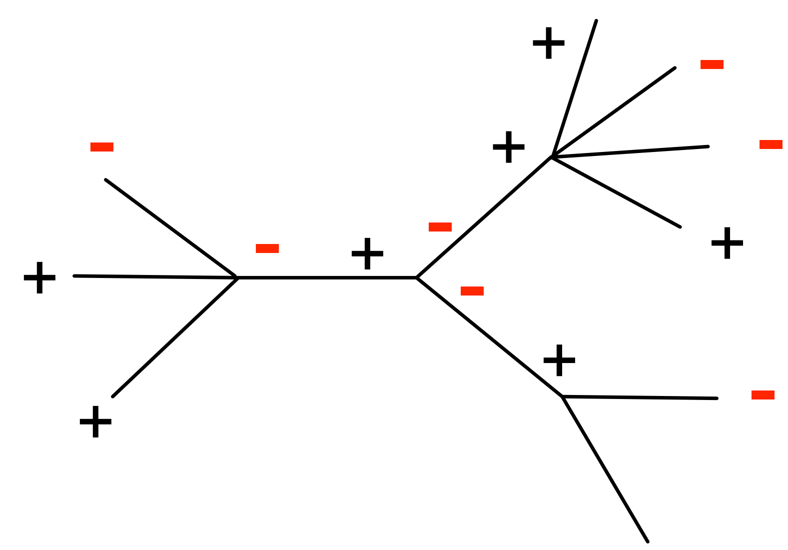
✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)

Britto, Cachazo, Feng '04



✓ CSW relations: compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]

Cachazo, Svrcek, Witten '04



Benefits and drawbacks of LO

Benefits of LO:

- fastest option; often the only one
- test quickly new ideas with fully exclusive description
- many working, well-tested approaches
- highly automated, crucial to explore new ground, but no precision

Benefits and drawbacks of LO

Benefits of LO:

- fastest option; often the only one
- test quickly new ideas with fully exclusive description
- many working, well-tested approaches
- highly automated, crucial to explore new ground, but no precision

Drawbacks of LO:

- large scale dependences, reflecting large theory uncertainty
- no control on normalization
- poor control on shapes
- poor modeling of jets

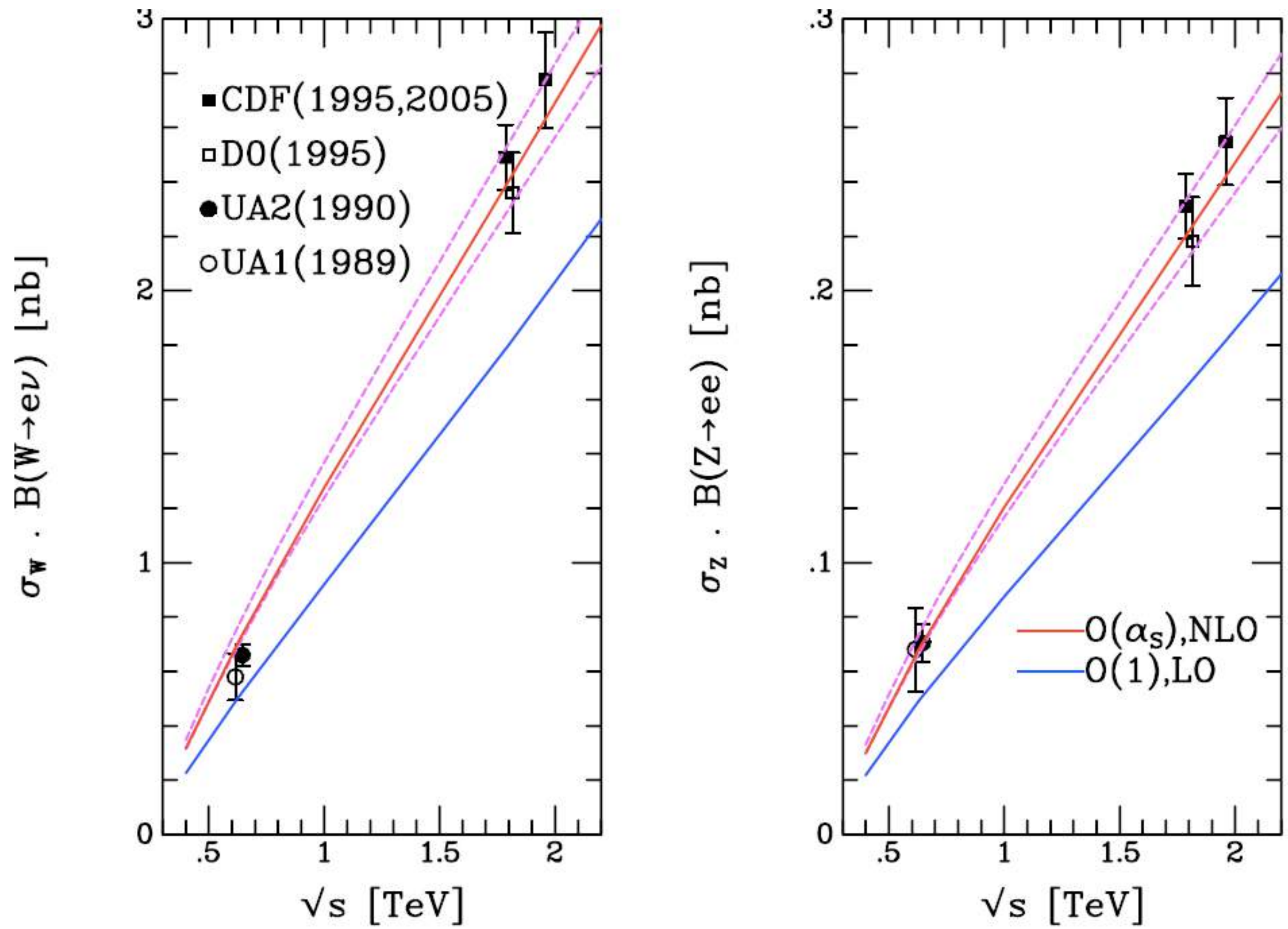
Example: W+4 jet cross-section $\propto \alpha_s(Q)^4$

Vary $\alpha_s(Q)$ by ±10% via change of Q ⇒ cross-section varies by ±40%

Is it necessary to go beyond LO?

Very early observation:

at least NLO corrections are needed to describe data



Drell Yan production is one of the first processes for which NLO corrections have been computed

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma_{\rm njets}^{\rm LO}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

- Instead, choosing a scale μ' one gets

$$\sigma_{\rm njets}^{\rm LO}(\mu') = \alpha_s(\mu')^n A(p_i, \epsilon_i, \ldots) = \alpha_s(\mu)^n \left(1 + n\, b_0\, \alpha_s(\mu) \ln \frac{\mu^2}{\mu'^2} + \ldots\right) A(p_i, \epsilon_i, \ldots)$$

So the change of scale is an NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

- Instead, choosing a scale μ' one gets

$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu') = \alpha_s(\mu')^n A(p_i, \epsilon_i, \ldots) = \alpha_s(\mu)^n \left(1 + n\, b_0\, \alpha_s(\mu) \ln \frac{\mu^2}{\mu'^2} + \ldots \right) A(p_i, \epsilon_i, \ldots)$$

  So the change of scale is an NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

- Notice that at Leading Order the normalization is not under control:

$$\frac{\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu)}{\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu')} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu')}\right)^n$$

NLO n-jet cross-section

Now consider n-jet cross-section at NLO. At scale μ the result reads

$$\sigma^{\rm NLO}_{\rm njets}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \dots) + \alpha_s(\mu)^{n+1}\left(B(p_i, \epsilon_i, \dots) - n b_0 \ln\frac{\mu^2}{Q_0^2}\right) + \dots$$

- So the NLO result compensates the LO scale dependence. The residual dependence is NNLO.

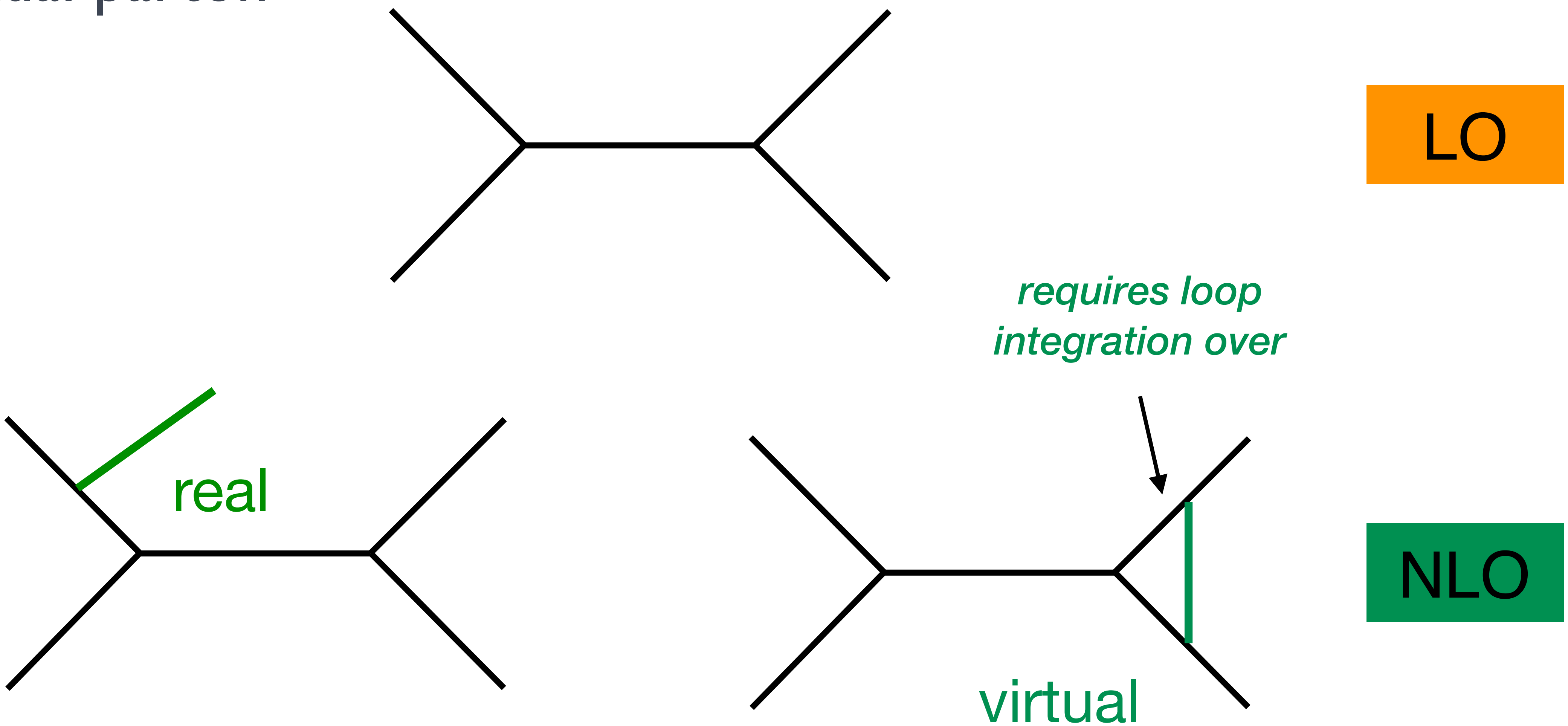
- Scale dependence and normalization start being under control only at NLO, since compensation mechanism kicks in

- Notice also that a good scale choice automatically resums large logarithms to all orders, while a bad one spuriously introduces large logs and ruins the PT expansion

- Scale variation is conventionally used to estimate theory uncertainty, but the validity of this procedure should not be overrated (see later)

NLO calculations

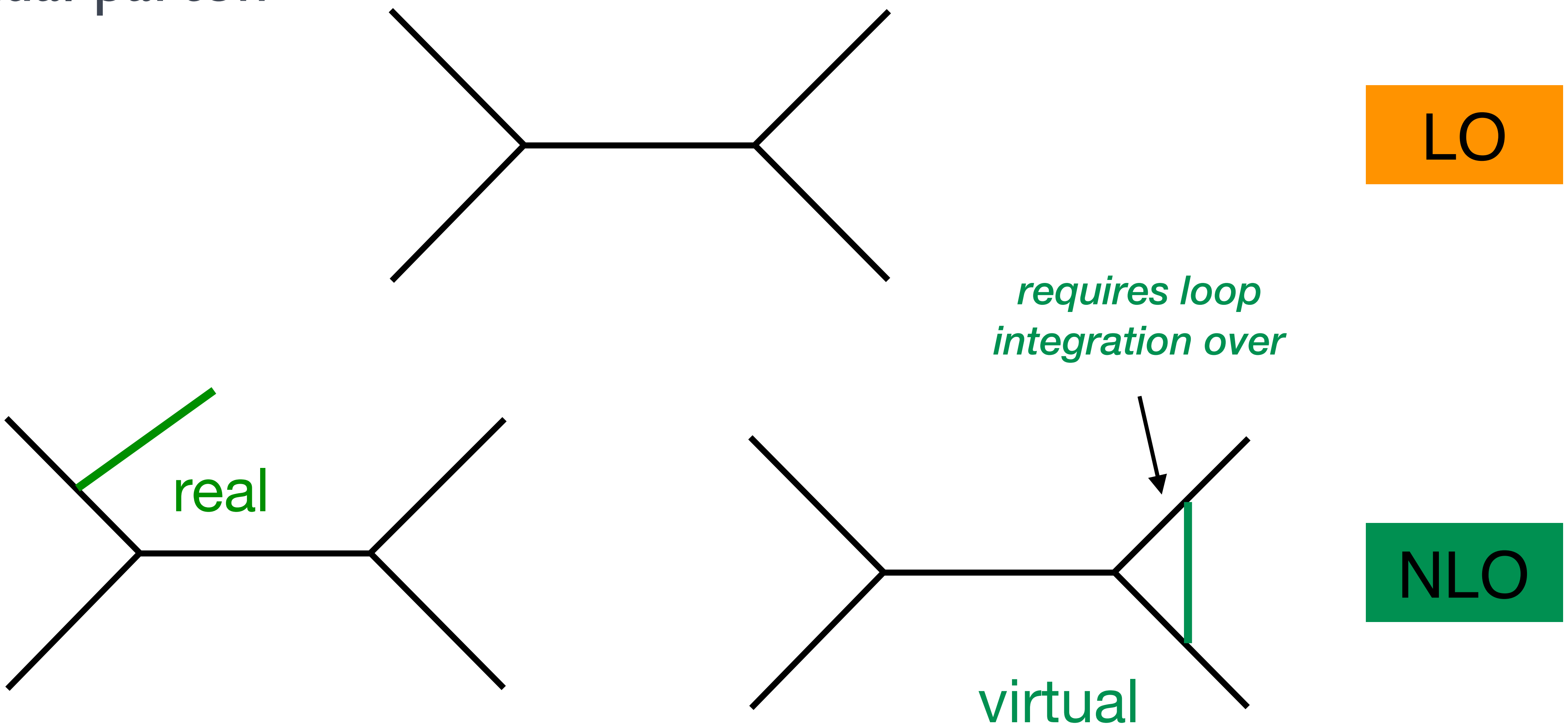
NLO accuracy requires to dress a process with one real or one virtual parton



Sample diagrams shown. All diagrams must be included.

NLO calculations

NLO accuracy requires to dress a process with one real or one virtual parton



Sample diagrams shown. All diagrams must be included.

We won't have time to do detailed NLO calculations, but let's look a bit more in detail at the issue of divergences/subtraction

Regularization procedures in QCD

Regularization: a way to make intermediate divergent quantities meaningful

- In QCD dimensional regularization is today the standard procedure, based on the fact that d-dimensional integrals are more convergent if one reduces the number of dimensions.

$$\int \frac{d^4 l}{(2\pi)^4} \to \mu^{2\epsilon} \int \frac{d^d l}{(2\pi)^d}\,, \quad d = 4 - 2\epsilon < 4$$

- N.B. to preserve the correct dimensions a mass scale μ is needed
- Divergences show up as intermediate poles 1/ε $\int_0^1 \frac{dx}{x} \to \int_0^1 \frac{dx}{x^{1-\epsilon}} = \frac{1}{\epsilon}$
- This procedure works both for UV divergences and IR divergences

Alternative regularization schemes: photon mass (EW), cut-offs, Pauli-Villard ...
Compared to those methods, dimensional regularizatiom has the big virtue that it leaves the regularized theory Lorentz invariant, gauge invariant, unitary etc.

Subtraction and slicing methods

- Consider e.g. an n-jet cross-section with some arbitrary infrared safe jet definition. At NLO, two divergent integrals, but the sum is finite

$$\sigma_{\rm NLO}^J = \int_{n+1} d\sigma_{\rm R}^J + \int_n d\sigma_{\rm V}^J$$

- Since one integrates over a different number of particles in the final state, real and virtual need to be evaluated first, and combined then

- This means that one needs to find a way of removing divergences before evaluating the phase space integrals

- Two main techniques to do this
  - *phase space slicing* ⇒ obsolete because of practical/numerical issues
  - *subtraction method* ⇒ most used in recent applications

Subtraction method

- The real cross-section can be written schematically as

$$d\sigma_R^J = d\phi_{n+1}|\mathcal{M}_{n+1}|^2 F_{n+1}^J(p_1, \ldots, p_{n+1})$$

where F^J is the arbitrary jet-definition

Subtraction method

- The real cross-section can be written schematically as

$$d\sigma_R^J = d\phi_{n+1} |\mathcal{M}_{n+1}|^2 F_{n+1}^J(p_1, \ldots, p_{n+1})$$

where F^J is the arbitrary jet-definition

- The matrix element has a non-integrable divergence

$$|\mathcal{M}_{n+1}|^2 = \frac{1}{x}\mathcal{M}(x)$$

where x vanishes in the soft/collinear divergent region

Subtraction method

- The real cross-section can be written schematically as

$$d\sigma_R^J = d\phi_{n+1}|\mathcal{M}_{n+1}|^2 F_{n+1}^J(p_1, \ldots, p_{n+1})$$

where F^J is the arbitrary jet-definition

- The matrix element has a non-integrable divergence

$$|\mathcal{M}_{n+1}|^2 = \frac{1}{x}\mathcal{M}(x)$$

where x vanishes in the soft/collinear divergent region

- IR divergences in the loop integration regularized by taking D=4-2ε

$$2\,\mathrm{Re}\{\mathcal{M}_V \cdot \mathcal{M}_0^*\} = \frac{1}{\epsilon}\mathcal{V}$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma^J_{\mathrm{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma^J_{\text{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

- Infrared safety of the jet definition implies

$$\lim_{x \to 0} F^J_{n+1}(x) = F^J_n$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma^J_{\rm NLO} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

- Infrared safety of the jet definition implies

$$\lim_{x \to 0} F^J_{n+1}(x) = F^J_n$$

- KLN cancelation guarantees that

$$\lim_{x \to 0} \mathcal{M}(x) = \mathcal{V}$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma^J_{\rm NLO} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

- Infrared safety of the jet definition implies

$$\lim_{x \to 0} F^J_{n+1}(x) = F^J_n$$

- KLN cancelation guarantees that

$$\lim_{x \to 0} \mathcal{M}(x) = \mathcal{V}$$

- One can then add and subtract the analytically computed divergent part

$$\sigma^J_{\rm NLO} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F^J_n + \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F^J_n + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

Subtraction method

- This can be rewritten exactly as

$$\sigma^J_{\rm NLO} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \left(\mathcal{M}(x) F^J_{n+1} - \mathcal{V} F^J_n\right) + \mathcal{O}(1) \mathcal{V} F^J_n$$

$\Rightarrow$ Now both terms are finite and can be evaluated numerically

- Subtracted cross-section must be calculated separately for each process (but mostly automated now). It must be valid everywhere in phase space
- Systematised in the seminal papers of Catani-Seymour (dipole subtraction, '96) and Frixione-Kunszt-Signer (FKS method, '96)
- Subtraction used in all recent NLO applications and public codes (Event2, Disent, MCFM, NLOjet++, MC@NLO, POWHEG ...)

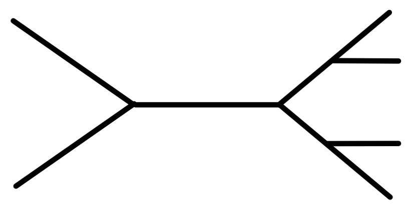
Ingredients at NLO

A full N-particle NLO calculation requires:

Ingredients at NLO

A full N-particle NLO calculation requires:

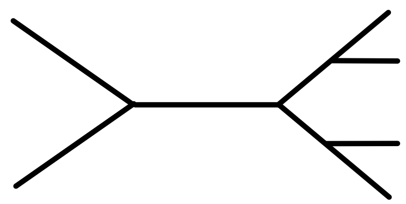
- tree graph rates with N+1 partons
  ➔ soft/collinear divergences



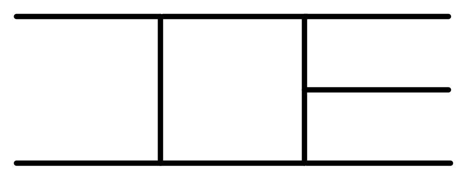
Ingredients at NLO

A full N-particle NLO calculation requires:

- tree graph rates with N+1 partons
  ➔ soft/collinear divergences



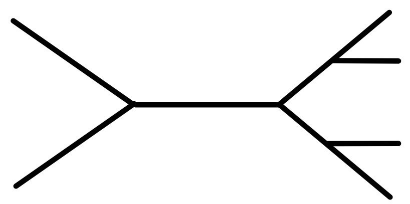
- virtual correction to N-leg process
  ➔ divergence from loop integration,
  use e.g. dimensional regularization



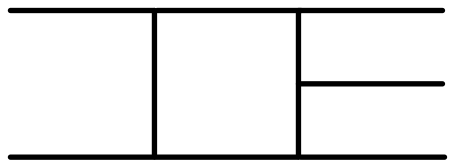
Ingredients at NLO

A full N-particle NLO calculation requires:

- tree graph rates with N+1 partons
  ➔ soft/collinear divergences



- virtual correction to N-leg process
  ➔ divergence from loop integration,
  use e.g. dimensional regularization

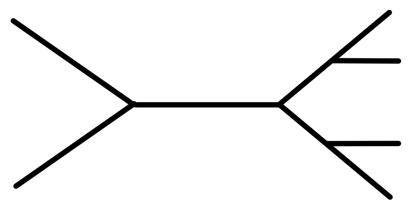


- set of subtraction terms

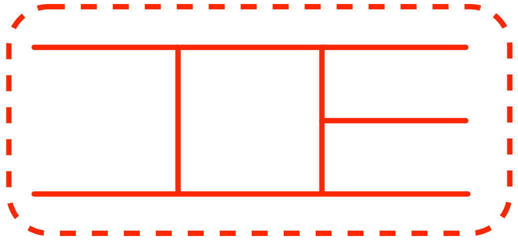
Ingredients at NLO

A full N-particle NLO calculation requires:

- ☑ tree graph rates with N+1 partons
  ➔ soft/collinear divergences



- ☐ virtual correction to N-leg process
  ➔ divergence from loop integration,
  use e.g. dimensional regularization



bottleneck for a very long time

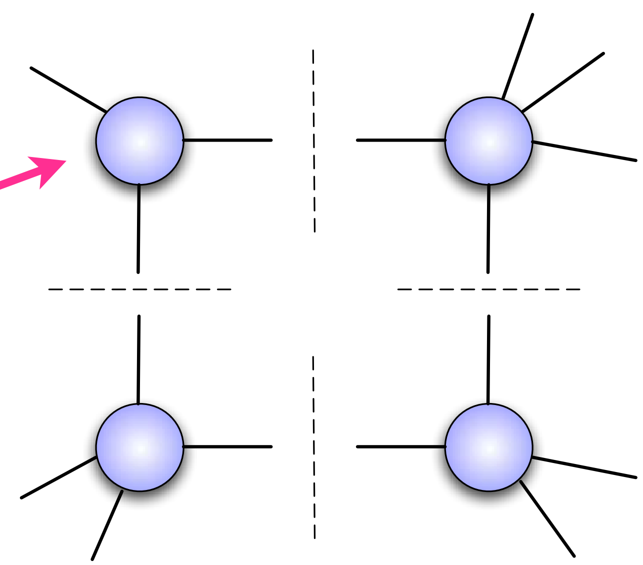
- ☑ set of subtraction terms

Virtual one-loop: two breakthrough ideas

Aim: NLO loop integral without doing the integration

1) "... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ..."

NB: non-zero because cut gives complex momenta



Britto, Cachazo, Feng '04

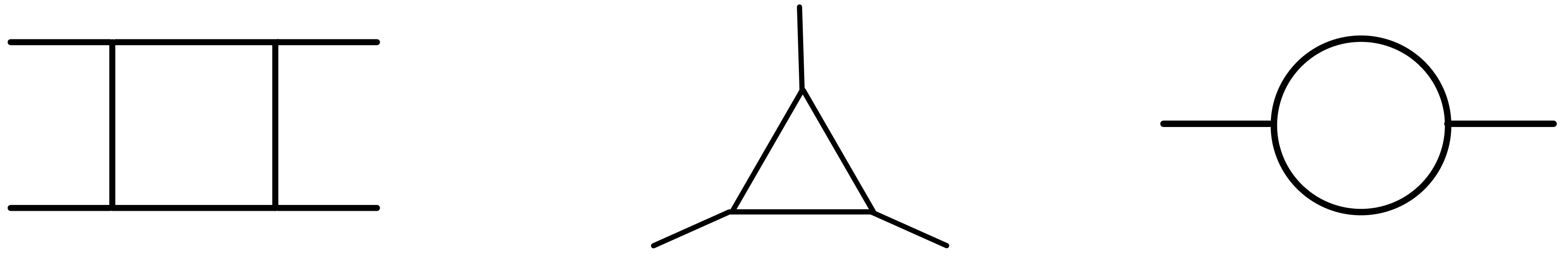
Quadrupole cuts: 4 on-shell conditions on 4 dimensional loop momentum) freezes the integration. But rational part of the amplitude, coming from $D=4-2\varepsilon$ not 4, computed separately

One-loop: two breakthrough ideas

Aim: NLO loop integral without doing the integration

2) The OPP method: "We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals...."

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right)$$



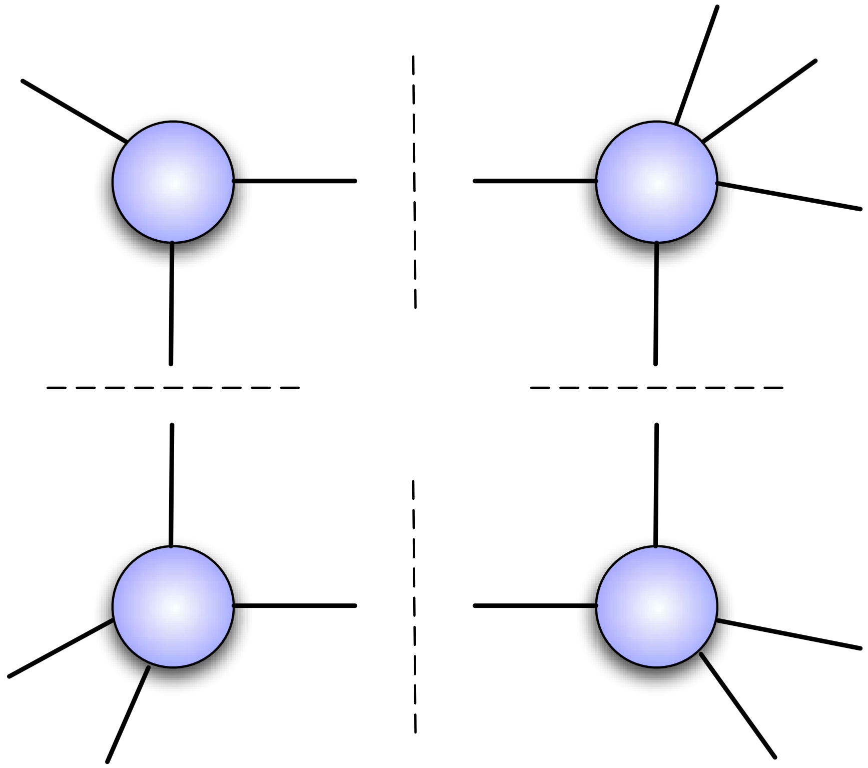
Ossola, Pittau, Papadopolous '06

Coefficients can be determined by solving system of equations: no loops, no twistors, just algebra!

Virtual (one-loop) amplitude

Bottleneck for a long time... but thanks to these and other theoretical breakthrough ideas

- connection between NLO amplitudes and LO ones
- input from supersymmetry/string theory
- sophisticated algebraic methods
- connections with formal theory and pure mathematics ...



the problem of computing NLO QCD corrections is now solved

Automated NLO (aka NLO revolution)

<u>Example</u>: single Higgs production processes (similar results available for all SM processes of similar complexity, backgrounds to Higgs studies)

Process		Syntax	Cross section (pb)	
Single Higgs production			LO 13 TeV	NLO 13 TeV
g.1	$pp \to H$ (HEFT)	`p p > h`	$1.593 \pm 0.003 \cdot 10^{1}$ $^{+34.8\%}_{-26.0\%}$ $^{+1.2\%}_{-1.7\%}$	$3.261 \pm 0.010 \cdot 10^{1}$ $^{+20.2\%}_{-17.9\%}$ $^{+1.1\%}_{-1.6\%}$
g.2	$pp \to Hj$ (HEFT)	`p p > h j`	$8.367 \pm 0.003 \cdot 10^{0}$ $^{+39.4\%}_{-26.4\%}$ $^{+1.2\%}_{-1.4\%}$	$1.422 \pm 0.006 \cdot 10^{1}$ $^{+18.5\%}_{-16.6\%}$ $^{+1.1\%}_{-1.4\%}$
g.3	$pp \to Hjj$ (HEFT)	`p p > h j j`	$3.020 \pm 0.002 \cdot 10^{0}$ $^{+59.1\%}_{-34.7\%}$ $^{+1.4\%}_{-1.7\%}$	$5.124 \pm 0.020 \cdot 10^{0}$ $^{+20.7\%}_{-21.0\%}$ $^{+1.3\%}_{-1.5\%}$
g.4	$pp \to Hjj$ (VBF)	`p p > h j j $$ w+ w- z`	$1.987 \pm 0.002 \cdot 10^{0}$ $^{+1.7\%}_{-2.0\%}$ $^{+1.9\%}_{-1.4\%}$	$1.900 \pm 0.006 \cdot 10^{0}$ $^{+0.8\%}_{-0.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.5	$pp \to Hjjj$ (VBF)	`p p > h j j j $$ w+ w- z`	$2.824 \pm 0.005 \cdot 10^{-1}$ $^{+15.7\%}_{-12.7\%}$ $^{+1.5\%}_{-1.0\%}$	$3.085 \pm 0.010 \cdot 10^{-1}$ $^{+2.0\%}_{-3.0\%}$ $^{+1.5\%}_{-1.1\%}$
g.6	$pp \to HW^{\pm}$	`p p > h wpm`	$1.195 \pm 0.002 \cdot 10^{0}$ $^{+3.5\%}_{-4.5\%}$ $^{+1.9\%}_{-1.5\%}$	$1.419 \pm 0.005 \cdot 10^{0}$ $^{+2.1\%}_{-2.6\%}$ $^{+1.9\%}_{-1.4\%}$
g.7	$pp \to HW^{\pm} j$	`p p > h wpm j`	$4.018 \pm 0.003 \cdot 10^{-1}$ $^{+10.7\%}_{-9.3\%}$ $^{+1.2\%}_{-0.9\%}$	$4.842 \pm 0.017 \cdot 10^{-1}$ $^{+3.6\%}_{-3.7\%}$ $^{+1.2\%}_{-1.0\%}$
g.8*	$pp \to HW^{\pm} jj$	`p p > h wpm j j`	$1.198 \pm 0.016 \cdot 10^{-1}$ $^{+26.1\%}_{-19.4\%}$ $^{+0.8\%}_{-0.6\%}$	$1.574 \pm 0.014 \cdot 10^{-1}$ $^{+5.0\%}_{-6.5\%}$ $^{+0.9\%}_{-0.6\%}$
g.9	$pp \to HZ$	`p p > h z`	$6.468 \pm 0.008 \cdot 10^{-1}$ $^{+3.5\%}_{-4.5\%}$ $^{+1.9\%}_{-1.4\%}$	$7.674 \pm 0.027 \cdot 10^{-1}$ $^{+2.0\%}_{-2.5\%}$ $^{+1.9\%}_{-1.4\%}$
g.10	$pp \to HZ j$	`p p > h z j`	$2.225 \pm 0.001 \cdot 10^{-1}$ $^{+10.6\%}_{-9.2\%}$ $^{+1.1\%}_{-0.8\%}$	$2.667 \pm 0.010 \cdot 10^{-1}$ $^{+3.5\%}_{-3.6\%}$ $^{+1.1\%}_{-0.9\%}$
g.11*	$pp \to HZ jj$	`p p > h z j j`	$7.262 \pm 0.012 \cdot 10^{-2}$ $^{+26.2\%}_{-19.4\%}$ $^{+0.7\%}_{-0.6\%}$	$8.753 \pm 0.037 \cdot 10^{-2}$ $^{+4.8\%}_{-6.3\%}$ $^{+0.7\%}_{-0.6\%}$
g.12*	$pp \to HW^{+}W^{-}$ (4f)	`p p > h w+ w-`	$8.325 \pm 0.139 \cdot 10^{-3}$ $^{+0.0\%}_{-0.3\%}$ $^{+2.0\%}_{-1.6\%}$	$1.065 \pm 0.003 \cdot 10^{-2}$ $^{+2.5\%}_{-1.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.13*	$pp \to HW^{\pm}\gamma$	`p p > h wpm a`	$2.518 \pm 0.006 \cdot 10^{-3}$ $^{+0.7\%}_{-1.4\%}$ $^{+1.9\%}_{-1.5\%}$	$3.309 \pm 0.011 \cdot 10^{-3}$ $^{+2.7\%}_{-2.0\%}$ $^{+1.7\%}_{-1.4\%}$
g.14*	$pp \to HZW^{\pm}$	`p p > h z wpm`	$3.763 \pm 0.007 \cdot 10^{-3}$ $^{+1.1\%}_{-1.5\%}$ $^{+2.0\%}_{-1.6\%}$	$5.292 \pm 0.015 \cdot 10^{-3}$ $^{+3.9\%}_{-3.1\%}$ $^{+1.8\%}_{-1.4\%}$
g.15*	$pp \to HZZ$	`p p > h z z`	$2.093 \pm 0.003 \cdot 10^{-3}$ $^{+0.1\%}_{-0.6\%}$ $^{+1.9\%}_{-1.5\%}$	$2.538 \pm 0.007 \cdot 10^{-3}$ $^{+1.9\%}_{-1.4\%}$ $^{+2.0\%}_{-1.5\%}$
g.16	$pp \to Ht\bar{t}$	`p p > h t t~`	$3.579 \pm 0.003 \cdot 10^{-1}$ $^{+30.0\%}_{-21.5\%}$ $^{+1.7\%}_{-2.0\%}$	$4.608 \pm 0.016 \cdot 10^{-1}$ $^{+5.7\%}_{-9.0\%}$ $^{+2.0\%}_{-2.3\%}$
g.17	$pp \to Htj$	`p p > h tt j`	$4.994 \pm 0.005 \cdot 10^{-2}$ $^{+2.4\%}_{-4.2\%}$ $^{+1.2\%}_{-1.3\%}$	$6.328 \pm 0.022 \cdot 10^{-2}$ $^{+2.9\%}_{-1.8\%}$ $^{+1.5\%}_{-1.6\%}$
g.18	$pp \to Hb\bar{b}$ (4f)	`p p > h b b~`	$4.983 \pm 0.002 \cdot 10^{-1}$ $^{+28.1\%}_{-21.0\%}$ $^{+1.5\%}_{-1.8\%}$	$6.085 \pm 0.026 \cdot 10^{-1}$ $^{+7.3\%}_{-9.6\%}$ $^{+1.6\%}_{-2.0\%}$
g.19	$pp \to Ht\bar{t}j$	`p p > h t t~ j`	$2.674 \pm 0.041 \cdot 10^{-1}$ $^{+45.6\%}_{-29.2\%}$ $^{+2.6\%}_{-2.9\%}$	$3.244 \pm 0.025 \cdot 10^{-1}$ $^{+3.5\%}_{-8.7\%}$ $^{+2.5\%}_{-2.9\%}$
g.20*	$pp \to Hb\bar{b}j$ (4f)	`p p > h b b~ j`	$7.367 \pm 0.002 \cdot 10^{-2}$ $^{+45.6\%}_{-29.1\%}$ $^{+1.8\%}_{-2.1\%}$	$9.034 \pm 0.032 \cdot 10^{-2}$ $^{+7.9\%}_{-11.0\%}$ $^{+1.8\%}_{-2.2\%}$

Automated NLO (aka NLO revolution)

Example: single Higgs production processes (similar results available for all SM processes of similar complexity, backgrounds to Higgs studies)

Process		Syntax	Cross section (pb)			
Single Higgs production			LO 13 TeV		NLO 13 TeV	
g.1	$pp \to H$ (HEFT)	`p p > h`	$1.593 \pm 0.003 \cdot 10^{1}$	$^{+34.8\%}_{-26.0\%}$ $^{+1.2\%}_{-1.7\%}$	$3.261 \pm 0.010 \cdot 10^{1}$	$^{+20.2\%}_{-17.9\%}$ $^{+1.1\%}_{-1.6\%}$
g.2	$pp \to Hj$ (HEFT)	`p p > h j`	$8.367 \pm 0.003 \cdot 10^{0}$	$^{+39.4\%}_{-26.4\%}$ $^{+1.2\%}_{-1.4\%}$	$1.422 \pm 0.006 \cdot 10^{1}$	$^{+18.5\%}_{-16.6\%}$ $^{+1.1\%}_{-1.4\%}$
g.3	$pp \to Hjj$ (HEFT)	`p p > h j j`	$3.020 \pm 0.002 \cdot 10^{0}$	$^{+59.1\%}_{-34.7\%}$ $^{+1.4\%}_{-1.7\%}$	$5.124 \pm 0.020 \cdot 10^{0}$	$^{+20.7\%}_{-21.0\%}$ $^{+1.3\%}_{-1.5\%}$
g.4	$pp \to Hjj$ (VBF)	`p p > h j j $$ w+ w- z`	$1.987 \pm 0.002 \cdot 10^{0}$	$^{+1.7\%}_{-2.0\%}$ $^{+1.9\%}_{-1.4\%}$	$1.900 \pm 0.006 \cdot 10^{0}$	$^{+0.8\%}_{-0.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.5	$pp \to Hjjj$ (VBF)	`p p > h j j j $$ w+ w- z`	$2.824 \pm 0.005 \cdot 10^{-1}$	$^{+15.7\%}_{-12.7\%}$ $^{+1.5\%}_{-1.0\%}$	$3.085 \pm 0.010 \cdot 10^{-1}$	$^{+2.0\%}_{-3.0\%}$ $^{+1.5\%}_{-1.1\%}$
g.6	$pp \to HW^{\pm}$	`p p > h wpm`	$1.195 \pm 0.002 \cdot 10^{0}$	$^{+3.5\%}$ $^{+1.9\%}$ [illegible]	$1.419 \pm 0.005 \cdot 10^{0}$	$^{+2.1\%}$ $^{+1.9\%}_{-1.4\%}$
g.7	$pp \to$ [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	$^{+1.2\%}_{-1.0\%}$
g.8*	$pp \to$ [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	$^{+0.9\%}_{-0.6\%}$
g.9	$pp \to$ [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	$^{+1.9\%}_{-1.4\%}$
g.10	$pp \to$ [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	$^{+1.1\%}_{-0.9\%}$
g.11*	$pp \to$ [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	$^{+0.7\%}_{-0.6\%}$
g.12*	$pp \to HW^{+}W^{-}$ (4f)	`p p > h w+ w-`	$8.325 \pm 0.139 \cdot 10^{-3}$	$_{-0.3\%}$ $_{-1.6\%}$	$1.065 \pm 0.003 \cdot 10^{-2}$	$_{-1.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.13*	$pp \to HW^{\pm}\gamma$	`p p > h wpm a`	$2.518 \pm 0.006 \cdot 10^{-3}$	$^{+0.7\%}_{-1.4\%}$ $^{+1.9\%}_{-1.5\%}$	$3.309 \pm 0.011 \cdot 10^{-3}$	$^{+2.7\%}_{-2.0\%}$ $^{+1.7\%}_{-1.4\%}$
g.14*	$pp \to HZW^{\pm}$	`p p > h z wpm`	$3.763 \pm 0.007 \cdot 10^{-3}$	$^{+1.1\%}_{-1.5\%}$ $^{+2.0\%}_{-1.6\%}$	$5.292 \pm 0.015 \cdot 10^{-3}$	$^{+3.9\%}_{-3.1\%}$ $^{+1.8\%}_{-1.4\%}$
g.15*	$pp \to HZZ$	`p p > h z z`	$2.093 \pm 0.003 \cdot 10^{-3}$	$^{+0.1\%}_{-0.6\%}$ $^{+1.9\%}_{-1.5\%}$	$2.538 \pm 0.007 \cdot 10^{-3}$	$^{+1.9\%}_{-1.4\%}$ $^{+2.0\%}_{-1.5\%}$
g.16	$pp \to Ht\bar{t}$	`p p > h t t~`	$3.579 \pm 0.003 \cdot 10^{-1}$	$^{+30.0\%}_{-21.5\%}$ $^{+1.7\%}_{-2.0\%}$	$4.608 \pm 0.016 \cdot 10^{-1}$	$^{+5.7\%}_{-9.0\%}$ $^{+2.0\%}_{-2.3\%}$
g.17	$pp \to Htj$	`p p > h tt j`	$4.994 \pm 0.005 \cdot 10^{-2}$	$^{+2.4\%}_{-4.2\%}$ $^{+1.2\%}_{-1.3\%}$	$6.328 \pm 0.022 \cdot 10^{-2}$	$^{+2.9\%}_{-1.8\%}$ $^{+1.5\%}_{-1.6\%}$
g.18	$pp \to Hb\bar{b}$ (4f)	`p p > h b b~`	$4.983 \pm 0.002 \cdot 10^{-1}$	$^{+28.1\%}_{-21.0\%}$ $^{+1.5\%}_{-1.8\%}$	$6.085 \pm 0.026 \cdot 10^{-1}$	$^{+7.3\%}_{-9.6\%}$ $^{+1.6\%}_{-2.0\%}$
g.19	$pp \to Ht\bar{t}j$	`p p > h t t~ j`	$2.674 \pm 0.041 \cdot 10^{-1}$	$^{+45.6\%}_{-29.2\%}$ $^{+2.6\%}_{-2.9\%}$	$3.244 \pm 0.025 \cdot 10^{-1}$	$^{+3.5\%}_{-8.7\%}$ $^{+2.5\%}_{-2.9\%}$
g.20*	$pp \to Hb\bar{b}j$ (4f)	`p p > h b b~ j`	$7.367 \pm 0.002 \cdot 10^{-2}$	$^{+45.6\%}_{-29.1\%}$ $^{+1.8\%}_{-2.1\%}$	$9.034 \pm 0.032 \cdot 10^{-2}$	$^{+7.9\%}_{-11.0\%}$ $^{+1.8\%}_{-2.2\%}$

✓ A solved problem

NLO automation

Various public tools developed: Blackhat+Sherpa, GoSam+Sherpa, Helac-NLO, Madgraph5_aMC@NLO, NJet+Sherpa, OpenLoops+Sherpa, Samurai, Recola ...

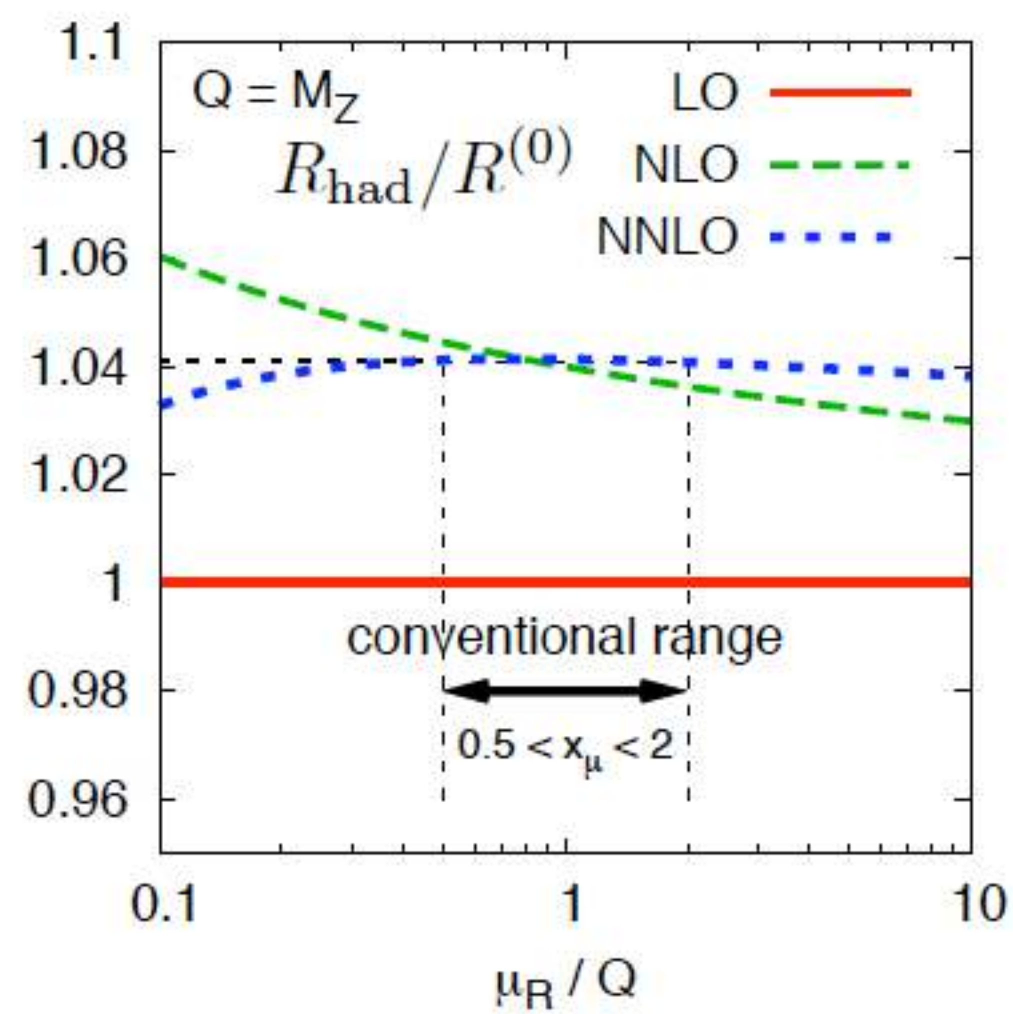
- Practical limitation: high-multiplicity processes still difficult because of numerical instabilities, need long run-time on clusters to obtain stable results (edge: 5-6 particles in the final state, depending on the process)
- Today focus on
  - ➡ automation of NLO for BSM signals
  - ➡ loop-induced processes: formally higher-order, but enhanced by gluon PDF
  - ➡ automation of NLO electroweak corrections (necessary to match accuracy of NNLO).

Comparison to NLO is the standard now in most LHC analyses

Uncertainties

The "unpleasant" feature that cross-sections depend on the choice of renormalization and factorization scale can be turned into something useful, i.e. a way to quantify the theoretical error

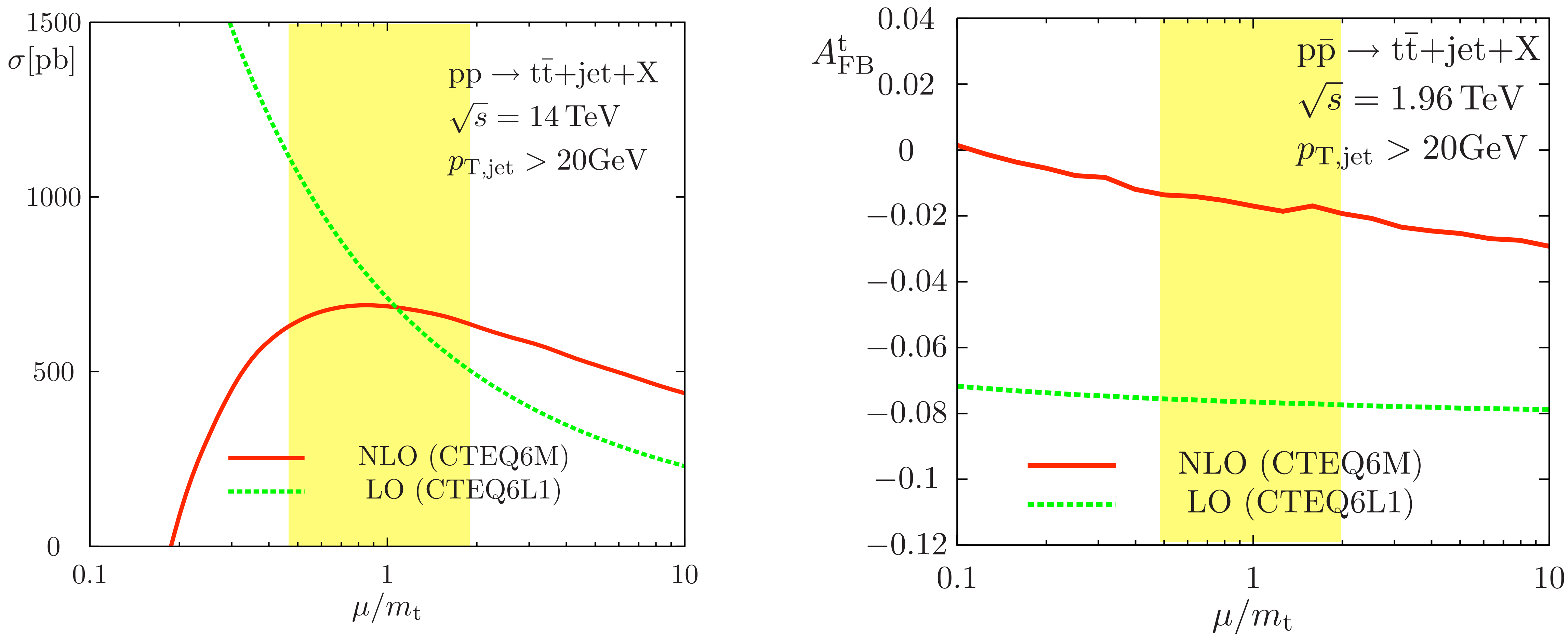
Example: R-ratio (again!)
Fix both scales to the scale at which the hard process occurs (Q) and vary them up and down by a factor 2



NB:
- the factor 2 is conventional
- it is a procedure that seemed to work well in practice
- in complicated processes large degree of freedom in the choice of the scale

1. LHC example of NLO: tt+1jet

Dittmaier, Kallweit, Uwer '08

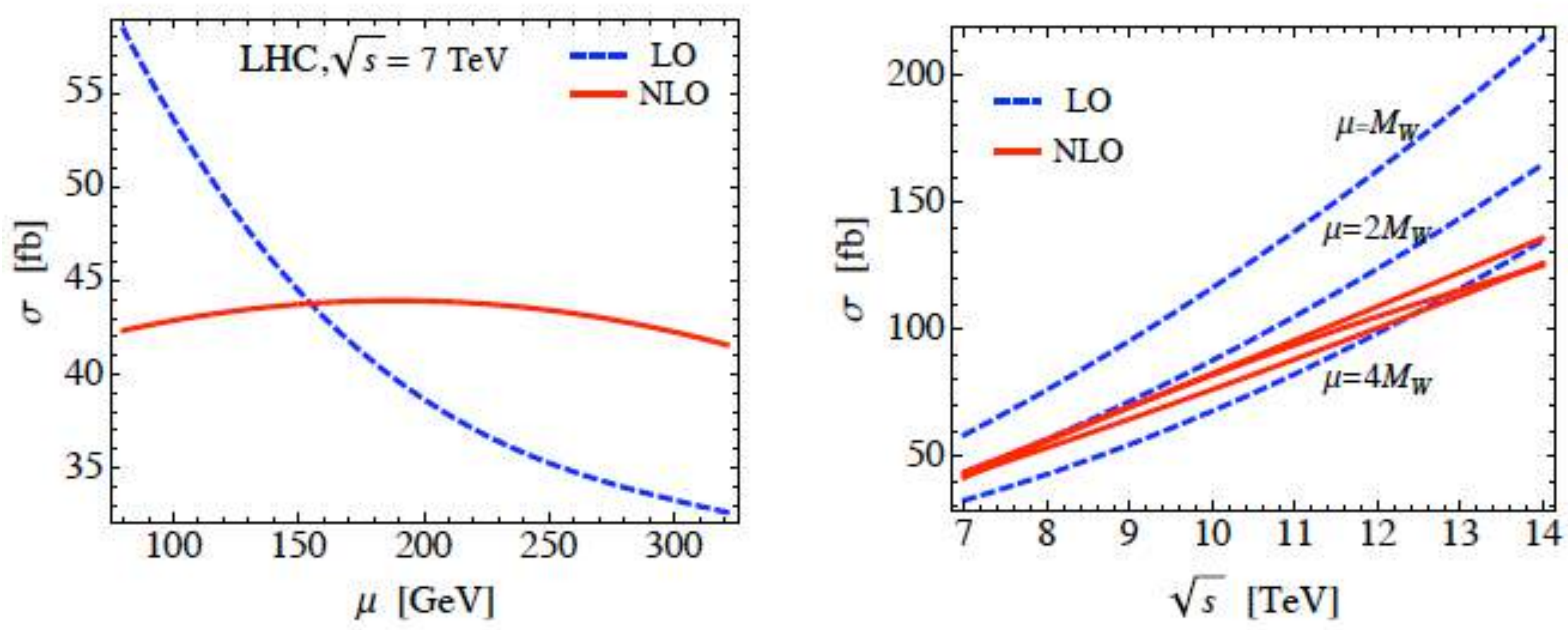


- improved stability of NLO result [but no decays]
- forward-backward asymmetry at the Tevatron compatible with zero
- LO scale uncertainty underestimates shift to NLO for the asymmetry

2. LHC example of NLO: WW+2jets

LO calculations: very large theoretical uncertainties

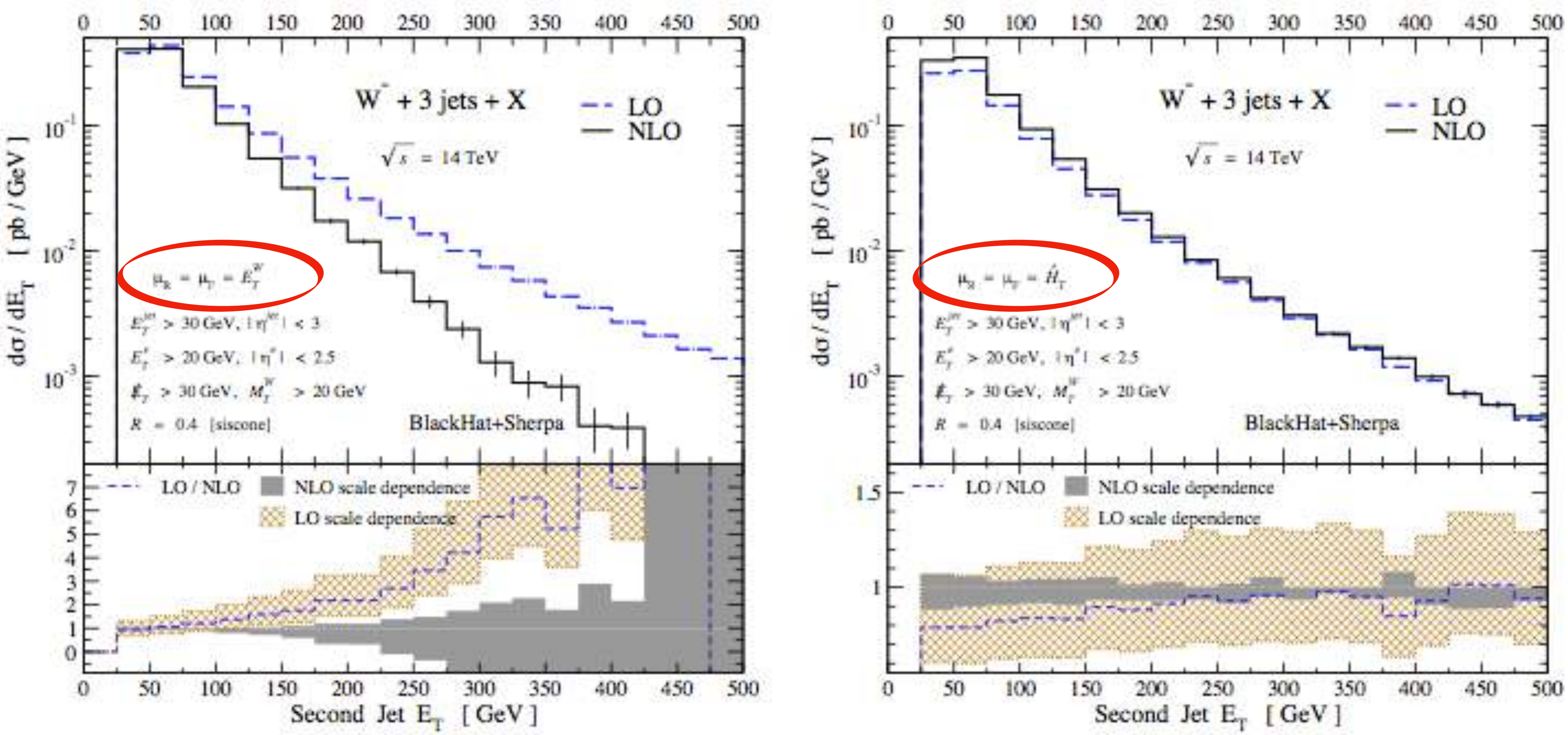
<u>Example</u>: cross-section for W^+W^- + 2 jet production at the LHC



Melia, et al. '11

3. LHC example of NLO: W+3jets

<u>Scale choice:</u> example of W+3 jets (problem more severe with more jets)



... large logarithms can appear in some distributions, invalidating even an NLO prediction.

Bern et al. '09

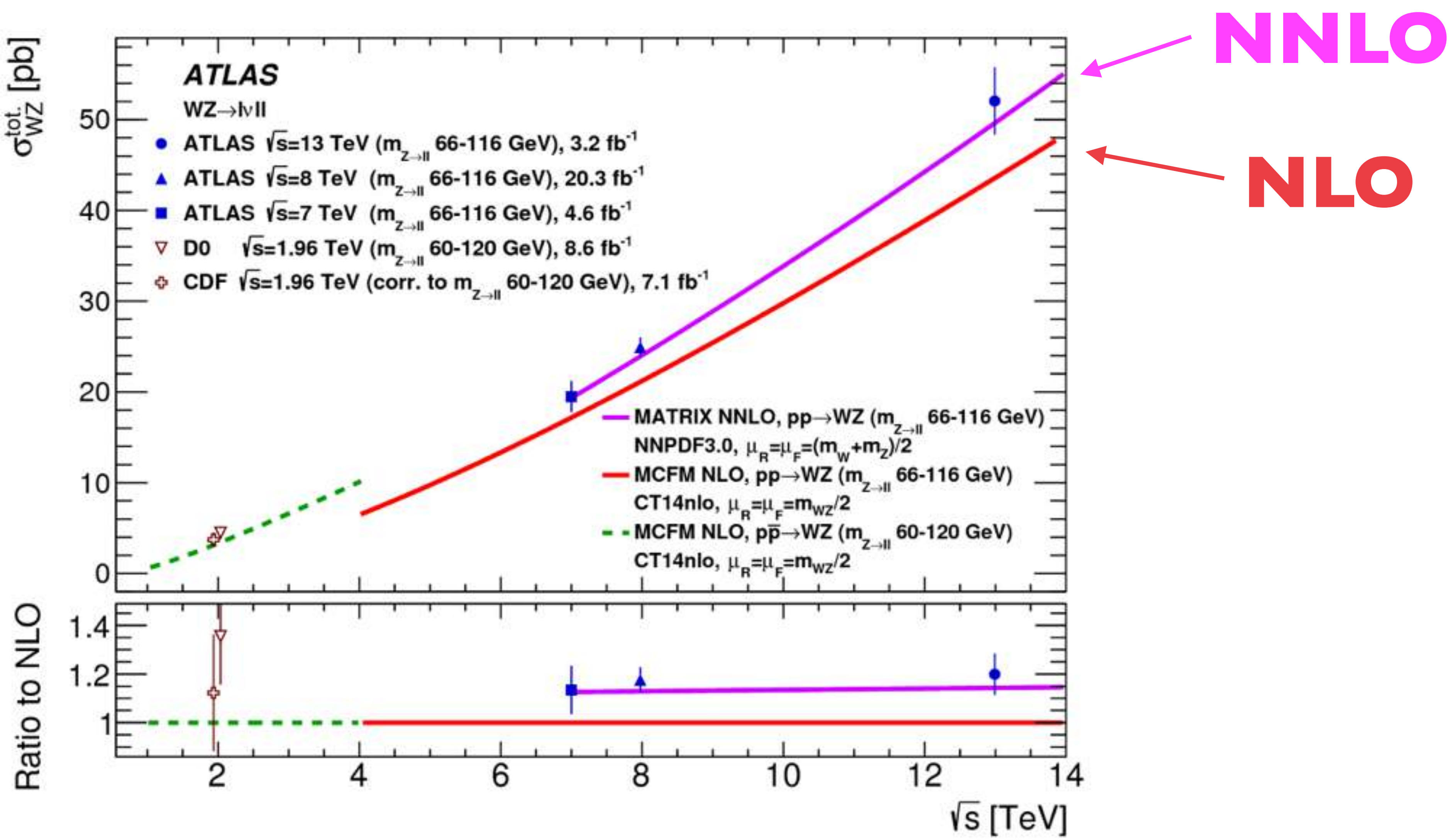
NLO revolution?

- few years ago: each NLO calculation resulted in a paper. Now, as for leading order, just run a code and get the results
- possibility to do precise studies of signal and backgrounds using the same tool (very practical + avoid errors)
- what lead to this remarkable progress? the fact that

1. leading order can be computed automatically and efficiently (e.g. via recursion relations)
2. one can reduce the one-loop to product of tree-level amplitudes
3. it was well understood how to subtract singularities
4. the basis of master integrals was known

But for item 2. everything was there since the time of Passarino-Veltman (even item 2. was understood, but no efficient/practical method exited).
We will later on compare this to the current status of NNLO

Is NNLO needed?



LHC data clearly already requires NNLO

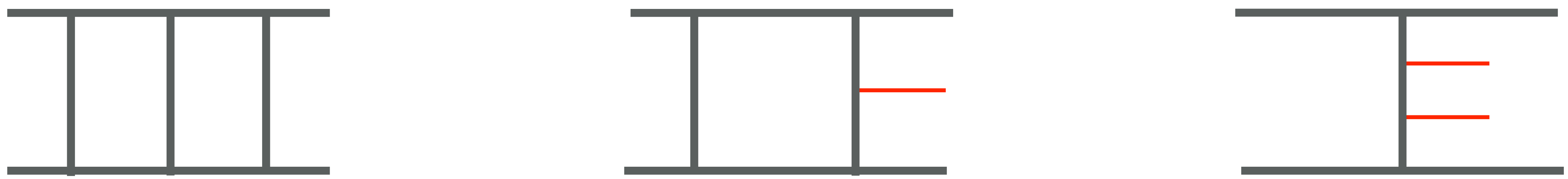
Same conclusion in all measurements examined so far

With more data NLO likely to be insufficient

Why is NNLO difficult

calculation of two-loop master integrals (when many scales are involved)

methods to cancel (overlapping) divergences before integration



$$\int d\Phi_n 2\mathrm{Re}|\mathcal{M}^{2-loop}\mathcal{M}^{tree}| \qquad \int d\Phi_n d\Phi_1 2Re|\mathcal{M}_{n+1}^{\mathrm{one-loop}}\mathcal{M}_{n+1}^{\mathrm{tree}}| \qquad \int d\Phi_n d\Phi_2 |\mathcal{M}^{\mathrm{tree}}|_{n+2}^2$$

$$\int d\Phi_n \left\{\left(a_4\frac{1}{\epsilon^4}+a_3\frac{1}{\epsilon^3}+\ldots+a_0\right)-\left(a_4\frac{1}{\epsilon^4}+a_3\frac{1}{\epsilon^3}+\ldots+b_0)\right)\right\}$$

Cancelation manifest after phase space integration, but to have fully differential results must achieve cancelation before integration

Ingredients for NNLO

At NNLO the situation is very different from NLO

1. leading order of very limited importance
2. no procedure to reduce two-loop to tree-level (unitarity approaches still face many outstanding issues)
3. subtraction of singularities far from trivial
4. basis set of master integrals not known, integrals not all/always known analytically

And all this even for simple processes (no full result exists for any $2 \to 3$ scattering process)

What changed in the last years (and is undergoing more changes)

1. technology to compute integrals
2. extension of systematic subtraction to NNLO

NNLO example: Drell-Yan

Drell-Yan processes: Z/W production (W → lν , Z → l^+l^-)

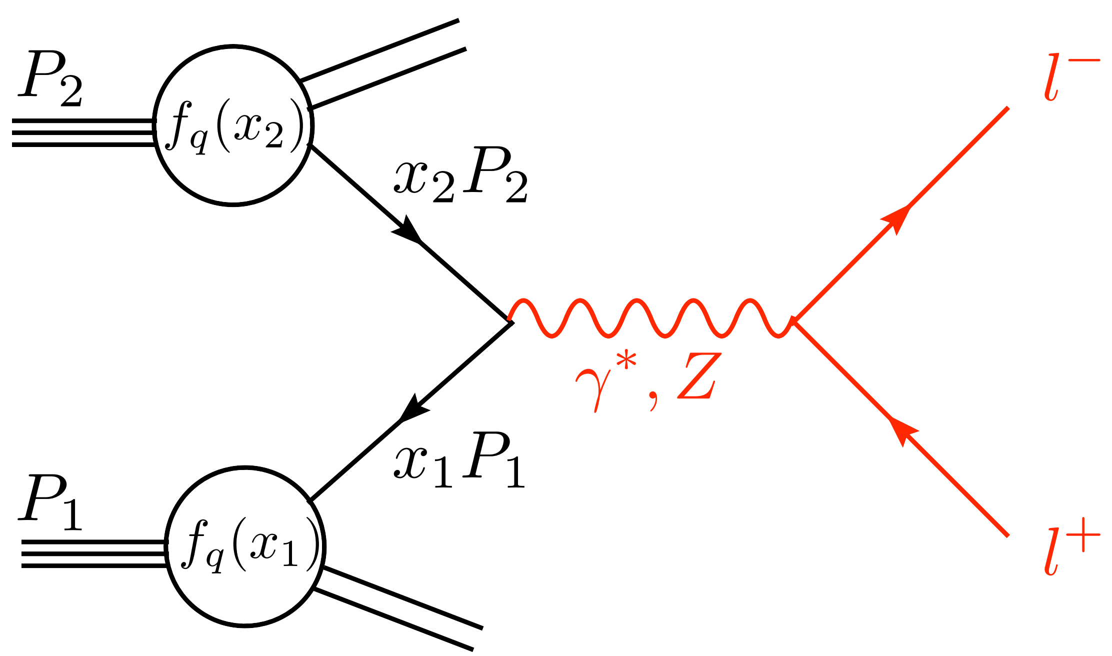
Very clean, golden-processes in QCD because

✓ dominated by quarks in the initial state

✓ no gluons or quarks in the final state (QCD corrections small)

✓ leptons easier experimentally (clear signature)

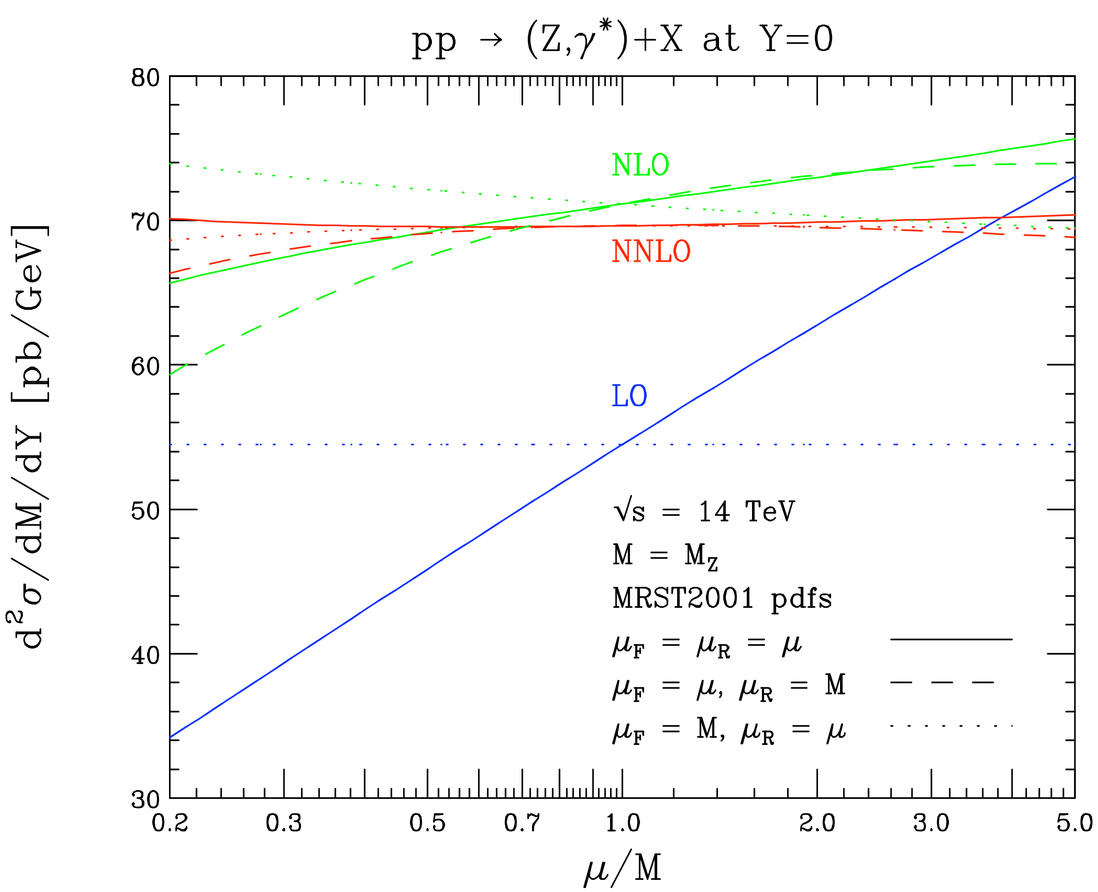
⇒ as clean as it gets at a hadron collider

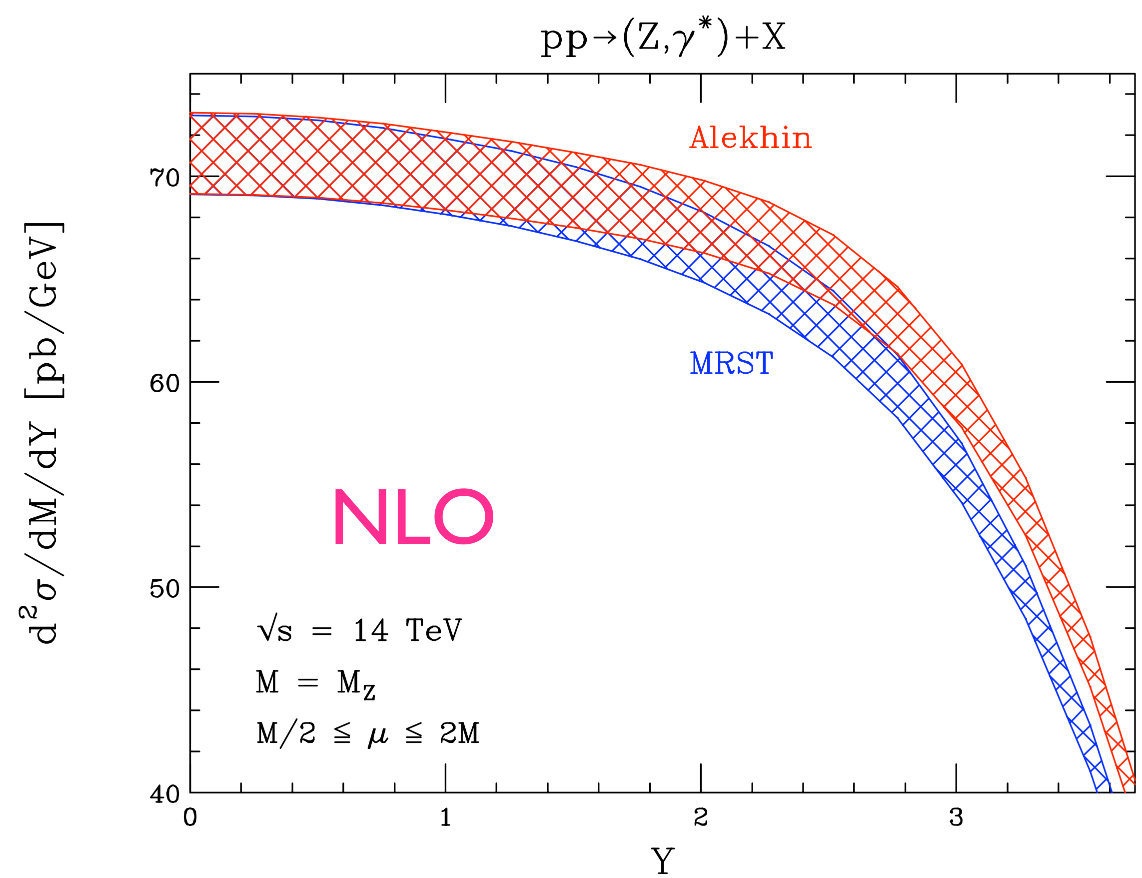


NNLO example: Drell-Yan

- most important and precise test of the SM at the LHC
- best known process at the LHC: spin-correlations, finite-width effects, γ-Z interference, fully differential in lepton momenta

Scale stability and sensitivity to PDFs



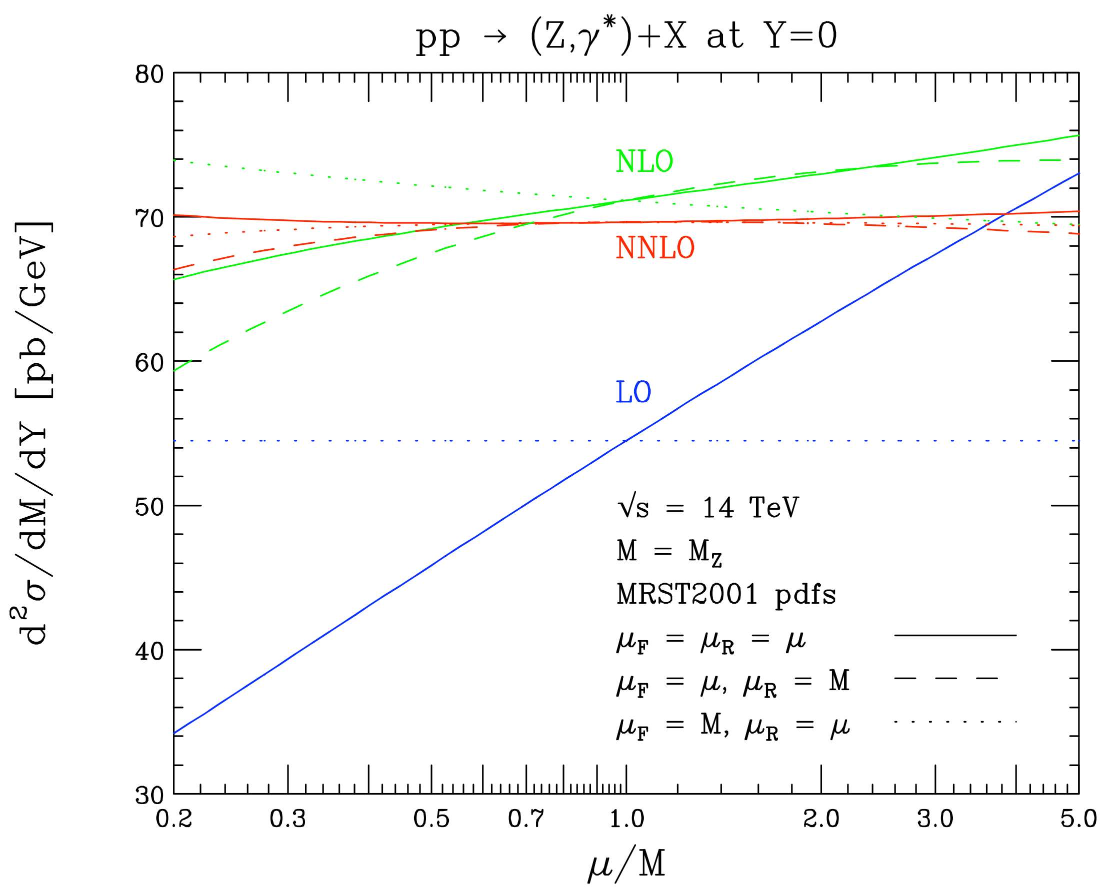


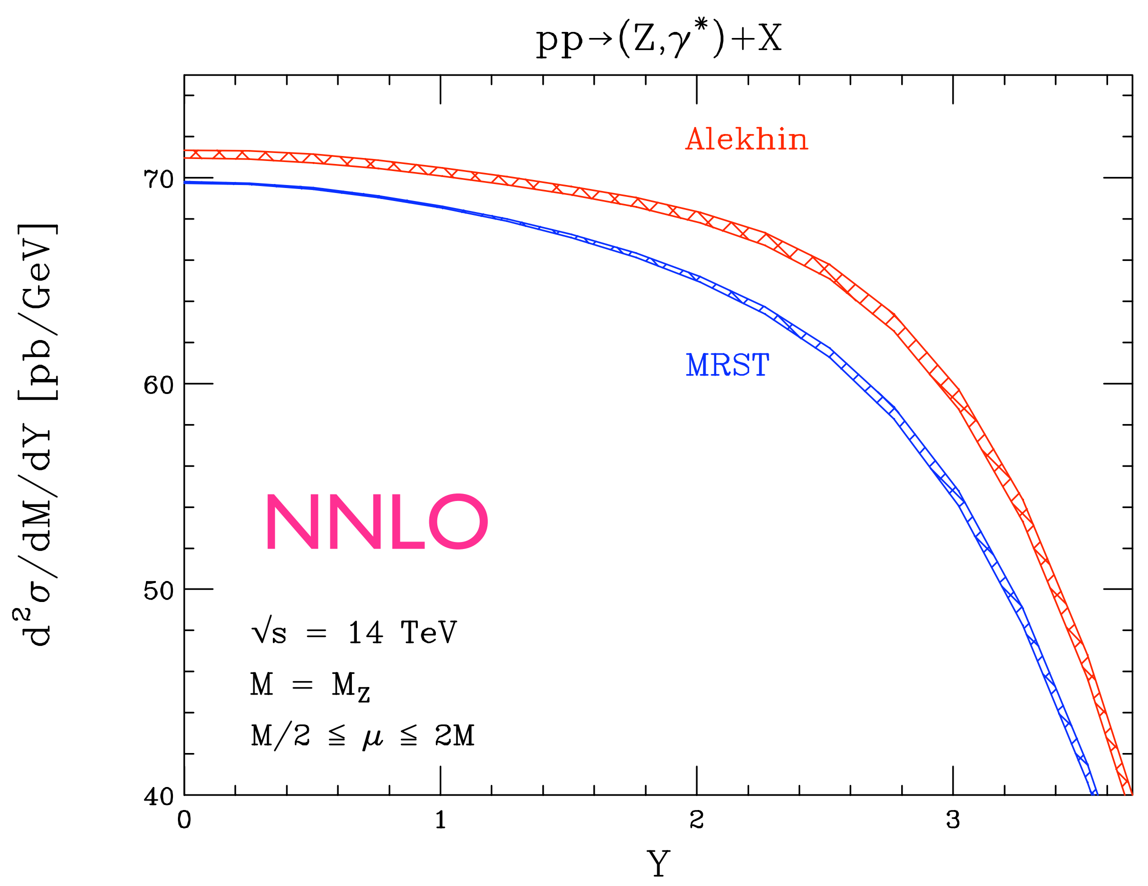
Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

NNLO example: Drell-Yan

- most important and precise test of the SM at the LHC
- best known process at the LHC: spin-correlations, finite-width effects, γ-Z interference, fully differential in lepton momenta

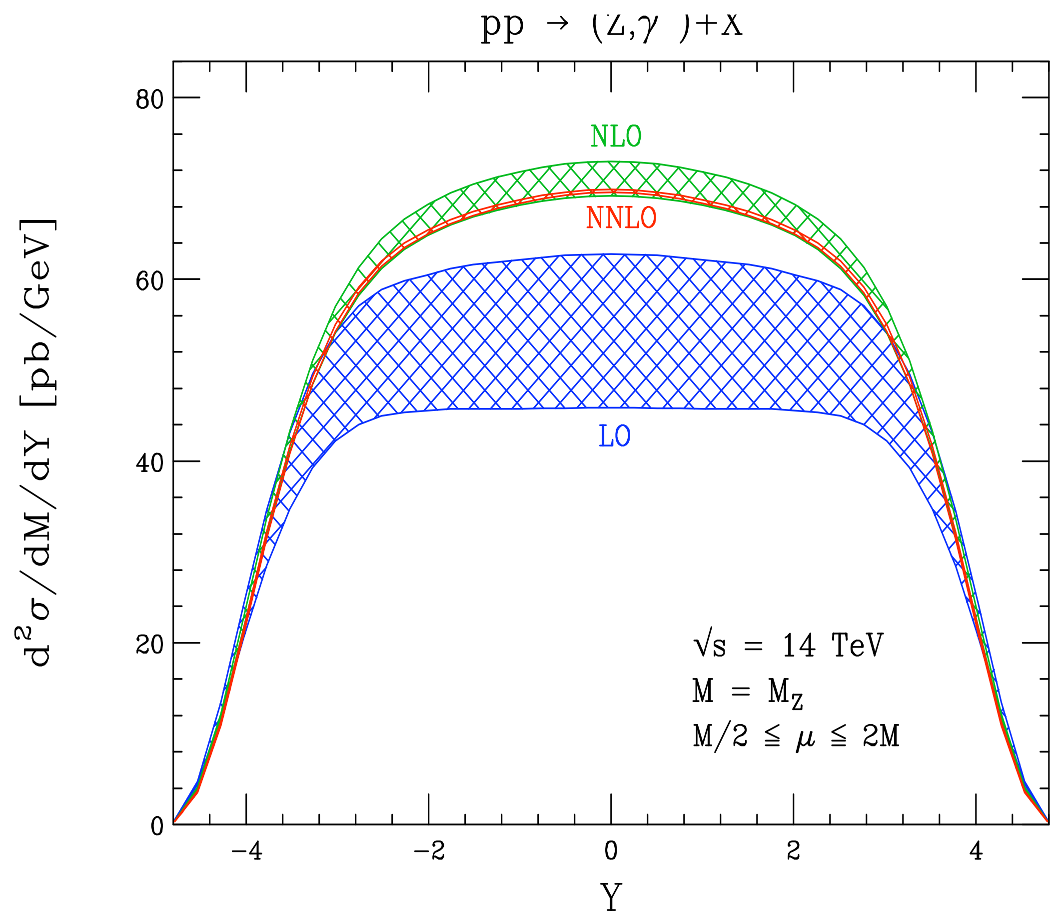
<u>Scale stability and sensitivity to PDFs</u>

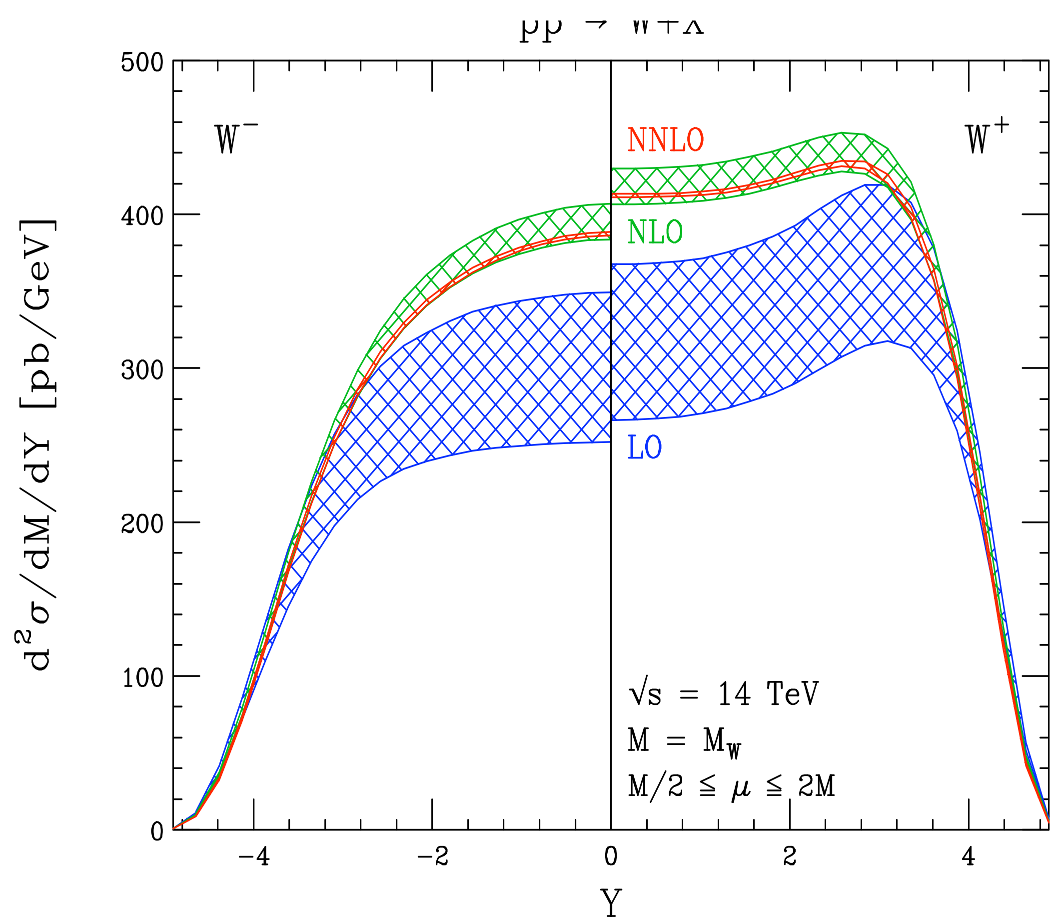




Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

Drell-Yan: rapidity distributions





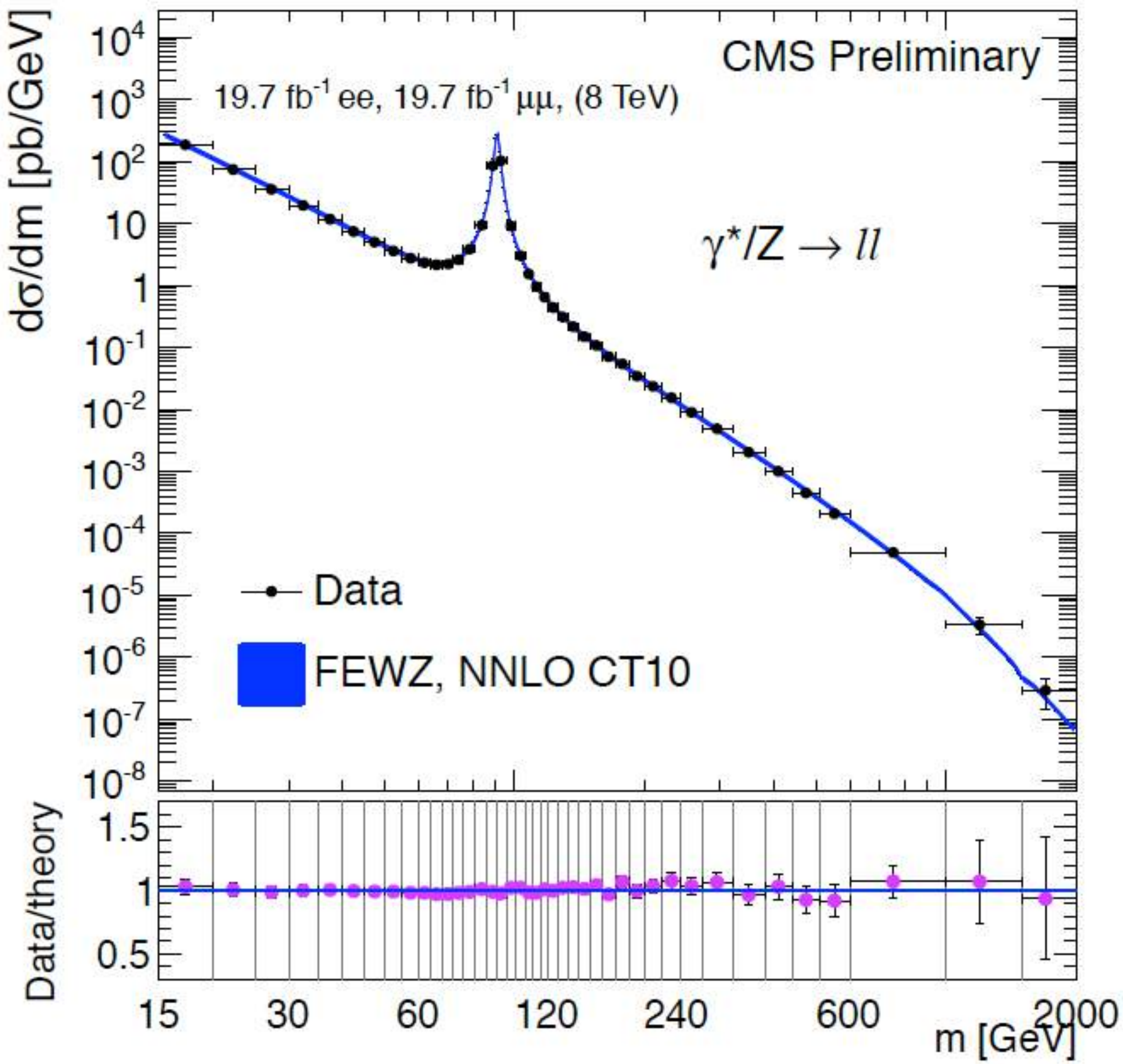
Anastasiou, Dixon, Melnikov, Petriello '03, '05; Melnikov, Petriello '06

☛ at the LHC: perturbative accuracy of the order of 1%

NNLO vs LHC data

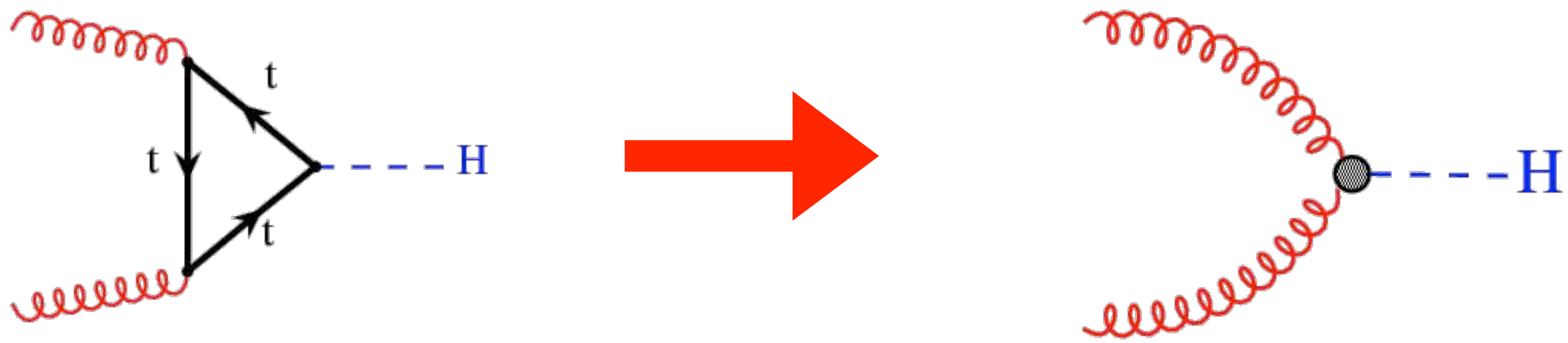
Impressive agreement between experiment and NNLO theory

CMS-PAS-SMP-14-003

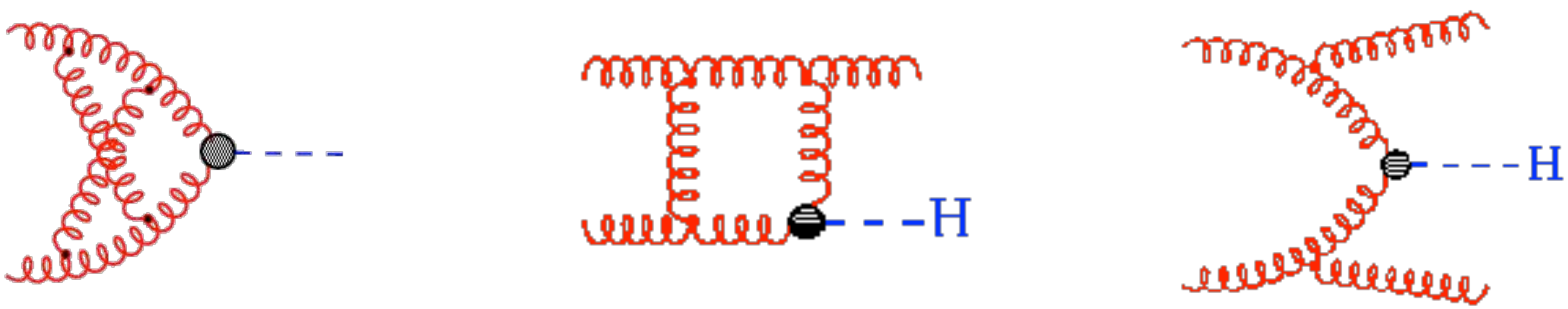


NNLO example: Higgs production

Inclusive Higgs production via gluon-gluon fusion in the large m_t-limit:

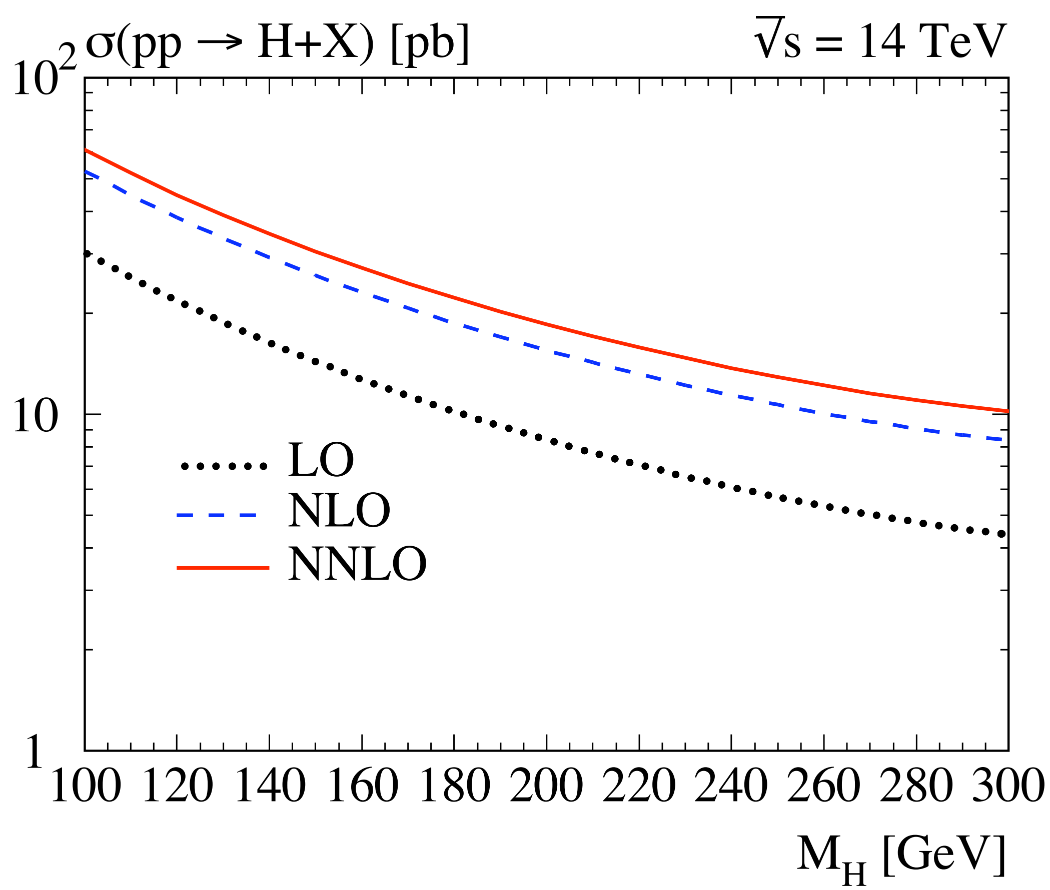


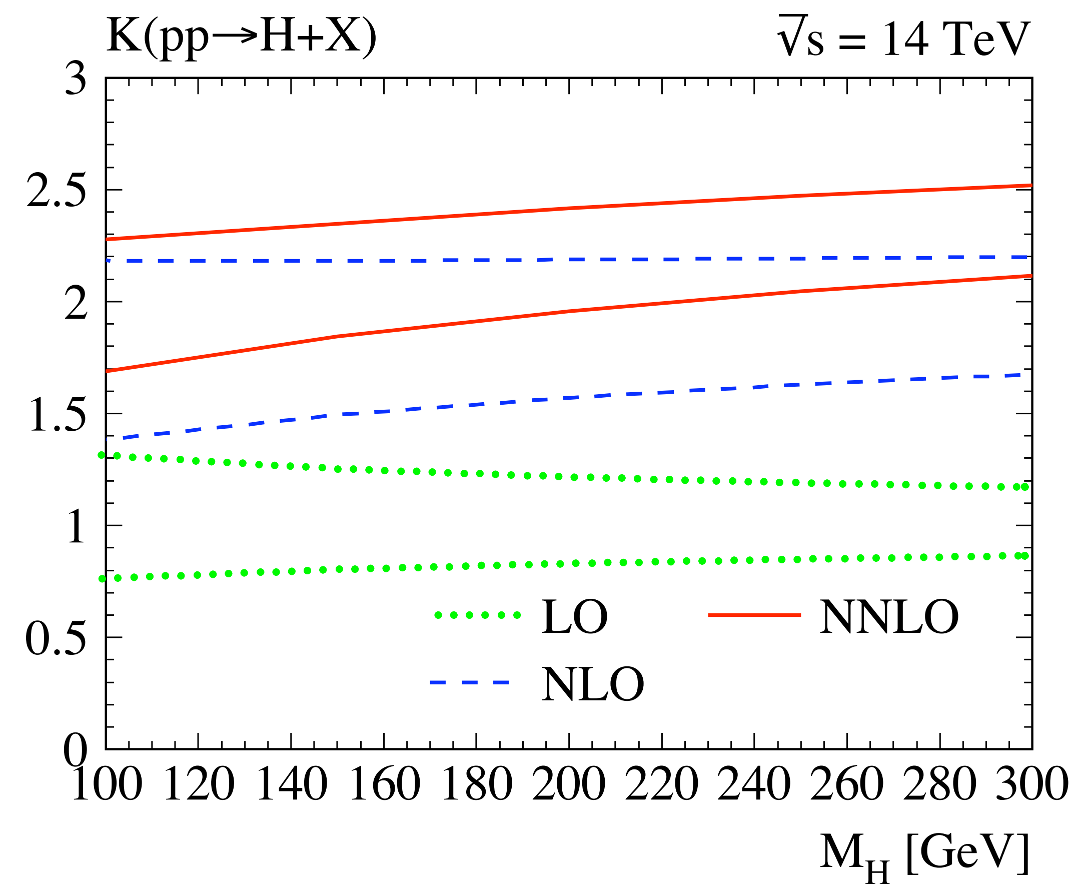
NNLO corrections known since few years now:



virtual-virtual real-virtual real-real

NNLO example: Higgs production

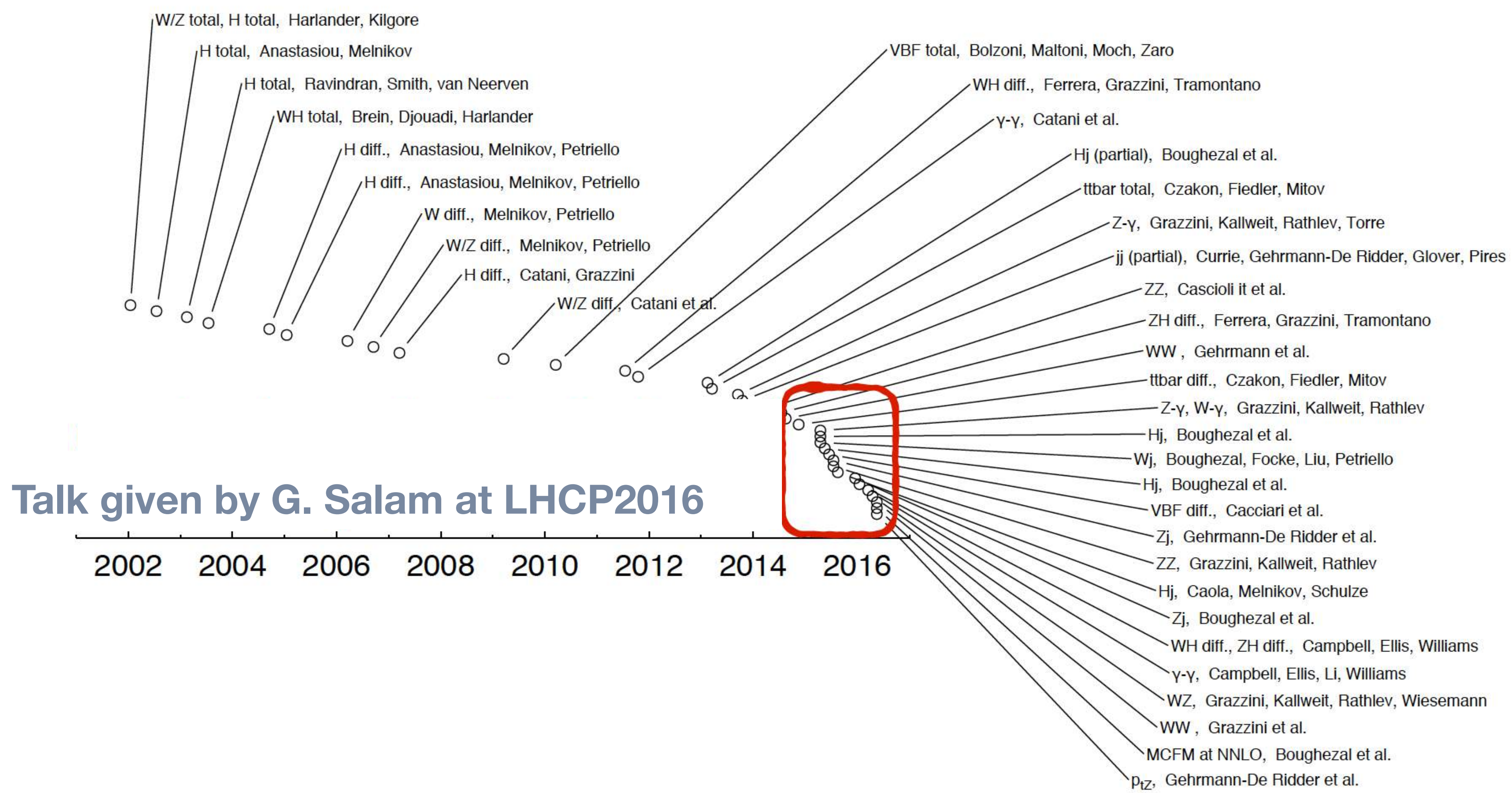




Kilgore, Harlander '02
Anastasiou , Melnikov '02

NNLO: the next challenge

An explosion of NNLO results in the last two years

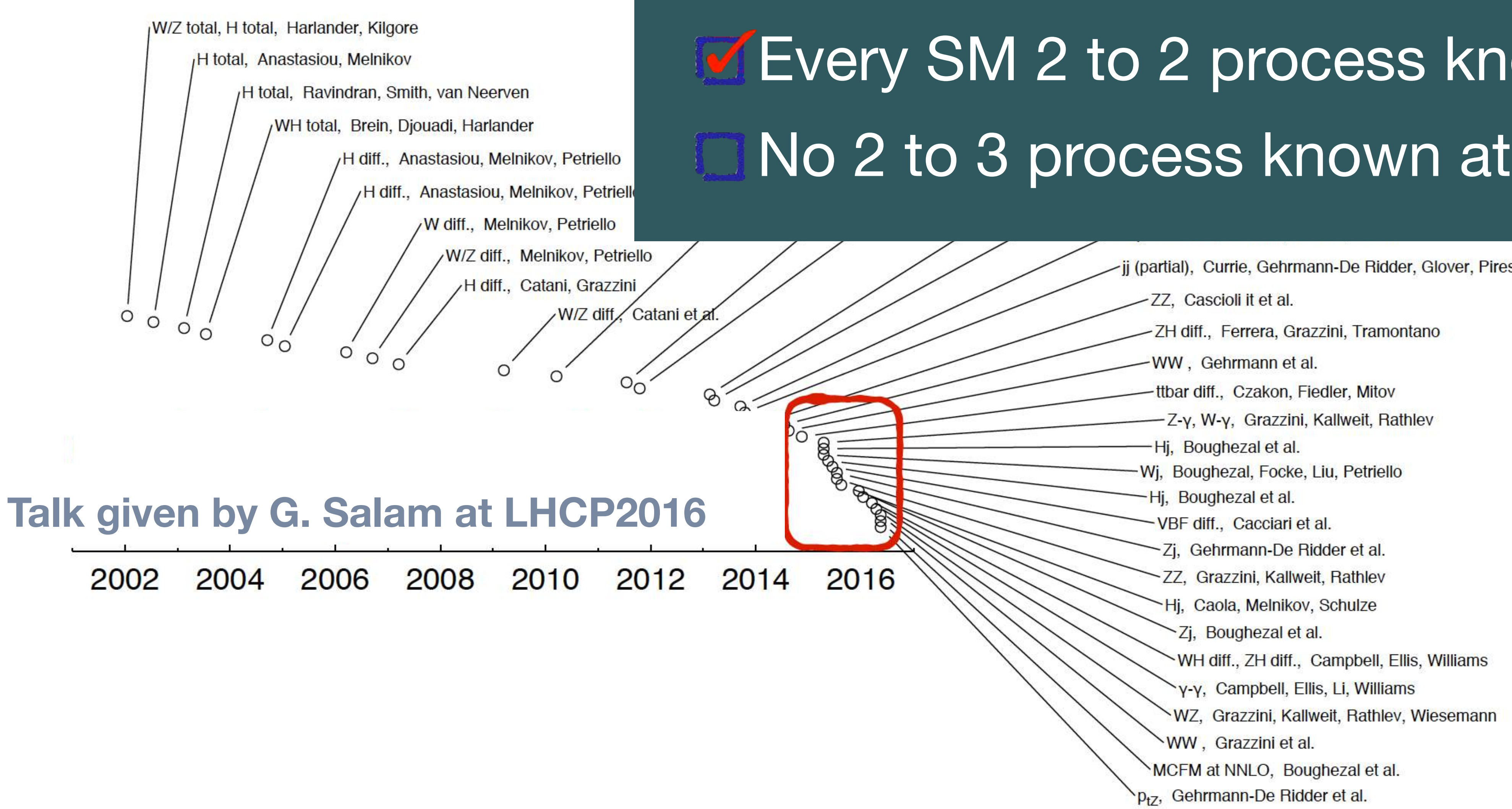


Things are developing rapidly, but a number of conceptual and technical challenges remain to be faced

NNLO: the next challenge

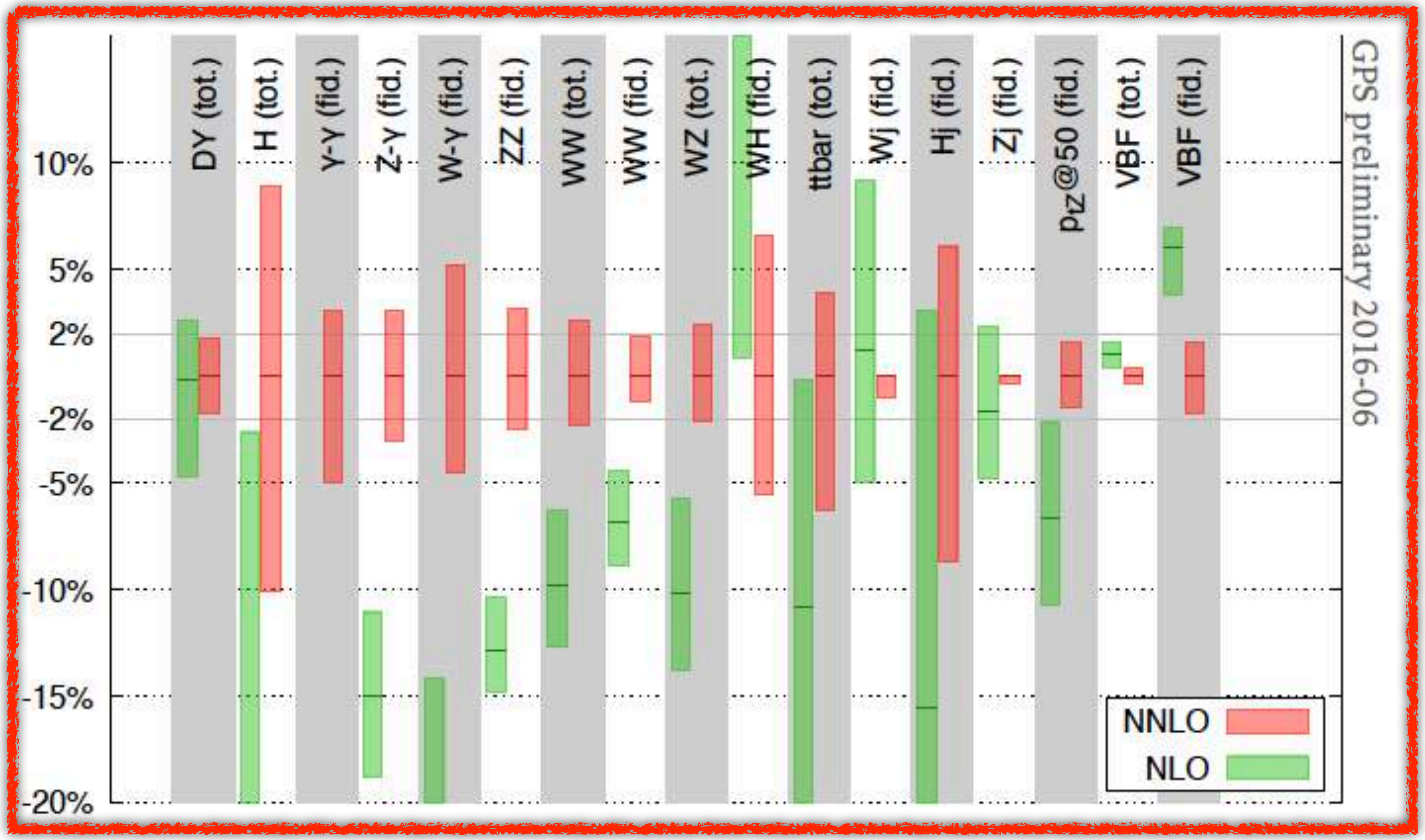
An explosion of NNLO results in the last two years

- ☑ Every SM 2 to 2 process known at NNLO
- ☐ No 2 to 3 process known at NNLO



Things are developing rapidly, but a number of conceptual and technical challenges remain to be faced

NNLO uncertainty?



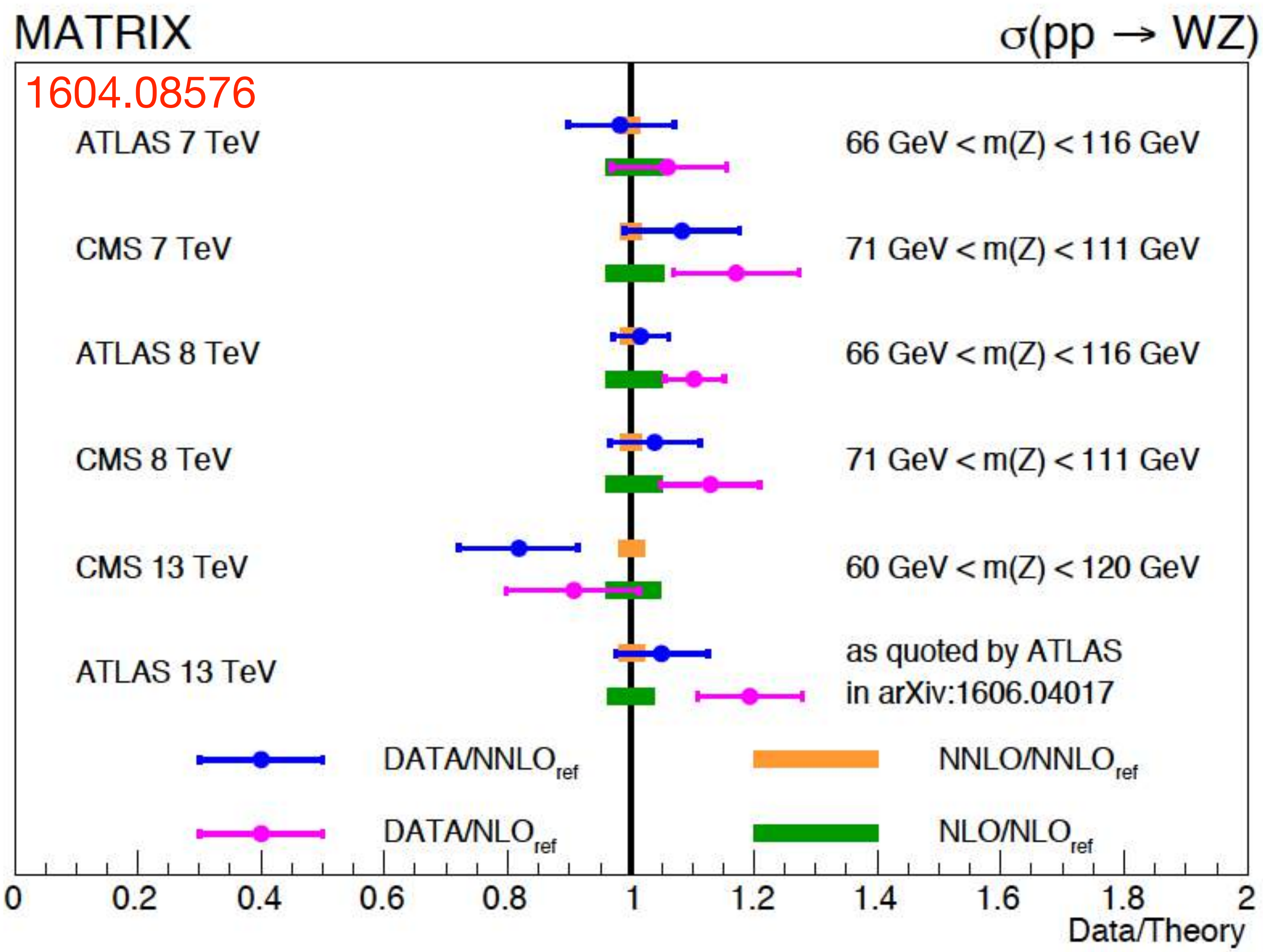
NNLO ***scale*** uncertainty bands of 1-2%.

Is the ***theory*** uncertainty indeed 1-2%?

NNLO vs LHC data

Example:

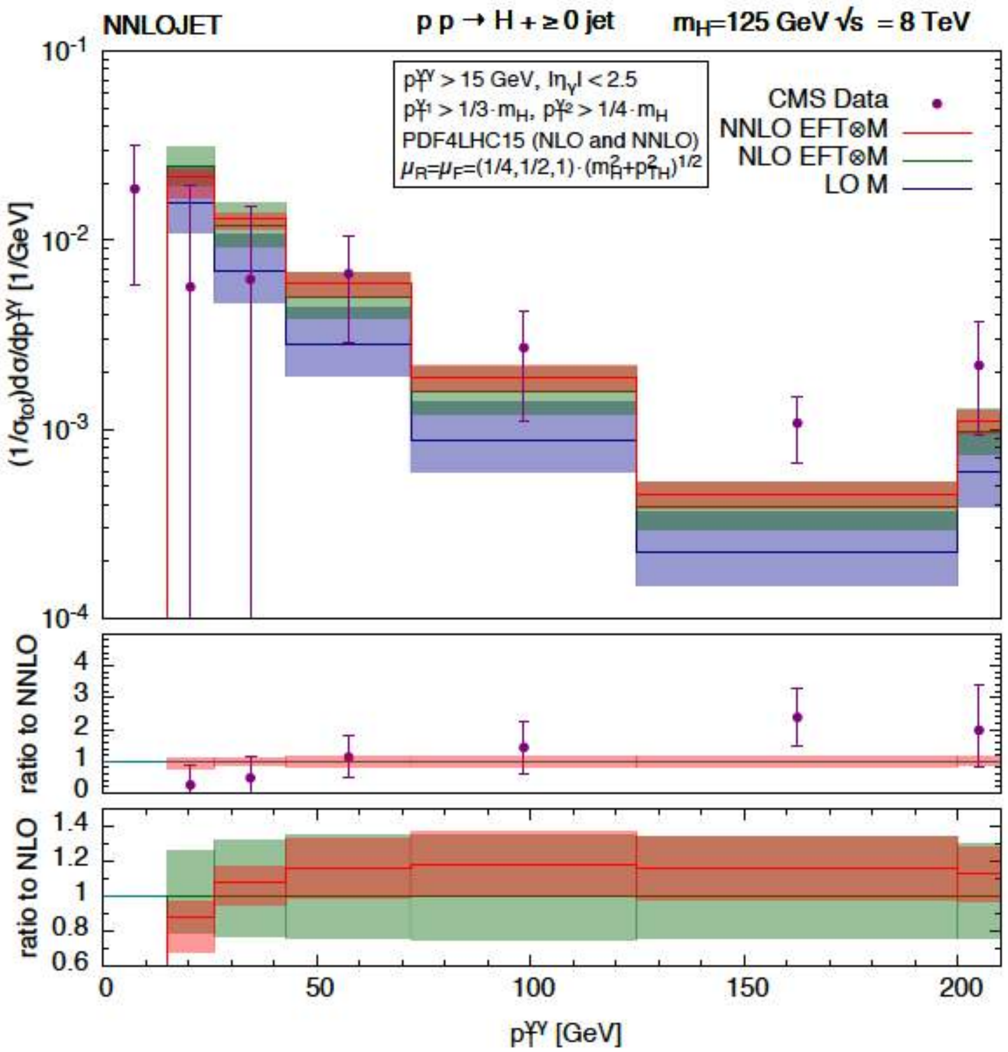
comparison of LHC data to NLO and NNLO for WZ production



Again, better agreement of LHC data with NNLO

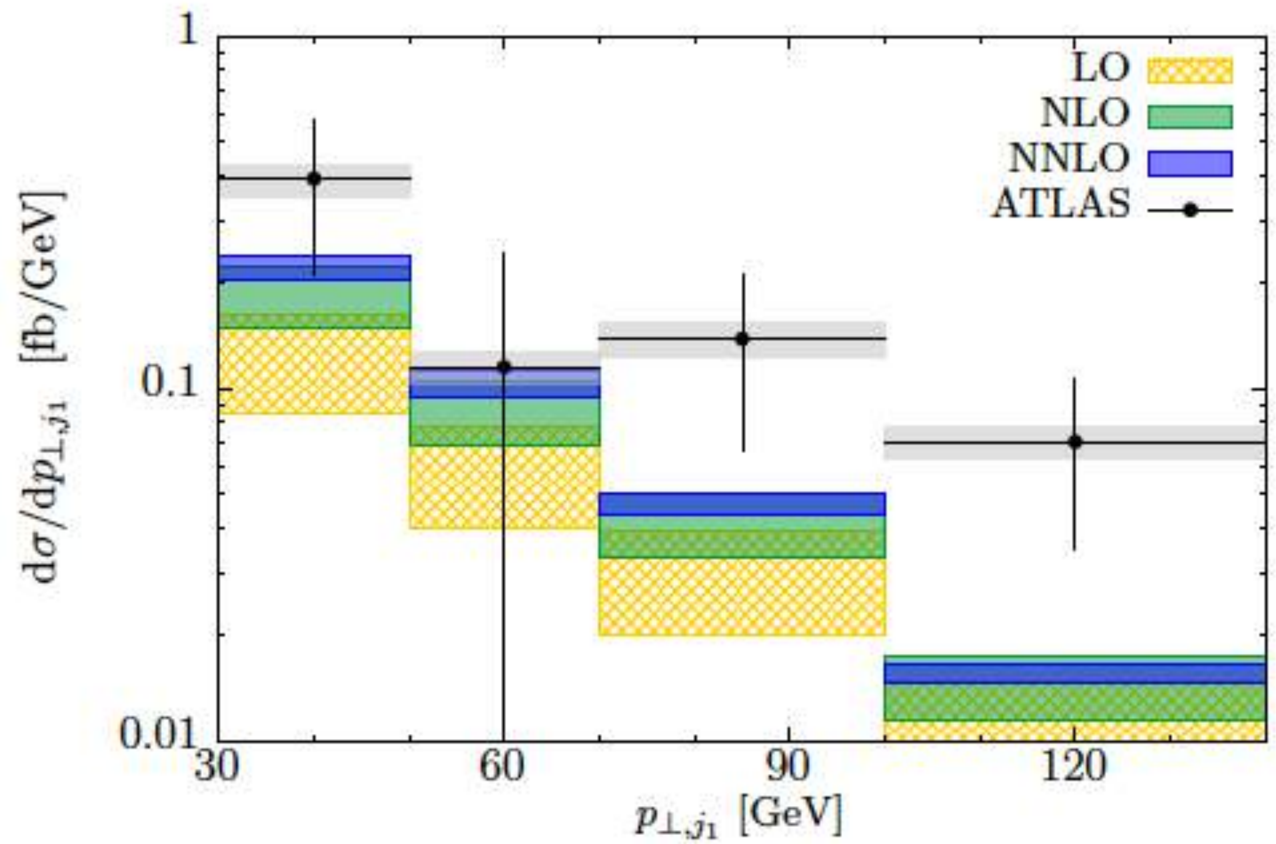
NNLO: Higgs +1jet

Decays of Higgs to bosons also included. Fiducial cross-sections compared to ATLAS and CMS data



Chen at al. 1607.08817

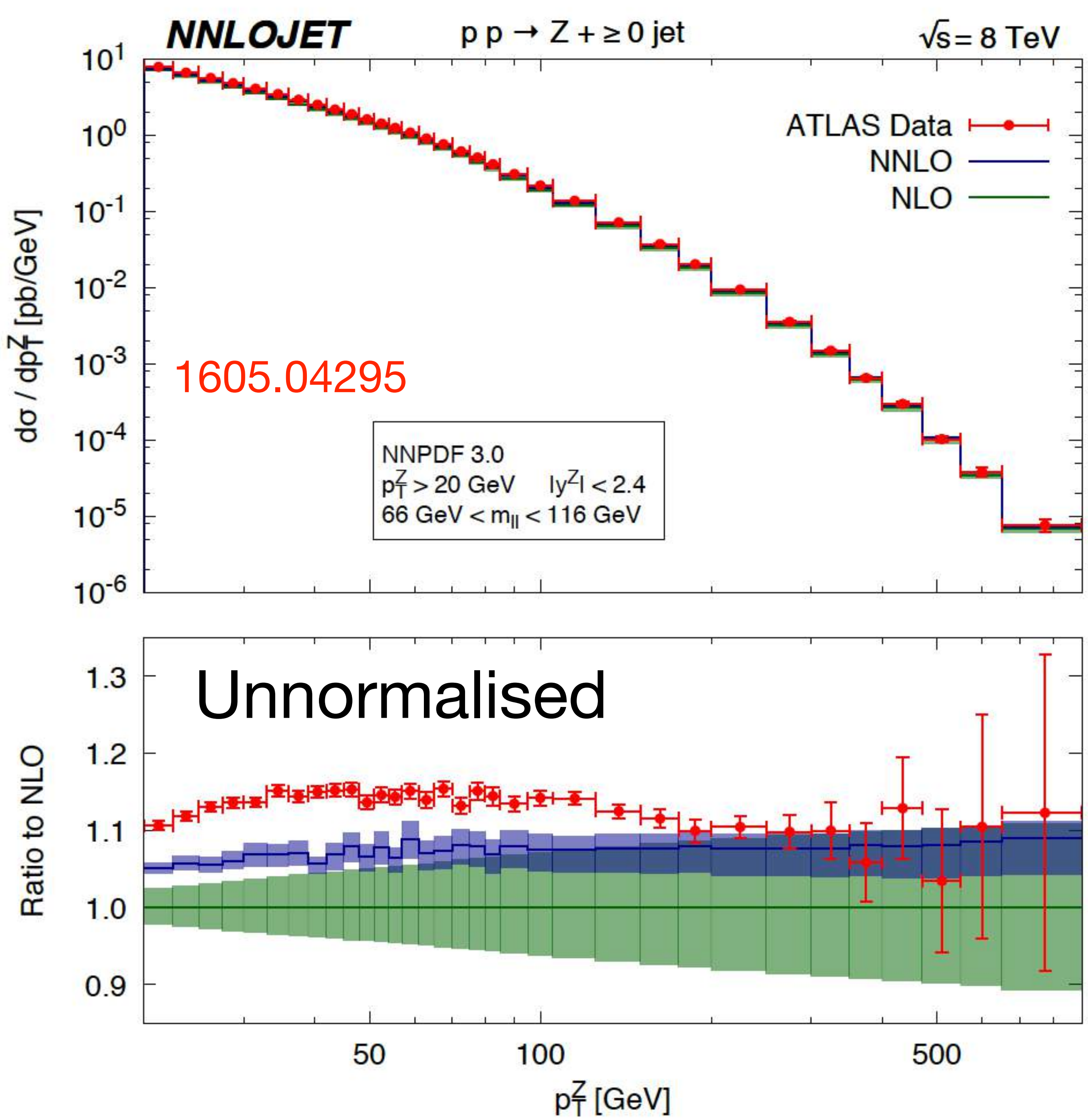
Caola, Melnikov, Schulze 1508.02684



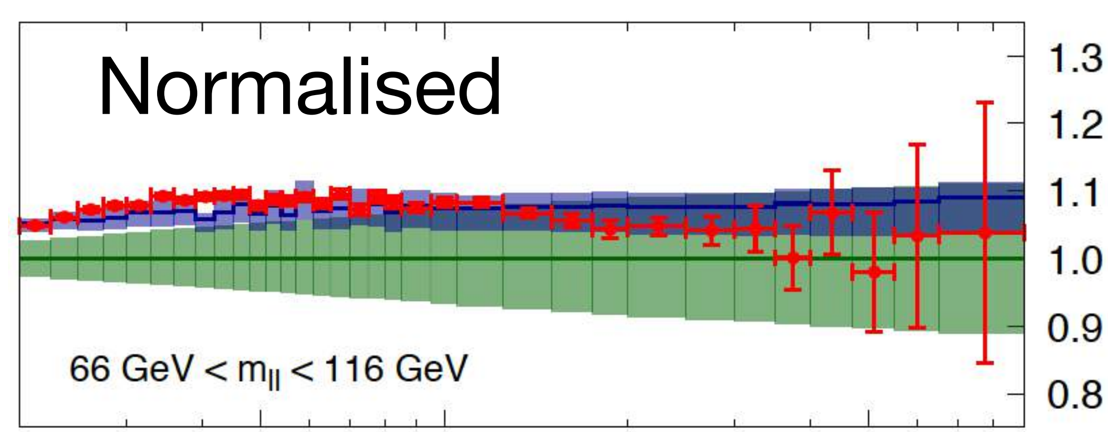
Good agreement on normalised distributions, less good agreement on unnormalised ones (but current data have large errors)

NNLO: Z + 1jet

Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan '16
Boughezal, Liu, Petriello '16
Boughezal, Ellis, Focke, Giele, Liu, Petriello '15

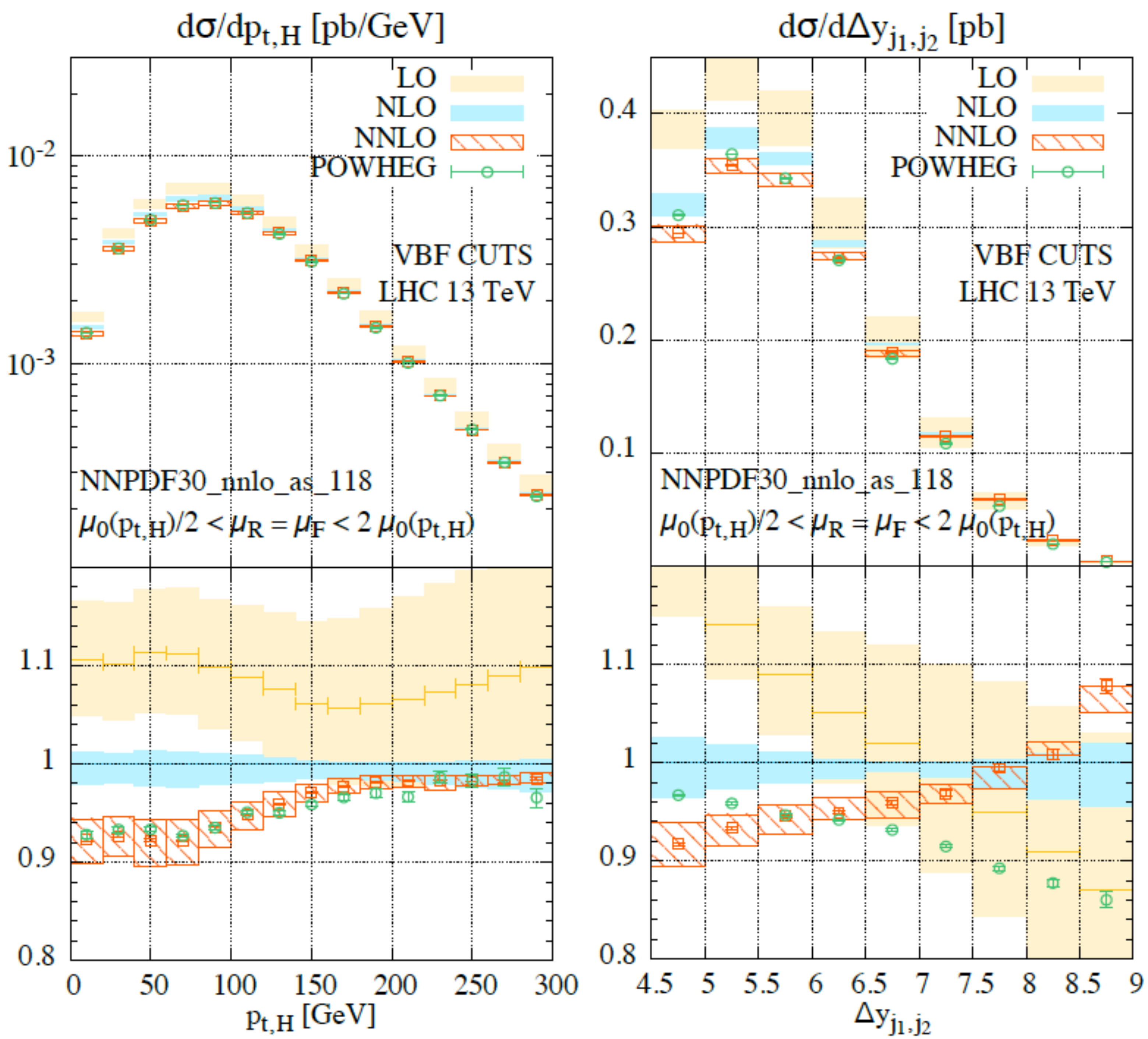


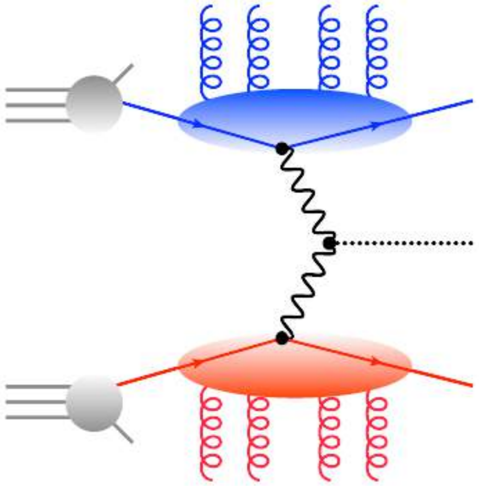
- inclusion of NNLO does not fully resolve tension between data and theory
- better agreement in normalised distribution
- remember 2-3% luminosity error on data



Fully differential VBFH at NNLO

Cacciari et al 1506.02660





- Allows to study realistic observables, with realistic cuts
- NNLO corrections much larger (10%) than expected (1%)
- Important for coupling measurements

N3LO

Two LHC processes known at N3LO

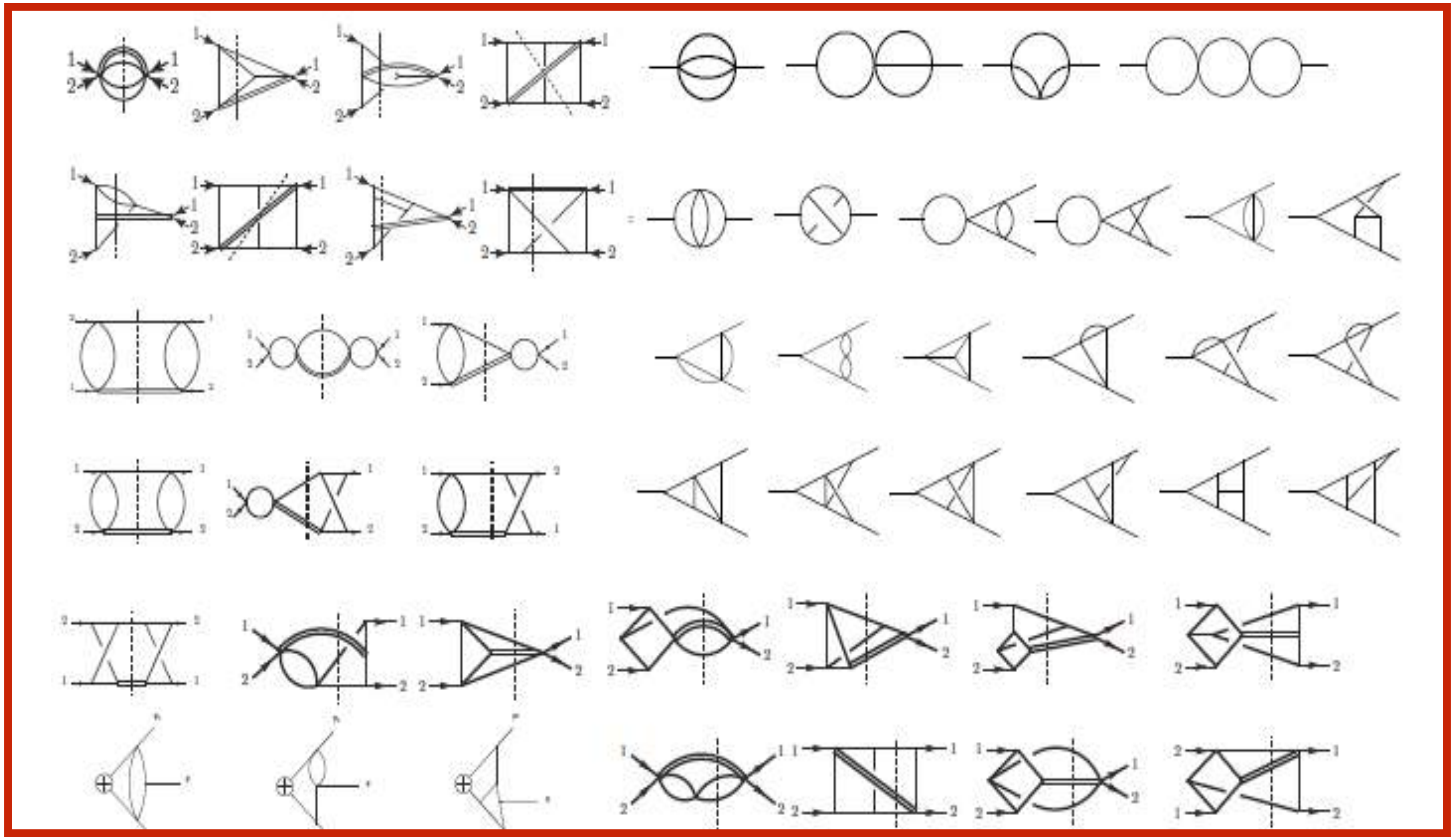
Gluon fusion Higgs production (in the large m_t effective theory)

Vector boson fusion Higgs production (in the structure function approximation, i.e. double DIS process)

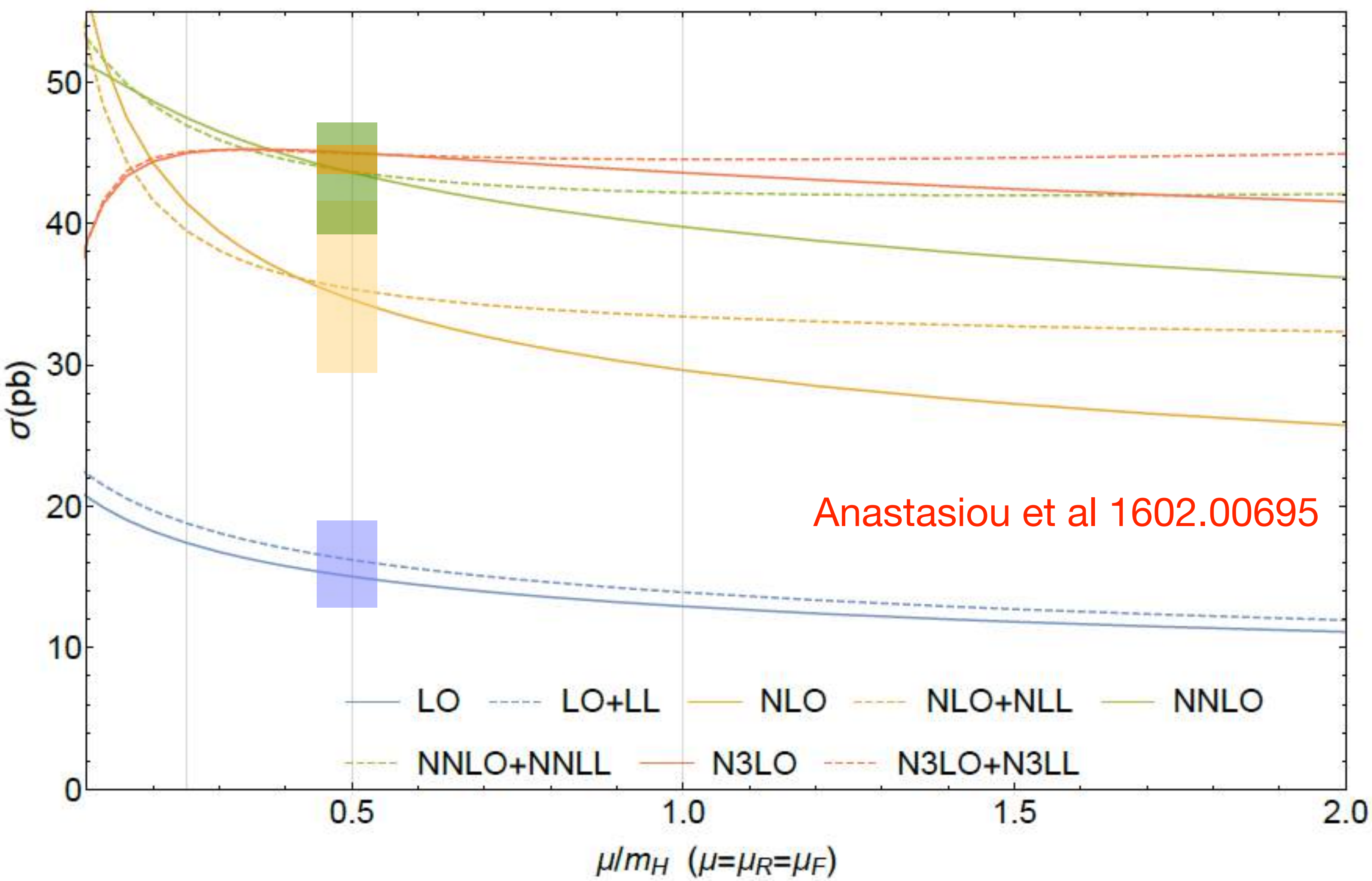
t

Higgs production at N3LO

- O(100000) interference diagrams (1000 at NNLO)
- 68273802 loop and phase space integrals (47000 at NNLO)
- about 1000 master integrals (26 at NNLO)

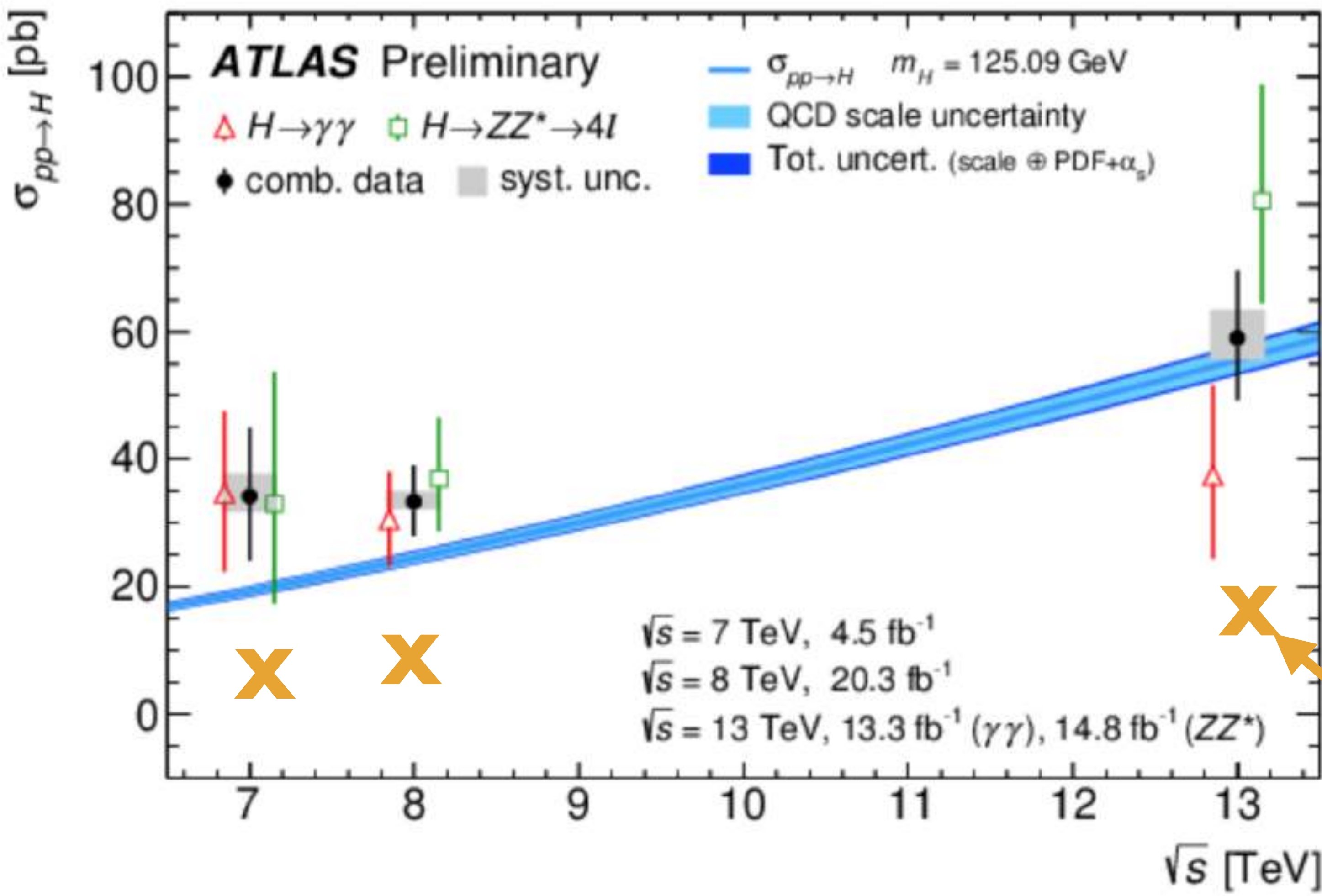


Higgs production at N3LO



- N^3LO finally stabilizes the perturbative expansion
- also matched to resummed calculation (essentially no impact on central value at preferred scale $m_H/2$)

Higgs production: theory vs data



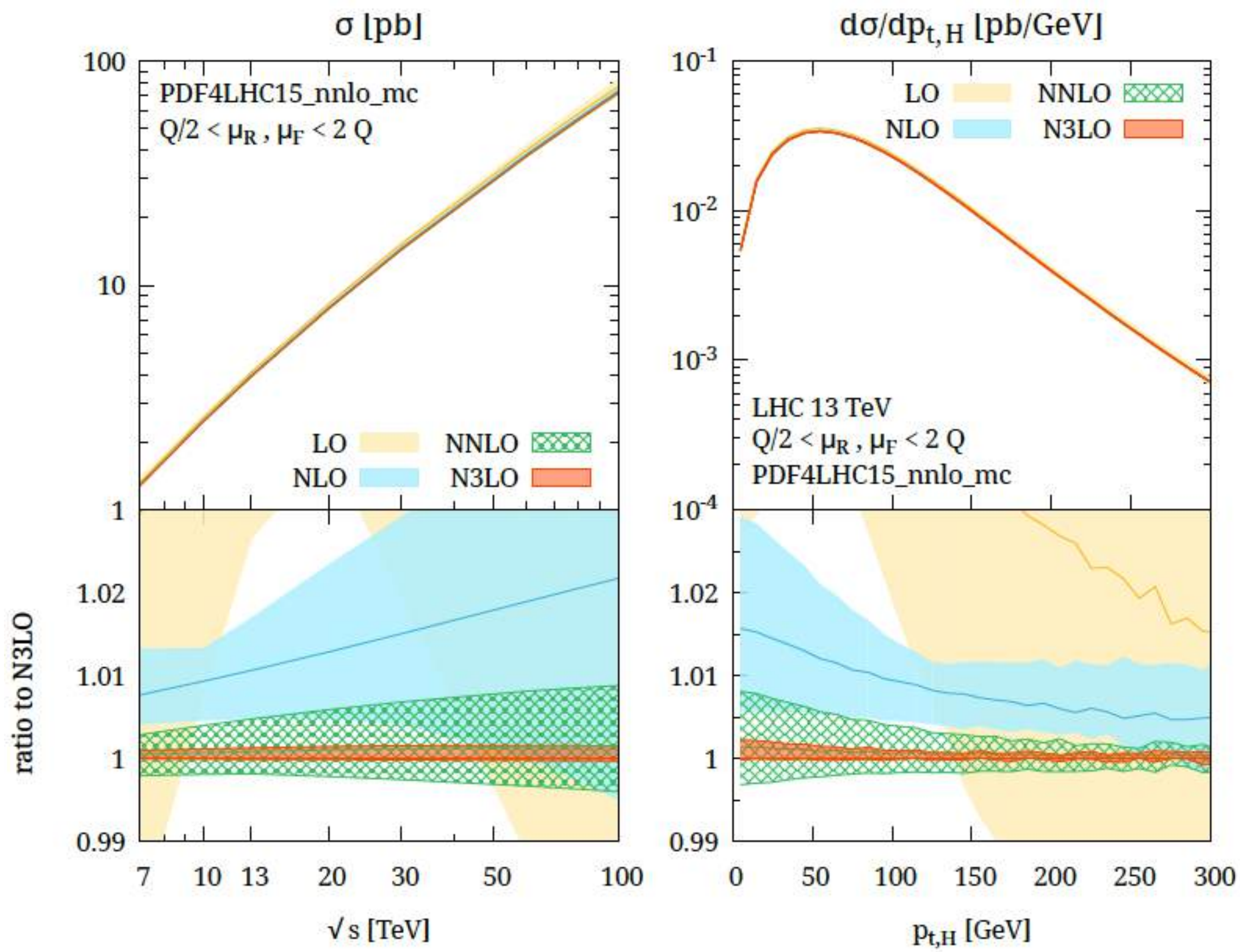
Theory 15 years ahead of experiment!

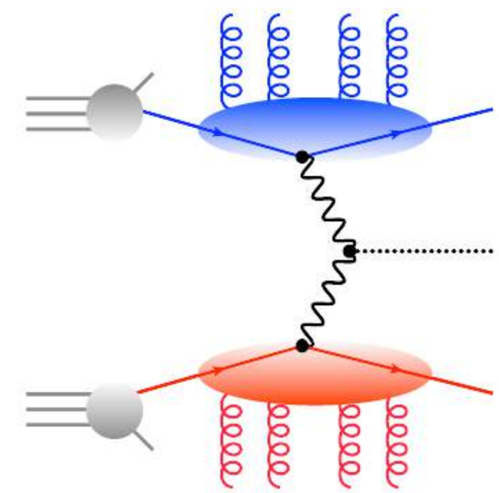
Theory predictions without higher orders

Next challenge: extend N^3LO accuracy to differential distributions (hard but within reach?)

… and inclusive VBF at N3LO

Dreyer & Karlberg 1606.00840





Again, NNLO was outside the NLO uncertainty band, while N^{3}LO band (with sensible scale) is fully contained in the NNLO band

Summary of perturbative calculations

- LO: fully automated. Edge: 10-12 particles in the final state
- NLO: also automated. Edge: 4-6 particles in the final state
- NNLO: the new frontier. Lots of new $2 \to 2$ processes in the last year ($2 \to 1$ more than10 years old). Currently no $2 \to 3$ calculation for the LHC
- NNNLO: fully inclusive Higgs production via gluon fusion (large m_t effective theory) and vector boson fusion (factorised approximation)

Higgs studies at the LHC

- The discovery of the Higgs boson at the LHC was a milestone in particle physics
- Higgs boson is the only fundamental scalar particle ever discovered. Its study at the LHC is new territory
- It is clear that this will be a long research program at the LHC [in comparison the b-quark was discovered forty years ago and, Belle II at SuperKEK, will now further study hadrons containing b-quarks]

An extremely rich program

Precision measurements
- mass, width
- spin, CP, couplings
- off-shell coupling, width interferometry
- differential distributions

Rare / beyond SM decays
- $H \rightarrow Z\gamma$
- $H \rightarrow \mu\mu$
- $H \rightarrow cc$
- $H \rightarrow \tau\mu$, τe, $e\mu$
- $H \rightarrow J/\Psi\gamma$, $\Upsilon\gamma$, …

Tool for discovery
- portal to BSM
- portal to hidden sector
- portal to DM

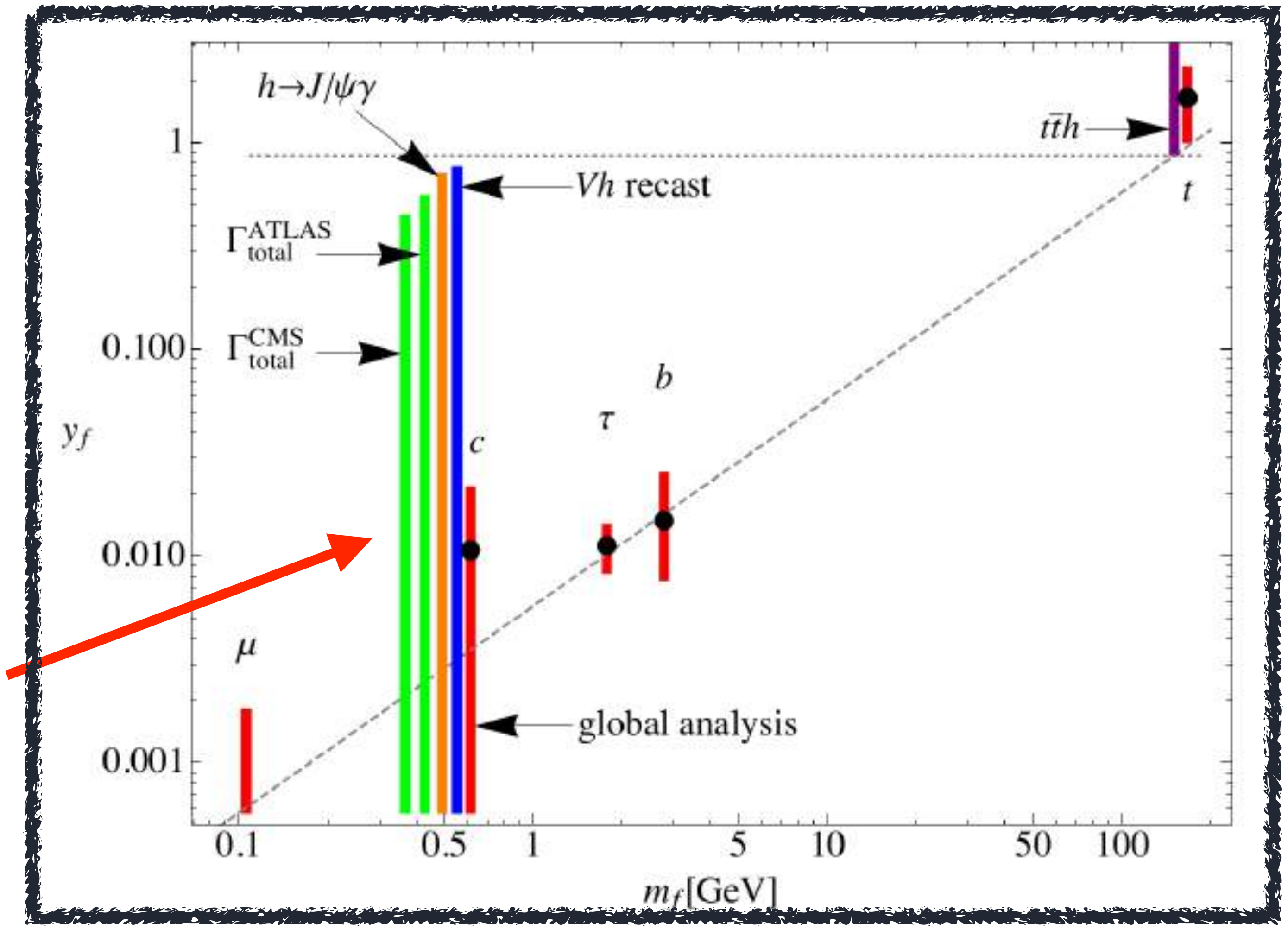
… and much more
- Higgs potential
- di-Higgs
- other FCNC decays
- …

SM minimal or not?
- 2HDM
- MSSM, NMSSM
- extra Higgs states, doubly-charged Higgs

Two examples, out of many, where theoretical precision brings new opportunities in the Higgs sector

1. Higgs coupling to light quarks

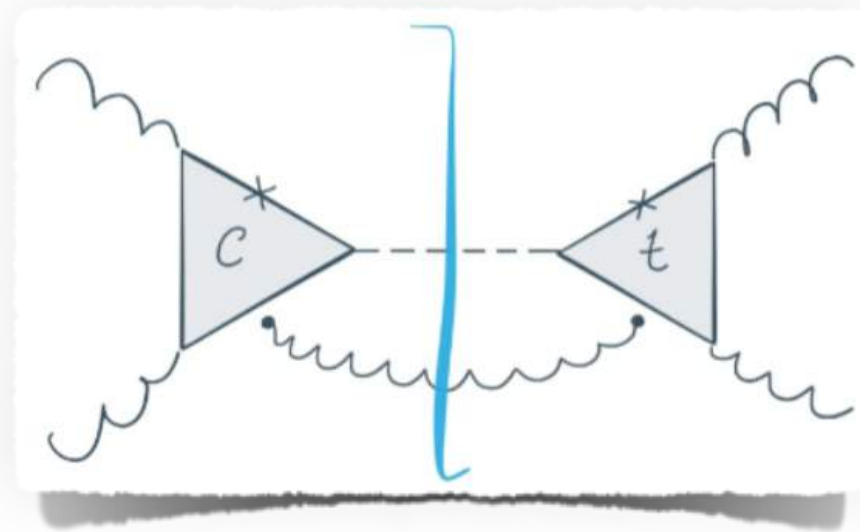
- couplings to 2nd (and 1st) generation notoriously very difficult because they are very small
- a number of ways to constraint the coupling of Higgs to charm:
  - rare exclusive Higgs decays
  - Higgs + charm production
  - constraint from VH (H →bb) including charm mis-tagging
  - constraint from Higgs width

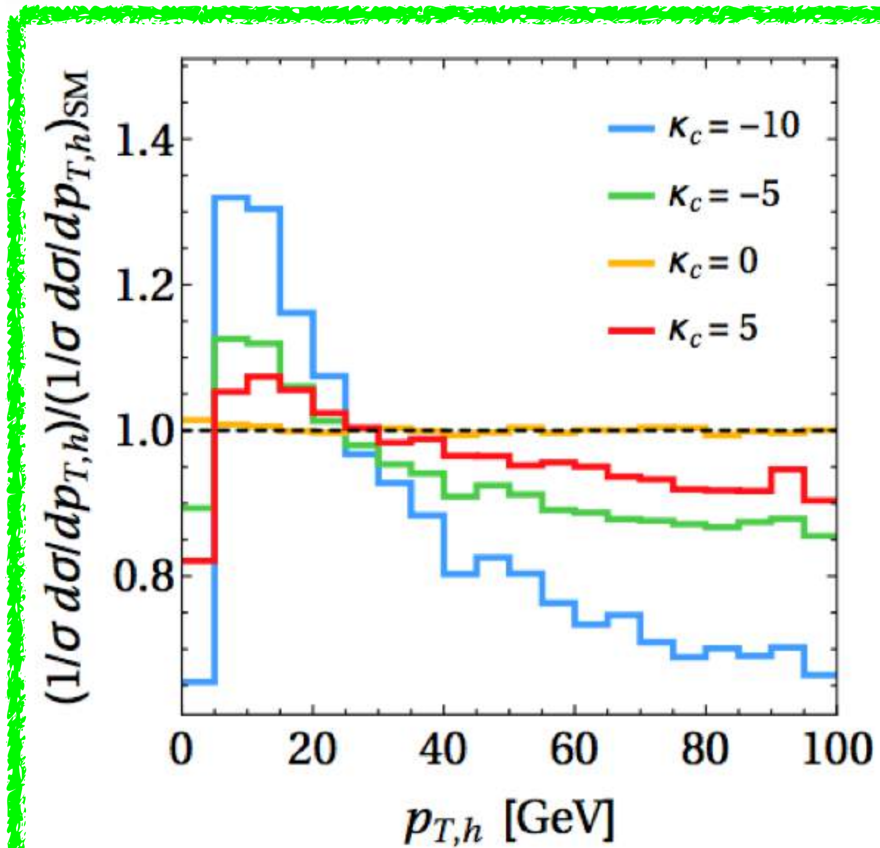


still largely unconstraint

1. Higgs coupling to light quarks

- Higgs produced dominantly via top-quark loop (largest coupling)
- but interference effects with light quarks are not negligible
- provided theoretical predictions are accurate enough (few%?), constraint on charm (and possible strange) Yukawa can be significantly improved



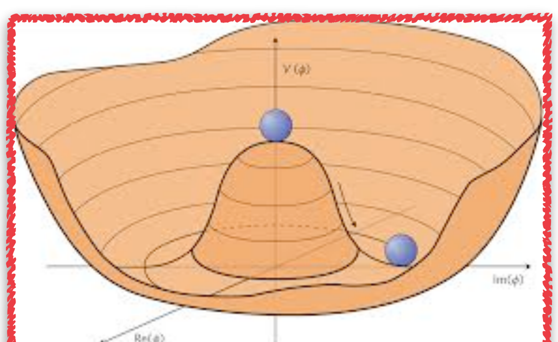


Bishara et al '16

2. The Higgs potential

The Higgs boson is responsible for the masses of all particles we know of. Its potential, linked to the Higgs self coupling, is predicted in the SM, but we have not tested it so far

$$V_{\rm SM} = \frac{m_h}{2} h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

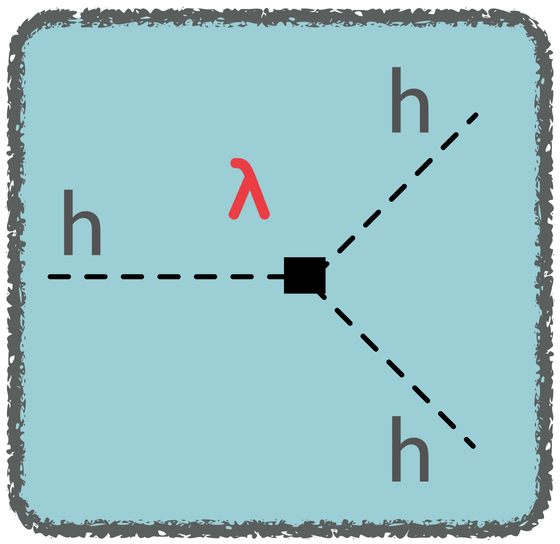


- [x] Single Higgs
  done O(45pb)
- [ ] Double Higgs
  very hard
  O(45fb)
- [ ] Triple Higgs
  out of reach
  O(0.1fb)

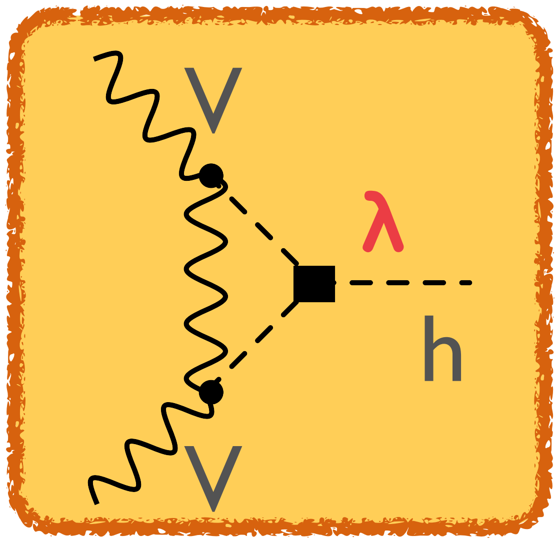
Bounds on λ today from LHC data still very loose (about a factor 10)

2. The Higgs potential

Traditionally: suggested to measure it through the production of two Higgs bosons (but difficult because of very small production rates)



Double Higgs



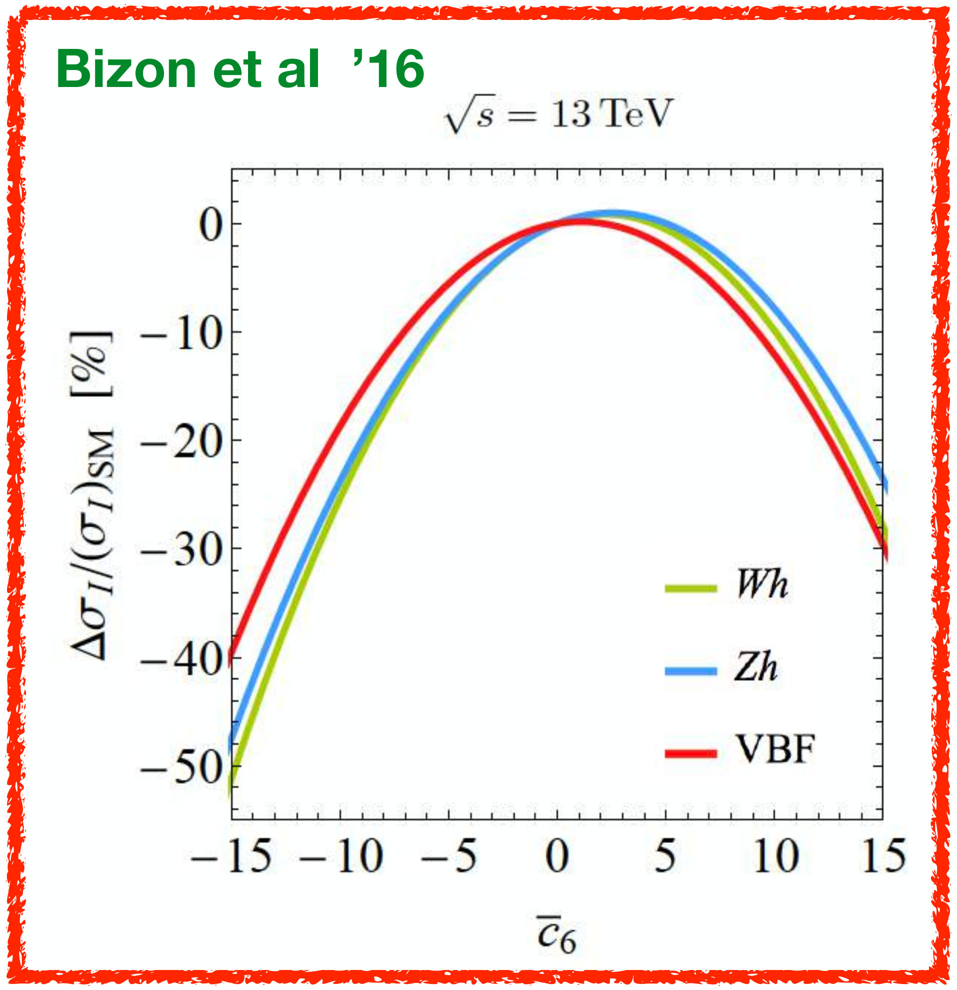
Single Higgs

New idea: **exploit indirect sensitivity to λ of single Higgs production**
Provides a wealth of new measurements (many production processes, many kinematic distributions), but theory and measurements must be accurate enough

2. The Higgs potential

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{\bar{c}_6}{2v^2}\mathcal{O}_6$$

$$\mathcal{O}_6 = -\lambda_{\rm SM}(H^\dagger H)^3$$



See also
De Grassi et al 1702.01737
Di Vita et al 1704.01953
Maltoni et al 1709.08649
Di Vita et al 1711.03978
[…]

New idea: exploit indirect sensitivity to λ of single Higgs production
Provides a wealth of new measurements (many production processes, many kinematic distributions) to be used in a global fit (but theory must be accurate enough)

Recap

In this lecture we have

- Played around with LHC kinematics
- Looked at the LO calculation of di-jet production
- Understood the challenges to perform higher-order calculations
- Reviewed the status of higher-order calculations
- Looked at two examples of ideas where precision can be used to extract information in a new way