Focus on WIMPs

Historical aside - so far all DM searches grunted & failed.

2 previous examples - 1. Neptune discovered through wobbles in orbit of Uranus.

   Original DM! Found in 1846 in 30 min of searching within 1° of prediction of Urban Le Verrier.


   Although originally thought to be planet Vulcan.

Freeze out: why we like WIMPs. (See Ch. 3 for more details)

   (For more details see Bernstein - Kusuki Theory in孕育 variance)

   If DM has more than gravitational interactions here is a nice story.

   Symplectic $XX \rightarrow \frac{f}{e}$ in $\gamma$ form.

   After by slow DM produced along with SM particles now in thermal sym

   Phase space density of DM $f(p, t)$ (NB $f \propto 1/p^4$ & density $\propto 1/p^2$)

   Does Liouville eqn: $\frac{df}{dt} = H$

   Bottom:

   $\frac{\partial}{\partial t} f \frac{\partial}{\partial p} f = C \sum_{\text{DM}} f$

   Often don't need to worry about still phase space distribution, some $x$ chance

   (Method of pseudo-chemical potential)
Frankel: $ds^2 = +dt^2 - a^2(t) (dr^2 + r^2 d\theta^2)$

Assumed $k = 0$.

$H \equiv \frac{\dot{a}}{a}$.

From the Friedmann equation:

$\frac{\dot{a}}{a} = \frac{1}{2} H^2 - \frac{k}{a^2}$. 

The universe is homogeneous and isotropic, so $k = 0$. Thus:

$\frac{\dot{a}}{a} = \frac{1}{2} H^2$. 

From the Friedmann equation, we have:

$H^2 = \frac{8\pi G}{3} \rho$.

For a flat universe ($k = 0$):

$H^2 = \frac{8\pi G}{3} \rho$.

For a closed universe ($k > 0$):

$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$.

For an open universe ($k < 0$):

$H^2 = \frac{8\pi G}{3} \rho + \frac{k}{a^2}$.
In general, there are no solutions to the non-interacting Liouville eqn:
\[ L(t) = 0 = \frac{\partial f}{\partial t} - \frac{\partial}{\partial p} \left( \frac{\partial f}{\partial p} \right). \]

But for \( m=0 \), \( m \to \infty \) has solutions. So for very high or very low temperatures we can show:

Assume \( f(y) = e^{(y - y(0))E} \), a chemical pd.

Then \( L(y) = 0 \Rightarrow \dot{E} - EP + \frac{e}{a} \frac{dE}{dp} \frac{dp}{E} = 0 \)

\[ \dot{E} = \frac{1}{a} \frac{dp}{E} + \frac{E}{p} \frac{dp}{E}. \]

\( m = 0 \): \( E = p \), \( \dot{p} = p \left( 1 - \frac{\dot{a}}{a} \frac{p}{\beta} \right) \Rightarrow \) bh \( E = 0 \) \( \beta = \text{const} \cdot a \).

So massive particles stay in equilibrium at \( T \sim \frac{1}{a} \).

Finally, \( E = m + \frac{p^2}{2m} \), \( \dot{p} = \frac{p^2}{2m} \left( 1 - 2 \frac{\dot{a}}{a} \frac{p}{\beta} \right) \Rightarrow \) bh \( E = m \beta + \text{const} \).

\( \beta \dot{a} = \frac{1}{2} a \ddot{a} \), \( \ddot{a} = a \).

So massive particles stay in equilibrium \( T \sim \frac{1}{a^2} \).

Since we don't usually care about full phase space, we can take instead:

\[ N = \int \frac{d^3p}{(2\pi)^3} f(p,t) \] (integrated by parts)

Recall Radial \( \pi a^3 \) mols. \( \pi a^3 \text{mol} \)

\[ ds^2 = -dt^2 + a^2(t) \left( dx^2 + x^2 dx^2 + dy^2 \right) \text{Pfv.} \]

\[ \dot{N} + 3Hn = g \int \frac{d^3p}{(2\pi)^3} \mathcal{F}(E) \]
\begin{align*}
\sum_{\text{spins}} \frac{d^3 p_1}{(2\pi)^3} \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\
\times \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4}
\end{align*}

Assume \( f_1 \) or \( \bar{f}_1 \) or BE is thermal equilibrium.
Assume \( T \ll E \) so \( \bar{f}_1 \) \& MB \simeq 1.
Assume \( 3,4 \) (SM states) are kept in equilibrium by other reactions.
\[ \langle \text{out}_1 \rangle = \int \text{out}_1 d^4 \eta \langle \text{in}_1 \rangle \]

Then \( \hat{n} + 3\text{H} = \langle \text{in}_1 \rangle \left( \hat{n}_{\text{H}_2} - \hat{n}_{\text{H}_2} \right) \)

Related balance \( \Rightarrow \hat{n}_{\text{H}_2} = \frac{\hat{n}_{\text{H}_2}}{\hat{n}_{\text{H}_2}} \).

Finally, since \( \hat{A}_1 \subset \text{XH} \),
\[ \hat{n} + 3\text{H} = \langle \text{in}_1 \rangle \left( \frac{\hat{n}}{\hat{n}} - \frac{\hat{n}}{\hat{n}} \right) \]

Typically done numerically.

\( \hat{n} = \alpha^2 \) (even \( \mu \) interacts)

so useful to define \( Y = \hat{n} / \hat{s} \)

Also \( \hat{s} = 3 \text{A}^2 \) const.

\[ \frac{dY}{d\tau} = \frac{\hat{n}}{s} - \frac{3}{s} \hat{n} = \frac{\hat{n}}{s} + 3\text{H} \Rightarrow \frac{dY}{d\tau} = \langle \text{in}_1 \rangle < \text{Y}^2 > \]

\( < \text{Y}^2 > \) Confining density

\( Y = \frac{m}{A} \)

\[ \frac{dY}{d\tau} = \frac{\hat{n}}{s} - \frac{3}{s} \hat{n} = \frac{\hat{n}}{s} + 3\text{H} \Rightarrow \frac{dY}{d\tau} = \langle \text{in}_1 \rangle < \text{Y}^2 > \]
\[
\frac{dx}{dt} = \frac{dx}{dt} \frac{dt}{dt} \quad \text{In RD} \quad H = \frac{\rho}{3} \frac{1}{T^2} \quad \text{at} \quad T \approx \frac{1}{\sqrt{\lambda}}
\]

So \( \frac{a}{a} \sim \frac{1}{T} \Rightarrow \quad a \sim T^{-2} \)

\[
\Rightarrow \quad \frac{dT}{dt} \sim T^{-3} \Rightarrow \frac{dT}{dt}
\]

Putting in factors
\[
\frac{dx}{dt} = \frac{1}{3} g \frac{\rho_0}{g^2} \frac{m^2}{M_p} \frac{1}{s} \frac{dt}{dx}
\]

Finally
\[
\frac{dx}{dY} = \frac{\rho_0}{g} \frac{M_p}{\hbar} \frac{1}{s} \left( \frac{\rho_0}{g} \right)^{-1} \quad \text{or} \quad Y = \frac{\rho_0}{g} \frac{M_p}{\hbar} \frac{1}{s} \left( \frac{\rho_0}{g} \right)^{-1}
\]

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The plot that launched a thousand papers: "Christopher Maltby".

Equation between \( NSN \) and \( H \).

For simplicity assume \( NSN = \text{const} \).

Early \( NSN \sim T^3 \), \( H \sim T^2 \). So expansion not matched \( T \sim \text{equilibrium} \).

One \( T \) \( \sim M \) had to do \( f \rightarrow x \), still do \( x \rightarrow f \).

\( \sim (MT)^2 e^{-MT} \) \( H \sim T^2 \)

Equation wins, falls out of equilibrium Freeze Out.
Early on, $T$ large, a small: $\nabla < \frac{\nu}{\rho}$, $\frac{\nu}{\rho} \ll T$

Putting $Ne = \frac{\nu}{\rho}$, $\frac{\nu}{\rho} \ll T$

One below $\text{max}$, $A$ does if $\rho$, distribution at $T = T_0 (x,\frac{\nu}{\rho})$

Later here, $\nabla < \frac{\nu}{\rho}$, ignore $\nabla$, $\frac{\nu}{\rho} \ll T_0 (x,\frac{\nu}{\rho})$

Consider here, $e^{\frac{\nu}{\rho} - T_0}$

[Really use B.E. numerically for details]

Approx: freeze at: $T_0 (x,\frac{\nu}{\rho}) = T_0$

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Take logs: $x_f \approx \log \left( \frac{M_x M_p \nu}{\rho} \right) + \log \text{correction, depending on } M_x$

What if had density in DM? $\frac{\nu}{\rho} = H = \frac{\nu}{\rho} = T_0$

$\text{result: } \frac{\nu}{\rho} = \frac{H}{T_0}$

So per: $L_x^2 = \frac{\nu}{\rho} \times S_0 \times M_x \times \frac{T_2}{T_3} \frac{1}{\nu} \frac{M_x}{T_3 \frac{\nu}{\rho} \frac{1}{\nu}}$

Since at only log survive to low range of $\frac{\nu}{\rho}$ $x_f \approx 25 \Rightarrow \frac{\nu}{\rho} \approx \frac{1}{25}$
I have dropped lots of numerical footnotes, πi's etc. (See Clue)

Putting Planck in $0.186 \text{ cm s}^{-1}$, Minkowski

\[
\frac{\text{Sah}}{\text{h}} = 0.1 \left(\frac{\text{a f t}}{25}\right) \left(\frac{g\beta}{80}\right) \left(\frac{3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}}{\text{cm}^2 \text{s}^-2}\right).
\]

\[
\text{OV} \sim \frac{a^2}{M^2} = 3 \times 10^{-26} \text{ cm}^2 \text{s}^-1 \quad \text{WIMP}
\]

Lee-Weinberg + Unnatural define mass range.

\[
<\text{OV}> \sim G^2 F M^2 \Rightarrow 0.12 \text{ cm}^2 \leq 0.12 \Rightarrow \text{OV} > \text{weinberg}
\]

\[
\Rightarrow m_x \approx 1 \text{ GeV}
\]

\[
<\text{OV}> \sim \frac{4 \pi (91)}{m_x^2} \Rightarrow m_x \lesssim 40-100 \text{ TeV}
\]

Find mass range for "WIMP".

 Thermal relic abundance set by annihilations in SM.

Notice lower end of range set by SM couplings to W/2.

If other gauge forces had DM couples to $5 \sim \frac{m^2 g_4}{\text{x}^2}$

0.5 Our DM can be lighter. But if instead still need to avoid cooling structure it no WDM $m_x \approx m \geq 3 \text{ keV}$ + BBN $> 1 \text{ MeV}$
Q: speed freeze out for LSPs? Why are they not a thermal relic? Where do they come from? (Asymmetric DM)

- Other idea: WIMP (thermal) decays into sterile neutrino (Super WIMP) Feyn...

Recap

DM stable, WIMP had relic gives correct abundance, NO particle in SM \rightarrow new physics.

What type of new physics?

Why stable if m > 100 GeV and coupled to SM?

LSPs

- Photon stable since lightest gauge
- Bidden "??" chiral partner.

DM stable because lightest particle chases the new symmetry

- Simplest example is \( \mathbb{Z}_2 \) SM even
- New odd

S. LSP stable Can only couple in SM + SM

Off \( \mathbb{Z}_2 \rightarrow \mathbb{Z}_2 \rightarrow U(1) \rightarrow SU(2) \rightarrow \text{gauge or global} \)

Cf: Weinberg may BSM models have new scale particle + partner to SM reason