

First day:

1. Using

$$if_{abc} = 2\text{Tr} \left(t^c [t^a, t^b] \right) \quad (1)$$

show that f_{abc} is anti-symmetric with respect to any pair of indices.

2. Prove the Fierz identity

$$\delta_j^i \delta_k^l = \frac{1}{N} \delta_k^i \delta_j^l + 2(t^a)_k^i (t^a)_j^l \quad (2)$$

3. Use the Fierz identity to derive the Casimirs C_F and C_A .

4. Show that

$$t^a t^b t^a = -\frac{1}{2N} t^b \quad (3)$$

and draw the corresponding colour diagram.

5. Simplify

$$if_{abc} t^a t^c \quad (4)$$

and draw the corresponding colour diagram.

6. Prove the following identity

$$\left[t^a, [t^b, t^c] \right] + \left[t^b, [t^c, t^a] \right] + \left[t^c, [t^a, t^b] \right] = 0. \quad (5)$$

Use now the above identity to prove the Jakobi identity

$$f_{abe} f_{cde} + f_{bce} f_{ade} + f_{cae} f_{bde} = 0 \quad (6)$$

7. Visualize the Jakobi identity using color diagrams.

8. Produce a $q\bar{q}$ pair via a vector current. Check that coherent radiation off the q and \bar{q} cancels when $q\bar{q}$ are produced by a colour singlet source (photon $\rightarrow q\bar{q}$) and adds up into radiation off a gluon in the case of a $g \rightarrow q\bar{q}$ splitting process.