Dark Matter Problems: Farinaldo Queiroz

Problem 1:

Remember that (Statistical Mechanics):

$$\rho_{BE} = \frac{g\pi^2}{30}T^4; \ \rho_{FD} = \frac{7g\pi^2}{8}T^4. \tag{1}$$

Therefore,

$$g_* = \sum_{i\,bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{i\,fermions}\left(\frac{T_i}{T}\right)^4. \tag{2}$$

Also remember that entropy, $s = (p+\rho)/T$, thus for radiation,

$$s = \frac{2\pi^2}{45} g_{*s} T^3 \tag{3}$$

where,

$$g_{*s} = \sum_{i\,bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{i\,fermions}\left(\frac{T_i}{T}\right)^3. \tag{4}$$

Compute the entropy of the universe today. Use: ($T_\gamma \sim 2.7$ K). Which particles are considered radiation today?

Problem 2:

a) Using the same logic for the energy density one can find that relativitic particles follow,

$$n = \frac{g_{eff}}{\pi^2}\zeta(3)T^3 \tag{5}$$

where $g_{eff} = g$ (bosons), $3/4g$ (fermions) with g being the degree of freedom of the individual particle species.

Let $Y \equiv n/s \sim 0.28 g_{eff}/g_{*s}$. This is the yield for a hot relic (radiation). The particle was a hot relic at decoupling but today is might be cold so $\rho = m.n = m.Y_{today}.s_{today}$.

You should have computed s_{today} in the previous question. Assume that $g_{eff} = 1.5$ and $g_{*s} = 10.75$ and they did not during the period of time which the hol relic became cold.

With these ingredients you should be able to compute ρ and find the abundance of this species using $\Omega = \rho/\rho_c$, where ρ_c is the critical density.

b) which particle could that be?

The answer to these problems will show you why the Standard Model needs to be extended. You will understand why physics beyond the Standard Model is so important.