# Dark Matter

## I. Evidence

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ICTP SAIFR | International Centre for Theoretical Physics South American Institute for Fundamental Research

*School on DM and neutrinos*
*ICTP-SAIFR, July 23, 2018*

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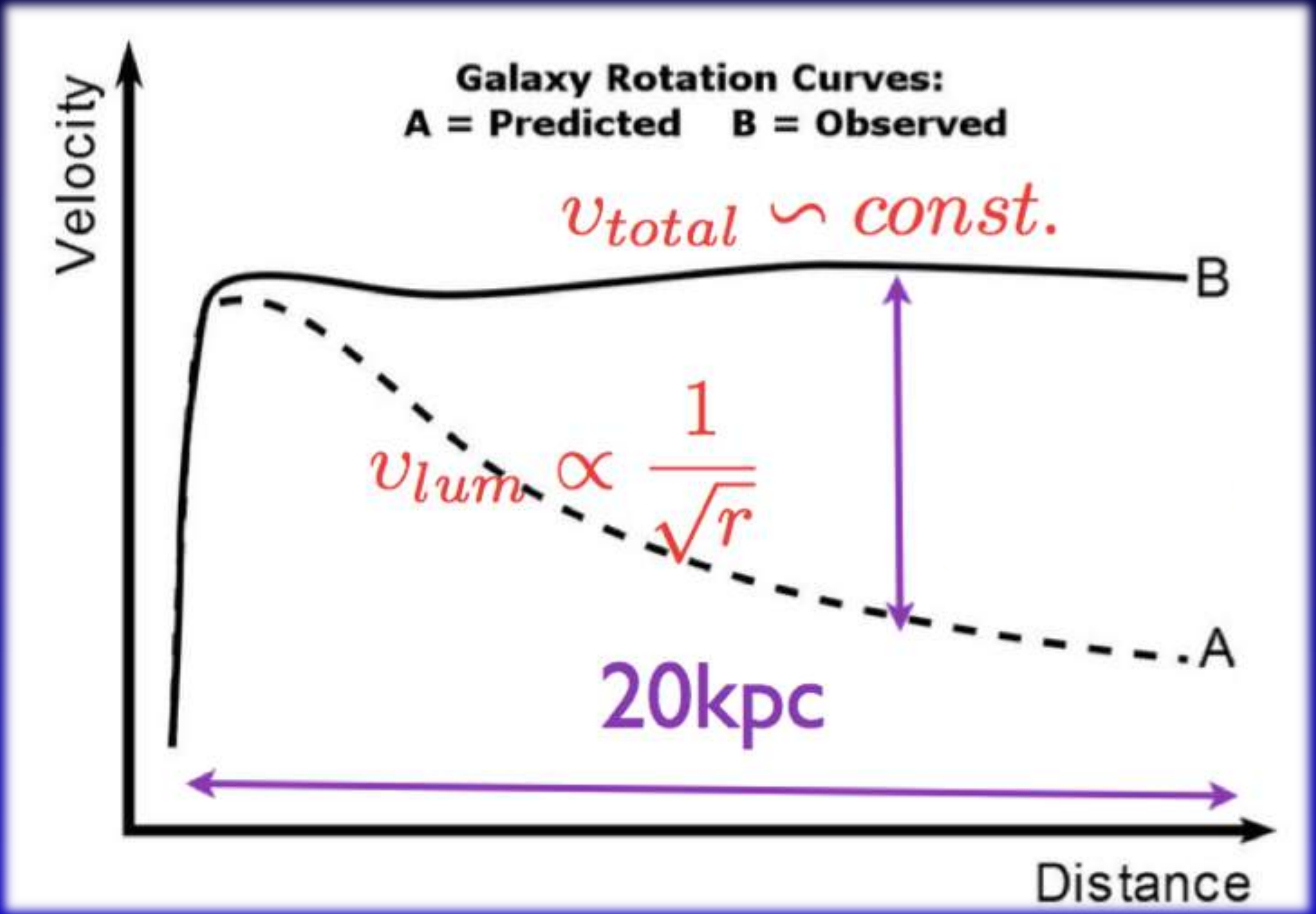
(with interesting twists)

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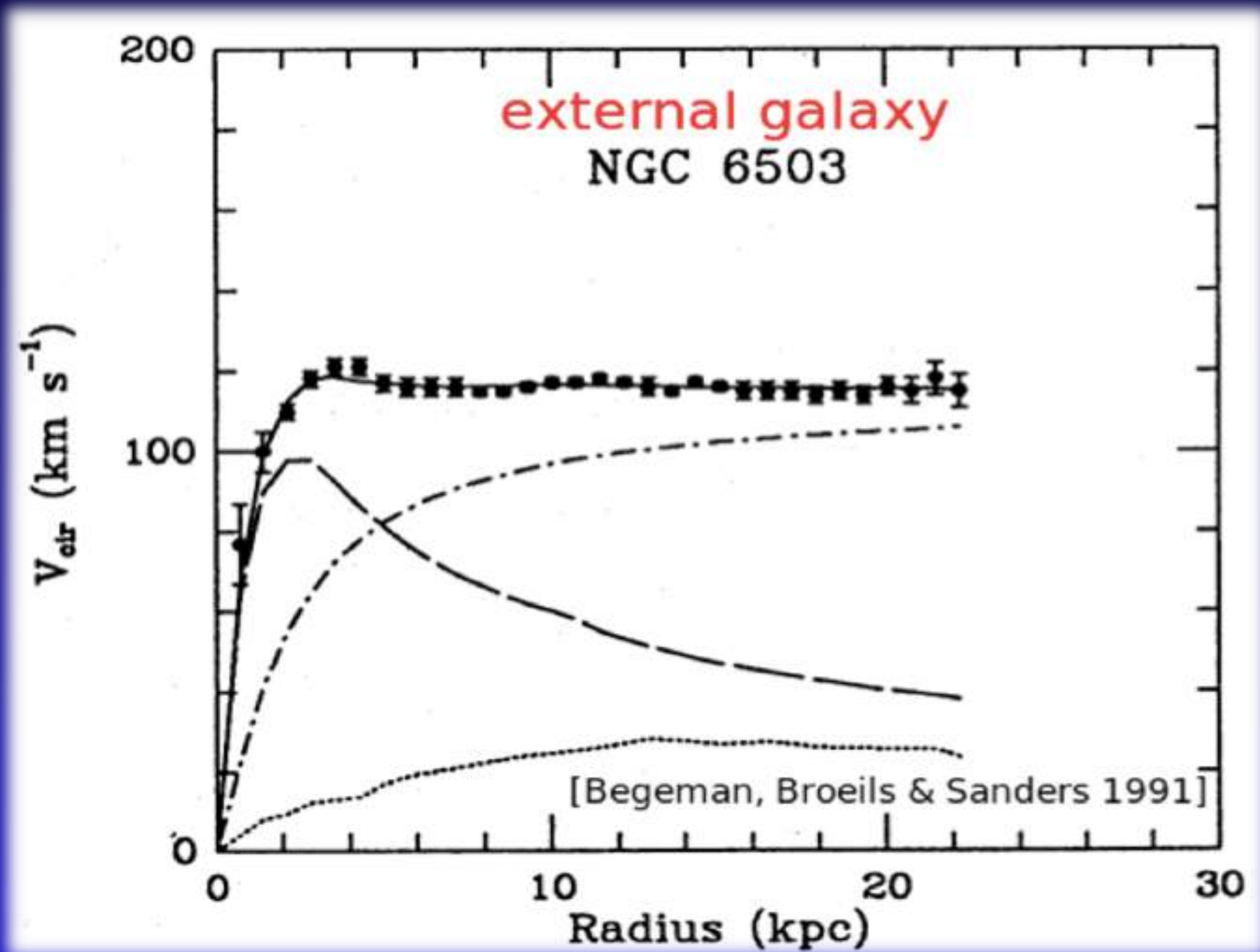


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# *Not only the disk: Jeans analysis*

Fluid continuity equation

$$\frac{\partial f}{\partial t} + \sum_{\alpha=1}^{6} \frac{\partial}{\partial w_\alpha}(f\dot{w}_\alpha) = 0$$

Collisionless Boltzmann Equation

$$\frac{\partial f}{\partial t} + \vec{\mathbf{v}} \cdot \vec{\nabla} f - \vec{\nabla}\Phi \cdot \frac{\partial f}{\partial \vec{\mathbf{v}}} = 0$$

(CBE)

# *Jeans analysis*

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$$\nu(\vec{\mathbf{x}}) \equiv \int f(\vec{\mathbf{x}}, \vec{\mathbf{v}}) d^3\vec{v}$$

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$$\nu \frac{\partial \bar{v}_j}{\partial t} \quad + \quad \bar{v}_i \nu \frac{\partial \bar{v}_j}{\partial x_i} \quad = \quad -\nu \frac{\partial \Phi}{\partial x_j} \quad - \quad \frac{\partial}{\partial x_i}(\nu \sigma_{ij}^2) \quad (j = 1, 2, 3)$$

$$\text{acceleration} + \begin{array}{c}\text{kinematic}\\ \text{viscosity}\end{array} \quad = \quad \text{gravity} \quad + \quad \text{pressure}$$

*Jeans analysis:*
*a practical example in a specific case*
*cylindrical symmetry*

$$\Sigma(z) = \int_{-z}^{z} \rho(z')dz'$$

$$\frac{1}{\nu}\frac{\partial}{\partial z}(\nu\sigma^2_{zz}) = -\frac{\partial\Phi}{\partial z} = -2\pi G\Sigma(z)$$

$$\Sigma(z) = -\frac{\sigma^2_{zz}}{2\pi G}\left(\frac{d}{dz}\ln\nu\right)$$

$$h^{-1} \equiv \frac{d}{dz}\ln\nu$$

From observations

$$\sigma^2_{zz} = (20\ \mathrm{km/s})^2$$

$$h = 300pc$$

$$\Sigma \approx 50\ M_\odot \mathrm{pc}^{-2}$$

# Some words about Gravitational lensing

## Gravitational lensing geometry

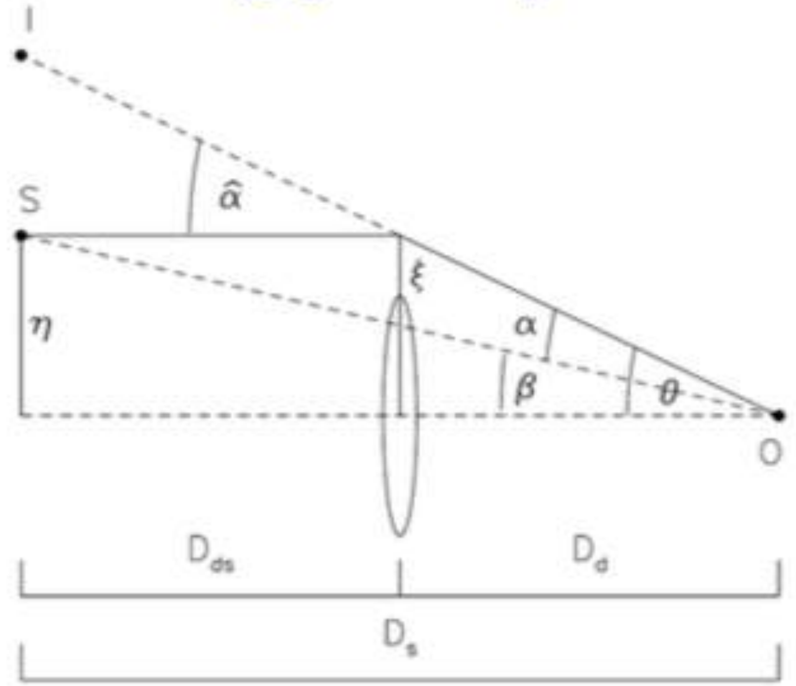
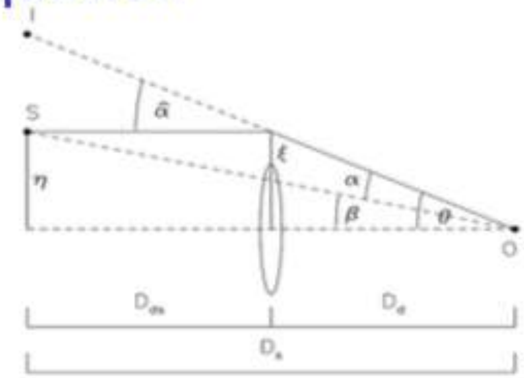



Figure from Narayan & Bartelmann (1996; arXiv:astro-ph/9606001)

- Parameters:
  - Angular diameter distances: $D_s$, $D_d$ and $D_{ds}$
  - Source position: $\beta = \eta / D_s$ - *Impact parameter*
  - Image position: $\theta = \xi / D_d$
  - Deflexion angle: $\alpha = \hat{\alpha}\ D_{ds} / D_s$

# Some words about Gravitational lensing

## The Lensing Equation




- $$\eta = \frac{D_s}{D_d}\xi - D_{ds}\hat{\alpha}(\xi)$$
- Using the angular quantities instead of the physical ones we get:
$$\beta = \theta - \frac{D_{ds}}{D_s}\hat{\alpha}(D_d\theta)$$
- Then by using the *physical* deflexion angle we arrive to the **Lens Equation**:
$$\beta = \theta - \alpha(\theta)$$

# Some words about Gravitational lensing

## The deflection angle

- From GR one can estimate the deflection angle due to a mass point:

$$\hat{\alpha} = \frac{4\ G\ M}{c^2 \xi}$$

- The projected angle is then:

$$\alpha = |\boldsymbol{\alpha}| = \frac{D_{ds}}{D_s} \frac{4\ G\ M}{c^2 D_d |\boldsymbol{\theta}|}$$

- Considering now a direction, as all the angles lies in the image-lens direction, we have:

$$\boldsymbol{\alpha} = \frac{D_{ds}}{D_s} \frac{4\ G\ M}{c^2 D_d |\boldsymbol{\theta}|} \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|}$$

# The Einstein angle

- The lens equation can be easily solved in this case.
- Lets first define a 'natural scale', the *Einstein angle* for this lens:

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{ds}}{D_s D_d}}$$

- The lens equation then becomes:

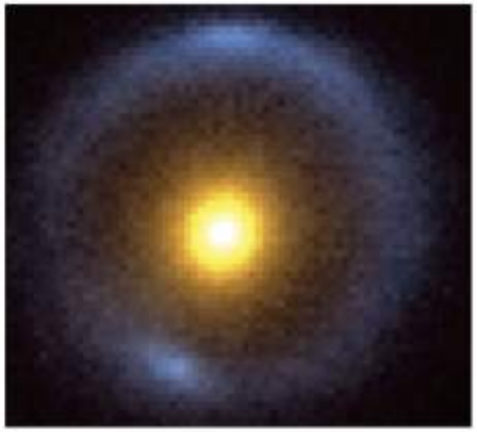
$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

- It has two solutions:

$$\theta_\pm = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right)$$

# *The Einstein angle*

- What is 'natural' about the *Einstein angle or radius* ?
- If $\beta = 0 \rightarrow \theta = \theta_E$



- The distance between two images (solutions) is $2\theta_E$
- The density inside the the Einstein radius is the critical lensing density

$$\Sigma_{cr} = \frac{c^2}{4\pi G}\frac{Ds}{D_d\ D_{ds}}$$

above which multiple imaging occur.

# Lensing regimes

## The lensing effect

- Gravitational lensing causes several effects on the images of the sources:
  1. Radial displacement
  2. Multiple imaging (given certain conditions)
  3. Magnification of the angular size $\xrightarrow{\textit{surface brightness conservation}}$ and flux
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- The prevalence/interest of one or more of those effects over the others have to do with the *lensing regime*.

# *Lensing regimes*

## Lensing regimes

- Different lensing regimes differentiate themselves by:
  1. The lens mass distribution
  2. The distances involved
  3. The impact parameter
- The main lensing regimes are:
  1. Micro lensing $\theta \leq \theta_E \quad dist. \sim kpc$
  2. Strong lensing $\theta \leq \theta_E \quad dist. \sim Gpc$
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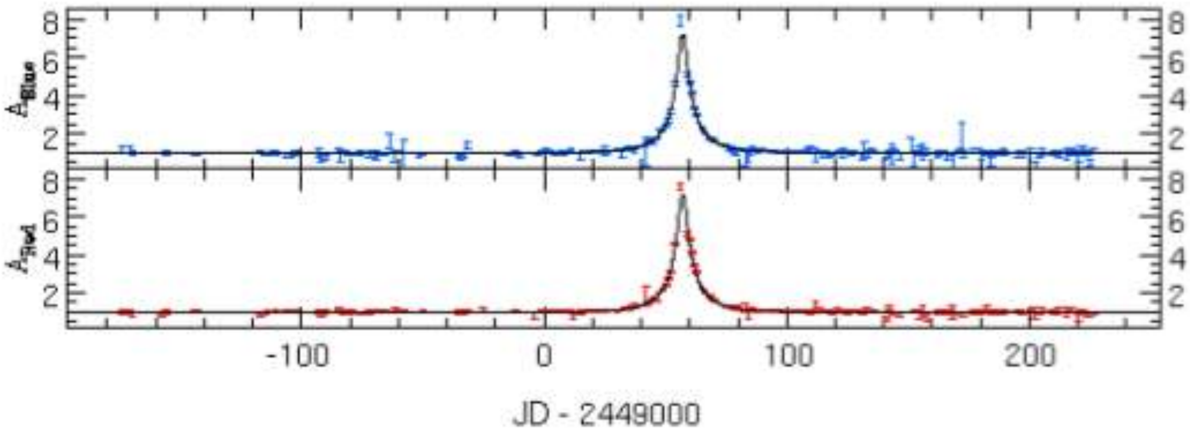
# *Micro Lenses*

## Micro lenses

- Typical *Einstein radius*

$$\theta_E = 0.902 mas \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_d}{10 kpc}\right)^{-1/2} \left(1 - \frac{D_d}{D_s}\right)^{-1/2}$$

- We cannot observationally resolve the multiple imaging, but we can measure the variation of the flux due to the magnification




- Note that microlensing should produce symmetric and achromatic light curves as above.

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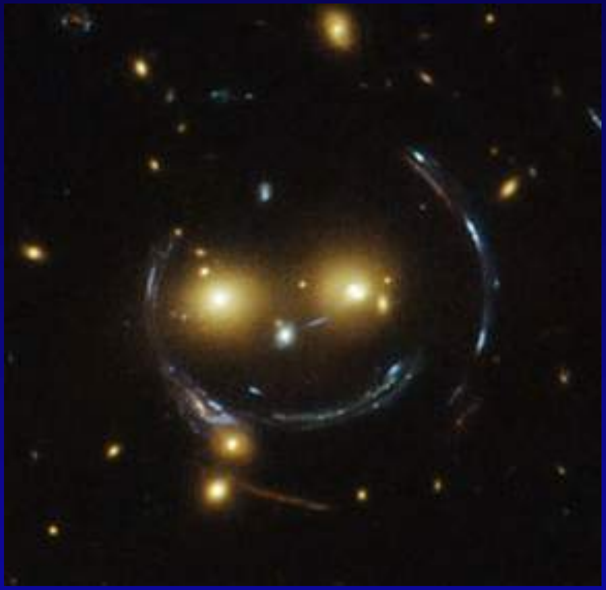

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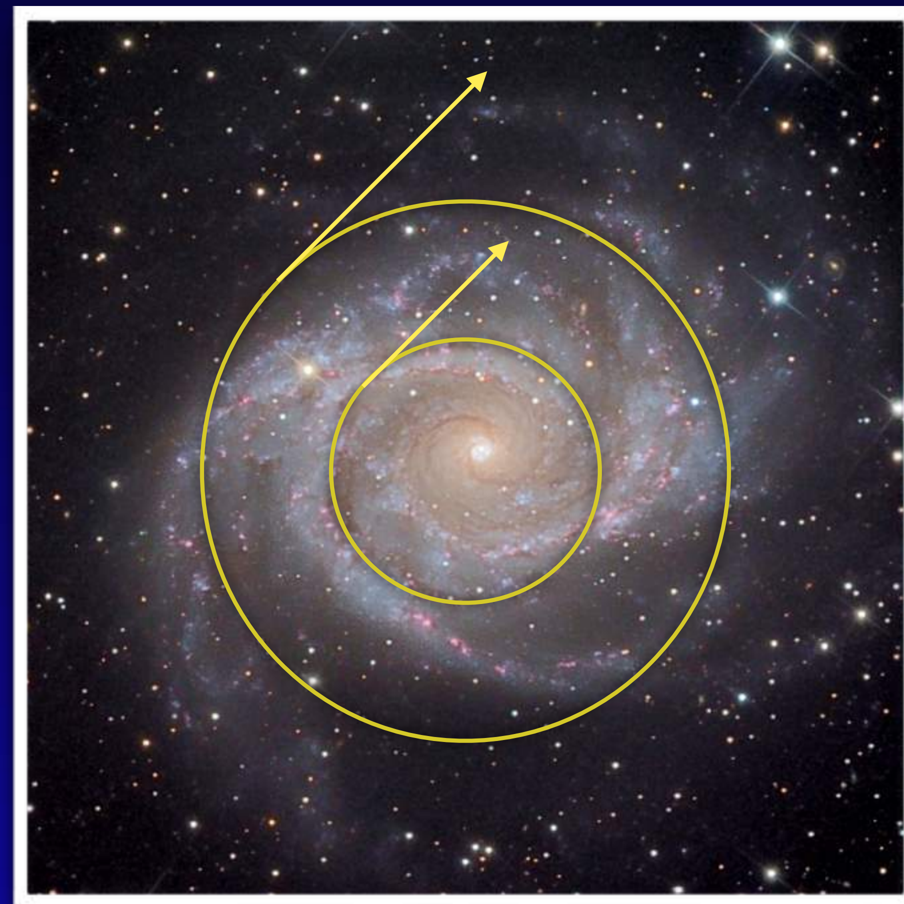

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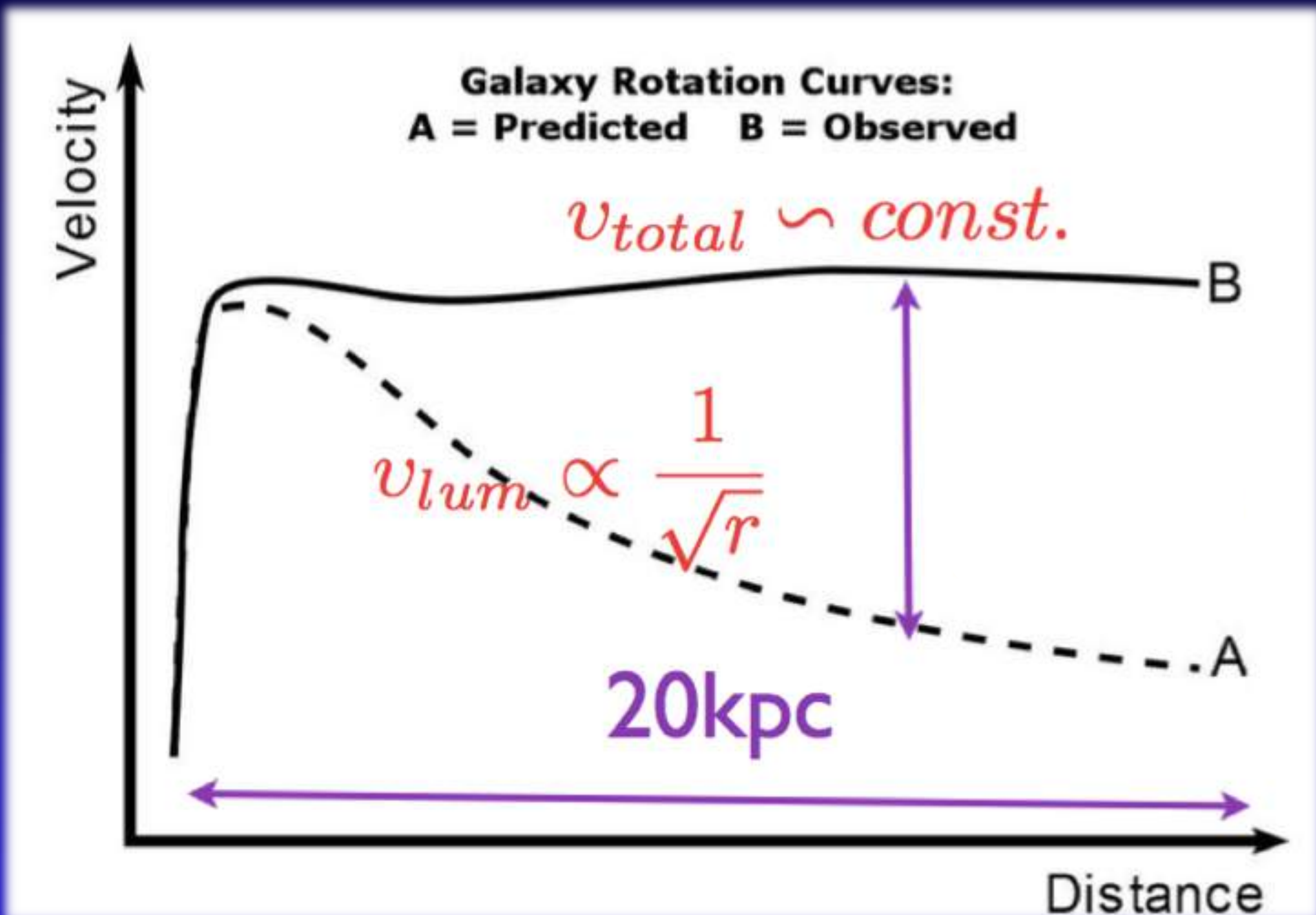


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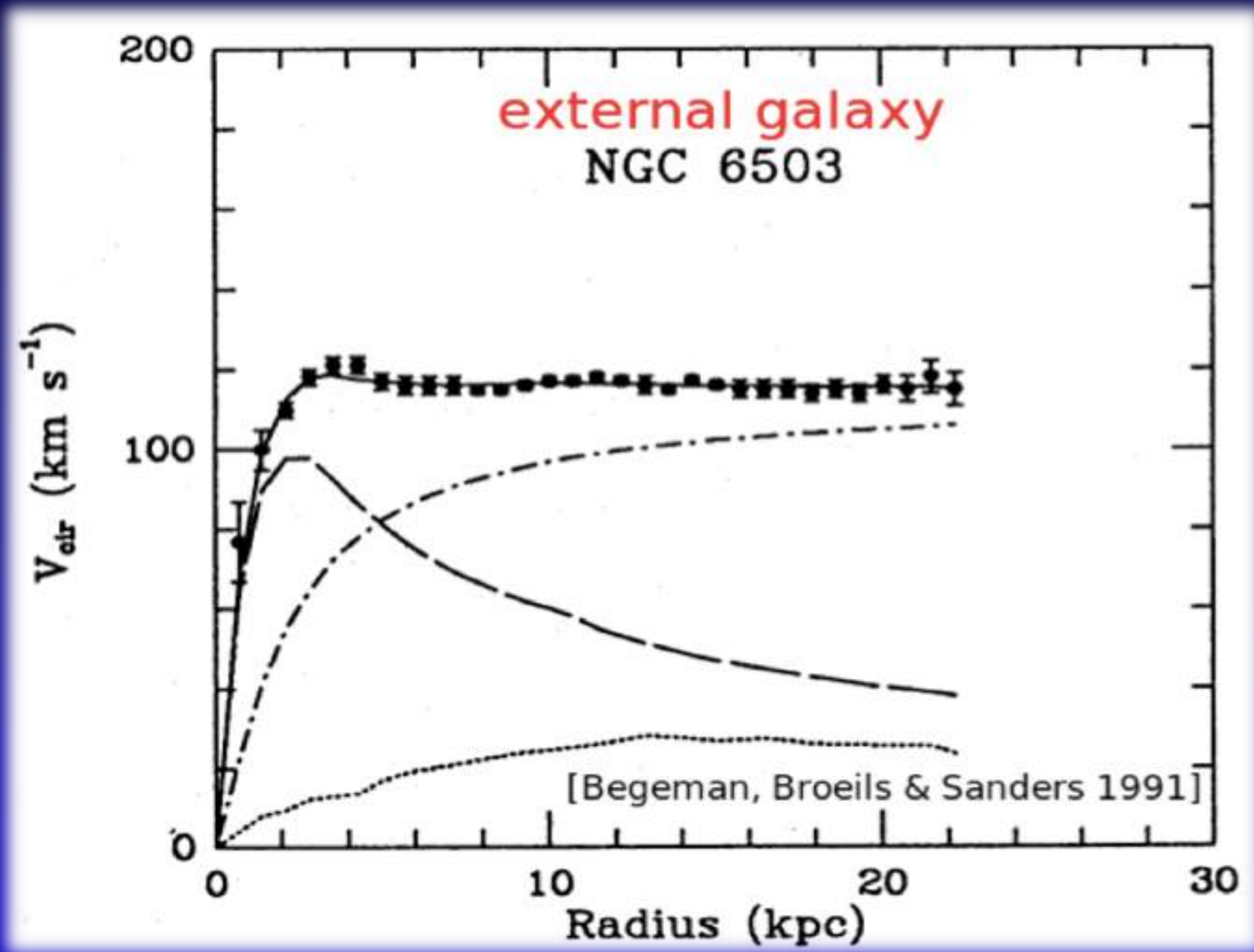


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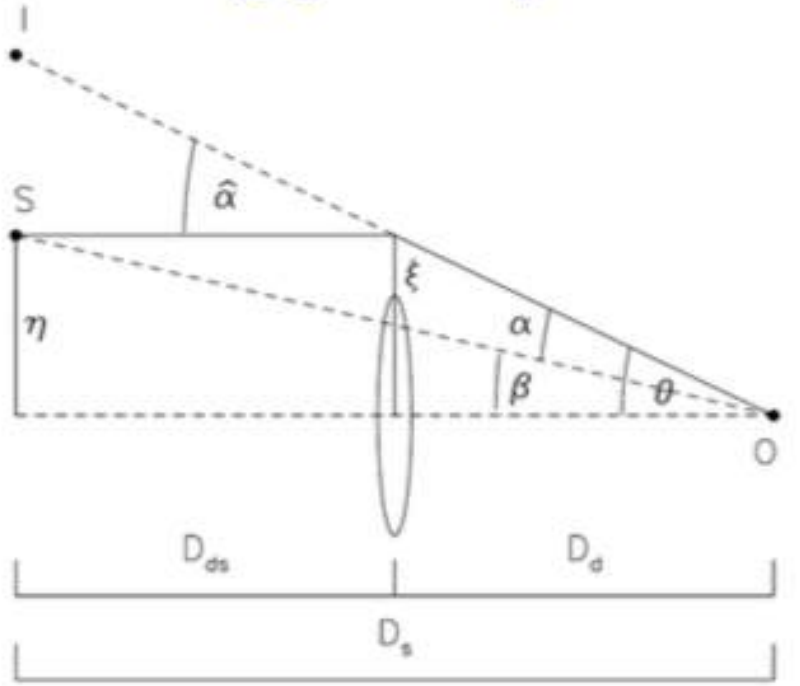


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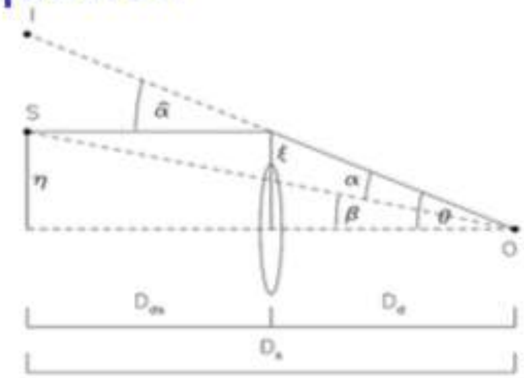


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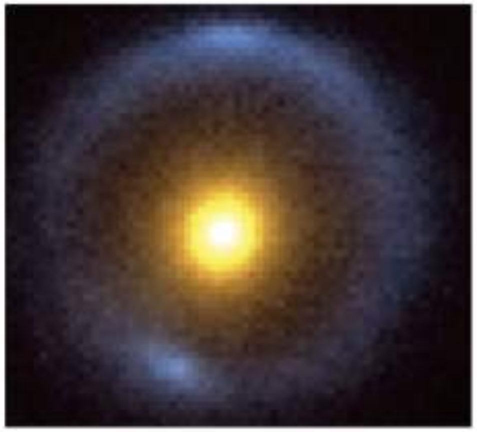

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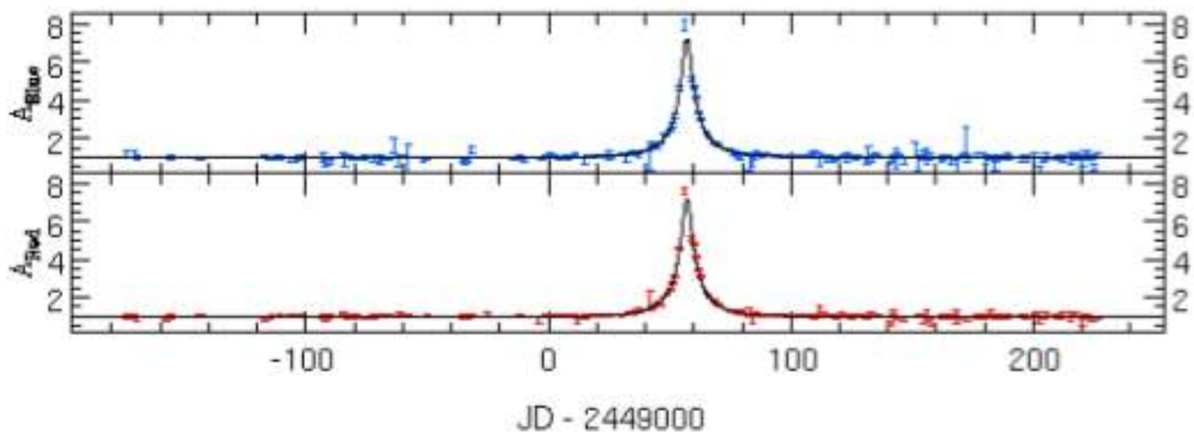


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