Exercises in support of Dark Matter course

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1. The bulge of a Milky–Way like Galaxy can be approximated, at the leading order, with a sphere of radius R_b=2 kpc, total mass M_b=10^{10} $M_\odot$, and constant density. Compute the Rotation Curve generated by the bulge in the plane of the Galactic Disk, neglecting the contributed of the Disk, at a distance R=4 kpc from the Galactic Center.

2. The disk of a Milky–Way like Galaxy can be approximated, at the leading order, with a cylinder of radius R_d=10 kpc, thickness along the z-axis h_d= 1kpc, and total mass M_d =5 $\times 10^{10}$ $M_\odot$, and constant density. Compute the contribution of the disk to the rotation curve (neglecting the effect of the bulge in the previous exercise) at a distance R = 20kpc in the central plane (at the origin of the z-axis, in the very middle of the stellar disk).

3. Neglecting the galactic bulge, determine whether the stellar disk is a collisionless system of stars, namely if stars can be considered as an ideal gas (with no sizable mutual interaction). The typical mass of an individual star is M_*=1$M_\odot$, and the typical velocity of stars is v_*=100 km/s

4. A Coma-like Galaxy Cluster has a typical radius R_c= 1Mpc, and counts within itself with approximately $\#_G$=1000 galaxies, of typical mass M_G=5$\times 10^{10}$ $M_\odot$ and with a velocity dispersion of approximately v_G=1000 km/s. Determine whether the Cluster is a collisionless system of galaxies, namely if galaxies can be considered as the particles of an ideal gas within the Cluster, similarly to what done above for stars in a Galaxy.

5. Using the Virial theorem, and the quantities contained in the previous exercise, determine the total gravitational mass of the Cluster described therein.

6. Consider a spherical distribution of dark matter with density ρ_{dm}(R). Then, the circular velocity induced by this distribution is:

$$v^2_{dm}(R) = \frac{GM_{dm}(R)}{R} \tag{1}$$

   with $M_{dm}(R)$ being the enclosed DM mass at each radius (integrate the density given above).

   Assuming I am in a region of the Galaxy where only the DM contribution matters (so by observing v_o, I am basically observing v_{dm}), is it possible to reconstruct the DM density at a given radius R_o, $\rho_{dm}(R_o)$, if I have only the observation of $v_{dm}(R_o)$? (Invert the equation above, and solve for ρ_{dm}).

   In a region of the galaxy where the main contribution to the rotation curve is given by Dark Matter (between 15kpc and 30 kpc), and the observed rotation curve is flat (constant value), what is the Dark Matter distribution (density function ρ_{dm}(R)) needed to support that rotation curve?