Introduction to quantum computing and simulability

Demonstrating a quantum advantage I

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Outline: Demonstrating a quantum advantage I

- Introduction and motivation;
- Linear optics;
  - Linear optics with bosons and fermions;
- BosonSampling;
- Random quantum circuits;
Review: **postBPP** and **postBQP**

- Postselection: The ability to condition acceptance on some particular (not-impossible) event, no matter how unlikely.
Review: postBPP and postBQP

- postBPP lives inside PH (in the 3rd level)
- postBQP lives outside PH!*

* Fine-print: actually, P^postBQP lives outside PH
Recipe for demonstrating quantum advantage

1 - Take a restricted model of quantum computing \( A \). e.g. circuits of commuting gates or linear optics

2 - Give it postselection, and see what comes out. (call it \text{postA})

3 - If \( A \) + post-selection includes quantum computing, then \text{postA} = \text{postBQP}

4 - Suppose there is a classical algorithm to efficiently simulate \( A \) (i.e. sample from same distribution). Then \text{postA} \subseteq \text{postBPP}.

5 - But then \text{postBQP} \subseteq \text{postBPP} and \text{PH} collapses!
Quantum advantage (AKA “quantum supremacy”)

• Show a quantum computer doing something a classical computer cannot.
  • Even if that “something” does not have any clear applications!

• Task: simulate some restricted quantum circuit;

  Weak but exact simulation!

• Postselection argument: there is no efficient classical algorithm for this, otherwise $\text{PH}$ collapses; 😃

• Bad news: a realistic quantum device can’t do it either! 😞
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Linear optics: states

- System of $m$ bosonic **modes** described by creation operators:
  \[
  \{ a_i^\dagger \}_i = 1, \ldots m
  \]

- where
  \[
  [a_i, a_j^\dagger] = \delta_{ij} \\
  [a_i, a_j] = 0 \\
  a_i \langle \rangle = 0
  \]
  Vacuum state

- $n$-boson states:
  \[
  |S\rangle = |s_1, s_2, \ldots s_m\rangle = \frac{(a_1^\dagger)^{s_1} (a_2^\dagger)^{s_2} \ldots (a_m^\dagger)^{s_m}}{\sqrt{s_1!s_2!\ldots s_m!}} | \rangle
  \]
Linear optics: dynamics

- Linear-optical transformations (or interferometers):

\[ U \in SU(m) \implies a_i \mapsto \sum_{j=1}^{m} U_{ij} a_j \]

- Very restricted set of operations!
  - The state space for \( n \) photons in \( m \) modes has dimension \( \binom{m+n-1}{n} \)
  - Also: not universal for QC!

- Alternative (Hamiltonian) characterization:

\[ H = \sum_{ij} h_{ij}(a_i a_j^\dagger + a_j a_i^\dagger) \]
Linear optics: dynamics

- Elementary two-mode linear-optical transformations:

  Beam splitter: \[
  \begin{pmatrix}
  \cos \theta & i \sin \theta \\
  i \sin \theta & \cos \theta
  \end{pmatrix}
  \]

  Phase shifter: \[
  \begin{pmatrix}
  1 & 0 \\
  0 & e^{i\phi}
  \end{pmatrix}
  \]
Linear optics: dynamics

- Any $m$-mode interferometer can be written as a circuit:
Linear optics: example (HOM effect)

- Consider a 50:50 (or balanced) beam splitter

\[ U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \]

- If one photons enters it...

Input: \[ |10\rangle = a_1^\dagger |\rangle \]

Output: \[ \frac{1}{\sqrt{2}} (a_1^\dagger + i a_2^\dagger) |\rangle \]

\[ = \frac{1}{\sqrt{2}} (|10\rangle + i|01\rangle) \]
Linear optics: example (HOM effect)

• Consider a 50:50 (or balanced) beam splitter

\[ U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \]

• Now if two photons enter…

Input: \[ |11\rangle = a_1^\dagger a_2^\dagger | \rangle \]

Output: \[ \frac{1}{2}(a_1^\dagger + ia_2^\dagger)(ia_1^\dagger + a_2^\dagger)| \rangle \]

\[ = \frac{i}{\sqrt{2}}(|20\rangle + |02\rangle) \]

Hong-Ou-Mandel effect!
Linear optics

- If we had a general 2-mode transformation:

\[ U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \]

- Two photons enter; what is the probability they leave separately?

Input: \[ |11\rangle = a_1^\dagger a_2^\dagger |\rangle \]

Output: \[ (aa_1^\dagger + ba_2^\dagger)(ca_1^\dagger + da_2^\dagger)| \rangle \]

\[ = (ad + bc)|11\rangle + |\text{collision terms}\rangle \]
Linear optics

- If we had a general 2-mode transformation:

\[ U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \]

- Probability of outcome \(|11\rangle\):

\[ \Pr(11) = |ad + bc|^2 = \left| \text{Per} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right|^2 \]

\[ \text{Per}(A) = \sum_{\sigma \in S_m} \prod_{i=1}^{m} a_{i,\sigma(i)} \]
Side note: Fermionic “linear optics”

- What if we had **fermions**, rather than bosons?

\[
U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}
\]

**Bosons**

\[
[a_i, a_j^\dagger] = \delta_{ij}
\]

\[
\text{Per} \begin{pmatrix} a & b \\ c & d \end{pmatrix}
\]

**Fermions**

\[
\{f_i, f_j^\dagger\} = \delta_{ij}
\]

\[
\text{Det} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1
\]
Side note: Fermionic “linear optics”

- What if we had **fermions**, rather than bosons?

\[ U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \]

- **HOM effect**
- **Pauli exclusion principle**
Linear optics

• General $n$-photon $m$-mode interferometer $U$:

* from now on: no-collision inputs/outputs
Linear optics

- Transition between two (no-collision) states.

\[ |S\rangle \rightarrow |T\rangle \]

- Probability:

\[ \Pr_{S\rightarrow T} = |\text{Per}(U_{S,T})|^2 \]

\( U_{S,T} \) : submatrix of \( U \) with rows/columns chosen according to \( S \) and \( T \)
Linear optics

Example:

**Input:** $|S\rangle = |110\rangle$

**Output:** $|T\rangle = |011\rangle$

**Interferometer:**

$U = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

$U_{S,T} = \begin{pmatrix} d & e \\ g & h \end{pmatrix}$

$Pr_{S\rightarrow T} = |dh + ge|^2$
Side note: Fermionic linear optics II

Transition: $|S\rangle \rightarrow |T\rangle$

Bosons: $\Pr_{S \rightarrow T} = |\text{Per}(U_{S,T})|^2$

Fermions: $\Pr_{S \rightarrow T} = |\text{Det}(U_{S,T})|^2$

Permanent and determinant are similar, but in terms of complexity, they are **very** different!
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• Random quantum circuits;
BosonSampling

• It is not hard to show that

\[
\text{Linear optics + postselection} = \text{postBQP}
\]

This outcome happens \(2/27\) of the times

\[
|00\rangle \rightarrow |00\rangle \\
|01\rangle \rightarrow |01\rangle \\
|10\rangle \rightarrow |10\rangle \\
|11\rangle \rightarrow -|11\rangle
\]
BosonSampling

- It is not hard to show that

  \[
  \text{Linear optics} + \text{postselection} = \text{postBQP}
  \]

  ⇒ Linear optics cannot be simulated on a classical computer!

- But this is about **exact** simulation - a real-world experiment can’t simulate this exactly **either**!

Time to bring out the (complexity-theoretic) big guns!
Consider the following $n$-photon $m$-mode experiment:

where $m = O(n^2)$ and $U$ is some uniformly-random matrix.
BosonSampling

- BosonSampling **task**: sample from that $n$-photon distribution;
  - Or in fact any sufficiently close to it!
  - **Not** a decision problem!

**Theorem** [Aaronson and Arkhipov]

If there was a classical algorithm capable of sampling efficiently from some distribution $D'$ such that

$$|D - D'| < \delta$$

in time $\text{poly}(n, 1/\delta)$, the polynomial hierarchy ($\text{PH}$) would collapse to its 3rd level!
BosonSampling

- Outline of result:
  1. Probabilities = permanents of submatrices of a random matrix;

\[
\begin{pmatrix}
\text{Uniform random } U
\end{pmatrix}
\]

Submatrices look like independent Gaussian matrices
BosonSampling

- Outline of result:
  1. Probabilities = permanents of submatrices of a random matrix;
  2. **Permanent-of-Gaussian conjecture**;

**Fact**: Permanent is $\#P$-hard in the worst case.

**Conjecture**: Gaussian matrices are among the hardest Permanents to compute;

Plausible conjecture that has **nothing** to do with linear optics!
BosonSampling

• Outline of result:

1. Probabilities = permanents of submatrices of a random matrix;

2. **Permanent-of-Gaussian conjecture**;

3. A classical simulation that samples from a distribution very close to the ideal one must get many of these **#P-hard** probabilities right!

4. Another minor conjecture + 90 pages of complexity-theoretic cannons shooting at the problem…

5. If an efficient classical simulation for approximate BosonSampling existed, **PH** would collapse to the 3rd level!
BosonSampling

- Some additional comments:
  - Result does **not** mean linear optics can compute permanents!
  - $m=O(n^2)$ means no-collision outcomes vastly more likely.
  - Randomness of $U$ avoids structures that classical algorithms could "exploit".
  - This sampling task has no known applications;
    - Except as a “demonstration of force” by quantum devices 😊
  - Best known classical algorithm takes time $O(n \ 2^n)$;
BosonSampling: pros and cons

• Cons
  • No error-correction, so it is unclear whether experiments could be performed to this level of accuracy!
  • Hard to verify that device is doing what it should;
    • Leandro has more to say about this!

• Pros
  • Much “easier” to implement than universal quantum computation;
    • Doesn’t require nonlinearities or adaptive measurements;
    • ~ 50-90 photon experiment?
  • New insights into foundations of q. computing and q. optics;
First experiments

• Four small-scale experiments reported in December 2012.
First experiments

- Four small-scale experiments reported in December 2012.
A zoo of intermediate quantum models

- Many other similar “quantum supremacy” results!
  - IQP; [BJS]
  - Constant-depth quantum circuits [TD]; These predate BosonSampling
  - Quantum approximate optimization algorithms; [Farhi, Harrow]
  - A version of the 1-clean-qubit model; [Morimae, Fujii, Fitzsimmons]
  - Generalized Clifford circuits [JVdN]
  - Random quantum circuits;
  - And more!
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Random quantum circuits

- Google’s Quantum AI Lab + John Martini’s group at UCSB
  - Another way to demonstrate the power of quantum computers.
- Meet Google’s 72-qubit quantum processor: Bristlecone!

  ![Image: Google AI blog]

- Goal: Demonstrate “Quantum advantage” on 72 qubits! (…soon)
The random quantum circuit model
The random quantum circuit model

- But what is known about the complexity of simulating this?
- Aaronson and Chen [Dec ’16]
  - Task: “Given as input a random quantum circuit $C$, generate a set of outputs such that at least 2/3 of them have above-median probability in the output distribution of $C$.”

Generate a list of samples where $\geq 2/3$ of them are among those above this height;
The random quantum circuit model

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  - Needs one further conjecture (similar to permanent-of-Gaussians in BosonSampling);
- Bouland, Fefferman, Nirkhe and Vazirani [March ’18!]
  - Made a major step towards improving the status of the required conjectures.
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Next episode:
Neat idea! Let’s build one already, what’s taking so long?