Introduction to quantum computing and simulability

Demonstrating a quantum advantage I

Daniel J. Brod

Leandro Aolita

Ernesto Galvão

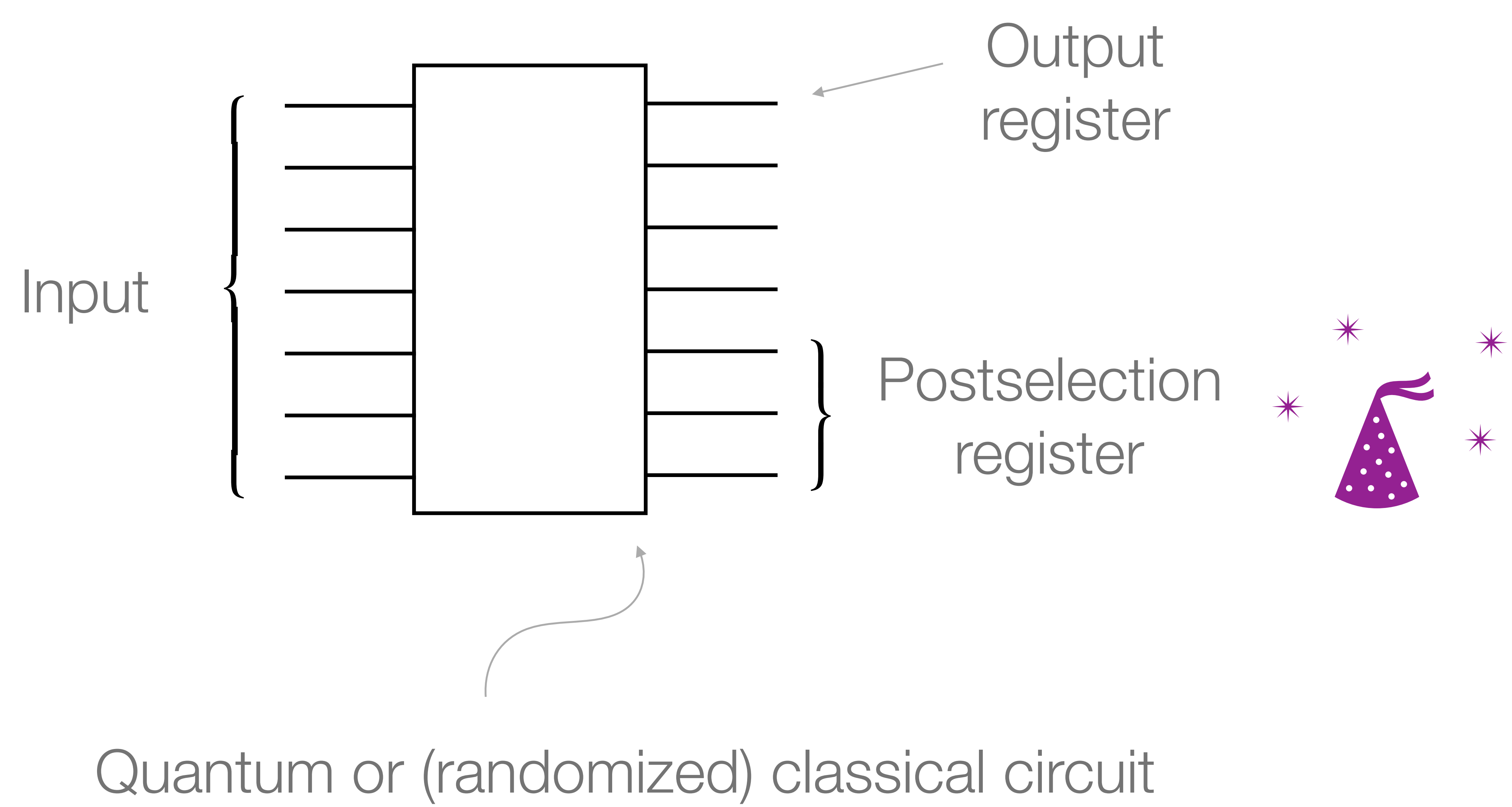
Instituto de Física
Universidade Federal Fluminense

Outline: Demonstrating a quantum advantage I

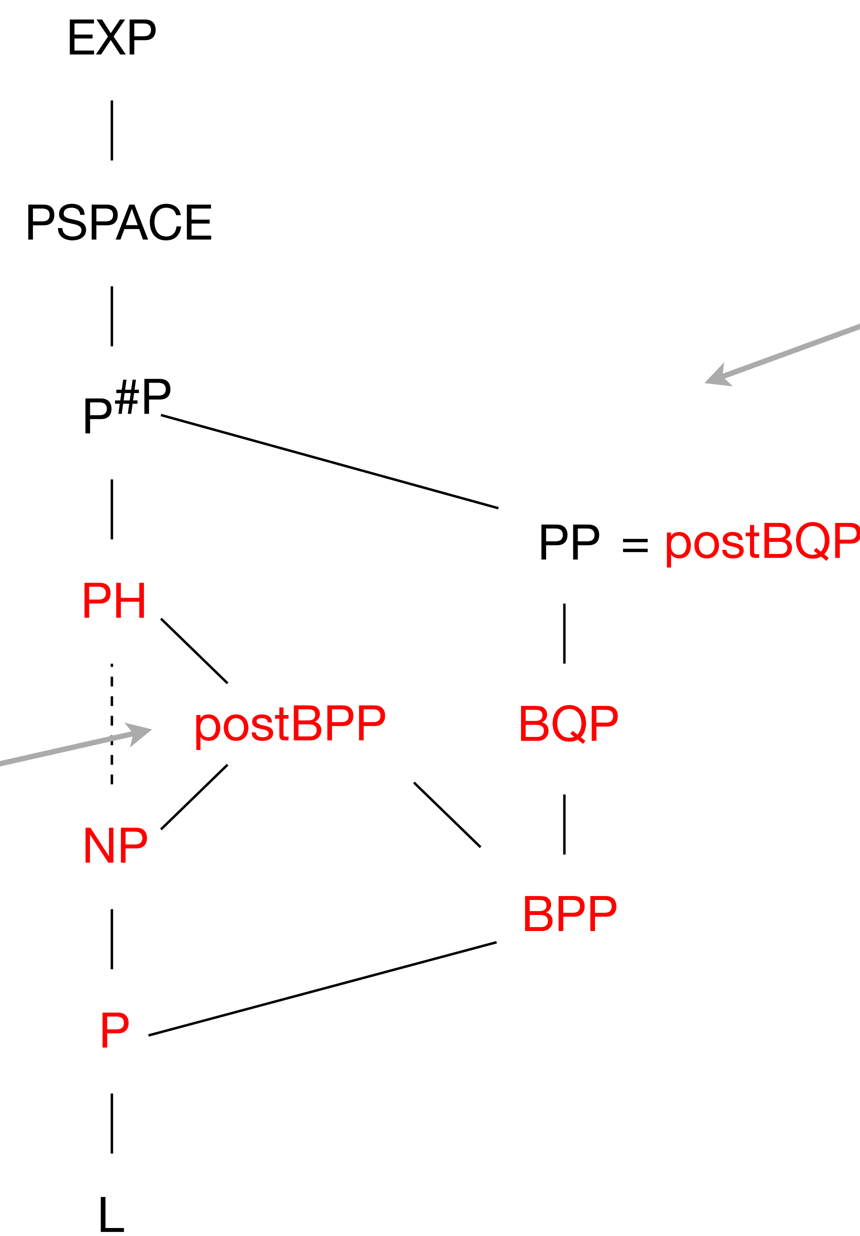
- Introduction and motivation;
- Linear optics;
  - Linear optics with bosons and fermions;
- BosonSampling;
- Random quantum circuits;

Review: **postBPP** and **postBQP**

- Postselection: The ability to condition acceptance on some particular (not-impossible) event, <u>no matter how unlikely</u>.



Review: **postBPP** and **postBQP**



postBPP lives inside **PH** (in the 3rd level)

postBQP lives outside **PH**!*

* Fine-print: actually, $\mathbf{P}^{\mathbf{postBQP}}$ lives outside **PH**

Recipe for demonstrating quantum advantage

1 - Take a restricted model of quantum computing **A.**
e.g. circuits of commuting gates or linear optics

2 - Give it postselection, and see what comes out.
(call it **postA**)

3 - If **A** + post-selection includes quantum computing,
then **postA** = **postBQP**

4 - Suppose there is a classical algorithm to efficiently
simulate **A** (i.e. sample from same distribution).
Then **postA** $\subseteq$ **postBPP**.

5 - But then **postBQP** $\subseteq$ **postBPP** and **PH collapses!**

Quantum advantage (AKA “quantum supremacy”)

- Show a quantum computer doing <u>something</u> a classical computer cannot.
  - Even if that “something” does not have any clear applications!
- **Task**: simulate some restricted quantum circuit;

Weak but **exact** simulation!

i.e. nonuniversal
(e.g. IQP, Linear optics)

- Postselection argument: there is no efficient classical algorithm for this, otherwise **PH** collapses; ☺
- Bad news: a realistic quantum device can’t do it either! ☹

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Linear optics: states

- System of m bosonic **modes** described by creation operators:

$$\{a_i^\dagger\}_{i=1,\ldots m}$$

- where

$$[a_i, a_j^\dagger] = \delta_{ij}$$

$$[a_i, a_j] = 0$$

$$a_i|\ \rangle = 0 \longleftarrow \text{Vacuum state}$$

- n-boson states:

$$|S\rangle = |s_1, s_2, \ldots s_m\rangle = \frac{\left(a_1^\dagger\right)^{s_1}\left(a_2^\dagger\right)^{s_2}\ldots\left(a_m^\dagger\right)^{s_m}}{\sqrt{s_1!s_2!\ldots s_m!}}|\ \rangle$$

Linear optics: dynamics

- Linear-optical transformations (or interferometers):

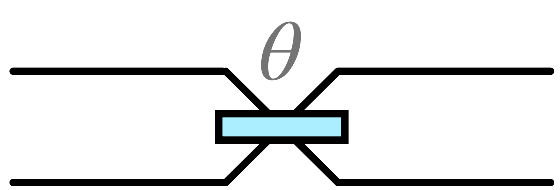
$$U \in SU(m) \qquad \Rightarrow \qquad a_i \mapsto \sum_{j=1}^{m} U_{ij}\, a_j$$

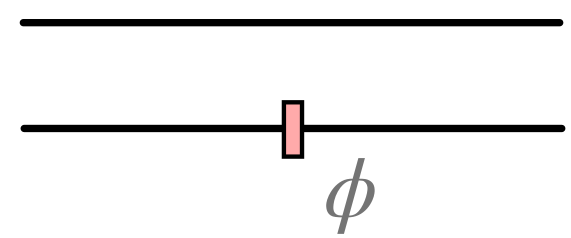
- **Very** restricted set of operations!
    - The state space for n photons in m modes has dimension $\binom{m+n-1}{n}$
    - Also: not universal for QC!

- Alternative (Hamiltonian) characterization: $H = \sum_{ij} h_{ij}(a_i a_j^\dagger + a_j a_i^\dagger)$

Linear optics: dynamics

- Elementary two-mode linear-optical transformations:





Beam splitter

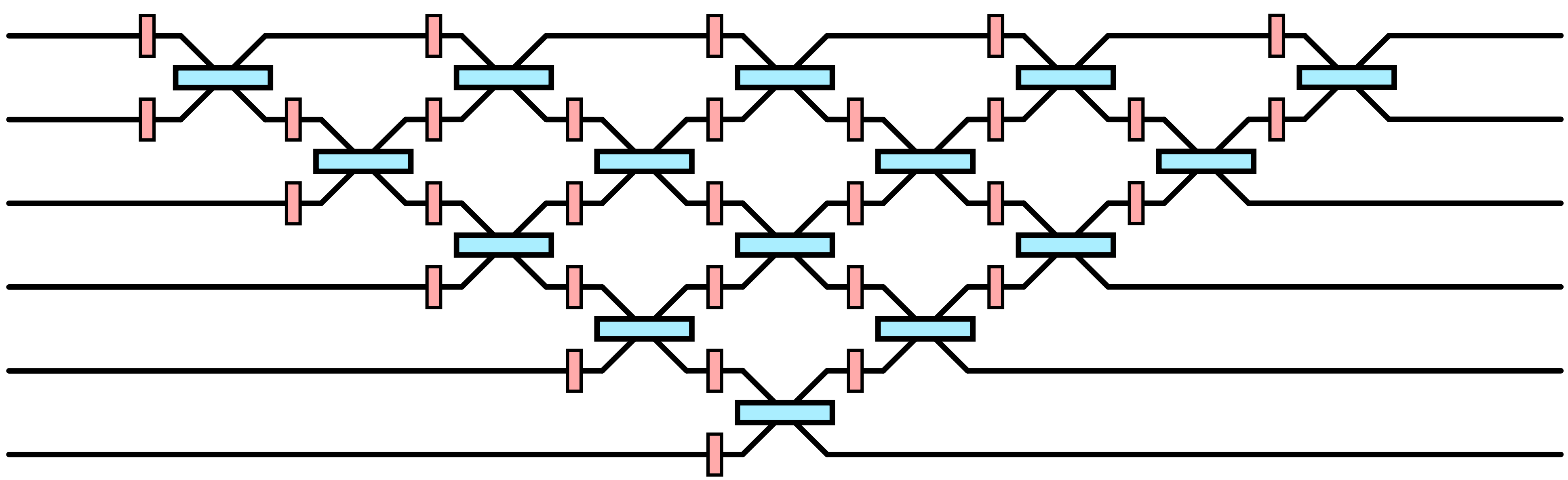
Phase shifter

$$\begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

Linear optics: dynamics

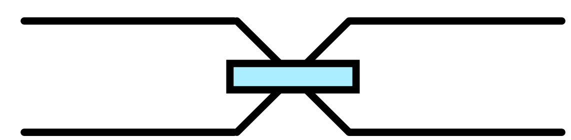
- Any m-mode interferometer can be written as a **circuit**:



Linear optics: example (HOM effect)

- Consider a 50:50 (or balanced) beam splitter

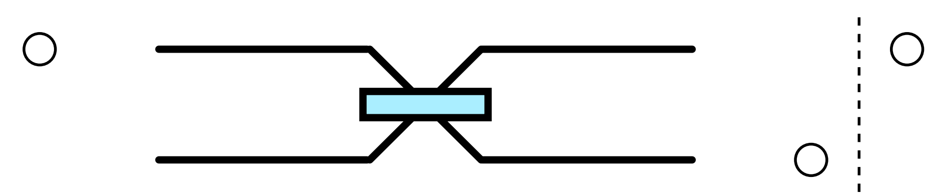
$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$



- If one photons enters it…

Input: $|10\rangle = a_1^\dagger |\ \rangle$

Output: $\frac{1}{\sqrt{2}}(a_1^\dagger + i a_2^\dagger)\,|\ \rangle$

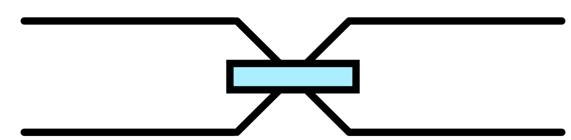


$$= \frac{1}{\sqrt{2}}(|10\rangle + i|01\rangle)$$

Linear optics: example (HOM effect)

- Consider a 50:50 (or balanced) beam splitter

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

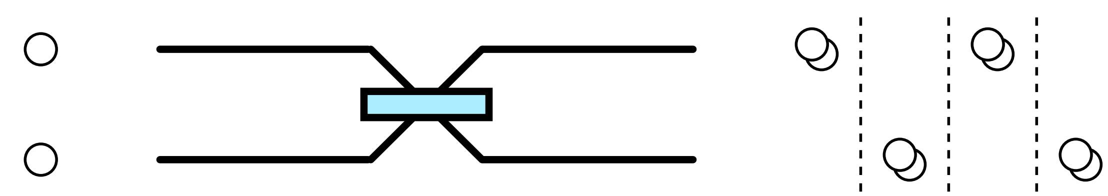


- Now if two photons enter…

Input: $|11\rangle = a_1^\dagger a_2^\dagger | \ \rangle$

Output: $\frac{1}{2}(a_1^\dagger + i a_2^\dagger)(i a_1^\dagger + a_2^\dagger) | \ \rangle$

$$= \frac{i}{\sqrt{2}}(|20\rangle + |02\rangle)$$

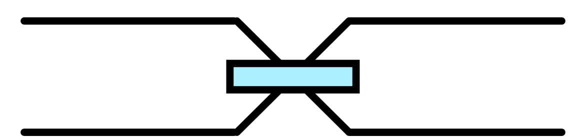


Hong-Ou-Mandel effect!

Linear optics

- If we had a general 2-mode transformation:

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$



- Two photons enter; what is the probability they leave separately?

Input: $|11\rangle = a_1^\dagger a_2^\dagger |\ \rangle$

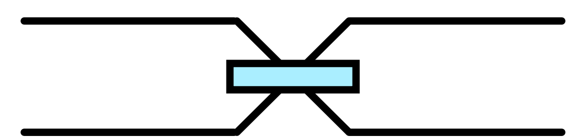
Output: $(aa_1^\dagger + ba_2^\dagger)(ca_1^\dagger + da_2^\dagger)|\ \rangle$

$$= (ad + bc)|11\rangle + |\text{collision terms}\rangle$$

Linear optics

- If we had a general 2-mode transformation:

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$



- Probability of outcome $|11\rangle$:

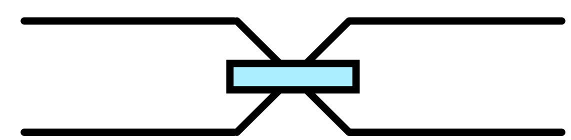
$$\Pr(11) = |ad + bc|^2 = \left| \mathrm{Per}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right|^2$$

$$\mathrm{Per}(A) = \sum_{\sigma \in S_m} \prod_{i=1}^{m} a_{i,\sigma(i)}$$

Side note: Fermionic “linear optics”

- What if we had **fermions**, rather than bosons?

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$



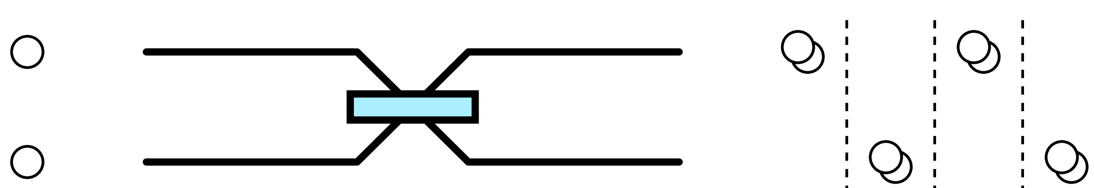
Bosons	Fermions
$[a_i, a_j^\dagger] = \delta_{ij}$	$\{f_i, f_j^\dagger\} = \delta_{ij}$
$\mathrm{Per}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$	$\mathrm{Det}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1$

Side note: Fermionic "linear optics"

- What if we had **fermions**, rather than bosons?

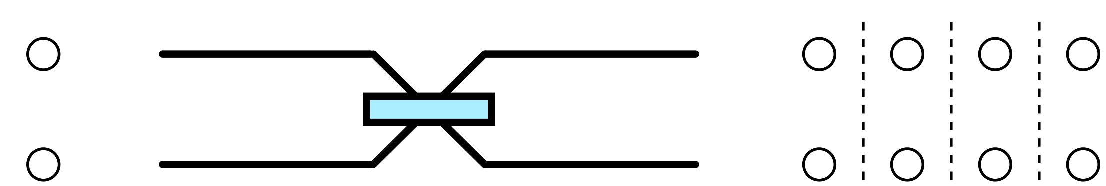
$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

Bosons



HOM effect

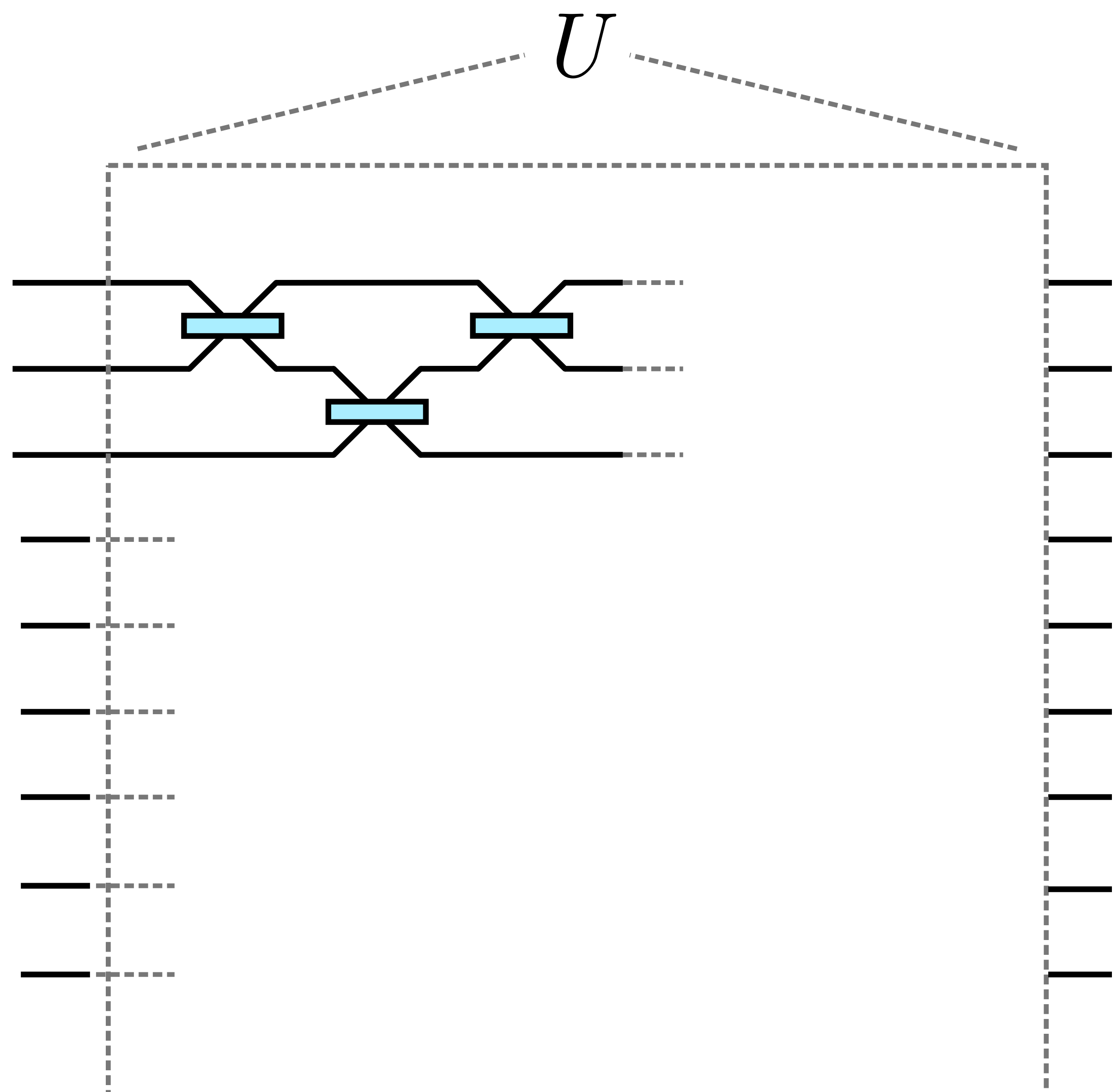
Fermions



Pauli exclusion principle

Linear optics

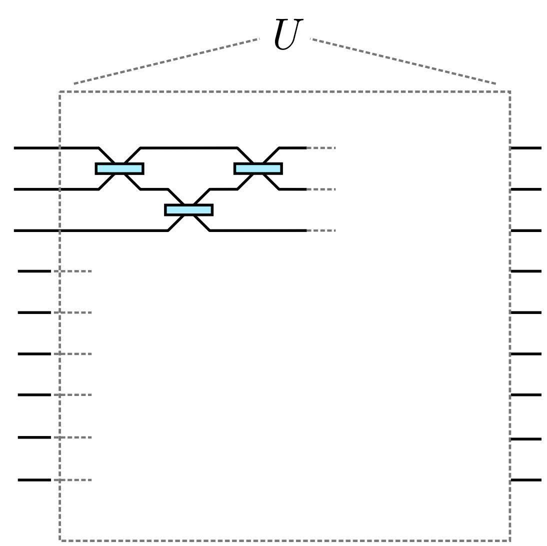
- General n-photon m-mode interferometer U:



* from now on: no-collision inputs/outputs

Linear optics

- Transition between two (no-collision) states.



$$|S\rangle \rightarrow |T\rangle$$

- Probability: $\Pr_{S \rightarrow T} = |\,\mathrm{Per}(U_{S,T})\,|^2$

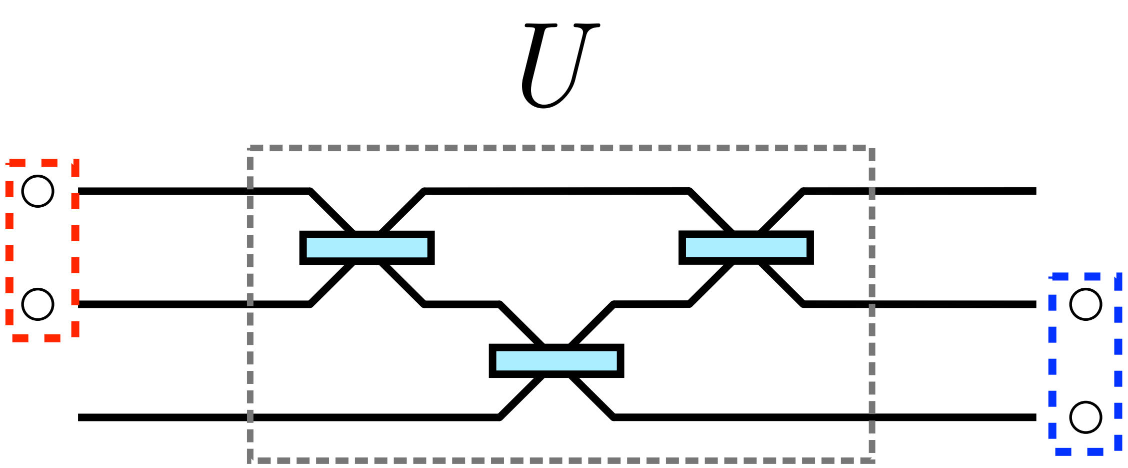
$U_{S,T}$: submatrix of U with rows/columns chosen according to S and T

Linear optics

Example:

Input: $|S\rangle = |110\rangle$

Output: $|T\rangle = |011\rangle$



Interferometer:

$$U = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$U_{S,T} = \begin{pmatrix} d & e \\ g & h \end{pmatrix}$$

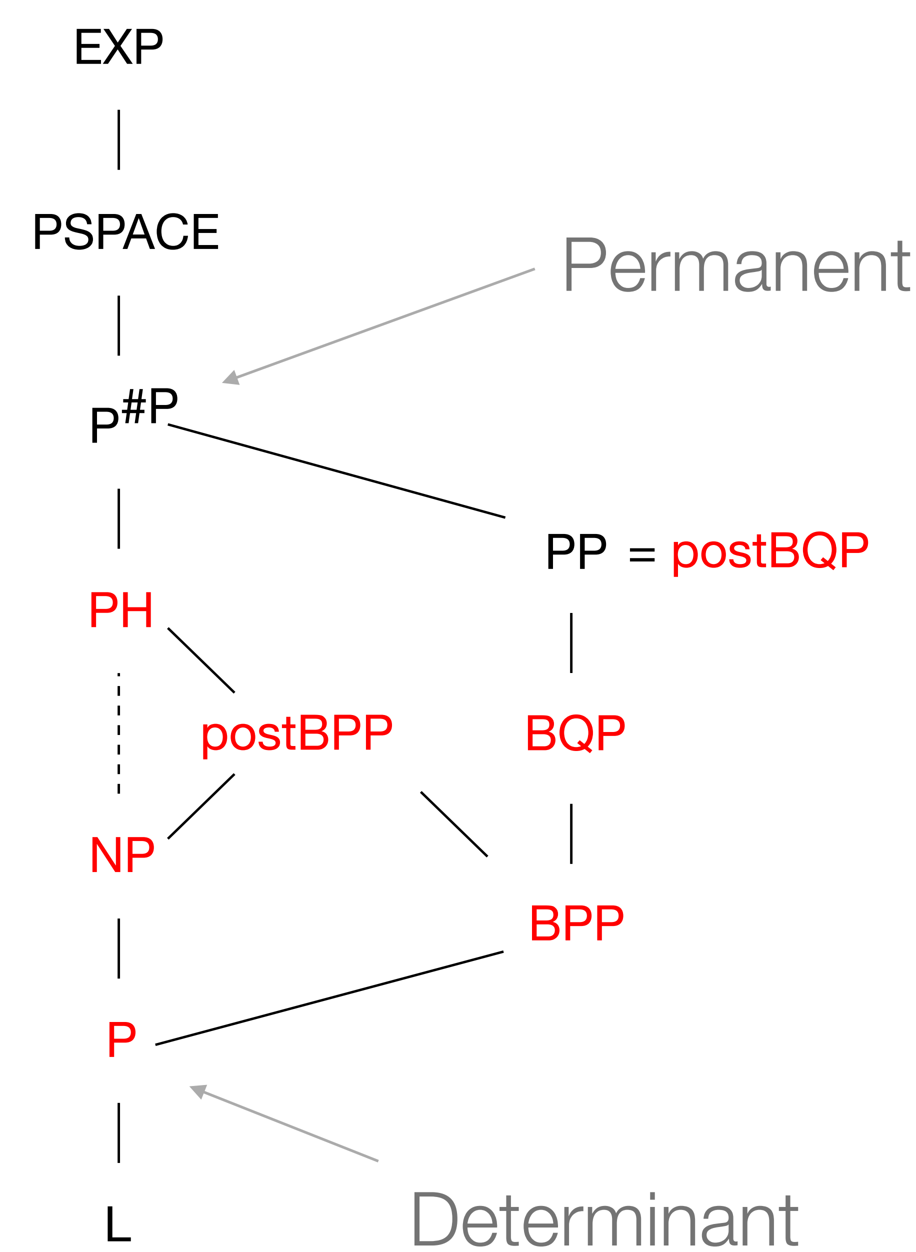
$$\Pr_{S \to T} = |dh + ge|^2$$

Side note: Fermionic linear optics II

Transition: $|S\rangle \rightarrow |T\rangle$

Bosons: $$\Pr_{S\rightarrow T} = |\mathrm{Per}(U_{S,T})|^2$$

Fermions: $$\Pr_{S\rightarrow T} = |\mathrm{Det}(U_{S,T})|^2$$



Permanent and determinant are similar.

but in terms of complexity, they are **very** different!

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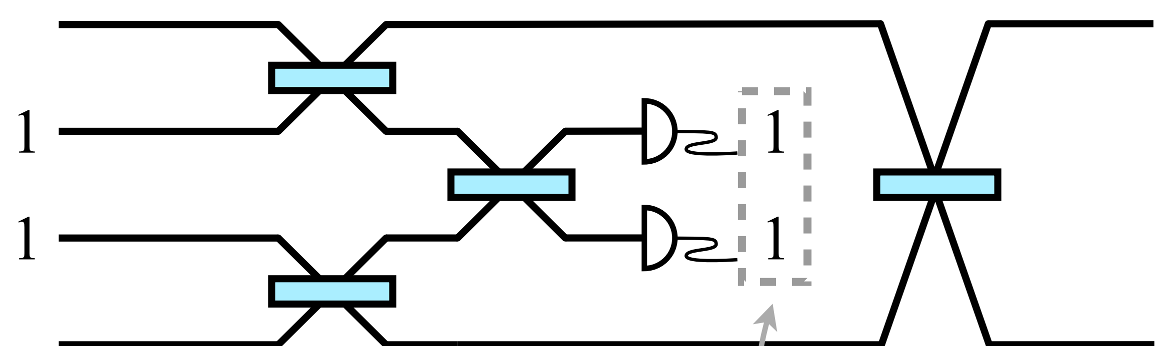
- Random quantum circuits;

BosonSampling

- It is not hard to show that

Linear optics + postselection = **postBQP**



$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |10\rangle$$
$$|11\rangle \rightarrow -|11\rangle$$

This outcome happens
2/27 of the times

BosonSampling

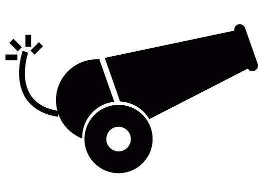
- It is not hard to show that

$$\text{Linear optics + postselection} = \textbf{postBQP}$$

⇒ Linear optics cannot be simulated on a classical computer!

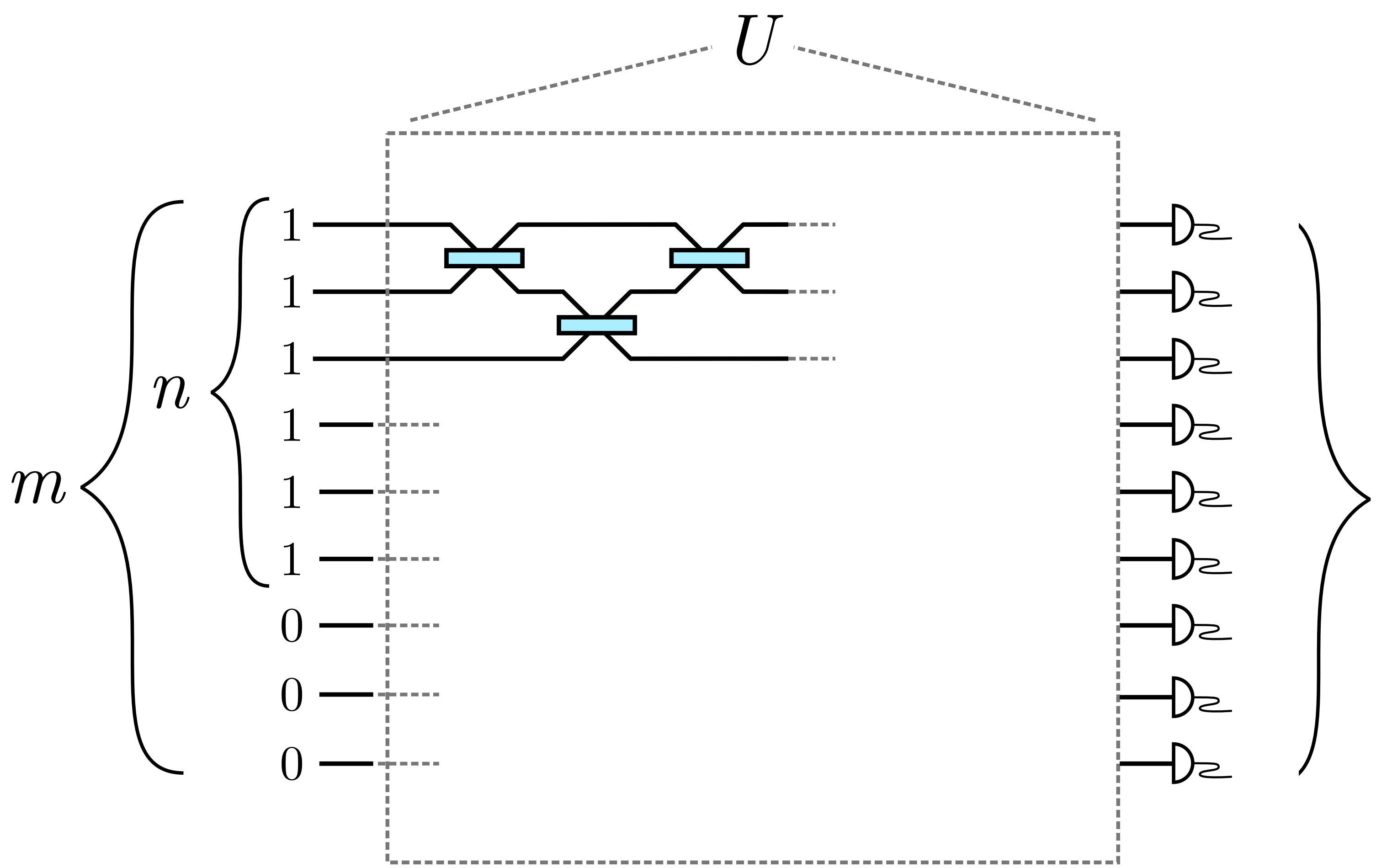
- But this is about **exact** simulation - a real-world experiment can't simulate this exactly **either**!

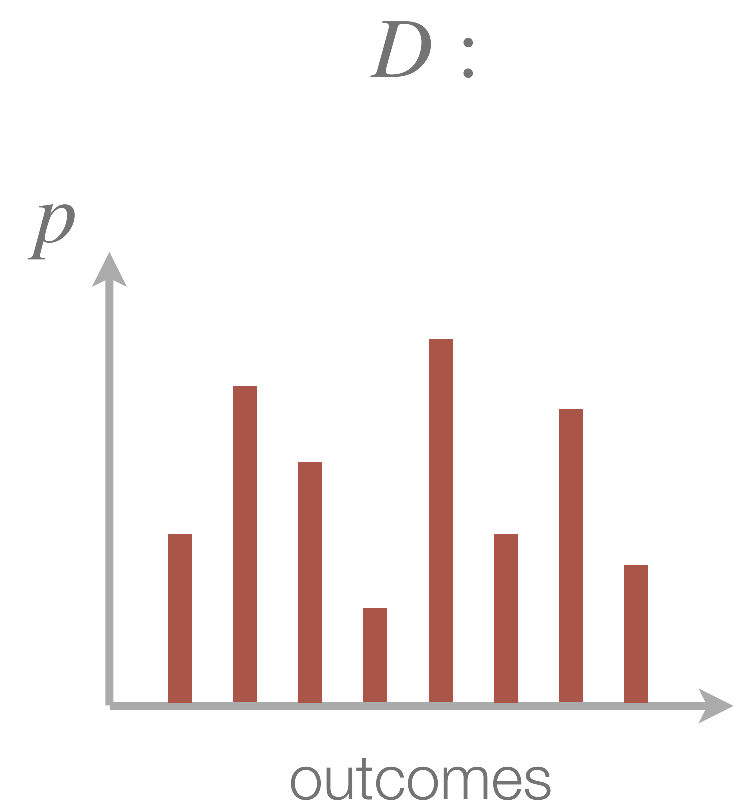
Time to bring out the (complexity-theoretic) big guns!



BosonSampling

- Consider the following n-photon m-mode experiment:





where $m = \mathrm{O}(n^2)$ and U is some uniformly-random matrix.

BosonSampling

- BosonSampling **task**: sample from that n-photon distribution;
  - Or in fact any sufficiently close to it!
  - **Not** a decision problem!

Theorem [Aaronson and Arkhipov]

If there was a classical algorithm capable of sampling efficiently from some distribution D' such that

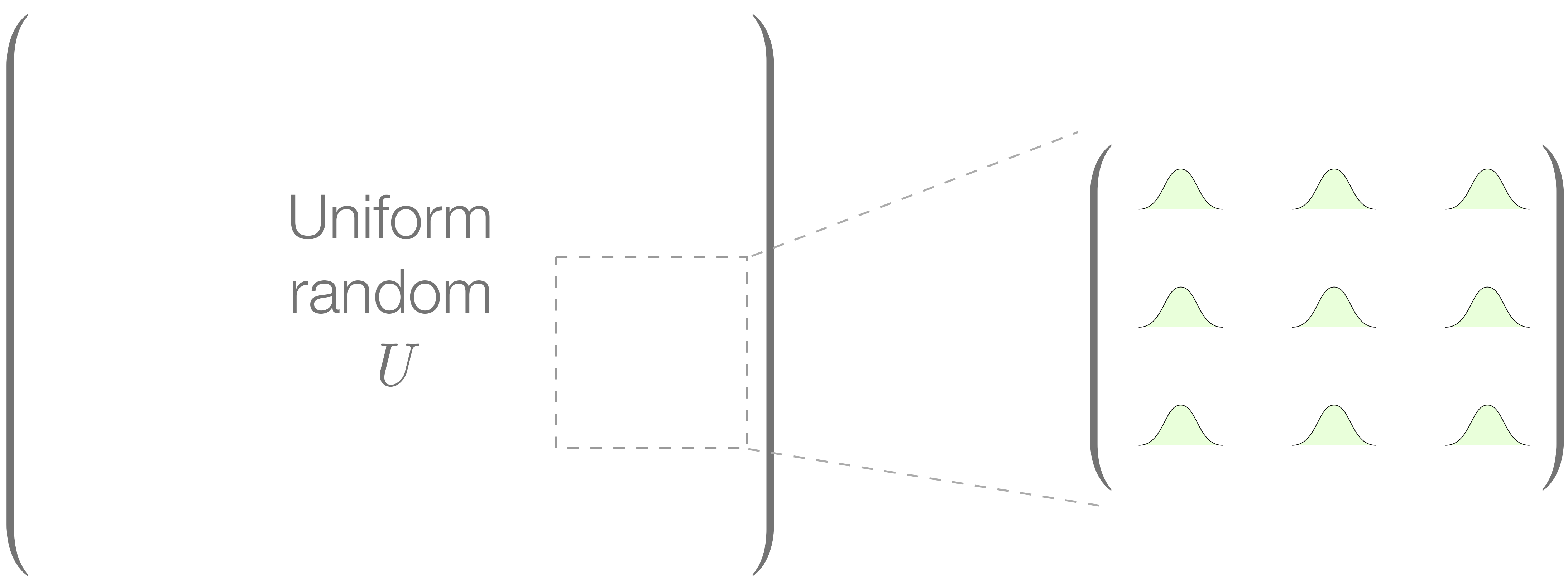
$$|D - D'| < \delta$$

in time poly(n, $1/\delta$), the polynomial hierarchy (**PH**) would collapse to its 3rd level!

BosonSampling

- Outline of result:

  1. Probabilities = permanents of submatrices of a random matrix;



Submatrices look like
independent Gaussian matrices

BosonSampling

- Outline of result:
  1. Probabilities = permanents of submatrices of a random matrix;
  2. **Permanent-of-Gaussian conjecture;**

Fact: Permanent is **#P**-hard in the worst case.

Conjecture: Gaussian matrices are among the hardest Permanents to compute;

Plausible conjecture that has **nothing** to do with linear optics!

BosonSampling

- Outline of result:

1. Probabilities = permanents of submatrices of a random matrix;
2. **Permanent-of-Gaussian conjecture;**
3. A classical simulation that samples from a distribution very close to the ideal one must get many of these **#P-hard** probabilities right!

....

4. Another minor conjecture + 90 pages of complexity-theoretic cannons shooting at the problem…

....

5. If an efficient classical simulation for approximate BosonSampling existed, **PH** would collapse to the 3rd level!

BosonSampling

- Some additional comments:
  - Result does **not** mean linear optics can compute permanents!
  - $m = \mathrm{O}(n^2)$ means no-collision outcomes vastly more likely.
  - Randomness of U avoids structures that classical algorithms could "exploit".
  - This sampling task has no known applications;
    - Except as a "demonstration of force" by quantum devices ☺
  - Best known classical algorithm takes time $\mathrm{O}(n\ 2^n)$;

BosonSampling: pros and cons

- Cons
  - No error-correction, so it is unclear whether experiments could be performed to this level of accuracy!
  - Hard to verify that device is doing what it should;
    - Leandro has more to say about this!
- Pros
  - Much “easier” to implement than universal quantum computation;
    - Doesn’t require nonlinearities or adaptive measurements;
    - ~ 50-90 photon experiment?
  - New insights into foundations of q. computing and q. optics;

First experiments

- Four small-scale experiments reported in December 2012.

Brisbane

Science AAAS

Photonic Boson Sampling in a Tunable Circuit

Matthew A. Broome,[1,2]* Alessandro Fedrizzi,[1,2] Saleh Rahimi-Keshari,[2] Justin Dove,[3] Scott Aaronson,[3] Timothy C. Ralph,[2] Andrew G. White[1,2]

Rome

nature photonics LETTERS
PUBLISHED ONLINE: 26 MAY 2013 | DOI: 10.1038/NPHOTON.2013.112

Integrated multimode interferometers with arbitrary designs for photonic boson sampling

Andrea Crespi[1,2], Roberto Osellame[1,2]*, Roberta Ramponi[1,2], Daniel J. Brod[3], Ernesto F. Galvão[3]*, Nicolò Spagnolo[4], Chiara Vitelli[4,5], Enrico Maiorino[4], Paolo Mataloni[4] and Fabio Sciarrino[4]*

Oxford

Science AAAS

Boson Sampling on a Photonic Chip

Justin B. Spring,[1]* Benjamin J. Metcalf,[1] Peter C. Humphreys,[1] W. Steven Kolthammer,[1] Xian-Min Jin,[1,2] Marco Barbieri,[1] Animesh Datta,[1] Nicholas Thomas-Peter,[1] Nathan K. Langford,[1,3] Dmytro Kundys,[4] James C. Gates,[4] Brian J. Smith,[1] Peter G. R. Smith,[4] Ian A. Walmsley[1]*

Vienna

LETTERS nature photonics
PUBLISHED ONLINE: 12 MAY 2013 | DOI: 10.1038/NPHOTON.2013.102

Experimental boson sampling

Max Tillmann[1,2]*, Borivoje Dakić[1], René Heilmann[3], Stefan Nolte[3], Alexander Szameit[3] and Philip Walther[1,2]*

First experiments

- Four small-scale experiments reported in December 2012.

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...meters with ...nic boson sampling

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For more about experiments, tune in tomorrow!

Ox...

Science AAAS

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nature photonics

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A zoo of intermediate quantum models

- Many other similar “quantum supremacy” results!
    - IQP; [BJS]
    - Constant-depth quantum circuits [TD];

  These predate BosonSampling

    - Quantum approximate optimization algorihtms; [Farhi, Harrow]
    - A version of the 1-clean-qubit model; [Morimae, Fujii, Fitzsimmons]
    - Generalized Clifford circuits [JVdN]
    - Random quantum circuits;
    - And more!

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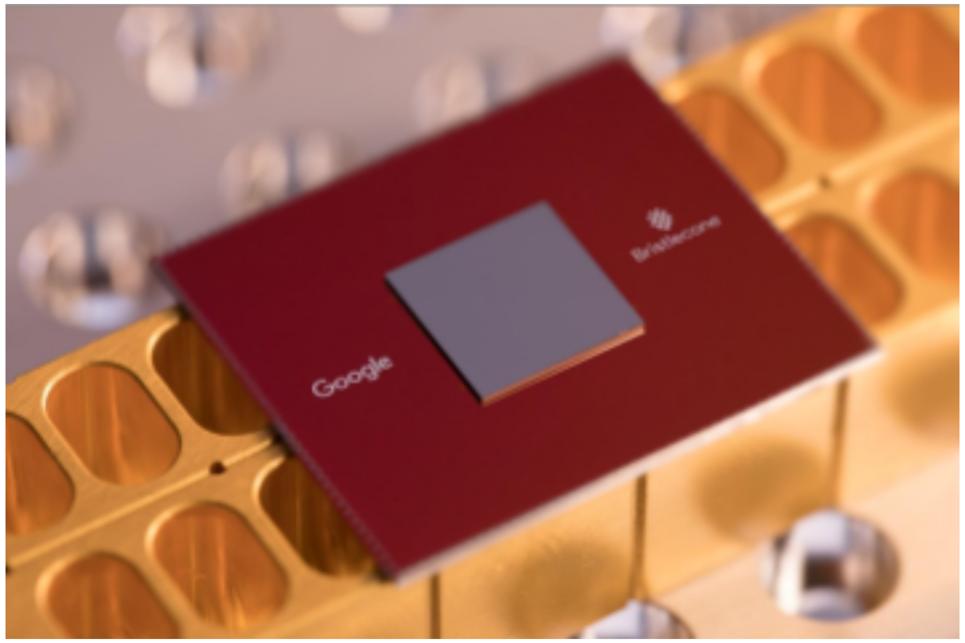
Random quantum circuits

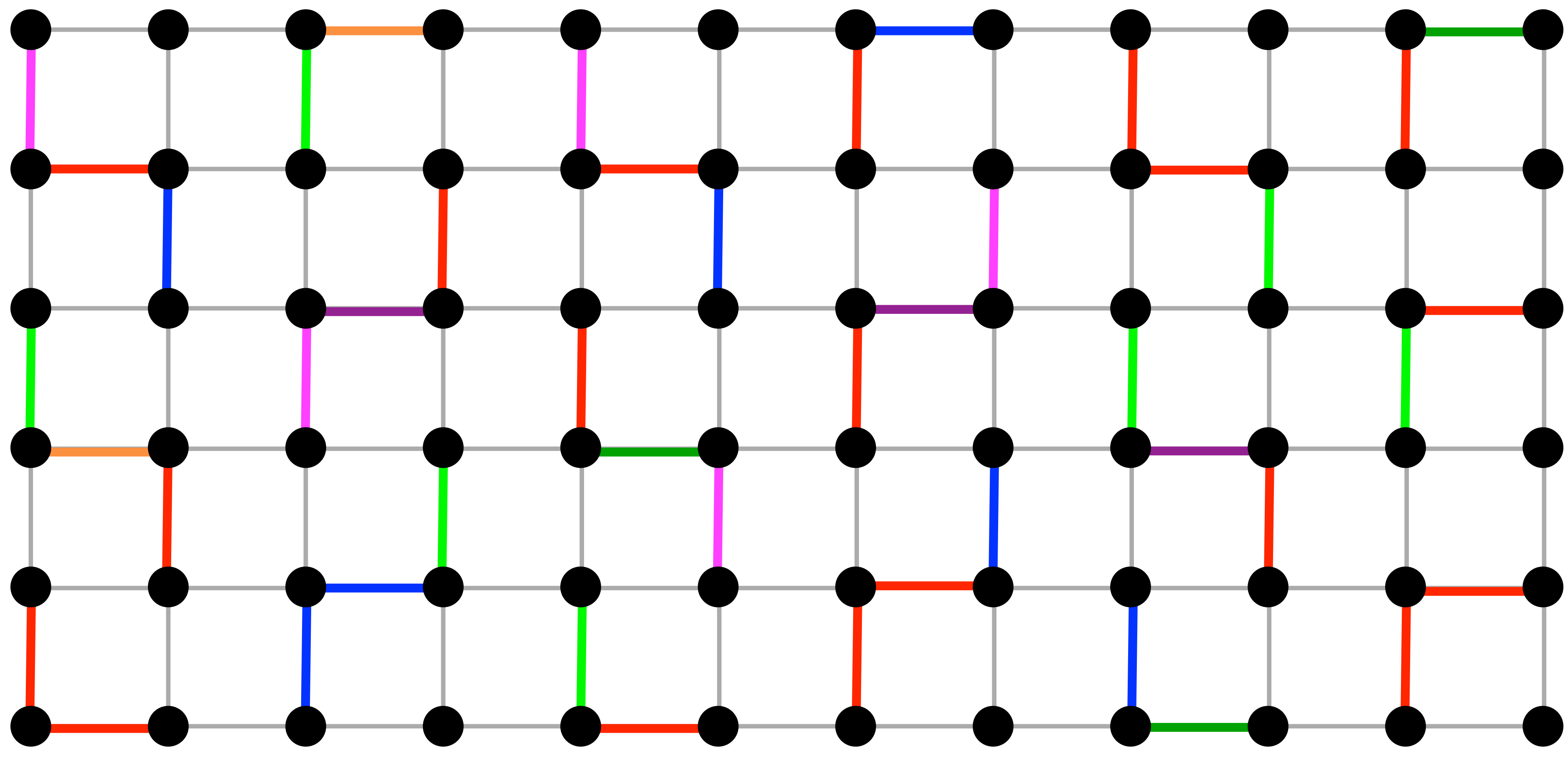
- Google's Quantum AI Lab + John Martini's group at UCSB
  - Another way to demonstrate the power of quantum computers.
- Meet Google's 72-qubit quantum processor: Bristlecone!



  - Goal: Demonstrate "Quantum advantage" on 72 qubits! (…soon)

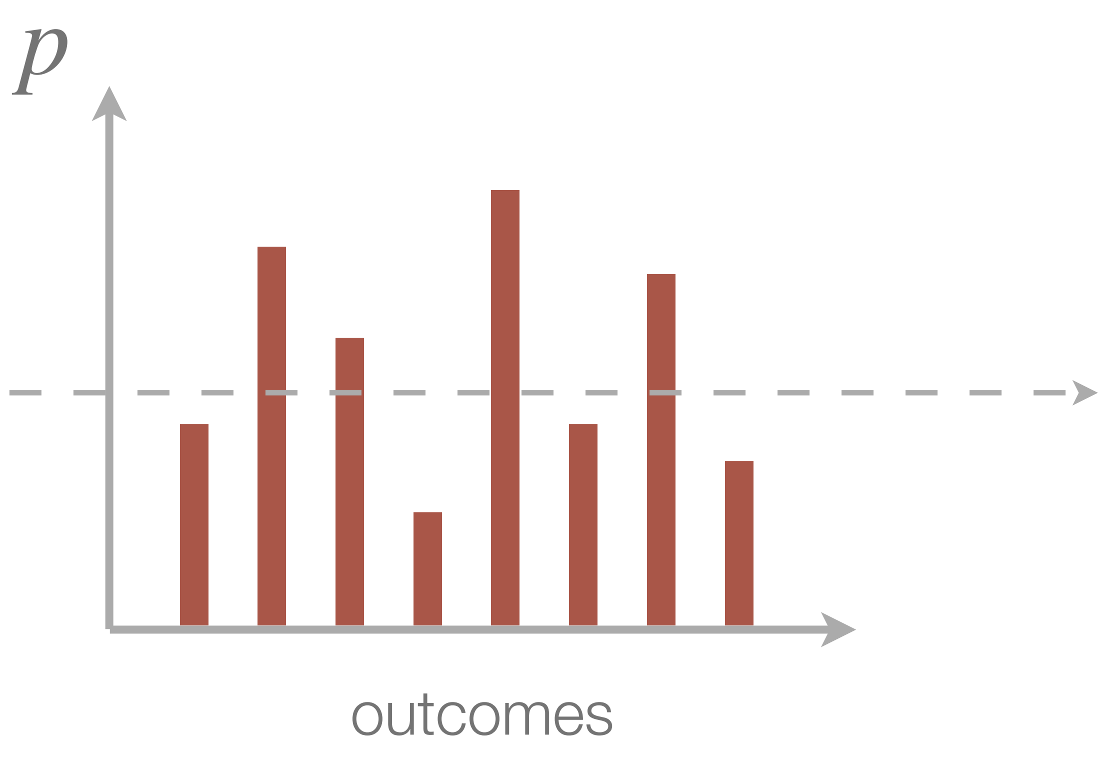
Image: Google AI blog

The random quantum circuit model



The random quantum circuit model

- But what is known about the complexity of simulating this?
- Aaronson and Chen [Dec ’16]
  - Task: “Given as input a random quantum circuit C, generate a set of outputs such that at least 2/3 of them have above-median probability in the output distribution of C ”;



Generate a list of samples where ≥ 2/3 of them are among those above this height;

The random quantum circuit model

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  - Task: "Given as input a random quantum circuit C, generate a set of outputs such that at least 2/3 of them have above-median probability in the output distribution of C";
  - Needs one further conjecture (similar to permanent-of-Gaussians in BosonSampling);
- Bouland, Fefferman, Nirkhe and Vazirani [March '18!]
  - Made a major step towards improving the status of the required conjectures.

The random quantum circuit model

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Next episode:
Neat idea! Let’s build one **already**, what’s taking so long?