FAPESP ICTP unesp IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

Introduction to quantum computing and simulability

Demonstrating a quantum advantage II

Daniel J. Brod

Leandro Aolita

Ernesto Galvão

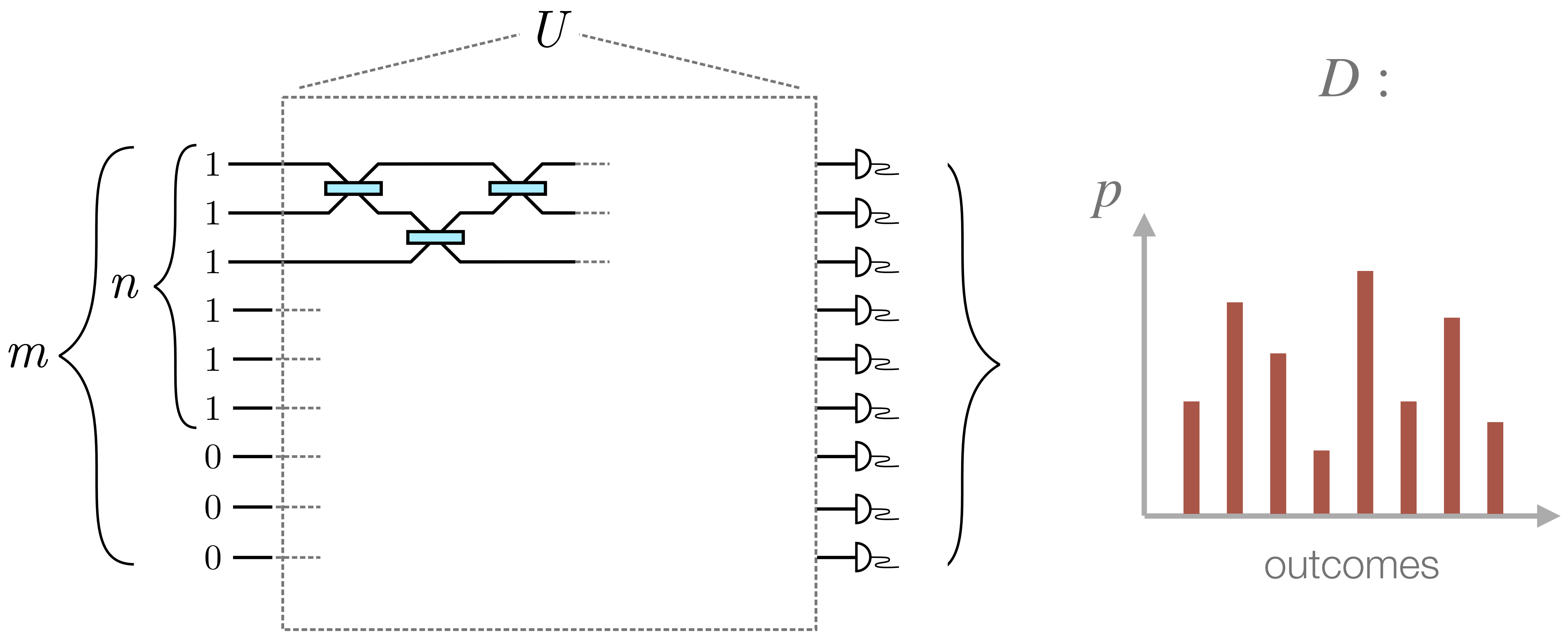
Instituto de Física
Universidade Federal Fluminense

Outline: Demonstrating a quantum advantage II

- BosonSampling: review and experiments;
- Experimental considerations;
    - Scattershot BosonSampling;
    - BosonSampling with losses;
- State-of-the-art: Simulations vs experiments;
    - BosonSampling;
    - Random quantum circuits;

BosonSampling

- Consider the following n-photon m-mode experiment:



where $m = O(n^2)$ and U is some uniformly-random matrix.

BosonSampling

Transition between two (no-collision) states

$$|S\rangle \to |T\rangle$$

Probability: $\displaystyle \Pr_{S\to T} = |\operatorname{Per}(U_{S,T})|^2$

$U_{S,T}$: submatrix of U with rows/columns chosen according to S and T

BosonSampling

- BosonSampling **task**: sample from that n-photon distribution;
  - Or in fact any sufficiently close to it!
  - **Not** a decision problem!
- Theorem[1]:

> If there was a classical algorithm capable of sampling efficiently from some distribution D' such that
>
> $$|D - D'| < \delta$$
>
> in time poly(n, $1/\delta$), the polynomial hierarchy (**PH**) would collapse to its 3rd level!

[1] Aaronson and Arkhipov, Theo. Comput. **4**, 143 (2013)

BosonSampling: pros and cons

- Cons
  - No error-correction, so theory still far from experimental reality;
  - Hard to verify that device is doing what it should;
  - No practical applications (so far!)
- Pros
  - Much “easier” to implement than universal quantum computation;
    - Doesn’t require nonlinearities or adaptive measurements;
    - ~ 50-90 photon experiment?
  - New insights into foundations of q. computing and q. optics;

First experiments

- Four small-scale experiments reported in December 2012.

Brisbane

Science AAAS

Photonic Boson Sampling in a Tunable Circuit

Matthew A. Broome,[1,2*] Alessandro Fedrizzi,[1,2] Saleh Rahimi-Keshari,[2] Justin Dove,[3] Scott Aaronson,[3] Timothy C. Ralph,[2] Andrew G. White[1,2]

Rome

nature photonics LETTERS

PUBLISHED ONLINE: 26 MAY 2013 | DOI: 10.1038/NPHOTON.2013.112

Integrated multimode interferometers with arbitrary designs for photonic boson sampling

Andrea Crespi[1,2], Roberto Osellame[1,2*], Roberta Ramponi[1,2], Daniel J. Brod[3], Ernesto F. Galvão[3*], Nicolò Spagnolo[4], Chiara Vitelli[4,5], Enrico Maiorino[4], Paolo Mataloni[4] and Fabio Sciarrino[4*]

Oxford

Science AAAS

Boson Sampling on a Photonic Chip

Justin B. Spring,[1*] Benjamin J. Metcalf,[1] Peter C. Humphreys,[1] W. Steven Kolthammer,[1] Xian-Min Jin,[1,2] Marco Barbieri,[1] Animesh Datta,[1] Nicholas Thomas-Peter,[1] Nathan K. Langford,[1,3] Dmytro Kundys,[4] James C. Gates,[4] Brian J. Smith,[1] Peter G. R. Smith,[4] Ian A. Walmsley[1*]

Vienna

LETTERS nature photonics

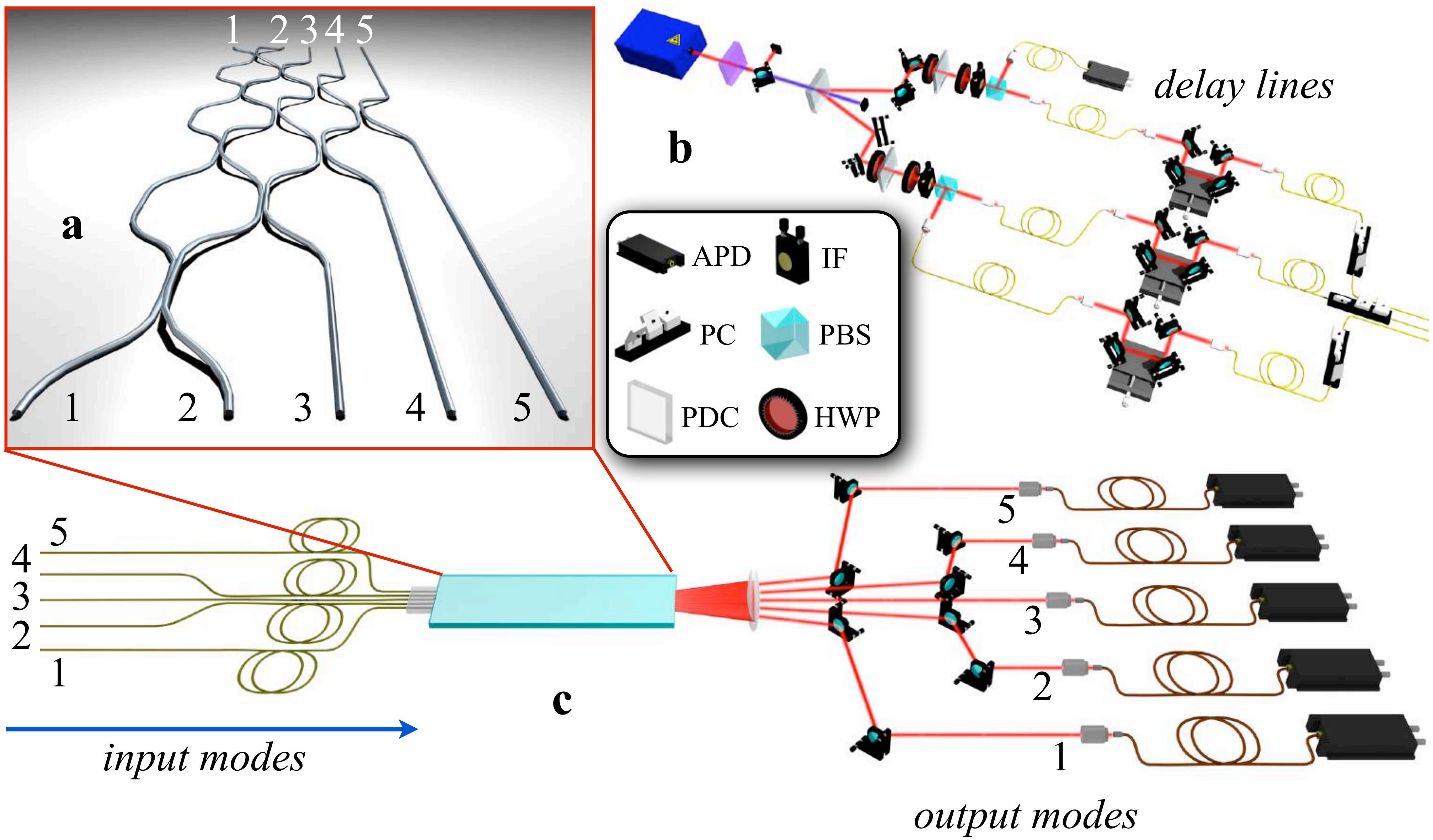
PUBLISHED ONLINE: 12 MAY 2013 | DOI: 10.1038/NPHOTON.2013.102

Experimental boson sampling

Max Tillmann[1,2*], Borivoje Dakić[1], René Heilmann[3], Stefan Nolte[3], Alexander Szameit[3] and Philip Walther[1,2*]

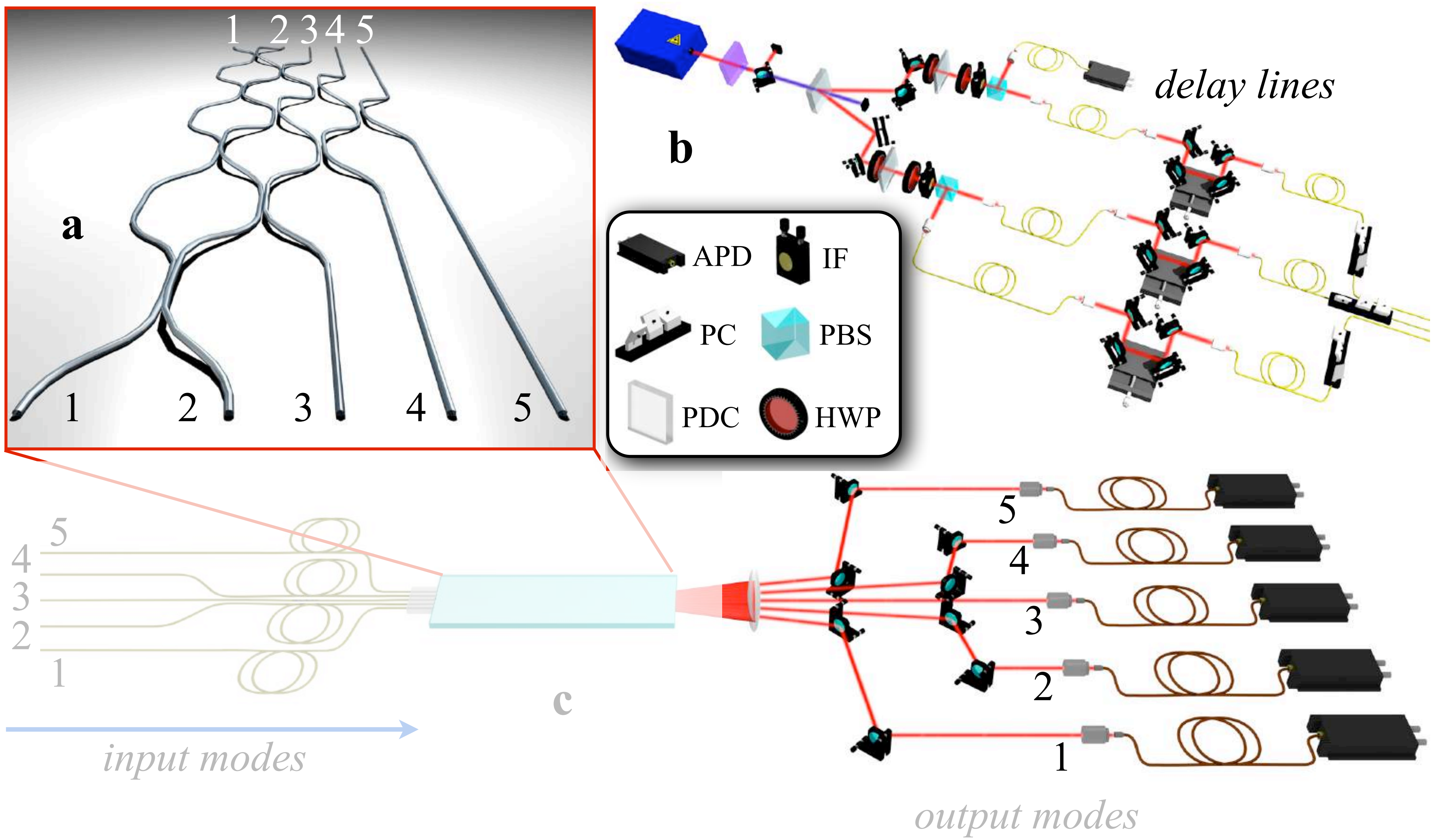
First experiments

- All with similar design, with 3 or 4 photons:



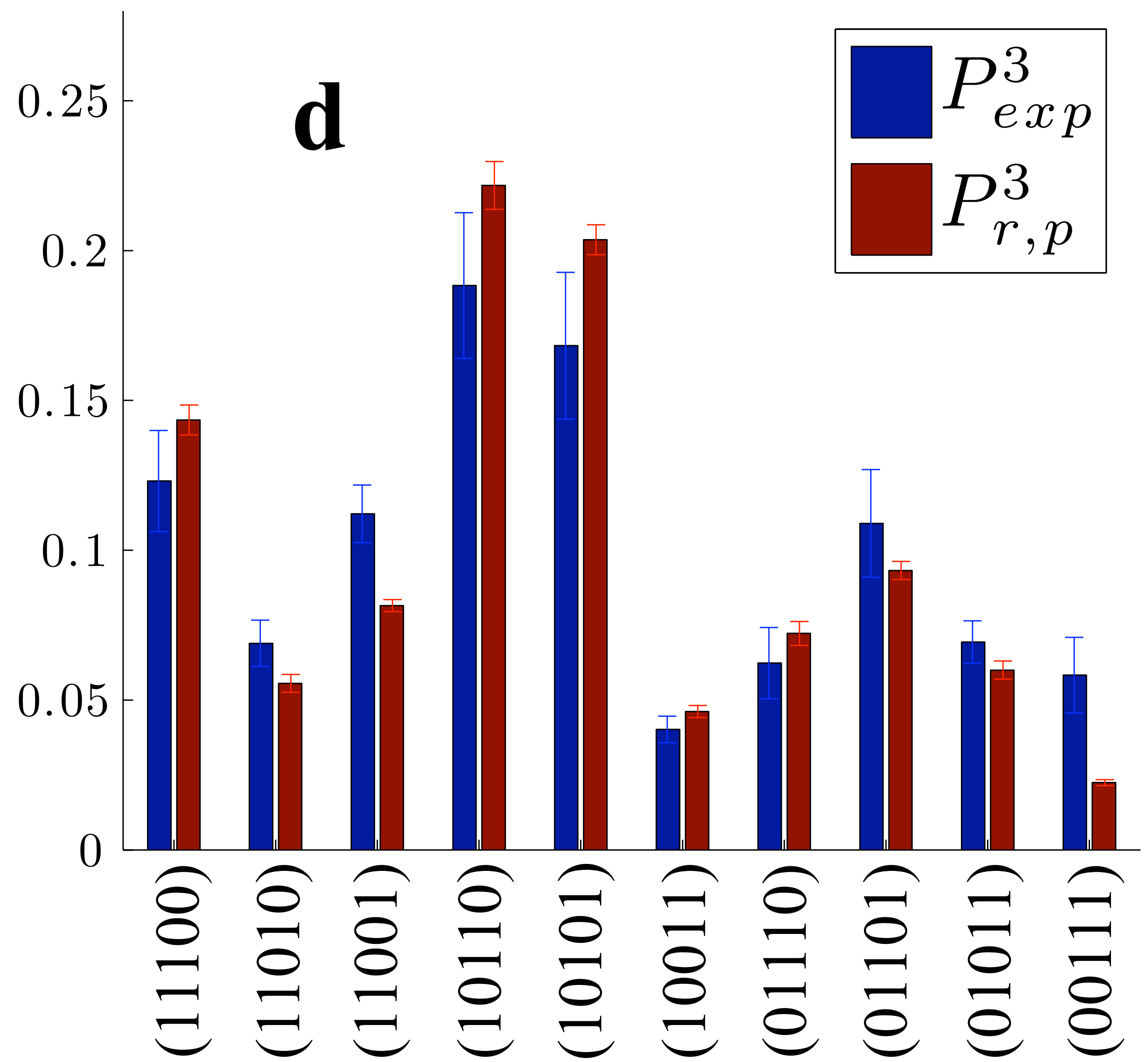
First experiments

- All with similar design, with 3 or 4 photons:



First experiments

- Goal: verify that permanent formula (hence quantum mechanics) works well for increasingly larger experiment sizes;



Outline: Demonstrating a quantum advantage II

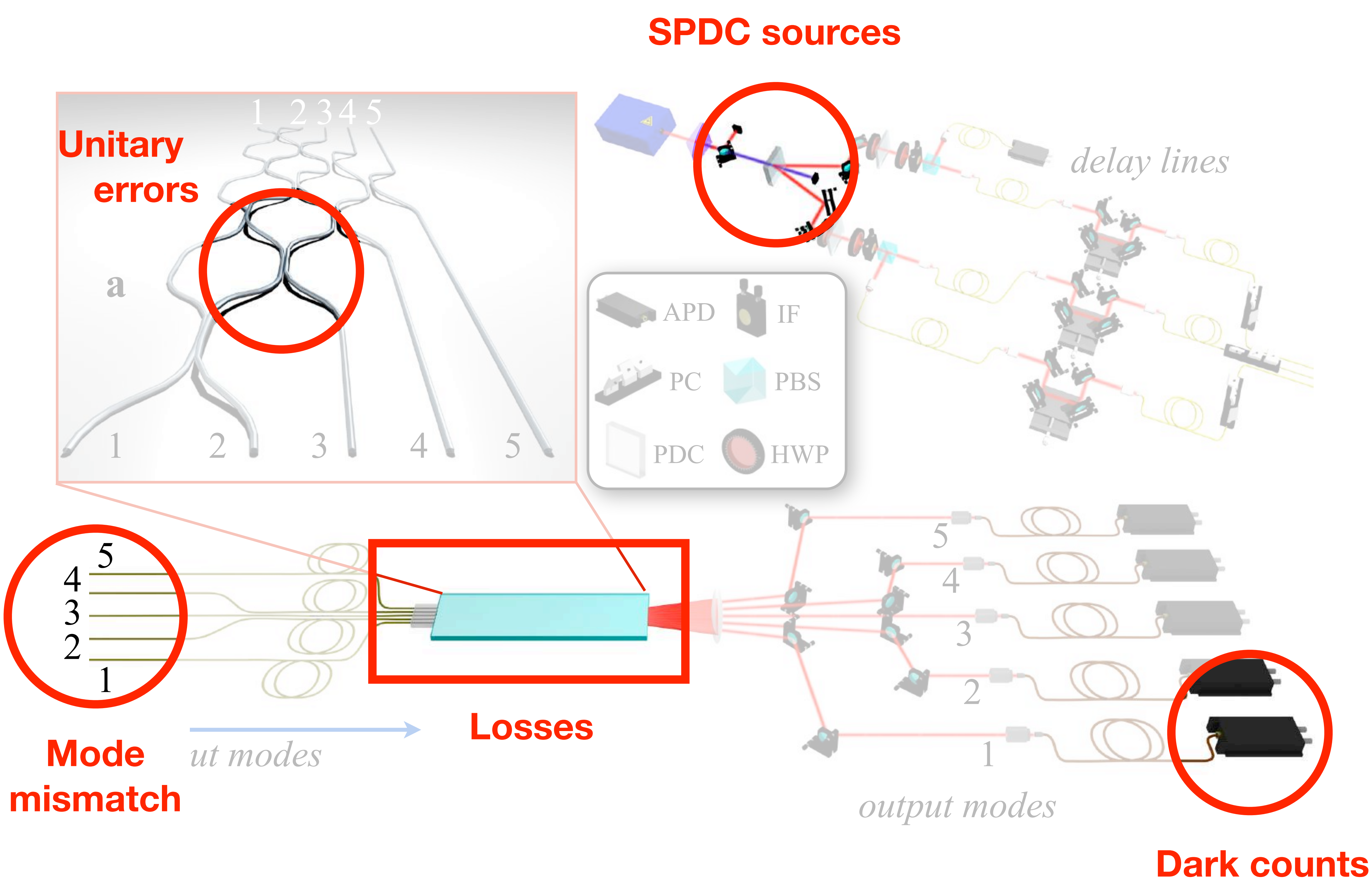
- BosonSampling: review and experiments;
- Experimental considerations;
  - Scattershot BosonSampling;
  - BosonSampling with losses;
- State-of-the-art: Simulations vs experiments;
  - BosonSampling;
  - Random quantum circuits;

BosonSampling: state-of-the-art

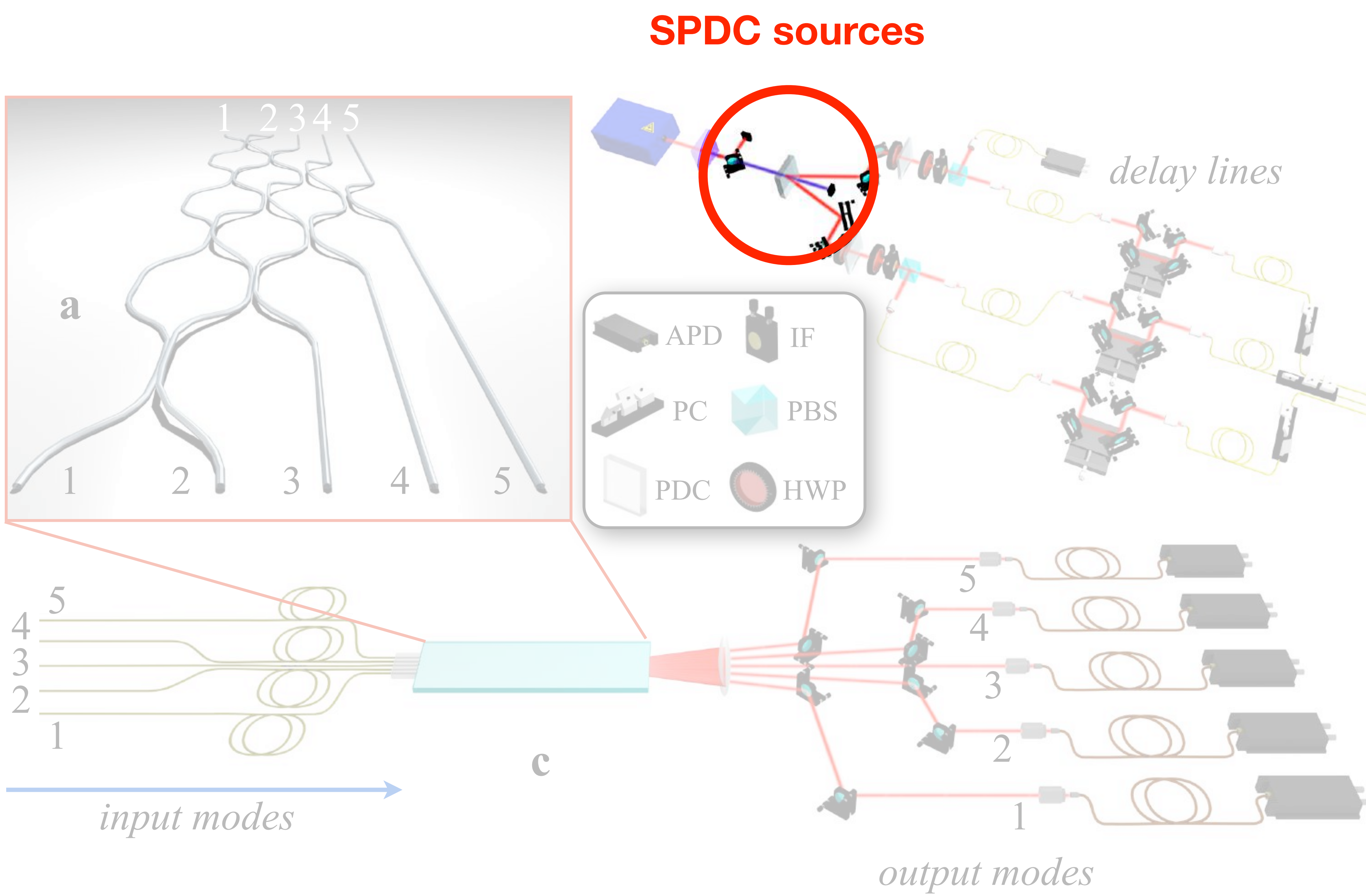
- Theory:
  - Variants of Bosonsampling to deal with experimental imperfections;
  - Proof of robustness of the model to errors;
  - Also advances on the side of classical simulations - sets the target!
- Experiments:
  - Use of time-bin encoding;
  - Improved quantum dot sources;
  - 5 photons in 16 modes by Jian-Wei Pan’s group[6];
    - 12-photon experiment promised for near future!

[6] Wang *et al*, PRL **120**, 230502 (2018)

Experimental imperfections

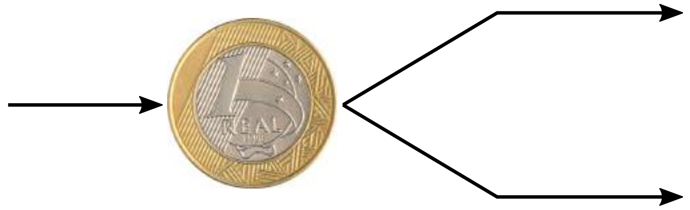


Experimental imperfections



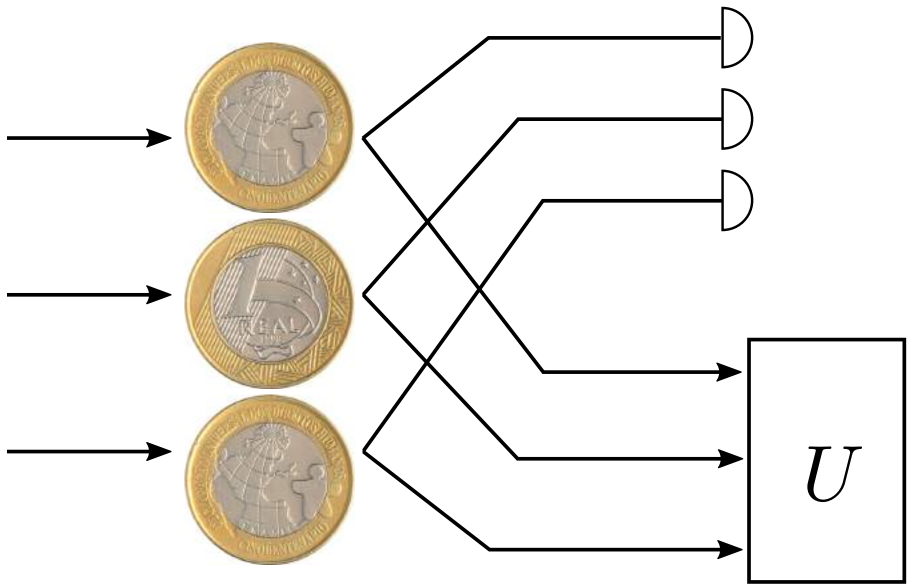
Probabilistic SPDC sources

- SPDC source:



$$|0,0\rangle + \epsilon\,|1,1\rangle + \ldots$$

- n SPDC sources



- If you want n heads by tossing n coins, it will take a long time!

Probabilistic SPDC sources

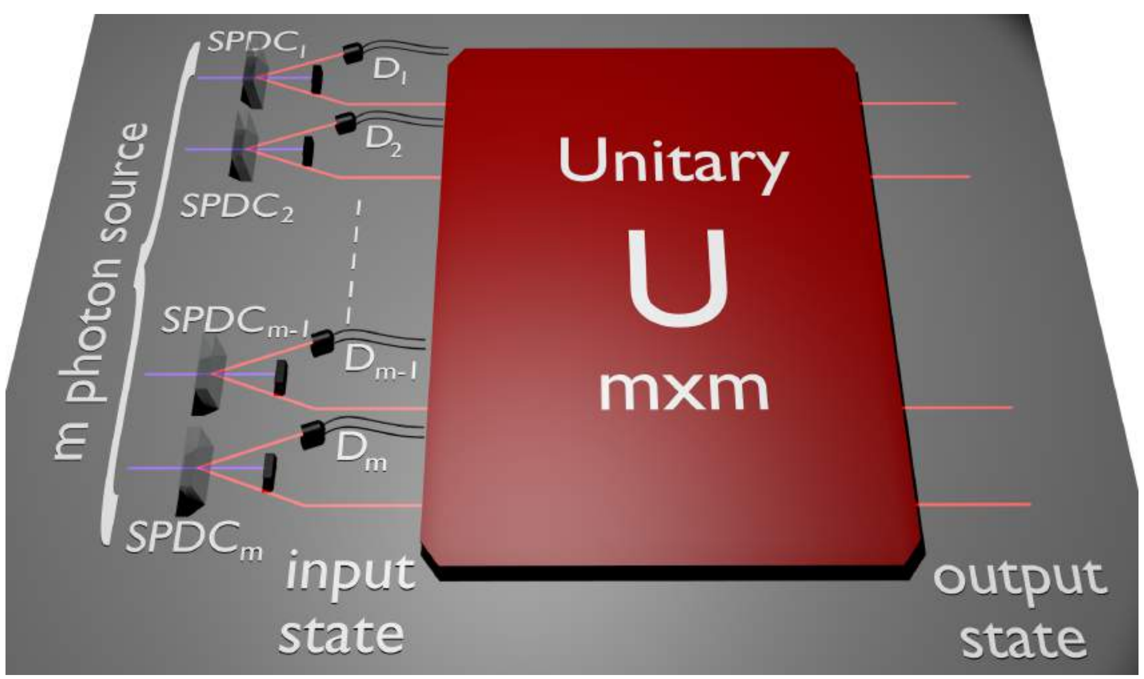
- If you want n heads by tossing n coins, it will take a long time!
- (Theoretical) Solution: Scattershot BosonSampling[7,8]
    - Toss $O(n^2)$ coins, and take whatever n heads you can get!

- You don't get to control the input state anymore - but that's ok.

[7] S. Kolthammer, unpublished,
[8] Lund *et al*, PRL **113**, 100502 (2014)]

Probabilistic SPDC sources

- If you want n heads by tossing n coins, it will take a long time!
- (Theoretical) Solution: Scattershot BosonSampling
  - Toss $O(n^2)$ coins, and take whatever n heads you can get!



RESEARCH ARTICLE

QUANTUM INFORMATION PROCESSING

Experimental scattershot boson sampling

Marco Bentivegna,[1] Nicolò Spagnolo,[1] Chiara Vitelli,[1,2] Fulvio Flamini,[1] Niko Viggianiello,[1] Ludovico Latmiral,[1] Paolo Mataloni,[1] Daniel J. Brod,[3] Ernesto F. Galvão,[4] Andrea Crespi,[5,6] Roberta Ramponi,[5,6] Roberto Osellame,[5,6] Fabio Sciarrino[1]*

5-fold counting increase due to scattershot approach!

[9] Bentivegna *et al*, Sci. Adv. **1**, e1400255 (2015)

Probabilistic SPDC sources

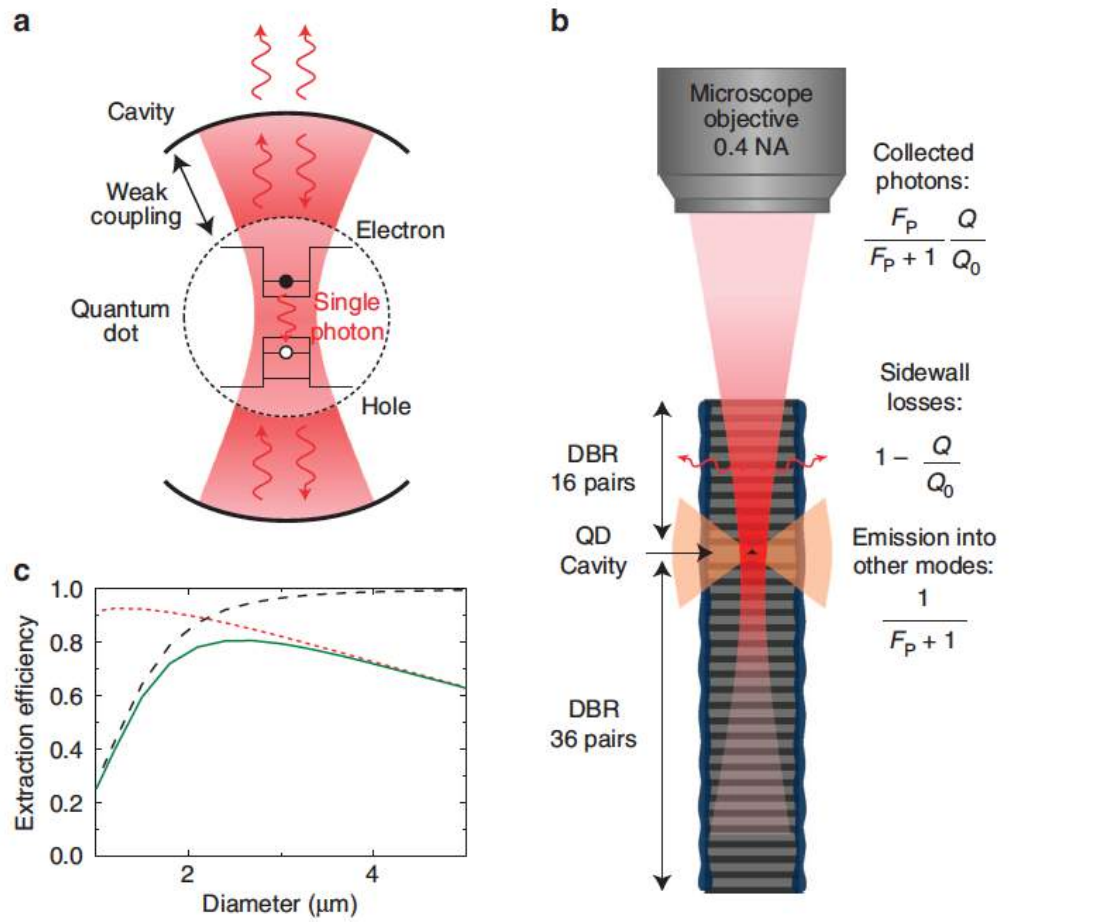
- (Experimental) Solution: …. Don't!

He *et al*, PRL **118**, 190501 (2017)

~200x increase in counting rates*

Loredo *et al*, PRL **118**, 130503 (2017)

Between 1 and 2 orders of magnitude increase*



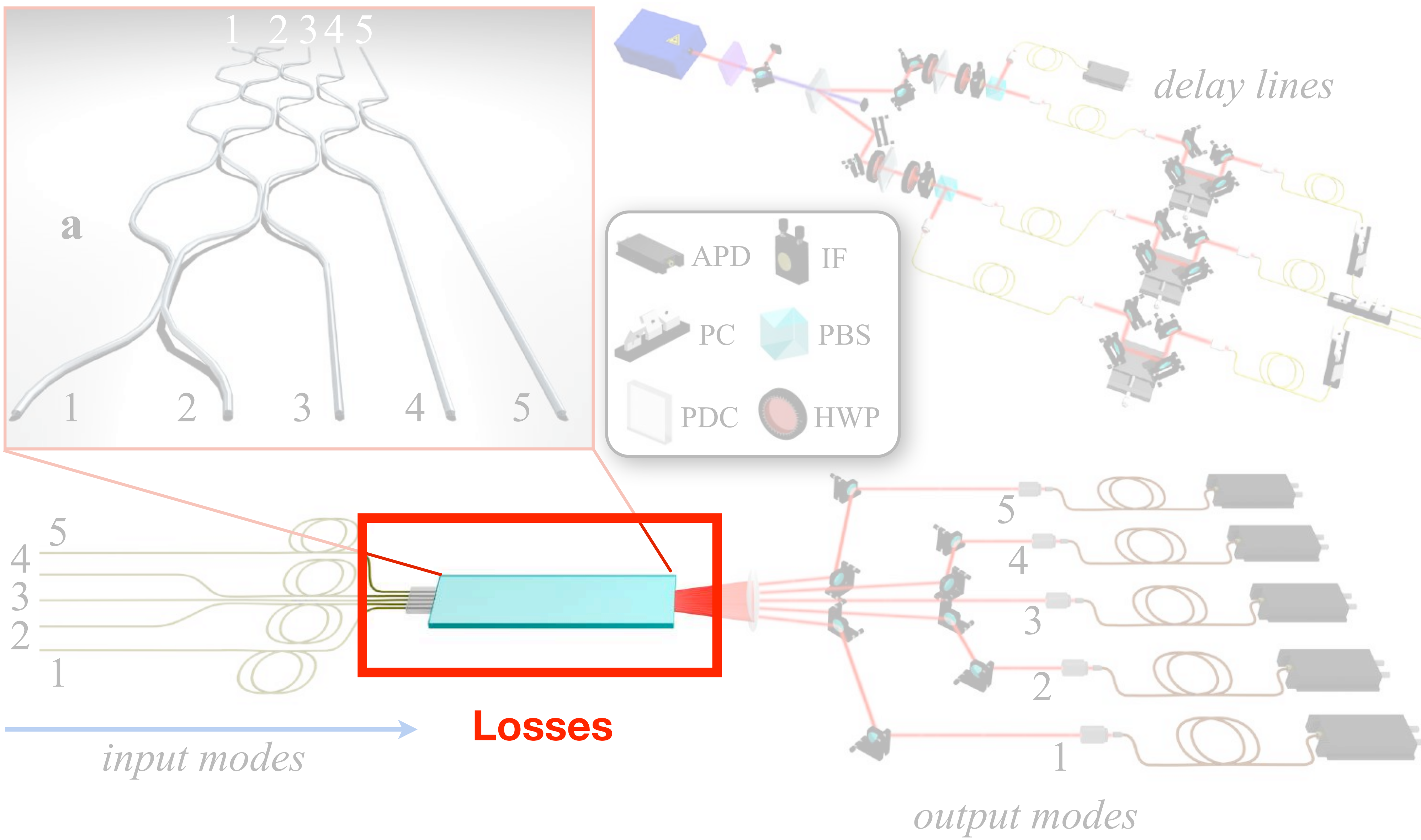
*Compared to non scattershot approach

[12] Gazzano *et al*, Nat. Comm. **4**, 1425 (2013)

Outline: Demonstrating a quantum advantage II

- BosonSampling: review and experiments;
- Experimental considerations;
  - Scattershot BosonSampling;
  - BosonSampling with losses;
- State-of-the-art: Simulations vs experiments;
  - BosonSampling;
  - Random quantum circuits;

Experimental imperfections



Losses

- (Theoretical) ~~solution~~ first step:
  - If only a **constant** number of photos is lost, it does not affect BosonSampling[13].
  - Not realistic at all, but good first step!

[13] Aaronson and Brod, PRA **93** 012335 (2016)

Losses

- BosonSampling with lost photons

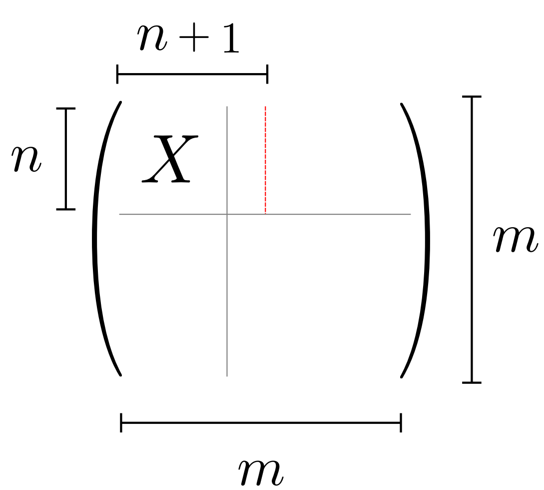
$$U = \begin{array}{ccc} & n & \\ n & \left(\begin{array}{c|c} X & \\ \hline & \end{array}\right) & m \\ & m & \end{array}$$

$$\Pr[S \to T] = |\mathrm{Per}(U_{S,T})|^2$$
$$= |\mathrm{Per}(X)|^2$$

Rows = output
Columns = input

Losses

- BosonSampling with lost photons
- Loss model:
  - We **know** we input $n+1$ photons but only n were detected
  - Extra assumption: Losses at the input*

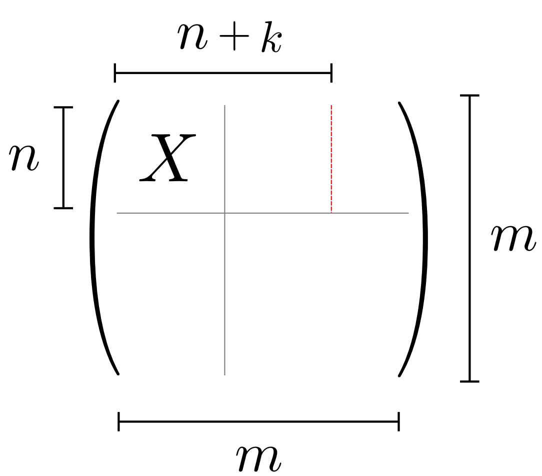


$$\Pr[S \to T] = \frac{1}{n+1} \sum_i |\mathrm{Per}(U_{S_i,T})|^2$$

*not strictly necessary!

Losses

- BosonSampling with lost photons
- Loss model:
  - We **know** we input $n+k$ photons but only n were detected
  - Extra assumption: Losses at the input.



$$\Pr[S \to T] = \frac{1}{|\Lambda|} \sum_{\bar{S} \in \Lambda} |\mathrm{Per}(U_{\bar{S},T})|^2$$

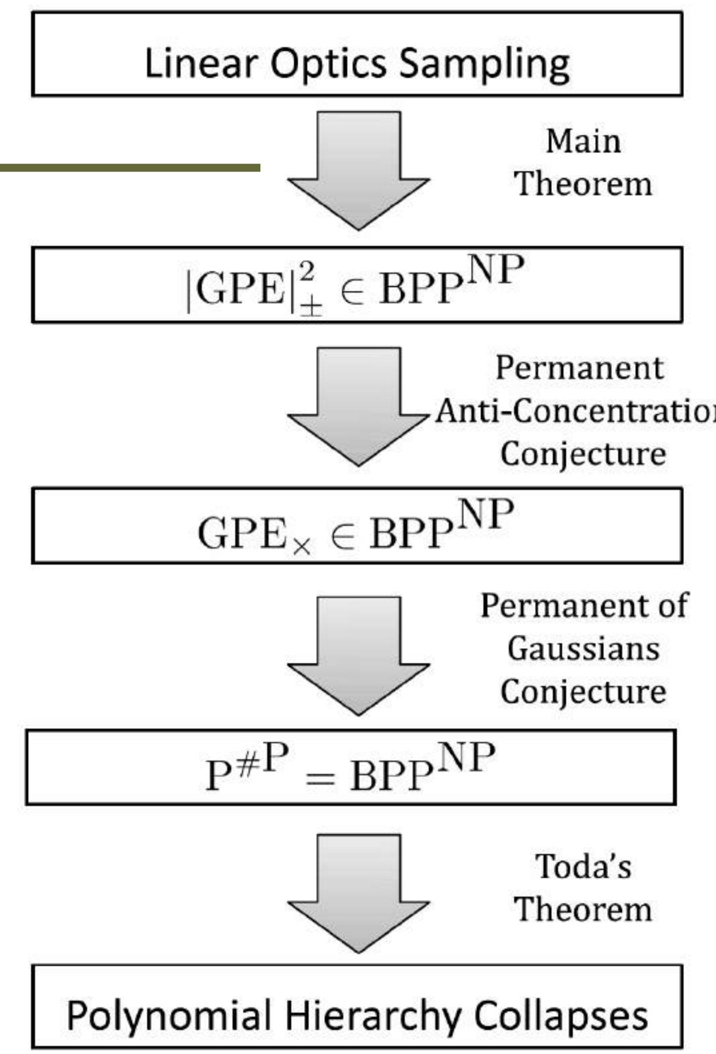
$^*|\Lambda| = \binom{n+k}{k}$

Losses

$$|\mathrm{Per}(X)|^2 \quad \text{(to error } \epsilon\text{)}$$

$$\varphi(A) := \frac{1}{|\Lambda|} \sum_{S \in \Lambda} |\mathrm{Per}(A_S)|^2 \quad \text{(to error } \epsilon\text{')}$$

$$A = \begin{array}{cc} & \overbrace{}^{n+k} \\ n & \left(\begin{array}{c|c} X & Y \end{array}\right) \end{array}$$



Losses

If this is Gaussian:

$$A = \begin{array}{cc} & \overbrace{\hphantom{X \quad Y}}^{n+k} \\ n & \left(\begin{array}{c|c} X & Y \end{array} \right) \end{array}$$

This is almost Gaussian for $c \approx 1$:

$$A[c] = \begin{array}{cc} & \overbrace{\hphantom{X \quad cY}}^{n+k} \\ n & \left(\begin{array}{c|c} X & cY \end{array} \right) \end{array}$$

$$\varphi(A[c]) = \frac{1}{|\Lambda|} \sum_{S \in \Lambda} |\mathrm{Per}(A[c]_S)|^2$$

Losses

If this is Gaussian:

$$A = \begin{array}{cc} & \overbrace{\hphantom{X \quad Y}}^{n+k} \\ n & \left(\begin{array}{c|c} X & Y \end{array}\right) \end{array}$$

This is almost Gaussian for $c \approx 1$:

$$A[c] = \begin{array}{cc} & \overbrace{\hphantom{X \quad cY}}^{n+k} \\ n & \left(\begin{array}{c|c} X & cY \end{array}\right) \end{array}$$

$$\varphi(A[c]) = \frac{\boxed{|\mathrm{Per}\, X|^2}}{|\Lambda|} + |c|^2 Q_1 + |c|^4 Q_2 + \ldots + |c|^{2k} Q_k$$

Losses

$$\varphi(A[c]) = \frac{|\mathrm{Per} X|^2}{|\Lambda|} + |c|^2 Q_1 + |c|^4 Q_2 + ... + |c|^{2k} Q_k$$

- Trick: Estimate φ for a few values of c, then find first coefficient by least squares method.
    - Problem: c must be close to 1, polynomial sampled in a small region.

Losses

$$\varphi(A[c]) = \frac{|\mathrm{Per} X|^2}{|\Lambda|} + |c|^2 Q_1 + |c|^4 Q_2 + ... + |c|^{2k} Q_k$$

- Trick: Estimate φ for a few values of c, then find first coefficient by least squares method.
  - Problem: c must be close to 1, polynomial sampled in a small region.

    $\Rightarrow$ Limit on estimation precision:

$$\epsilon' = O\left(\frac{\delta^{k+1/2} k^{k/2}}{n^{k/2}(n+k)^k}\epsilon\right)$$

Losses

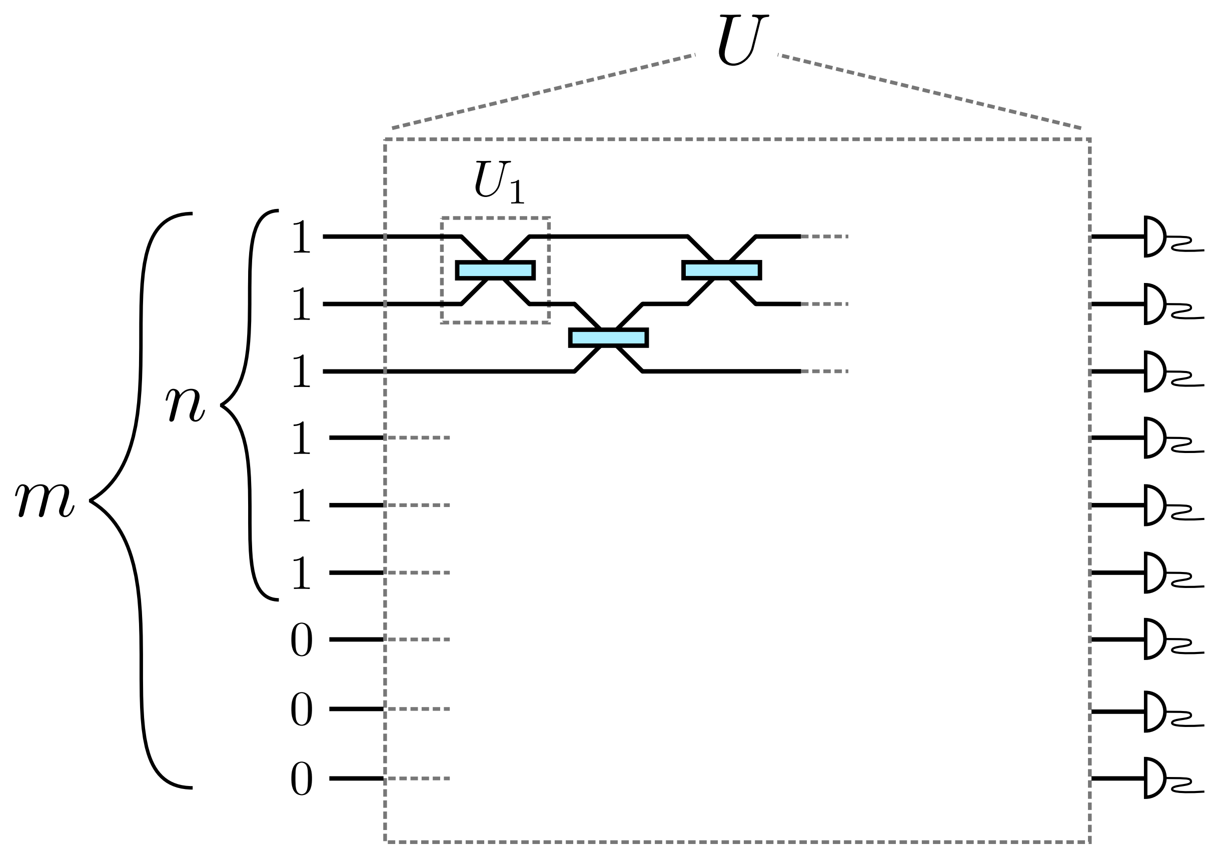
- **Main result:** BosonSampling with k lost photons, for constant k, is as hard as lossless BosonSampling.
  - (Roughly) equivalent to $\mathrm{O}(1/\mathrm{n})$ loss probability per photon
- Also applies to:
  - $\mathrm{O}(1/\mathrm{n})$ dark count probability
  - $\mathrm{O}(1/\mathrm{n})$ prob. of dark count **and** losses (unheralded!)
  - $\mathrm{O}(1/\mathrm{n})$ losses at the output

Losses

- (Theoretical) ~~solution~~ first step:
    - Still hard: $\mathrm{O}(1/\mathrm{n})$ probability of losses/dark counts (per photon/mode).
- This **cannot** take us all the way:
    - If all but $\mathrm{o}(\sqrt{n})$ of photons are lost, linear optics becomes classically simulable[14]

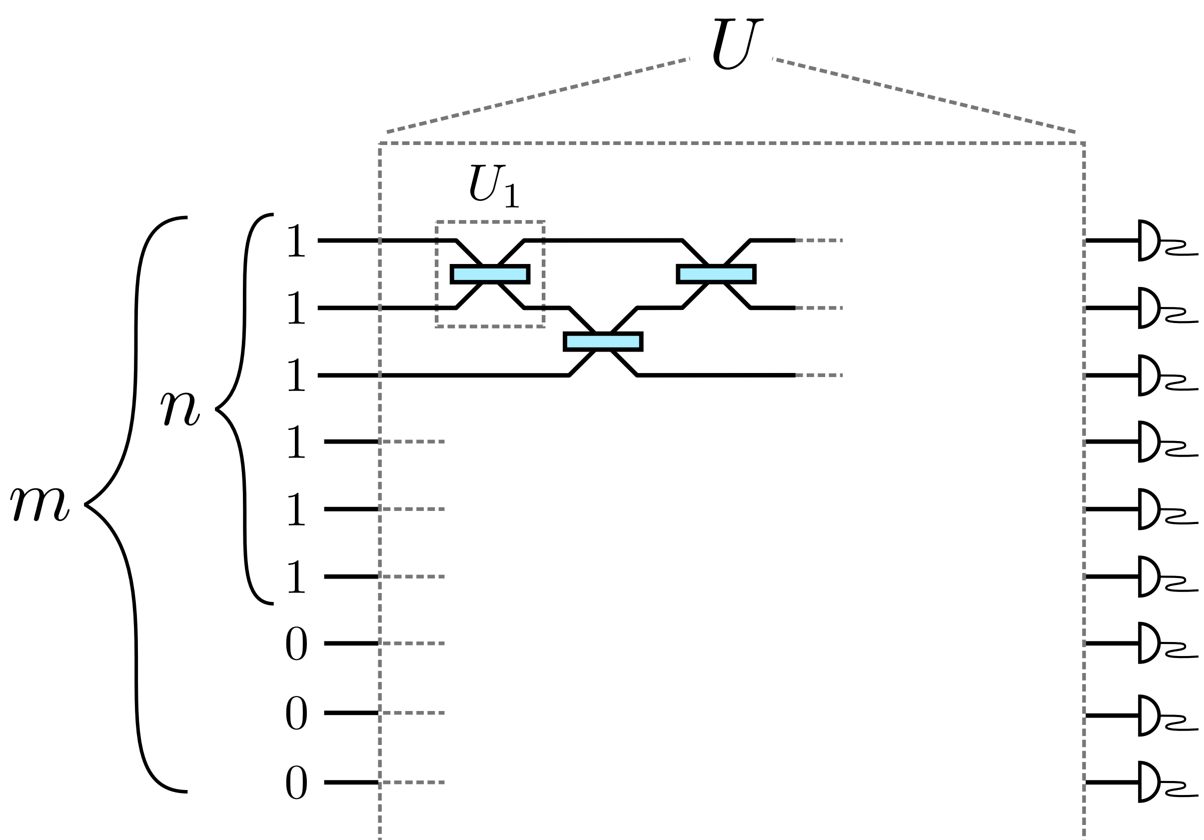
[14] Brod and Oszmaniec, New J. Phys. **20** 092002 (2018)

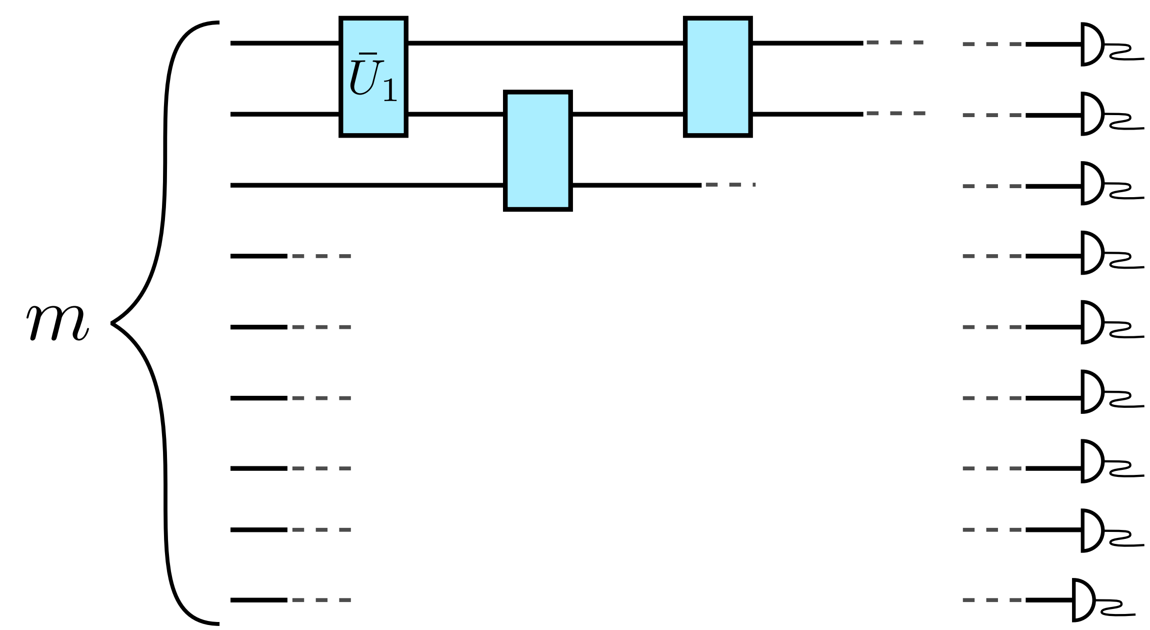
Losses



$$a_i^\dagger \mapsto \sum_j U_{ij} a_j^\dagger$$

Losses

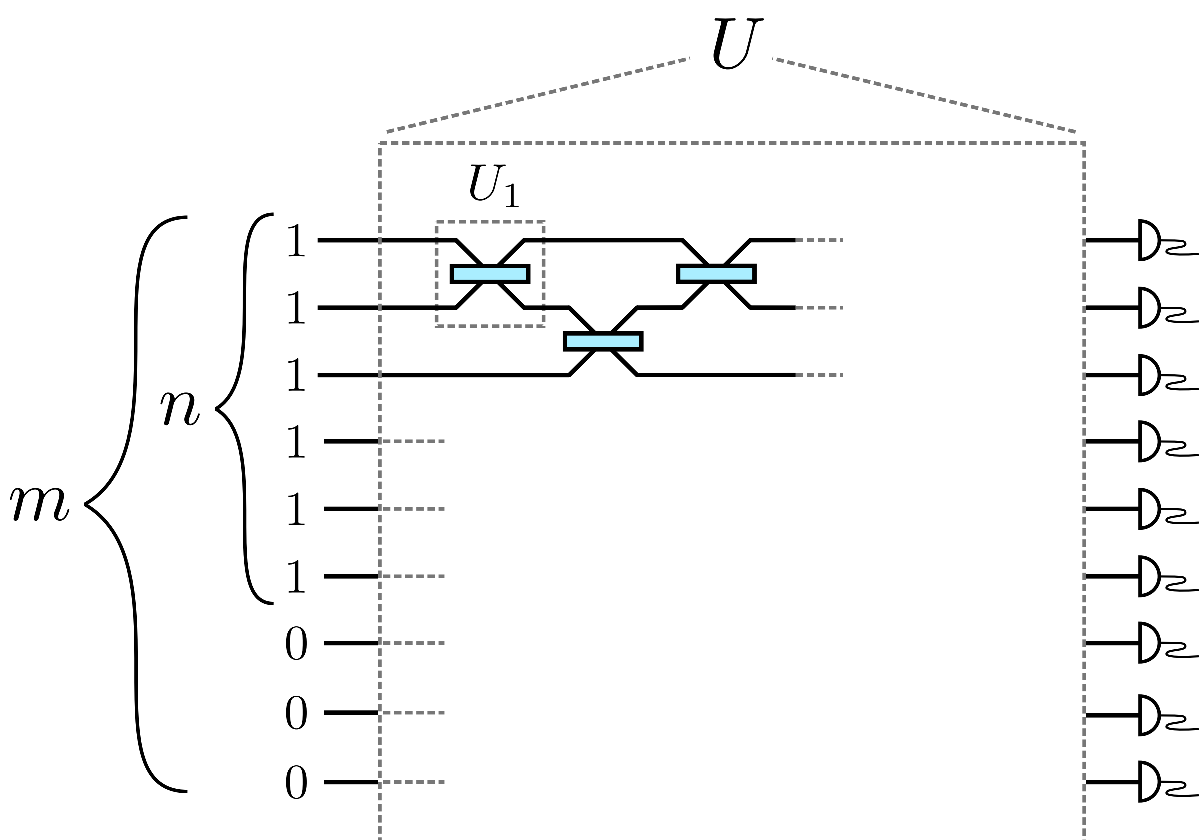


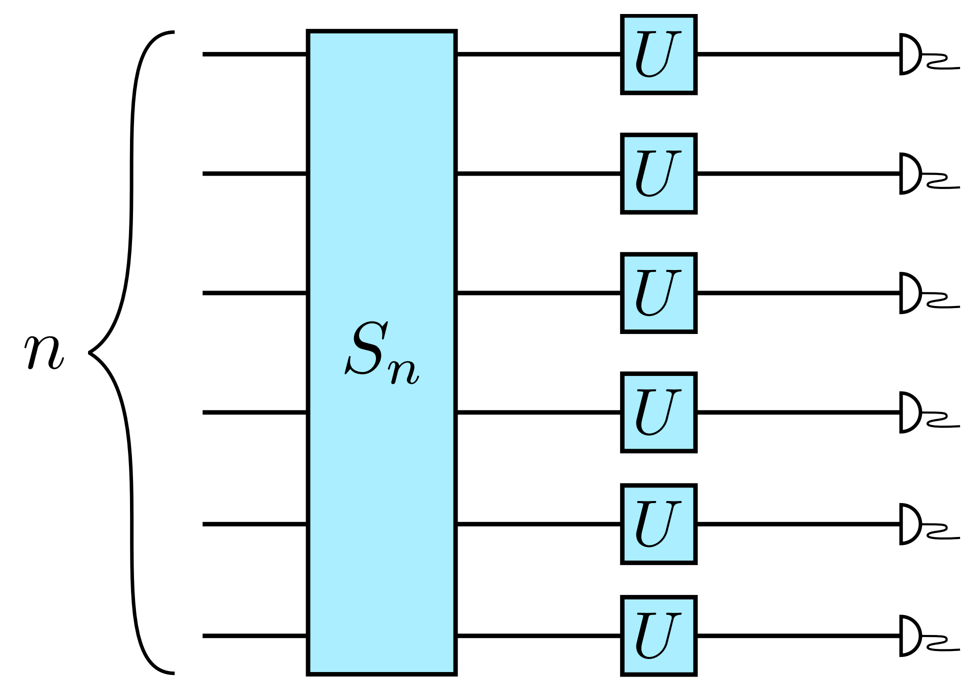


m n-level systems

("2nd quantization")

Losses

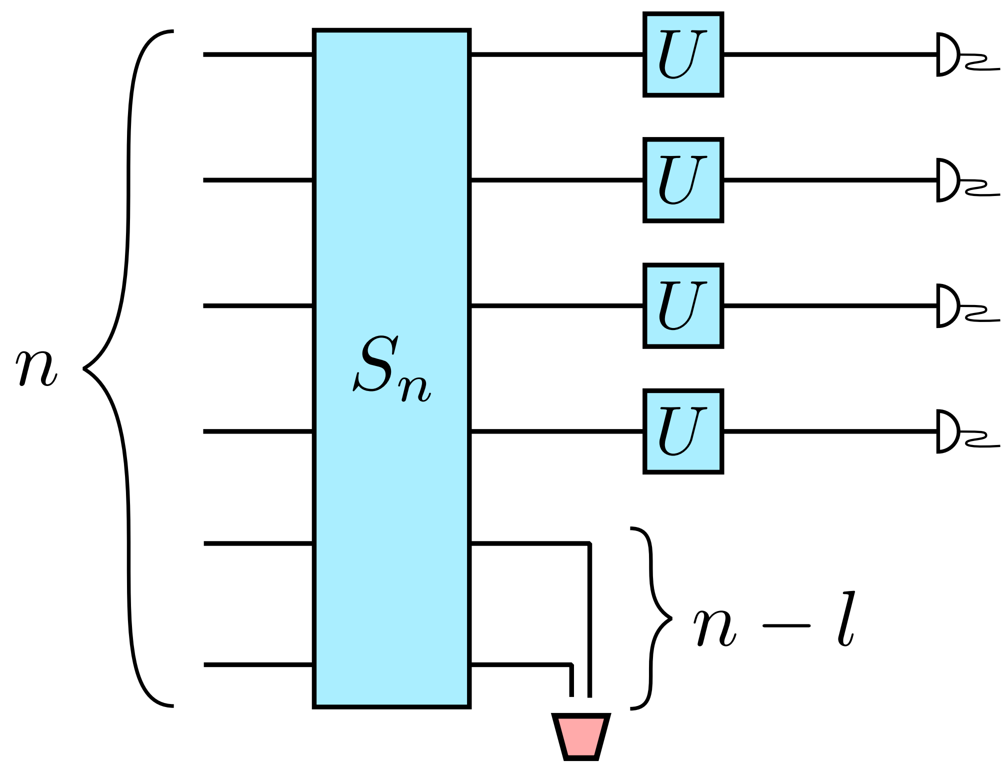




n m-level systems

(“1st quantization”)

Losses



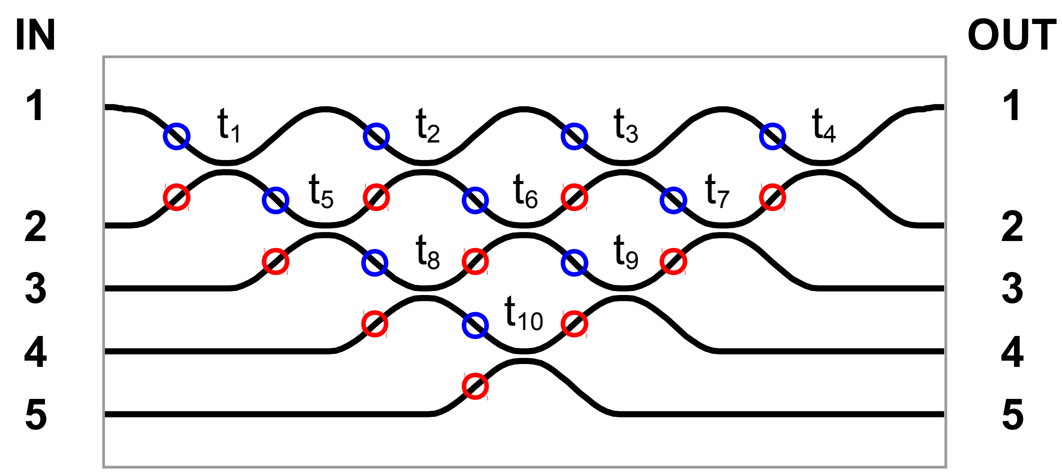
Losing photons degrades the entanglement of this state. Maybe, if too many photons are lost, the entanglement vanishes?

Losses

- If $l = \mathrm{o}(\sqrt{n})$, the entanglement goes to 0 for large n!
  - Caveat: does not quite qualify as classical simulation per original BosonSampling paper.
- Limits plausible hardness conjectures for lossy BosonSampling;
- Imposes more stringent constraints on physical realizations;

Losses

- How does this relate to realistic experiments?



- Result: If the shortest path from any input to any output has $C \log(n)$ beam splitters (for suitable C), on average $o(\sqrt{n})$ photons will be left.

Losses

- To summarize:
- BosonSampling is **still hard**:
  - $\mathrm{O}(1/\mathrm{n})$ probability of losses/dark counts (per photon/mode).
- BosonSampling **becomes easy**
  - $\mathrm{o}(\sqrt{n})$ photons left on average;
  - Also for a constant loss rate, **if** coupled with a dark count rate[15];
  - **Very** recent development suggests that BosonSampling becomes easy if a constant fraction of photons is lost[16].

[15] Rahimi-Keshari, Ralph, Caves, PRX **6** 021039 (2016)

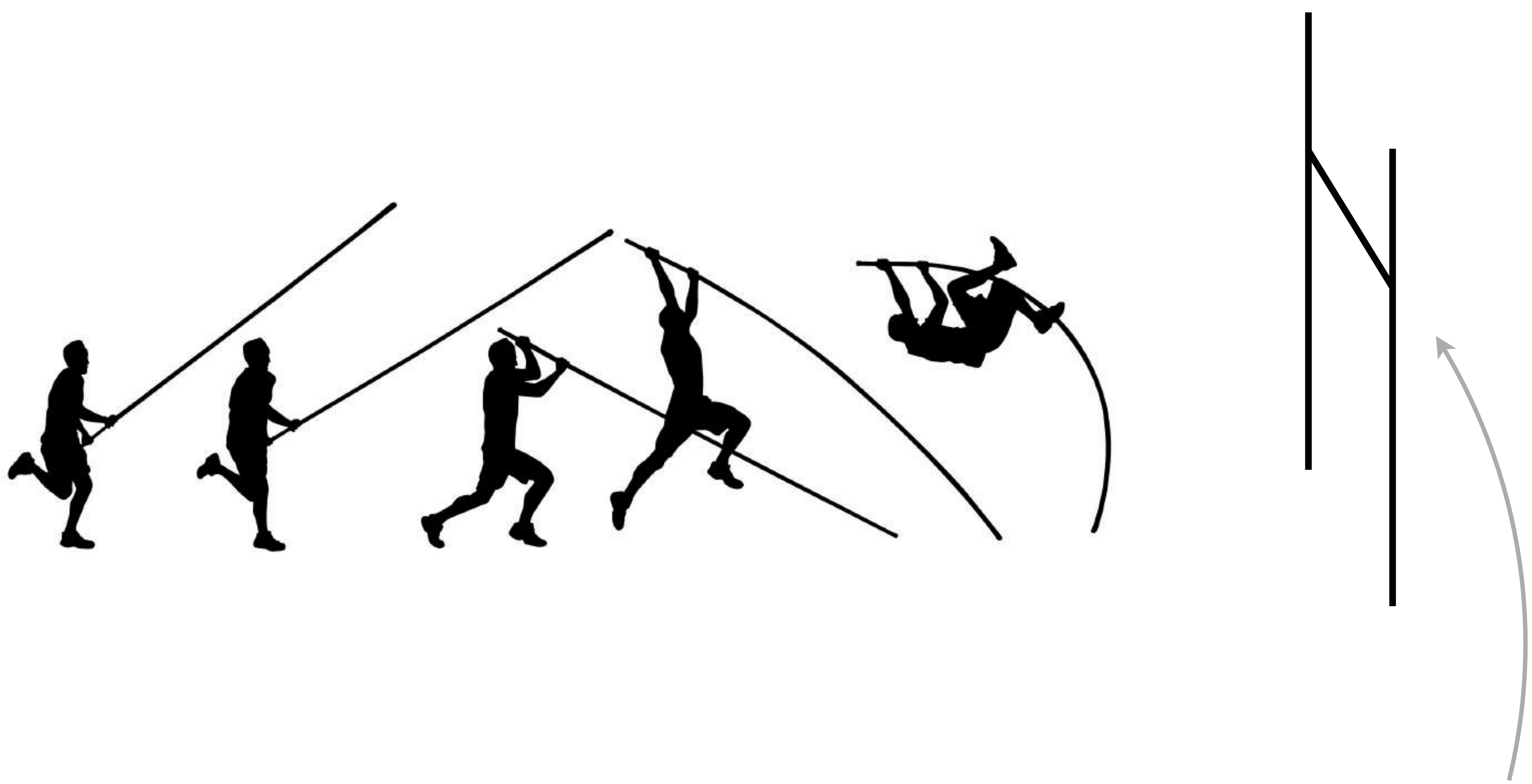
[16] Renema, Shchesnovich, Garcia-Patron arXiv:1809.01953 (2018)

Outline: Demonstrating a quantum advantage II

- BosonSampling: review and experiments;
- Experimental considerations;
  - Scattershot BosonSampling;
  - BosonSampling with losses;
- State-of-the-art: Simulations vs experiments;
  - BosonSampling;
  - Random quantum circuits;

Comparison with classical algorithms

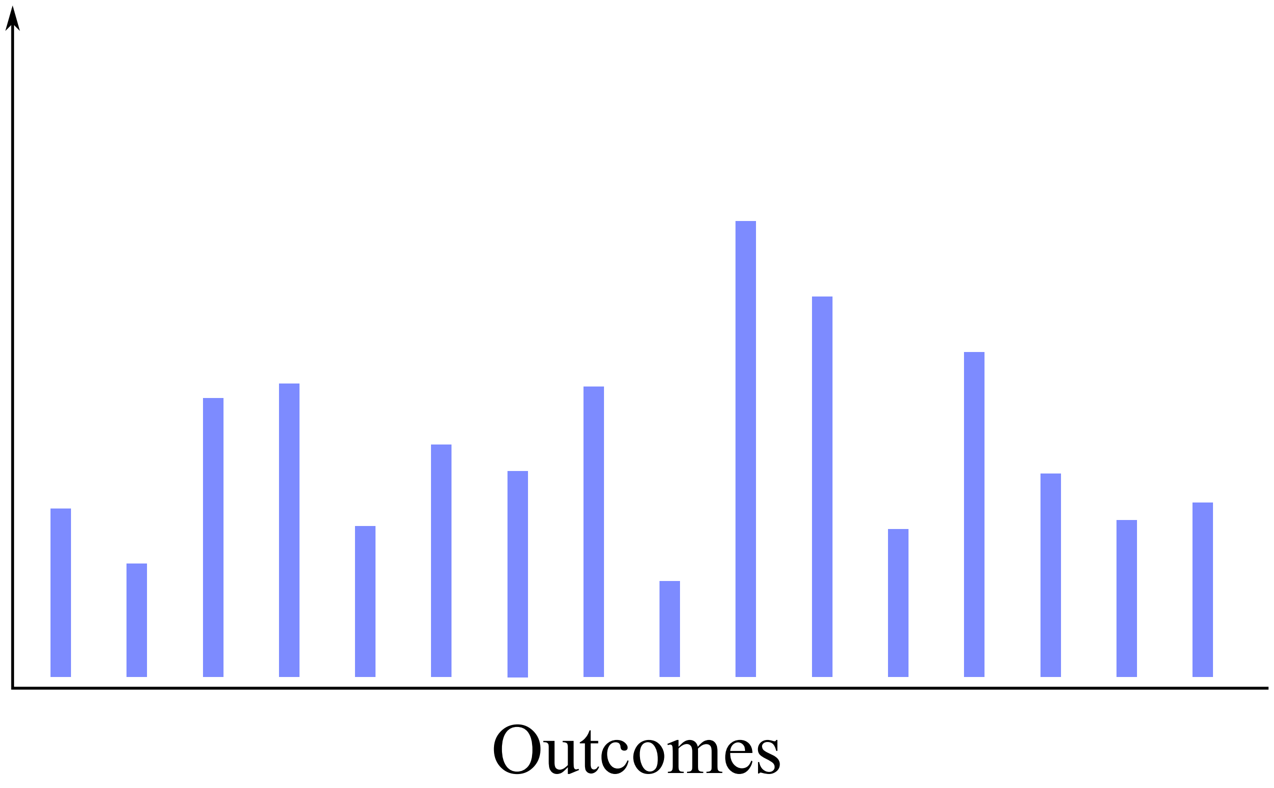
- Setting the bar!



Best classical algorithm sets the bar!

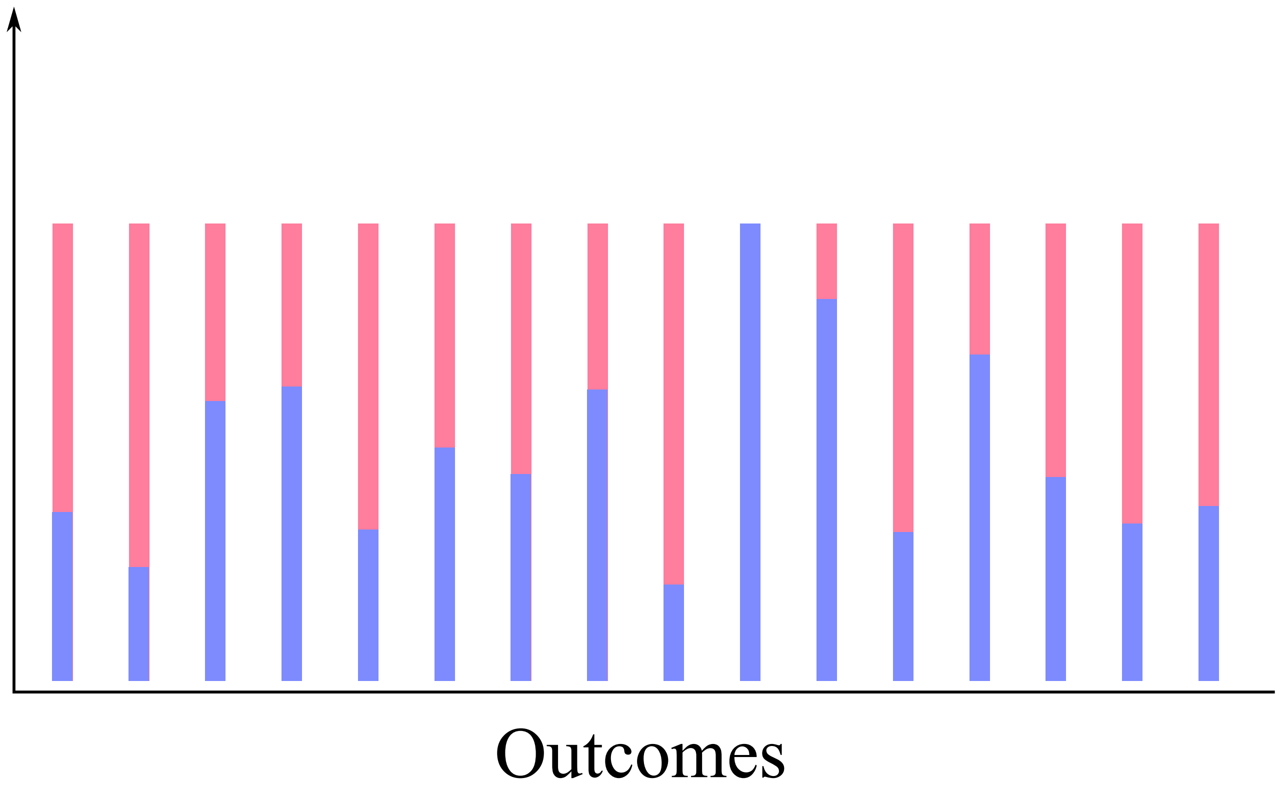
Classical algorithms: BosonSampling

- Common claim: “BosonSampling is ‘doubly’ hard, because Permanent is hard and there are exponentially many of them”.
- 1st counterexample: Rejection sampling



Classical algorithms: BosonSampling

- Common claim: “BosonSampling is ‘doubly’ hard, because Permanent is hard and there are exponentially many of them”.
- 1st counterexample: Rejection sampling



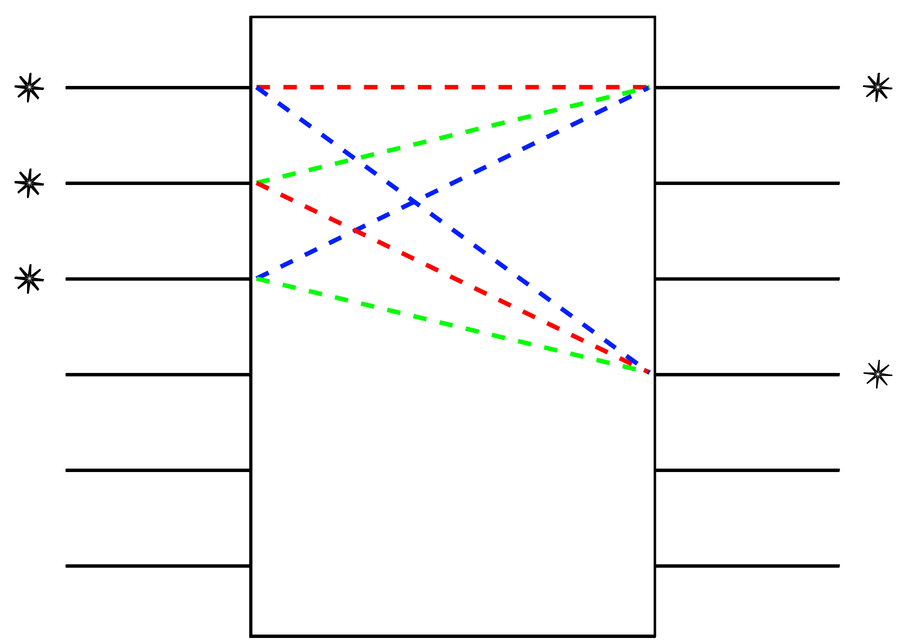
Classical algorithms: BosonSampling

- Common claim: “BosonSampling is ‘doubly’ hard, because Permanent is hard and there are exponentially many of them”.
- 2nd counterexample: Metropolised independence sampling[17];
  - Not **provably** correct;
  - Asymptotic scaling unknown;
  - Produces a sample from:
    - $n = 30$ in 30 mins
    - $n = 50$ in < 10 days (projected)

[17] Neville *et al*, Nat. Phys. **13**, 1153 (2017)

Classical algorithms: BosonSampling

- Common claim: “BosonSampling is ‘doubly’ hard, because Permanent is hard and there are exponentially many of them”.
- 3rd counterexample: Exact algorithm[18];



[18] Clifford and Clifford, Proc. 29th Annual ACM-SIAM SODA p.146 (2018)

Classical algorithms: BosonSampling

- Common claim: “BosonSampling is ‘doubly’ hard, because Permanent is hard and there are exponentially many of them”.
- 3rd counterexample: Exact algorithm
  - Runtime: $\mathrm{O}(mn\ 3^n)$;
    - Compare with $\mathrm{O}(n2^n)$ needed for a **single** permanent.
  - Can be improved to $\mathrm{O}(n2^n + \text{poly}(n,m))$!
    - Comparable with computing ~ 2 permanents!
    - Outputs probabilities as well as samples.

State of the art: BosonSampling

Current BosonSampling experiments:
~ 5 photons in 16 modes
Runtime: ?
Promise: 12 photons soon!

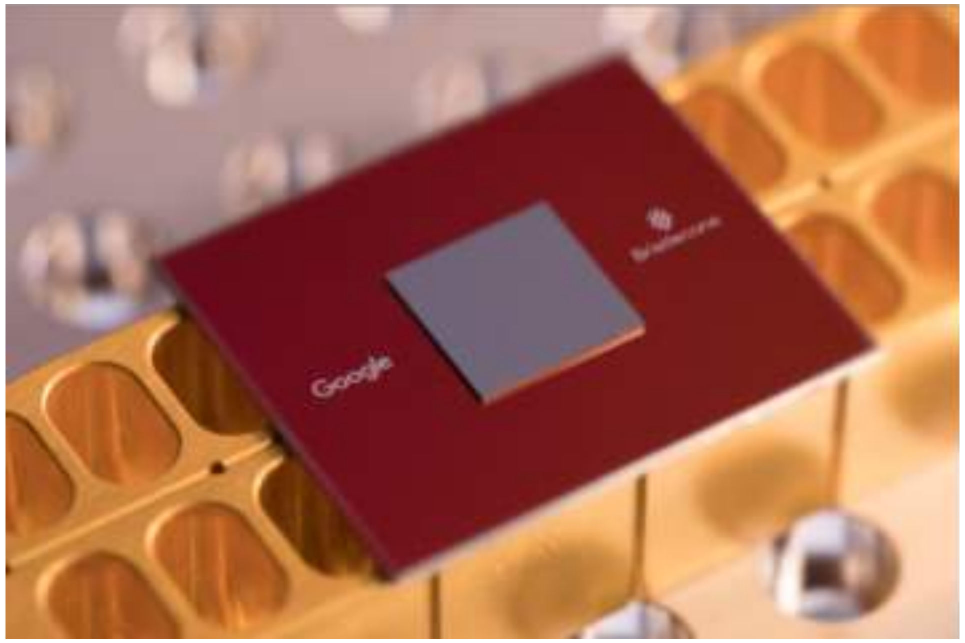
Best classical algorithms:
~ 50 photons in X modes
Runtime: ?
Best theoretical bounds:
90 photons

Outline: Demonstrating a quantum advantage II

- BosonSampling: review and experiments;
- Experimental considerations;
  - Scattershot BosonSampling;
  - BosonSampling with losses;
- State-of-the-art: Simulations vs experiments;
  - BosonSampling;
  - Random quantum circuits;

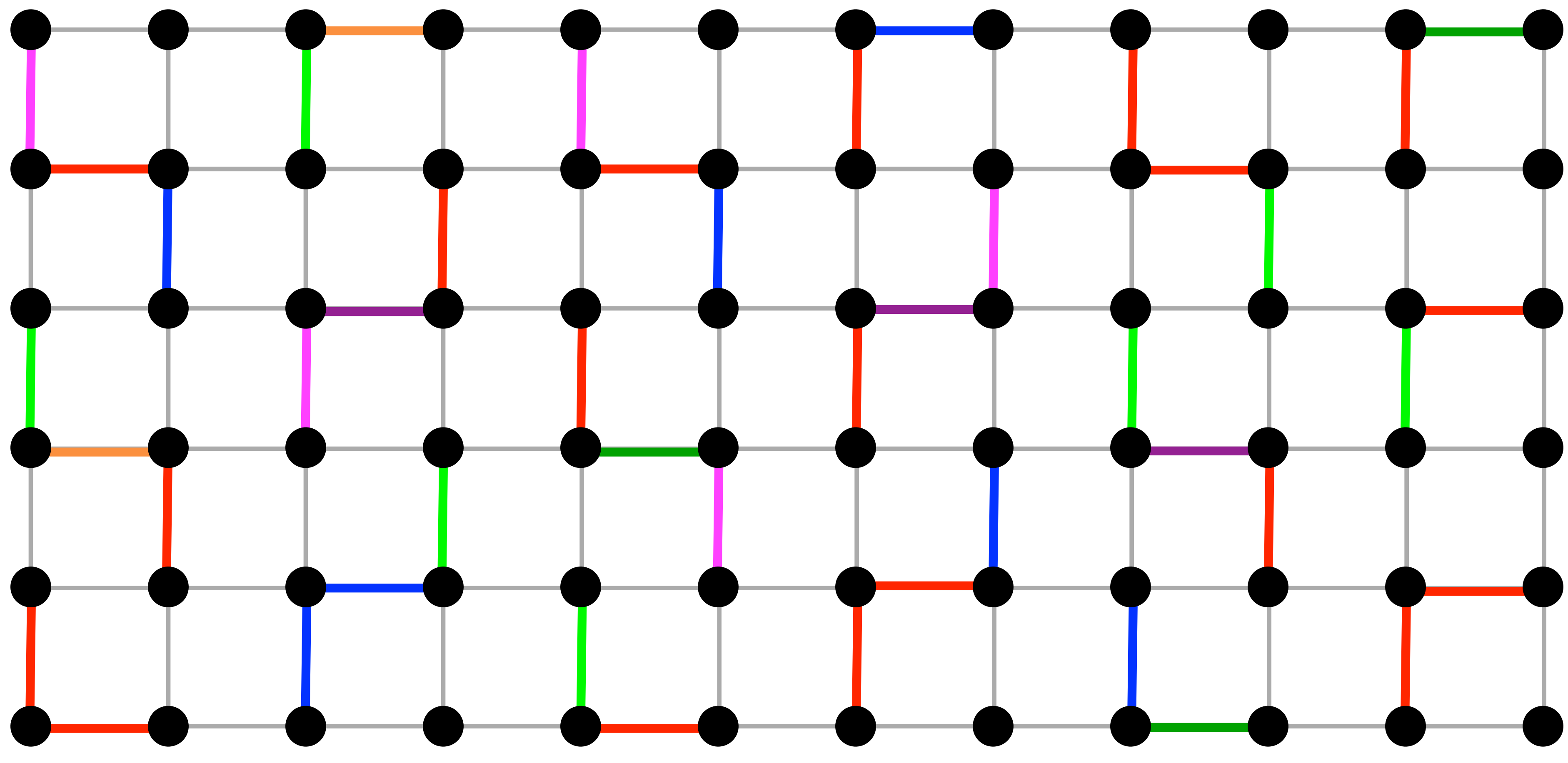
Classical algorithms: Random quantum circuits

- Another heavy-weight contender: Random quantum circuits!

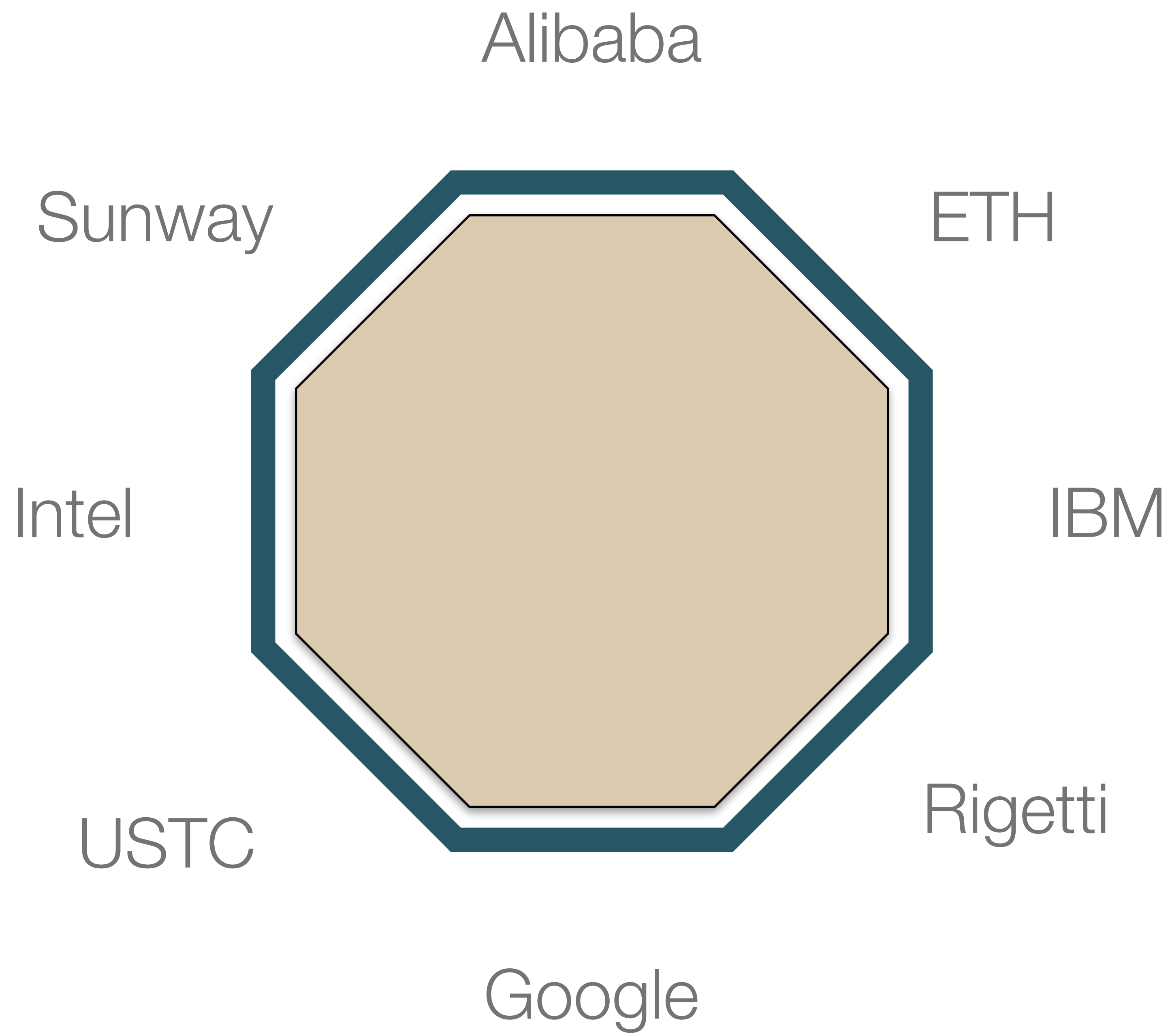


Google's promised 72-qubit processor "Bristlecone"

Classical algorithms: Random quantum circuits



Classical algorithms: Random quantum circuits



Classical algorithms: Random quantum circuits

Reference	General Technique	Qubits	Depth	# of Amplitudes
Intel [6]	Full amplitude-vector update	42	High	All
ETH [5]	Optimized full amplitude-vector update	5×9	25	All
IBM [7]	Tensor-slicing with minimized communication	7×7	27	All
		7×8	23	2^{37} out of 2^{56}
Google [8]	Preprocessing using undirected graphical model	7×8	30	1
USTC [9]	Qubit partition with partial vector update	8×9	22	1
Sunway [10]	Dynamic programming qubit partition	7×7	39	All
		7×7	55	1
Alibaba	Undirected graphical model with parallelization	9×9	40	1

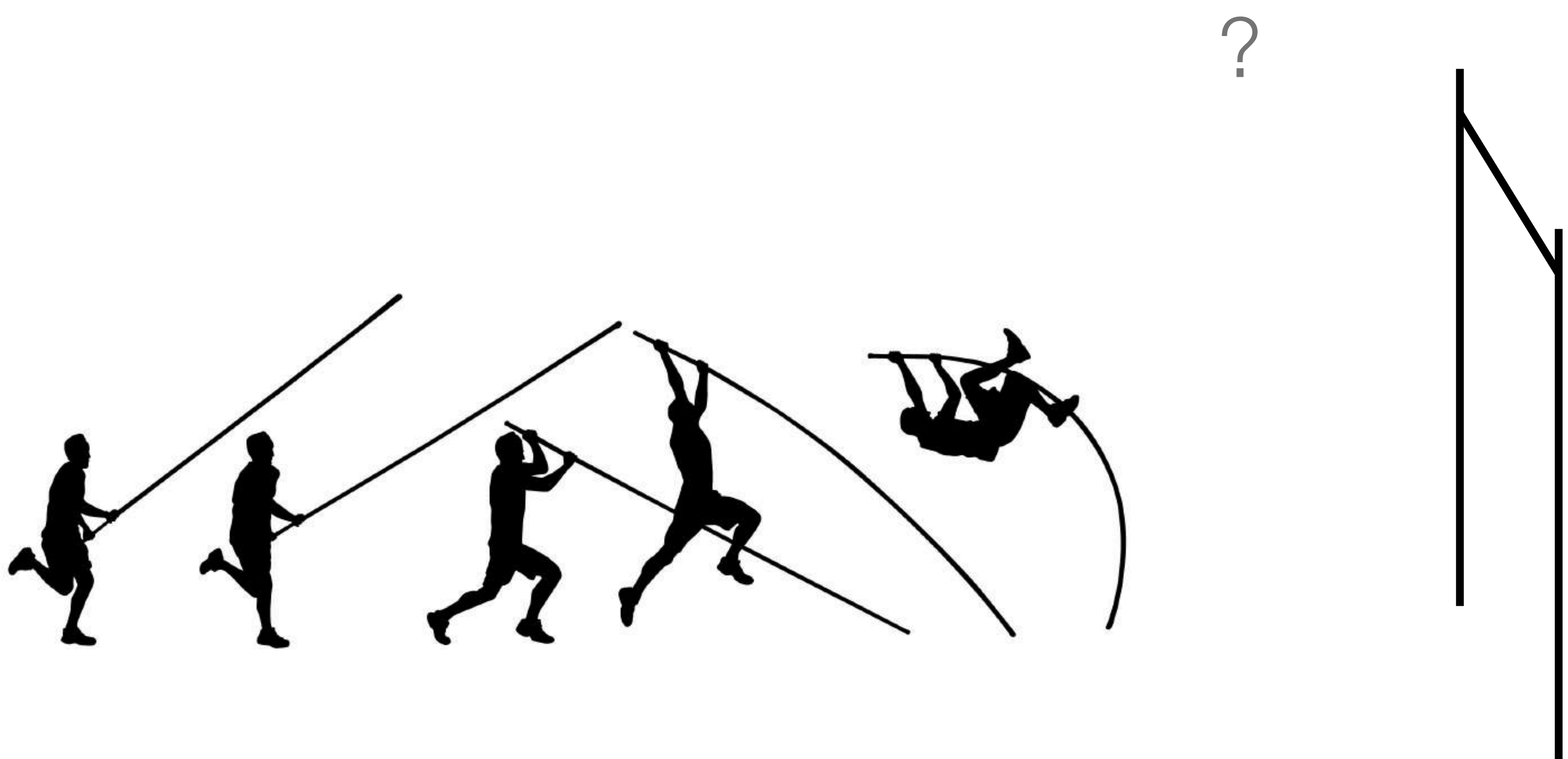
[?] Chen *et al*, arXiv:1805.01450

Classical algorithms: Random quantum circuits

Manufacturer	Name/Codename/Designation	Architecture	Layout	Socket	Fidelity	Qubits	Release date
Google	N/A	Superconducting	N/A	N/A	99.5% [1]	20 qb	2017
Google	N/A	Superconducting	7×7 lattice	N/A	99.7% [1]	49 qb [2]	Q4 2017 (planned)
Google	Bristlecone	Superconducting	6×12 lattice	N/A	99% (readout) 99.9% (1 qubit) 99.4% (2 qubits)	72 qb [3][4]	5 March 2018
IBM	IBM Q Experience 5	Superconducting	N/A	N/A	N/A	5 qb	2016 [1]
IBM	IBM Q Experience 16	Superconducting	2×8 lattice	N/A	N/A	16 qb [5]	17 May 2017
IBM	IBM Q 17	Superconducting	N/A	N/A	N/A	17 qb [5]	17 May 2017
IBM	IBM Q 20	Superconducting	N/A	N/A	N/A	20 qb [6]	10 November 2017
IBM	IBM Q 50 prototype	Superconducting	N/A	N/A	N/A	50 qb[6]	
Intel	17-Qubit Superconducting Test Chip	Superconducting	N/A	40-pin cross gap	N/A	17 qb [7][8]	10 October 2017
Intel	Tangle Lake	Superconducting	N/A	108-pin cross gap	N/A	49 qb [9]	9 January 2018
Rigetti	19Q	Superconducting	N/A	N/A	N/A	19 qb [10]	December 2017

https://en.wikipedia.org/wiki/List_of_quantum_processors (!)

State of the art: Random quantum circuits



Current RQC experiments:
~ 49 qubits
Promise: 72 photons soon!

Best classical algorithms:
~ 81 qubits in depth-40 random circuits

Conclusion slide

- It is unclear what is the best technology for quantum computing.
  - Superconducting qubits seem to be ahead - for now!
  - An actual universal quantum computer will likely be **hybrid**.
- The formalism of “quantum advantage/supremacy” gave us:
  - New tools to **understand** the power of different quantum systems;
  - An intermediate **milestone** for the field!
    - More (experimental/theoretical) **confidence** of the power of quantum devices;
    - Development of **new technologies** in the pursuit of this goal!
  - A flurry of activity - all this is from the last 5-6 years!