Lecture V: validating many-body quantum technologies (II)

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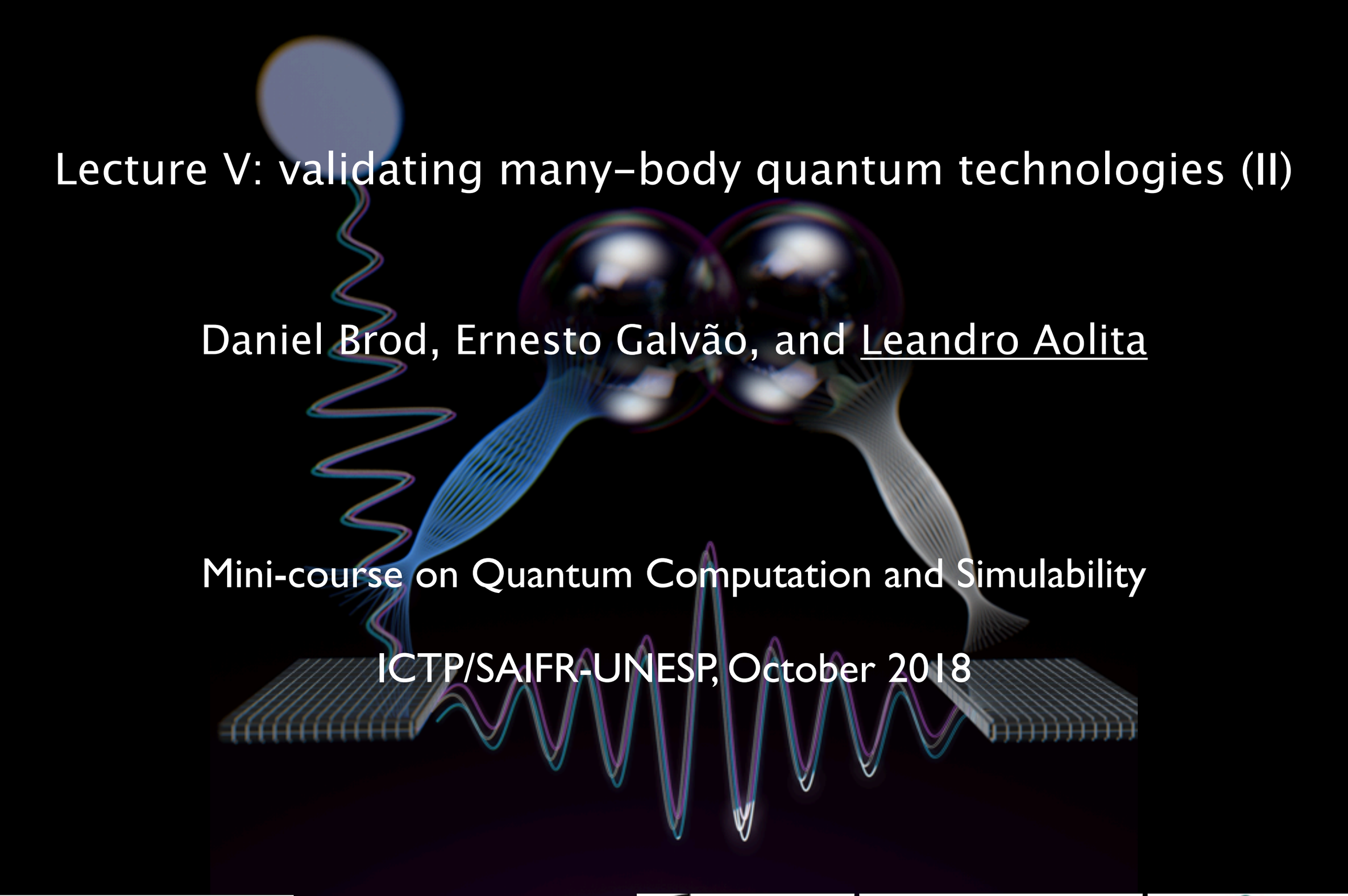

Mini-course on Quantum Computation and Simulability

ICTP/SAIFR-UNESP, October 2018

CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico
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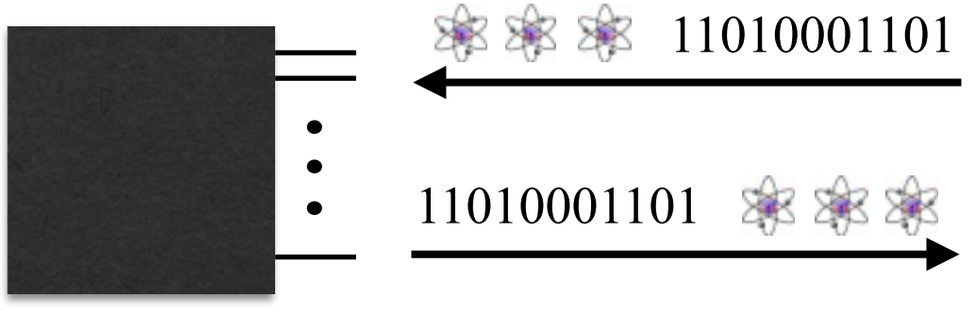
Certification of universal quantum computations

General quantum certification mindset

Untrusted prover Merlin



Skeptic certifier Arthur

- **Interactive tests:** prover and certifier exchange quantum messages until the certifier gets convinced.

Interactive proof (IPs): **(unbounded prover/classical certifier)** S. Goldwasser, S. Micali, and C. Rackoff, In Proceedings of the seventeenth annual ACM symposium on Theory of computing, pages 291–304. ACM New York, NY, USA, 1985.

Quantum interactive proofs (QIP): **(unbounded quantum prover/ BQP certifier)** A. Kitaev, J. Watrous, STOC '00: Proceedings of the thirty-second annual ACM symposium on Theory of computing, ACM, pp. 608 (2000); J. Watrous, Theor. Comput. Sci. 292 (3): 575 (2003).

Quantum-prover interactive proofs (QPIP): **(BQP quantum prover/ almost-classical certifier)** A. M. Childs, D. W. Leung, and M. A. Nielsen, Phys. Rev. A **71**, 032318 (2005)**;** D. Aharonov, M. Ben-Or, and E. Eban, arXiv: 0810.5375; A. Broadbent, J. Fitzsimons, and E. Kashefi, Proceedings of the 50th Annual IEEE Symposium on Foundations of Computer Science (FOCS 2009), 517 (2009); J. Fitzsimons and E. Kashefi, arXiv: 1203.5217.

- **Non-interactive tests:** certifier sends a classical input and prover returns an output that convinces him.

Quantum-prover interactive proofs with a single classical message (QPIP(1)): (BQP quantum prover/ classical* certifier): M. Cramer et al., Nat. Commun. **1**, 149 (2010); G. Toth et al., Phys. Rev. Lett. **105**, 250403 (2010); S. S. T. Flammia and Y.-K. Liu, Phys. Rev. Lett., **106,** 230501 (2011); M. P. da Silva, O. Landon-Cardinal, and D. Poulin, Phys. Rev. Lett., **107**, 210404 (2011); T. Moroder et al., New J. Phys. **14**, 105001 (2012); T. Baumgratz, D. Gross, M. Cramer, and M. B. Plenio, Phys. Rev. Lett. **111**, 020401 (2013); L. Aolita, C. Gogolin, M. Kliesch, and J. Eisert, Nat. Comms. **6**, 8498 (2015); M. Hayashi and T. Morimae, Phys. Rev. Lett., **115,** 220502 (2016).

↓

More practical

Interactive tests

Quantum-prover IPs

- A BQP prover can efficiently convince an almost-classical certifier :-)
- Require fully fledged fault-tolerant universal quantum computers :-(

Two main flavours:

Measurement-based quantum computing

- The certifier sends single-qubit states which include **traps** to test the prover.
- The prover stores the cluster state **blindly** and implements measurements instructed by the certifier.

A. Broadbent, J. Fitzsimons, and E. Kashefi, Proceedings of the 50th Annual IEEE Symposium on Foundations of Computer Science (FOCS 2009), 517 (2009); J. Fitzsimons and E. Kashefi, Phys. Rev. A **96**, 012303 (2017); T. Kapourniotis, E. Kashefi, and A. Datta, arXiv: 1403.1438; M. Hayashi and T. Morimae, Phys. Rev. Lett., **115,** 220502 (2016).

Certifier with a constant-size quantum circuit

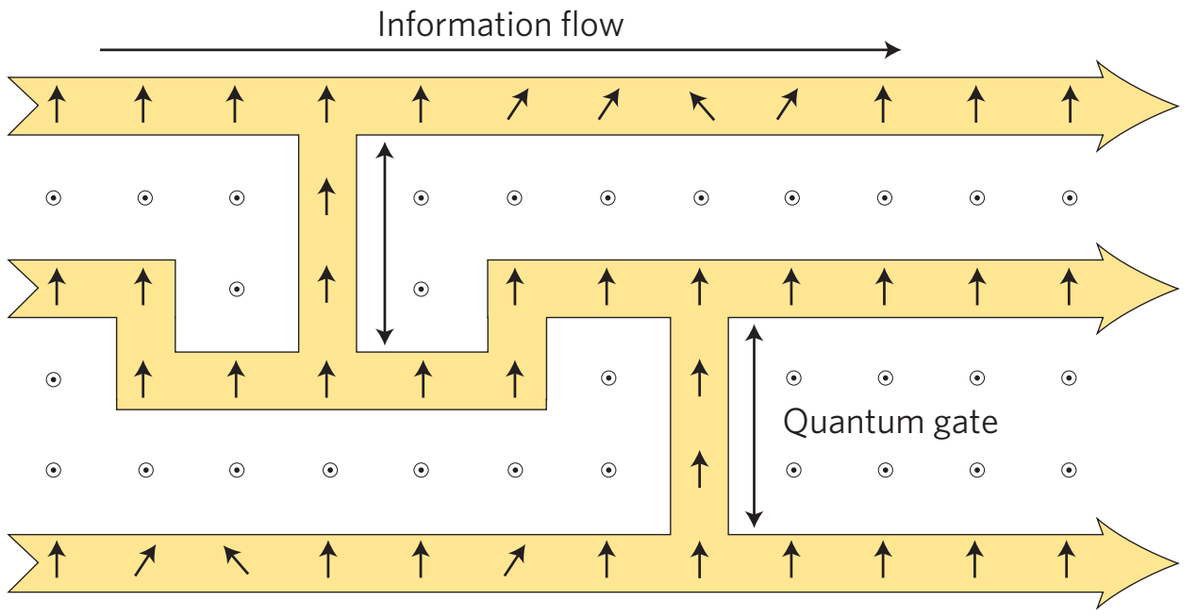
- The certifier sends **authenticated** qubits.
- The prover stores these qubits and either sends them back to the certifier, who **decodes and processes**, or processes himself instructed by the certifier.

D. Aharonov, M. Ben-Or, and E. Eban, arXiv: 0810.5375; D. Aharonov and U. Vazirani, arXiv: 1206.3686.

Measurement-based quantum-prover IPs

Conventional MBQC:

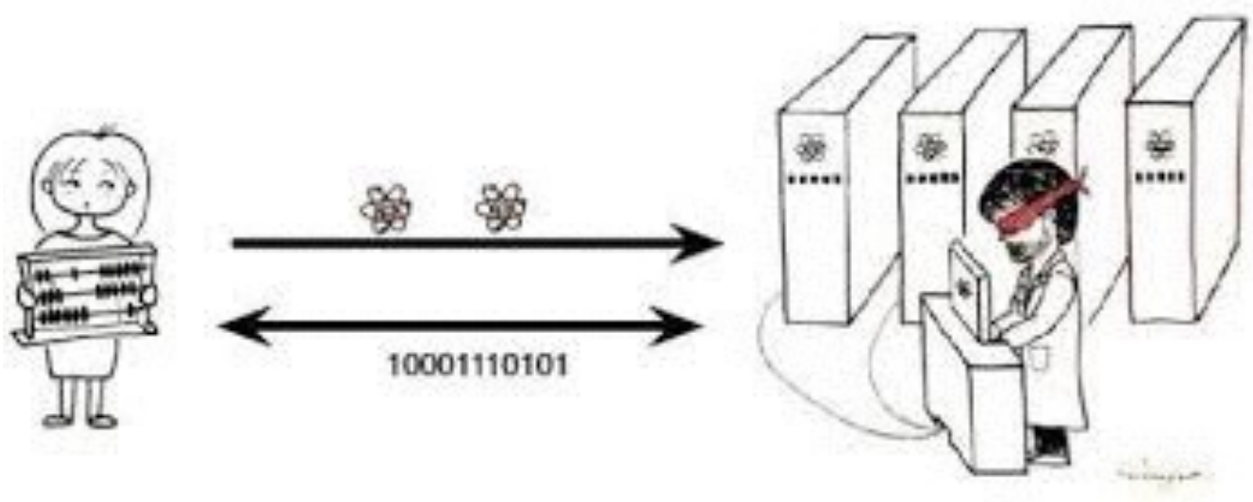
- Information processed by local measurements on a cluster state.
- Randomness compensated by adaptiveness.
- Measurement graph associated to an underlying circuit.



R. Rausendorf and H. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).

Blind MBQC:

- The **measurement graph** is **hidden** to Merlin.
- He performs measurements in directions given by Arthur but is **blind** to the **underlying circuit**.
- The hidden **graph contains traps** for him.



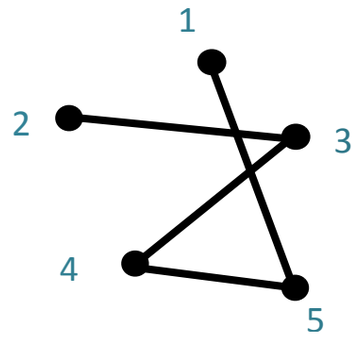
A. Broadbent, J. Fitzsimons, and E. Kashefi, Proceedings of the 50th Annual IEEE Symposium on FOCS (2009), 517 (2009); J. Fitzsimons and E. Kashefi, arXiv: 1203.5217.

Graph states (in a minute)

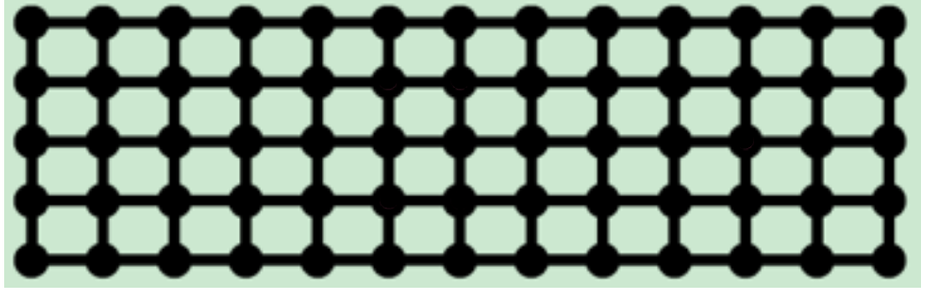
Definition:

Given a set of **vertices connected** by a pattern of **edges**, associate to **each vertex the state** $|+\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and apply a **controlled-*Z* gate** to each pair of **connected vertices**.

$$|G_5\rangle = CZ_{15}CZ_{23}CZ_{34}CZ_{45}|+\rangle_1 \otimes |+\rangle_2 \otimes |+\rangle_3 \otimes |+\rangle_4 \otimes |+\rangle_5$$



The cluster (state):



If a qubit is initialised in $|0\rangle$ *instead of* $|+\rangle$ *it does not get entangled to the graph*

M. Hein, J. Eisert, and H. J. Briegel, Phys. Rev. A **69,** 062311 (2004); M. Hein et al., In Proceedings of the International School of Physics "Enrico Fermi" on Quantum Computers, Algorithms and Chaos (2006).

Blind MBQC

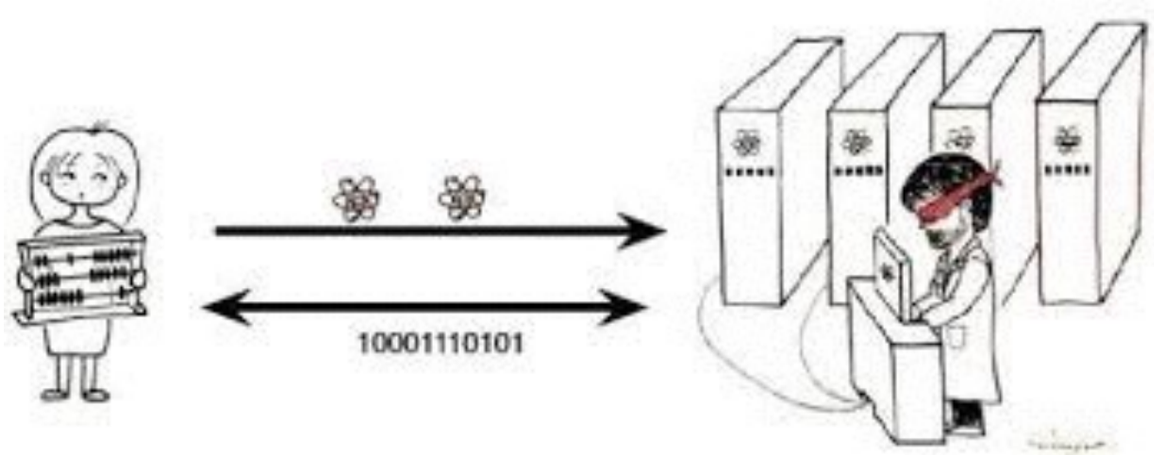
Preliminaries:

1. Depending on the computation, Arthur chooses a graph *G*.
2. For each vertex *v*, he prepares a qubit in:
   - an **input** state $|+_{\theta_v}\rangle$,
   - a **dummy** state $|z_v\rangle$, or
   - a **trap** state $|+_{\varphi_v}\rangle$,

(chosen randomly)

with $z_v \in \{0,1\}$, $|+_{\varphi_v}\rangle := \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\,\varphi_v}|1\rangle\right)$ and $|+_{\theta_v}\rangle := \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\,\theta_v}|1\rangle\right)$.

- The **inputs encode the computation**.
- The **dummies surround the traps**, isolating them.
- The **traps test Merlin**'s procedure. The **traps' positions** in *G* are **random**.



(Arthur) (Merlin)

Blind MBQC

Idea of the protocol:

1. Arthur sends G and the qubits to Merlin.
2. Merlin applies CZs according to G.
3. Merlin measures each qubit v in a basis B_ν given by Arthur and returns him the outcome. (with B_ν and $B_{\nu+1}$ correlated with z_ν, φ_ν, and θ_ν)
4. With the ν-th outcome, Arthur chooses $B_{\nu+1}$.
5. **If all trap** measurements yield the **correct** outcome, Arthur **accepts**. Otherwise, he rejects.

Then, if Arthur accepts,

$$\mathbb{P}_{\text{incorrect}} \leq \epsilon := \left(1 - \frac{N_{\text{T}}}{N}\right)^{\text{d}}.$$

d: constant (maximal weight of correctable Pauli error)

N_{T}: number of trap qubits

N: number of qubits

$(\Rightarrow F \geq 1 - \epsilon)$

- Universal quantum computations efficiently certifiable :-)
- Fault-tolerance and universality required :-(
- Dummy and trap qubits required :-(

***For the moment, way out of reach* :-(**

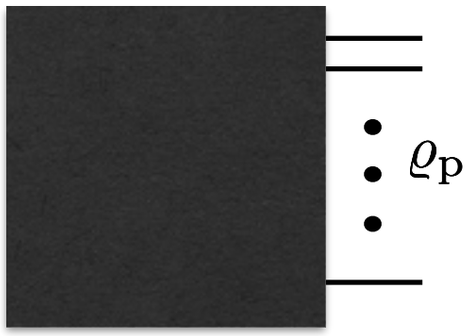
Non-interactive (measurement only!) tests

Non-interactive certification mindset

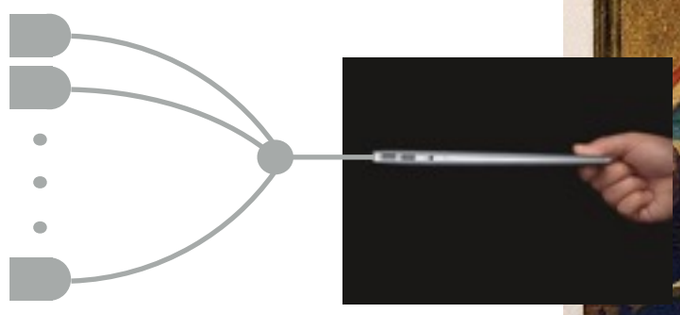
Untrusted quantum prover Merlin

Skeptic almost-classical certifier Arthur

$F(\varrho_{\mathrm{t}}, \varrho_{\mathrm{p}})$?



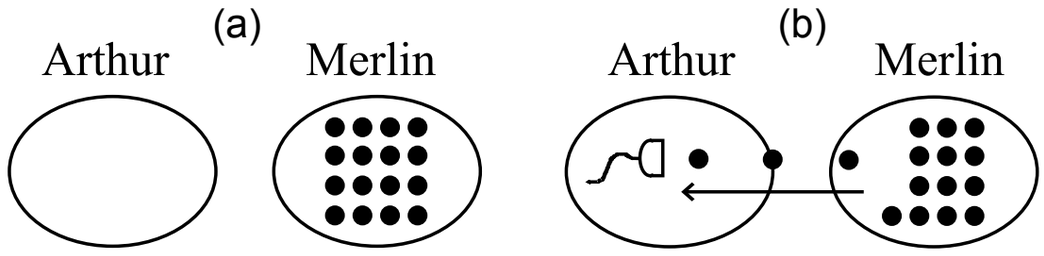
ϱ_{t} and C



- Similar mindset to QPIPs but with a single quantum interaction.
- No restriction on type of quantum noise, preparation totally unknown.
- ~~Only assumption: i.i.d. preparations~~ $\varrho_{\mathrm{p}}^{\otimes C}$.

Verifiable measurement-only blind quantum computing with stabilizer testing

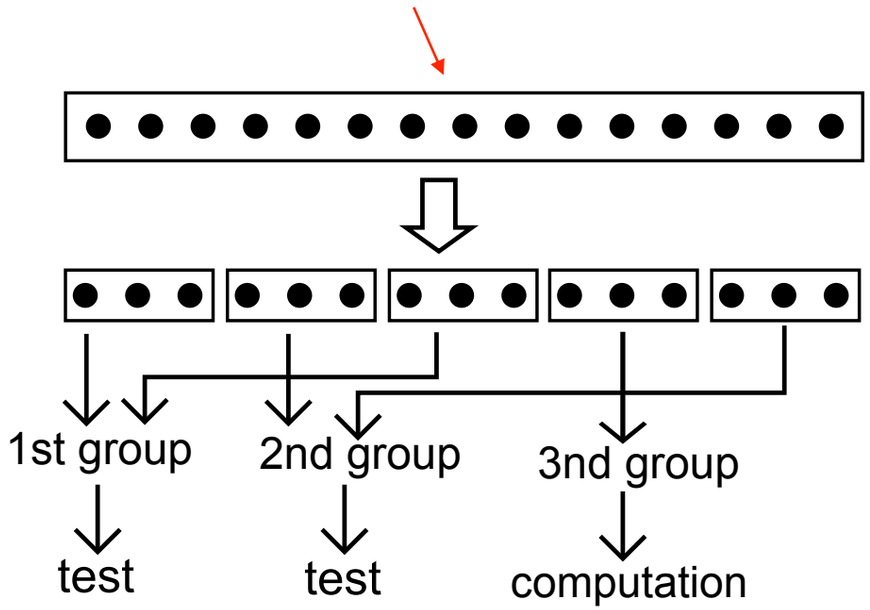
Non-interactive certification mindset:



Idea of the protocol:

1. Merlin prepares ***2k* + 1 copies of the *N*-qubit graph state** $|G\rangle$ and sends them one by one to Arthur.
2. Arthur randomly groups the $2k+1$ copies intro **3 groups**: **2 test groups** (of k N-qubit blocks each) and **1 computation group** (of one N-qubit block).
3. On each **test block**, Arthur runs one of **2 certification tests**. On the **computation block**, he runs the **MBQC**.
4. If **all *2k* tests** are **passed**, Arthur **accepts the outcomes** of the computation block.

$|G\rangle^{\otimes 2k+1}$ (if Merlin is honest)



M. Hayashi and T. Morimae, Phys. Rev. Lett., **115,** 220502 (2016).

The certification test: stabilizer testing (ground-state witnessing!)

Stabilizer operators:

$$X_i|+\rangle^{\otimes N} = |+\rangle^{\otimes N},\ \forall\ 1 \leq i \leq N$$

$$\Rightarrow \bigotimes_{e\in G} CZ_e\, X_i \bigotimes_{e\in G} CZ_e \bigotimes_{e\in G} CZ_e|+\rangle^{\otimes N} = \bigotimes_{e\in G} CZ_e|+\rangle^{\otimes N}$$

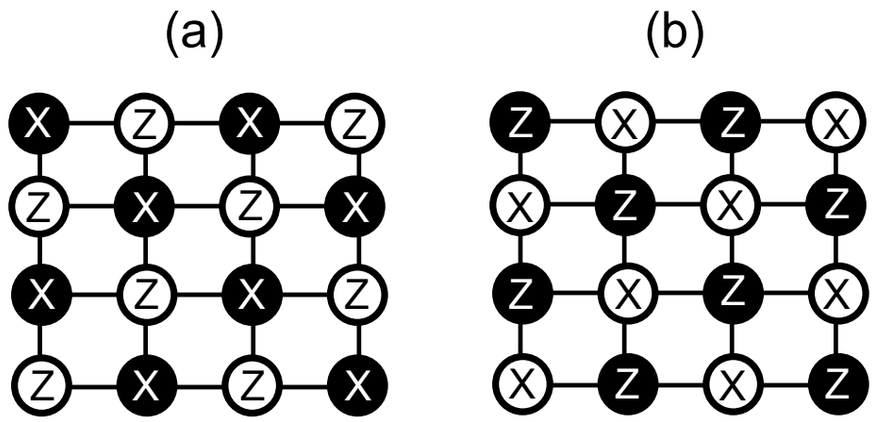
$$X_i \bigotimes_{j\in\mathcal{N}_i} Z_j |G\rangle = |G\rangle$$

(*i*-th generator of the Stabilizer group)

$|G\rangle$ is the unique state stabilized by all N generators of the stabilizer!!!

Stabilizer tests:

- Test 1: On each **black** qubit measure *X*. On each **white** quit measure *Z*. Accept if all outcomes coincide with stabilizer predictions.
- Test 2: On each **white** qubit measure *X*. On each **black** quit measure *Z*. Accept if all outcomes coincide with stabilizer predictions.



Theorem 1 *Assume that* $\alpha > \frac{1}{2k+1}$. *If the test is passed, with significance level* α, *we can guarantee that the resultant state* σ *of the third group satisfies*

$$(F = 1 - \mathbb{P}_{\text{incorrect}}) \longleftarrow \langle G|\sigma|G\rangle \geq 1 - \frac{1}{\alpha(2k+1)}. \quad)$$

Significance level: Maximum passing probability for an incorrect $(2k+1)N$ - qubit preparation

State of the computation block conditioned on all other $2k$ blocks having passed the test

- Universal quantum computations efficiently certifiable :-)
- No dummy or trap qubits required :-)
- Does not assume i.i.d. :-)
- Fault-tolerance and universality required :-(
- Only linear overhead in terms of copies of the state :-)

M. Hayashi and T. Morimae, Phys. Rev. Lett., **115,** 220502 (2016).

Classical verification of universal quantum computers!!! (???)

U. Mahadev, arXiv:1804.01082

QUANTUM COMPUTING

Graduate Student Solves Quantum Verification Problem

56 |

Urmila Mahadev spent eight years in graduate school solving one of the most basic questions in quantum computation: How do you know whether a quantum computer has done anything quantum at all?

Key assumption:

Quantum computers cannot break LWE
(leading candidate for post-quantum cryptography)

Conclusions of Lecture V:

- Interactive proofs
- Measurement only verification
- Classical verification of quantum computers?

Close family at Rio & São Paulo:

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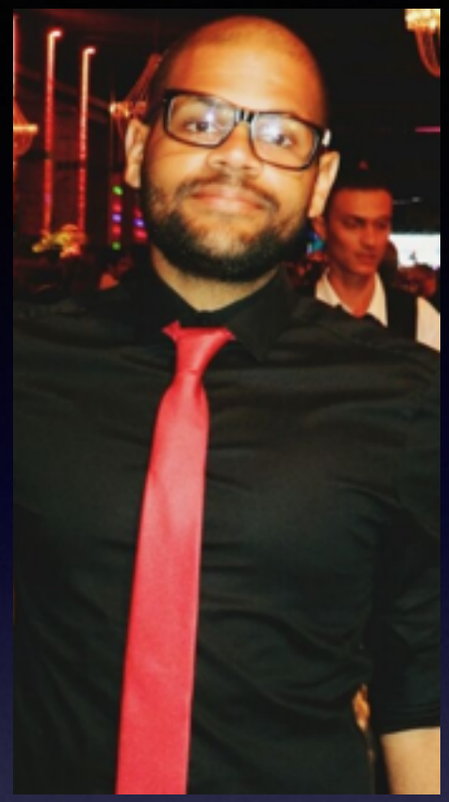
Postdoc

Carolina Gigliotti
PhD student



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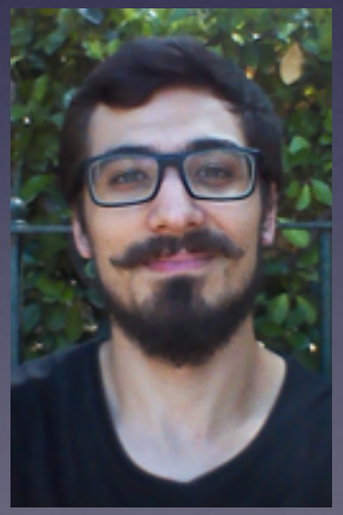

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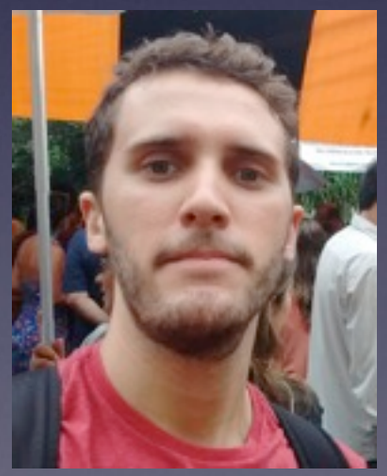

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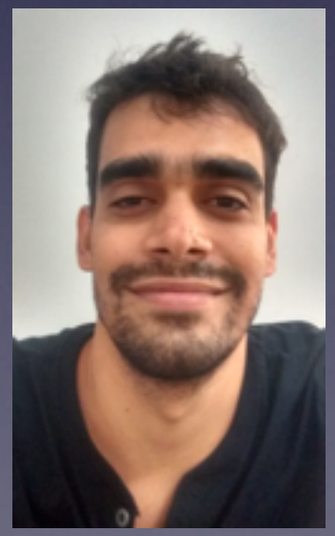

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Thank you for your attention!