Extreme-mass-ratio inspirals, in which a stellar-mass black hole or neutron star spirals into a supermassive black hole, are unique sources of gravitational waves. The small object executes hundreds of thousands of intricate orbits before plunging into the black hole, mapping the black hole's spacetime geometry with unparalleled precision. To model these systems, researchers use a combination of methods collectively called gravitational self-force theory. In these lectures I survey some of those methods, particularly focusing on the basics of black hole perturbation theory and matched asymptotic expansions.