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Outline

- Brief Review of Modified Gravity Theories
- Moffat’s Modified Gravity Theory and the rotation curve of the Milky Way
- Data Sets and Morphologies
- Results
- Conclusions
Review of Modified Gravity Theories

MOdified Newtonian Dynamics (MOND, M. Milgrom 1983)
The acceleration of a point particle:

\[ a = \frac{MG}{R^2} \nu \left( q, \frac{MG}{R^2 a_0} \right). \]

\( q \) is an adimensional parameter depending on the orbit’s shape and \( a_0 \) is a free parameter. For \( a \gg a_0 \) the theory behaves as Newtonian Mechanics, while for \( a \leq a_0 \) the theory becomes scale invariant and \( a \simeq \eta(q) \left( \frac{MGa_0}{R} \right)^{1/2} \).

Tensor-Vector-Scalar theory of gravity (TeVeS, J.D. Bekenstein 2004)

\[ S = S_{\tilde{g}} + S_A + S_\phi + S_m \]

Gravity is described by a metric \( g_{\mu \nu} \) as in RG plus a vector field \( A_\mu \) and a scalar field \( \phi \).
Problems for MOND and TEVES

- **MOND**
  - Different values of $a_0$ are needed to explain rotation curves of galaxies (Randriamampandry & Carignan MNRAS 439, 2132 (2014)).
  - The theory can not explain the gravitational lensing effect (Clowe et al ApJL, 648, L109 (2006)).
  - The theory is not able to explain the observed matter power spectrum (Dodelson IJMP D 20, 2749 (2011)).

- **TeVeS**
  - The theory is not able to explain the observed matter power spectrum (Dodelson IJMP D 20, 2749 (2011)).
  - It is not possible to reconcile gas profile and strong–lensing measurements in well known cluster systems (Nieuwenhuizen et al MNRAS 476, 3393 (2018)).

- **Open Discussion**
  - According to Clowe et al 2006, TeVeS and MOG have difficulties to explain the bullet cluster, while Brownstein & Moffat 2007 claim the opposite.
Moffat’s MOG

The MOdified Gravity theory (also named as Scalar-Tensor-Vector Gravity) is a covariant modification of General Relativity.

- MOG was proposed by J. Moffat in 2006.
- Two scalar fields and one vector field are added to RG.
- It has been used to describe observations of the Solar System (Moffat IJMP D16, 2075(2008)) and rotation curves of spiral galaxies (Moffat & Rahvar MNRAS 436, 1439 (2013)), without the need of dark matter.
- There are claims that MOG can fit both Bullet and the Train Wreck merging clusters (Brownstein & Moffat, MNRAS 382, 29 (2007); Israel & Moffat Galaxies 6, 41 (2018)).
- There are also articles that apply the MOG to gravitational waves, black holes, binary pulsars, lensing, globular clusters, motion of satellite galaxies, gravitational stability of galactic disks, N-body simulations, galactic sun’s motion. cosmological data
MOG action

\[ S_{\text{MOG}} = S_G + S_\phi + S_S + S_M. \]

\[ S_G = -\frac{1}{16\pi} \int \frac{1}{G} (R + 2\Lambda) \sqrt{-g} d^4x, \]

\[ S_\phi = -\frac{1}{4\pi} \int \omega \left[ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{2} \mu^2 \phi_\mu \phi^\mu + V_\phi(\phi_\mu \phi^\mu) \right] \sqrt{-g} d^4x, \]

\[ S_S = -\int \frac{1}{G} \left[ \frac{1}{2} g^{\alpha\beta} \left( \frac{\nabla_\alpha G \nabla_\beta G}{G^2} + \frac{\nabla_\alpha \mu \nabla_\beta \mu}{\mu^2} \right) + \frac{V_G(G)}{G^2} + \frac{V_\mu(\mu)}{\mu^2} \right] \sqrt{-g} d^4x. \]

where \( B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu \), \( \omega \) is an adimensional coupling constants, \( G \) represents the gravitational coupling strength, and \( \mu \) is the mass of the vector field \( \phi \), \( V_i \) are the self interaction potentials associated with each of the fields. For simplicity: \( V_\phi(\phi_\mu \phi^\mu) = V(G) = V(\mu) = 0. \)
The MOG weak field approximation

The equations for $\phi_\mu, G, \mu$ and the metric are solved considering perturbations around a Minkovskey space-time

\[
\begin{align*}
g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}. \\
\phi_\mu &= \phi_\mu(0) + \phi_\mu(1), \\
G &= G(0) + G(1) \\
\mu &= \mu(0) + \mu(1)
\end{align*}
\]

In a Minkovskey space-time $\phi_\mu(0) = 0$ and $G(0) = \text{constant}$. Besides, $\mu = \text{constant}$ is fixed. The energy-momentum tensor is expressed as:

\[
T_{\mu\nu} = T_{\mu\nu}(0) + T_{\mu\nu}(1).
\]
The MOG weak field approximation

After a lot of algebra the following equation is obtained:

\[ \nabla \left( \nabla \Phi_{\text{eff}} - \kappa \omega \nabla \phi^0 \right) = 4\pi G_0 \rho. \]

\[ \nabla \Phi_N \]

\[ \Phi_{\text{eff}}(\vec{x}) = -G_N \int \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} (1 + \alpha - \alpha e^{-\mu|\vec{x} - \vec{x}'|}) d^3x' \]

where \( G = G_N(1 + \alpha) \).

- For scales lower than \( \mu^{-1} \), the repulsive force cancels a part of the attractive force and newtonian gravity is recovered.
- For scales larger than \( \mu^{-1} \), the repulsive force becomes weaker and a newtonian force with a larger gravitational constant is obtained.
Estimates can be obtained of $\alpha$ and $\mu$ from the solutions for spherical symmetry (Moffat & Toth, Class. Quant. Grav. 26, 085002 (2009)).

$$\alpha = \frac{M}{(\sqrt{M} + E)^2} \left( \frac{G_\infty}{G_N} - 1 \right)$$

$$\mu = \frac{D}{\sqrt{M}}$$

$$G_\infty \simeq 20G_N$$

$$r_0 = \frac{1}{\mu}$$

$$M_0 = \alpha^2 M$$

In all versions of the theory $\alpha$ and $\mu$ are taken as constants.
The Milky Way is a complex system formed by stars, gas and dark matter gravitationally bound together.

The galaxy has three main baryonic components: disk, bulge and gas.
Observationally inferred morphologies

The gravitational potential of our galaxy receives contributions from baryons, and presumably from dark matter, separately

\[ \phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disk}} + \phi_{\text{gas}} + \phi_? \].
The Galkin compilation comprises the velocity measurements of 2701 objects at $R > 2.5$ kpc
- 2095 → Gas (HI, HII, C0, giant molecular clouds)
- 506 → Stars (open clusters, planetary nebulae, cepheids, carbon stars)
- 100 → masers (molecular clouds, comets, planetary and stellar atmospheres)

The Huang compilation comprises 43 data obtained from a binning of:
- 16000 → red clump giants selected from LSS-GAC y SDSSIII/APOGEE surveys
- 5700 → Halo K stars selected from the SDSS/SEGUE survey
**Data Sets**

**Huang compilation:**
- 43 data
- $r=[4.59,98.97]$ kpc

**Galkin compilation:**
- 2701 objects
- $r=[2.5,24.81]$ kpc
Photometric data are used to trace each **barionic component**.

This technique has been used to show that an extra component is needed to explain the observed rotation curve of the MW (Iocco et al., Nature Physics, 2015).
Parameters $\alpha$ and $\mu$

\[
\Phi_{\text{MOG}}(\vec{x}) = -G_N \int \frac{\rho_b(\vec{x}') + \rho_d(\vec{x}') + \rho_g(\vec{x}')}{|\vec{x} - \vec{x}'|} \left(1 + \alpha - \alpha e^{-\mu |\vec{x} - \vec{x}'|}\right) d^3x'
\]

- $(\alpha, \mu)^{SG} = (8.89, 4.2 \times 10^{-2})$, best fit obtained by Moffat & Rahvar MNRAS 436, 1439 (2013) to fit spiral galaxies;

- $(\alpha, \mu)^{MW} = (15.01, 3.13 \times 10^{-2})$, obtained by Moffat considering $M_{\text{MW}}^{\text{Moffat}} = 4 \times 10^{10} M_\odot$;

- $(\alpha, \mu)^C$, considering the mass of the Milky Way we obtain for each one of our observationally inferred morphologies.

\[
\alpha = \frac{M}{(\sqrt{M} + E)^2} \left(\frac{G_\infty}{G_N} - 1\right) \quad \mu = \frac{D}{\sqrt{M}}
\]
<table>
<thead>
<tr>
<th>baryonic morphology</th>
<th>Newton $\tilde{\chi}^2$</th>
<th>MW $\tilde{\chi}^2$</th>
<th>SG $\tilde{\chi}^2$</th>
<th>C $\tilde{\chi}^2$</th>
<th>$(\alpha, \mu)^C$</th>
<th>$M_{\text{C}}^{\text{MW}} \times 10^{10} , M_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[disk] [bulge]</td>
<td>Huang -- galkin</td>
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<tr>
<td>1 [44][40] G2</td>
<td>31.83 -- 10.69</td>
<td>4.50 -- 4.25</td>
<td>4.68 -- 4.25</td>
<td>8.59 -- 5.96</td>
<td>(15.79, 2.43 $\times 10^{-2}$)</td>
<td>6.6$^{+0.6}_{-0.4}$</td>
</tr>
<tr>
<td>2 [44][40] E2</td>
<td>30.80 -- 9.89</td>
<td>4.11 -- 3.83</td>
<td>4.25 -- 3.83</td>
<td>8.00 -- 5.39</td>
<td>(15.80, 2.41 $\times 10^{-2}$)</td>
<td>6.7$^{+0.7}_{-0.6}$</td>
</tr>
<tr>
<td>3 [44][45]</td>
<td>32.90 -- 8.51</td>
<td>3.36 -- 3.10</td>
<td>3.43 -- 3.10</td>
<td>6.85 -- 4.37</td>
<td>(15.83, 2.39 $\times 10^{-2}$)</td>
<td>6.8$^{+0.7}_{-0.6}$</td>
</tr>
<tr>
<td>4 [44][46]</td>
<td>29.85 -- 9.45</td>
<td>3.71 -- 3.51</td>
<td>3.79 -- 3.51</td>
<td>7.47 -- 5.03</td>
<td>(15.83, 2.39 $\times 10^{-2}$)</td>
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</tr>
<tr>
<td>5 [44][47]</td>
<td>35.73 -- 11.40</td>
<td>4.93 -- 4.66</td>
<td>5.16 -- 4.66</td>
<td>9.21 -- 6.51</td>
<td>(15.77, 2.44 $\times 10^{-2}$)</td>
<td>6.6 $\pm 0.6$</td>
</tr>
<tr>
<td>6 [44][48]</td>
<td>28.67 -- 13.65</td>
<td>6.17 -- 6.00</td>
<td>6.48 -- 6.00</td>
<td>13.00 -- 8.43</td>
<td>(15.74, 2.47 $\times 10^{-2}$)</td>
<td>6.4$^{+0.6}_{-0.5}$</td>
</tr>
<tr>
<td>7 [39][40] G2</td>
<td>33.84 -- 12.69</td>
<td>5.51 -- 5.45</td>
<td>5.74 -- 5.44</td>
<td>9.86 -- 7.37</td>
<td>(15.79, 2.42 $\times 10^{-2}$)</td>
<td>6.6$^{+0.6}_{-0.4}$</td>
</tr>
<tr>
<td>8 [39][40] E2</td>
<td>32.65 -- 11.72</td>
<td>5.02 -- 4.90</td>
<td>5.20 -- 4.90</td>
<td>9.14 -- 6.65</td>
<td>(15.80, 2.41 $\times 10^{-2}$)</td>
<td>6.7$^{+0.7}_{-0.6}$</td>
</tr>
<tr>
<td>9 [39][45]</td>
<td>30.19 -- 10.04</td>
<td>4.06 -- 3.93</td>
<td>4.17 -- 3.93</td>
<td>7.72 -- 5.23</td>
<td>(15.84, 2.38 $\times 10^{-2}$)</td>
<td>6.9$^{+0.7}_{-0.6}$</td>
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<tr>
<td>10 [39][46]</td>
<td>31.62 -- 11.22</td>
<td>4.54 -- 4.50</td>
<td>4.66 -- 4.50</td>
<td>8.53 -- 6.22</td>
<td>(15.83, 2.39 $\times 10^{-2}$)</td>
<td>6.9$^{+0.7}_{-0.6}$</td>
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<tr>
<td>11 [39][47]</td>
<td>35.10 -- 13.56</td>
<td>6.06 -- 5.98</td>
<td>6.33 -- 5.97</td>
<td>10.64 -- 8.10</td>
<td>(15.77, 2.44 $\times 10^{-2}$)</td>
<td>6.6 $\pm 0.6$</td>
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<tr>
<td>12 [39][48]</td>
<td>38.46 -- 16.32</td>
<td>7.66 -- 7.74</td>
<td>8.03 -- 7.74</td>
<td>15.79 -- 10.60</td>
<td>(15.73, 2.47 $\times 10^{-2}$)</td>
<td>6.4$^{+0.6}_{-0.5}$</td>
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<tr>
<td>13 [49][40] G2</td>
<td>33.70 -- 12.39</td>
<td>5.43 -- 5.29</td>
<td>5.66 -- 5.28</td>
<td>9.80 -- 7.17</td>
<td>(15.79, 2.42 $\times 10^{-2}$)</td>
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<tr>
<td>14 [49][40] E2</td>
<td>32.54 -- 11.45</td>
<td>4.94 -- 4.76</td>
<td>5.15 -- 4.76</td>
<td>9.09 -- 6.47</td>
<td>(15.81, 2.41 $\times 10^{-2}$)</td>
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<tr>
<td>15 [49][45]</td>
<td>30.14 -- 9.82</td>
<td>4.02 -- 3.83</td>
<td>4.14 -- 3.83</td>
<td>7.71 -- 5.11</td>
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<td>16 [49][46]</td>
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<td>8.49 -- 6.06</td>
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$\tilde{\chi}^2_{5\sigma} = 2.41$ for Huang, $\tilde{\chi}^2_{5\sigma} = 1.14$ for galkin.
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<th>$\tilde{\chi}^2$ Test</th>
<th>Best fit parameters</th>
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</table>
Best fit values for $\alpha$ y $\mu$

**Representative Morphology**

- $\alpha \in [14.44, 16.43]$, $\mu \in [2.29, 2.68] \times 10^{-2}$ kpc$^{-1}$
- $(\alpha, \mu)_{BF} = (16.4, 2.68 \times 10^{-2})$ with $\tilde{\chi}^2_{Huang} = 8.60$

**Best Fit Morphology**

- $\alpha \in [14.67, 16.59]$, $\mu \in [2.13, 2.52] \times 10^{-2}$ kpc$^{-1}$
- $(\alpha, \mu)_{BF} = (16.59, 2.52 \times 10^{-2})$ with $\tilde{\chi}^2_{Huang} = 2.78$

\[\alpha = \frac{M}{(\sqrt{M} + E)^2 \left( \frac{G_{\infty}}{G_N} - 1 \right)} \quad \mu = \frac{D}{\sqrt{M}}\]
Representative morphology
**Best Fit morphology**

![Graph showing best fit parameters for different models and data sets.](image-url)
Summary and Conclusions

- We have performed a test of Moffat’s MOG theory in the Milky Way using two recent compilations of data for the observed Rotation Curve, and adopting a complete set of observationally inferred morphologies for the stellar and gas components.

- We have also modified the key parameters of the theory, in order to match them to the baryonic mass of the Milky Way of each of our baryonic models.

- We conclude that modifying the gravitational potential according to the MOG theory, does not explain the discrepancy between the observed rotation curve, and that generated by the baryons only, in the Milky Way.

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Best Fit morphology with only 12 data sets from galkin