JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

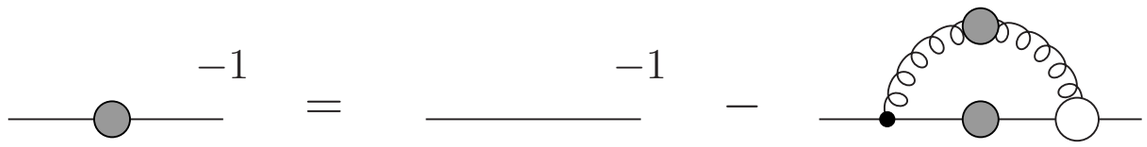
ICTP-SAIFR, Sao Paulo, Brazil

Mass generation in hadrons and QCD phase diagram from DSEs

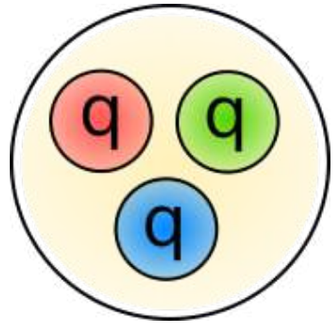
work together with Gernot Eichmann, Walter Heupel, Jan Lücker, Helios Sanchis-Alepuz, Christian Welzbacher, Richard Williams

Overview

1. Mass from nothing



2. Ordinary and exotic hadrons



Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

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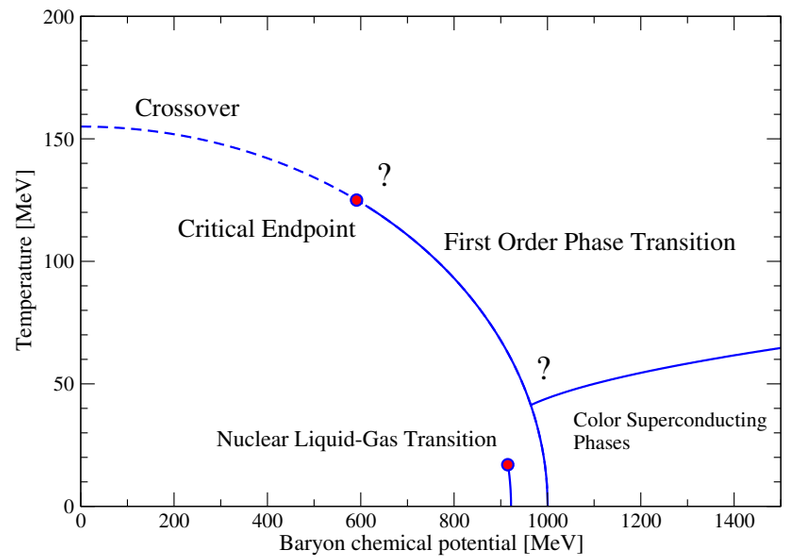
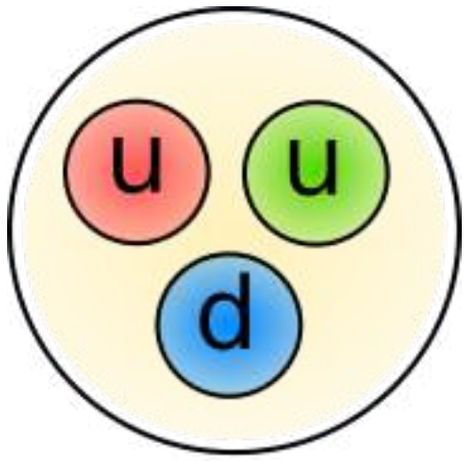


Properties of QCD: Dynamical mass generation



m_{proton} = 938 MeV

Dynamical quark masses via weak force

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000

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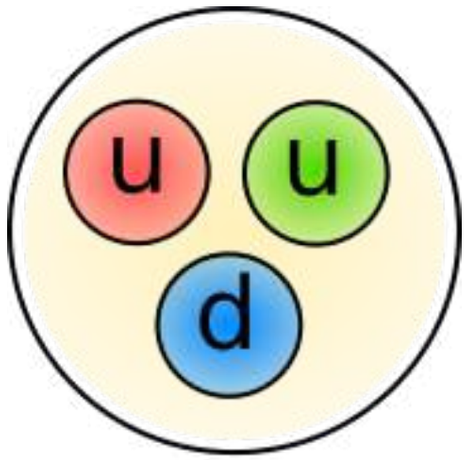

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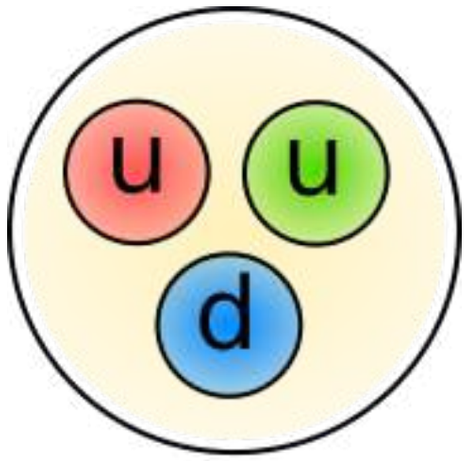

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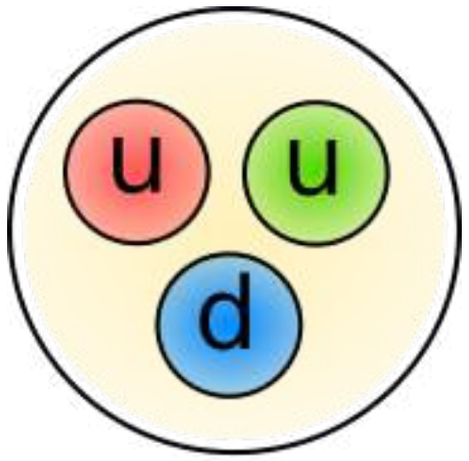

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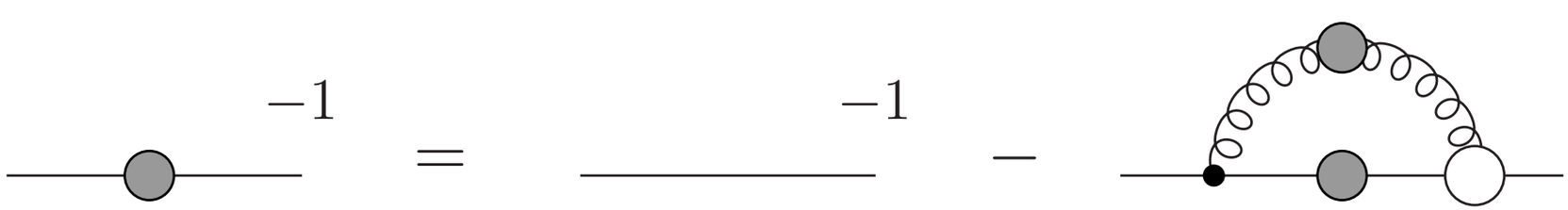
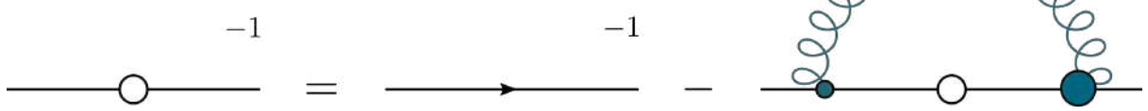


Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
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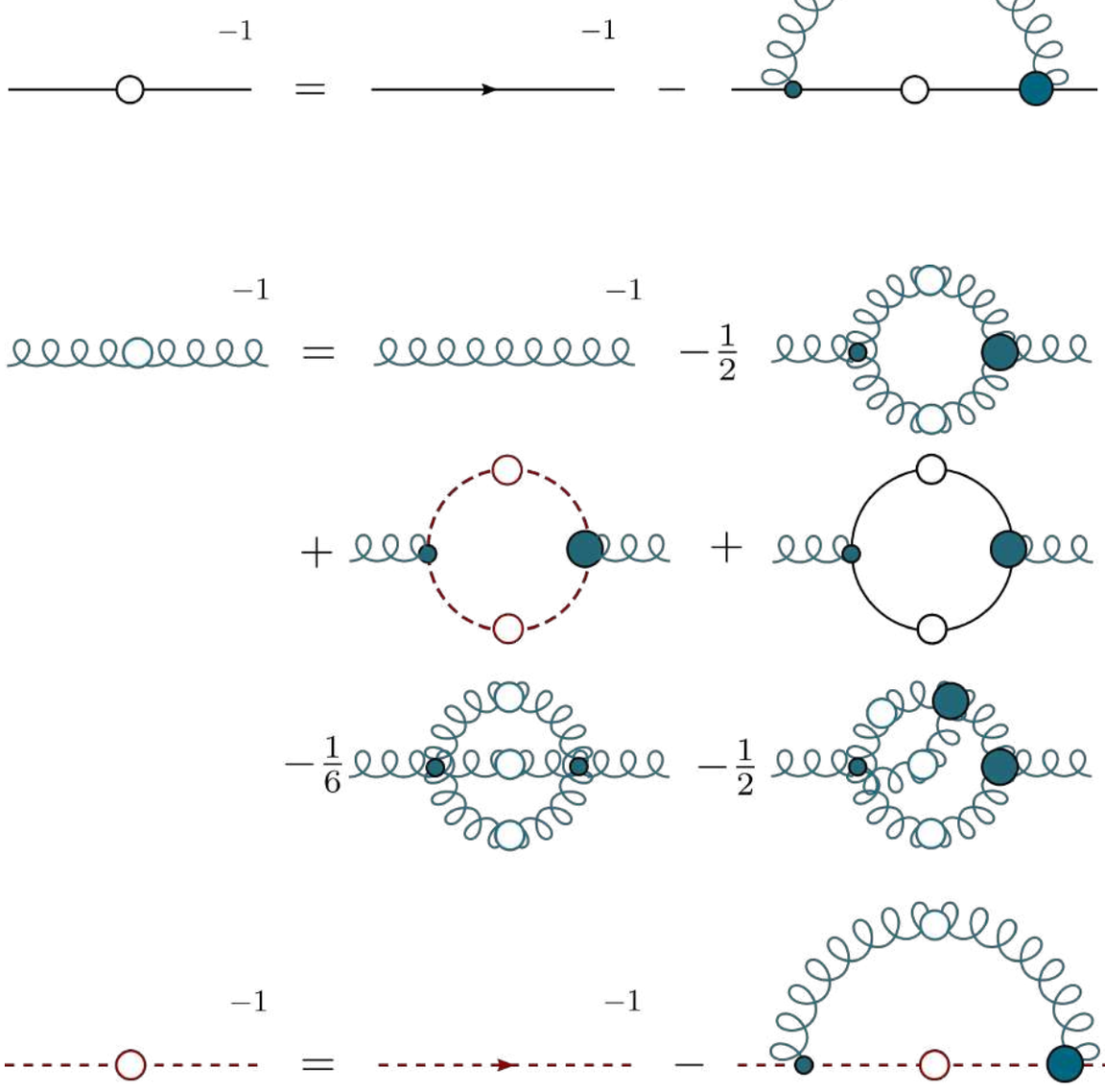

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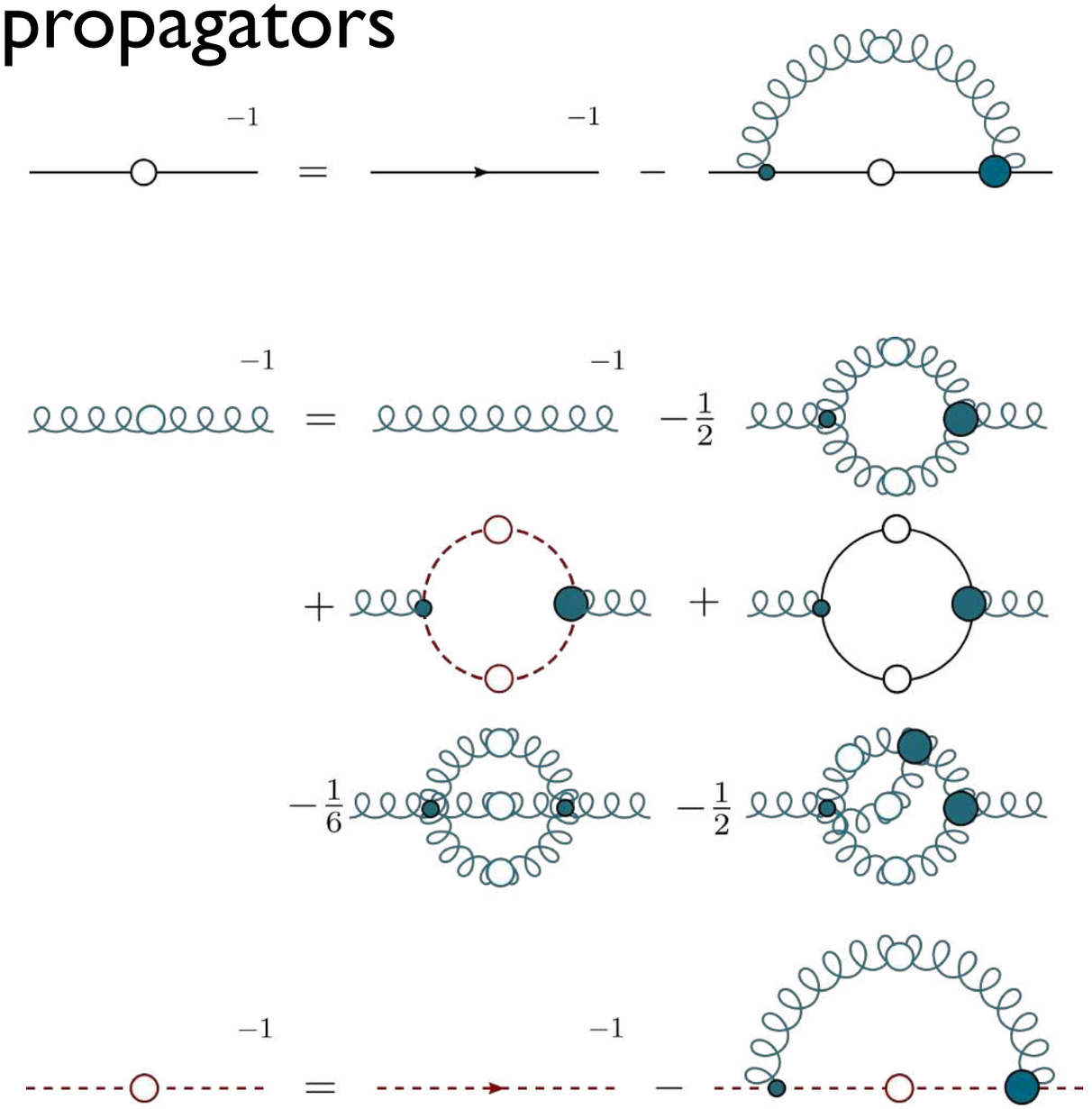

vertices

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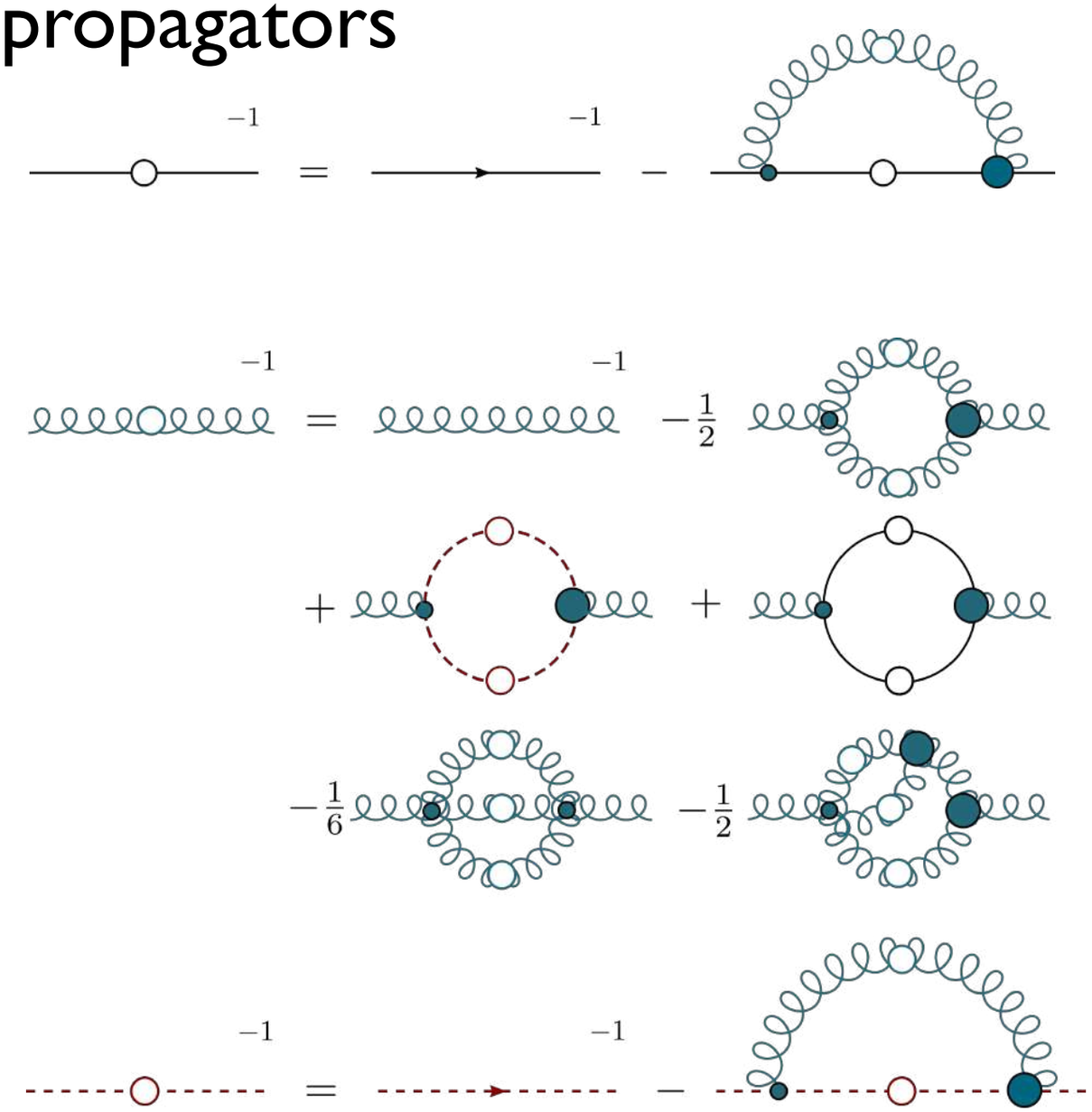

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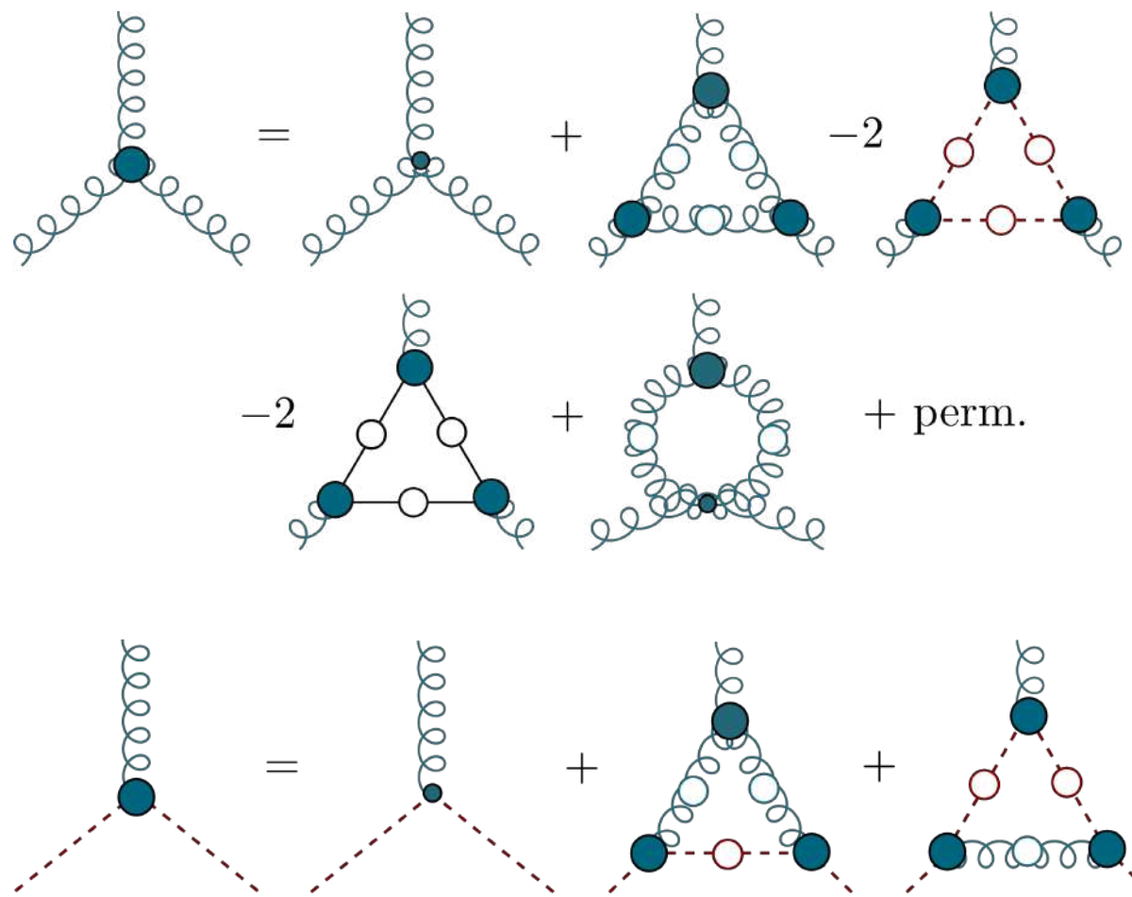

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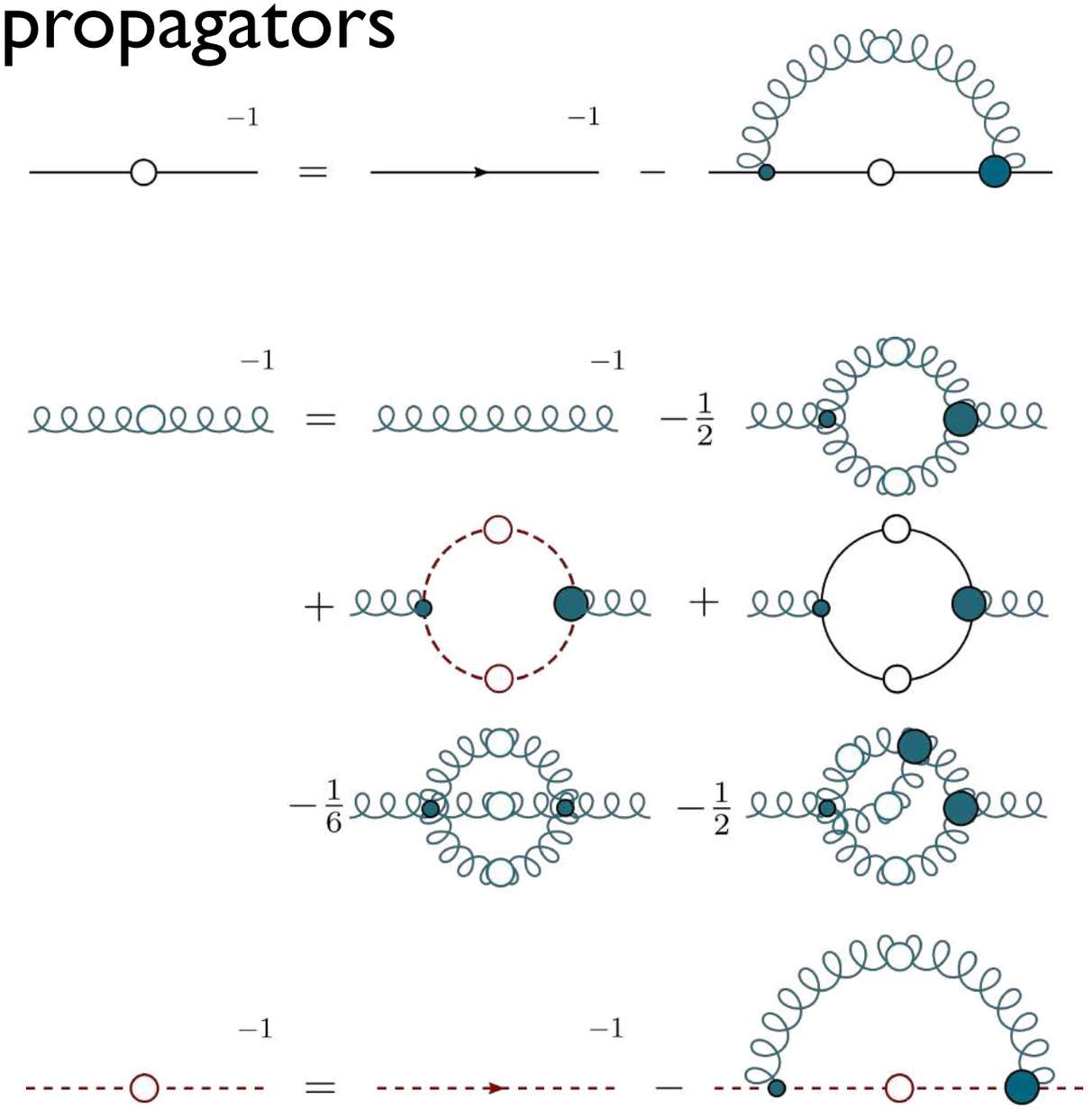

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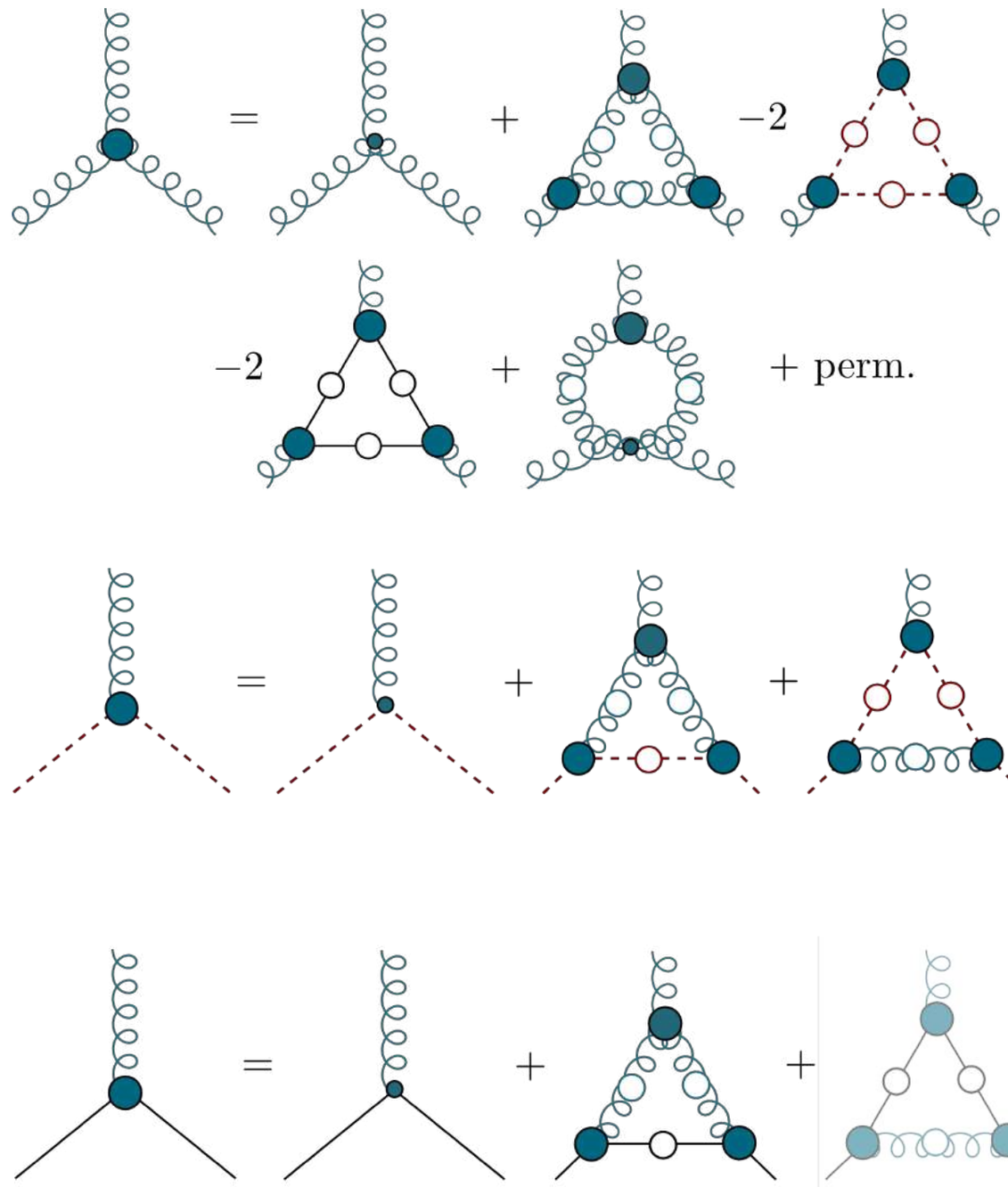

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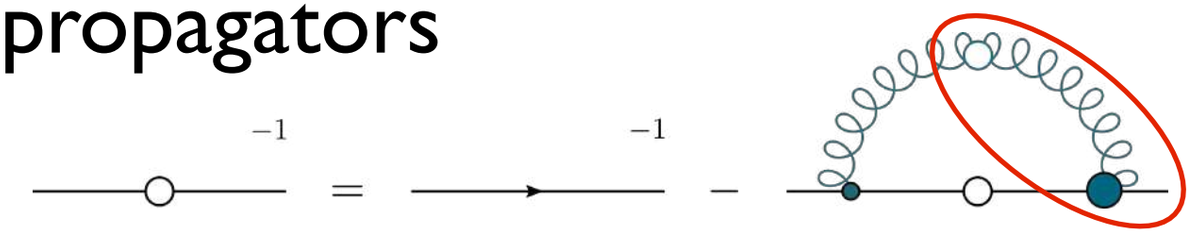

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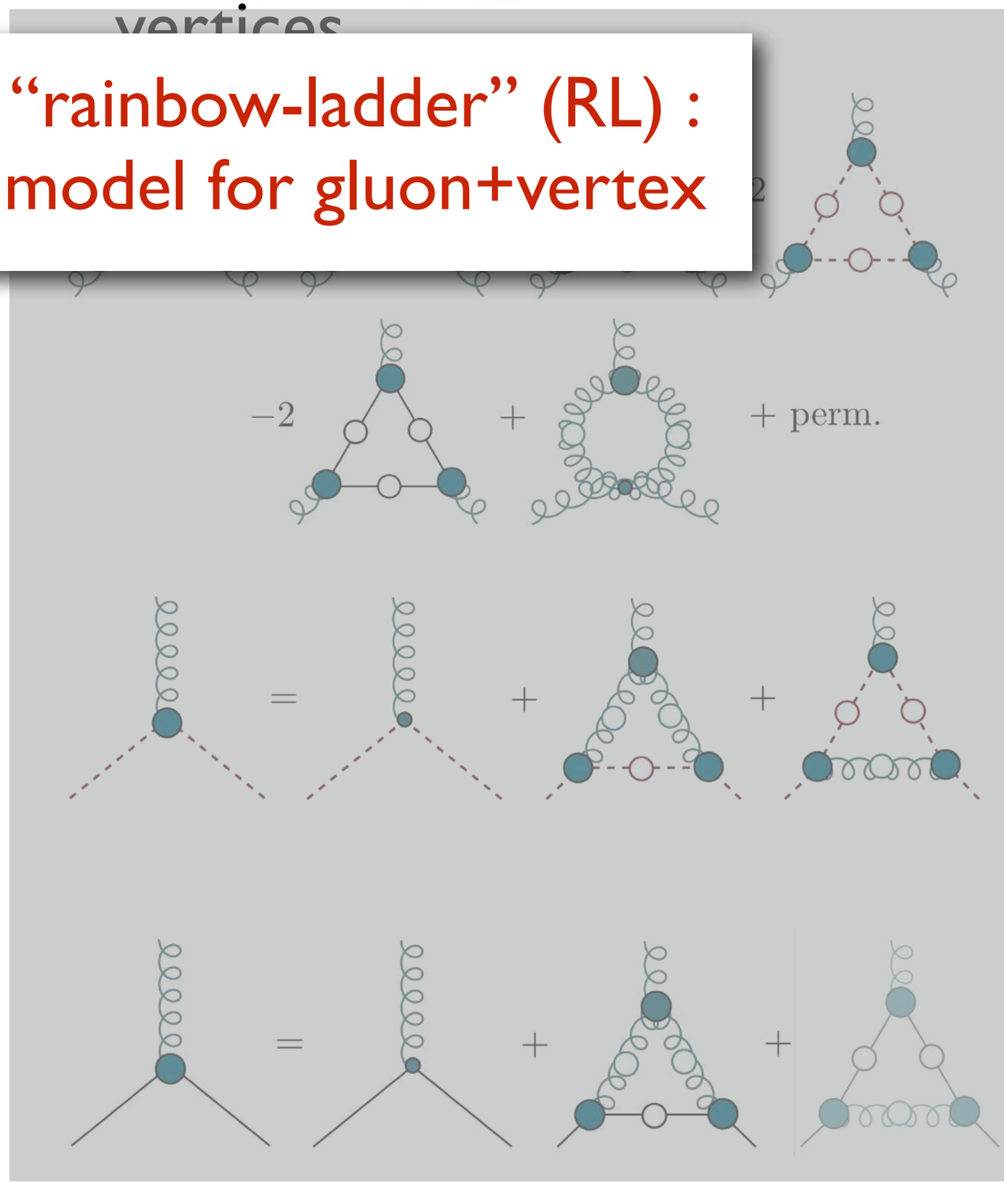
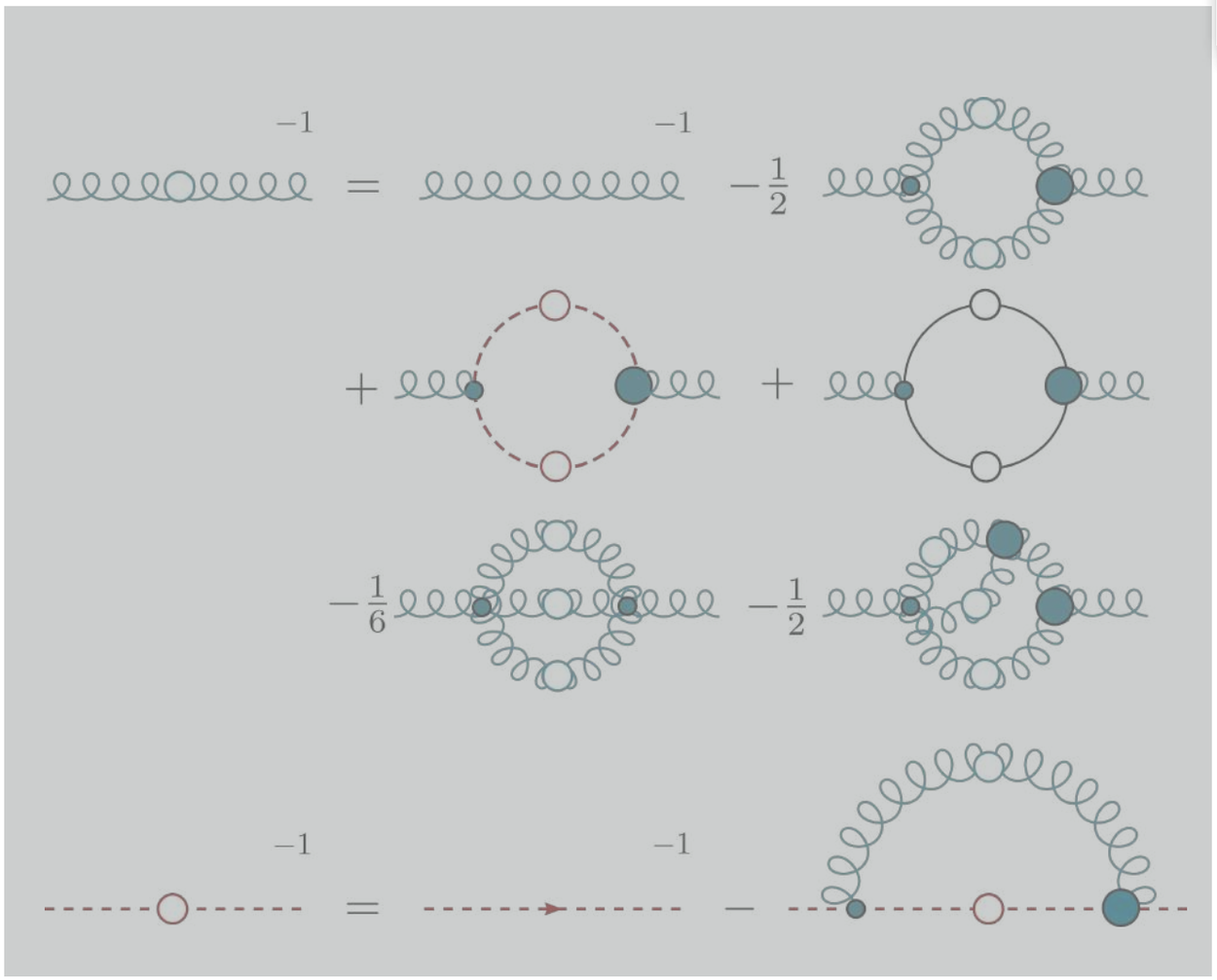


"rainbow-ladder" (RL) : model for gluon+vertex



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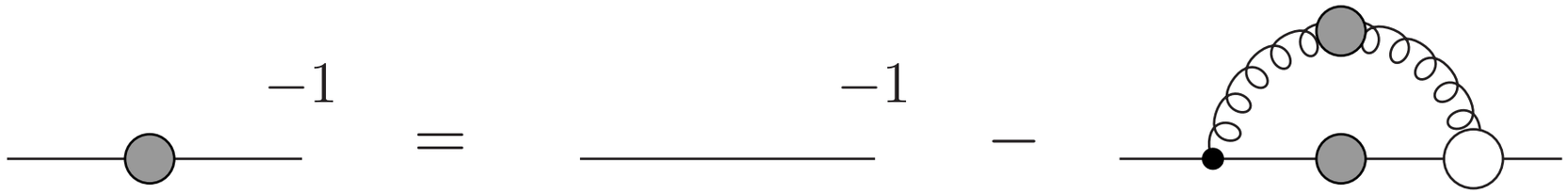
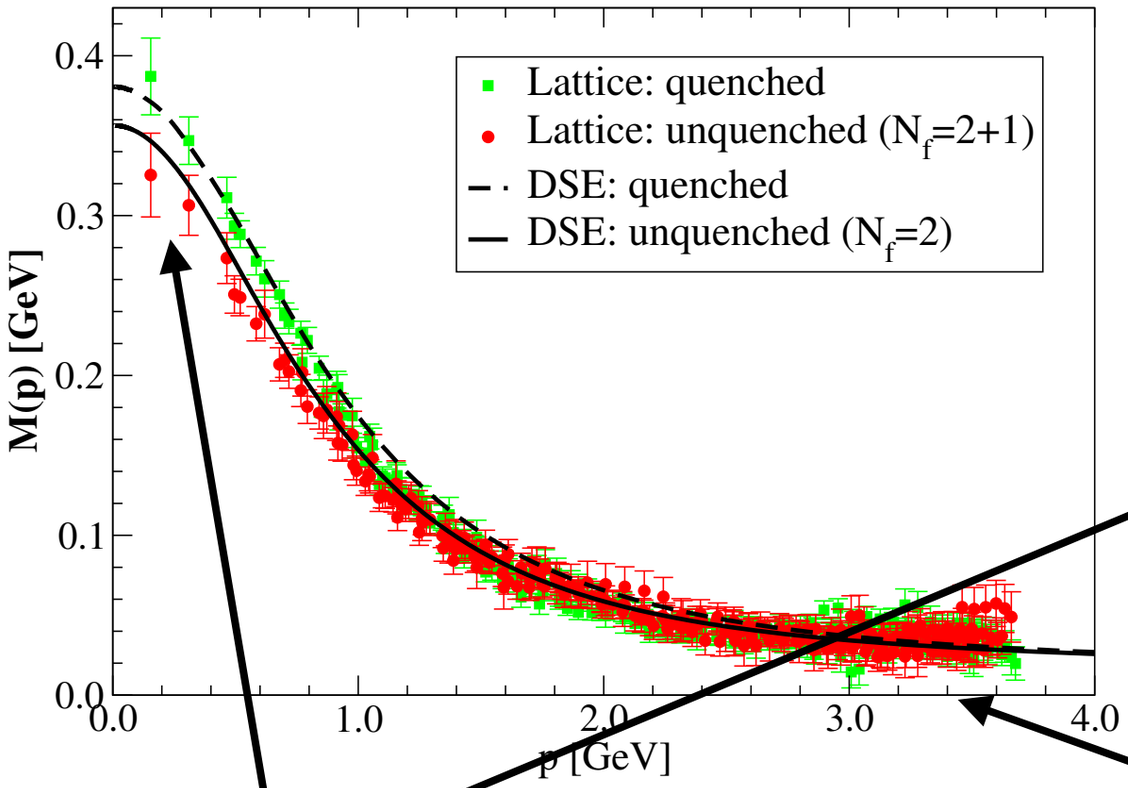
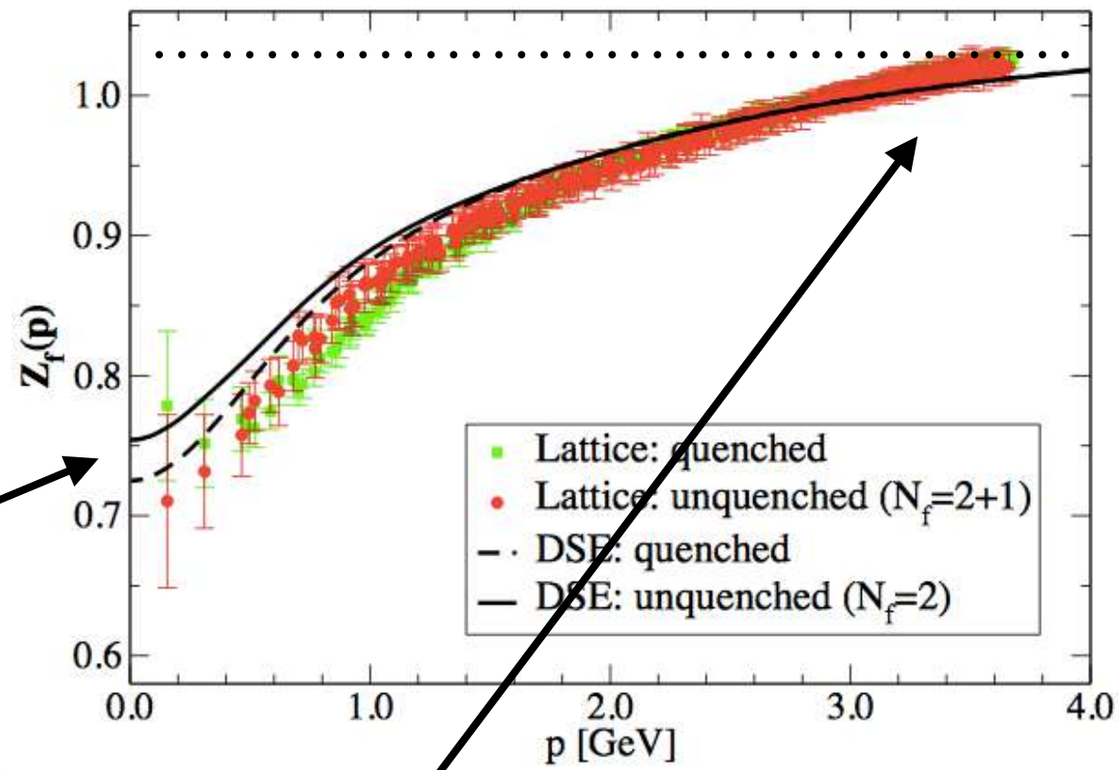
Quarks: mass from interaction



$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





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large mass; very composite

'current quark':
- small mass; non-composite

→ see talk by Fernando Serna

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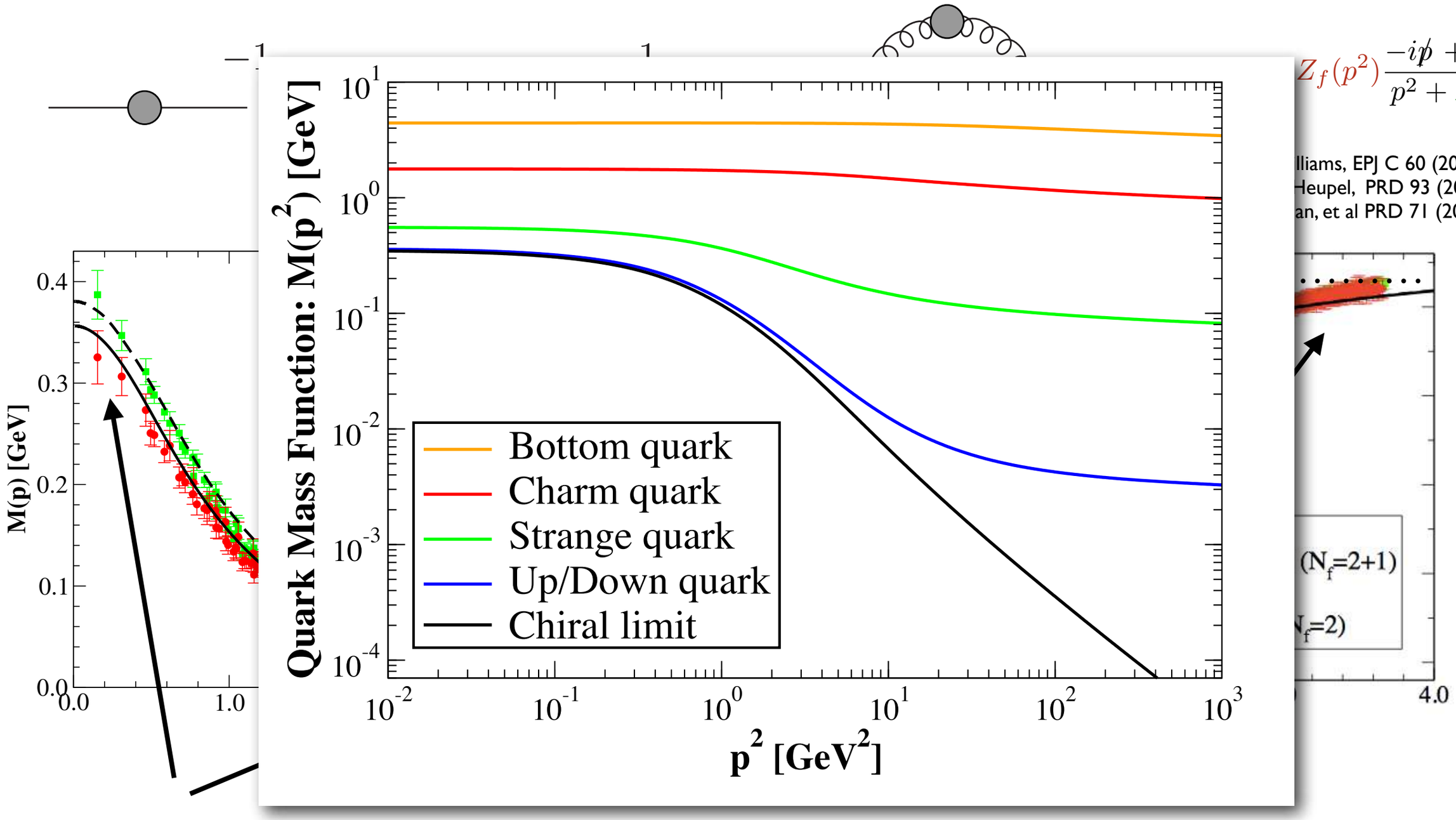

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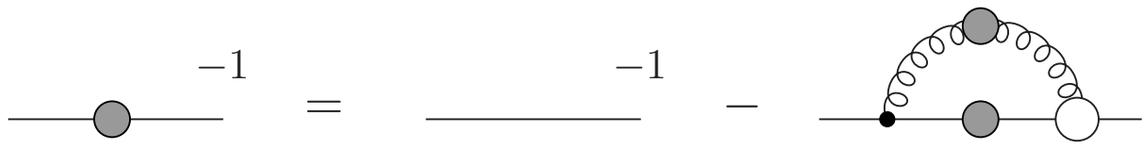

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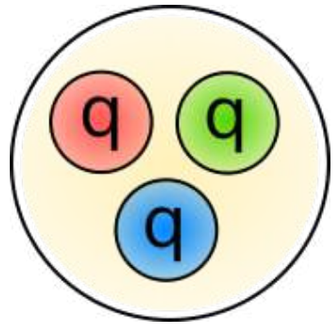

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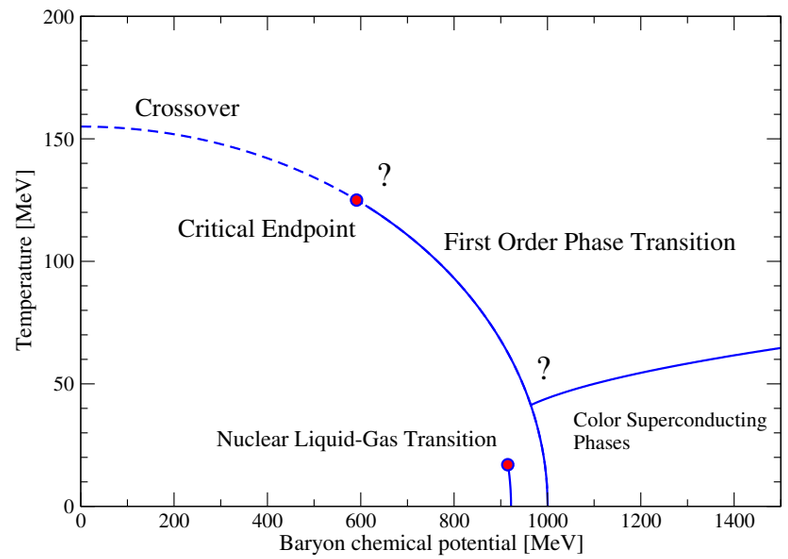
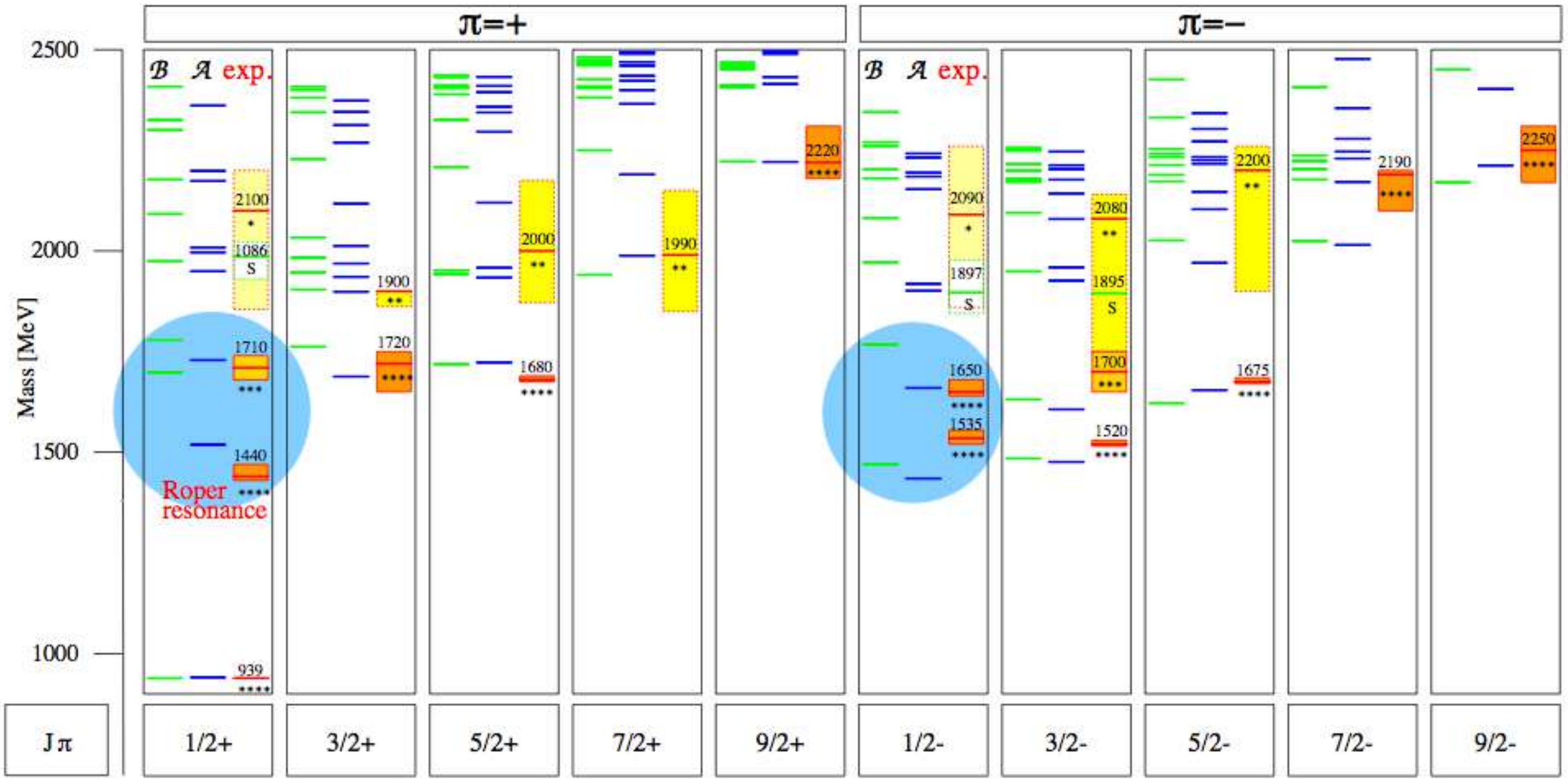


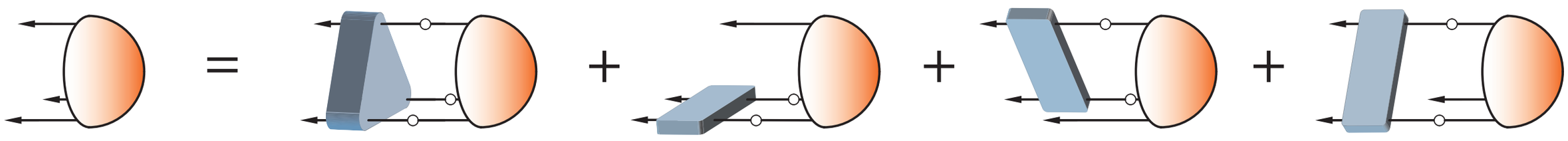
Light baryon spectrum - quark model



Loring, Metsch, Petry, EPJA 10 (2001) 395

- 'missing resonances'
- level ordering: $N_{\frac{1}{2}+}$ vs. $N_{\frac{1}{2}-}$

Faddeev - equation



- irreducible three-body forces: may be neglected Sanchis-Alepuz, Williams, work in progress…

- two-body interactions:
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- quark-diquark approximation Oettel, Hellstern, Alkofer and Reinhardt, PRC 58 (1998) 2459

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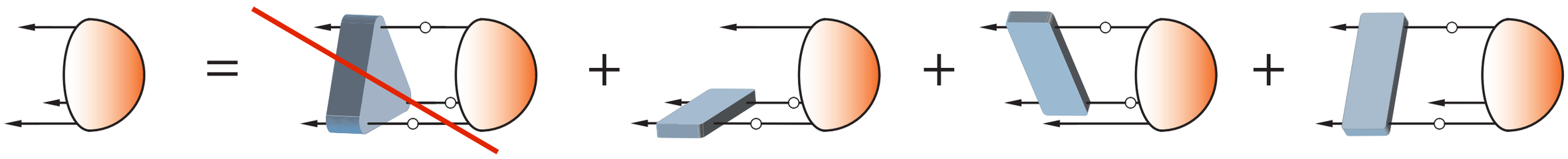

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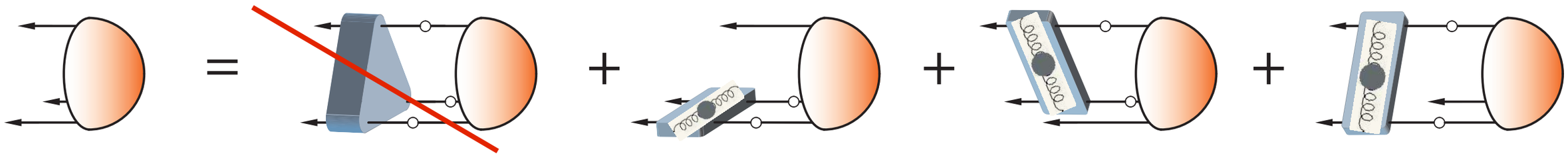

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Light baryon spectrum:

- 3 parameters + $m_{u,d,s}$
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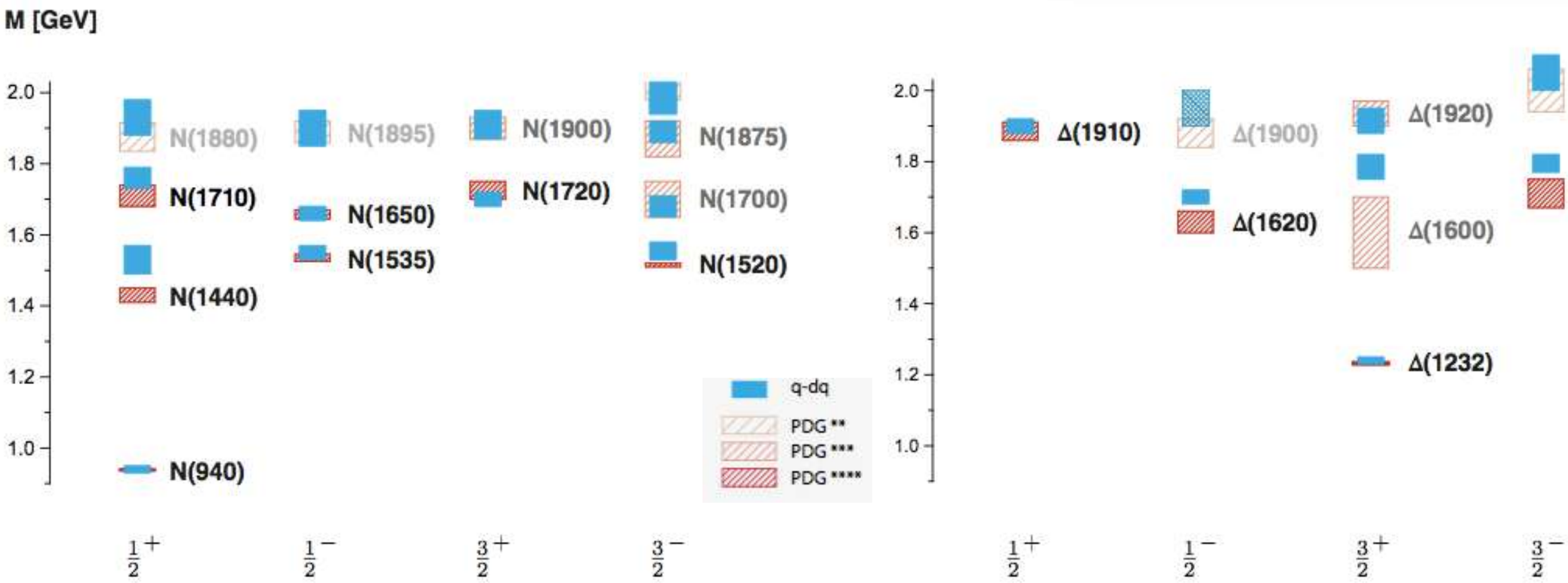

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
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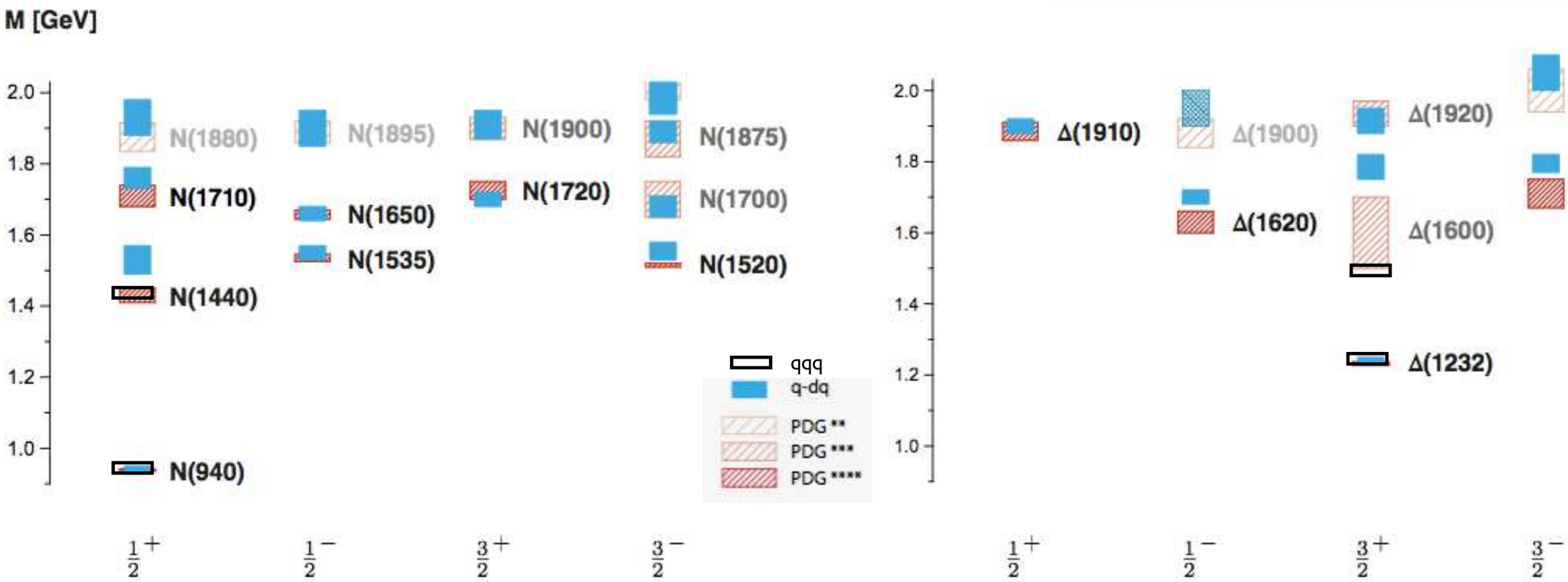

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- spectrum in one to one agreement with experiment
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- three-body agrees with diquark-quark where applicable

Relativistic nucleon

$$J^P = \left(\frac{1}{2}\right)^+$$

non-relativistic

three quarks with spin 1/2: $S = 1/2 \text{ or } S = 3/2$

parity $P = (-1)^L$: $L = 0 \text{ or } L = 2$

relativistic

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64 components in wave function

8 s-wave
36 p-wave
20 d-wave

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
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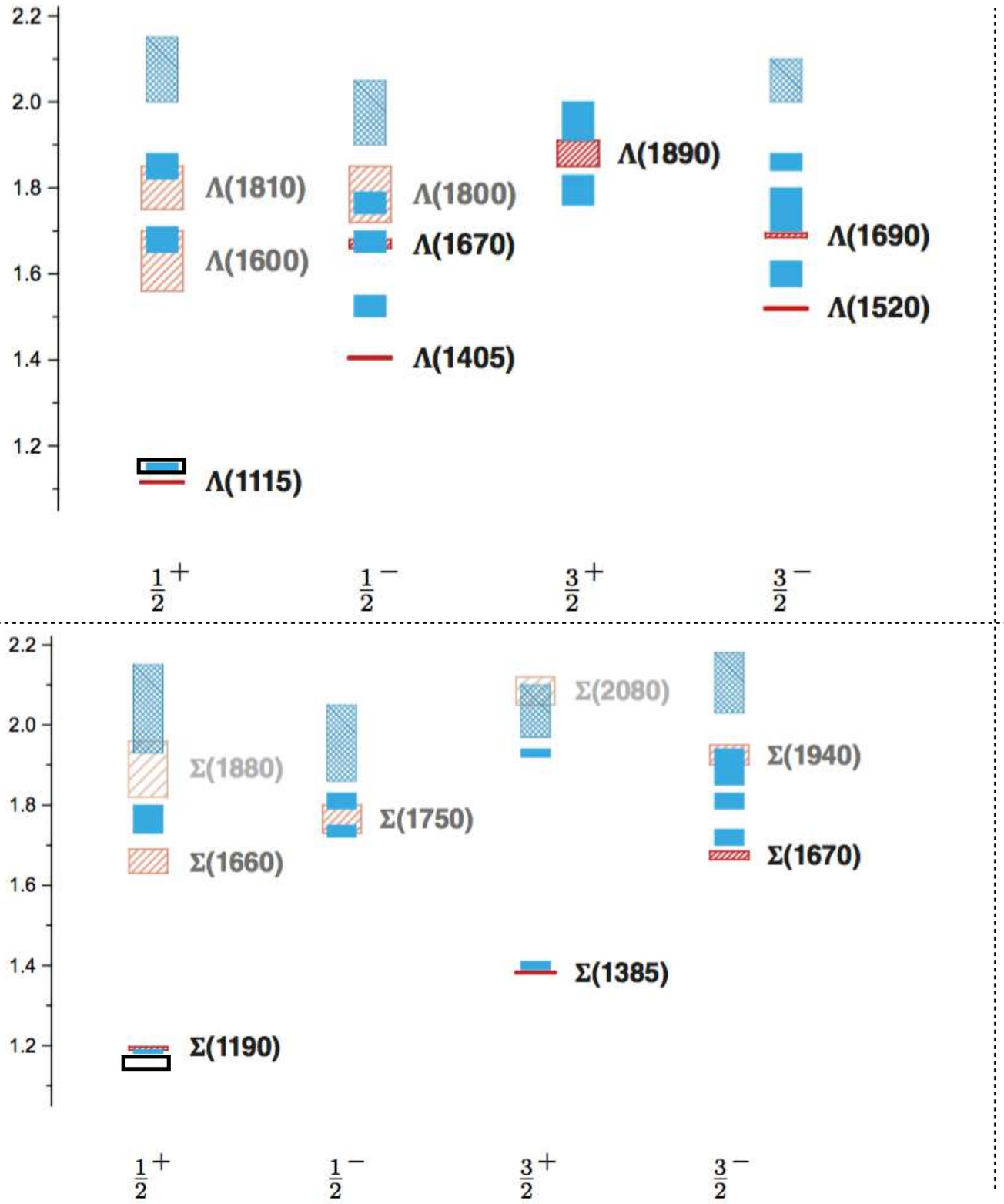

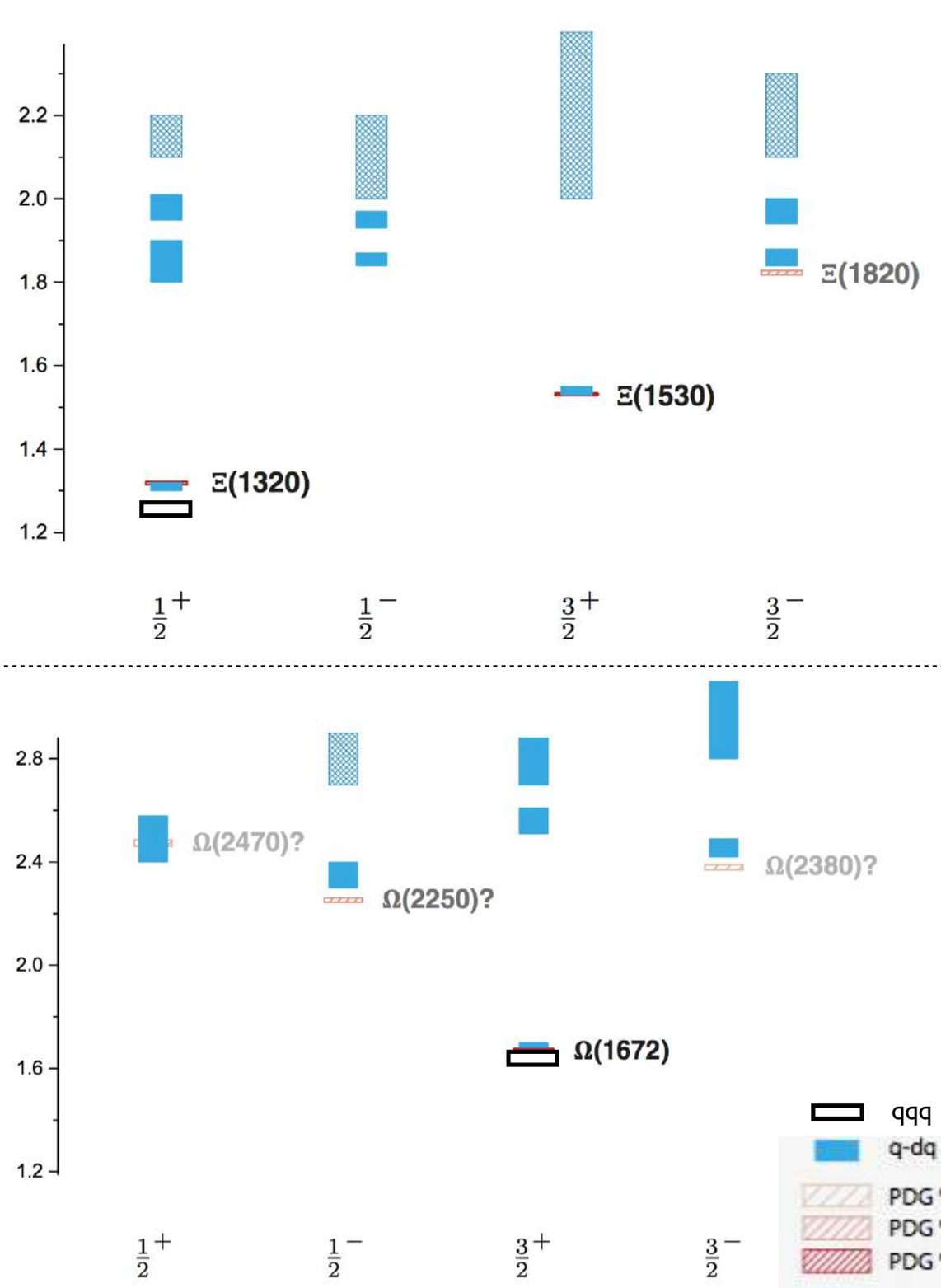

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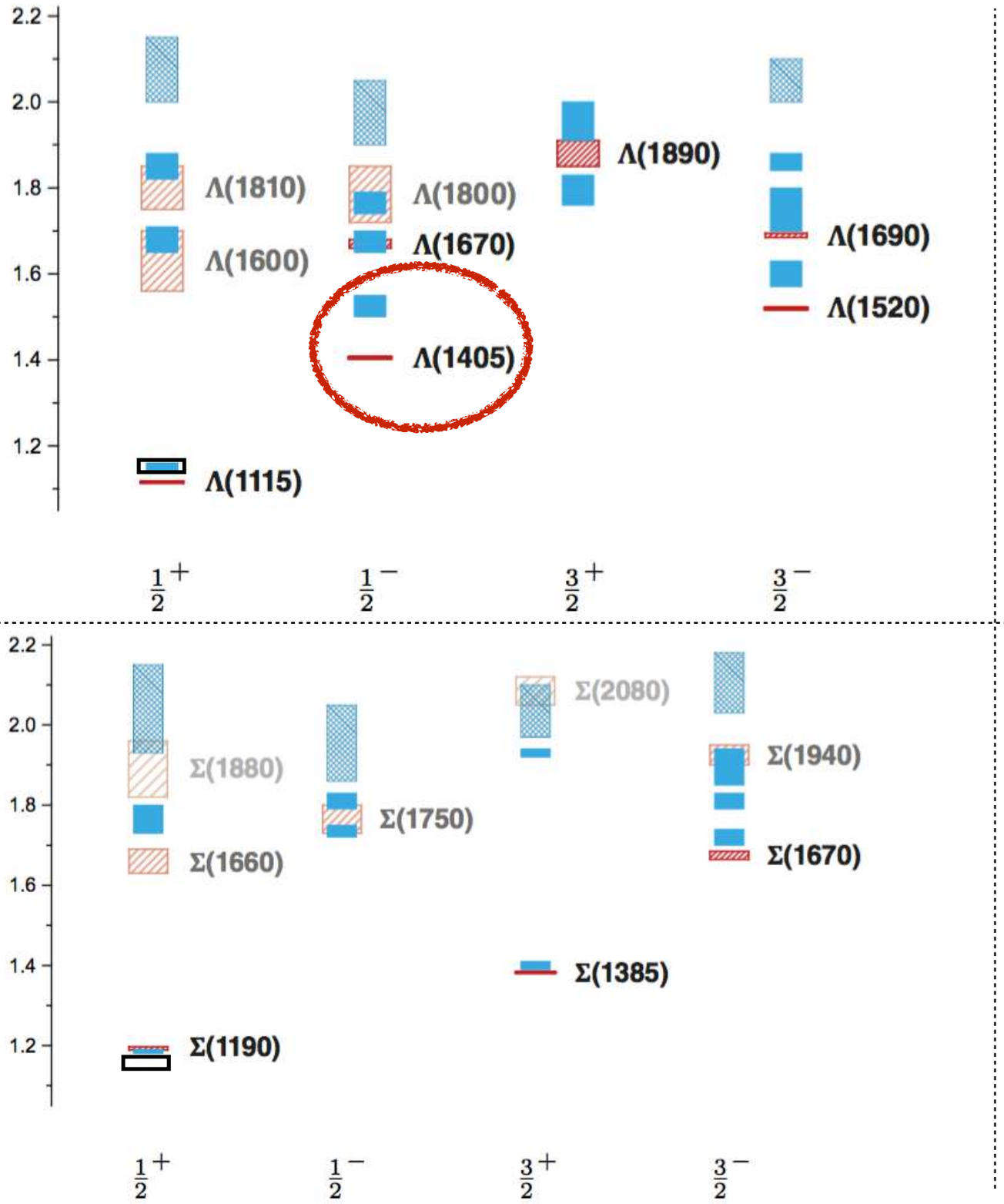

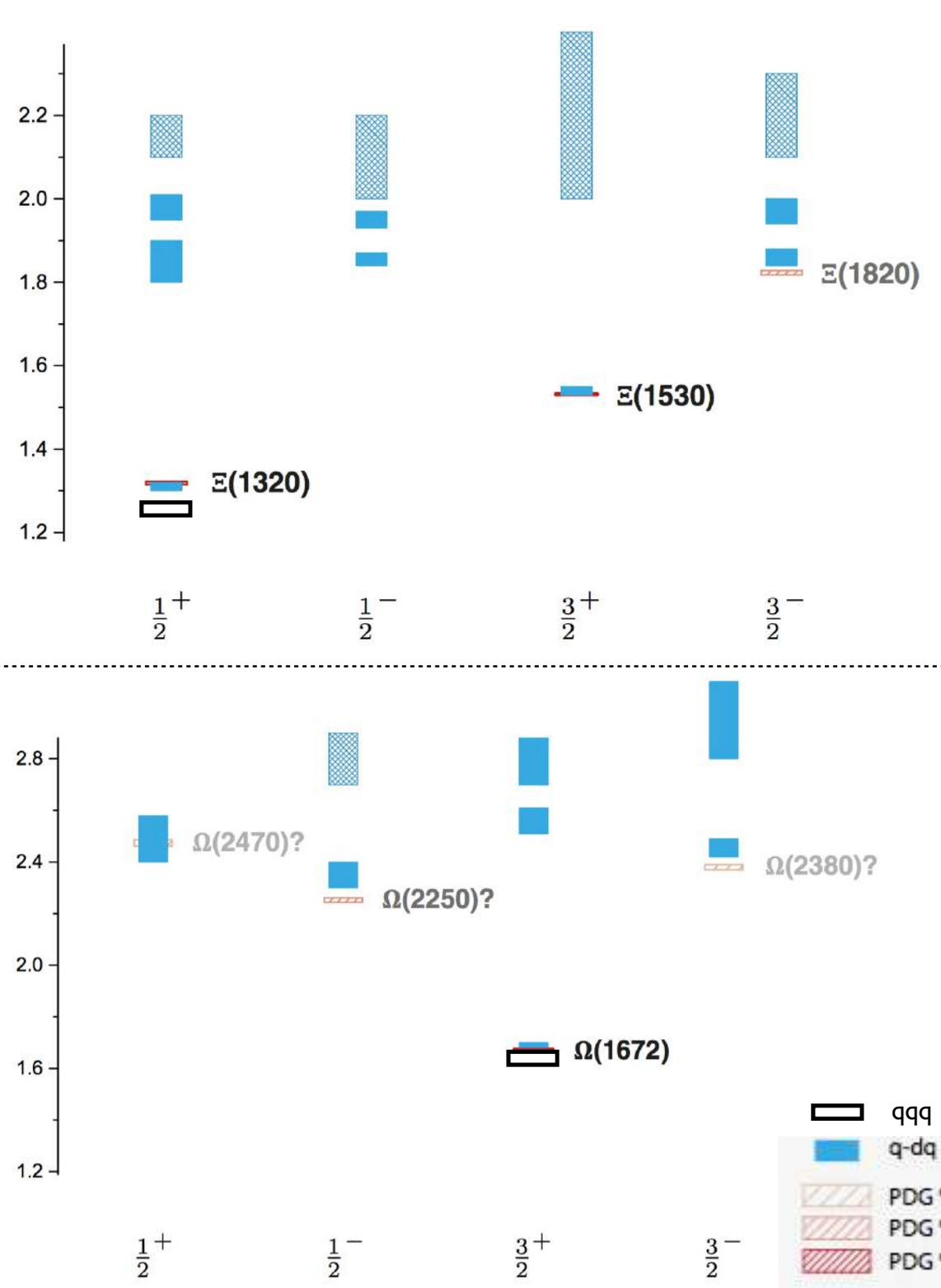

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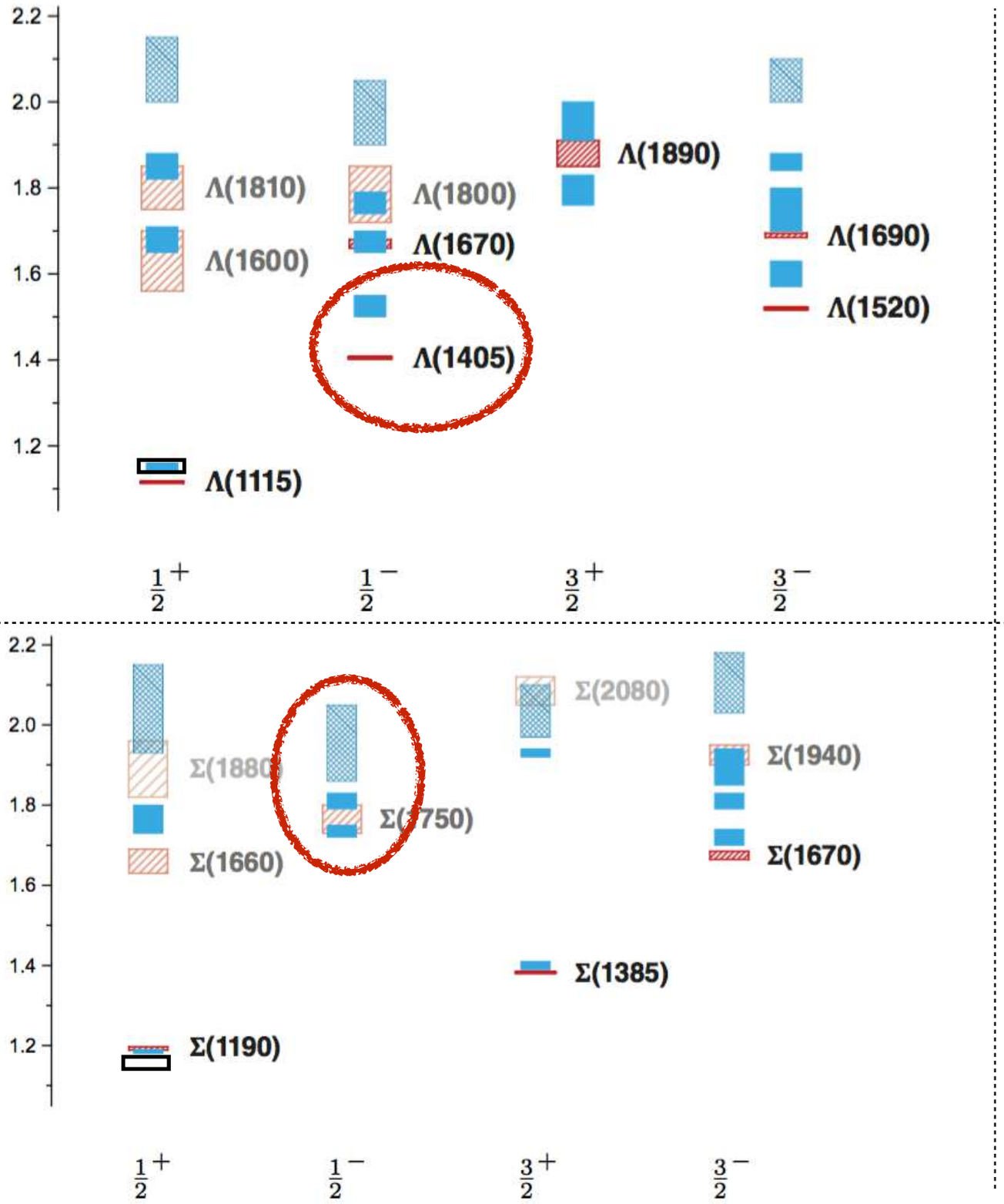

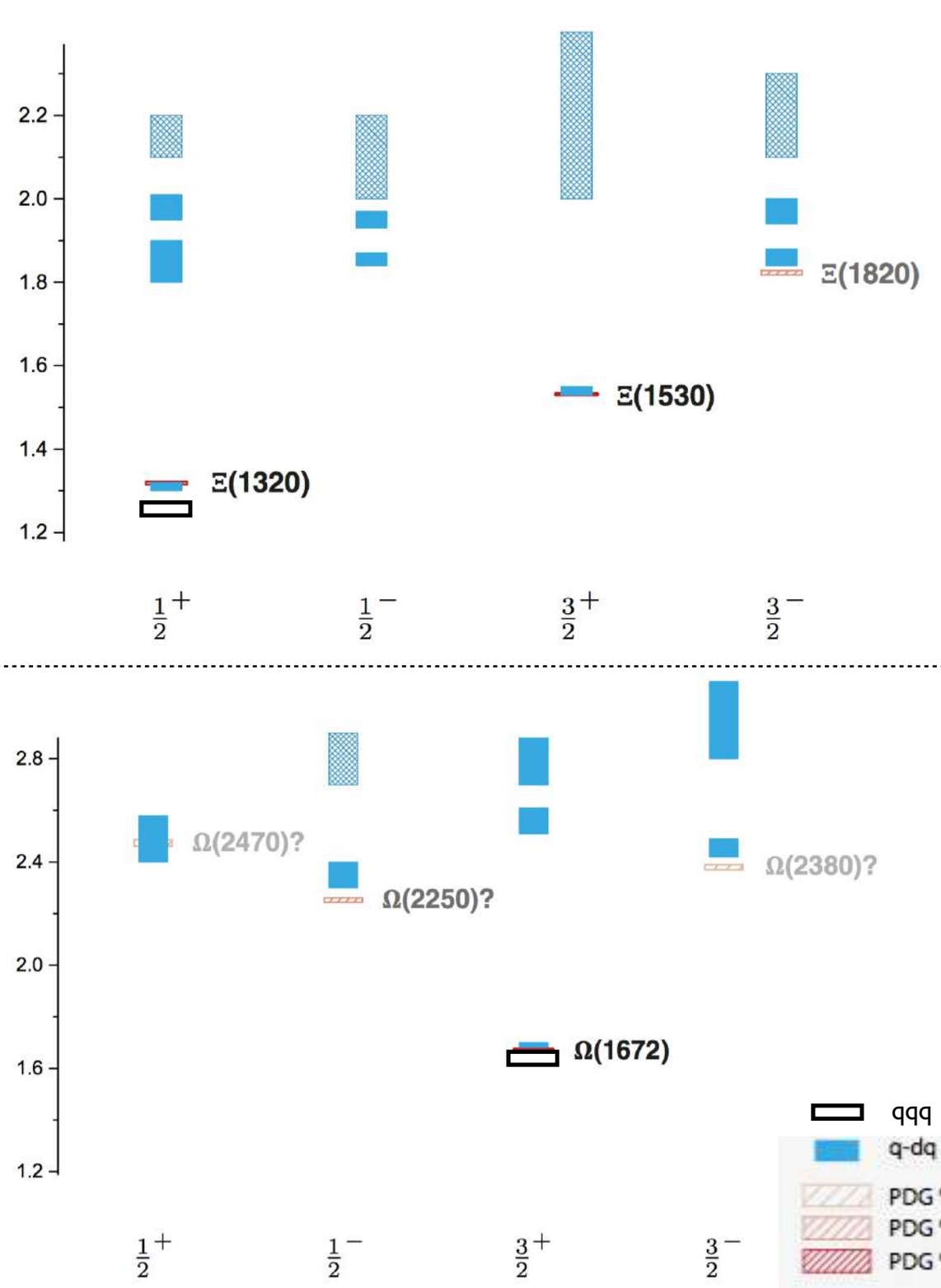

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Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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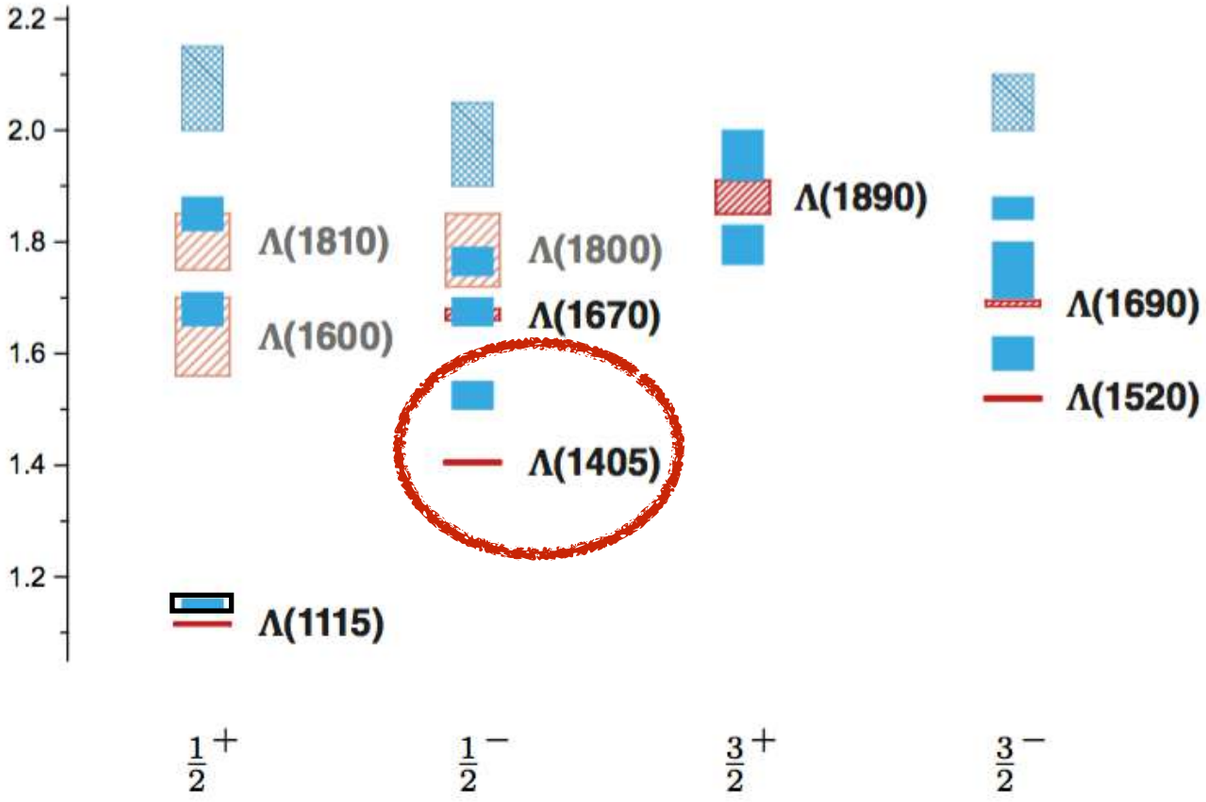
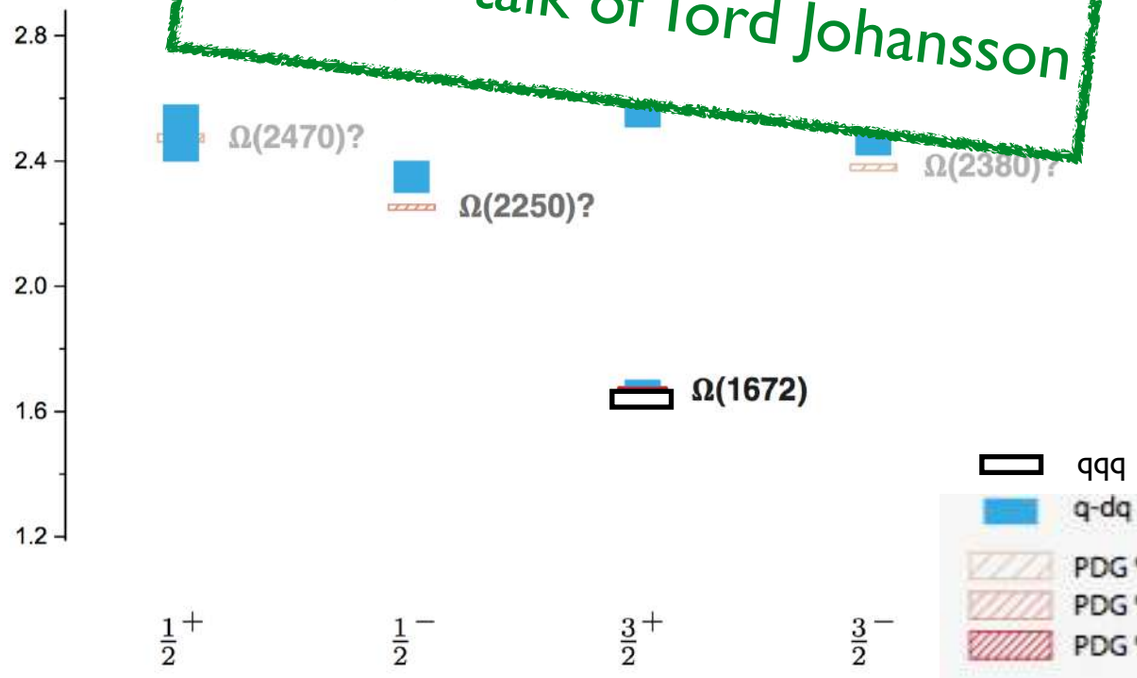


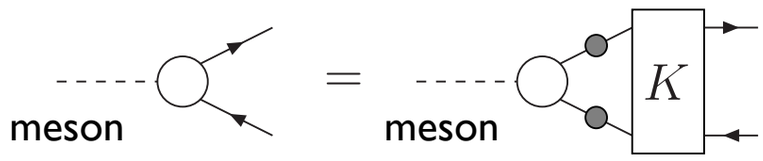
Prediction for PANDA

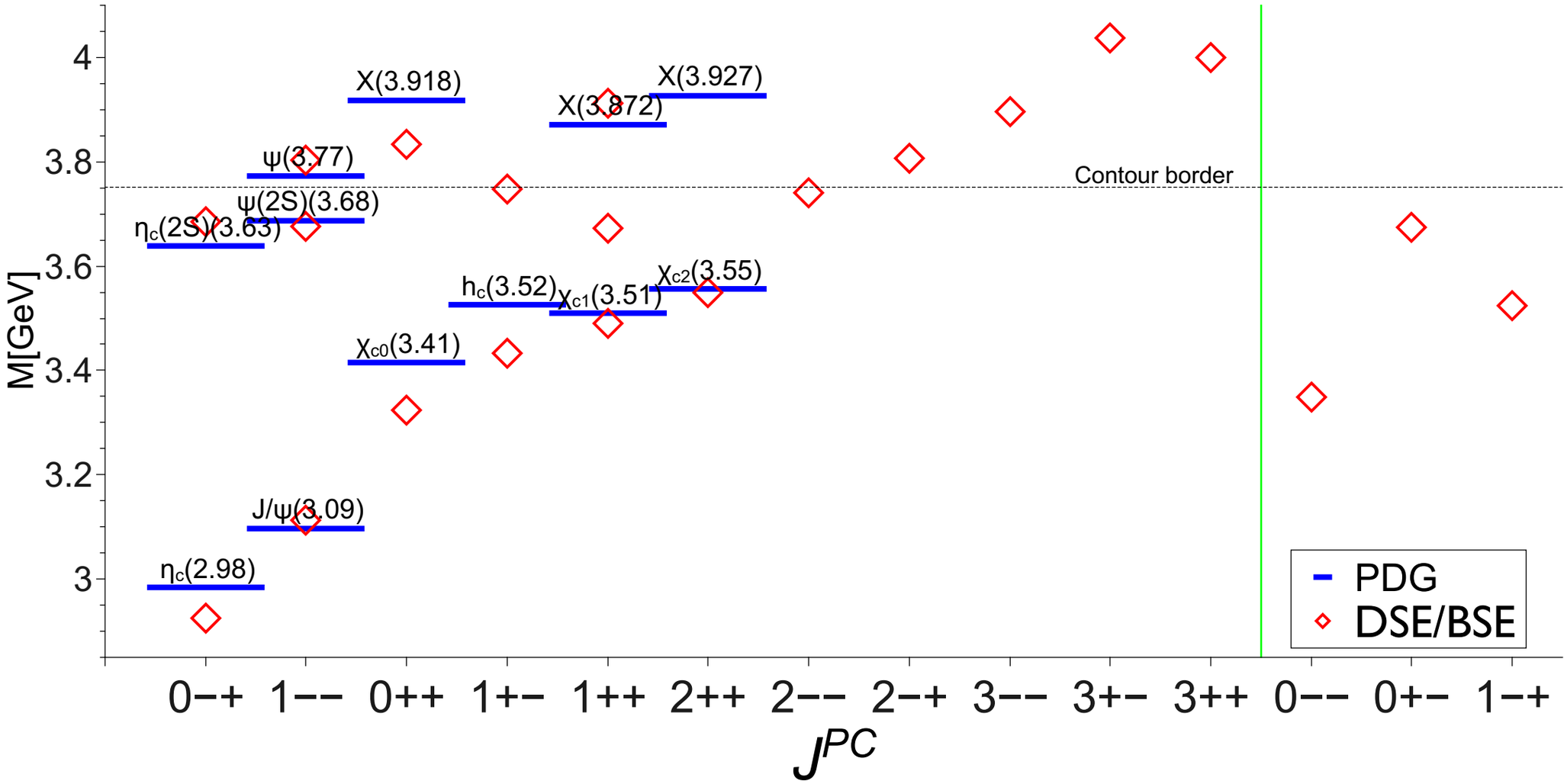
→ talk of Tord Johansson



Eichmann, CF, in preparation
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Charmonium spectrum - rainbow ladder

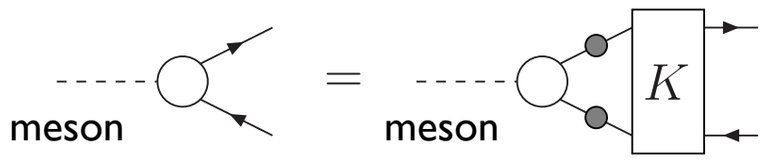


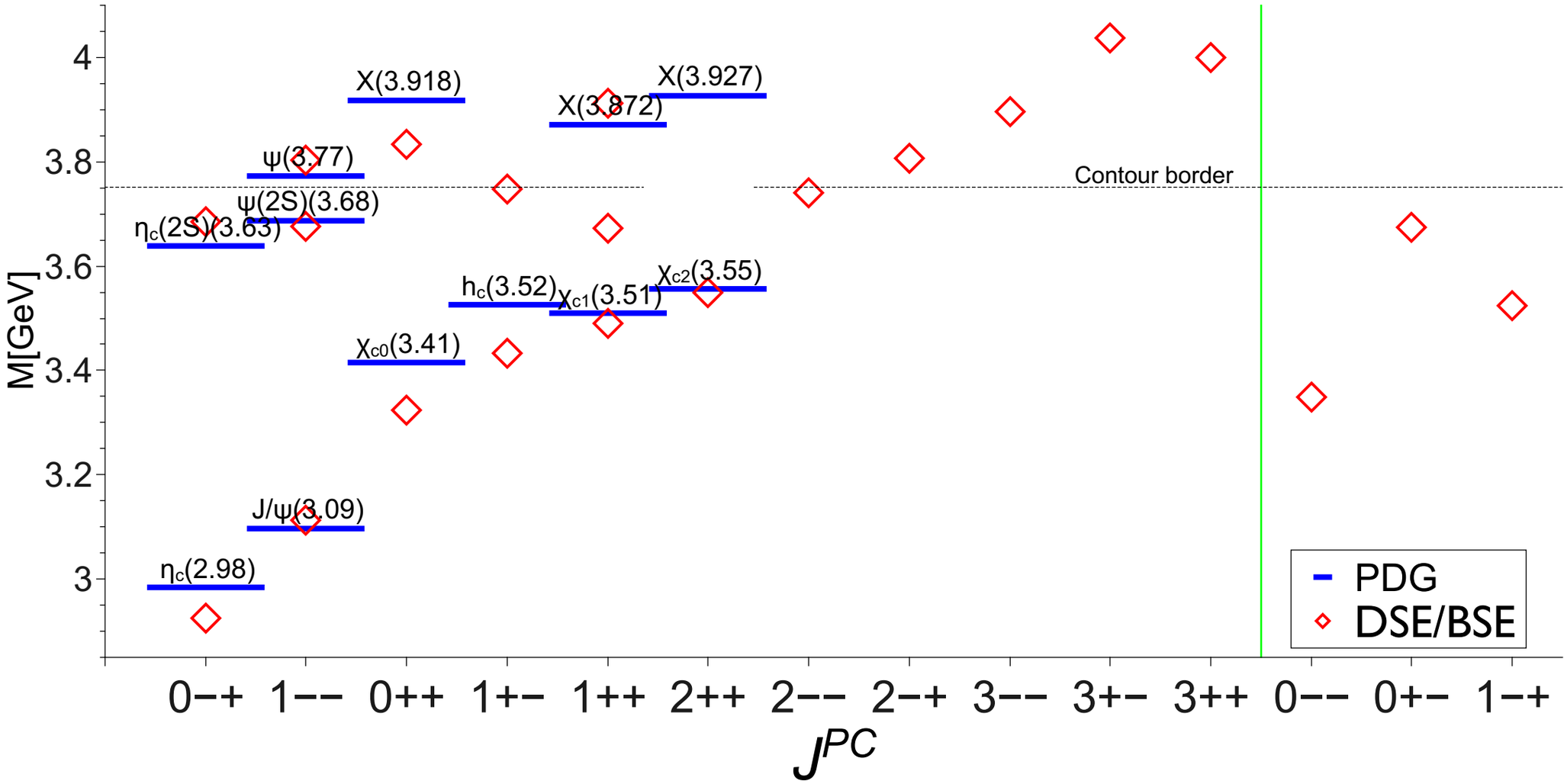


CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: $1^{--}, 2^{++}, 3^{--}, \ldots$
- acceptable channels: $0^{-+}, 1^{++}$
- deficiencies in other channels: missing spin-structure

Charmonium spectrum - rainbow ladder





CF, Kubrak, Williams, EPJA 51 (2015)
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- good channels: $1^{--}, 2^{++}, 3^{--}, \ldots$
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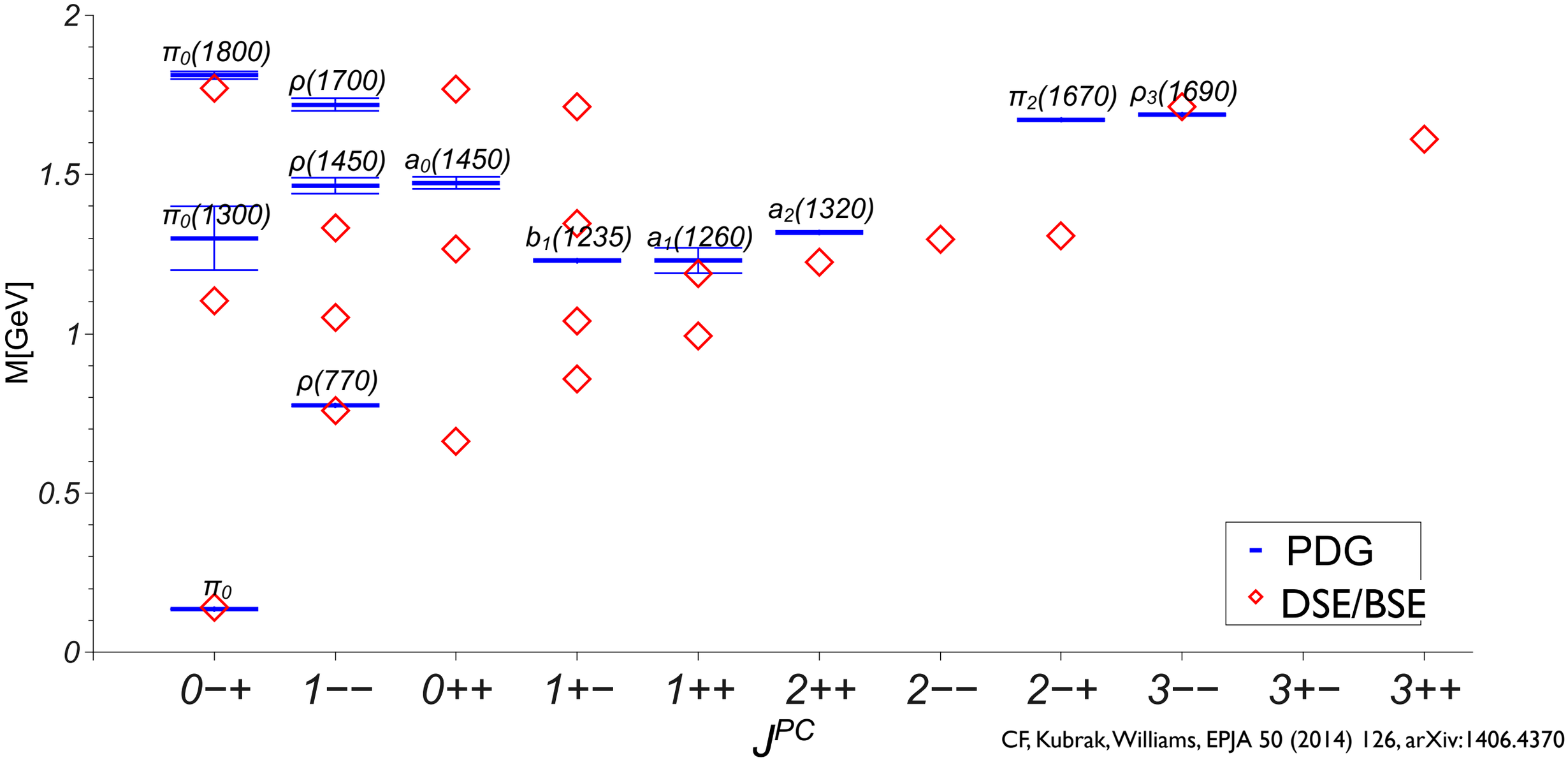
Light meson spectrum - rainbow ladder

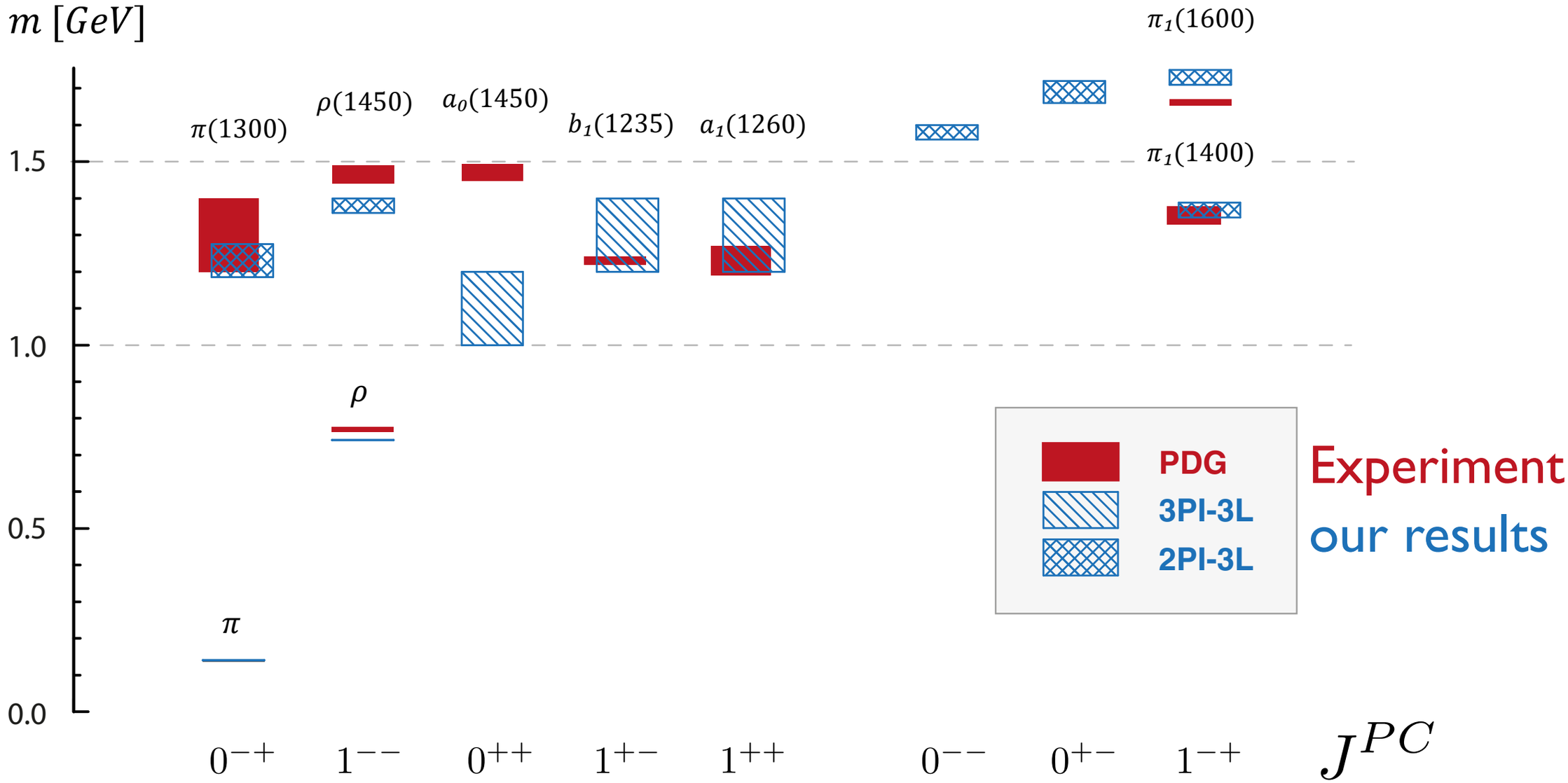


CF, Kubrak, Williams, EPJA 50 (2014) 126, arXiv:1406.4370

Williams, CF, Heupel, PRD93 (2016) 034026

- good channels (ground state): 0^{-+}, 1^{--}
- acceptable channels (ground state) : 2^{++}, 3^{--},...
- clear deficiencies in other channels and excited states

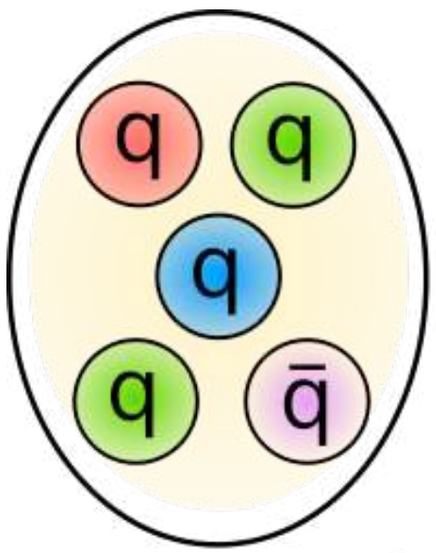
Light meson spectrum - full 3PI-calculation



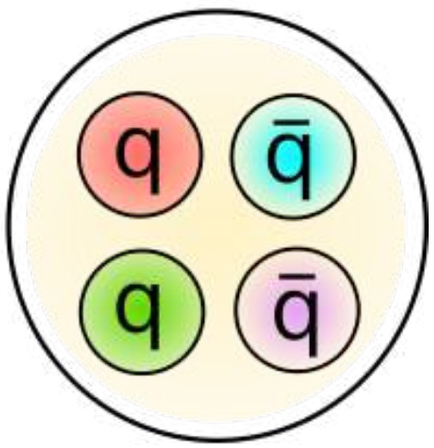
Williams, CF, Heupel, PRD93 (2016) 034026

- good agreement with experiment in most channels
- special channels:
  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
  scalar 0^{++} : complicated channel…

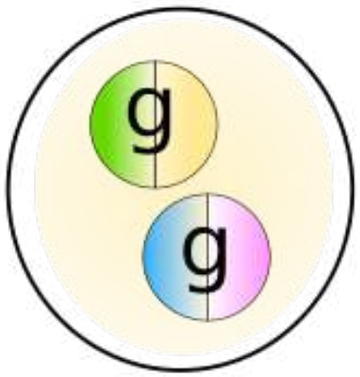
Tetraquarks in the light meson sector



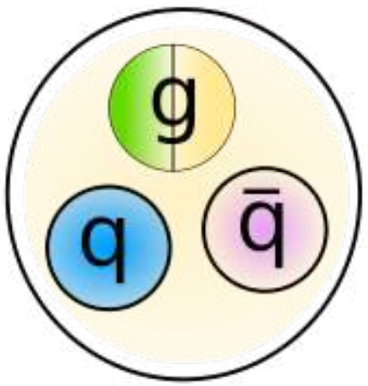
Pentaquark



Tetraquark

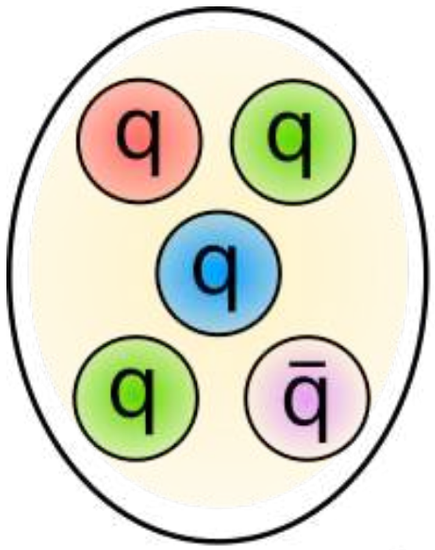


Glueball

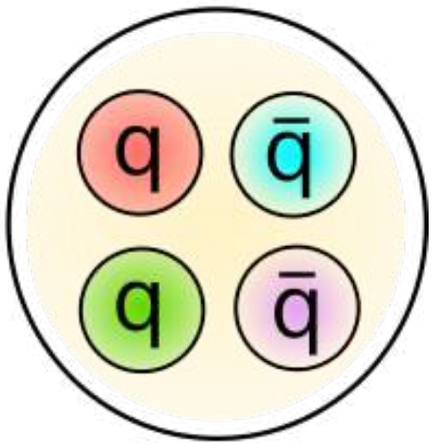


Hybrid

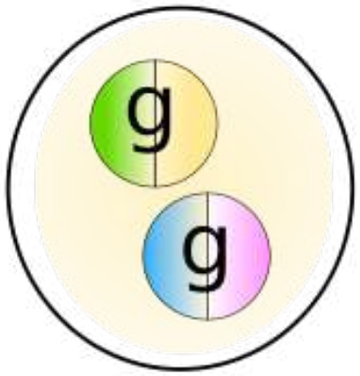
Tetraquarks in the light meson sector



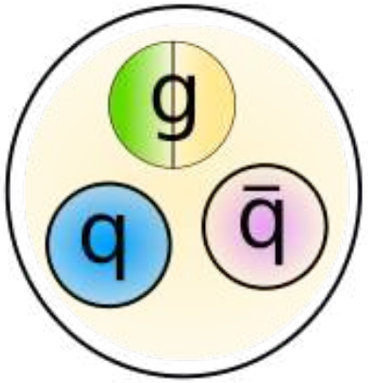
Pentaquark



Tetraquark

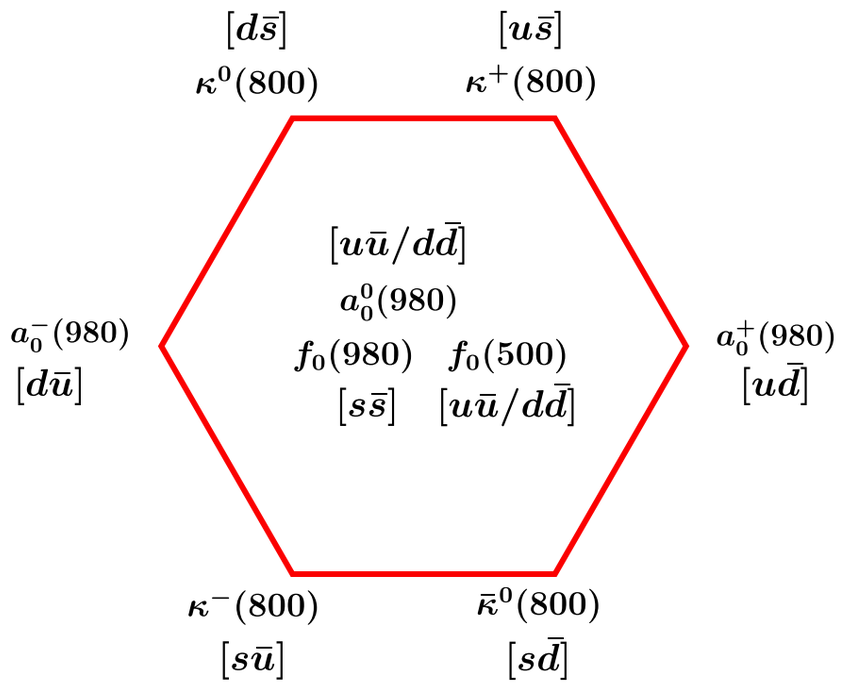


Glueball

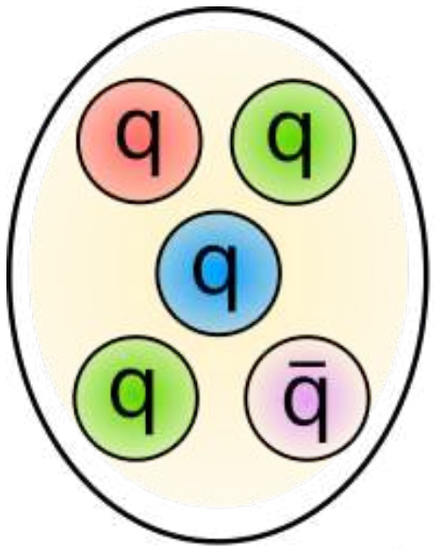


Hybrid

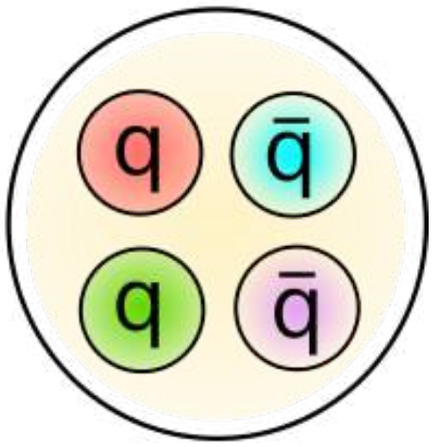
Light meson sector:
Scalars!



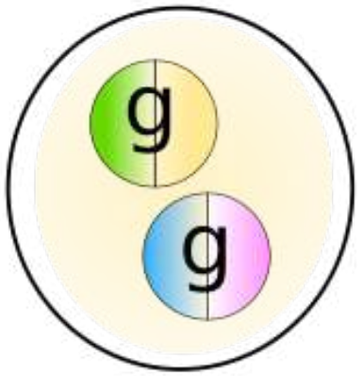
Tetraquarks in the light meson sector



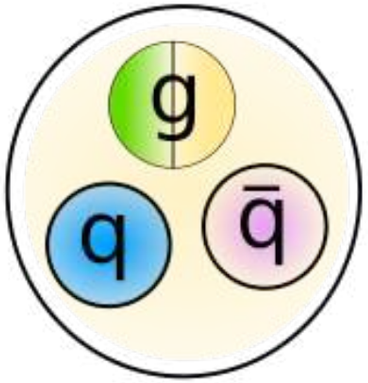
Pentaquark



Tetraquark



Glueball



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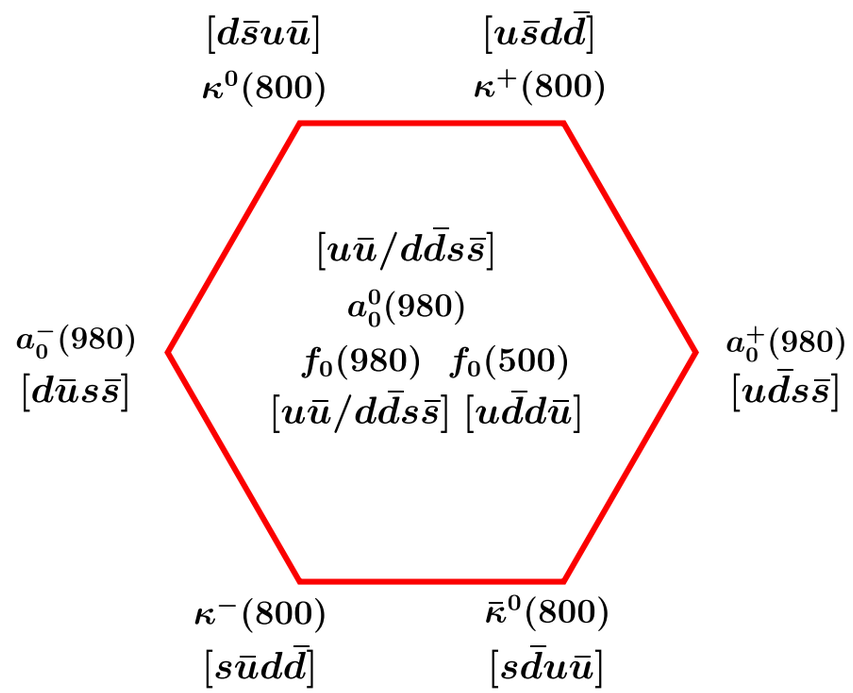
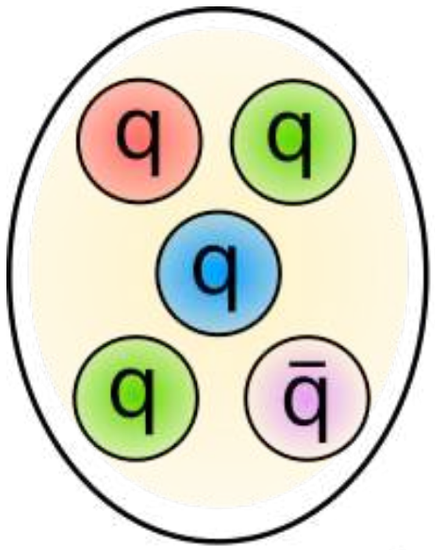
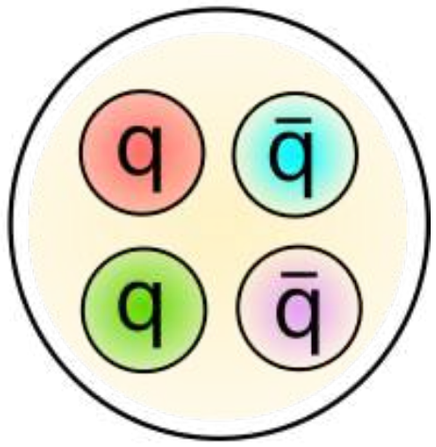


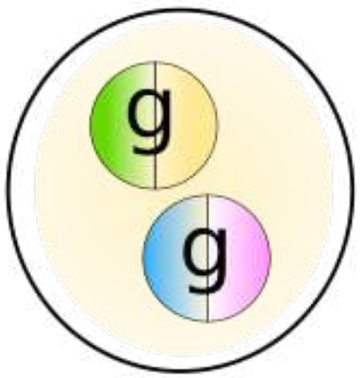
Tetraquarks in the light meson sector



Pentaquark



Tetraquark



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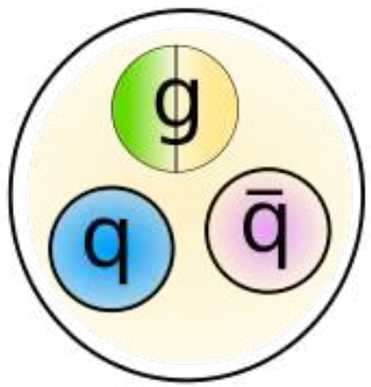
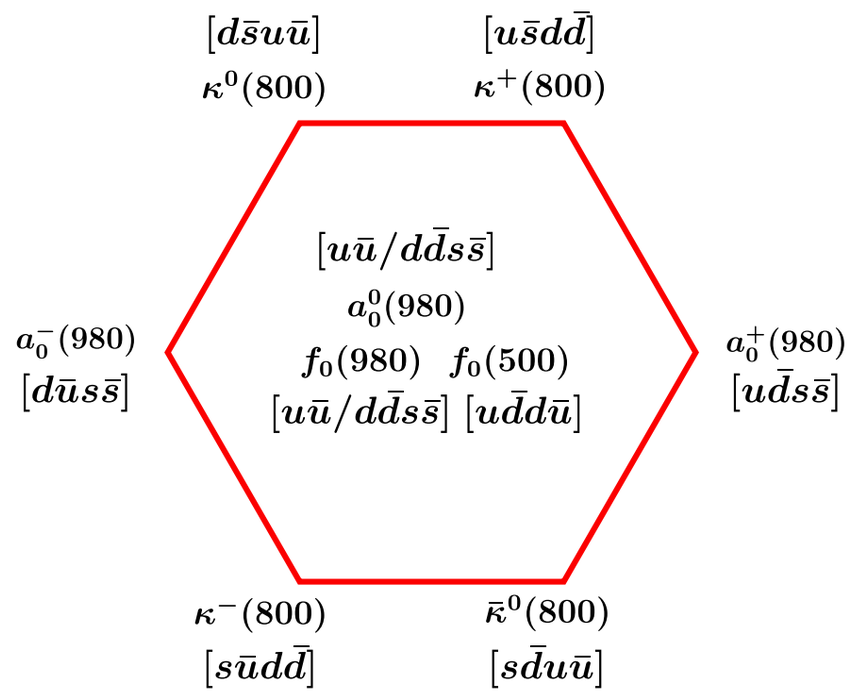


Hybrid

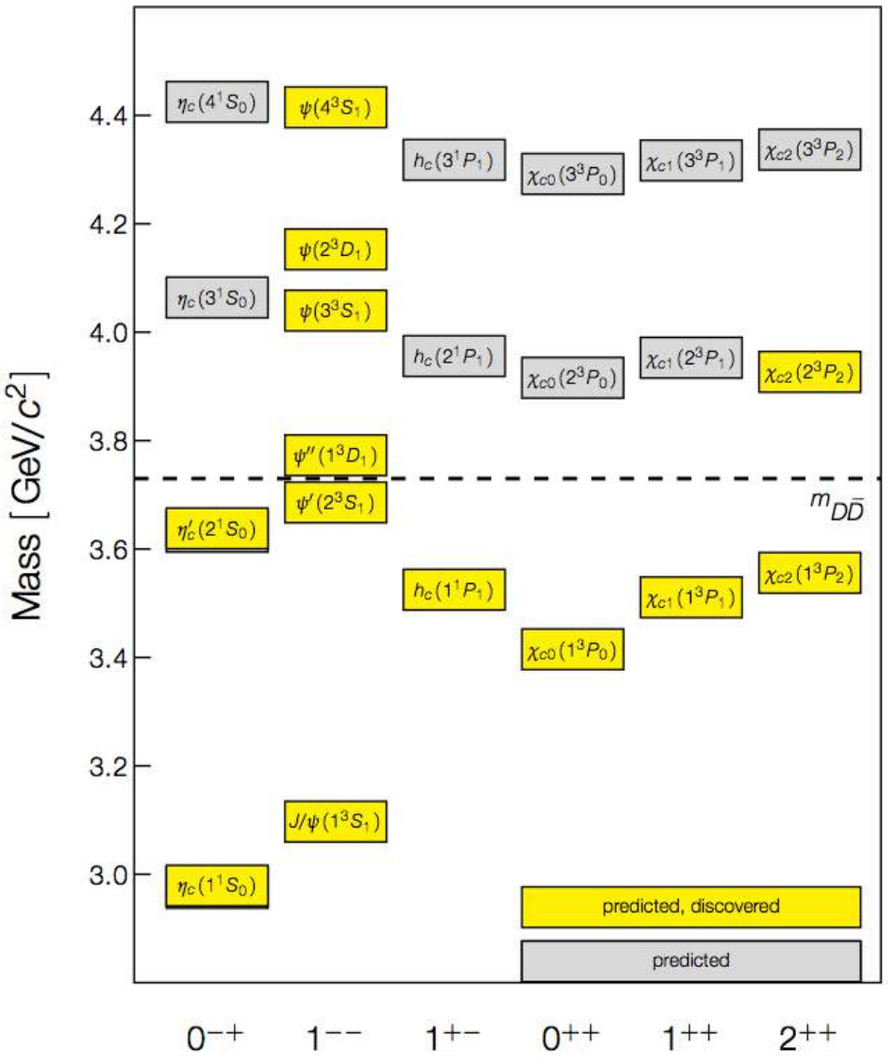
Light meson sector: Scalars!



$$f_0(500) \to \pi\pi$$

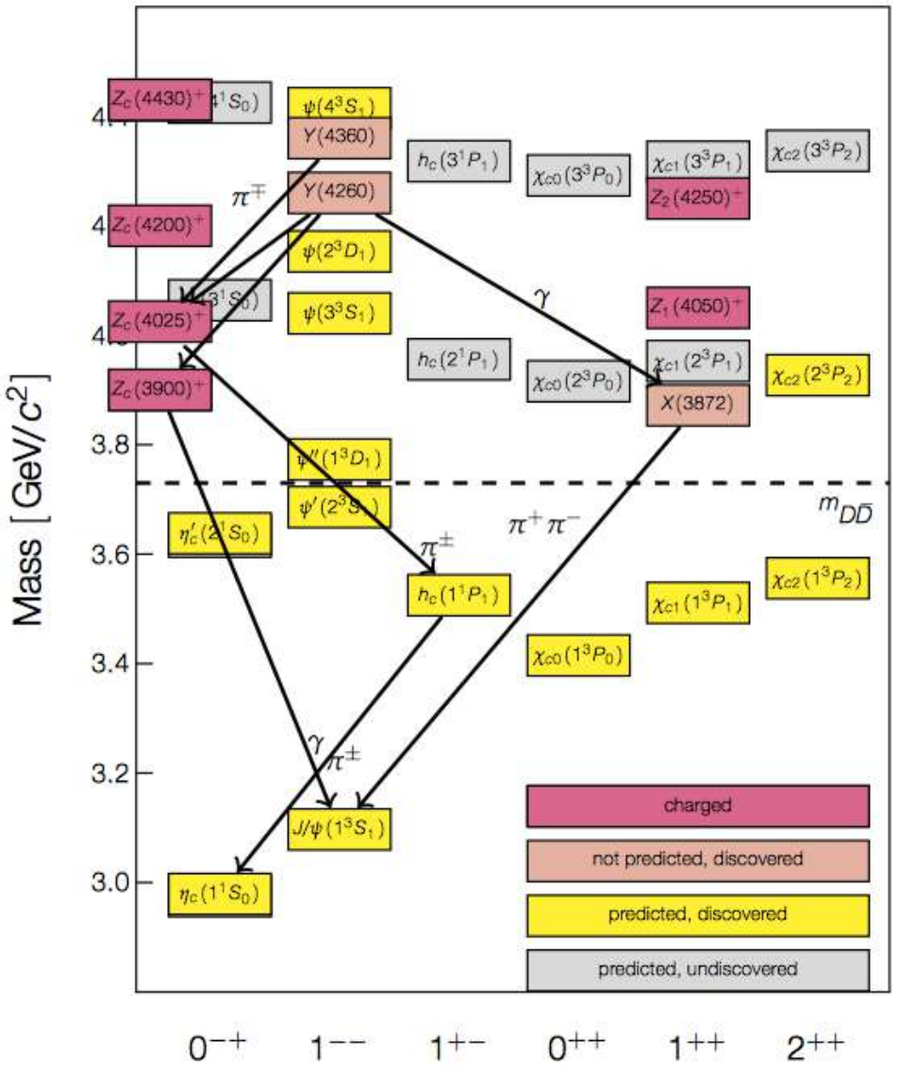
large decay width

Tetraquark candidates in charmonium region



Expected states: $c\bar{c}$

Tetraquark candidates in charmonium region



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Many new unexpected state found in experiments such as Belle, BABAR, BES, LHCb, etc

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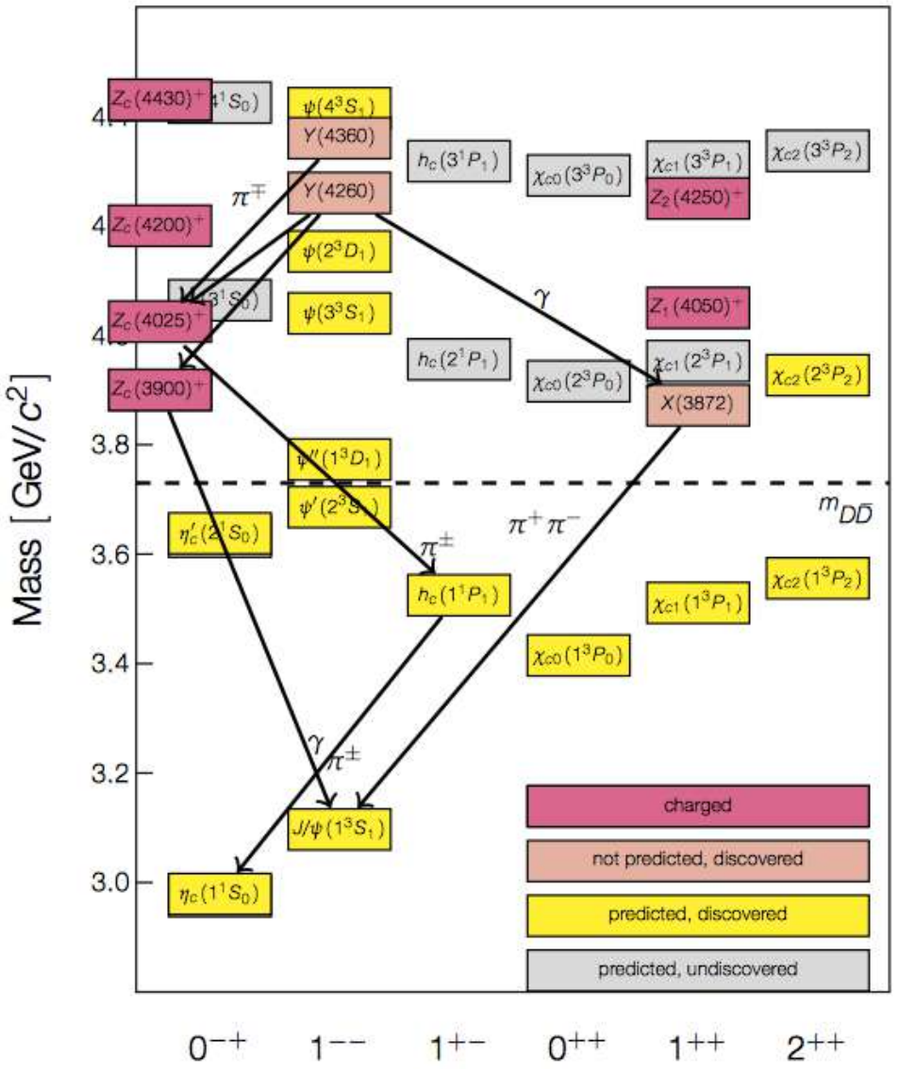

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charged: cannot be $c\bar{c}$, but $c\bar{c}u\bar{d}$

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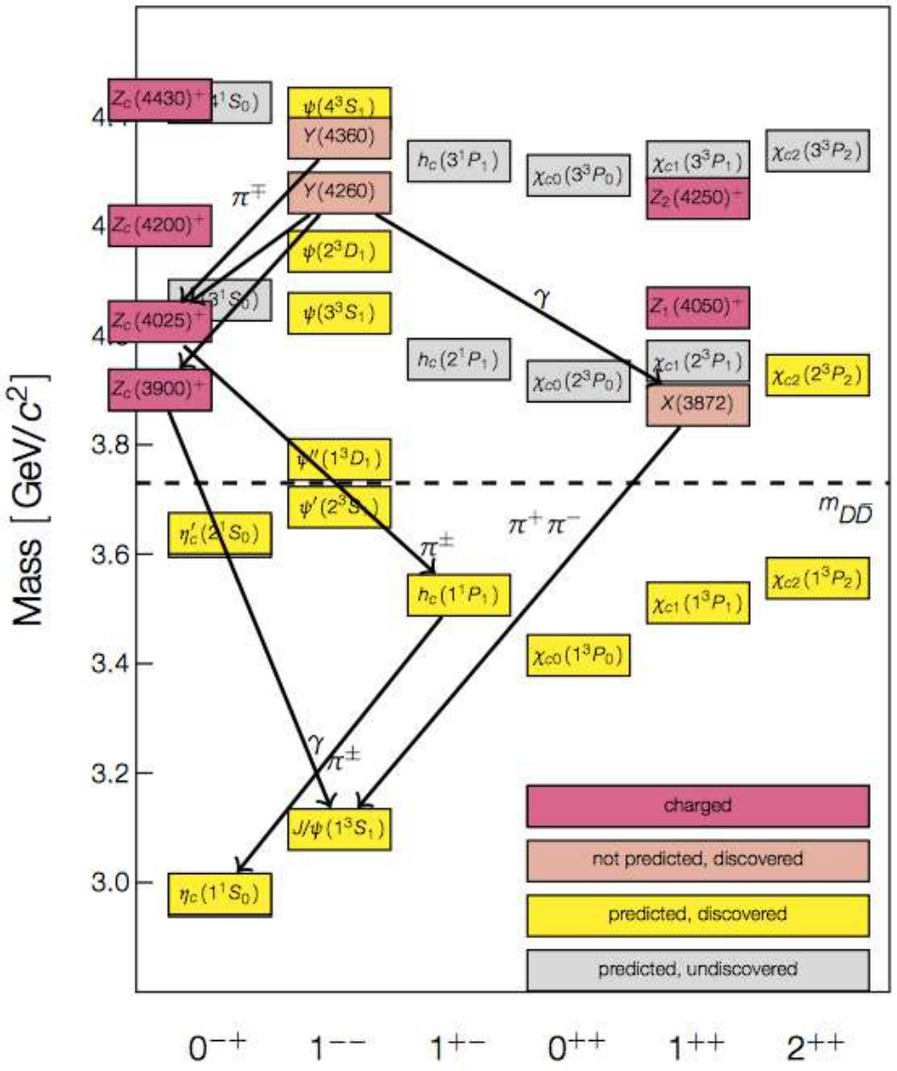

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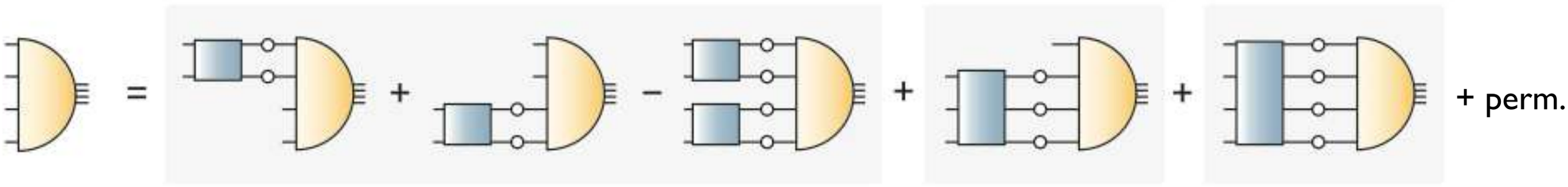
Related to details of QCD interaction of quarks and gluons

Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

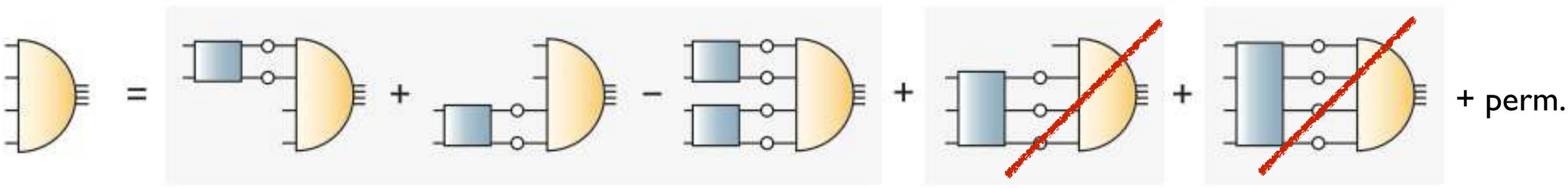
Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Solve four-body equation without/with possibility of internal clustering
- Key elements: quark propagator and interaction kernels

Tetraquarks from the four-body equation

Exact equation:



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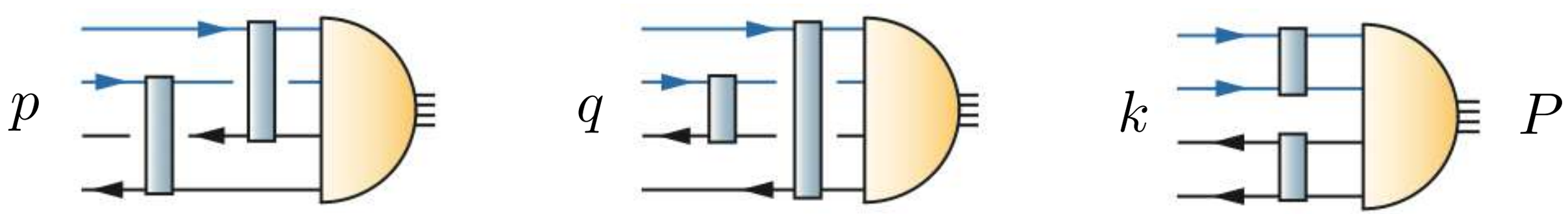
- Solve four-body equation without/with possibility of internal clustering
- Key elements: quark propagator and interaction kernels

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

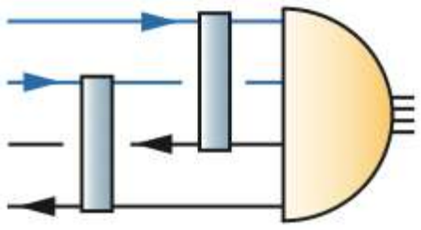
9 Lorentz scalars (built from P,p,q,k)

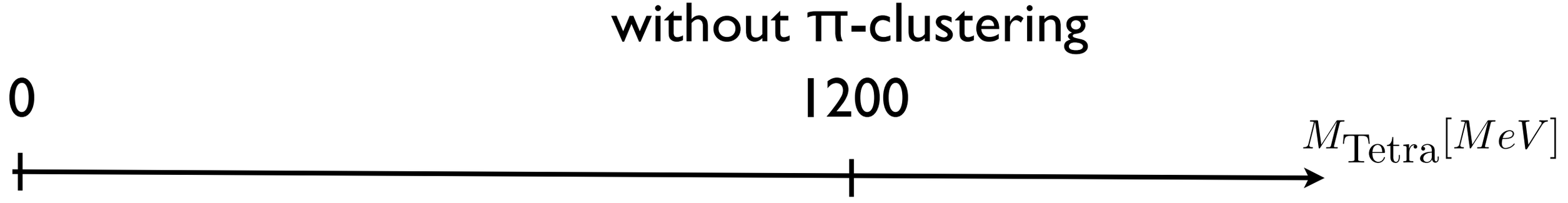
256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

- reasonable approximation: keep s-waves only;
  → 16 tensor structures

Bound state vs resonance: scalar tetraquarks

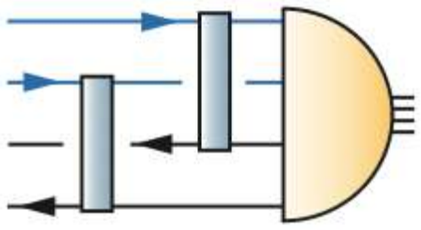




Bound state of
four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: scalar tetraquarks



without π-clustering

0 400 1200 $M_{\mathrm{Tetra}}[MeV]$

with π-clustering

Two-pion resonance

Bound state of
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Eichmann, CF, Heupel, PLB 753 (2016) 282-287

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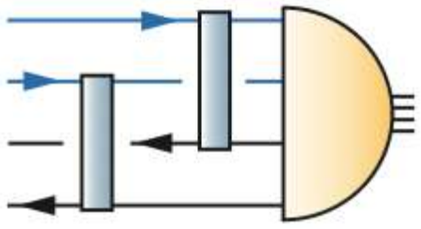

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Two-pion resonance

Bound state of
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⟶ identify with $f_0(500)$ (‘σ-meson’)

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

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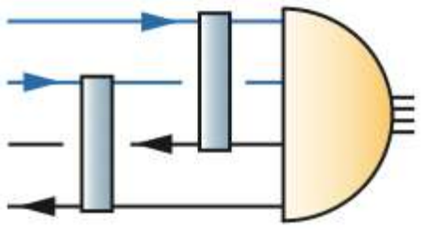


without π-clustering

0 400 1200 $M_{\rm Tetra}[MeV]$

with π-clustering

Two-pion resonance

Bound state of
four massive quarks

⟶ identify with $f_0(500)$ ('σ-meson')

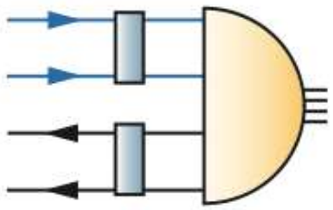
with strange quarks: $m_\kappa \sim 750\,\mathrm{MeV}$

$m_{a_0,f_0} \sim 1080\,\mathrm{MeV}$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

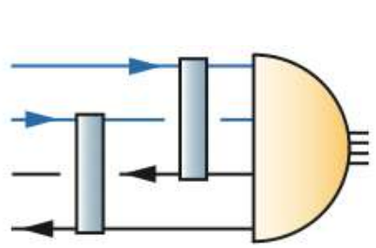
Outlook: heavy-light systems

Dynamical situation in **S4**-doublet:



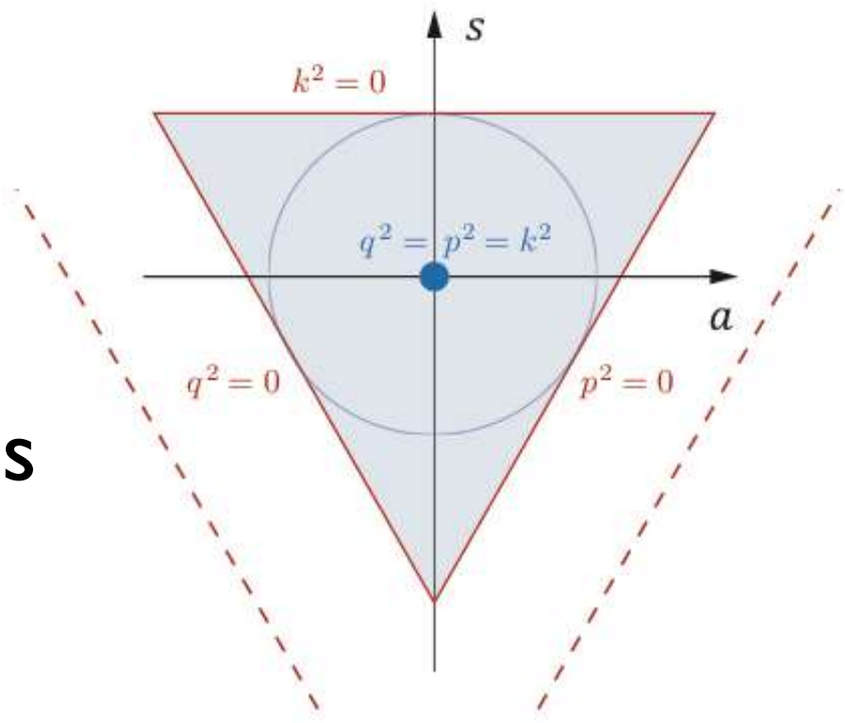
heavy-light diquarks

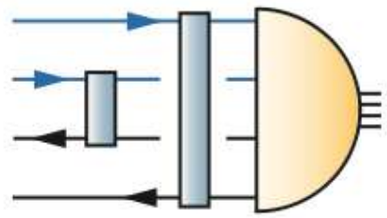
$$cq - \bar{c}\bar{q}$$



meson-molecules

$$c\bar{q} - \bar{c}q$$





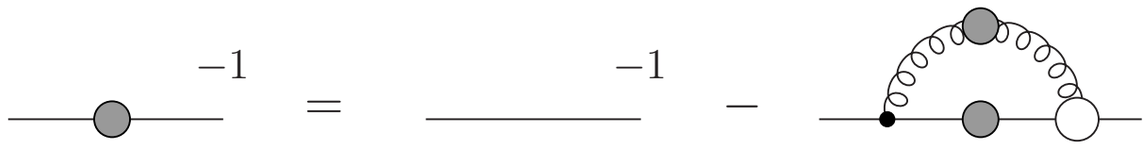
hadro-charmonium

$$c\bar{c} - q\bar{q}$$

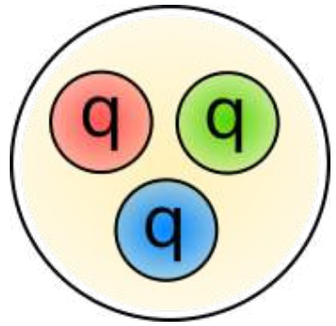
Dynamical decision of most important clustering!

Overview

1. Mass from nothing



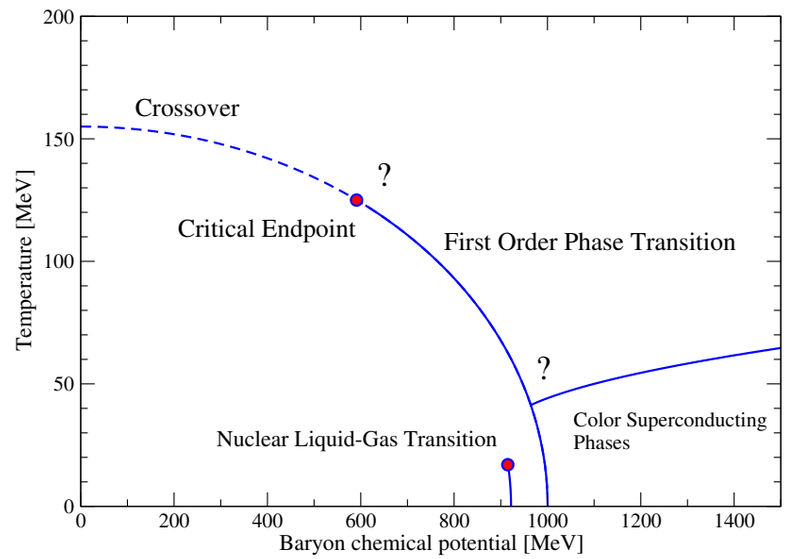
2. Ordinary and exotic hadrons



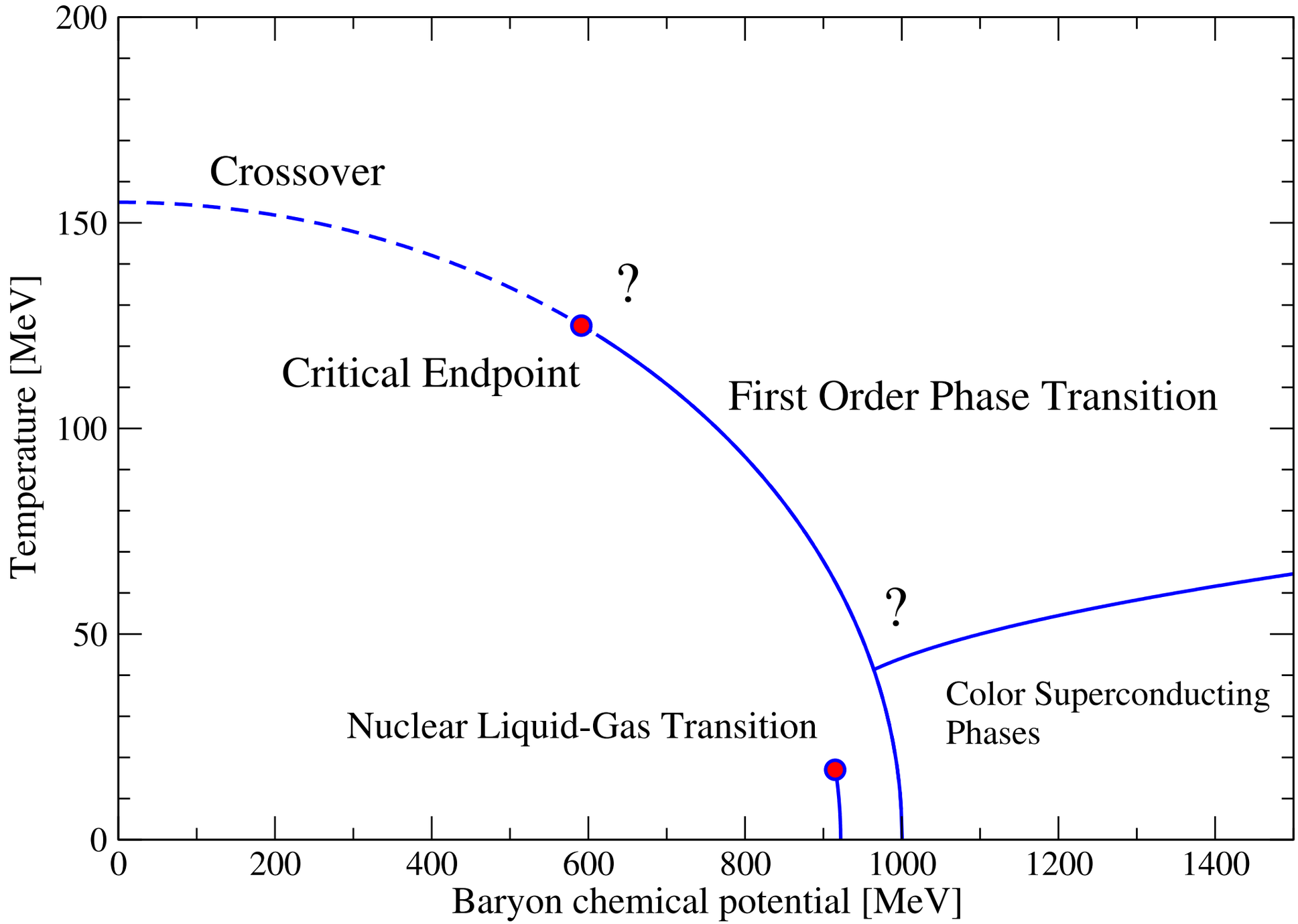
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

3. Phase transition and the early universe

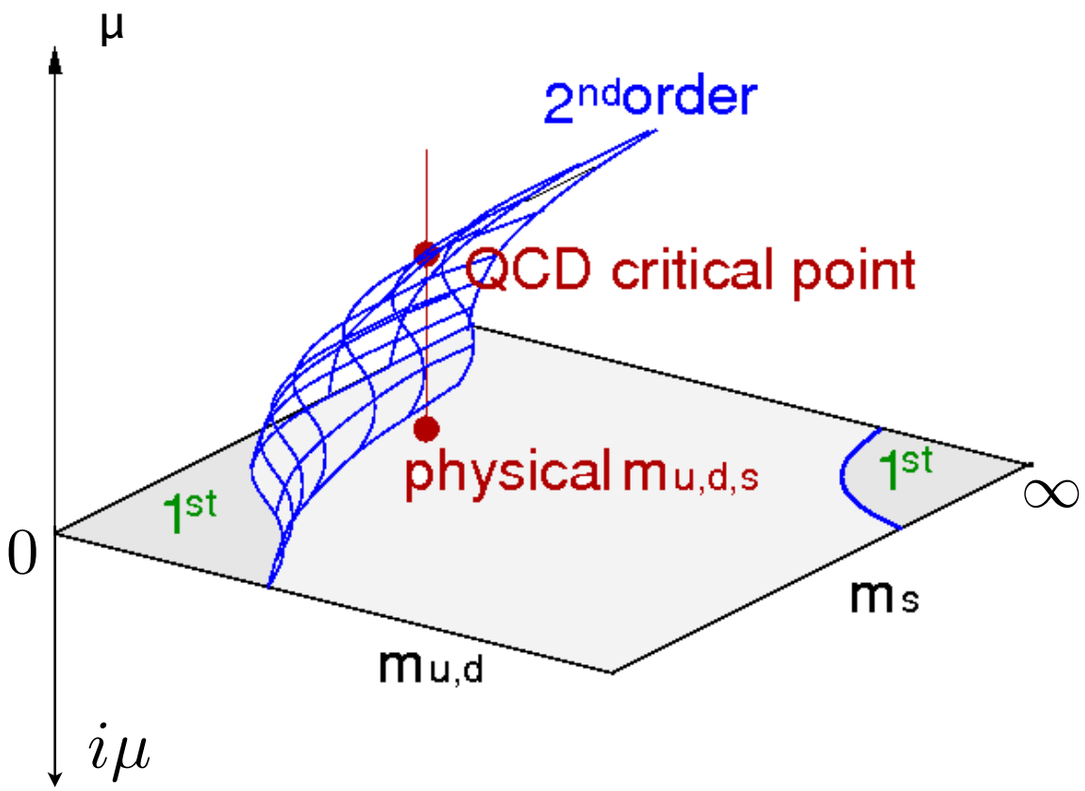
Review: CF, PPNP in press [1810.12938]



QCD phase transitions



Chiral transition line from analytic continuation



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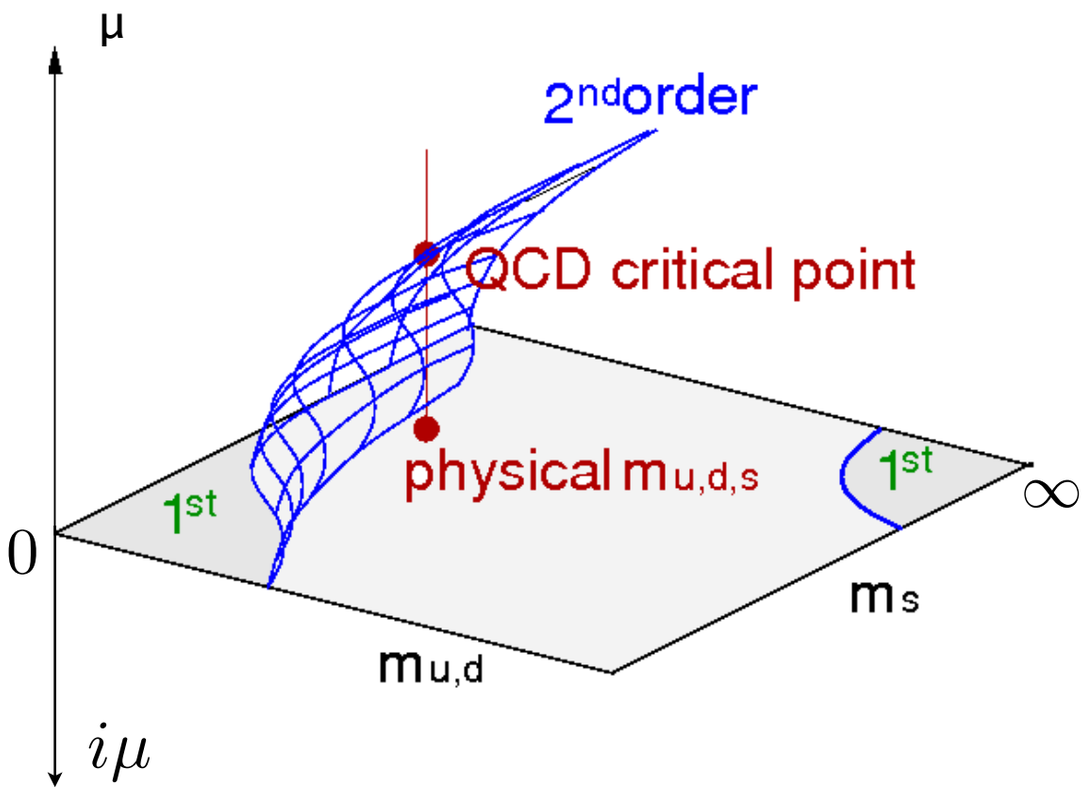
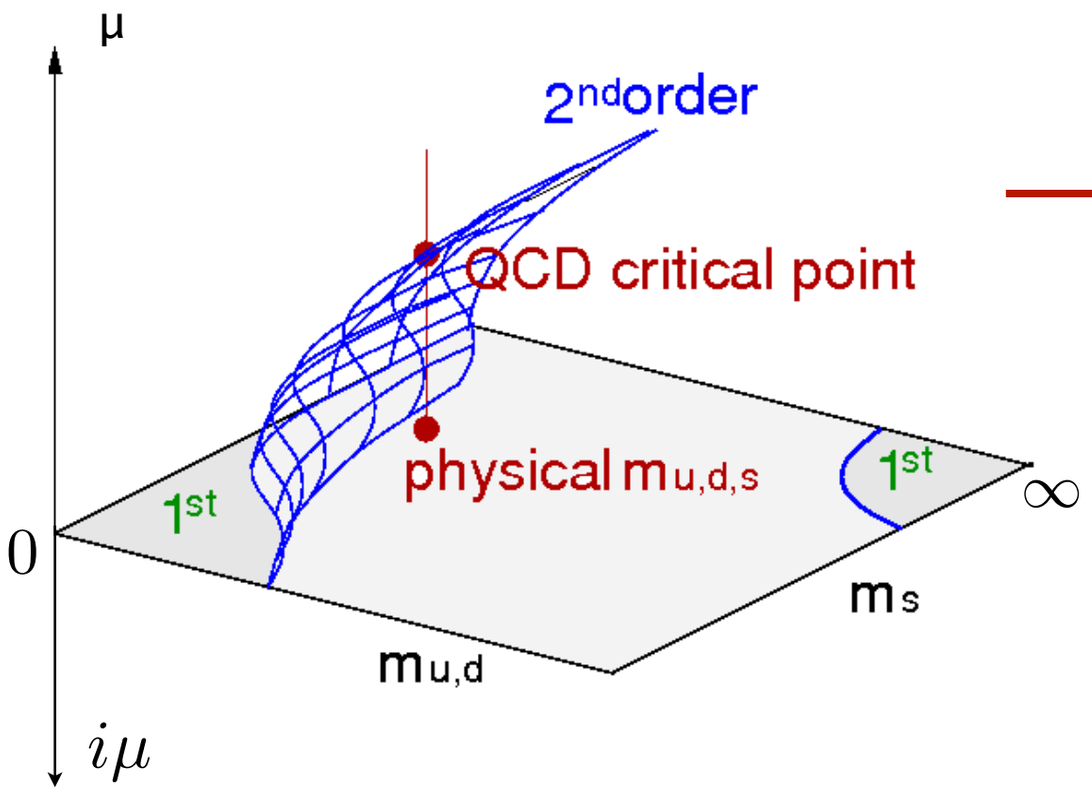
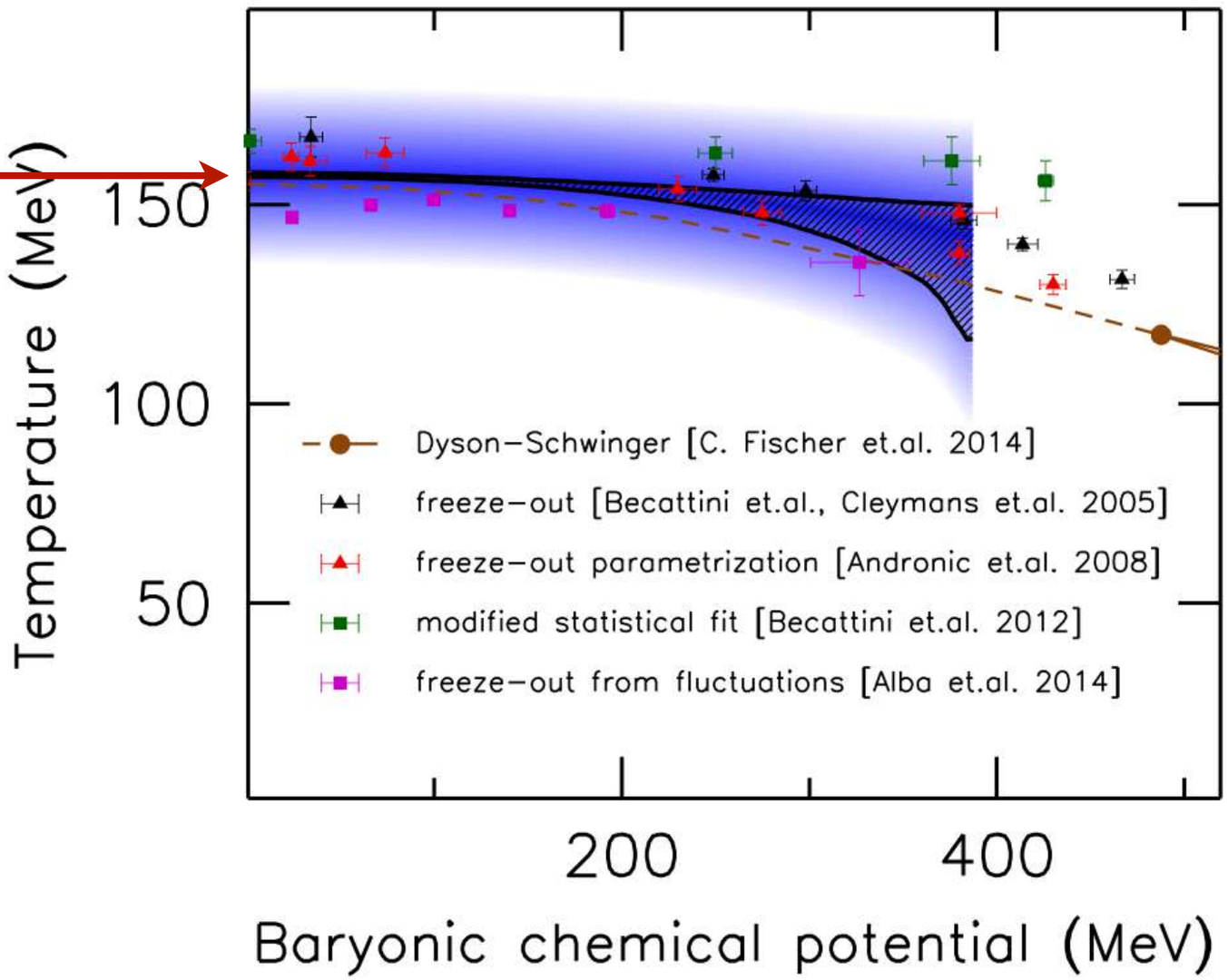


Lattice method:

- Det. crossover at imaginary μ and extrapolate to real μ
- Control systematics

Chiral transition line from analytic continuation





Lattice method:

- Det. crossover at imaginary μ and extrapolate to real μ
- Control systematics

Bellwied, Borsanyi, Fodor, Günther,
Katz, Ratti and Szabo, PLB 751 (2015) 559

HOT-QCD: similar results

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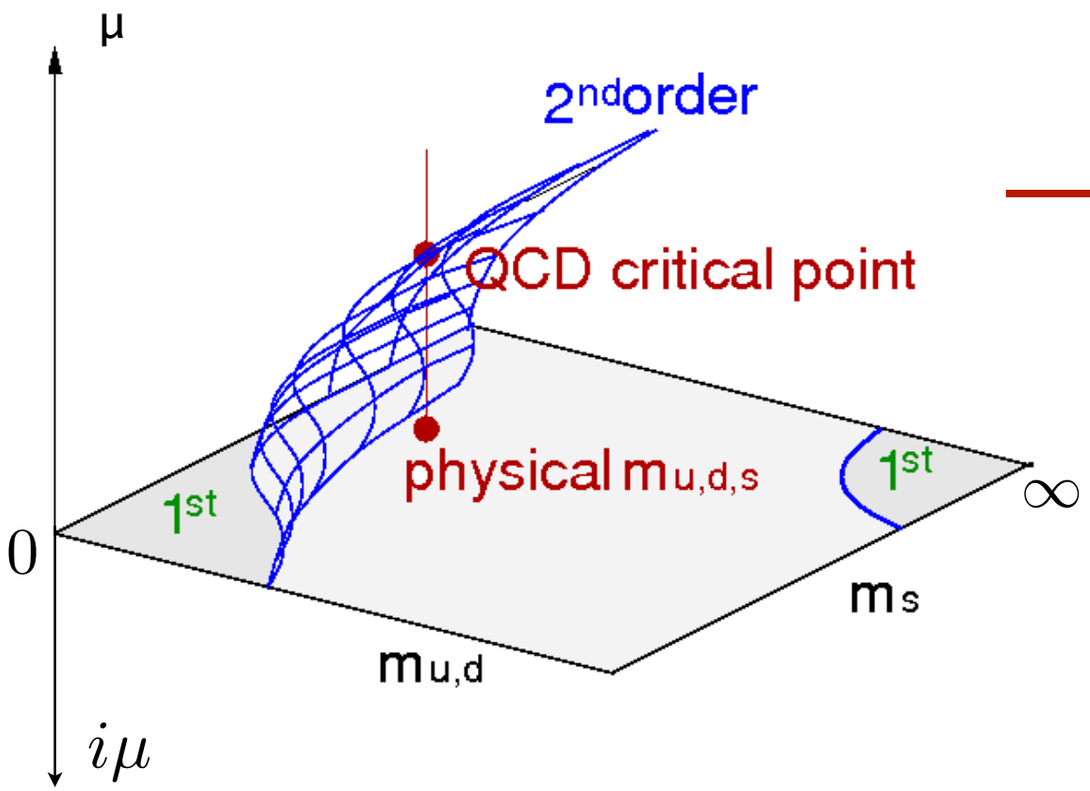

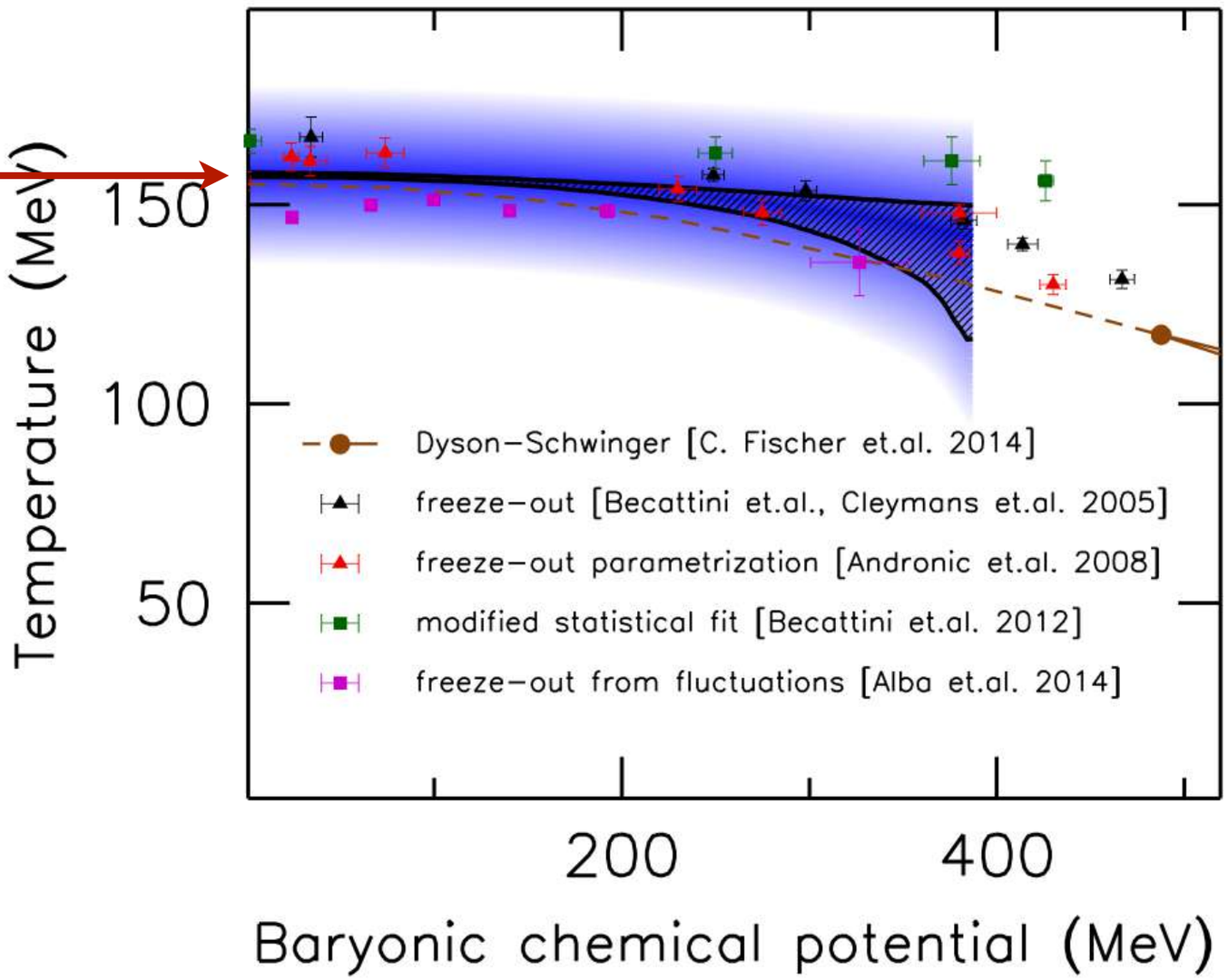

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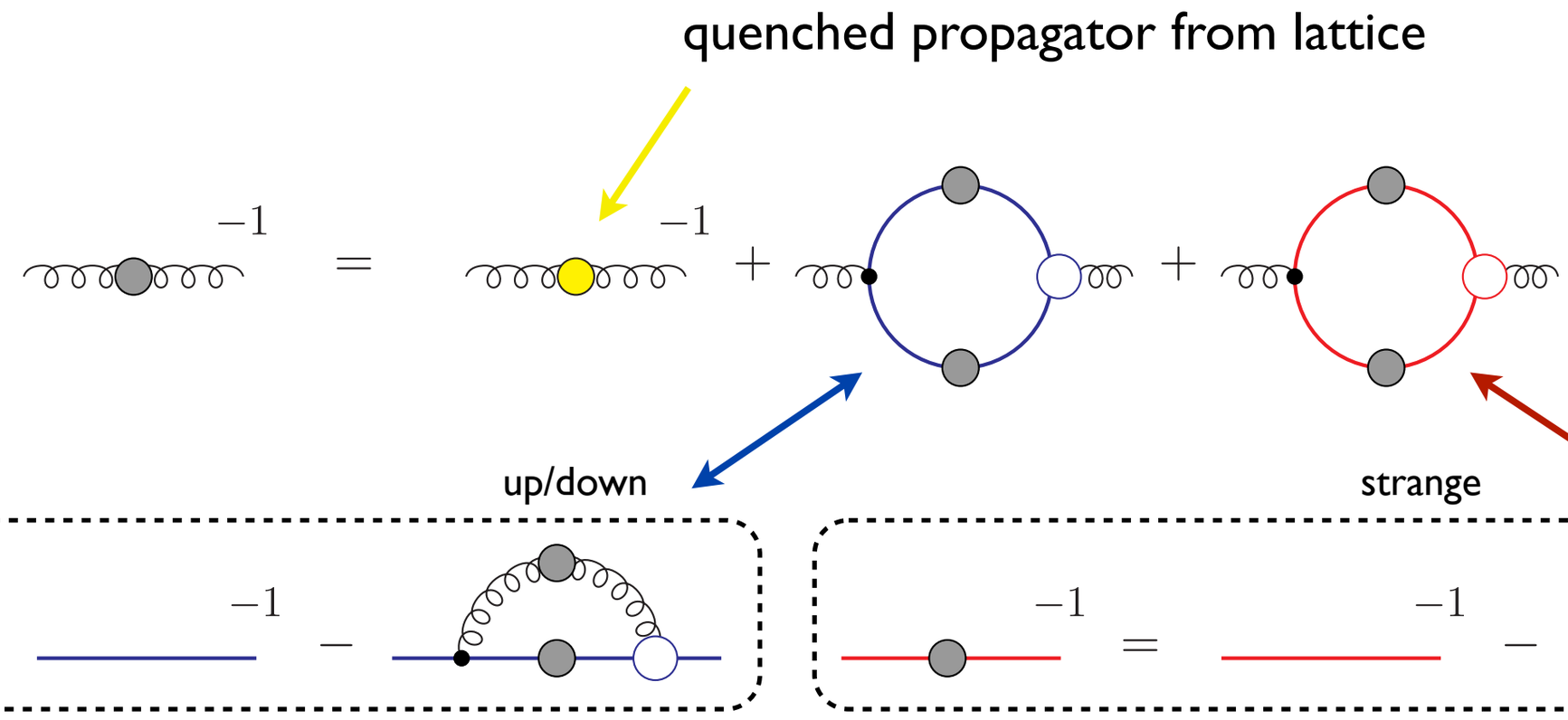
Katz, Ratti and Szabo, PLB 751 (2015) 559

HOT-QCD: similar results

Main result: no CEP for $\mu_B/T < 2\text{-}3$

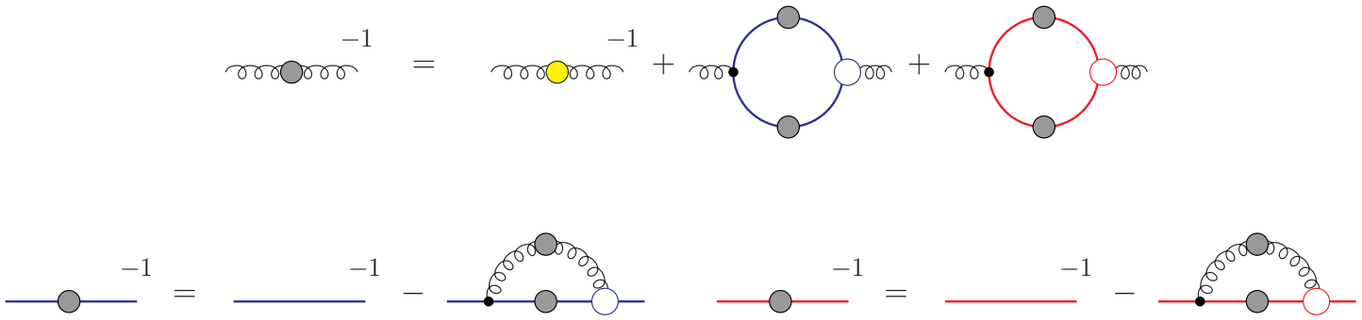
N_f=2+1-QCD with DSEs



- allows for systematic variation of $m_{u/d}$ and m_s
- quark-gluon vertex:
  ansatz built along STI and known UV/IR behavior

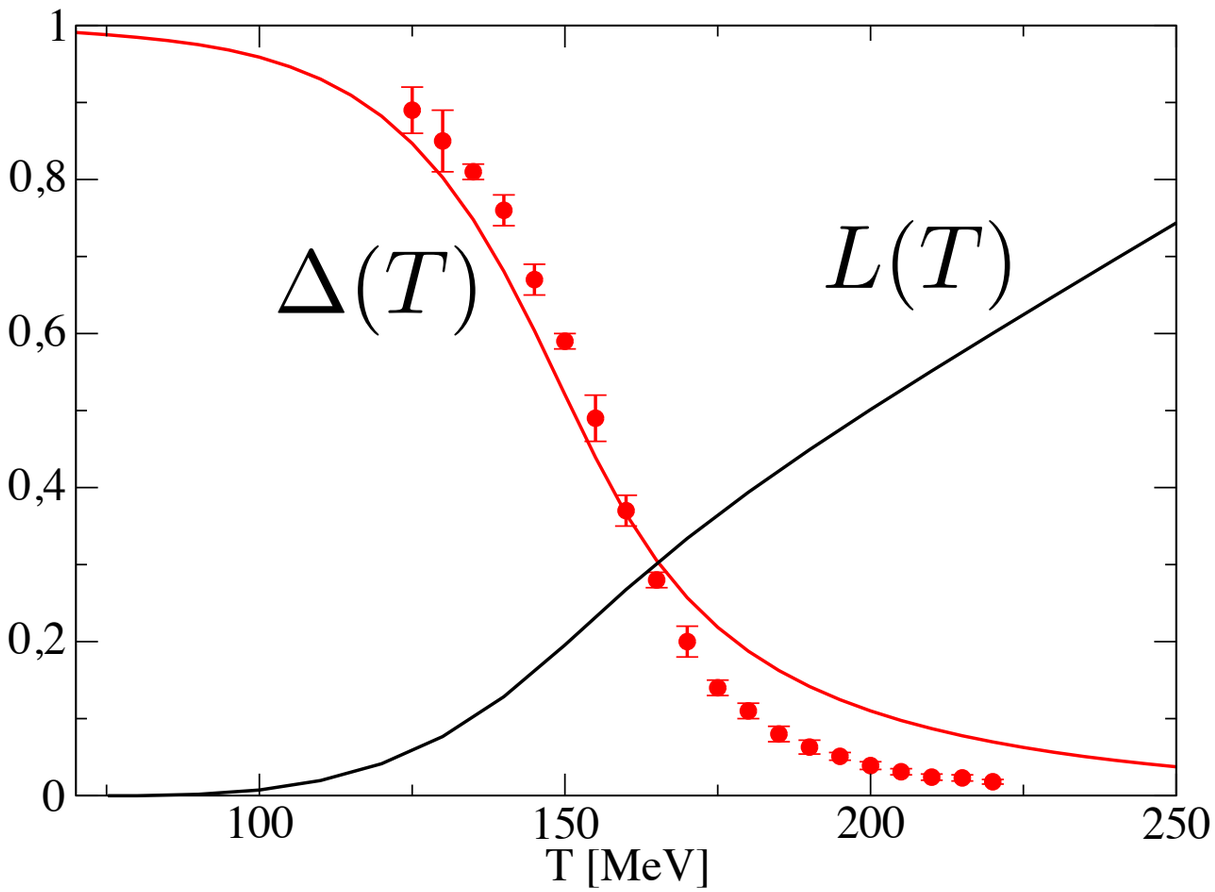
→ T,μ,m-dependent

N_f=2+1, zero chemical potential



$$\Delta(T) \sim M(T)$$

$$L(T) \sim \exp(-E/T)$$



Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

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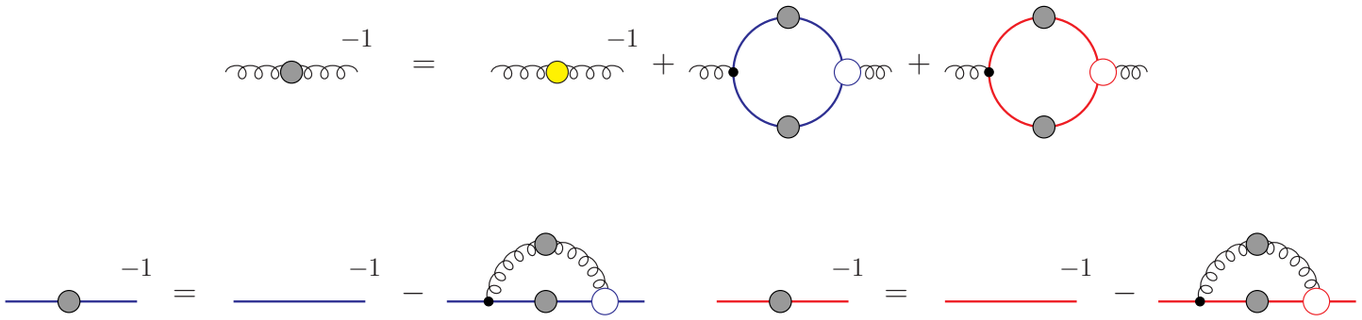

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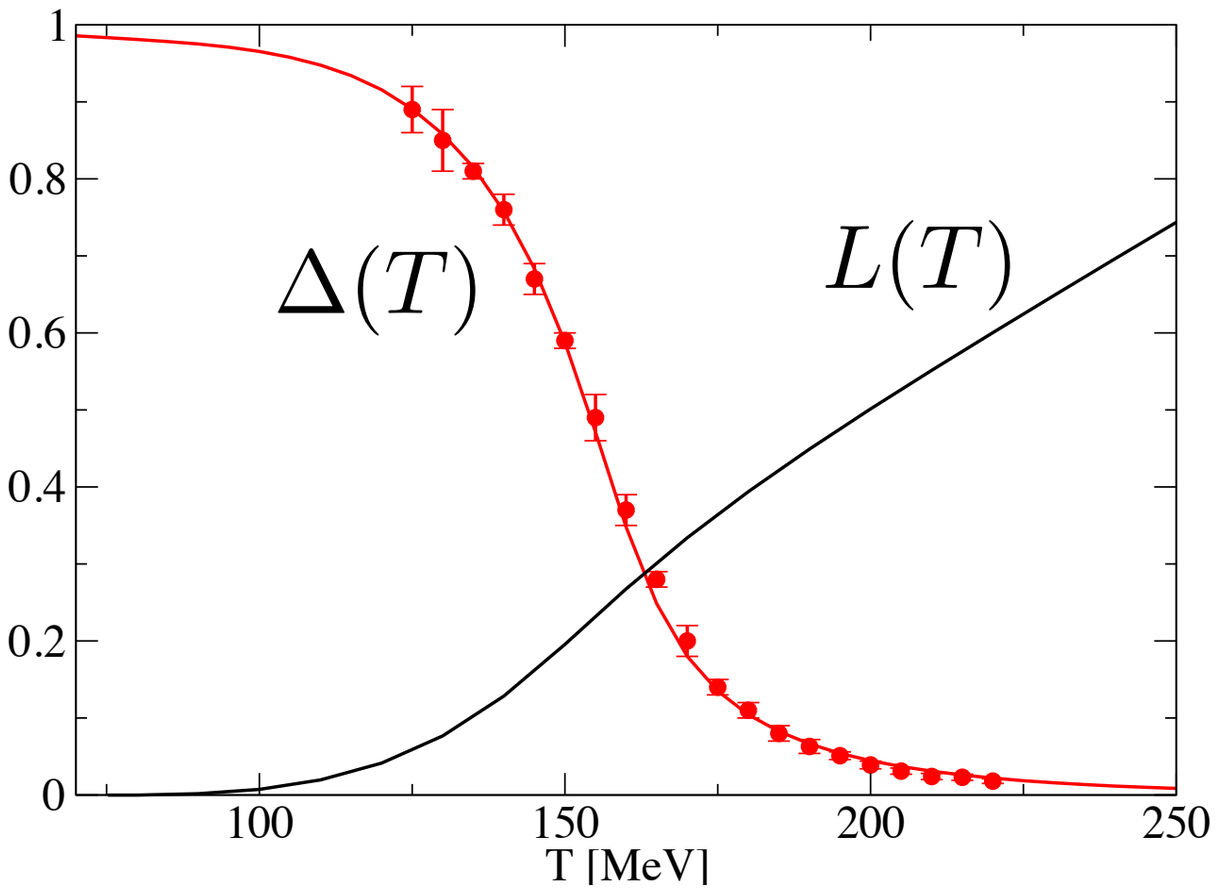

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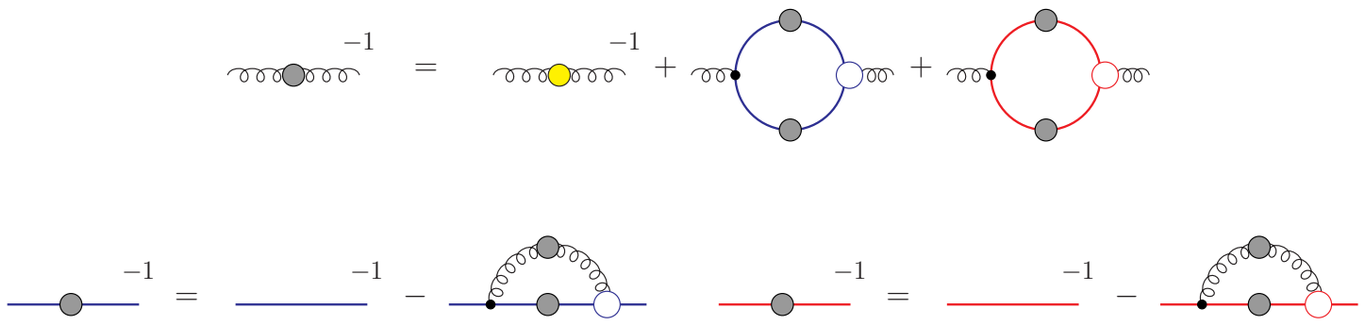

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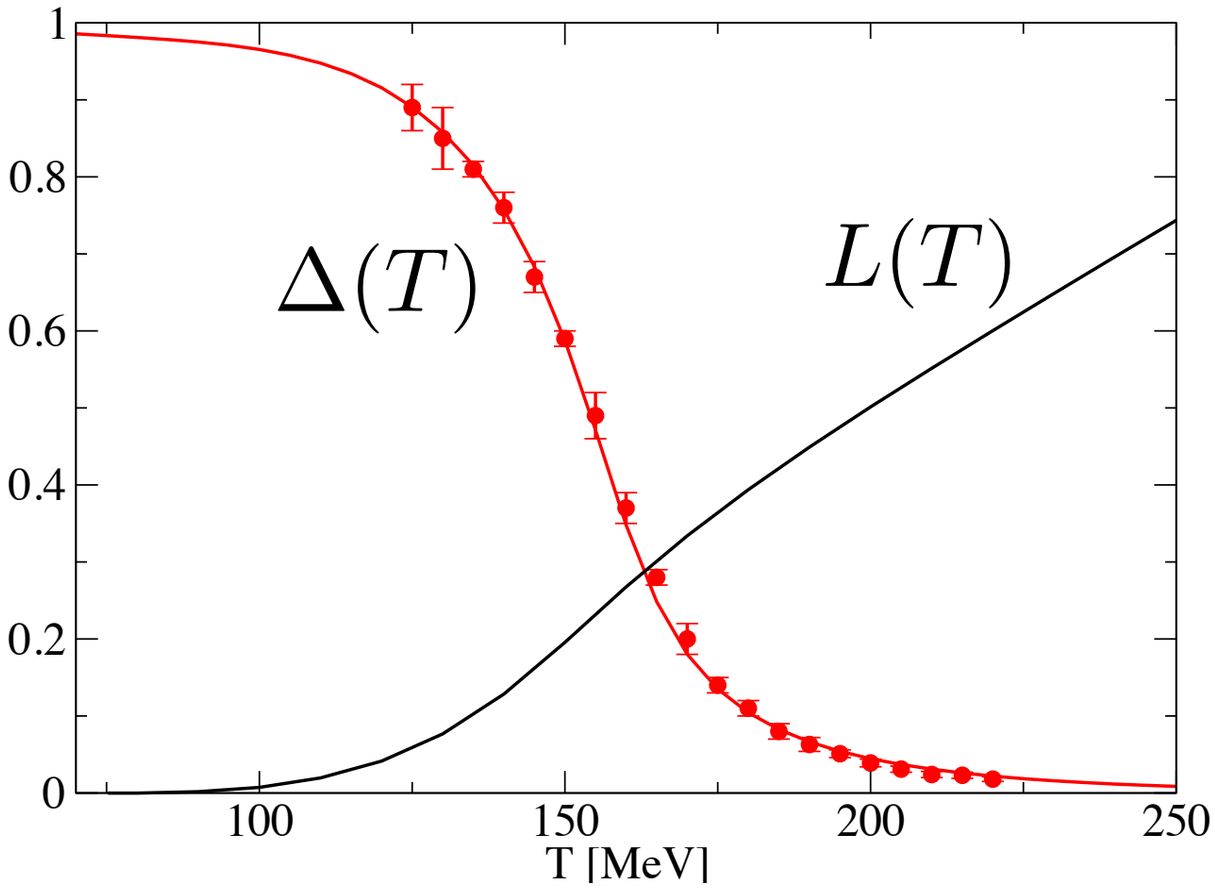

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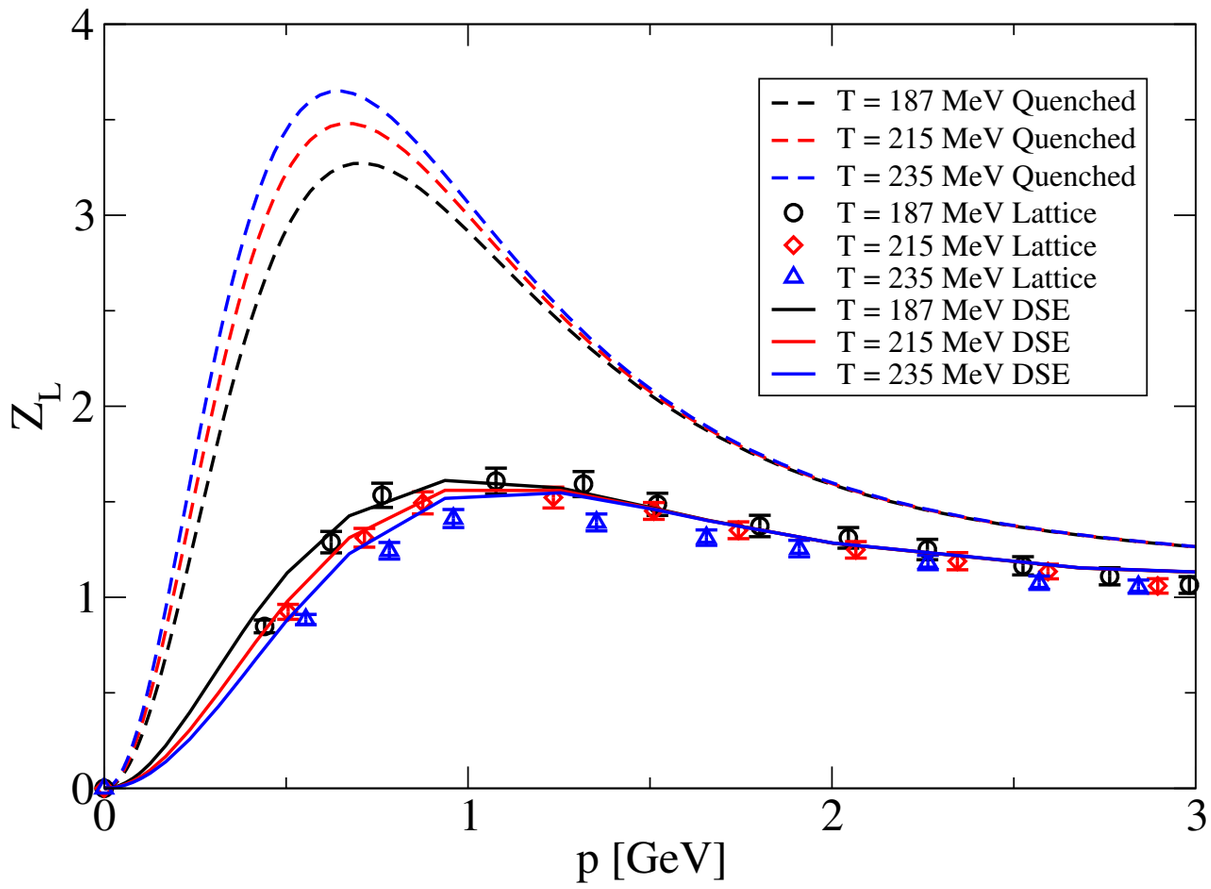
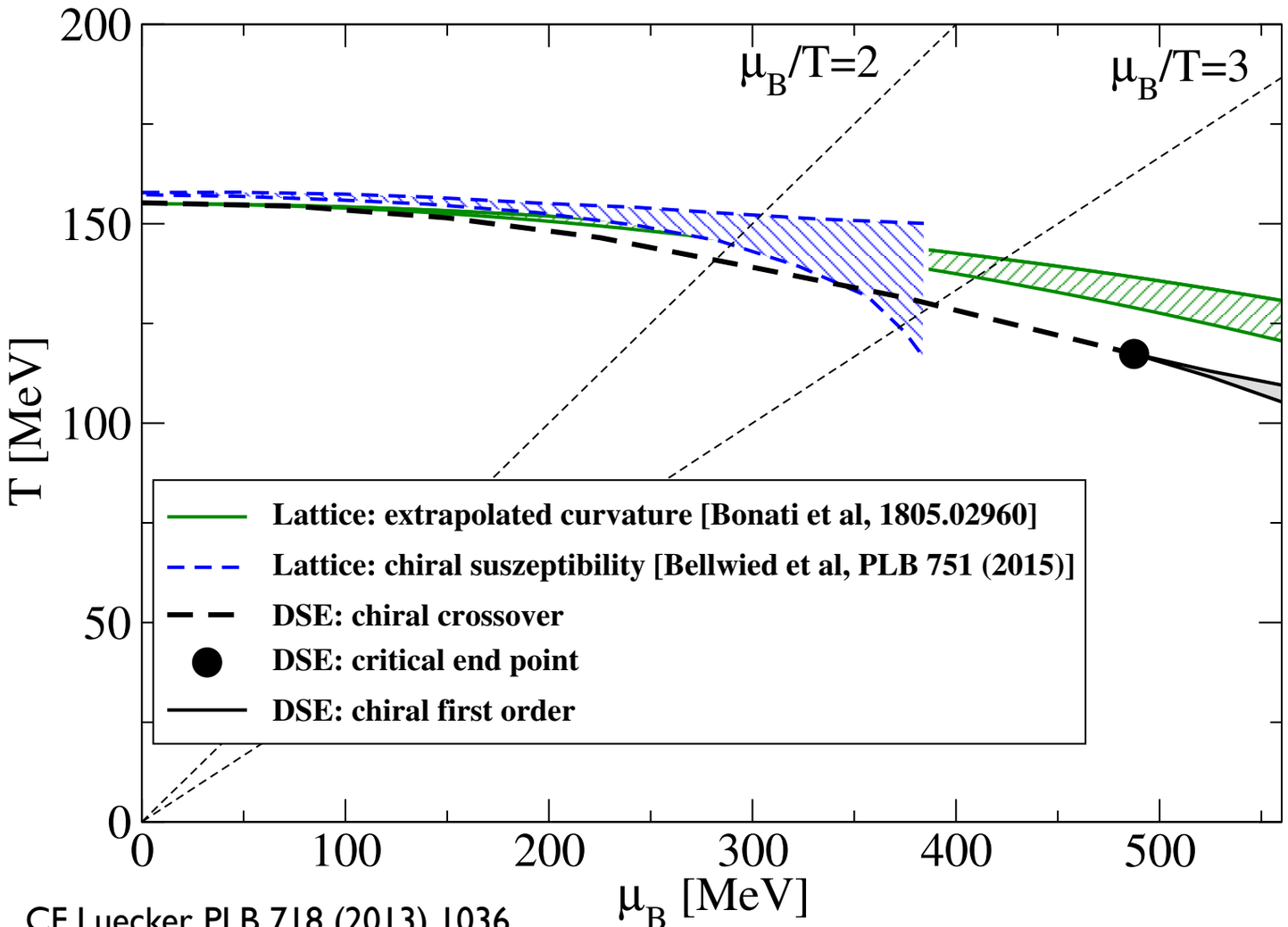


Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

- quantitative agreement: DSE prediction verified by lattice

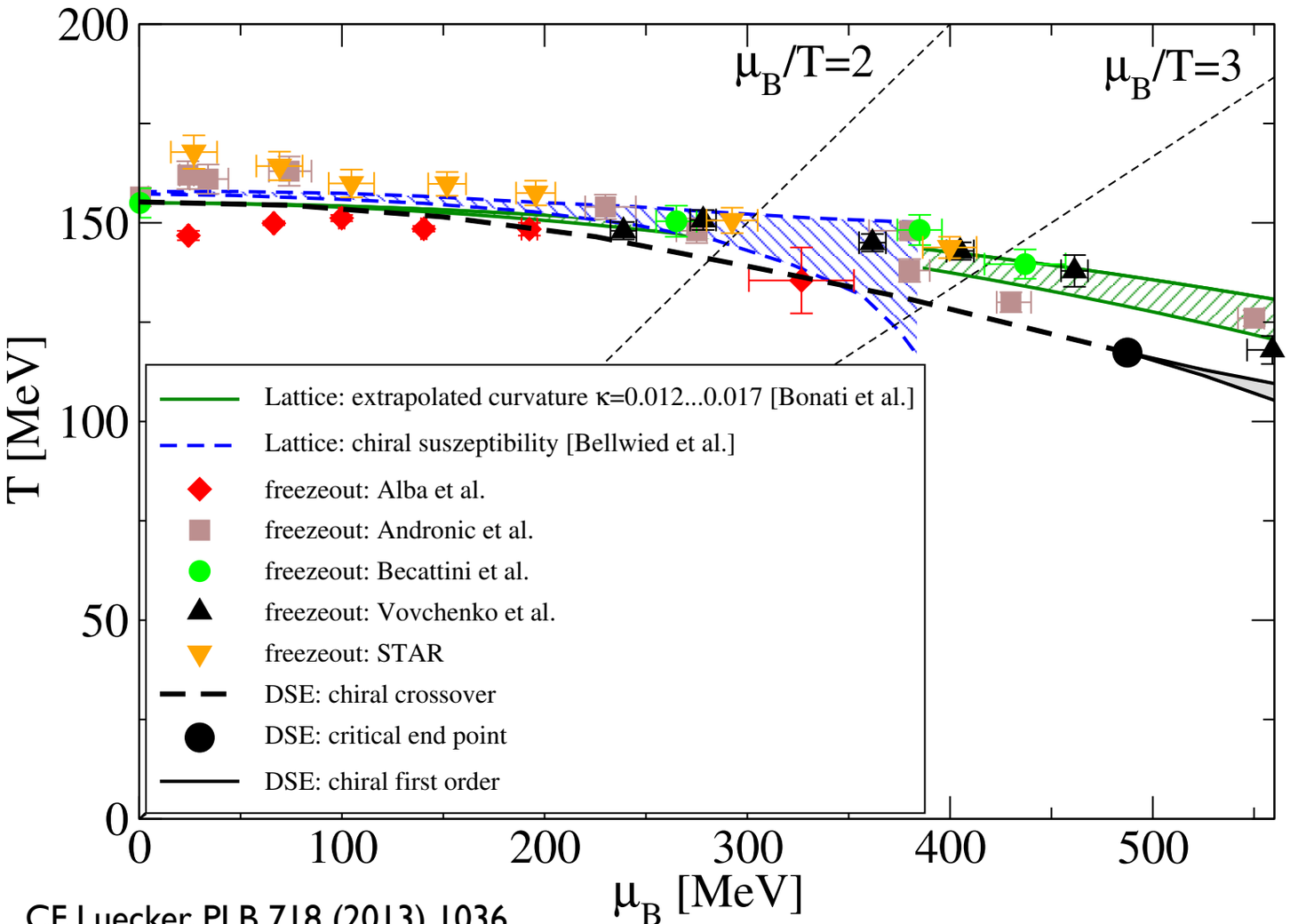
QCD phase diagram and heavy ion collisions



CF, Luecker, PLB 718 (2013) 1036,
CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

- combined lattice/DSE evidence: no CEP at $\mu_B/T < 2$-3
- CEP @ $(T, \mu_B)=(117,488)$ MeV
- Yang-Mills sector crucial !

QCD phase diagram and heavy ion collisions



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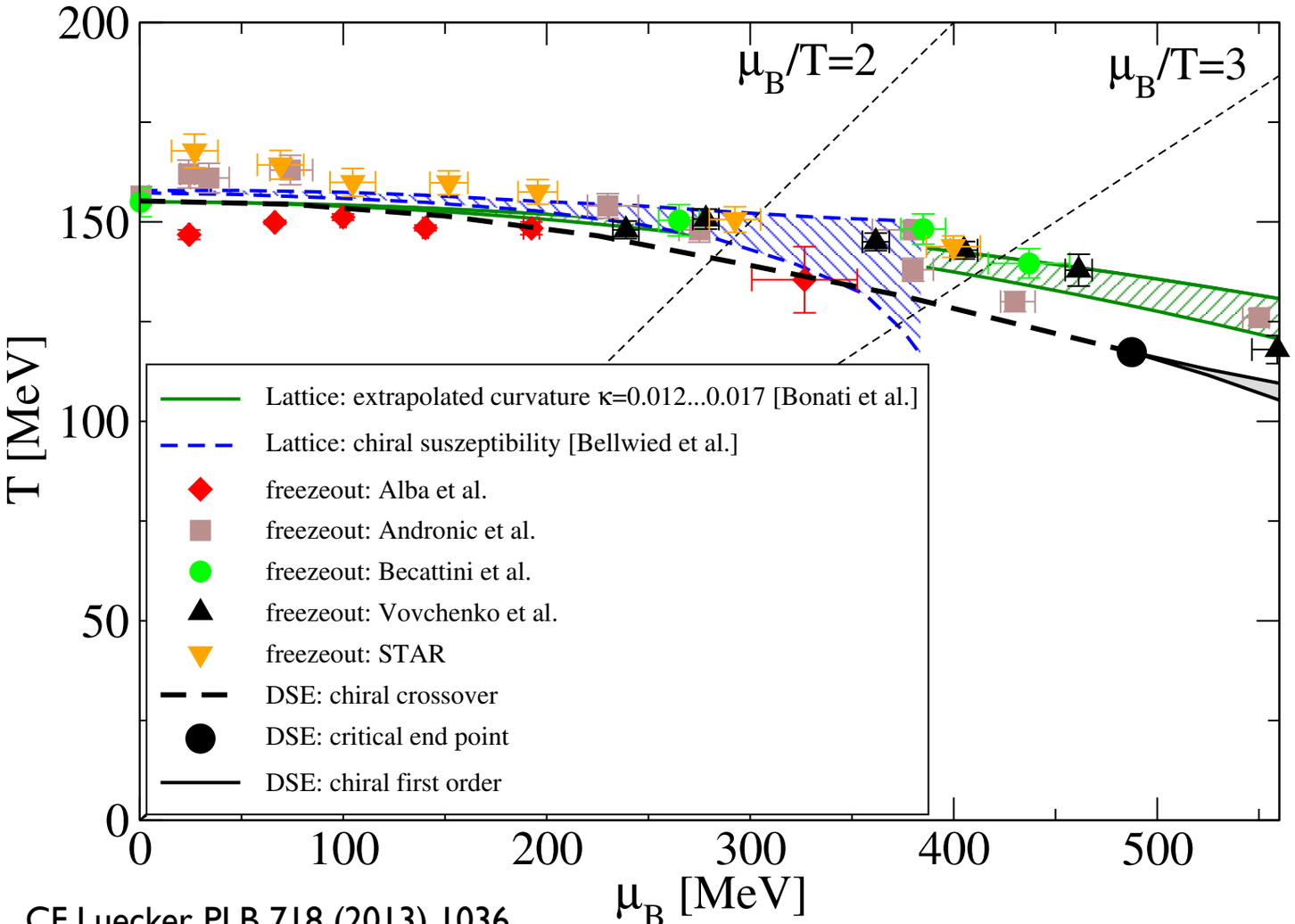


CF, Luecker, PLB 718 (2013) 1036,
CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

heavy ion collisions

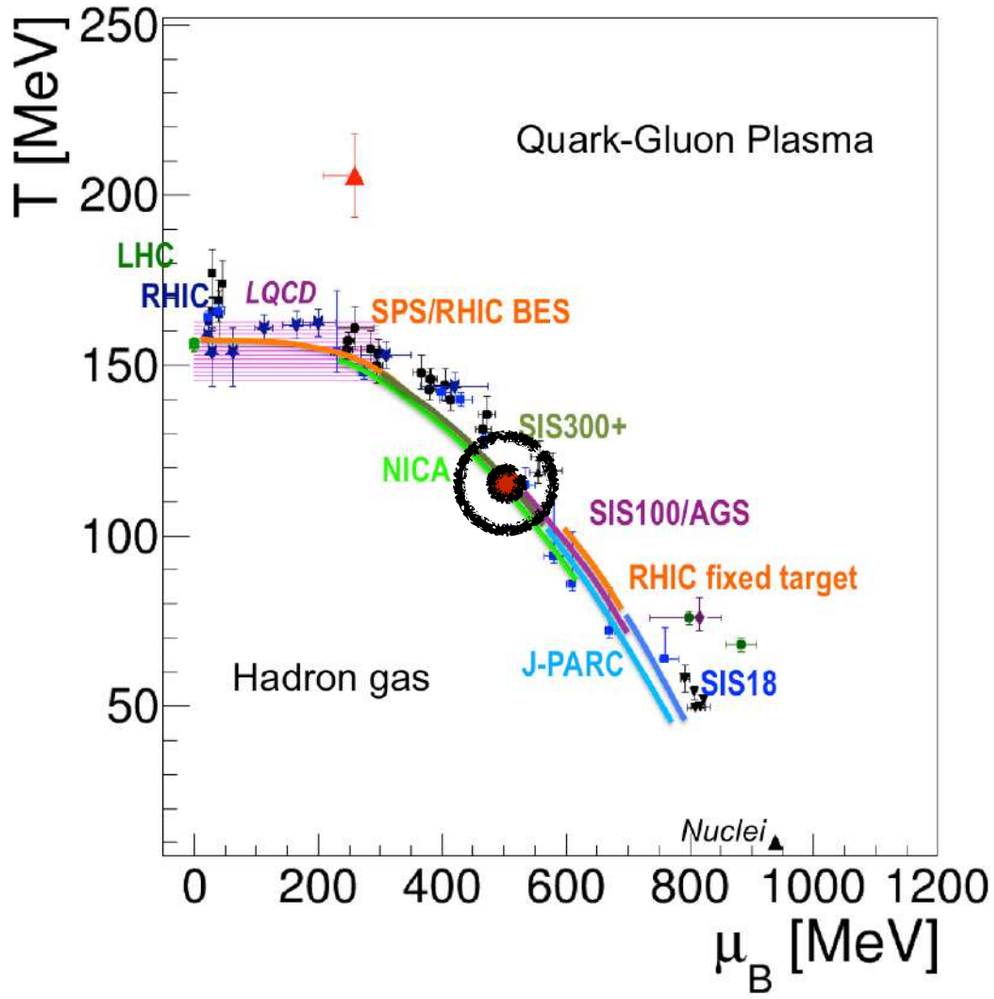


Figure adapted from talk of T. Galatyuk, Erice 2016

- combined lattice/DSE evidence: no CEP at $\mu_B/T < 2$-3
- CEP @ $(T, \mu_B)=(117,488)$ MeV
- Yang-Mills sector crucial !

Summary

Main goals:

- one framework for all areas of hadron physics:
  - discussed: baryons, tetraquarks, QCD phase diagram
  - not discussed: mesons, glueballs, hybrids,
    electromagnetic form factors
    anomalous magnetic moments
- access to DXSB, confinement,…

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

CF, PPNP in press [1810.12938]

Main challenge:

- systematic control over error budget:
  intrinsic + cp to other methods like lattice QCD

Main results:

- NOT high precision physics
- BUT competitive contributions in many areas

Backup Slides

Contact with experiment: fluctuations

X.~Luo and N.~Xu, Nucl. Sci. Tech. 28 (2017) no.8, 112 [arXiv:1701.02105 [nucl-ex]].

Quark chemical potentials related to those of conserved charges:

$$\mu_u = \mu_B/3 + 2\mu_Q/3$$
$$\mu_d = \mu_B/3 - \mu_Q/3$$
$$\mu_s = \mu_B/3 - \mu_Q/3 - \mu_S$$

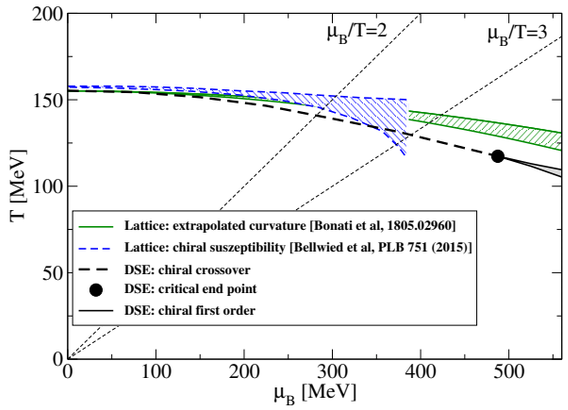
Serve to calculate susceptibilities:

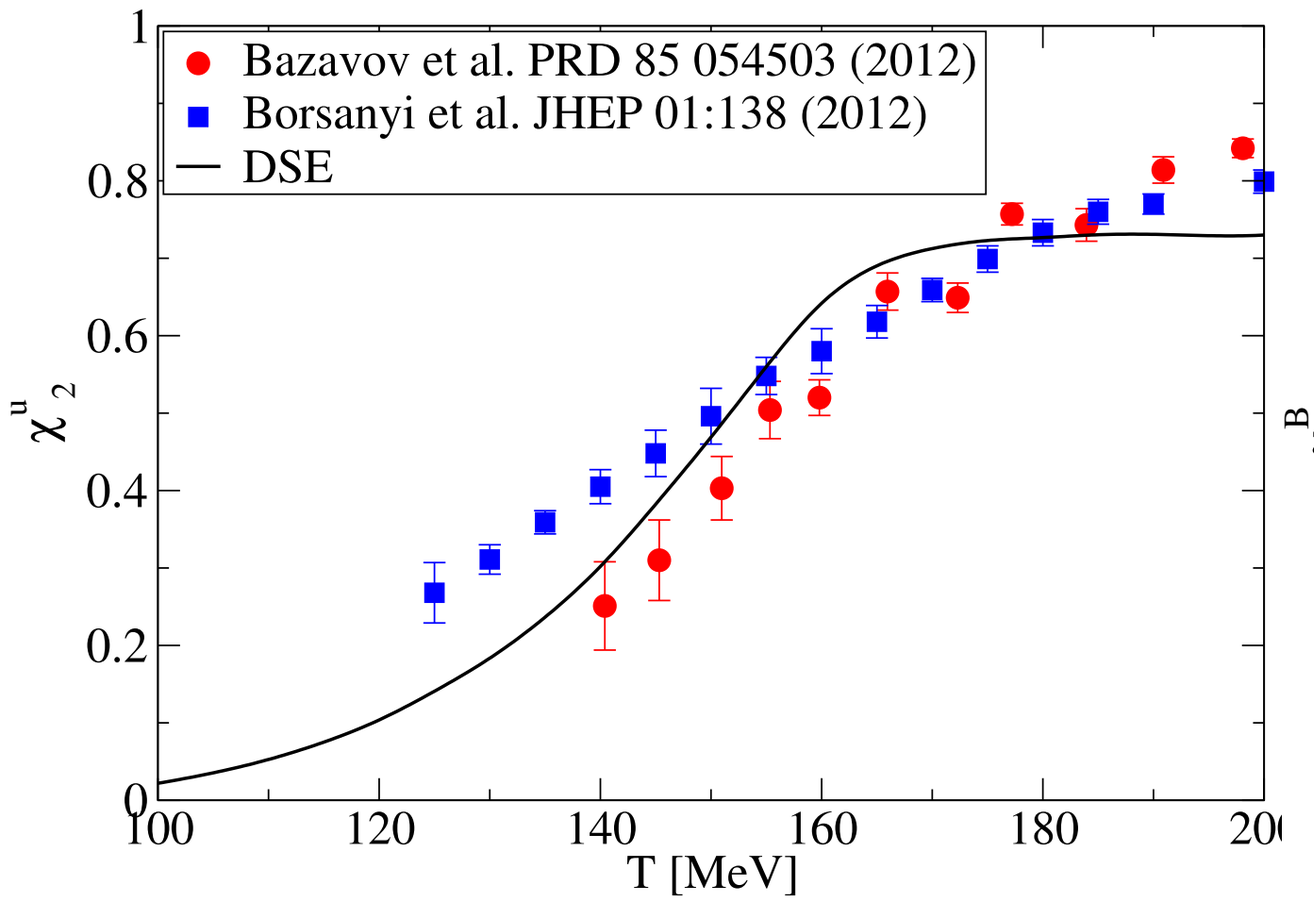
$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

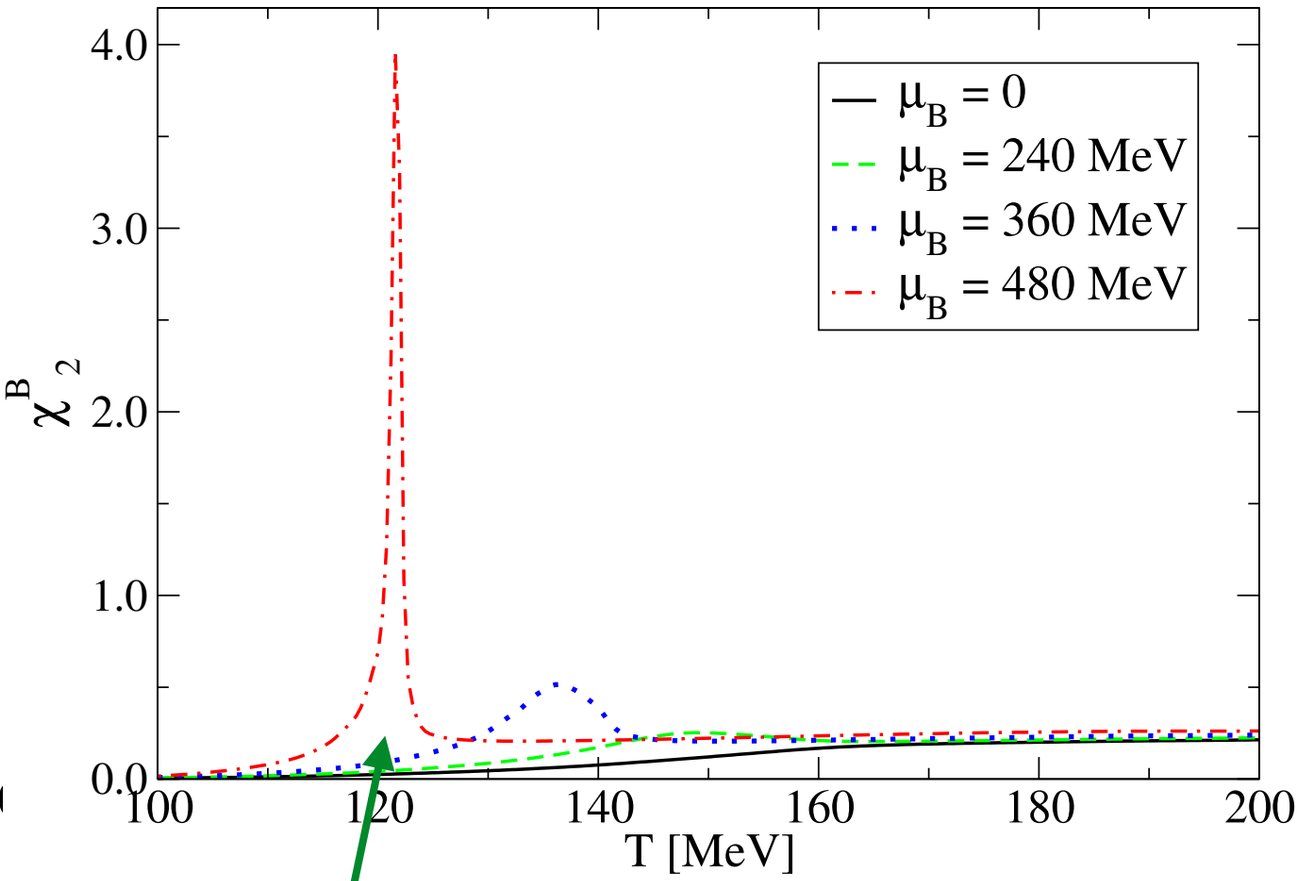
Related to cumulants, which can be extracted from experiment:

$$C^{BSQ}_{lmn} = VT^3 \chi^{BSQ}_{lmn}$$

Results for fluctuations





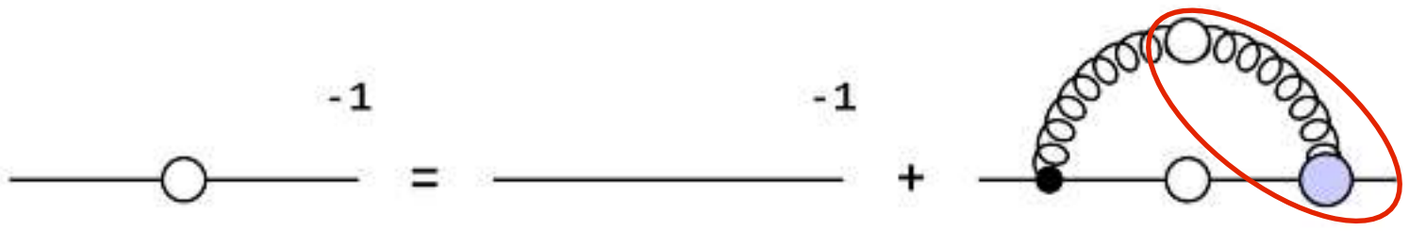


- first result: works !

location of CEP

Isserstedt et al. in preparation

Rainbow-ladder model for quark-gluon interaction



Combine gluon with quark-gluon vertex:

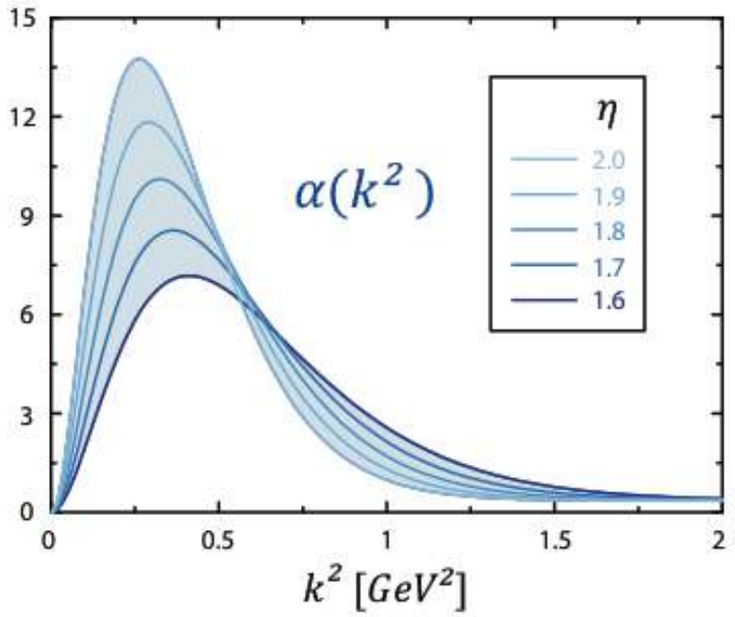
$$\Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

$$\sim \gamma^\mu \tau(k^2)$$ "approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

Rainbow-ladder model for quark-gluon interaction



Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

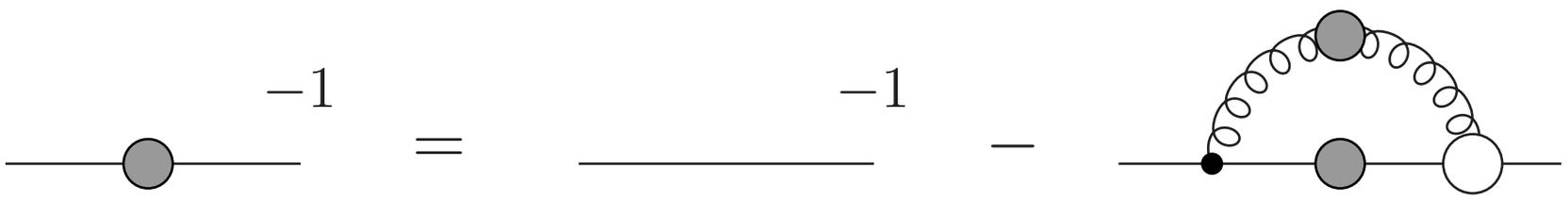
effective coupling

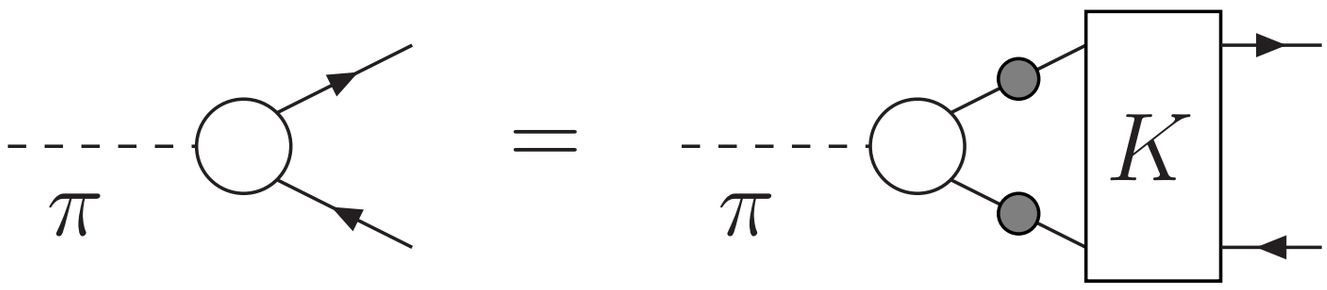
$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale Λ from f_π, masses m_u=m_d, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein

Theoretical Tools II: DSEs and BSEs





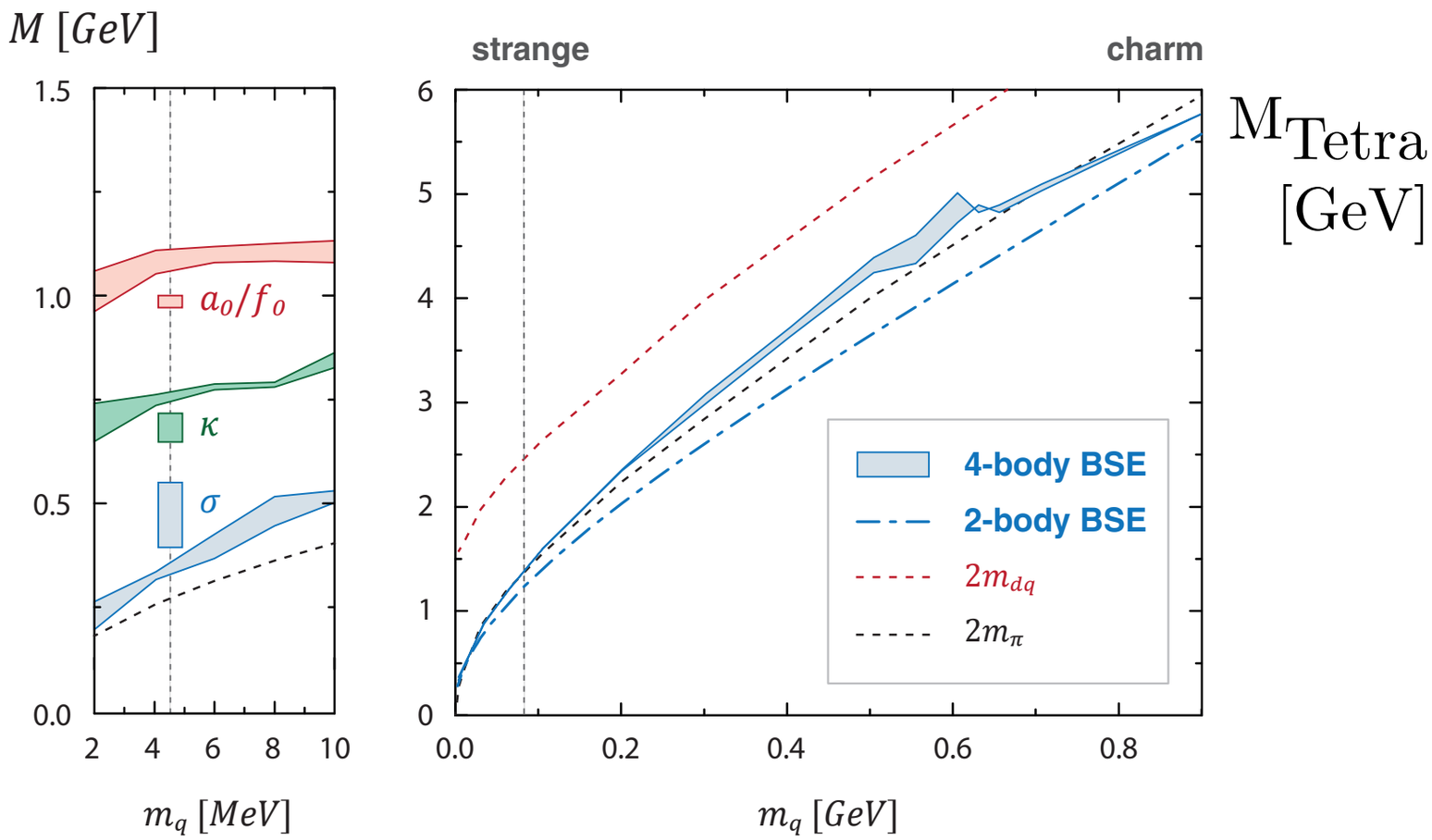
Kernel K uniquely related to quark-DSE via axWTI

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying, gauge dependent quark/gluon dynamics
- Need approximations for dressed quark-gluon vertex

Mass evolution of tetraquark



- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

$$m_\sigma \sim 350\,\mathrm{MeV}$$
$$m_\kappa \sim 750\,\mathrm{MeV}$$
$$m_{a_0,f_0} \sim 1080\,\mathrm{MeV}$$

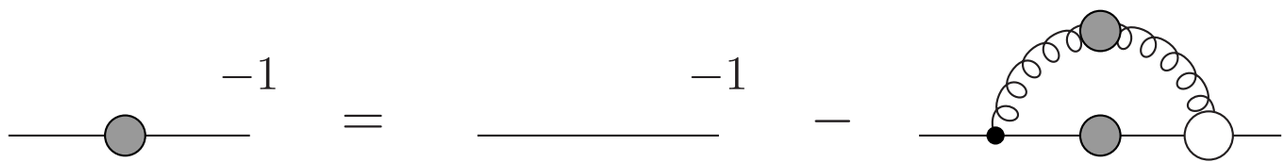
$$m_{ss\bar{s}\bar{s}} \sim 1.5\,\mathrm{GeV}$$
$$m_{cc\bar{c}\bar{c}} \sim 5.7\,\mathrm{GeV}$$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

QCD order parameters from propagators

Chiral order parameter:

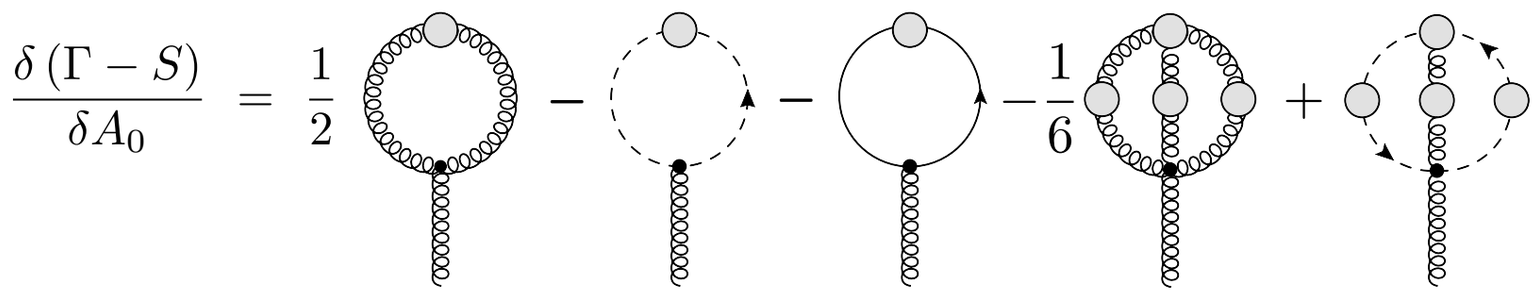
$$\langle \bar{\Psi}\Psi \rangle = Z_2 N_c Tr_D \frac{1}{T} \sum_\omega \int \frac{d^3p}{(2\pi)^3} S(\vec{p},\omega)$$



Deconfinement:

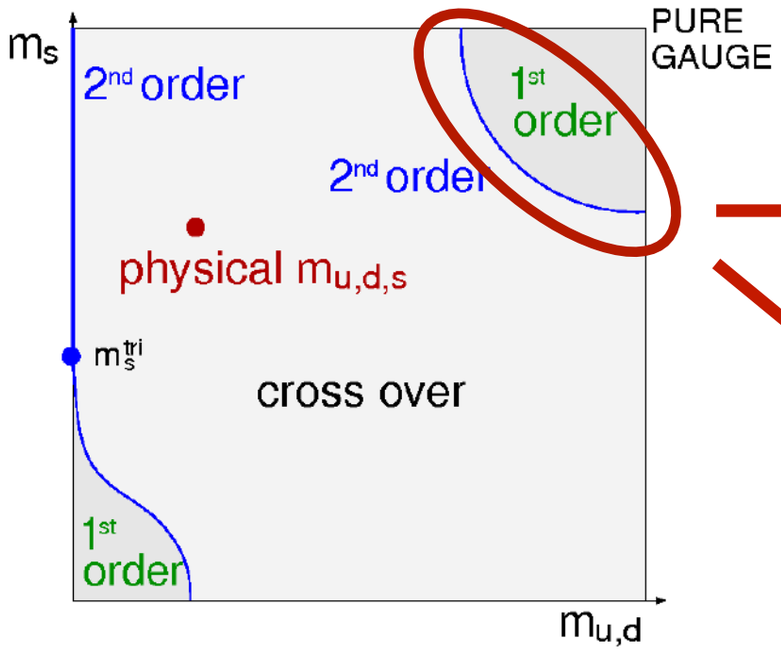
- Polyakov loop potential

$$L = \frac{1}{N_c} Tr\, e^{ig\beta A_0}$$



Braun, Gies, Pawlowski, PLB 684, 262 (2010)
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)
Fister, Pawlowski, PRD 88 045010 (2013)
CF, Fister, Luecker, Pawlowski, PLB 732 (2013)

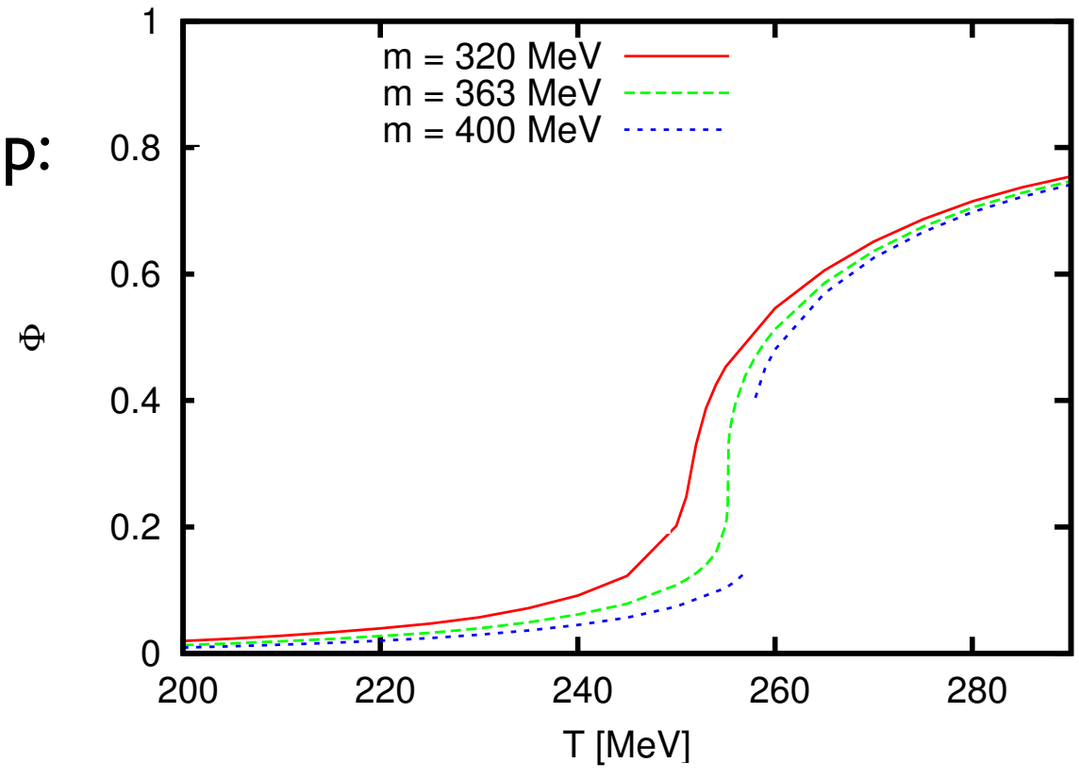
Critical line/surface for heavy quarks

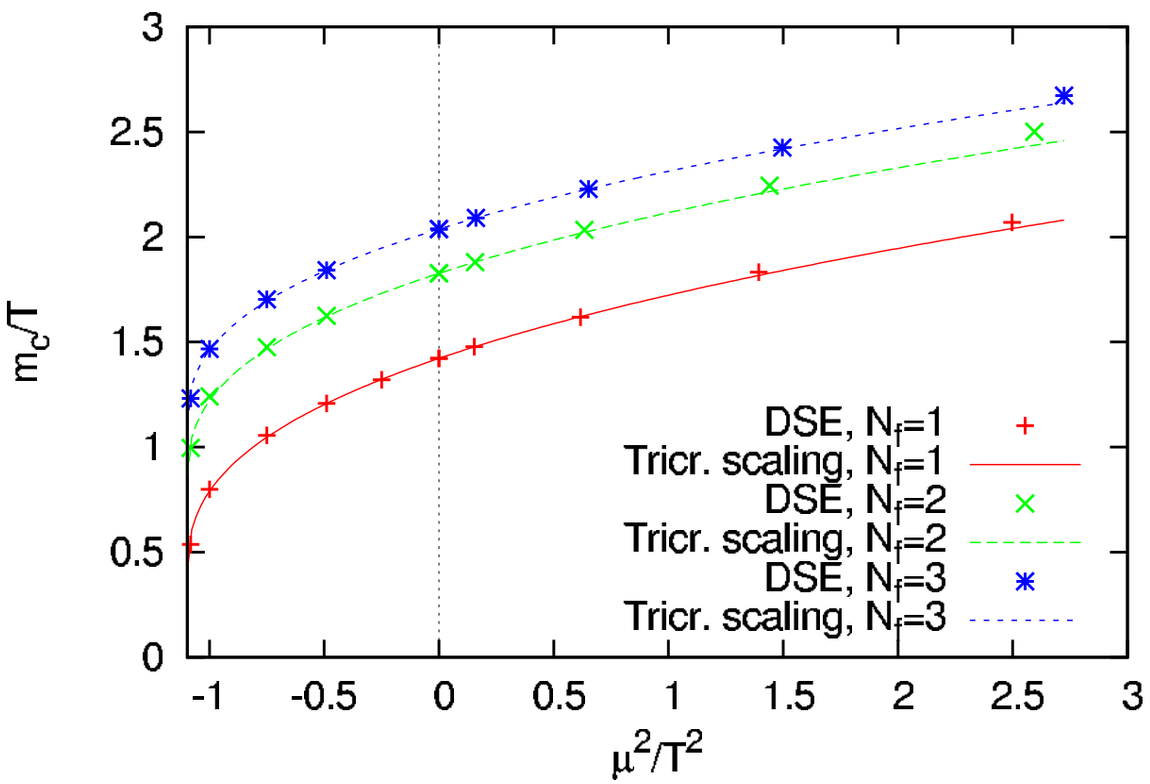


- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen

Lattice:
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

Polyakov Loop:





CF, Luecker, Pawlowski, PRD 91 (2015) 1

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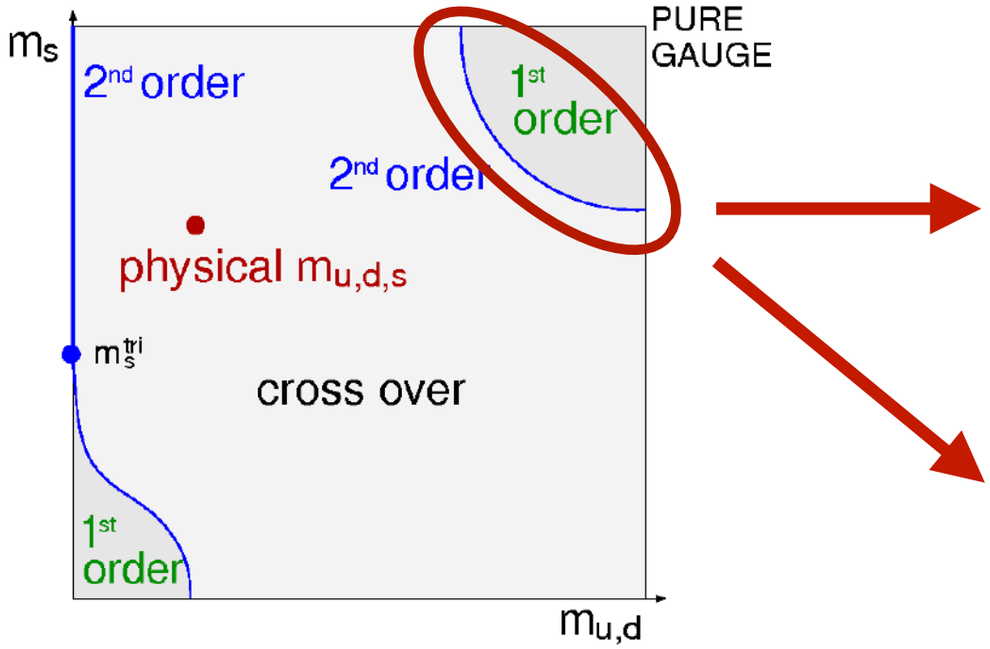

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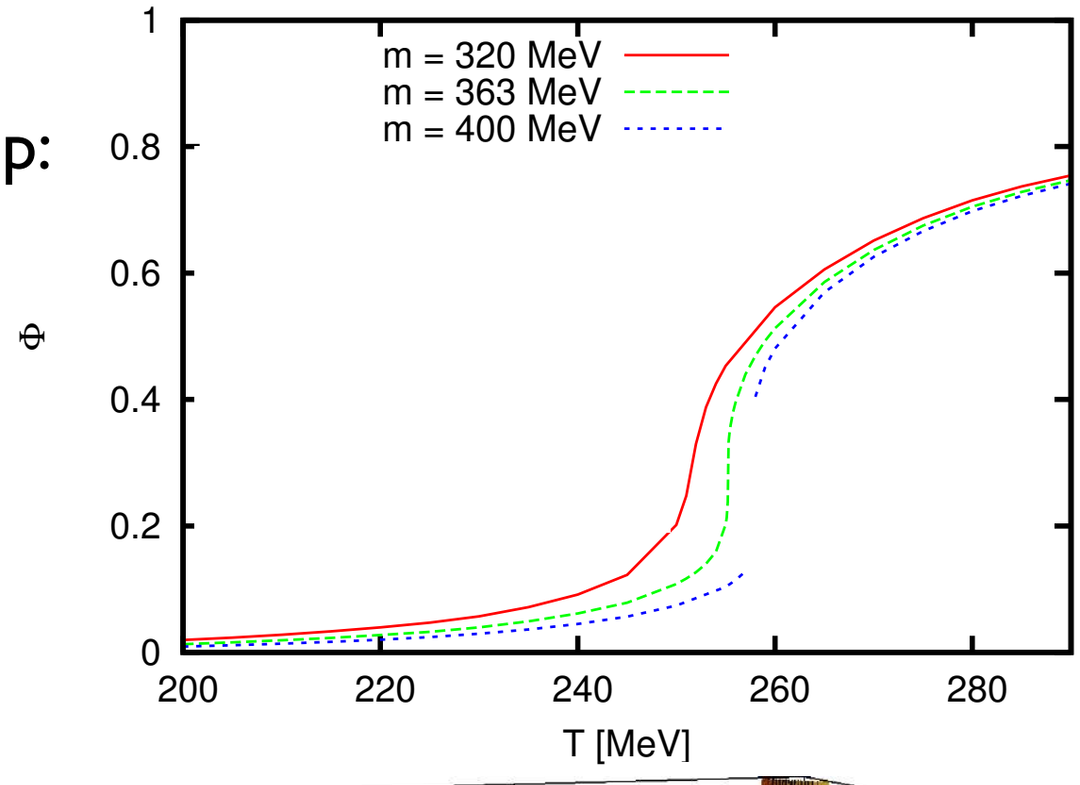

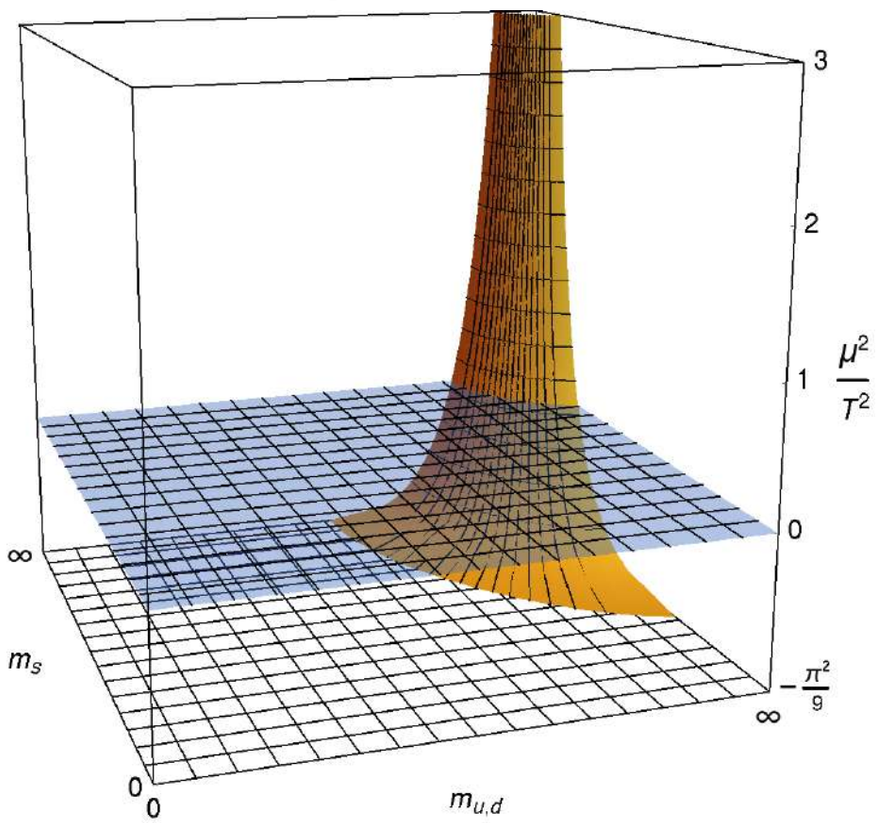

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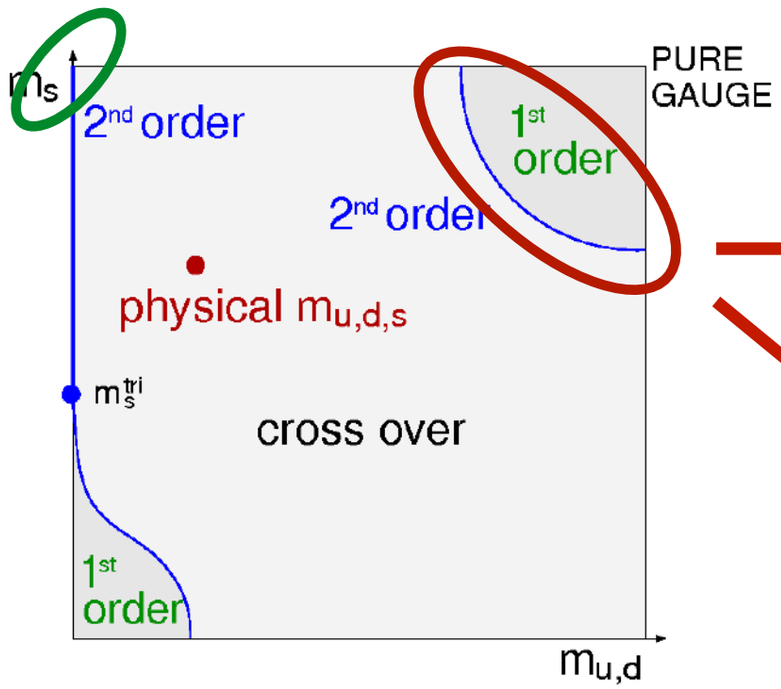
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

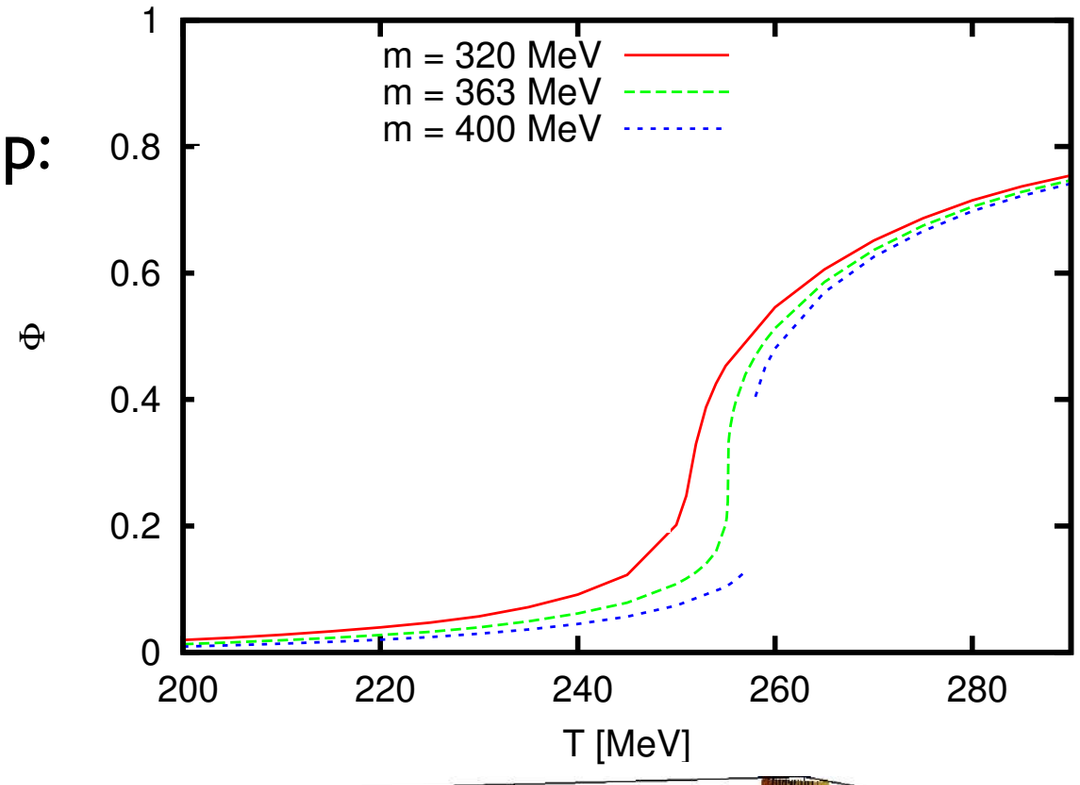
CF, Luecker, Pawlowski, PRD 91 (2015) 1

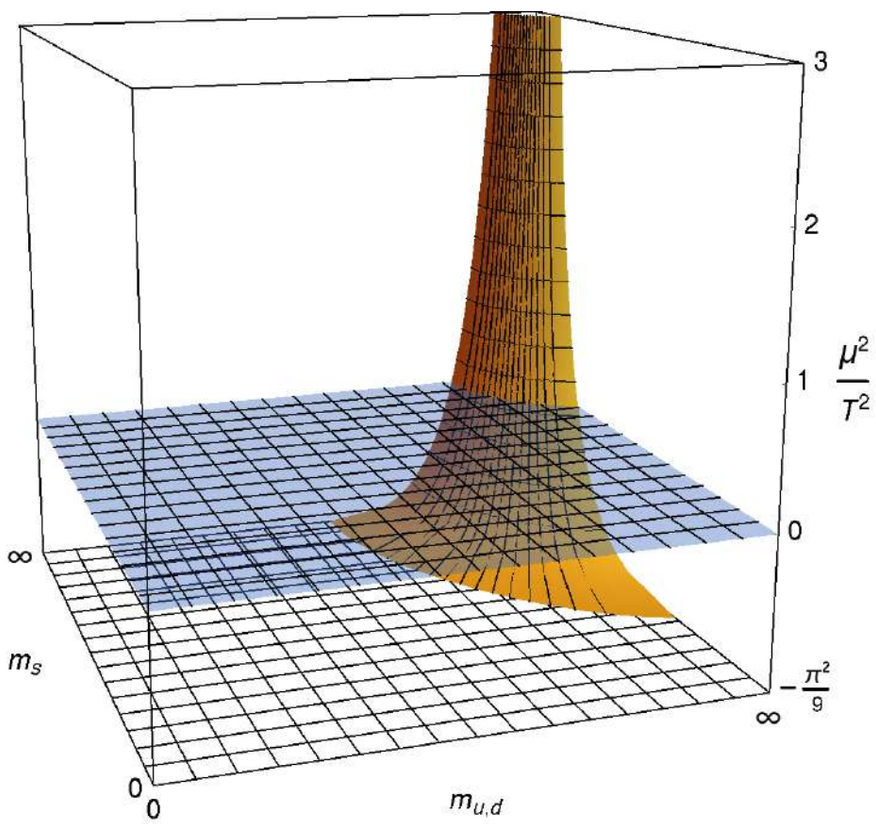
Critical line/surface for heavy quarks

Nf=2: CF and Mueller, PRD 84 (2011) 054013



Polyakov Loop:





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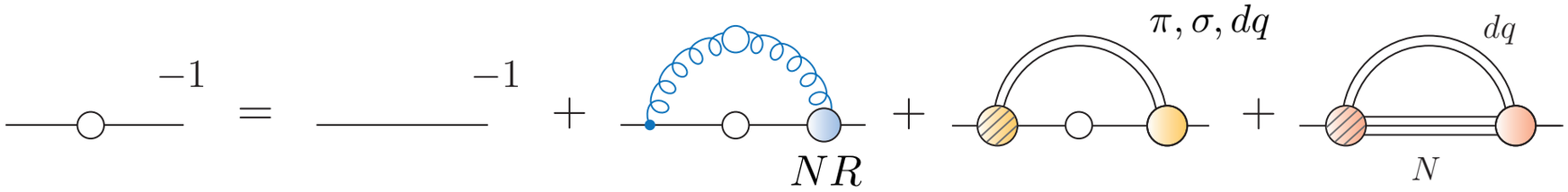
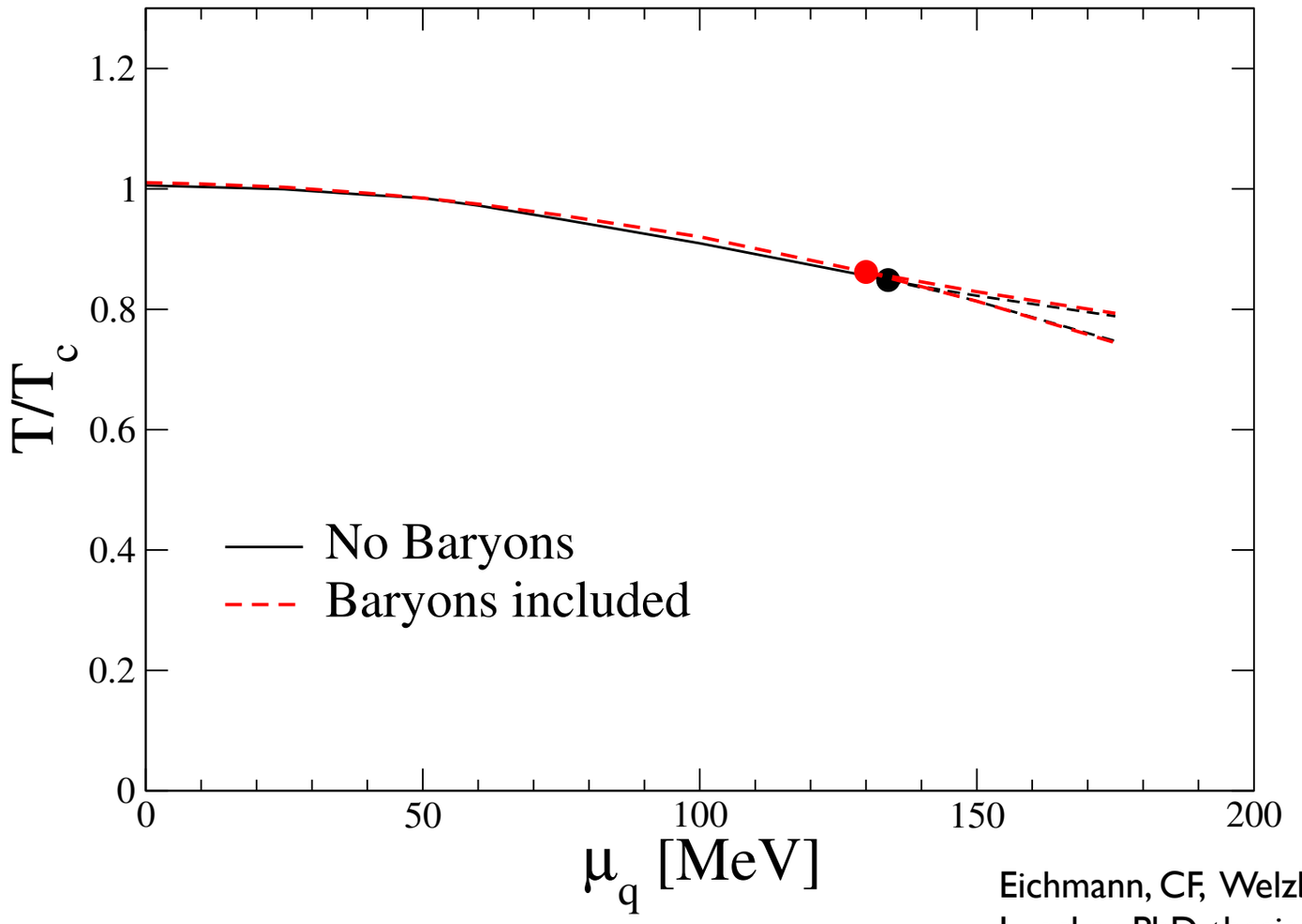
CF, Luecker, Pawlowski, PRD 91 (2015) 1

Hadron effects on the CEP - results (N_f=2)





Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]
Luecker, PhD-thesis, JLU Giessen, 2013

- Zero chemical potential: no effect
- almost no effect on location of CEP (mesons: similar)

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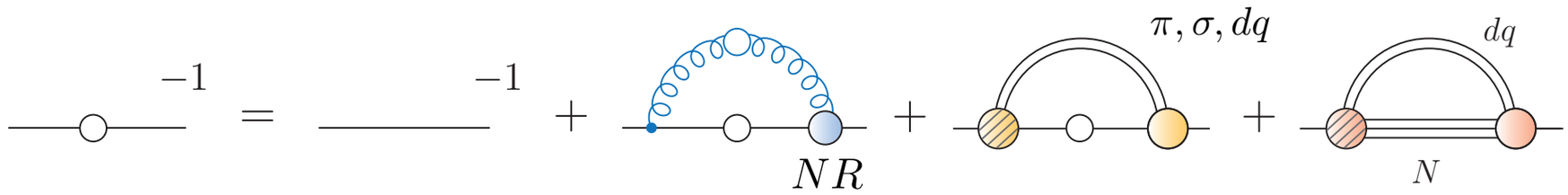

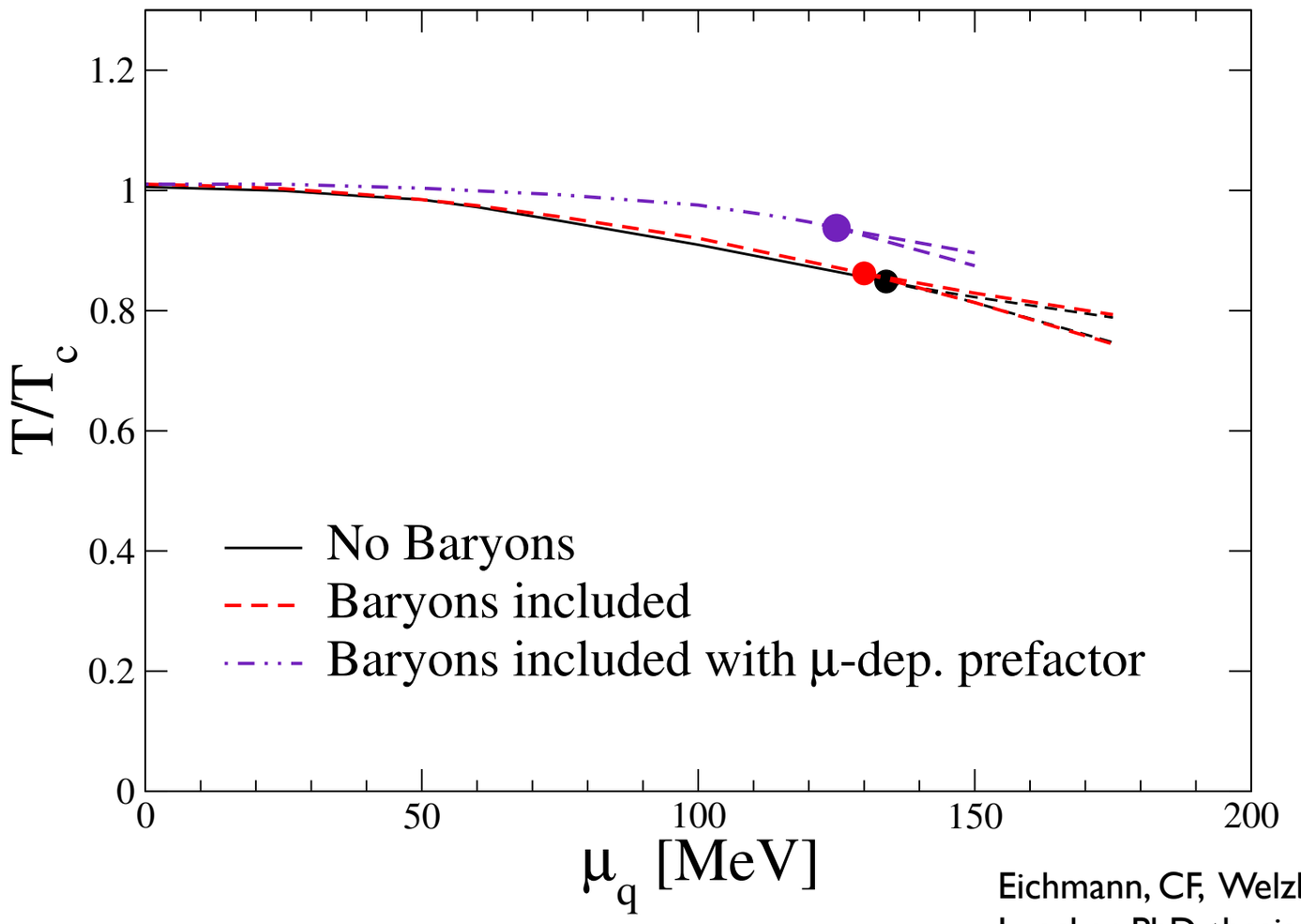

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Luecker, PhD-thesis, JLU Giessen, 2013

- Zero chemical potential: no effect
- almost no effect on location of CEP (mesons: similar)
- But: strong μ-dependence of baryon wave function may change situation…