UBA
Universidad de Buenos Aires
Argentina virtus robur et studium

Preparatory School for StatPhys 2019

July 1-5, 2019

São Paulo, Brazil

ICTP-SAIFR/IFT-UNESP

FAPESP ICTP unesp IFT - UNESP

Pablo Balenzuela (Buenos Aires, Argentina)
Mauro Copelli (Recife, Brazil)
Maxi San Miguel (Mallorca, Spain)
Bill Bialek (Princeton, USA)
Gabriel Mindlin (Buenos Aires, Argentina)

StatPhys 27

BA 2019

International Conference on Statistical Physics

Buenos Aires, Argentina, 8-12 July, 2019

TOPICS

- General and mathematical aspects
- Out-of-equilibrium aspects
- Quantum fluids and condensed matter
- Disordered and glassy systems
- Biological physics
- Soft matter
- Nonlinear physics
- Interdisciplinary and complex systems

PLENARY SPEAKERS

- Alexandre Arenas
- Bulbul Chakraborty
- Sharon Glotzer
- Martin Hairer
- Jorge Kurchan
- Cristina Marchetti
- Michele Parrinello
- Juan Pablo Paz
- Hajime Tanaka
- Angelo Vulpiani

2019 Boltzmann Medal & Young Scientist Award Ceremonies

Over 30 Invited Speakers - Oral and Poster Sessions - Satellite Meetings

LOCAL ORGANIZING COMMITTEE

Silvina Ponce Dawson
DF, FCEN-UBA; IFIBA, UBA-CONICET

Gustavo Lozano
DF, FCEN-UBA; IFIBA, UBA-CONICET

Pablo Balenzuela
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July 2nd, 2019
Total Eclipe in Argentina

Av. Alicia M de Justo 1502, Auditorios UCA Puerto Madero, Bs. As.

*Quantum flows image credit to Pablo Mininni

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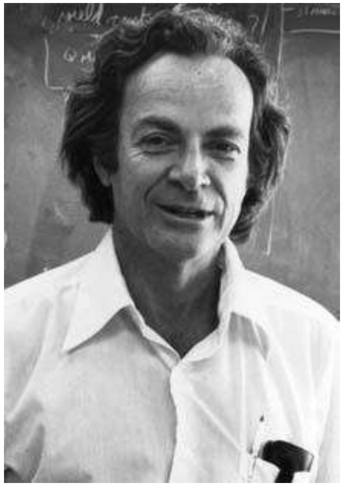

"If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis that all things are made of atoms — little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied."

And behaviour
emerges from
the spiking activity
of interacting neurons…

And social behavior
from the interacion
between agents...

Many issues addressed by contemporary statistical mechanics
deals with the **interaction between out of equilibrium, nonlinear units.**

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The units: what dynamics can we expect from a low dimensional, nonlinear system?

The interaction: what emerges out of the interaction between these nonlinear units?

Our first course:
nonlinear dynamics

The origin of dynamics

Newton, and a prescription that changed science

The rate of change of variables at an instant, defined by the variables at **that** instant

$$\frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{1}{m}\vec{F}\left(x, \frac{d\vec{x}}{dt}\right)$$

$$\left(\vec{x}(t=0), \frac{d\vec{x}}{dt}(0)\right)$$

We then measure a finite amount of information, and unveil the dynamics forever

$$\vec{x}(t=\Delta t) \approx \vec{x}(t=0) + \frac{d\vec{x}}{dt}(t=0)\Delta t,$$

$$\frac{d\vec{x}}{dt}(t=\Delta t) \approx \frac{d\vec{x}}{dt}(t=0) + \frac{1}{m}\vec{F}\left(\vec{x}(t=0), \frac{d\vec{x}}{dt}(t=0)\right)\Delta t.$$

$$\vec{x}(t=2\Delta t) \approx \vec{x}(t=\Delta t) + \frac{d\vec{x}}{dt}(t=\Delta t)\Delta t,$$

$$\frac{d\vec{x}}{dt}(t=2\Delta t) \approx \frac{d\vec{x}}{dt}(t=\Delta t) + \frac{1}{m}\vec{F}\left(\vec{x}(t=\Delta t), \frac{d\vec{x}}{dt}(t=\Delta t)\right)\Delta t.$$

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A “Newtonian” approach to neuroscience

$$
\begin{cases}
C\dfrac{dV}{dt} = I - g_K n^4 (V - E_K) - g_{Na} m^3 h (V - E_{Na}) - g_i (V - E_i) \\
\dfrac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V) n \\
\dfrac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V) m \\
\dfrac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V) h
\end{cases}
$$

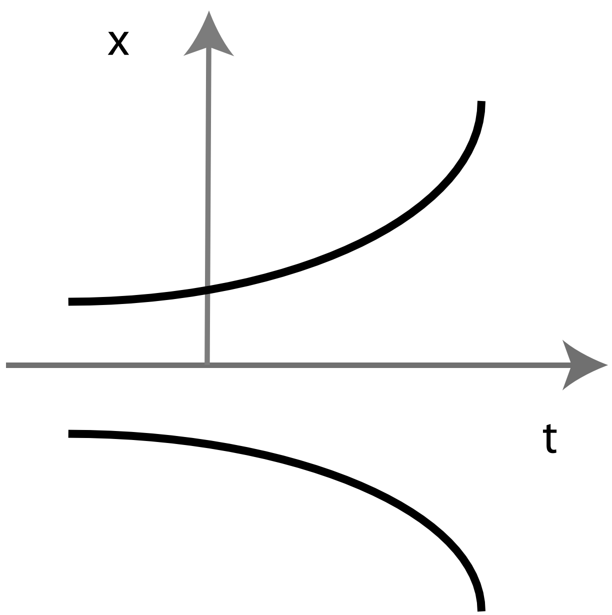
The “Newtonian labyrinth”: a closed expression for the two body problem.

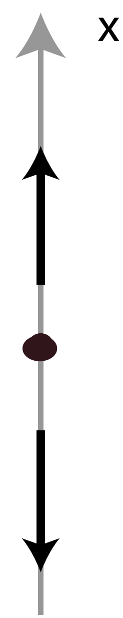
$$
\vec{F_g} = \frac{-GMm}{r^2} e_r,
$$

$$
\begin{cases}
\dfrac{d\phi}{dt} = l/\mu r^2 \\
\mu \dfrac{d^2 r}{dt^2} = F(r) + l^2/\mu r^3
\end{cases}
$$

$$
\frac{d^2 u}{d\phi^2} = -u - \frac{\mu}{l^2 u^2} F
$$

$$
\frac{1}{r} = \frac{c}{1 + \epsilon \cos(\phi)},
$$

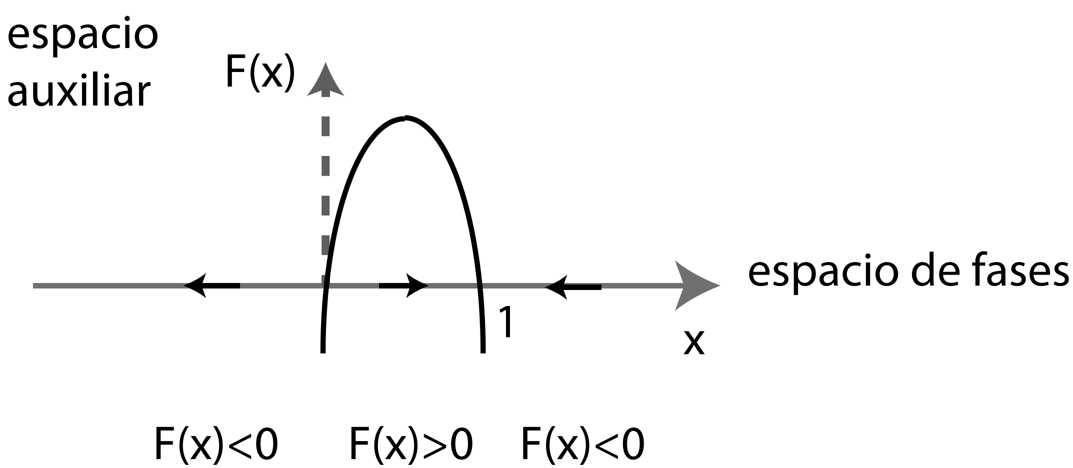




$$\frac{dx}{dt} = ax$$

$$x(t) = Ke^{at}$$

$$\boxed{\frac{dx}{dt} = ax\left(1 - \frac{x}{N}\right).}$$



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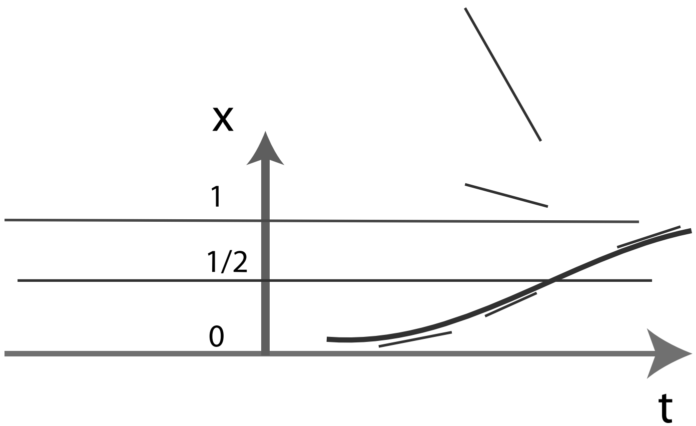
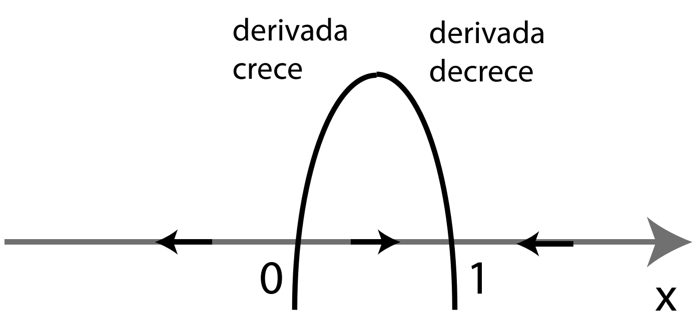


$$x(t) = \frac{Ke^{at}}{1 + Ke^{at}},$$



A qualitative argument…

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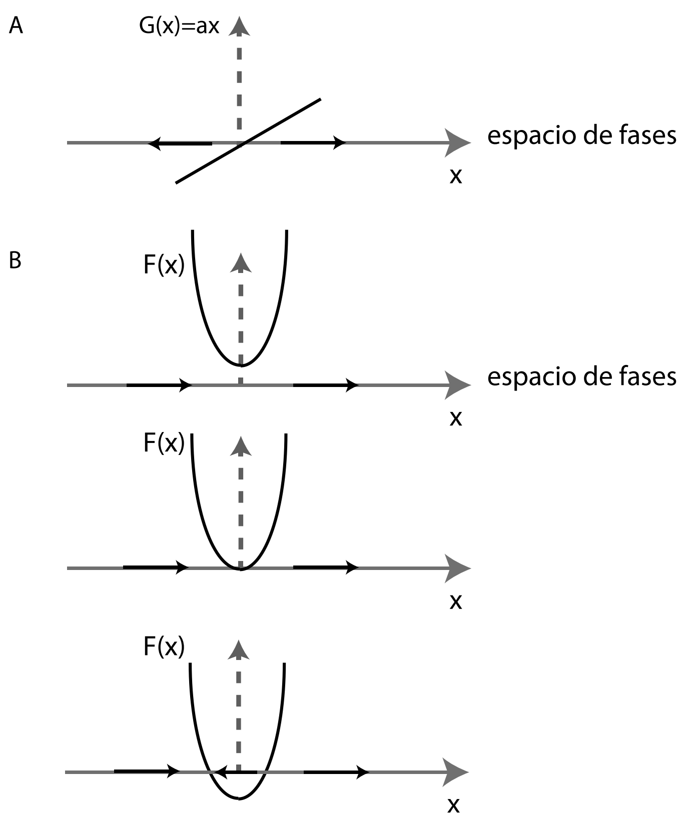
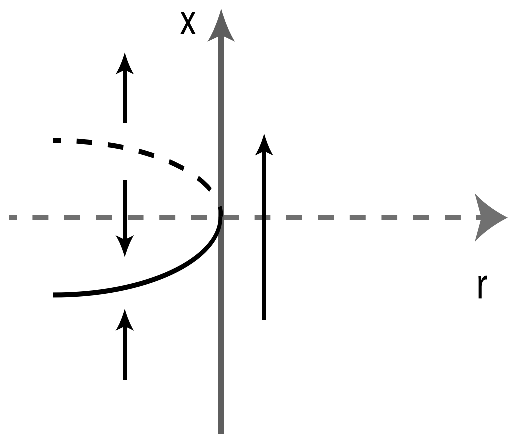
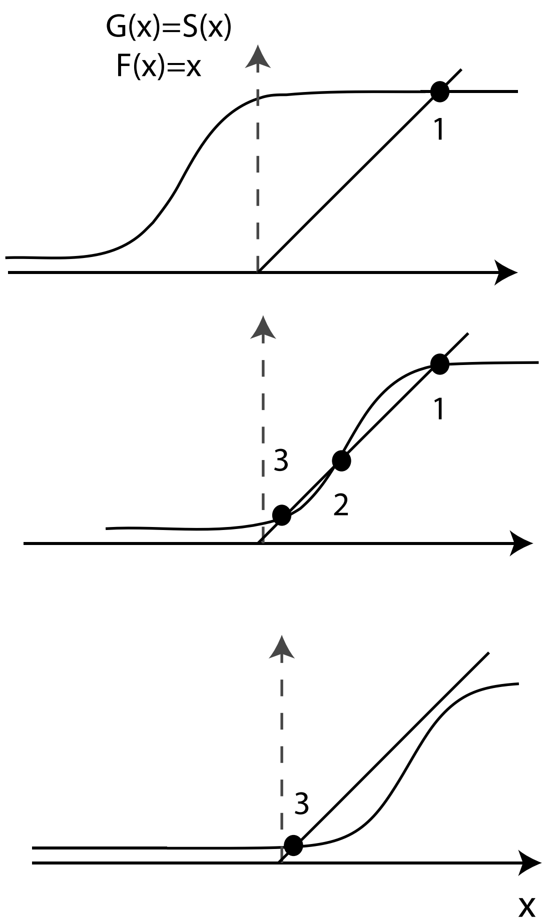


$$\frac{dx}{dt} = r + x^2 \quad .$$

$$r \in \mathbb{R}$$



$$\frac{dx}{dt} = r + x^2.$$

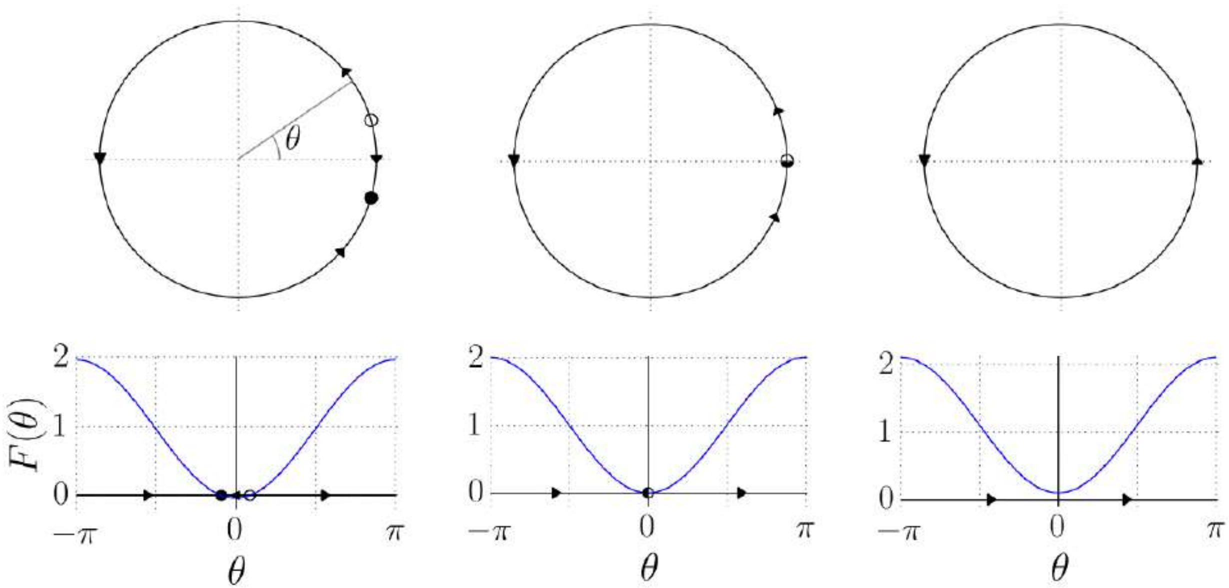


$$\frac{dx}{dt} = -x + S(\rho + cx)$$

$$S(x) = \frac{1}{1 + e^{-x}}$$

Dynamics is driven
By the vector field and
The topology of the
phase space

$$\frac{dx}{dt} = F(x) = \omega - \cos(x)$$
$$r \in \mathbb{R}, \quad x \in S^1$$



(a) $\omega \lesssim 1$ (b) $\omega = 1$ (c) $\omega > 1$

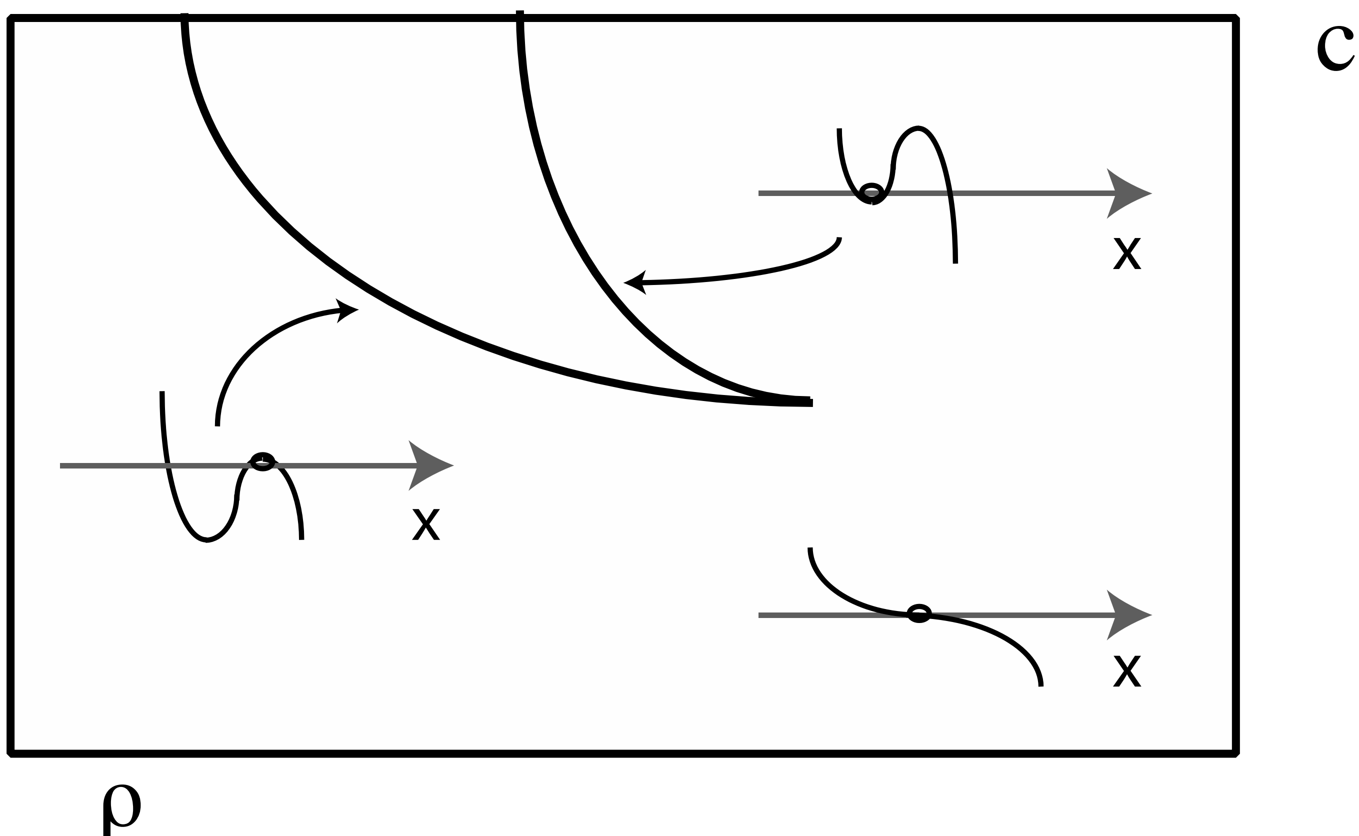
$$\frac{dx}{dt} = F(x) = \omega - \cos(x)$$
$$r \in \mathbb{R}, \quad x \in S^1$$

$$\frac{dx}{dt} = \omega - \cos(x) = 1 + \epsilon - \left\{1 - \frac{x^2}{2!} + \cdots\right\} = \epsilon + \frac{x^2}{2!} + O(4)$$
$$\frac{dx}{dt} \approx \epsilon + \frac{x^2}{2!}.$$

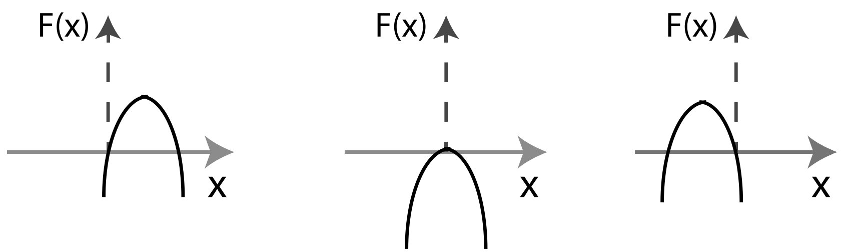
Normal forms: simple vector fields we can obtain algorithmically,
and are representative of large classes of systems

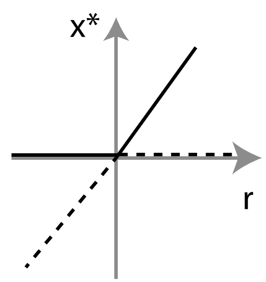
$$\frac{dx}{dt} = -x + S(\rho + cx)$$

$$S(x) = \frac{1}{1+e^{-x}}$$

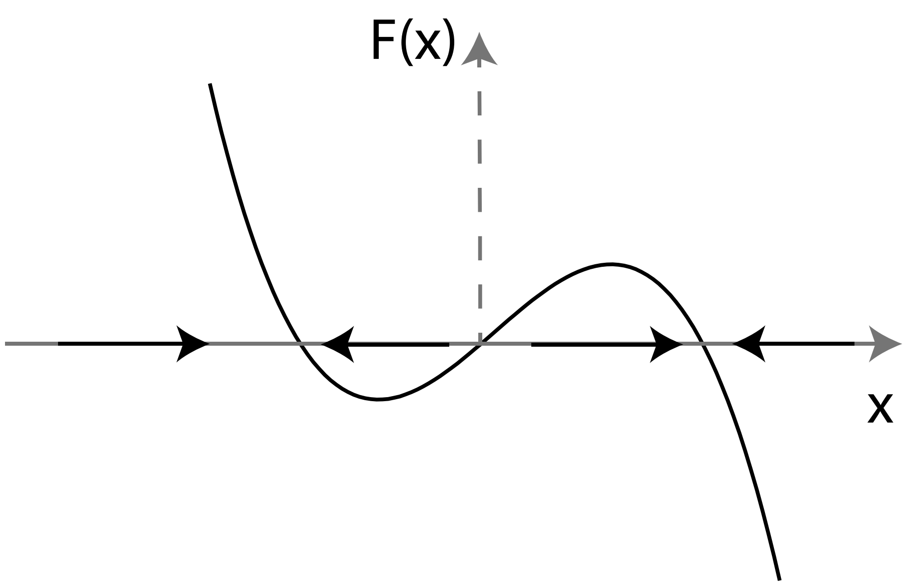


$$\boxed{\frac{dx}{dt} = rx - x^2}$$





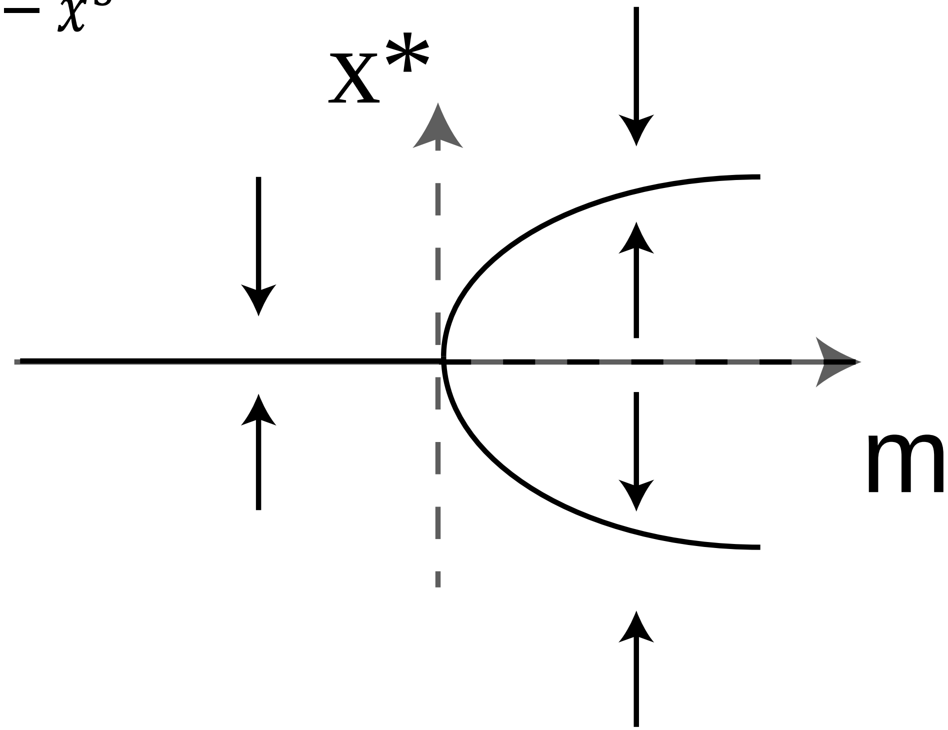
$$\frac{dx}{dt} = \mu x - x^3$$

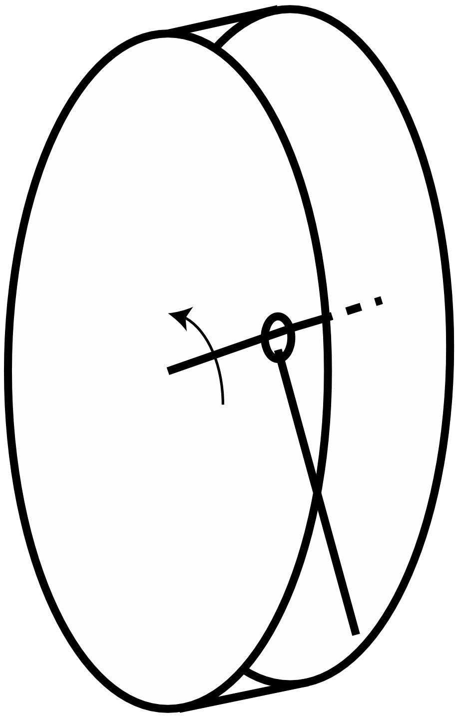


$$\frac{d(-x)}{dt} = \mu(-x) - (-x)^3 \Rightarrow$$

$$\frac{d\varepsilon}{dt} = \mu\varepsilon - 3x^2|_{x=\pm\sqrt{\mu}}\varepsilon = -2\mu\varepsilon$$

$$\frac{dx}{dt} = \mu(x) - x^3$$





$$mL^2\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + mgL\sin(\theta) = \Gamma - \mathrm{k}\theta.$$

$$\frac{d\theta}{dt} = \frac{d\theta}{Td\tau},$$

$$\frac{d^2\theta}{dt^2} = \frac{d^2\theta}{T^2 d\tau^2}$$

$$\frac{L}{gT^2}\frac{d^2\theta}{d\tau^2} + \frac{b}{mgLT}\frac{d\theta}{d\tau} + \sin(\theta) = \frac{\Gamma}{mgL} - \frac{k\theta}{mgL}.$$

$$\frac{b}{T} = mgL \Longrightarrow T = \frac{b}{mgL}$$

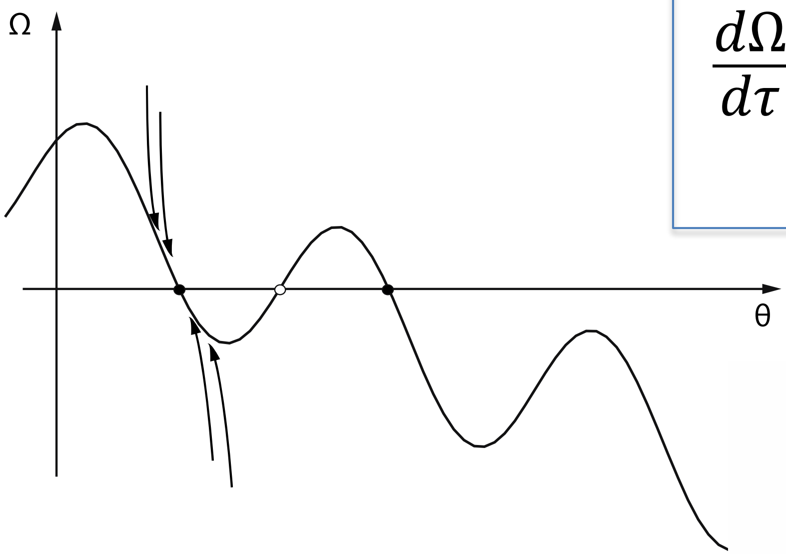
$$\frac{L}{gT^2} = \frac{L^3 m^2 g}{b^2}$$

$$m^2 \ll \frac{b^2}{gL^3}$$

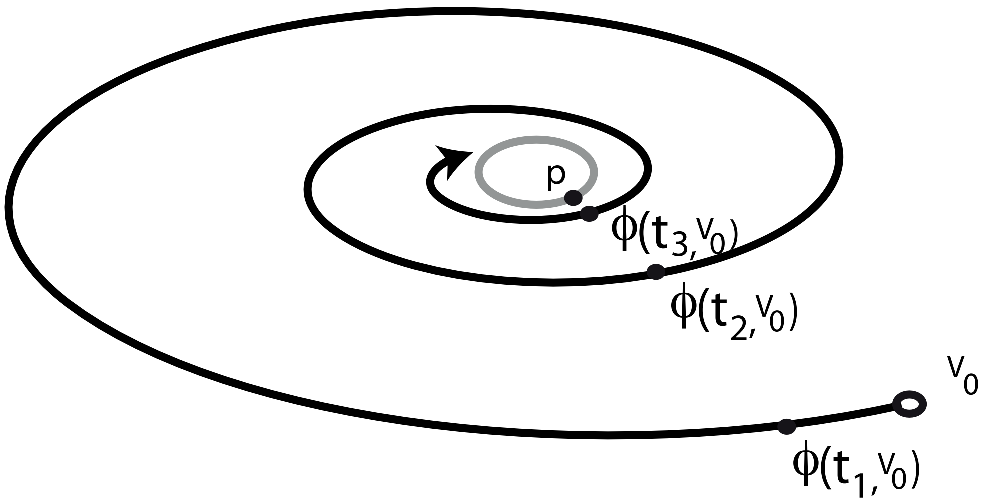
$$\boxed{\frac{d\theta}{d\tau} = \frac{\Gamma}{mgL} - \frac{k}{mgL}\theta - \sin(\theta)}$$

$$\frac{d\theta}{d\tau} = \Omega$$

$$\frac{d\Omega}{d\tau} = \frac{1}{\varepsilon}\left(-\Omega + \frac{\Gamma}{mgL} - \frac{k}{mgL}\theta - \sin(\theta)\right)$$

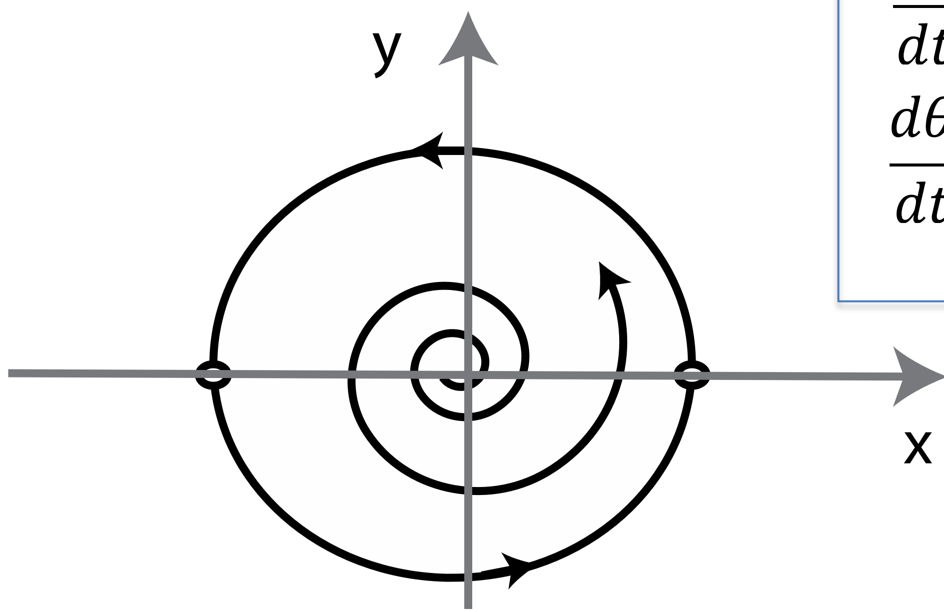


$$-\Omega + \frac{\Gamma}{mgL} - \frac{k}{mgL}\theta - \sin(\theta) = 0$$



$$\frac{dr}{dt} = r(1-r)$$

$$\frac{d\theta}{dt} = 1$$



$$\frac{dr}{dt} = r(1-r)$$

$$\frac{d\theta}{dt} = sin^2\theta + (r-1)^2$$

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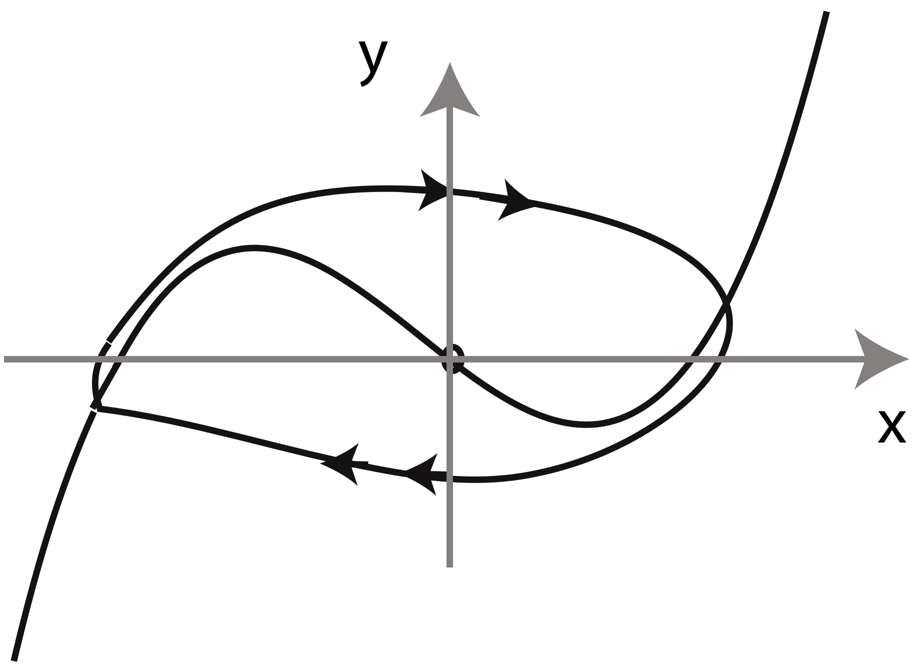
$$\boxed{\frac{d^2x}{dt^2} + \mu(x^2-1)\frac{dx}{dt} + kx = 0}$$

$$\frac{d^2x}{dt^2} + \mu(x^2-1)\frac{dx}{dt} = \frac{d}{dt}\left(\frac{dx}{dt} + \mu\left(\frac{x^3}{3} - x\right)\right)$$

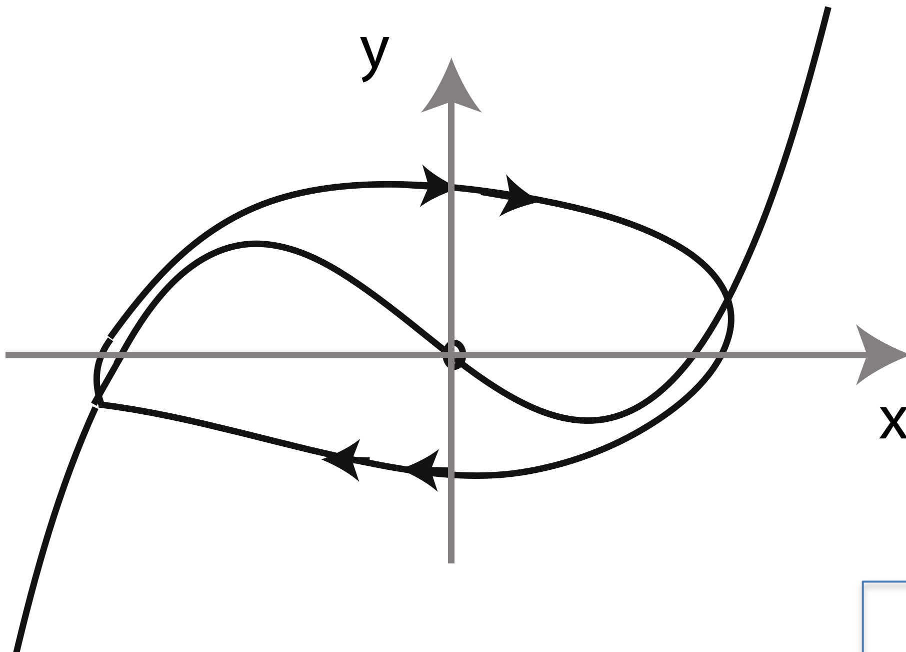
$$w = \frac{dx}{dt} + \mu\left(\frac{x^3}{3} - x\right)$$



$$\frac{dw}{dt} = -kx$$

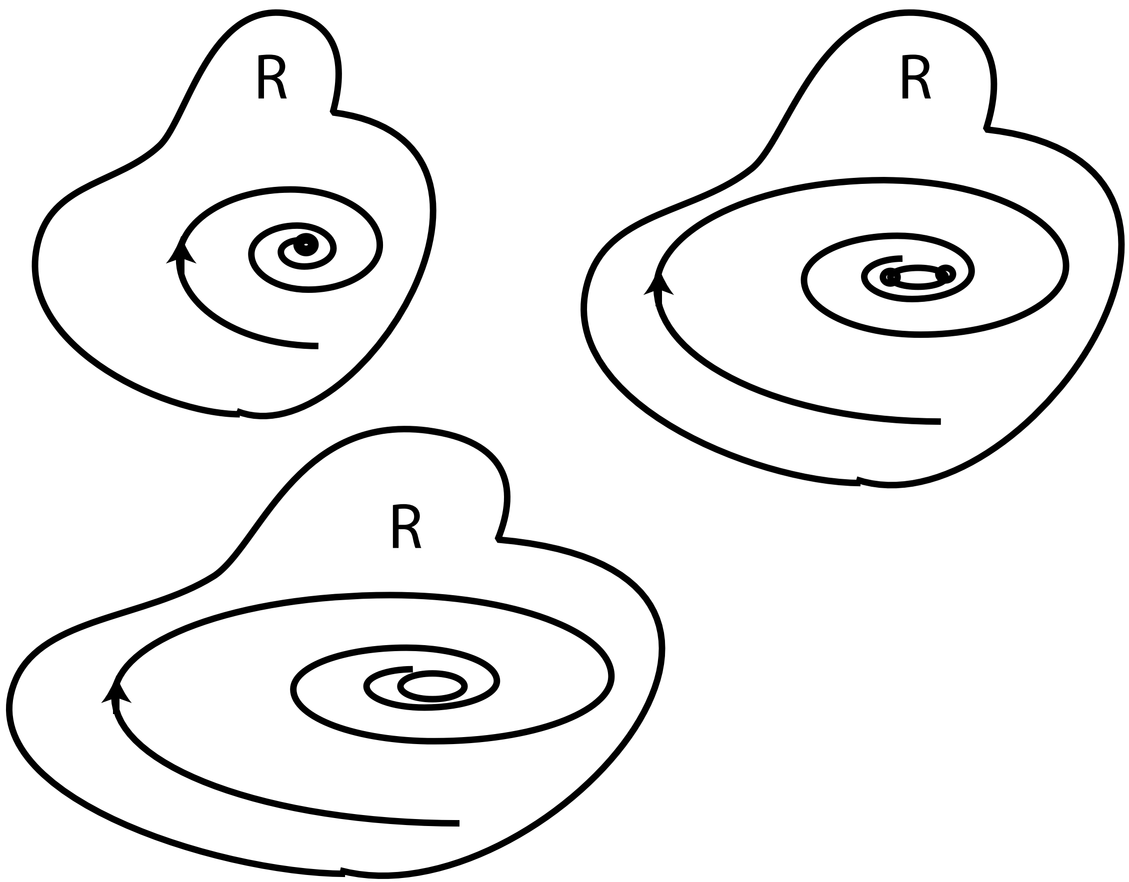
$$\frac{dx}{dt} = w - \mu\left(\frac{x^3}{3} - x\right)$$

$$= \mu\left(\frac{w}{\mu} - \left(\frac{x^3}{3} - x\right)\right)$$



$$\frac{dy}{dt} = -\frac{kx}{\mu}$$

$$\frac{dx}{dt} = \mu\left(y - \left(\frac{x^3}{3} - x\right)\right)$$


R
R
R

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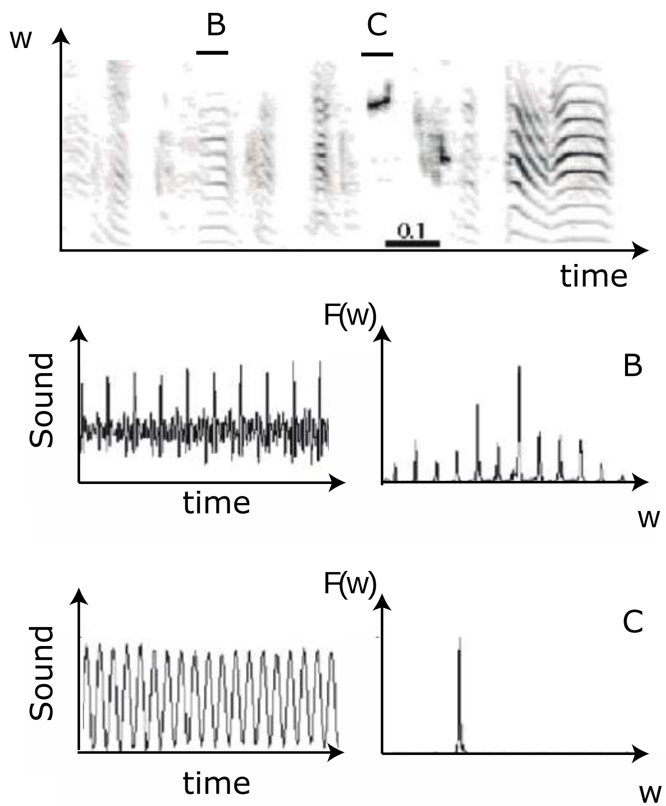

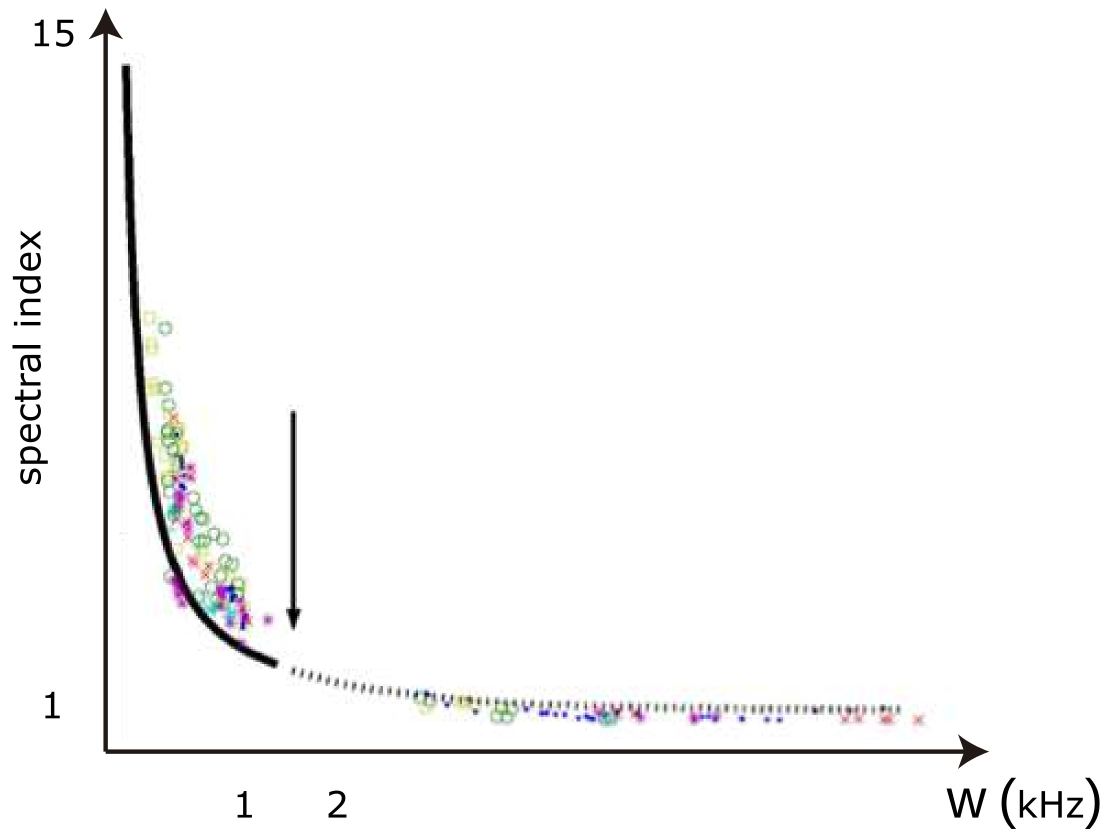
15
1
spectral index
1
2
W (kHz)

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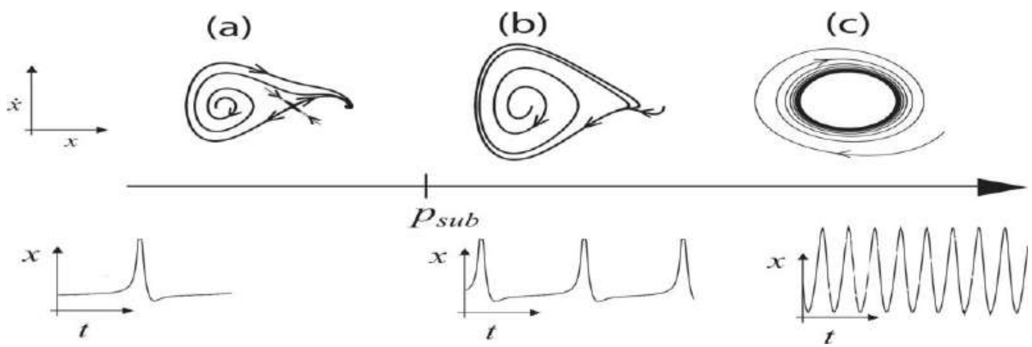
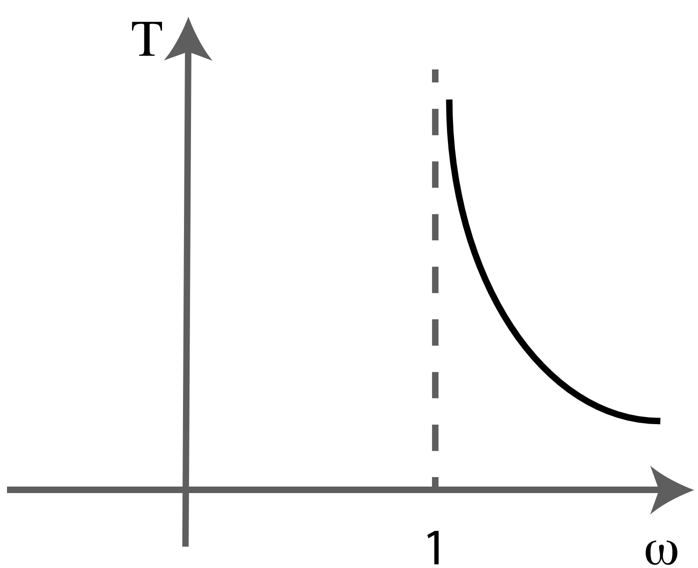


$$\frac{d\rho}{dt} = \rho(1-\rho)$$

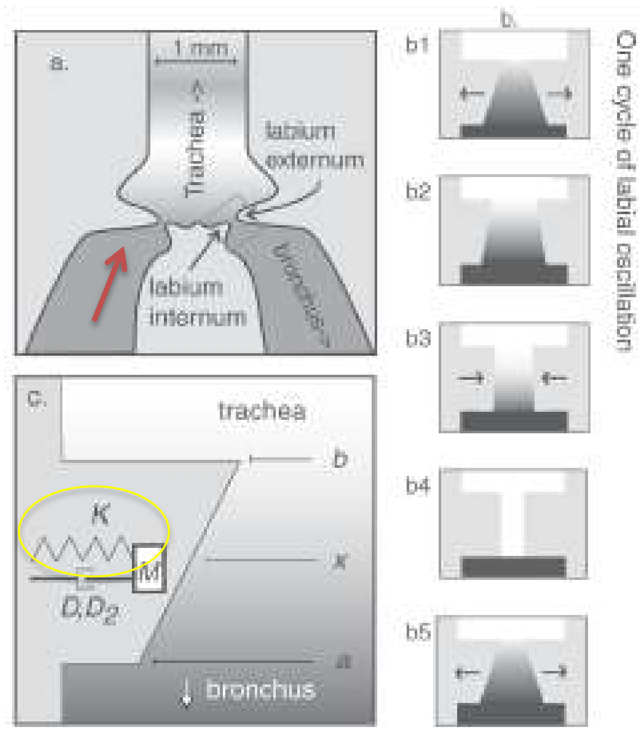
$$\frac{d\phi}{dt} = \omega - sin\phi,$$



$$T = \int_0^T dt = \int_0^{2\pi} \frac{dt}{d\theta} d\theta,$$

$$T = \frac{2\pi}{\sqrt{\omega^2 - 1}},$$

Our first steps in the field: the basic mechanics of labial motion



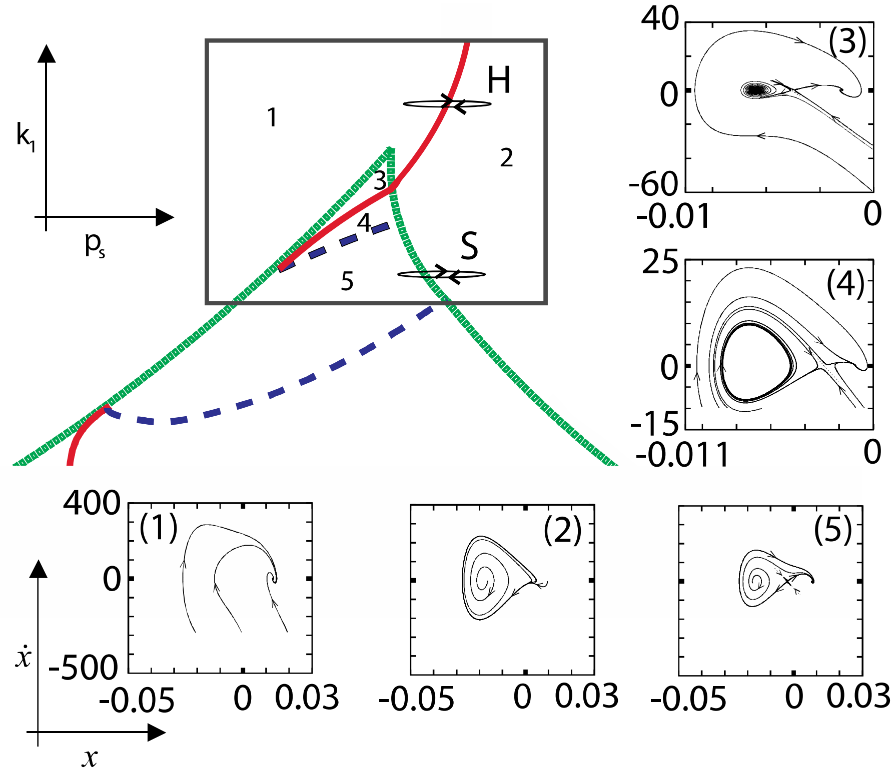
$$\frac{dx}{dt} = y,$$

$$\frac{dy}{dt} = (1/m)\left[-k(x)x - b(y)y - cx^2y + a_{lab}p_s\left(\frac{\Delta a + 2\tau\tau}{a_{01} + x + \tau y} \right) \right].$$

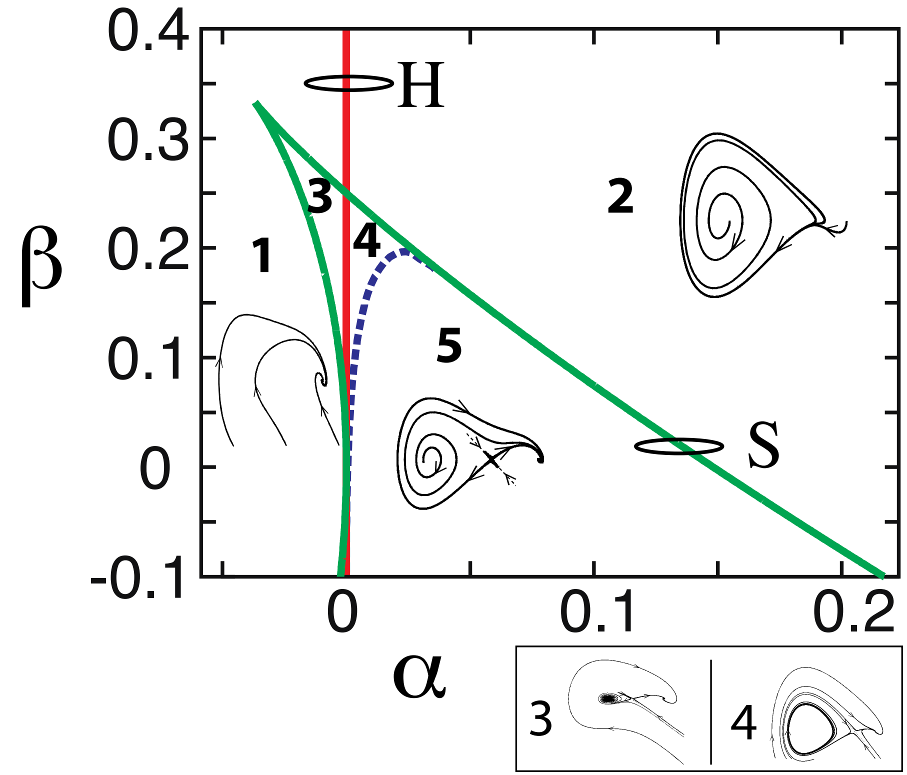
Two time dependent parameters
Control many features of the vocalizations:
air sac pressure and s.v. tension

Normal form reduction

Complete model

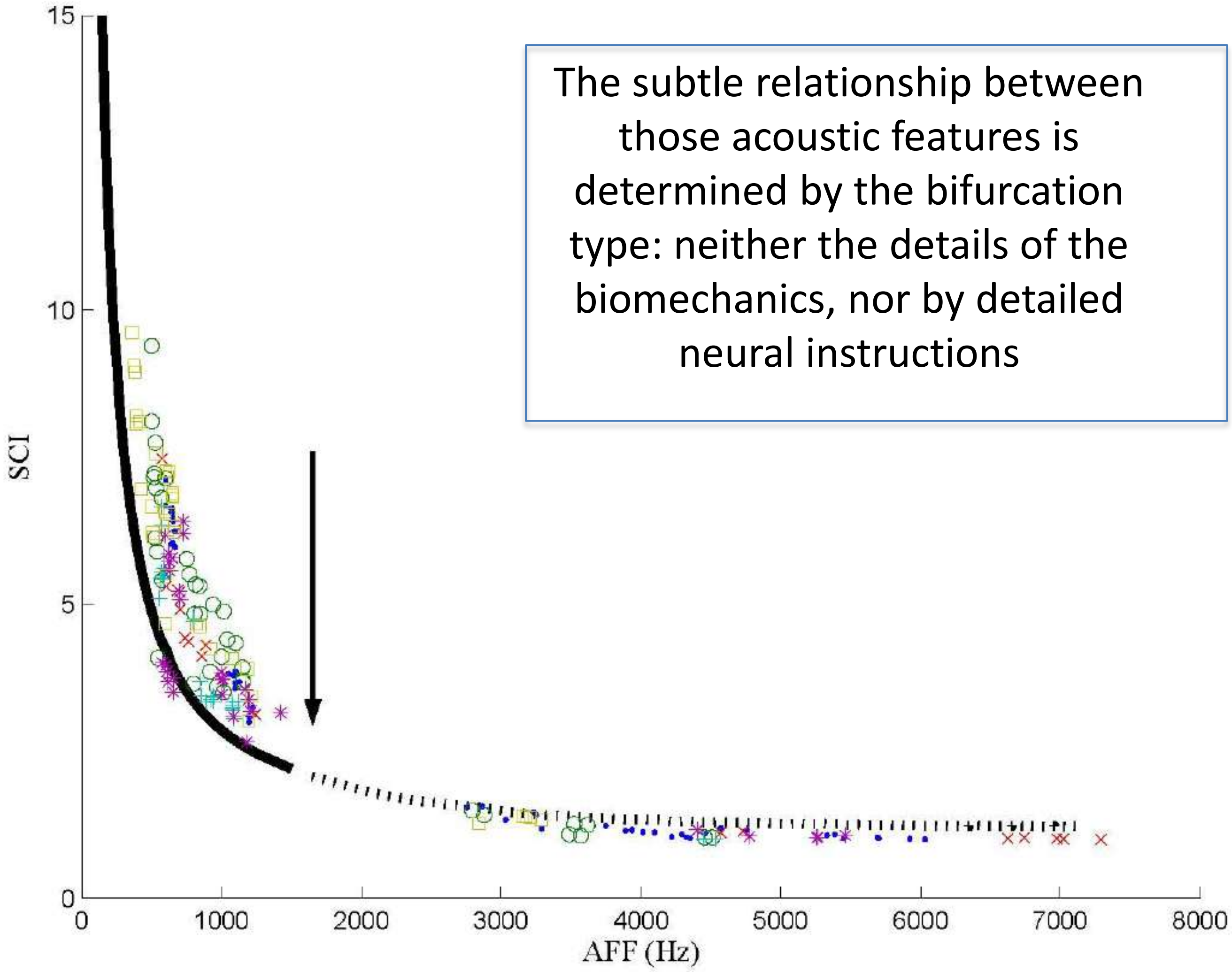


Normal form



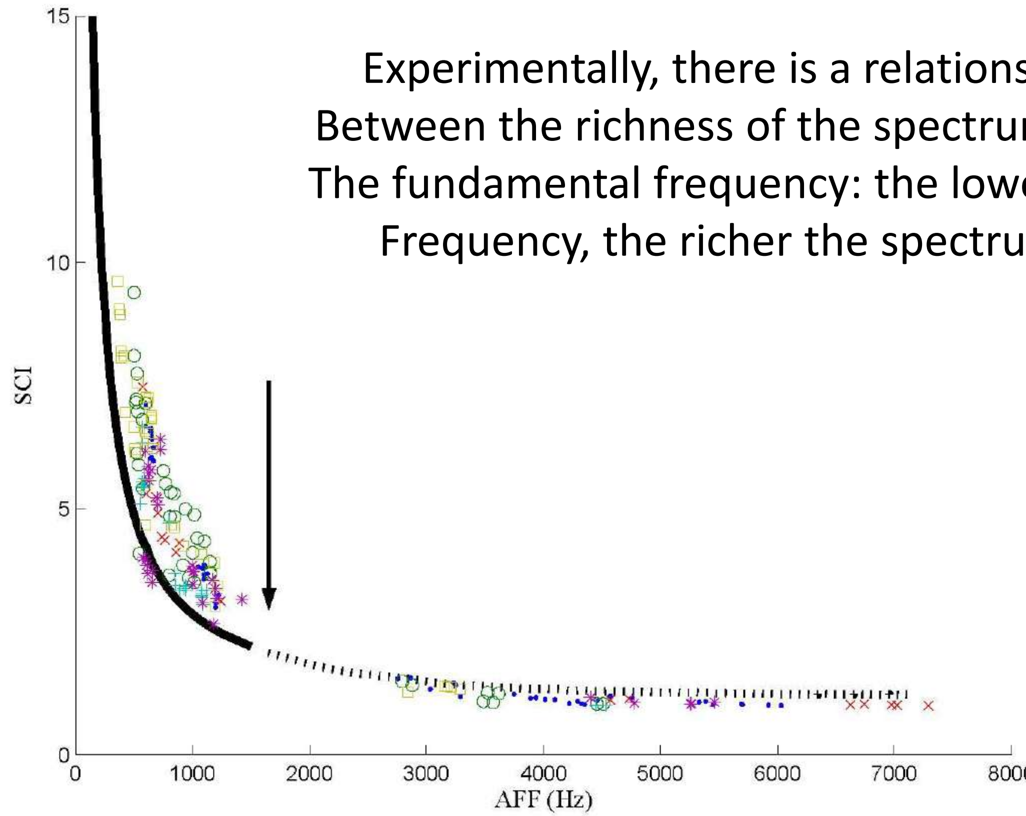
$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -a(t)g^2 - b(t)g^2 x - g^2 x^3 - g x^2 y + g^2 x^2 - g x y$$



The subtle relationship between those acoustic features is determined by the bifurcation type: neither the details of the biomechanics, nor by detailed neural instructions

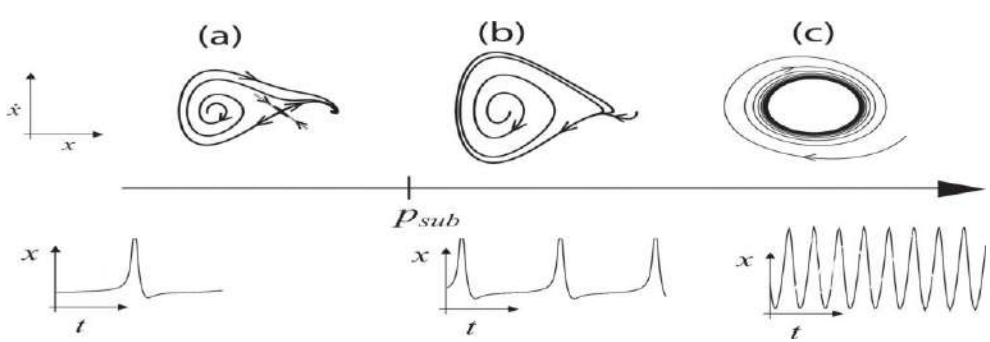
Experimentally, there is a relationship Between the richness of the spectrum and The fundamental frequency: the lower the Frequency, the richer the spectrum.

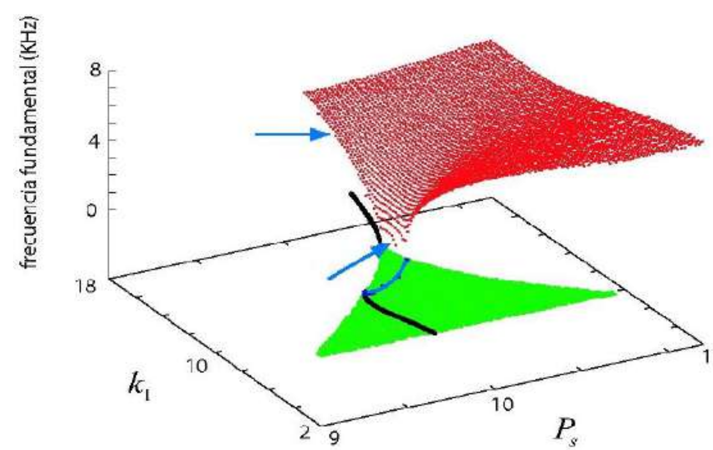


The spectral Index Content (SCI) is defined to Reflect how the energy is distributed in the spectrum compared to the energy in the Fundamental frequency component

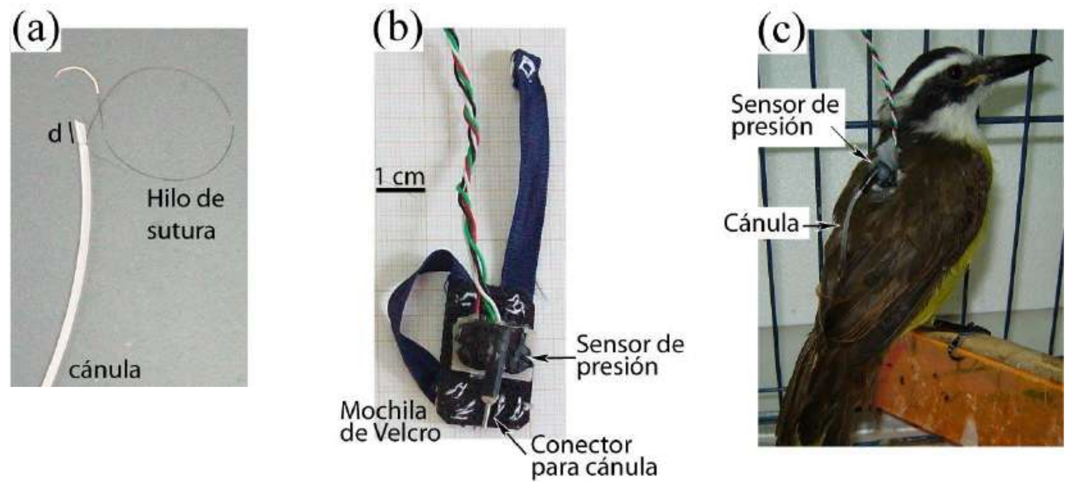
$$SCI = \left(\sum_i w_i e_i / E\right) \Big/ AFF$$

Signature of a snilc?

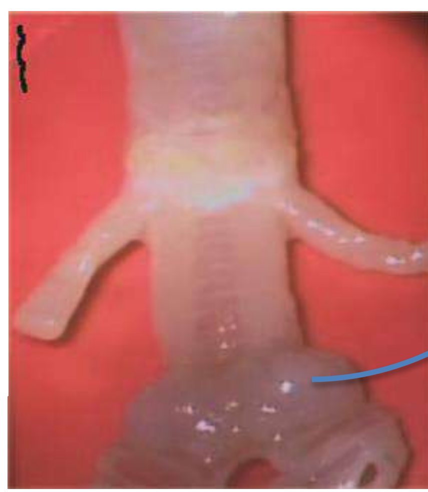




Direct mesurement of pressure and muscle activity



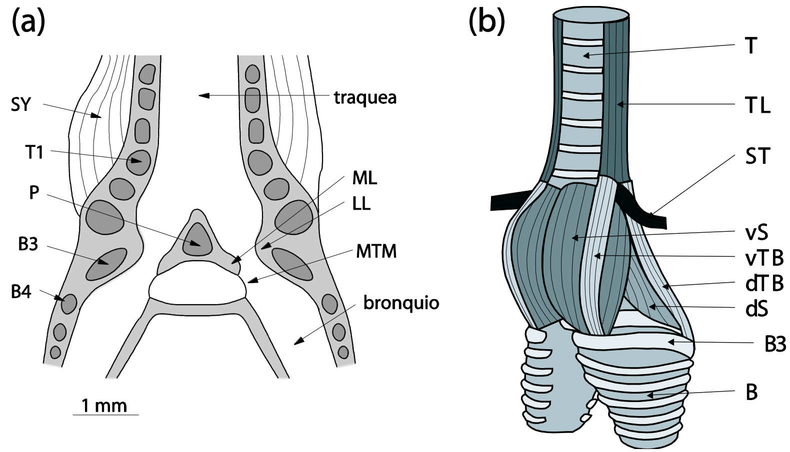
Air sac Pressure

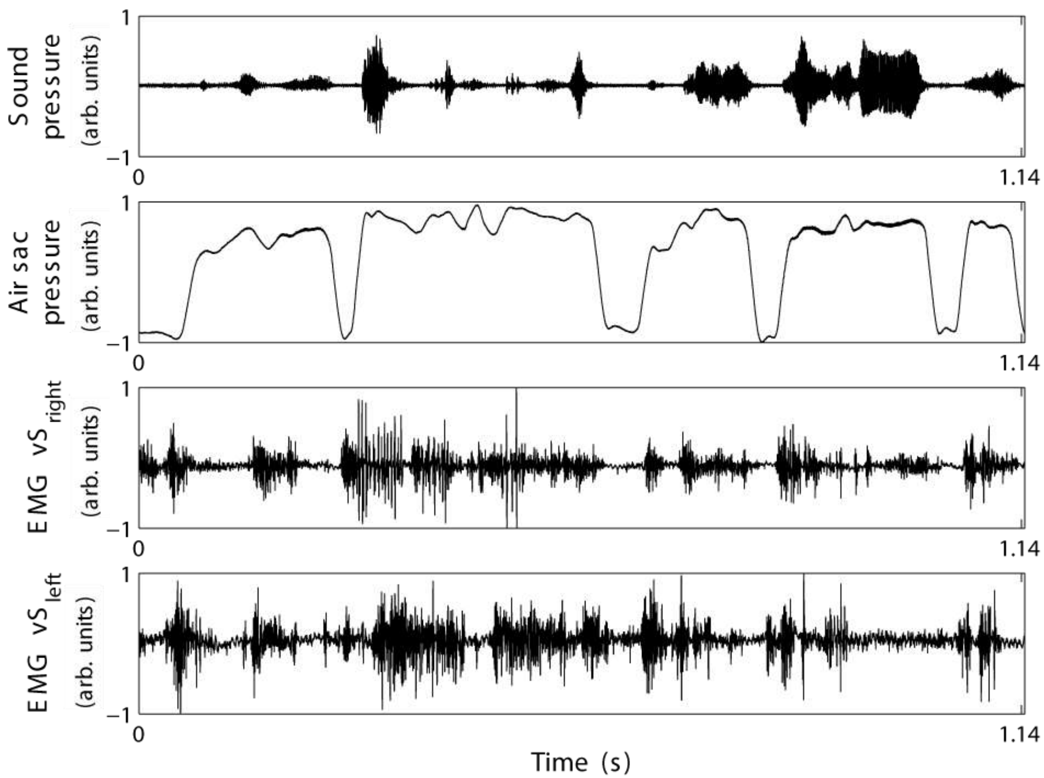


Tension
(activity of vS muscle)

With Franz Goller, 2003-present

Not that simple, these instructions…





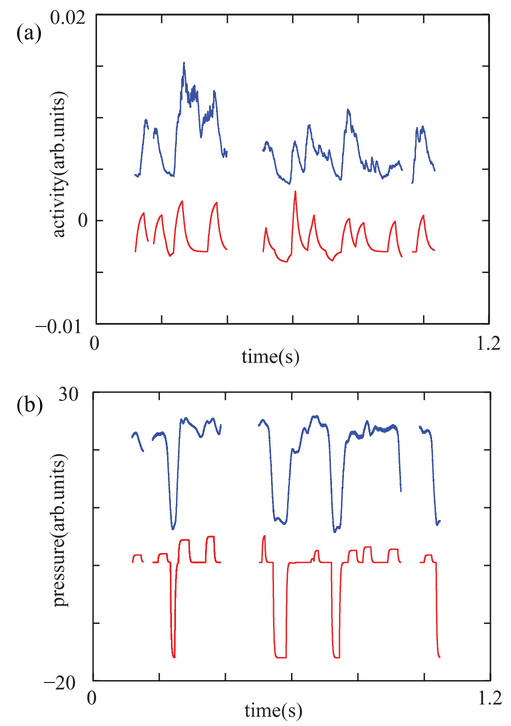
sound

pressure

Tension (right)

Tension (left)

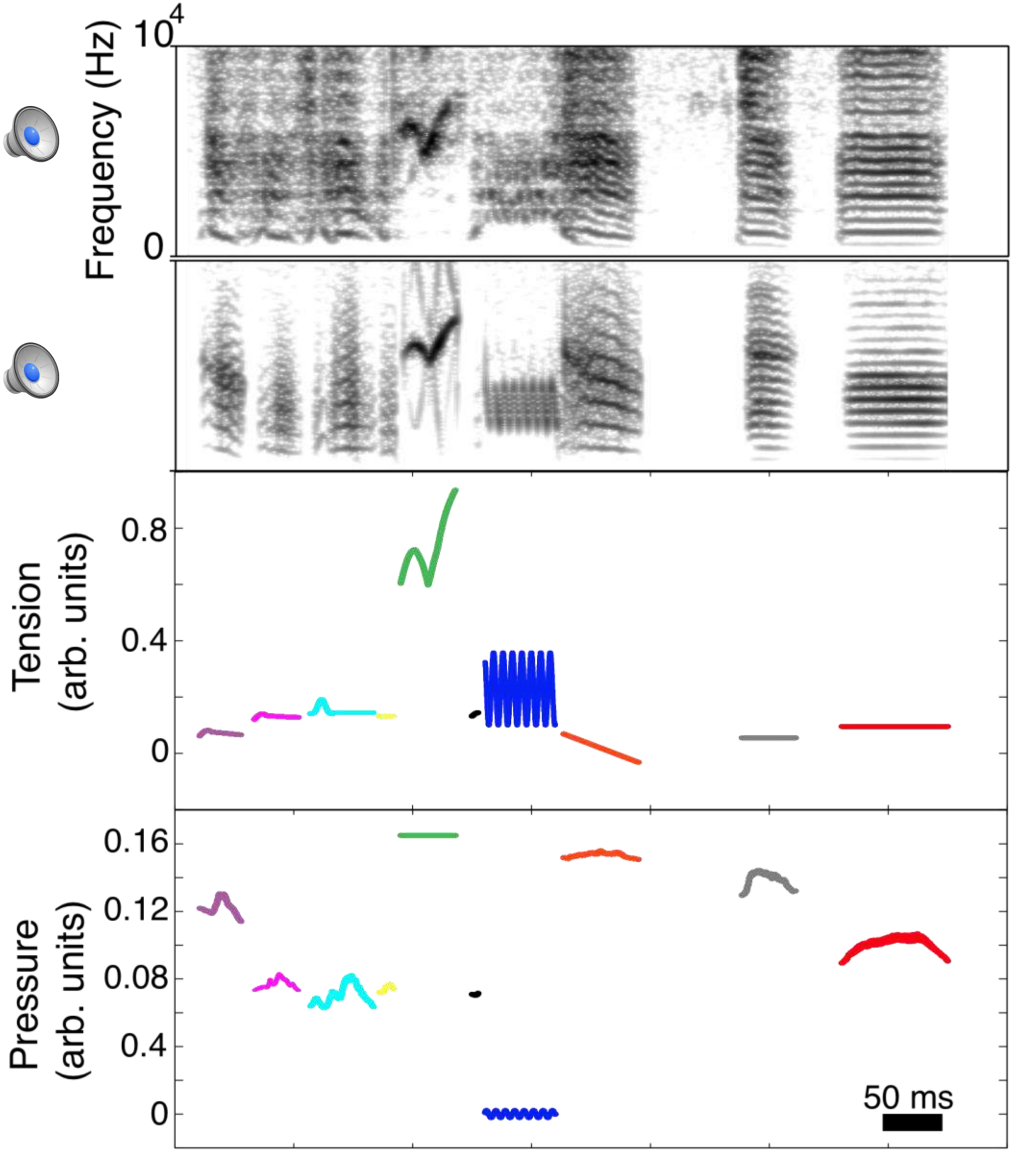
Reconstructed Instructions, Compared with the measured ones



Strategies to test the model

BOS

SYN



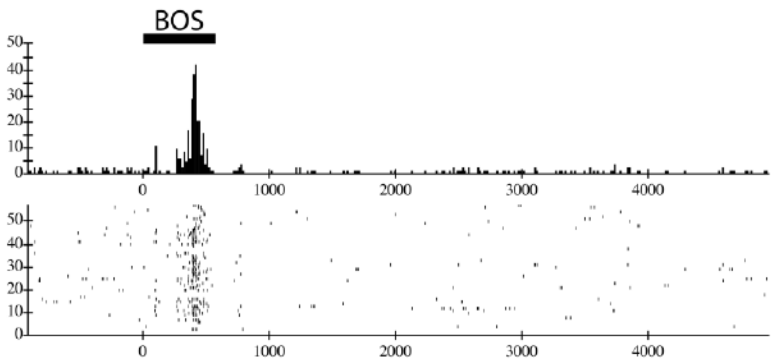
To get a synthetic song, we fit α (pressure) and β (tension) in the normal form model so that BOS and SYN share spectral Features (Fundamental, Spectral content)

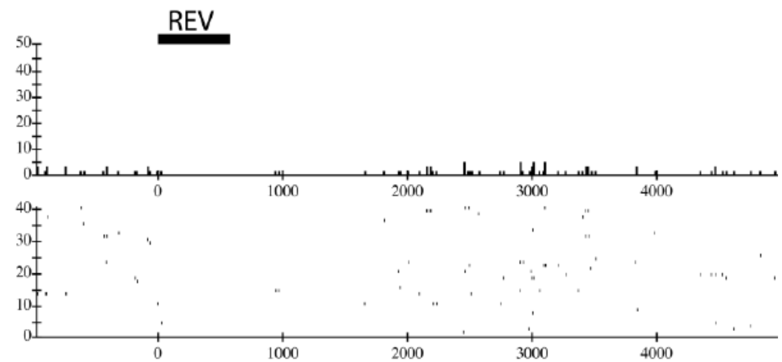
From Dan’s Lab

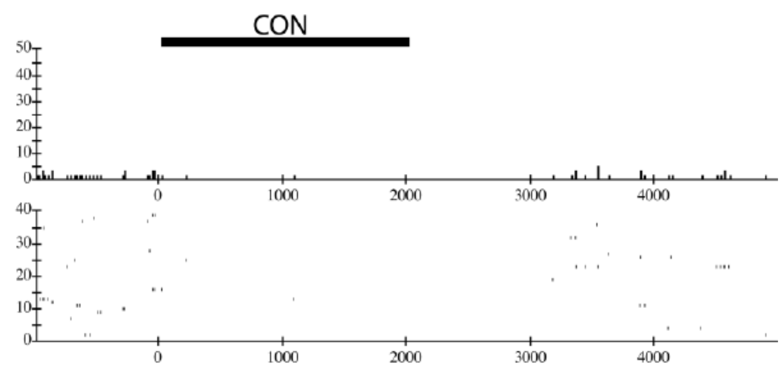
From Dan’s Lab

Testing the model

Neurons in HVC respond selectively to the bird’s own song (BOS)

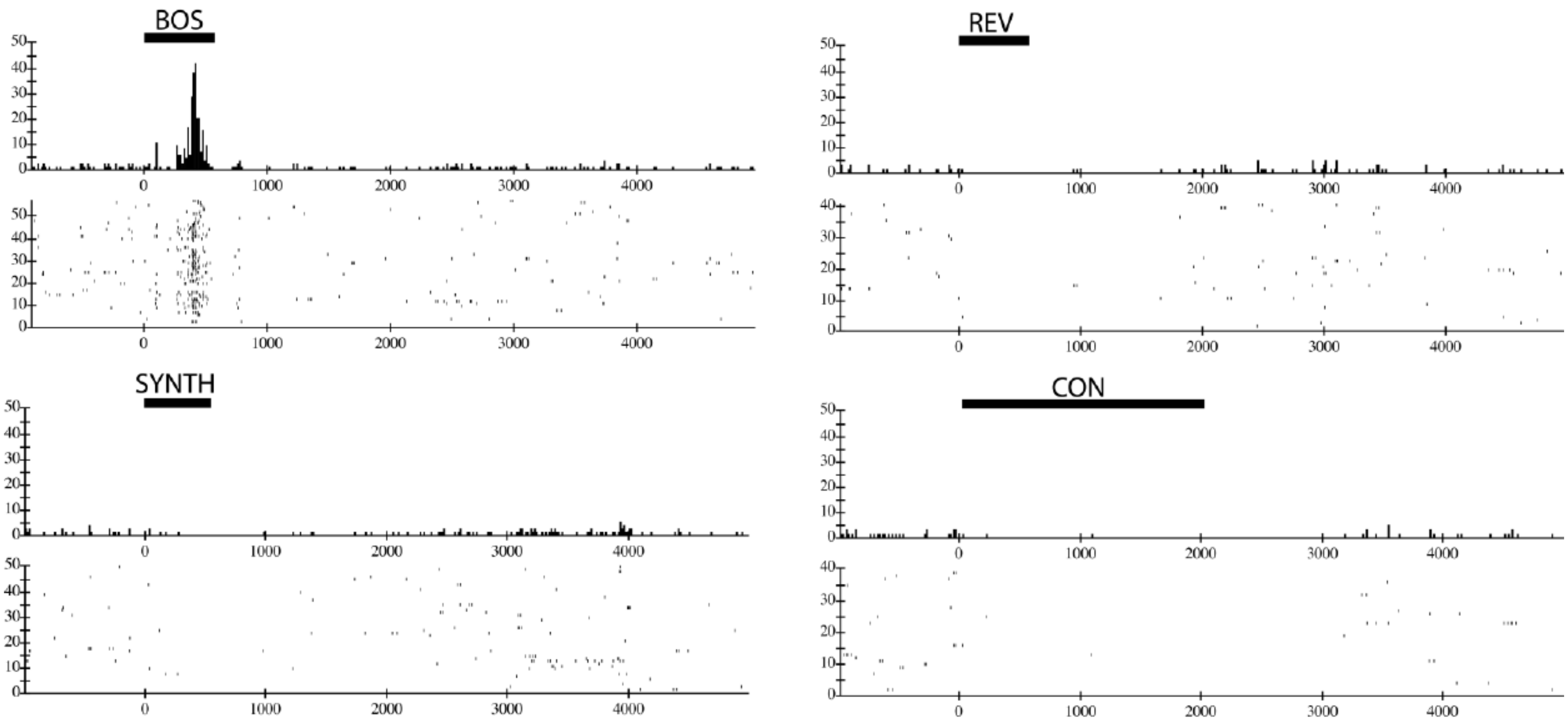




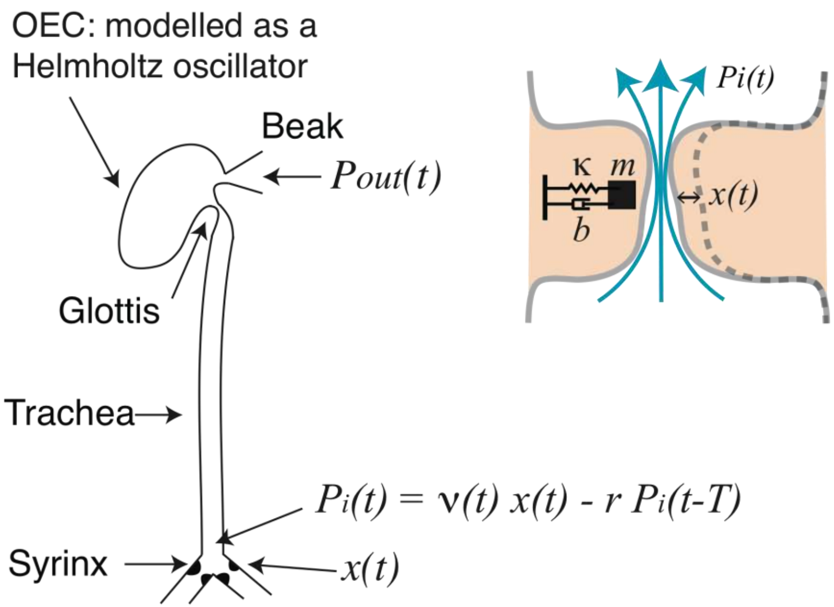


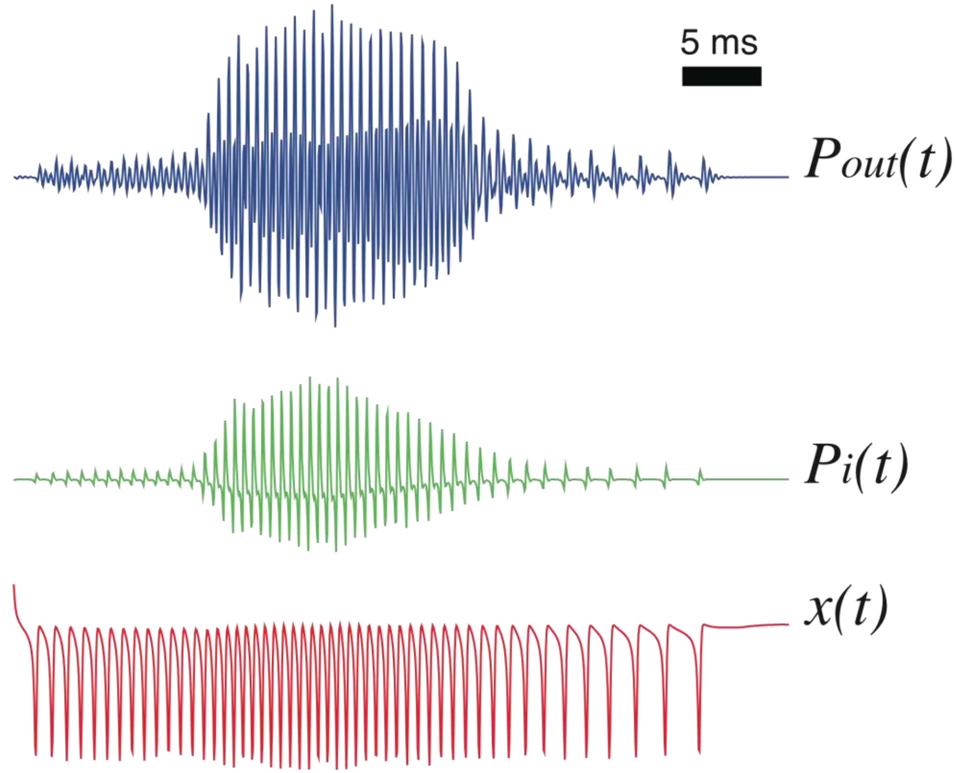
Testing the model

Neurons in HVC respond selectively to the bird's own song (BOS)



A more detailed modeling





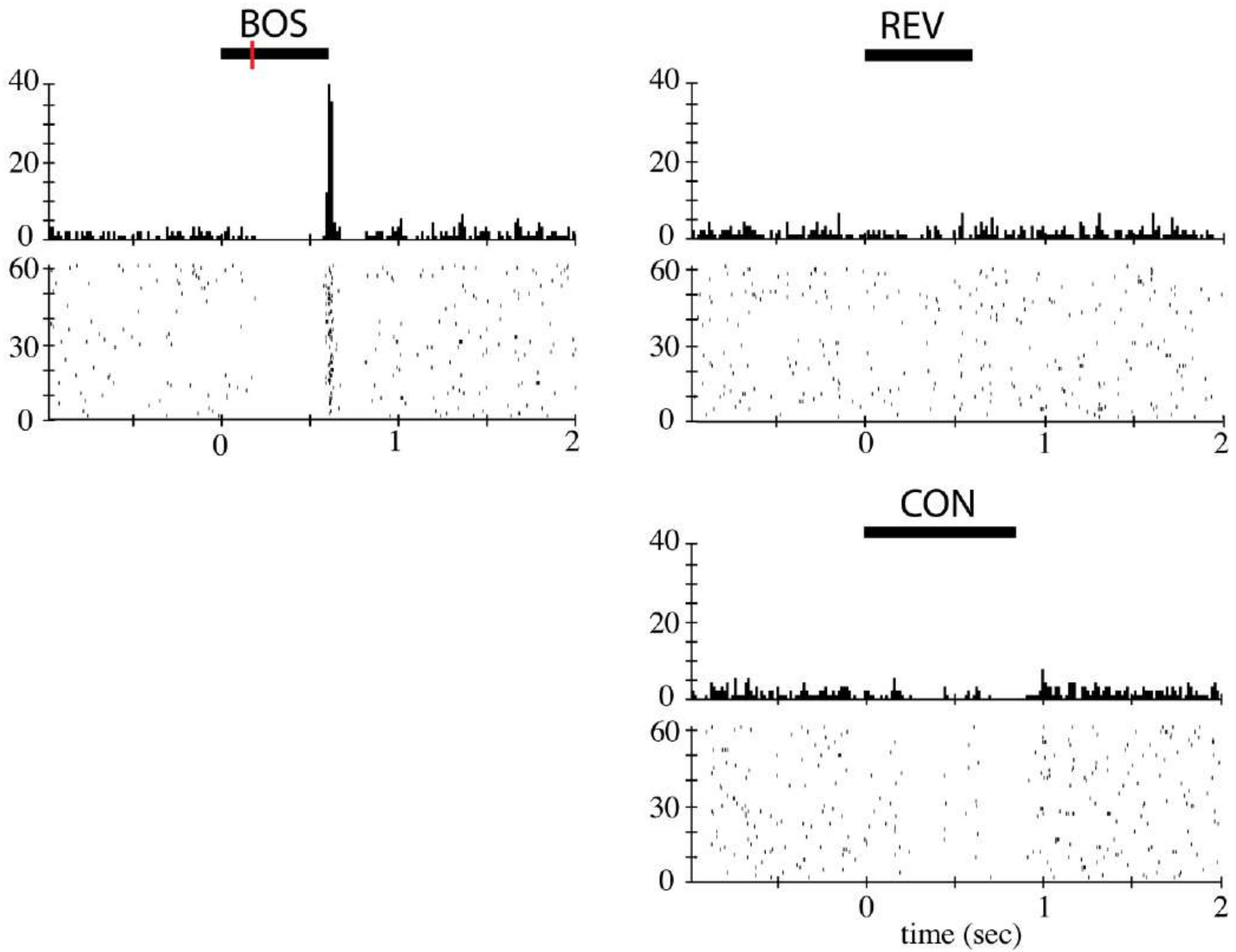
$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -\alpha(t)\gamma^2 - \beta(t)\gamma^2 x - \gamma^2 x^3 - \gamma x^2 y + \gamma^2 x^2 - \gamma x y$$

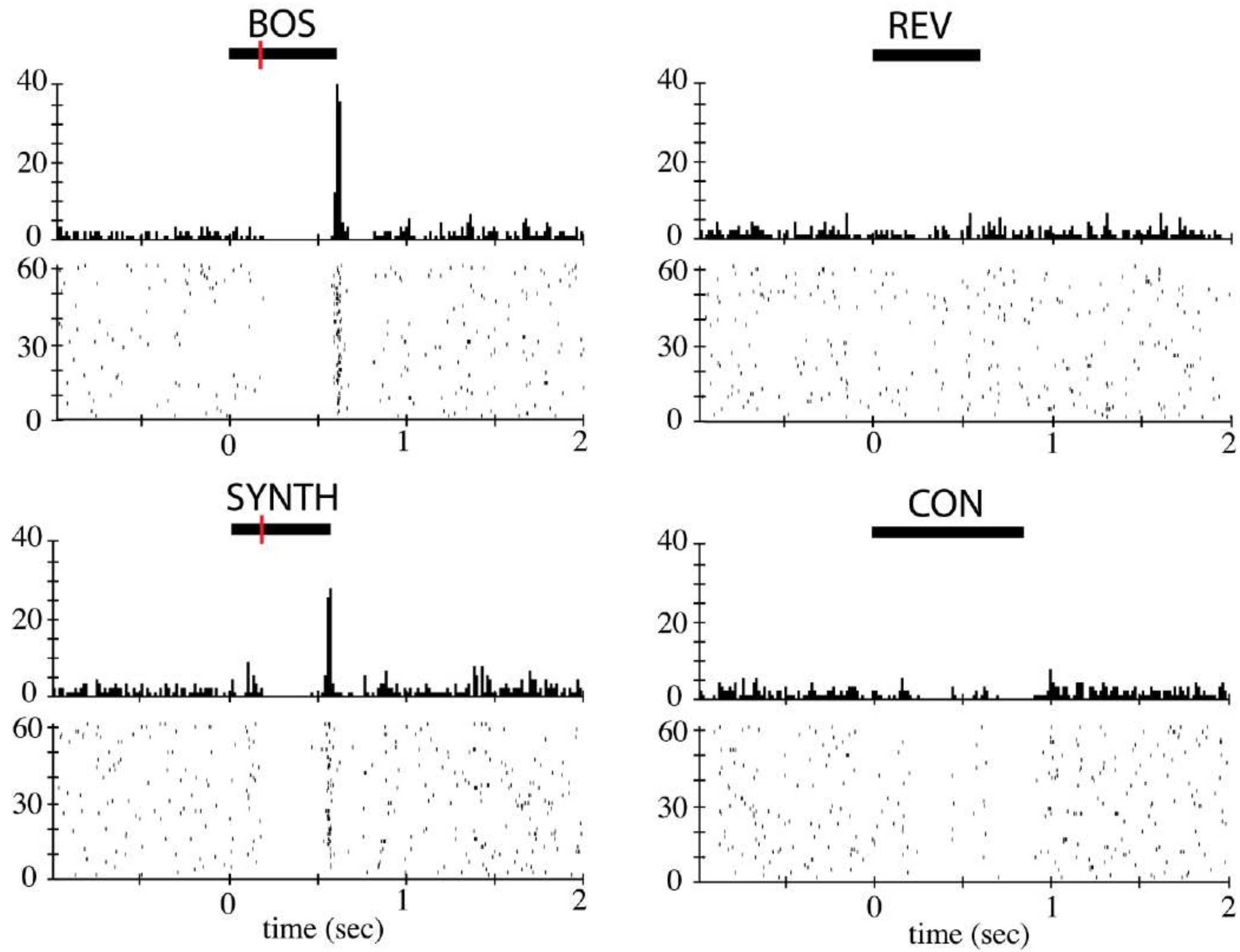
✓ More detailed modeling of the vocal tract (not just 3 tubes). Oropharingeal cavity as a resonator

✓ Intrinsic noise in the activity of the syringeal muscles

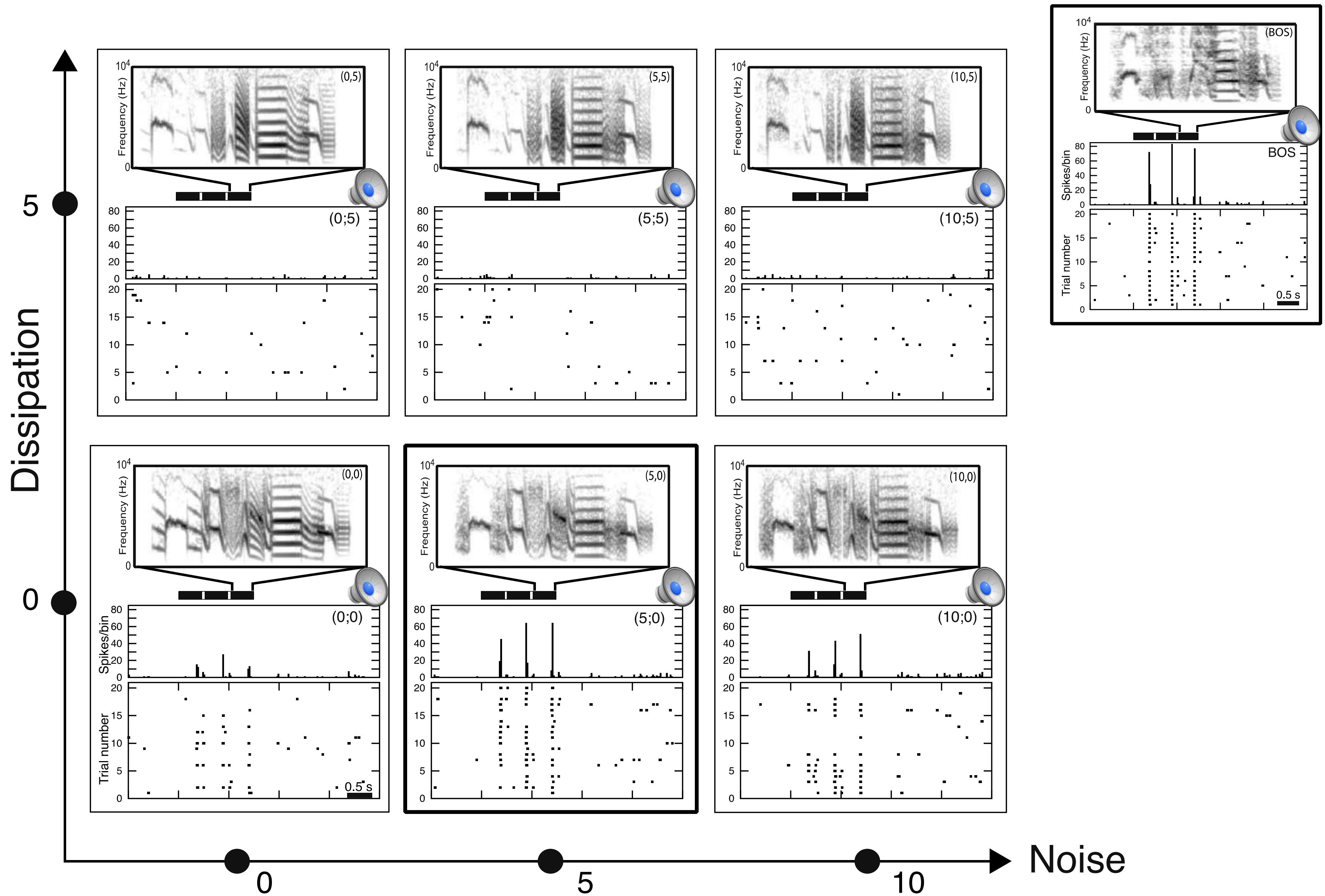
A more detailed modeling



A more detailed modeling



A strategy for studying a hierarchy of importance for the elements in the model

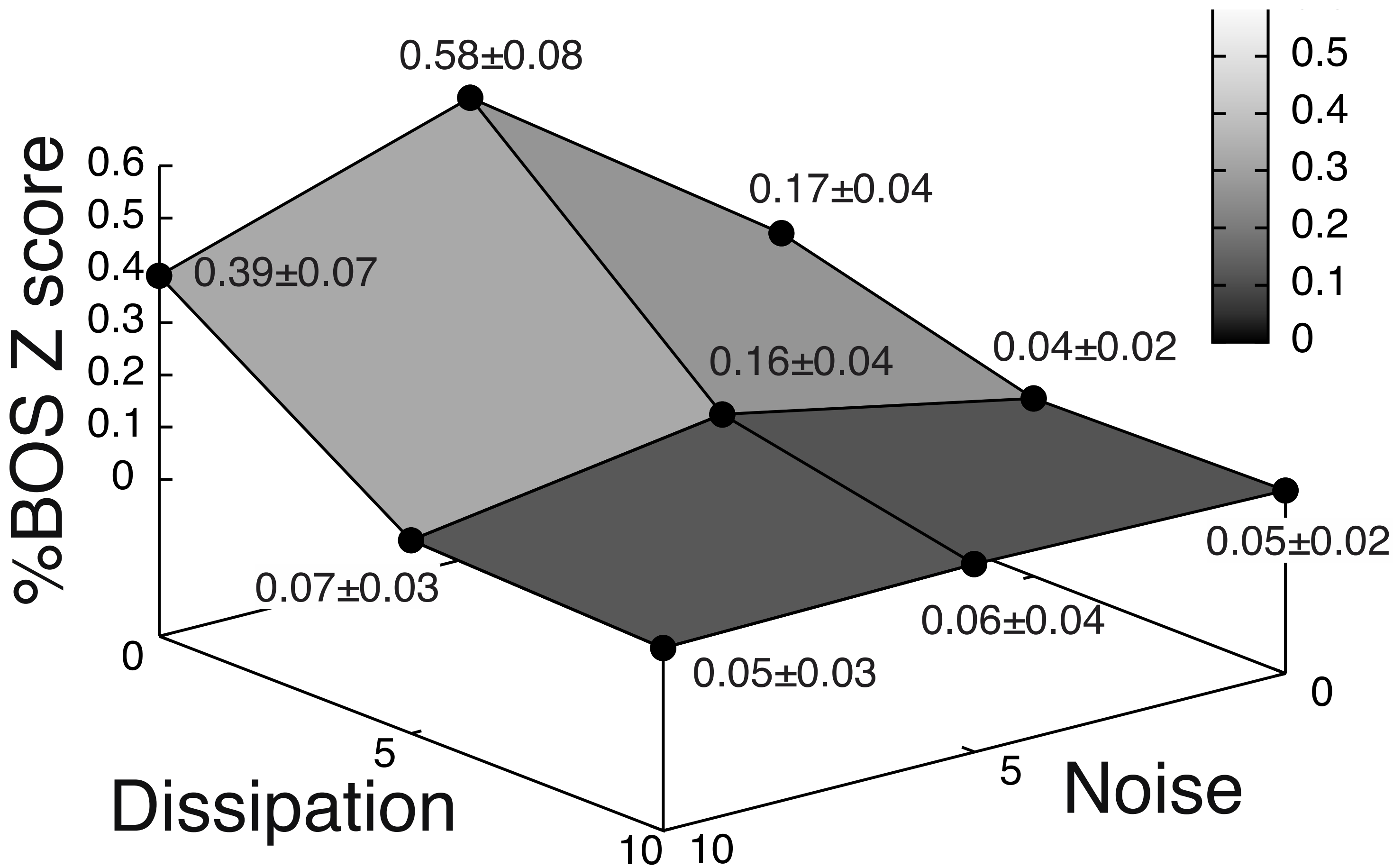


Tuning surface

Grouped data: 5 birds

The resonant cavity is very relevant (can be controlled by the birds while singing)

The noise has a particular value that maximize the responses



Nature 2013, Amador et al.

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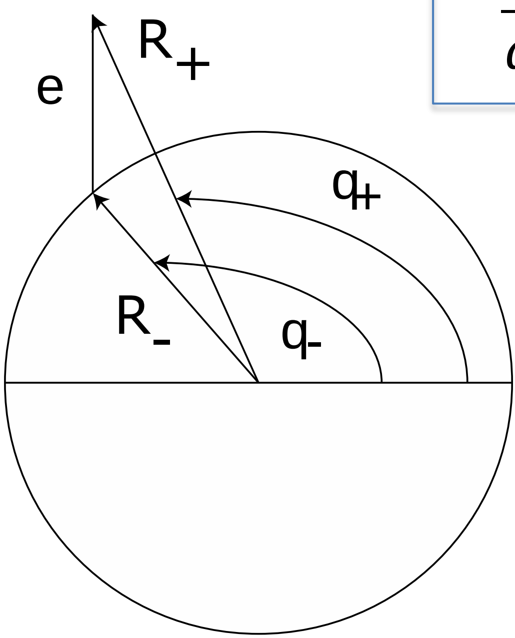
Beyond periodic orbits

$$\frac{dz}{dt} = (1 + i\eta)z - |z|^2 z + i\epsilon \sum_{n=-\infty}^{n=\infty} \delta(t - nT).$$

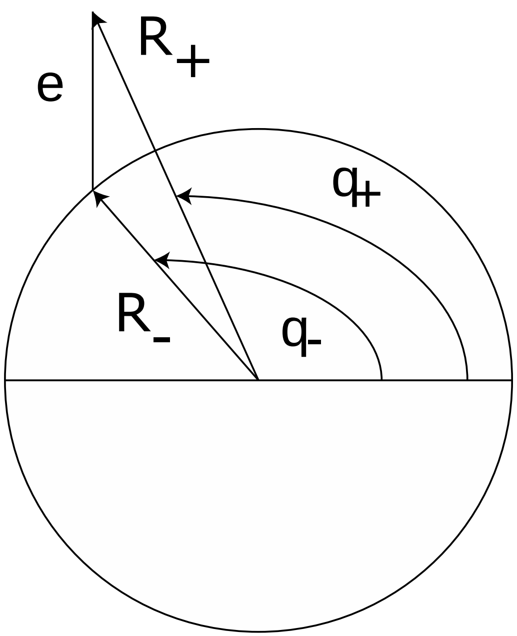


$$\frac{dR}{dt} = R(1 - R^2)$$

$$\frac{d\theta}{dt} = \eta$$

$$R(t) = \frac{1}{\sqrt{1 + \left(\frac{(1 - {R_0}^2)}{{R_0}^2}\right) e^{-2t}}}$$

$$\theta(t) = \theta_0 + \eta t$$



$$R_+ = \sqrt{{R_-}^2 + 2R_- \epsilon \sin(\theta_-) + \epsilon^2},$$

$$\theta_+ = tan^{-1}(\tan(\theta_-) + \epsilon/(R_- \cos(\theta_-)))$$

$$R_+ \approx R_n + \epsilon \sin(\theta_n)$$

$$\theta_+ \approx \theta_n + \epsilon \cos(\theta_n)/R_n$$

And after the autonomous evolution,

$$R_{n+1} = 1 + (R_n - 1 + \epsilon \sin(\theta_n))e^{-2T}$$

$$\theta_{n+1} = \theta_n + \eta T + \epsilon \cos(\theta_n)/R_n$$

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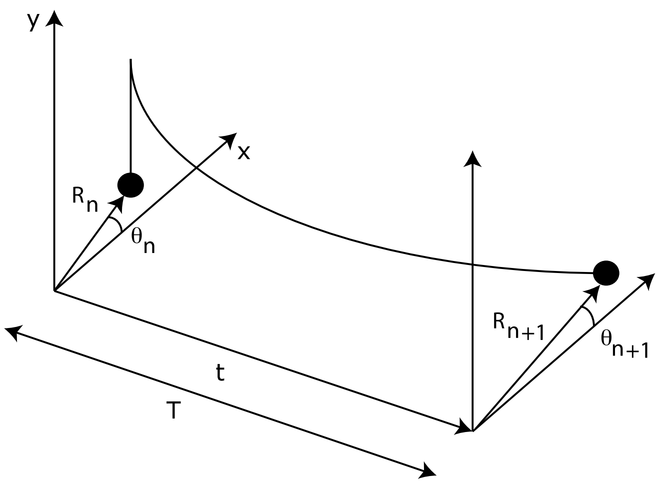

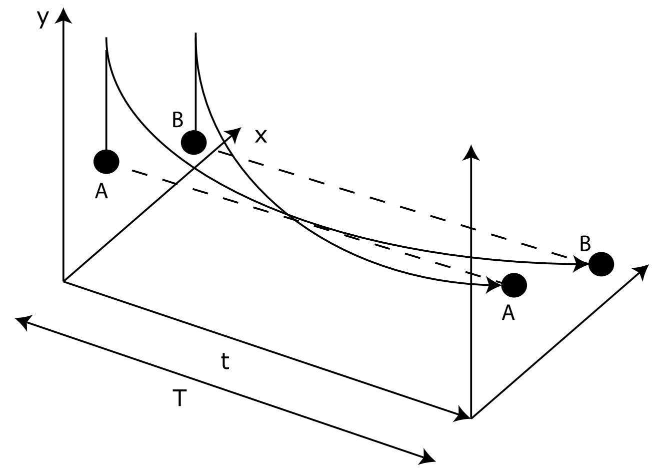

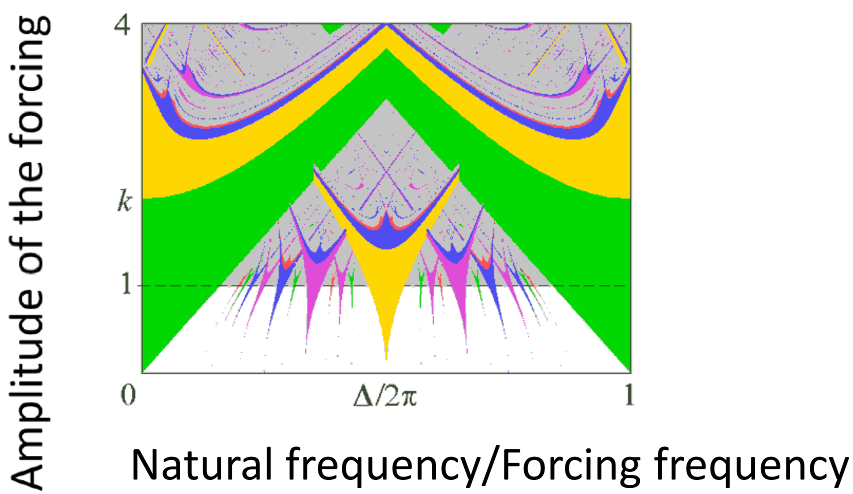

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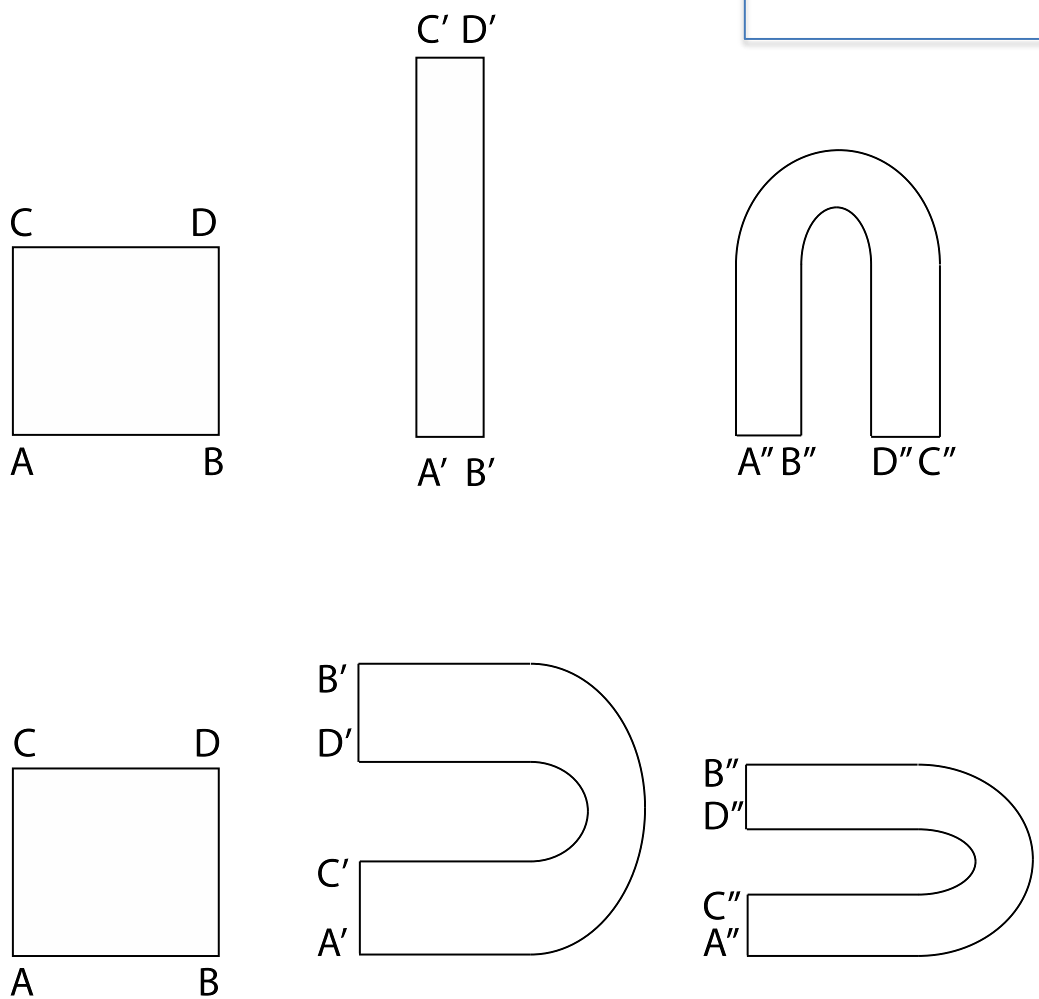
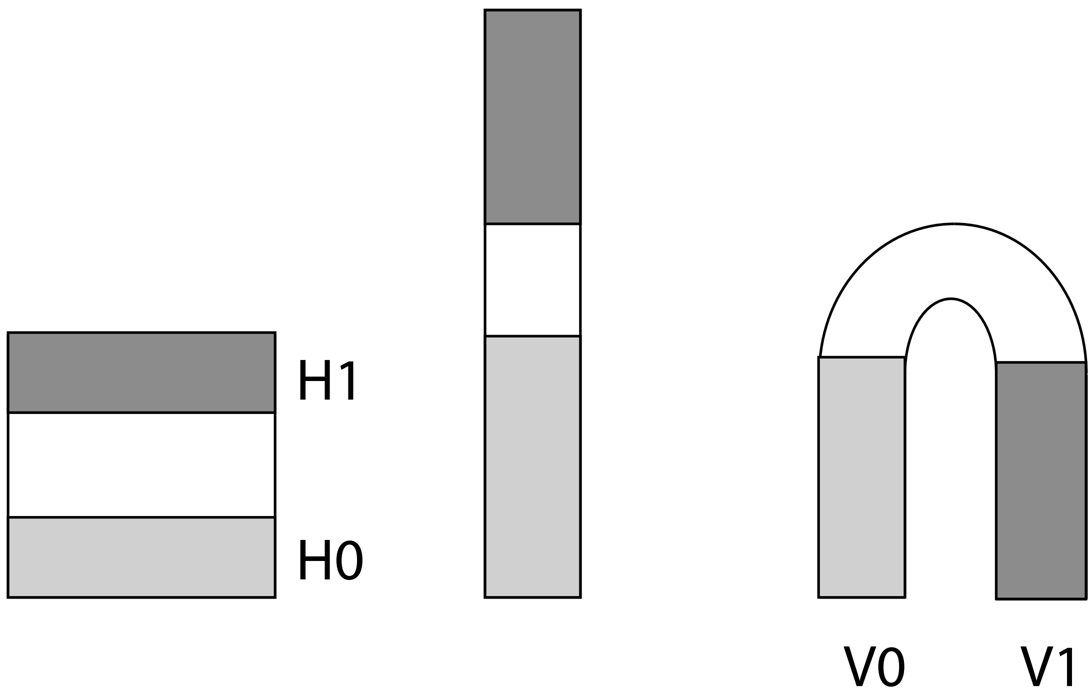
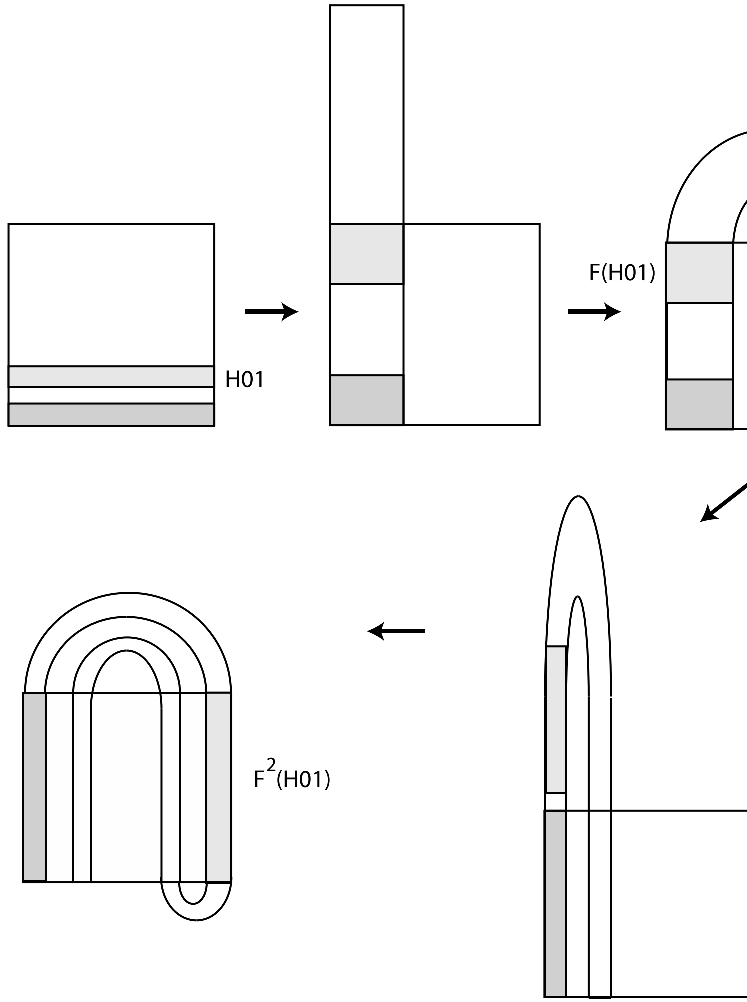
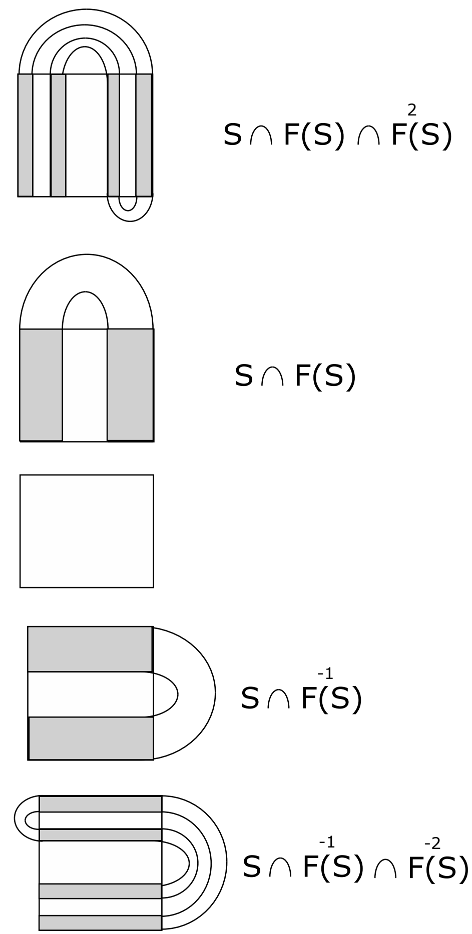
A caricature by Smale





H1
H0
V0
V1


H01
F(H01)
F
2
(H01)


$S \cap F(S) \cap F^2(S)$
$S \cap F(S)$
$S \cap F^{-1}(S)$
$S \cap F^{-1}(S) \cap F^{-2}(S)$

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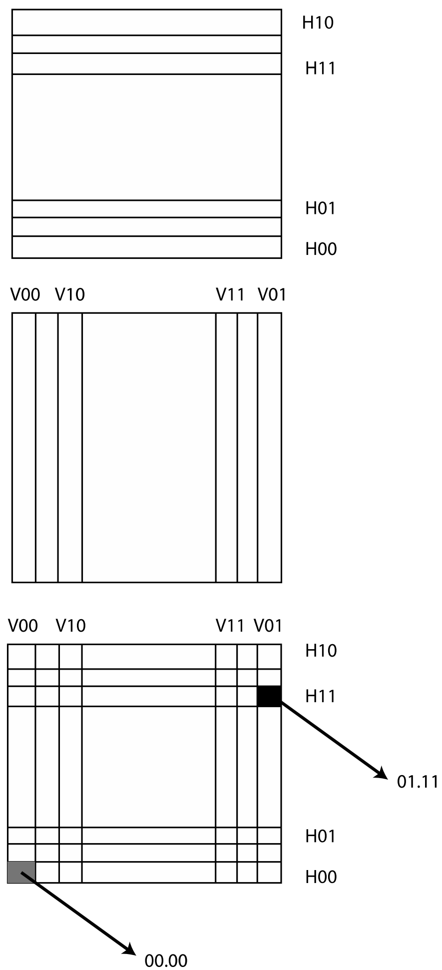
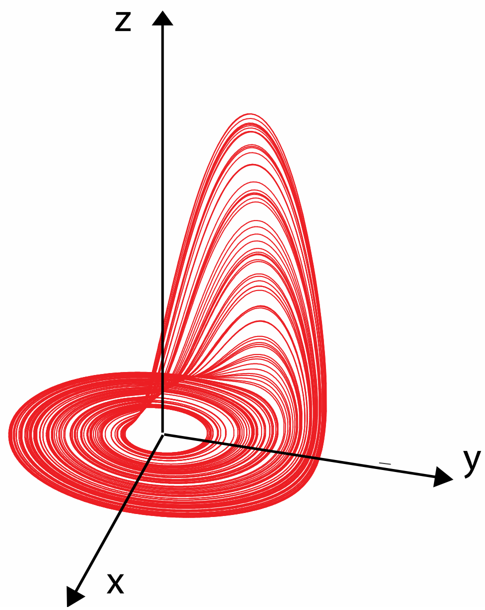
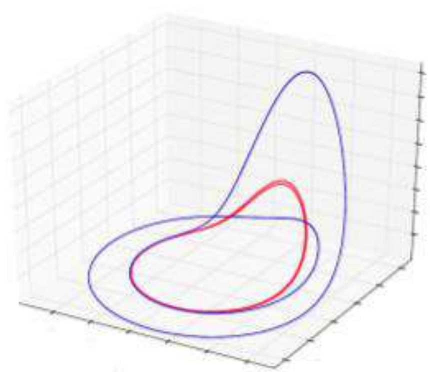


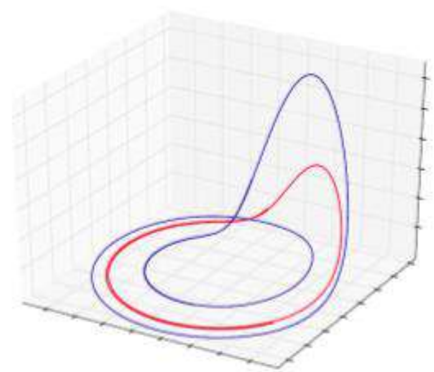


$$\frac{dx}{dt} = -y - z$$

$$\frac{dy}{dt} = x + 0.1\,y$$

$$\frac{dz}{dt} = 0.1 + z(x - 14),$$





	0, 1_1	1, 1_2	01, 2	011, 3_1	001, 3_2
0, 1_1	0	0	0	0	0
1, 1_2	0	0	1	1	1
01, 2	0	1	1	2	2
011, 3_1	0	1	2	2	3
001, 3_2	0	1	2	3	2

Rossler

	0, 1_1	1, 1_2	01, 2	011, 3_1	001, 3_2
0, 1_1	0	0	0	0	0
1, 1_2	0	0	0	0	0
01, 2	0	0	1	1	1
011, 3_1	0	0	1	2	1
001, 3_2	0	0	1	1	2

Lorenz

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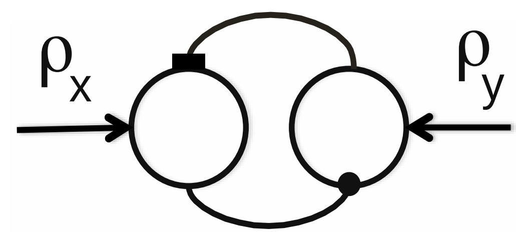
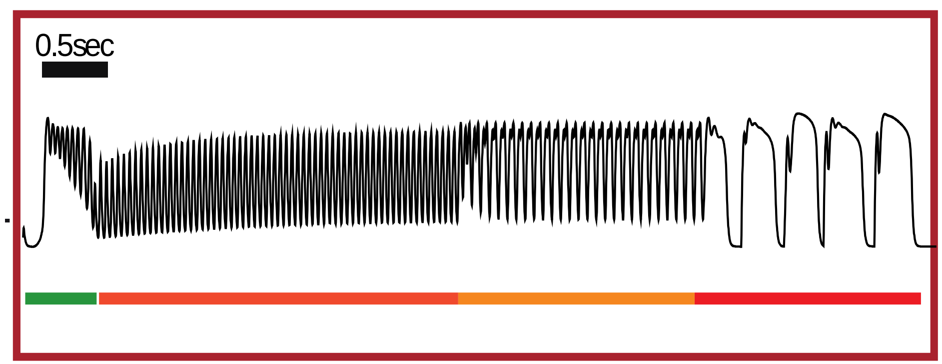
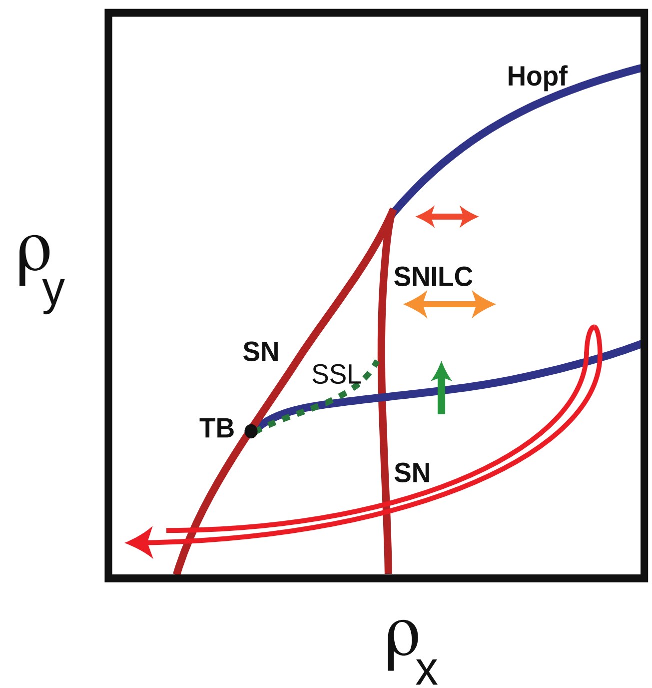


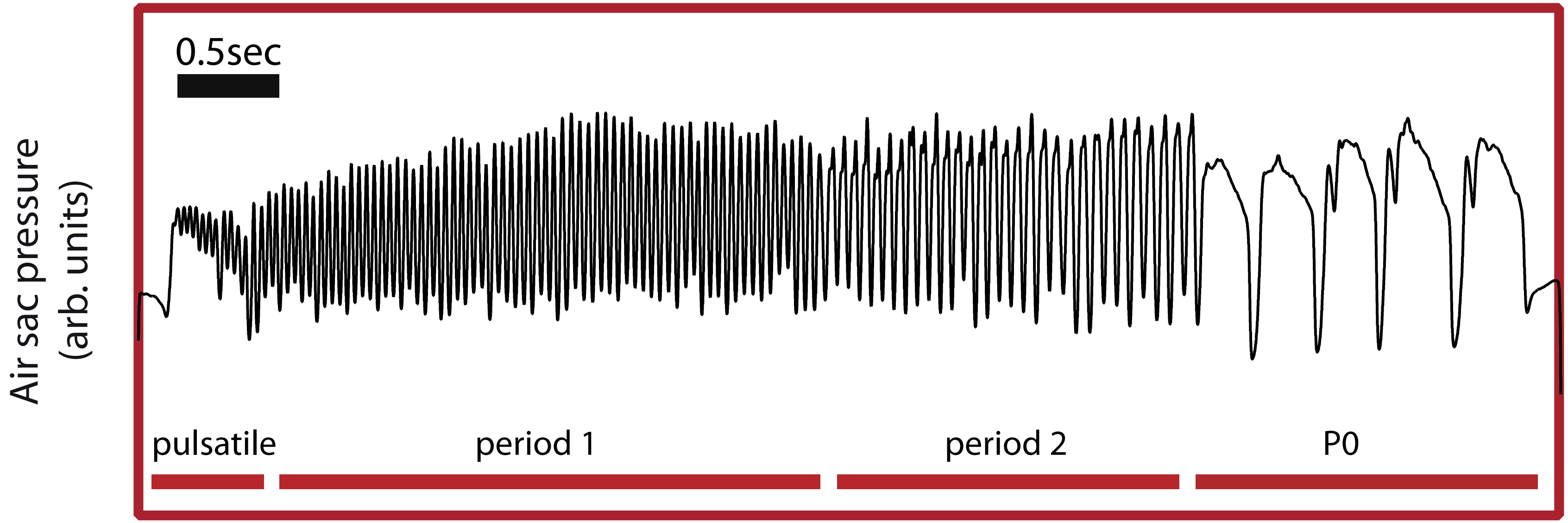




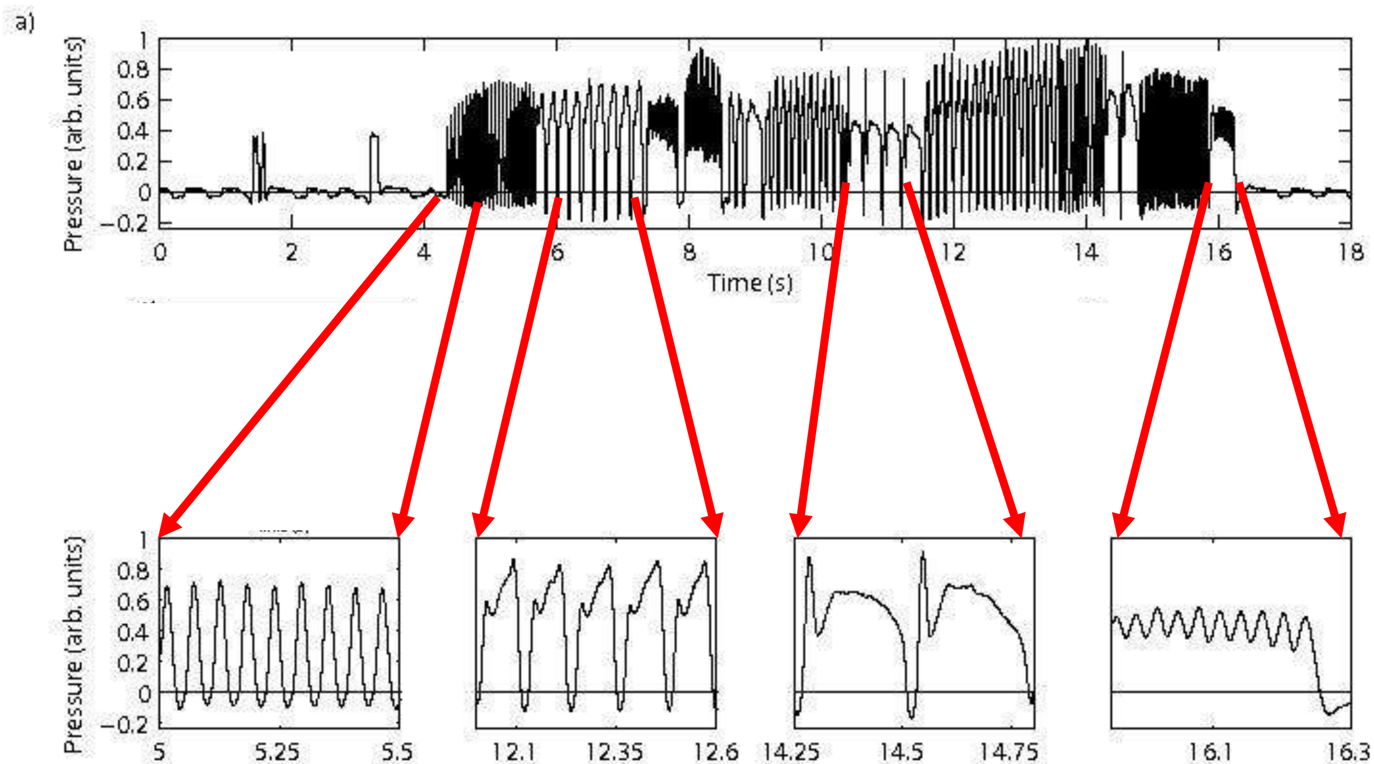
What dynamics can we expect from a low dimensional, nonlinear system?

How does low dimensional dynamics emerge in nature?

Part 2


0.5sec
Air sac pressure
(arb. units)
pulsatile
period 1
period 2
P0

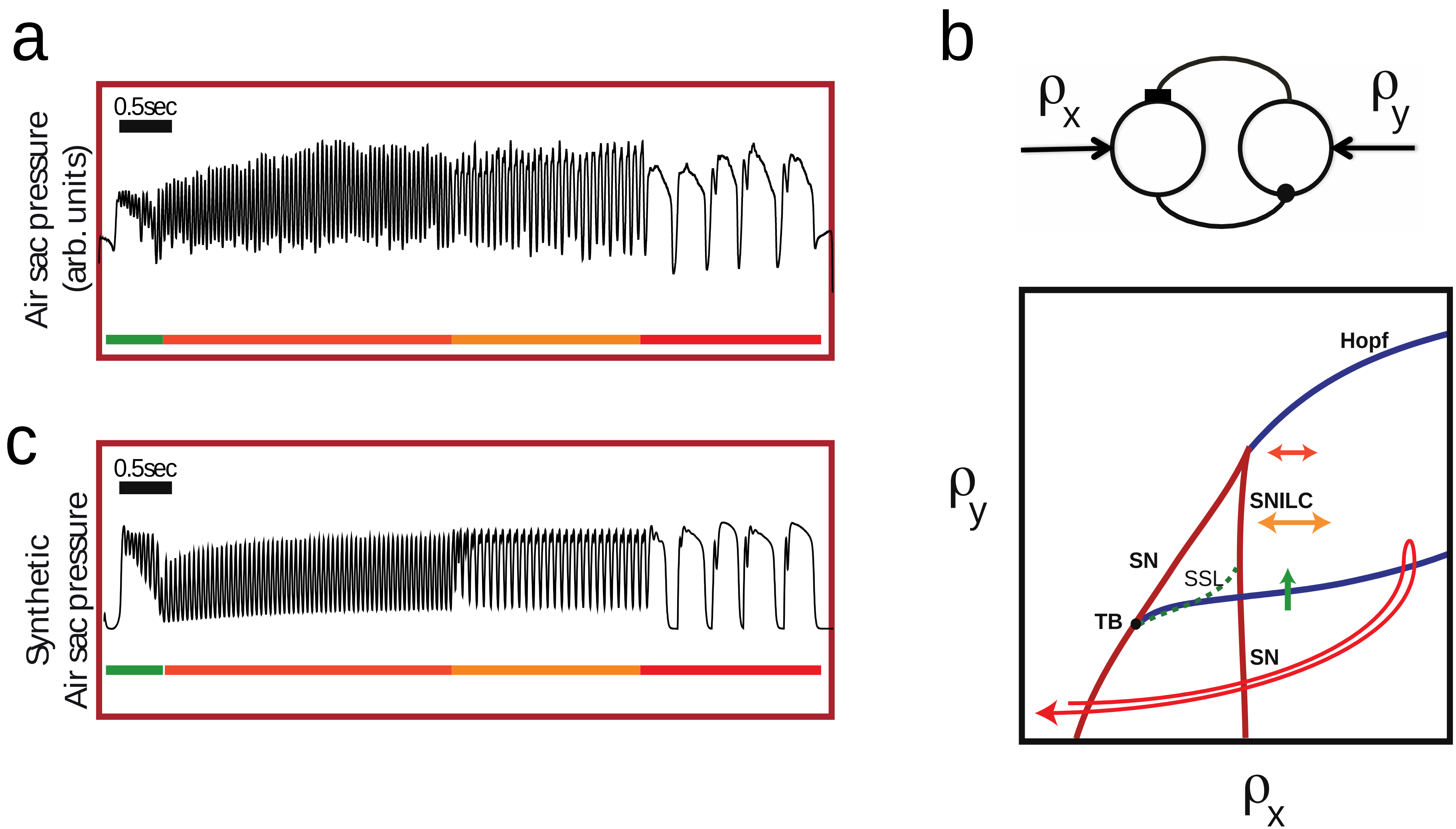
They are not that complex either…



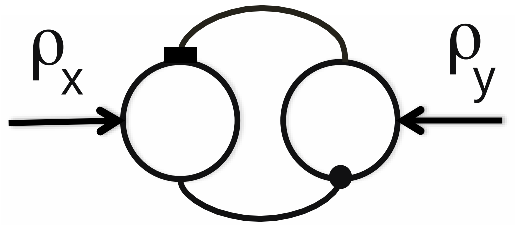
How complex are these gestures?

Can we find them as solutions of simple ODE dynamical model?

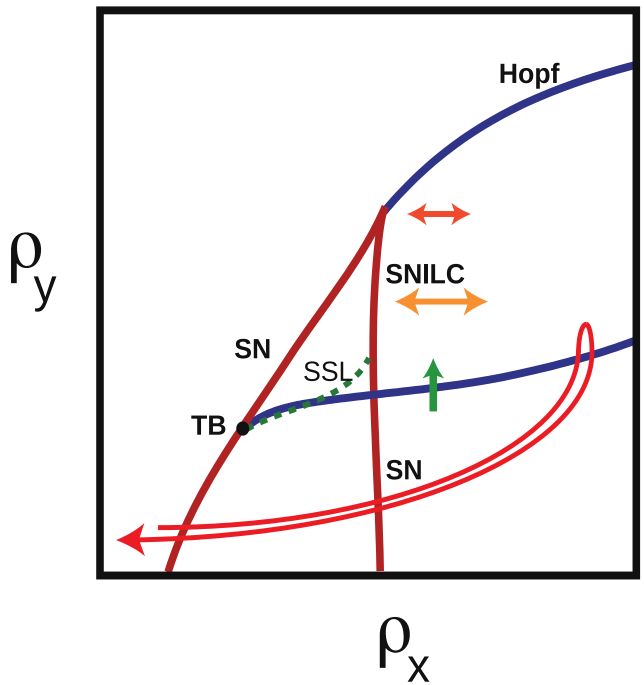
And they are the solutions of a
Low dimensional dynamical system



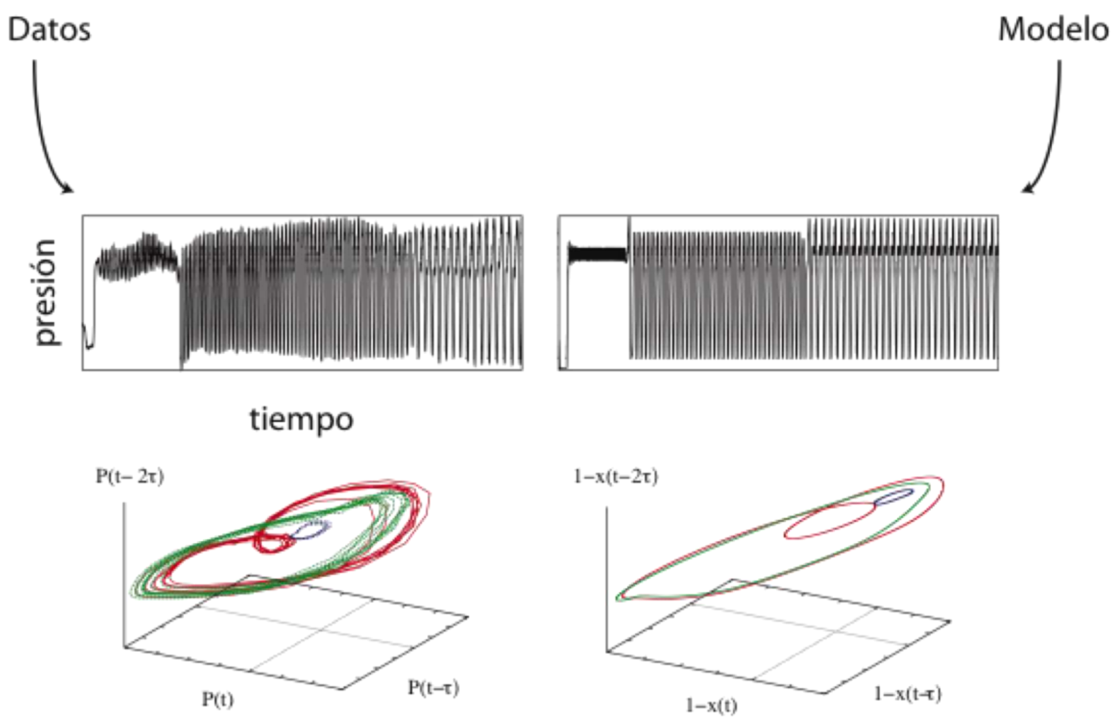
And they are the solutions of a
Low dimensional dynamical system



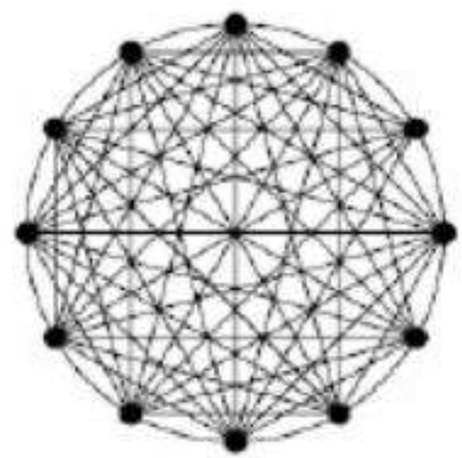
How do we obtain
low dimensional dynamics
from a large set of excitable units?



And it is really low dimensional…



Let us first consider only
Excitatory neurons, all to all coupling



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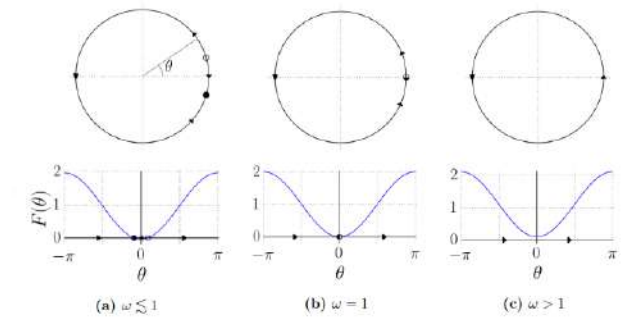
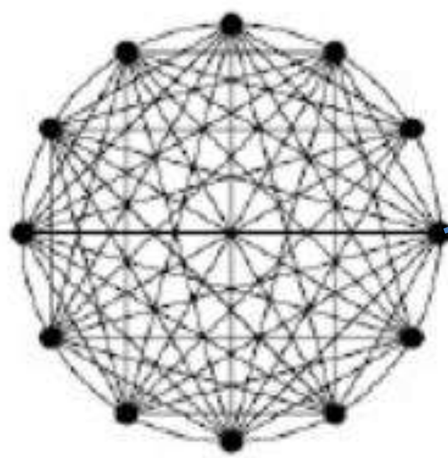
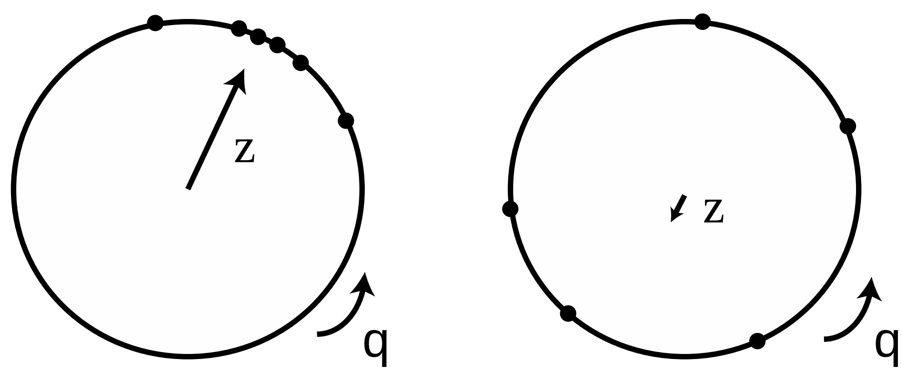




Each unit, a phase oscillator

Plotting all the phases together, the set is represented by a cloud. Here, two examples.



$$z = re^{i\psi} \equiv \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}.$$

This order parameter, z, describes the synchronicity

$$\frac{d\theta_i}{dt} = \omega_i - \cos(\theta_i) + \frac{K}{N}\sum_{j=1}^{N}(1 - \cos(\theta_j)),$$
$$i = 1,2, \dots, N.$$

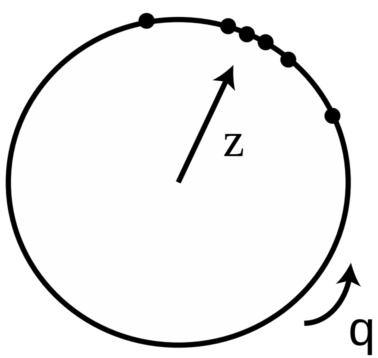
With z, we can write the System of equations as

$$z = re^{i\psi} \equiv \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}.$$

$$\frac{d\theta_i}{dt} = \omega_i - \cos(\theta_i) + K(1 - \mathrm{Re}(z)),$$
$$i = 1,2, \dots, N.$$

But the real question is whether we can write **equations for the order parameter**, which describes macroscopically the set of oscillators.

The strategy, to work with the distribution of phases



$$\int_0^{2\pi} f(\omega,\theta,t)\, d\theta = g(\omega),$$

We start with the continuity equation for the distribution

$$\boxed{\frac{\partial f}{\partial t} + \frac{\partial}{\partial \theta}\left(f\,\frac{d\theta}{dt}\right) = 0.}$$

We write the velocity in terms of z,

$$\frac{d\theta}{dt} = w - \left(\frac{e^{i\theta} + e^{-i\theta}}{2}\right) + k(1 - Re(z))$$

And we write the "continuous" version of z.

$$z = \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} d\omega f e^{i\theta}.$$

We write a mode expansion for the distribution,

$$f = \left(\frac{1}{\sqrt{2\pi}}\right)\left\{\sum_{n=-\infty}^{\infty} a_n(\omega, t)e^{in\theta}\right\},$$

$$a_n = \left(\frac{1}{\sqrt{2\pi}}\right)\left\{\int_{-\infty}^{\infty} f(\theta, \omega, t)e^{-in\theta}\, d\theta\right\},$$

And inserting these into the equation of continuity, we would get equations (an infinite set of them!) for the amplitudes. Unless we can justify a reduction in the number of equations, we won´t make much progress.

Inserting
f and ***dϑdt***

$$\boxed{\frac{\partial f}{\partial t}+\frac{\partial f}{\partial \theta}\frac{d\theta}{dt}+f\frac{\partial}{\partial \theta}\left(\frac{d\theta}{dt}\right)=0}$$

$$\frac{\partial f}{\partial t}=\sum_{n=-\infty}^{n=\infty}\frac{\partial a_n}{\partial t}\frac{e^{in\theta}}{\sqrt{2\pi}}$$

$$\frac{d\theta}{dt}\frac{\partial f}{\partial \theta}=\sum_{n=-\infty}^{n=\infty}\left(\omega-\frac{\left(e^{i\theta}+e^{-i\theta}\right)}{2}+k\left(1-Re(z)\right)\right)ina_n\frac{e^{in\theta}}{\sqrt{2\pi}}$$

$$\frac{\partial\left(\frac{d\theta}{dt}\right)}{\partial \theta}f=\sum_{n=-\infty}^{n=\infty}\frac{\left(-ie^{i\theta}+ie^{-i\theta}\right)}{2}a_n\frac{e^{in\theta}}{\sqrt{2\pi}}$$

Inserting
f and ***dϑdt***

$$\boxed{\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \theta}\frac{d\theta}{dt} + f\frac{\partial}{\partial \theta}\left(\frac{d\theta}{dt}\right) = 0}$$

$$\sum_{n=-\infty}^{n=\infty}\left(\frac{\partial a_n}{\partial t} + in\Big(\omega + k\big(1 - Re(z)\big)\Big)a_n - \frac{i}{2}na_{n-1} - \frac{i}{2}na_{n+1}\right)\frac{e^{in\theta}}{\sqrt{2\pi}} = 0$$

If we define $\alpha_n = \dfrac{\sqrt{2\pi}}{g(\omega)}a_n(\omega, t)$

Now one of the modes stands out:

$$\int_{-\infty}^{\infty} g(\omega)\alpha_1(\omega,t)d\omega = \int_{-\infty}^{\infty} d\omega \int_0^{2\pi} f(\theta,\omega,t)e^{-i\theta}\, d\theta$$
$$\equiv z^*$$

In other words, if we know how this mode evolves, and we can solve the first integral, we will have a macroscopic description of network.

$$\frac{\partial \alpha_n}{\partial t} = -in\left(\omega + k\left(1 - Re(z)\right)\right)\alpha_n + \frac{i}{2}n(\alpha_{n-1} + \alpha_{n+1})$$

Let us assume that (Ott´s ansatz)

$$\boxed{a_n = a_1^n}$$

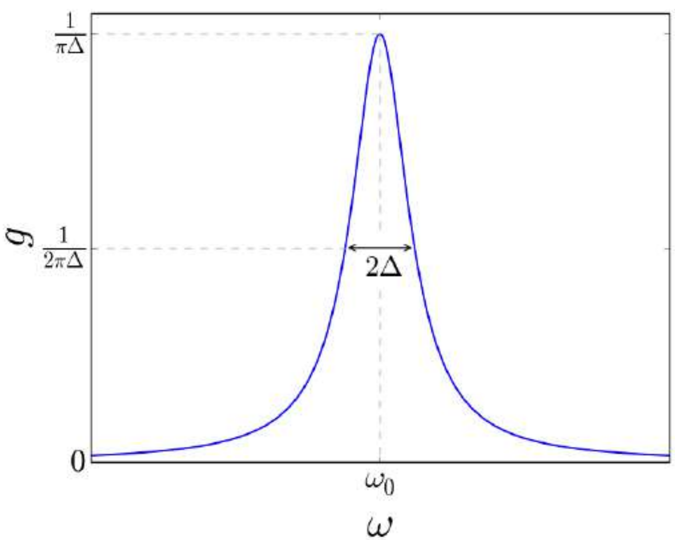
$$\frac{\partial \alpha_n}{\partial t} = n{\alpha_1}^{n-1}\frac{\partial \alpha_1}{\partial t} = -in\left(\omega + \left(1 - Re(z)\right)\right){\alpha_1}^{n} + \frac{i}{2}n({\alpha_1}^{n-1} + {\alpha_1}^{n+1})$$

$$\Rightarrow \boxed{\frac{\partial \alpha_1}{\partial t} = -i\left(\omega + \left(1 - Re(z)\right)\right)\alpha_1 + \frac{i}{2}(1 + {\alpha_1}^{2})}$$

So, this is a partial differential equation satisfied by the amplitude of the first model.

1. Its integral over the frequencies, gives the order parameter
2. If we know this, we know all the amplitudes

$$z^*(t) = \int_{-\infty}^{\infty} g(\omega)\alpha(\omega,t)d\omega$$



$$g(\omega) = \frac{\Delta}{\pi}\frac{\alpha(\omega,t)}{(\omega-\omega_0+i\Delta)(\omega-\omega_0+i\Delta)}$$

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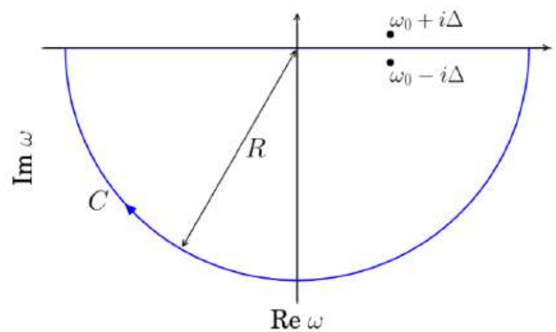
$$z^*(t) = \int_{-\infty}^{\infty} g(\omega)\alpha(\omega,t)d\omega = \alpha(\omega_0 - i\Delta, t)$$



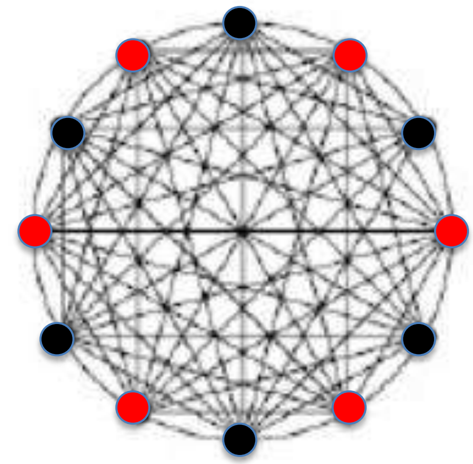
$$\boxed{\frac{dz}{dt} = i\Big((\omega_0 + i\Delta) + k\big(1 - Re(z)\big)\Big)z - \frac{i}{2}(1+z^2)}$$

$$\phi(t) \equiv \int_{-\infty}^{\infty} f(\omega, \theta, t) \left.\frac{d\theta}{dt}\right|_{\theta=\pi} d\omega$$

This is the flow of units crossing $\vartheta=\pi$, i.e. spiking

$$\boxed{\phi(t) = \frac{1}{\pi}\left(\frac{1+Re(z)}{|1+z|^2} - \frac{1}{2}\right)\left(\omega_0 + 1 + k\big(1 - Re(z)\big)\right) + \frac{\Delta Im(z)}{|1+z|^2}}$$

A really intuitive macroscopic quantity (how many spikes there are, globally, per unit of time), computed as a function Of the order parameter.

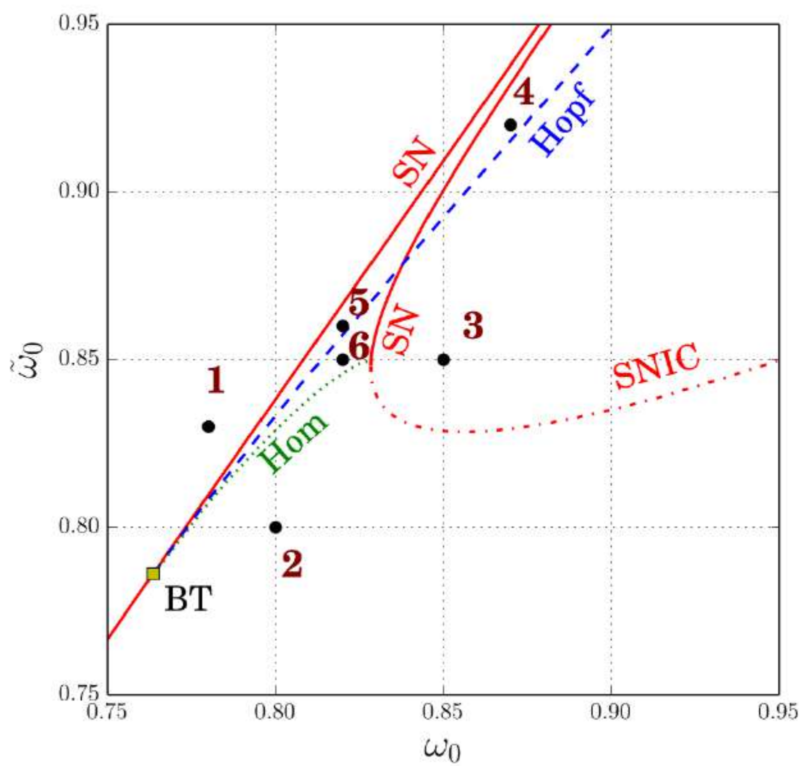


$$\frac{d\theta^E{}_i}{dt} = \omega_i - \cos(\theta^E{}_i) + \sum_{j=1}^{N} \frac{K_E}{N}\left(1-\cos\left(\theta^E{}_j\right)\right) - \sum_{l=1}^{N_I} \frac{K_I}{N}\left(1-\cos\left(\theta^I{}_j\right)\right),$$

$$\frac{d\theta^I{}_i}{dt} = \omega_i - \cos(\theta^I{}_i) + \sum_{j=1}^{N} \frac{\widehat{K_E}}{N}\left(1-\cos\left(\theta^E{}_j\right)\right) - \sum_{l=1}^{N_I} \frac{\widehat{K_I}}{N}\left(1-\cos\left(\theta^I{}_j\right)\right),$$

$$\frac{dz}{dt} = i\Big((\omega_0 + i\Delta) + k\big(1 - Re(z)\big) - K_I\big(1 - Re(z_I)\big)\Big)z - \frac{i}{2}(1 + z^2)$$

$$\frac{dz_I}{dt} = i\Big((\omega_0{}^I + i\Delta^I) + k\big(1 - Re(z_I)\big) - K_I\big(1 - Re(z)\big)\Big)z_I - \frac{i}{2}(1 + z_I{}^2)$$



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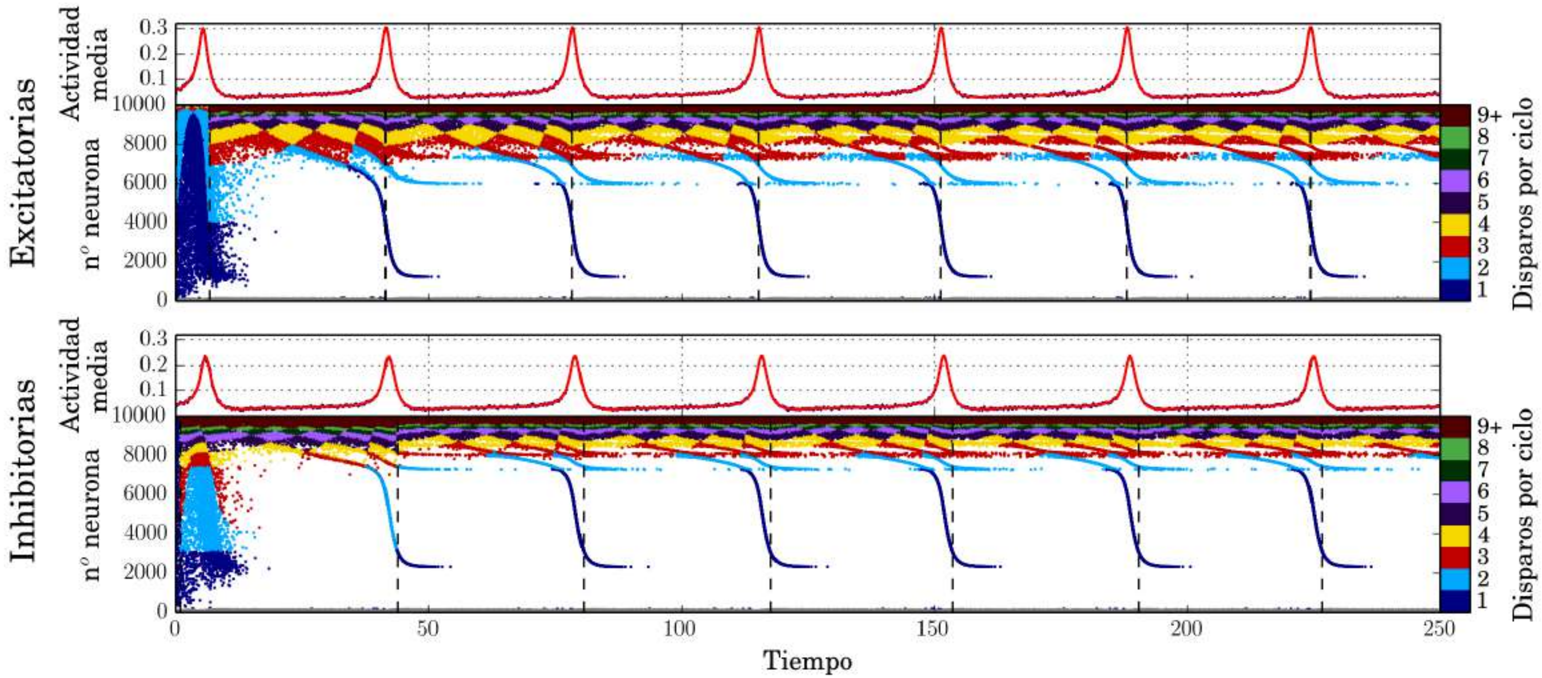
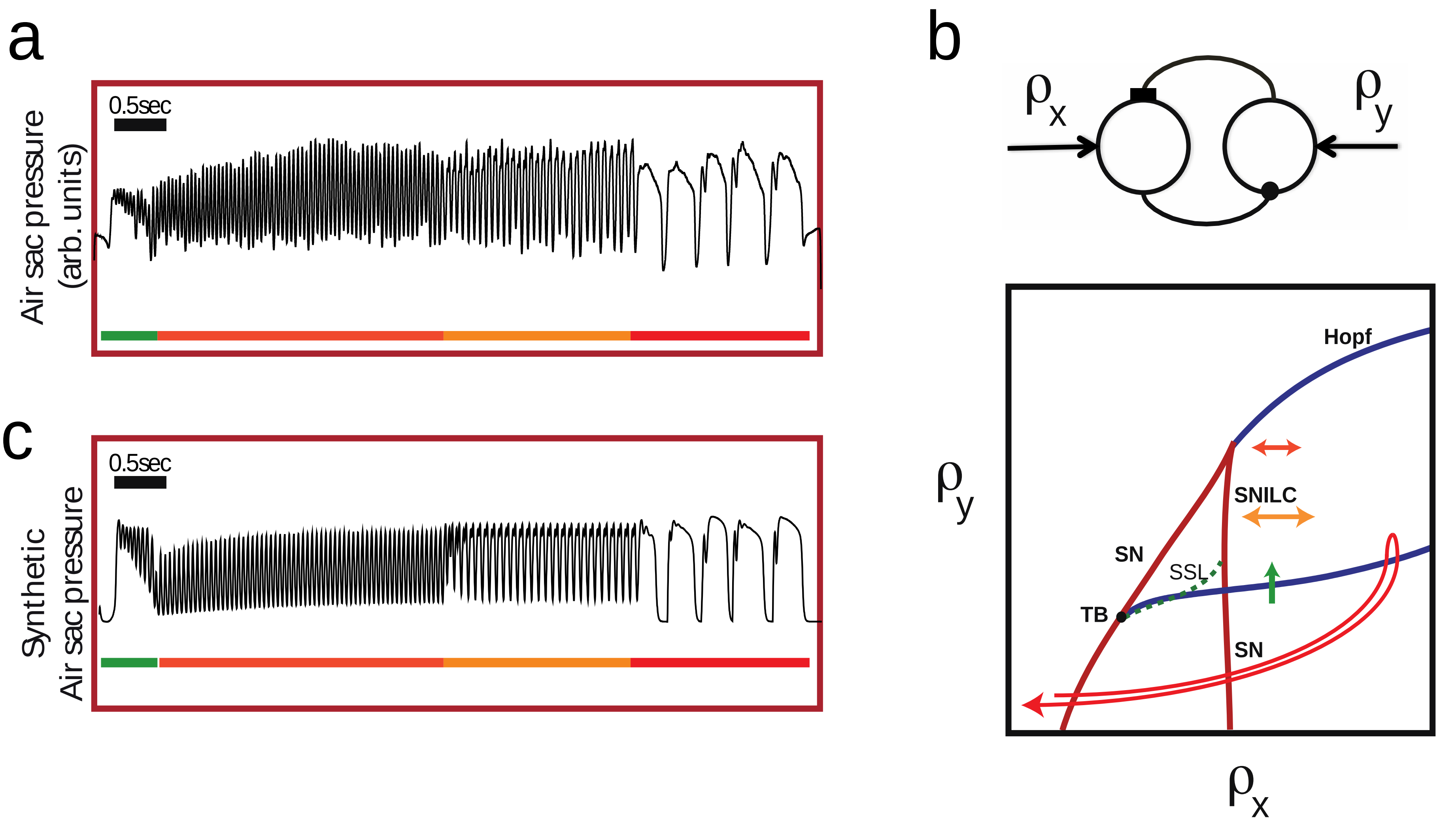
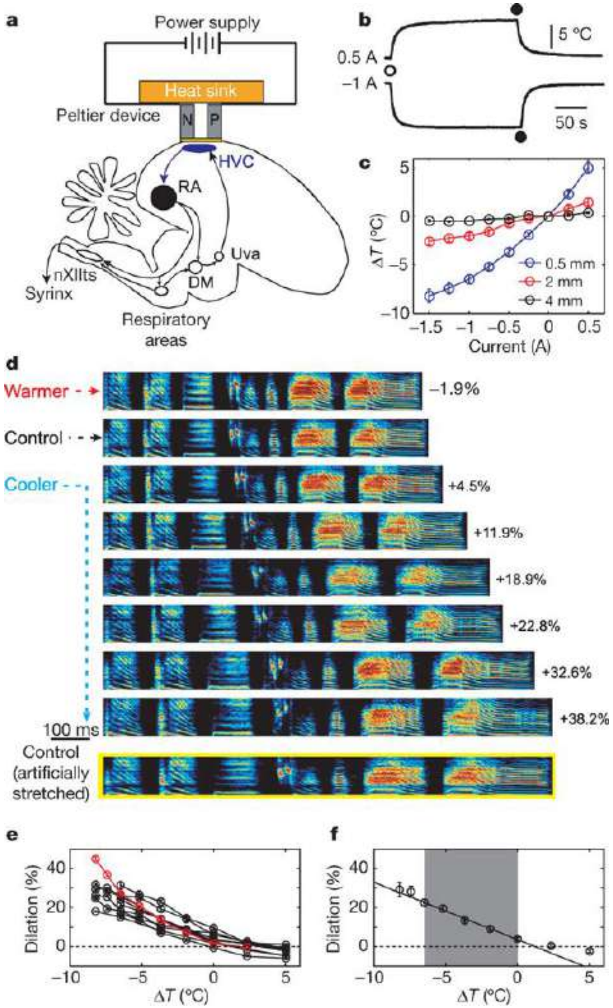


The same dynamical elements that we needed for the phenomenological model, in order to reproduce the physiological patterns when the respiratory system is forced by the telencephalon



A continuous representation of time at the telencephalon:

"Cool" experiments

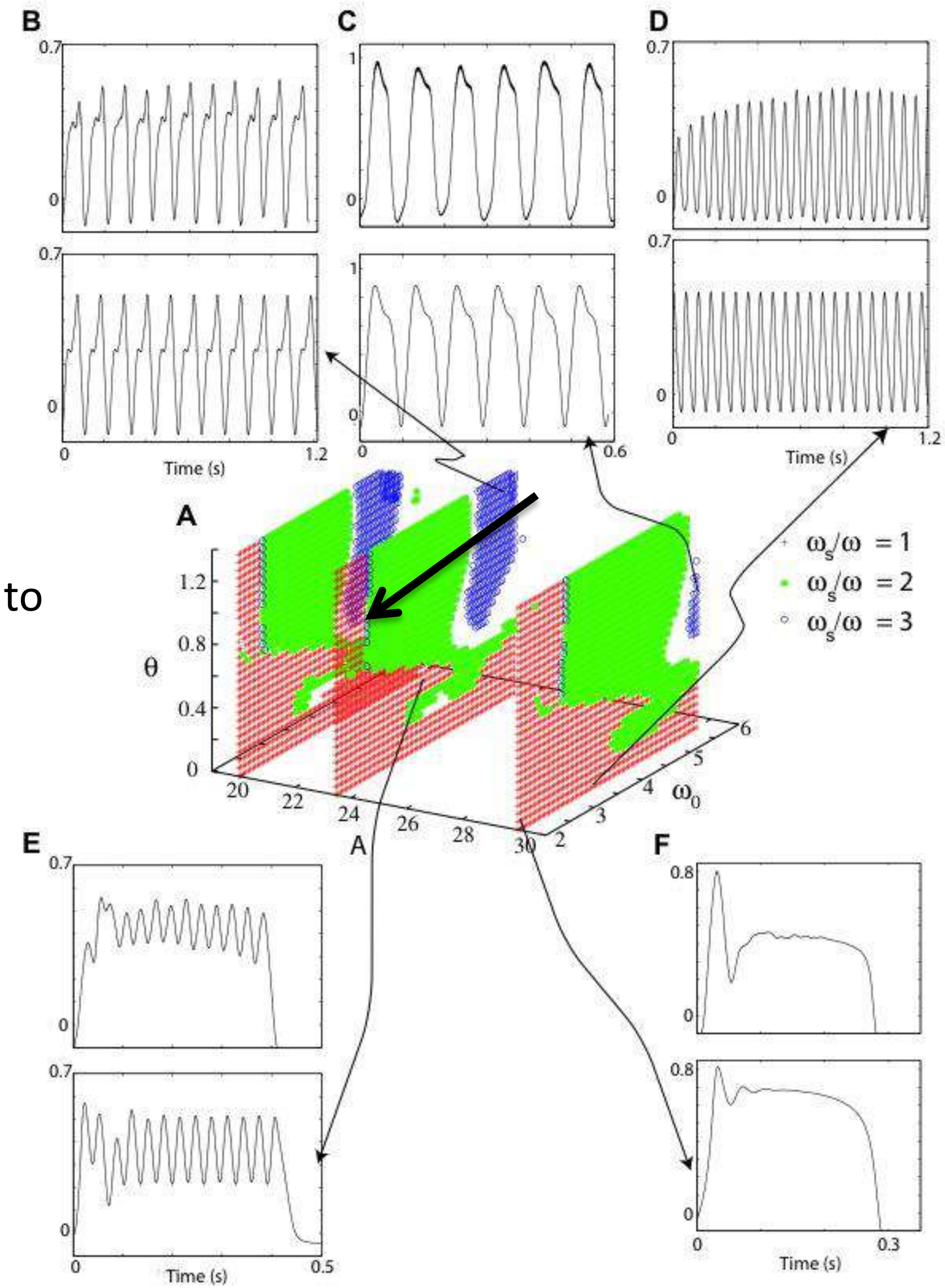


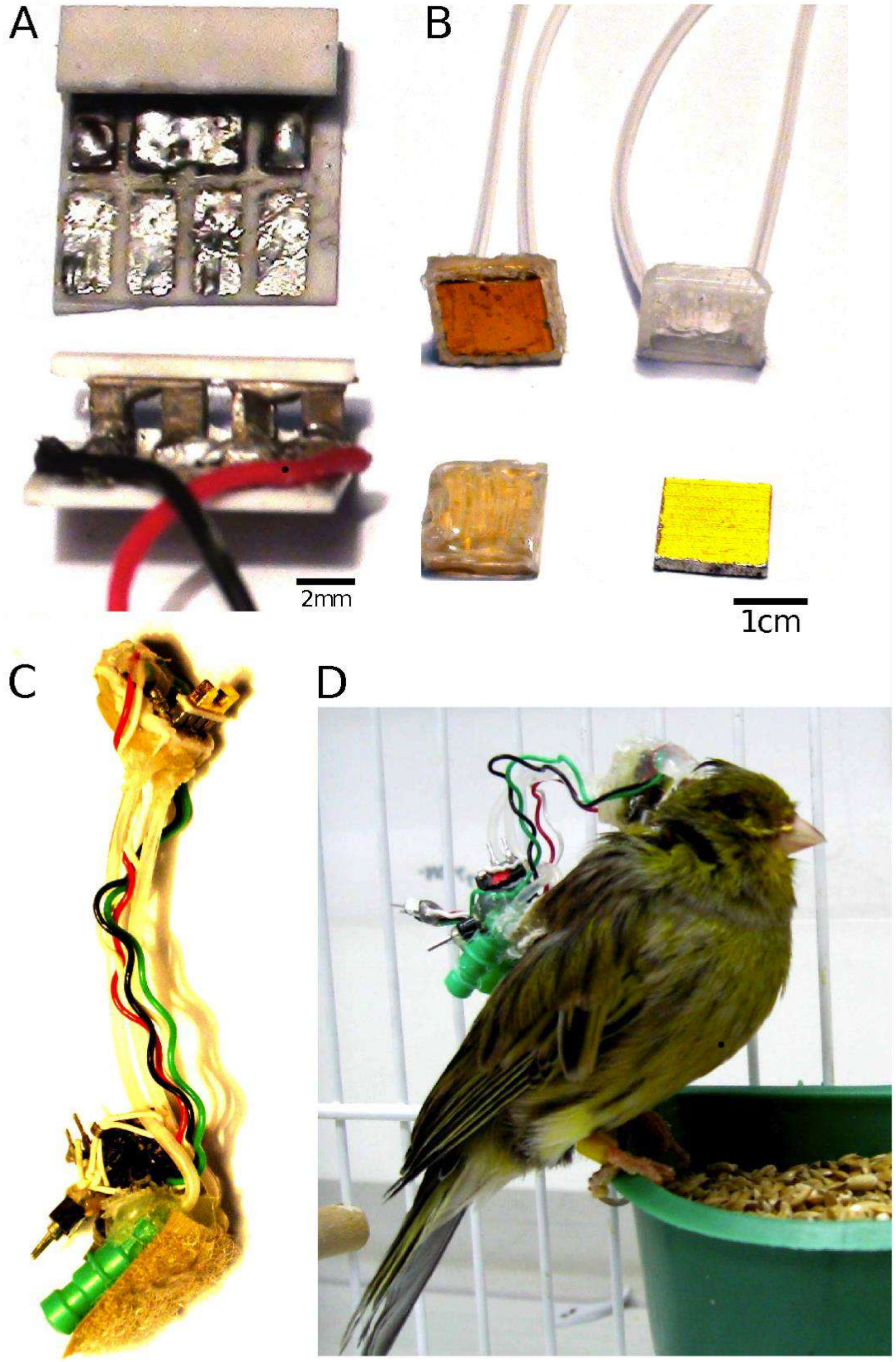
Long & Fee

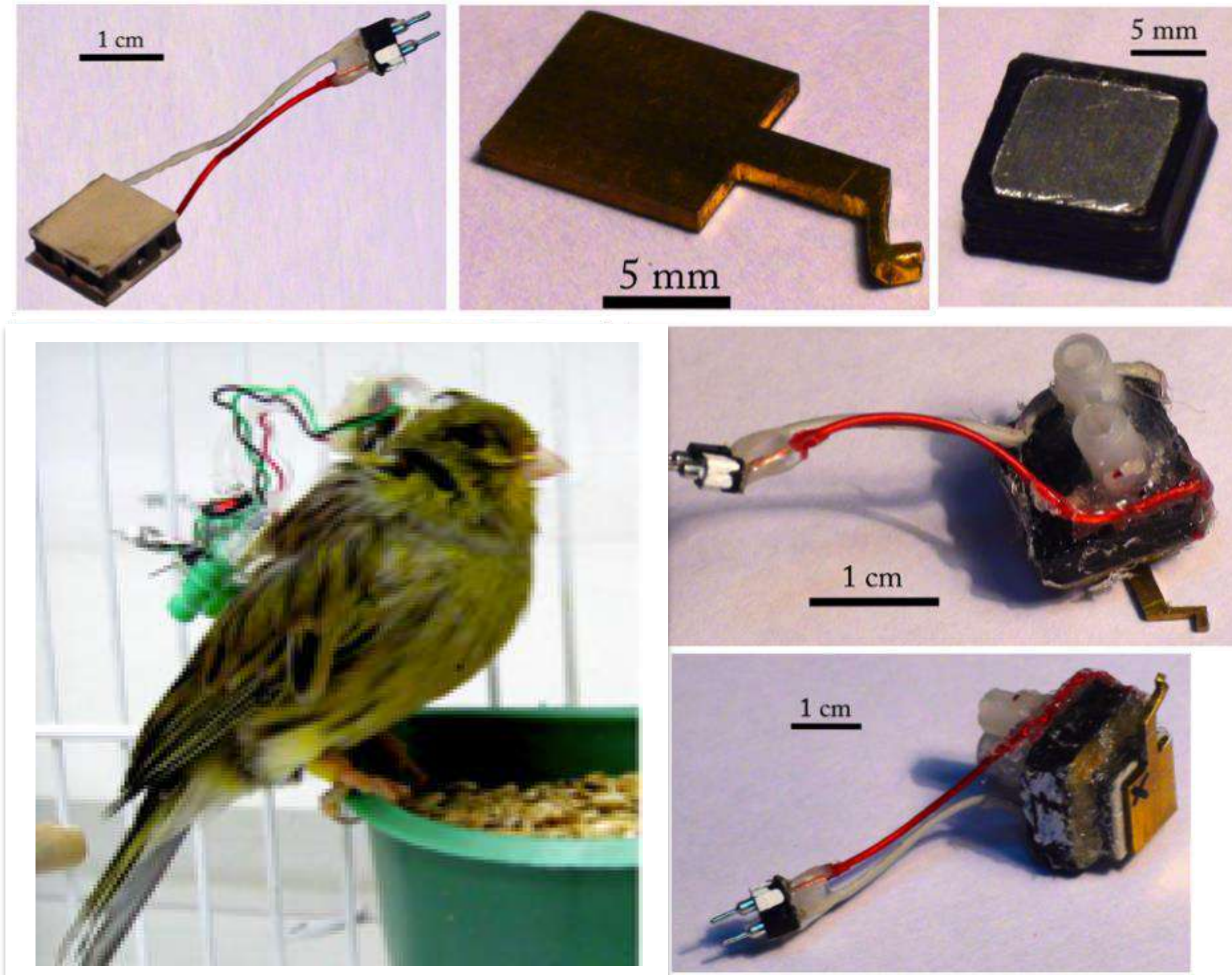
Consistent with one Time scale…

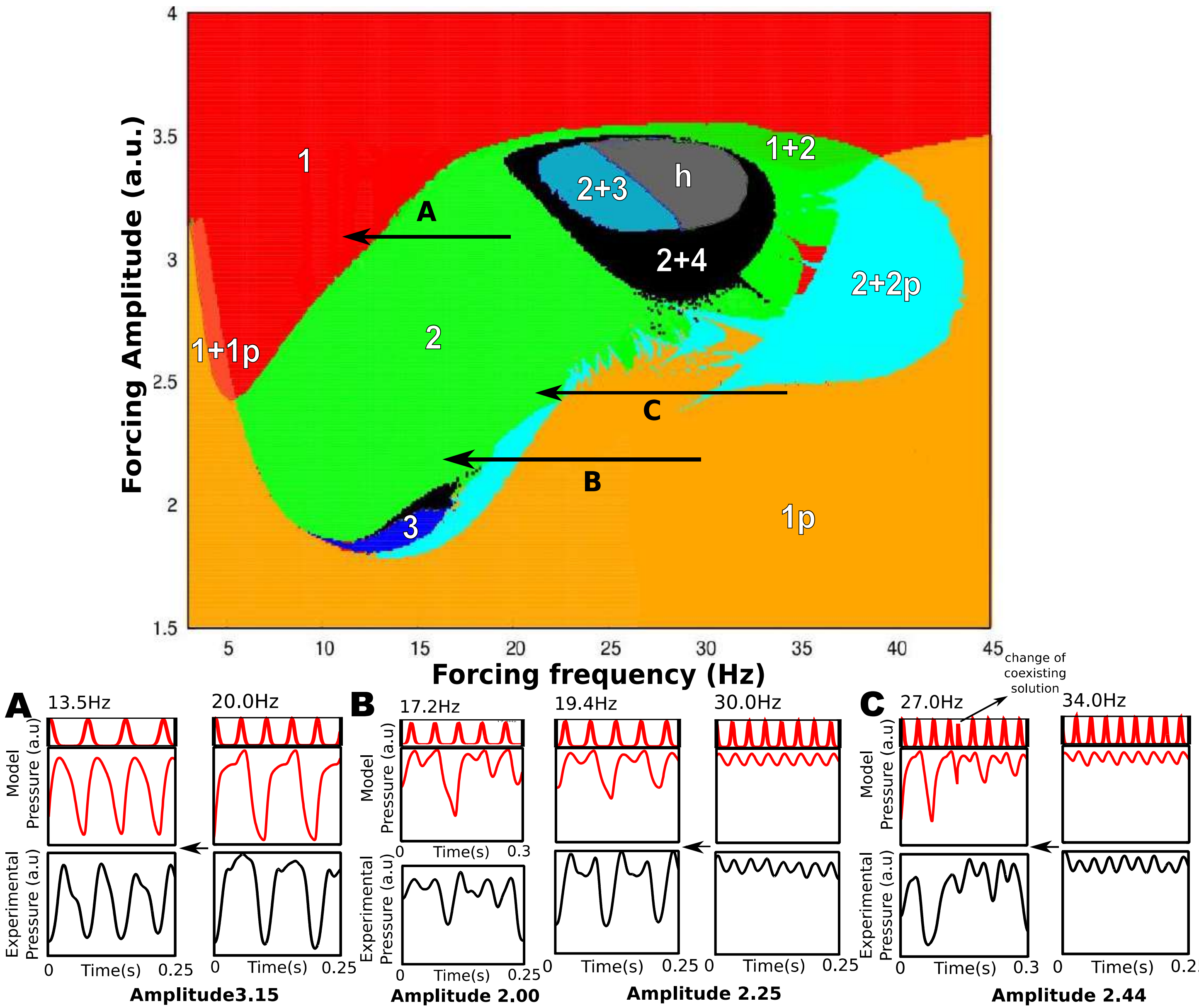
In canaries, we obtained the physiological parmeters forcing a nonlinear structure

It would be nice to be able to Change the frequency…

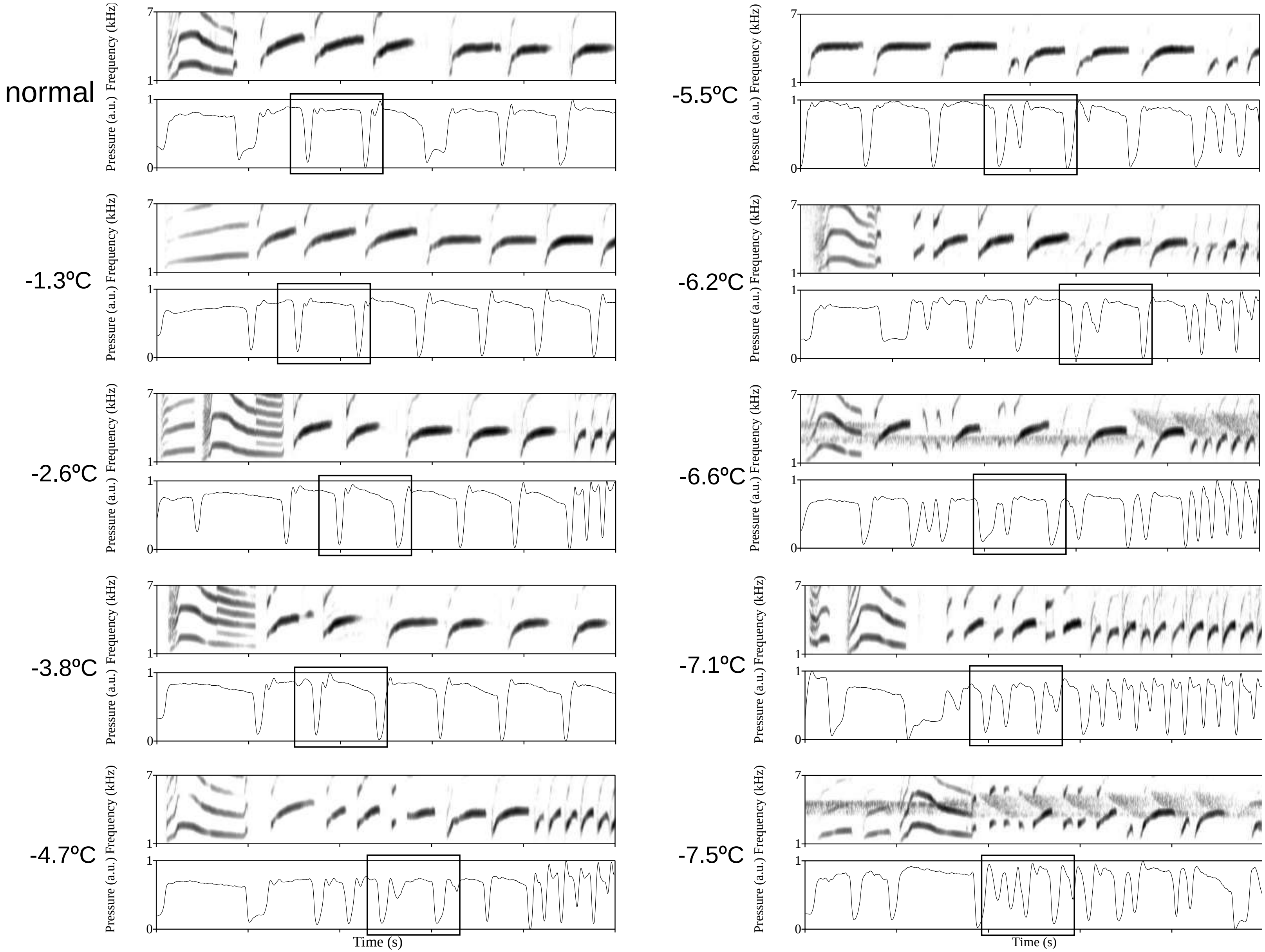


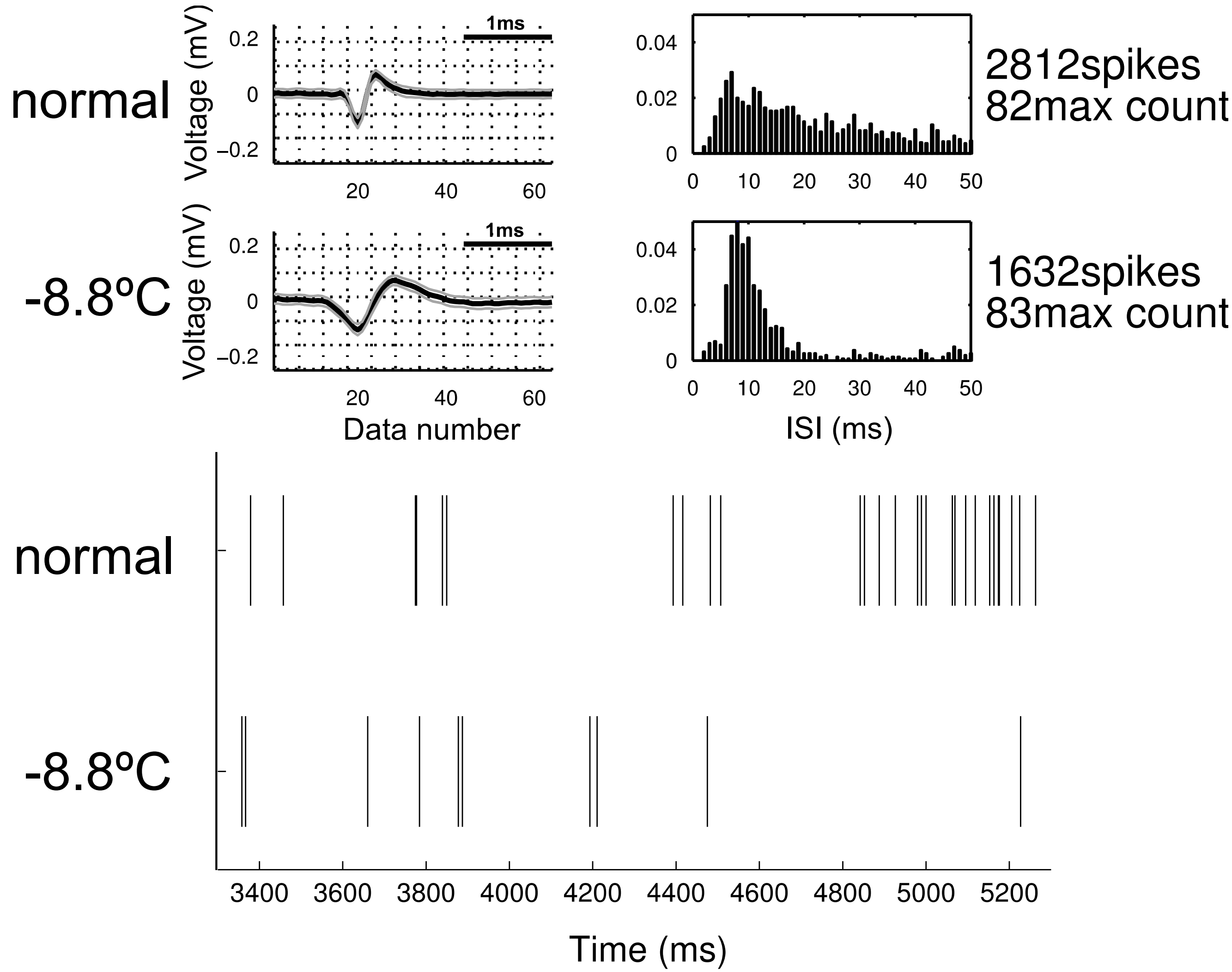

A
B
2mm
1cm
C
D


1 cm
5 mm
5 mm
1 cm
1 cm

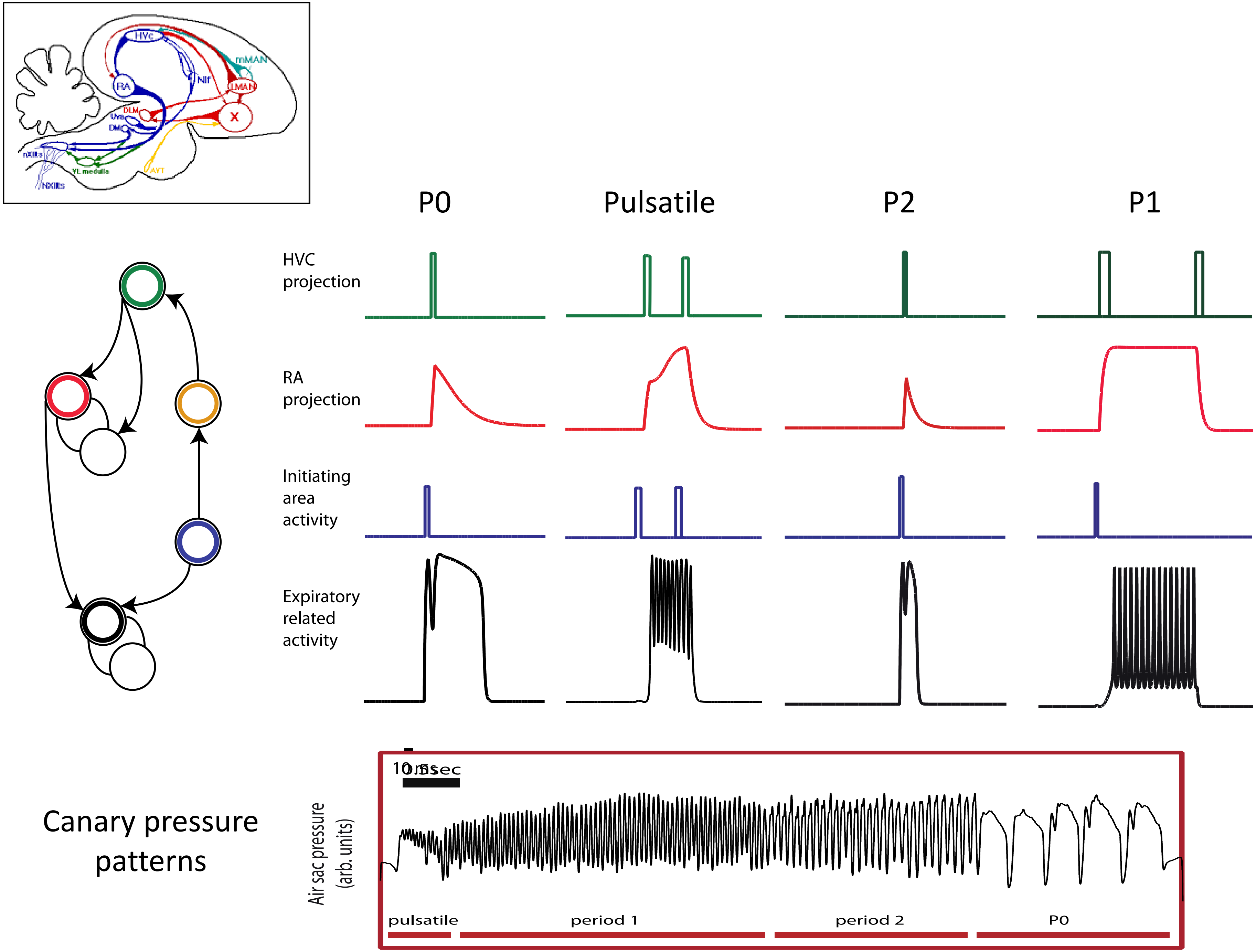


1
1+2
h
2+3
A
2+4
2+2p
2
1+1p
C
B
3
1p
Forcing Amplitude (a.u.)
Forcing frequency (Hz)
change of coexisting solution
A
13.5Hz
20.0Hz
Model Pressure (a.u)
Experimental Pressure (a.u)
Time(s)
Amplitude3.15
B
17.2Hz
19.4Hz
30.0Hz
Amplitude 2.00
Amplitude 2.25
C
27.0Hz
34.0Hz
Amplitude 2.44

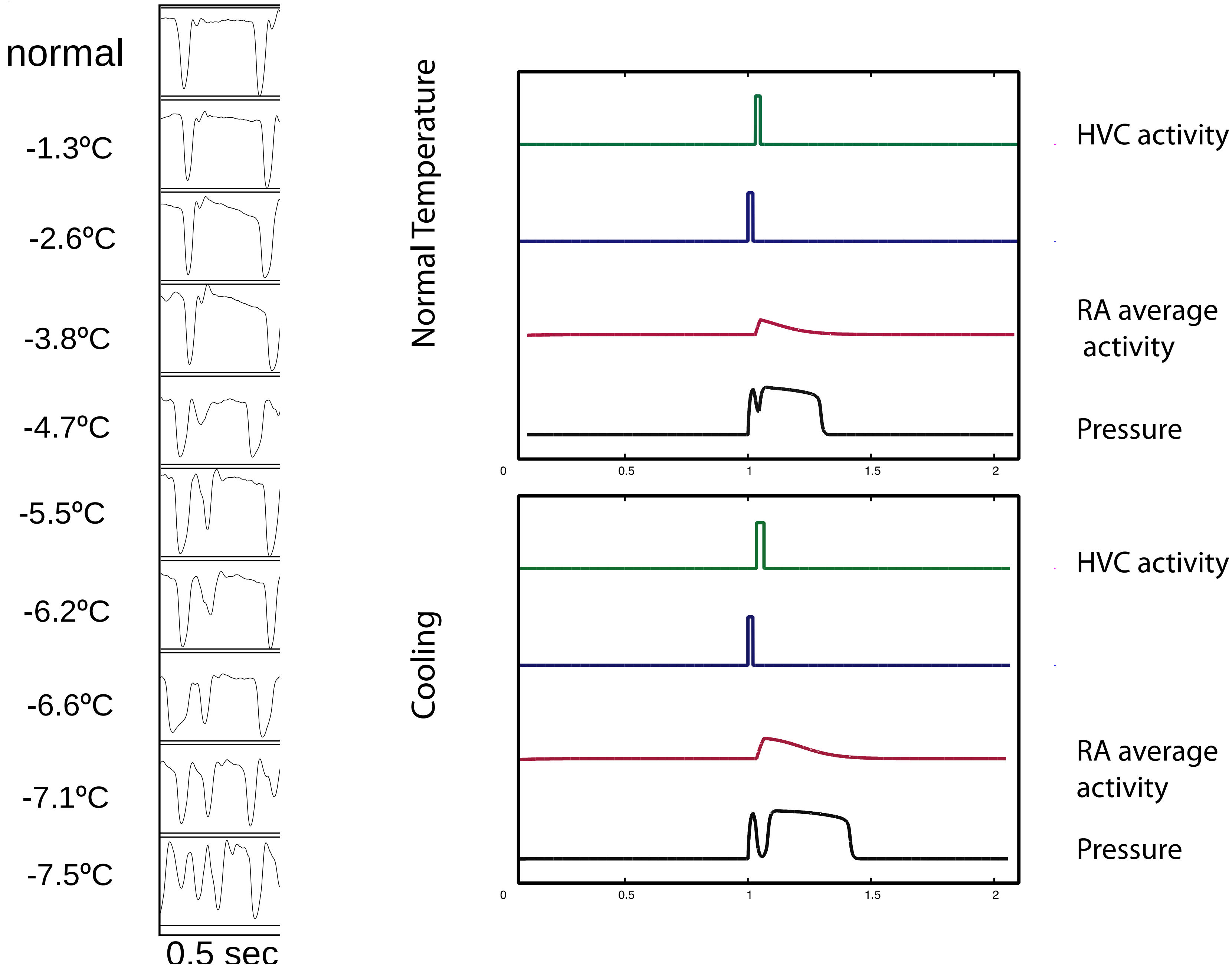

normal
-1.3ºC
-2.6ºC
-3.8ºC
-4.7ºC
-5.5ºC
-6.2ºC
-6.6ºC
-7.1ºC
-7.5ºC
Frequency (kHz)
Pressure (a.u.)
Time (s)



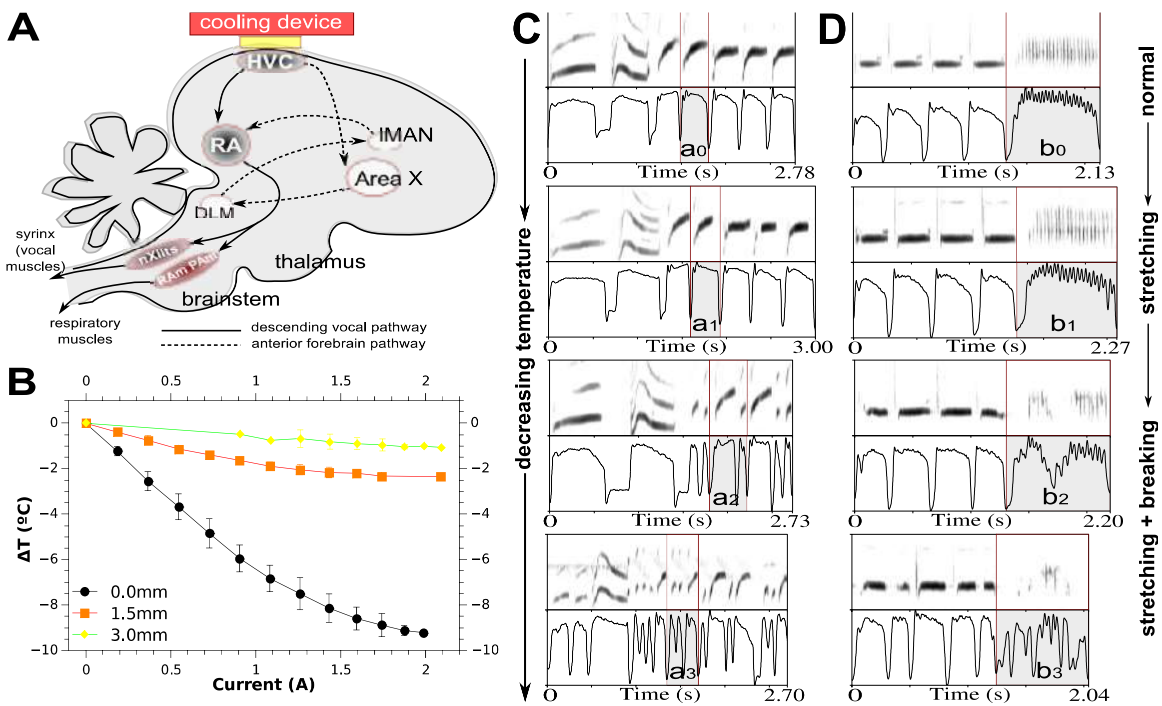
normal
-8.8ºC
Voltage (mV)
1ms
0.2
0
−0.2
20
40
60
Data number
0.04
0.02
0
10
30
50
ISI (ms)
2812spikes
82max count
1632spikes
83max count
3400
3600
3800
4000
4200
4400
4600
4800
5000
5200
Time (ms)



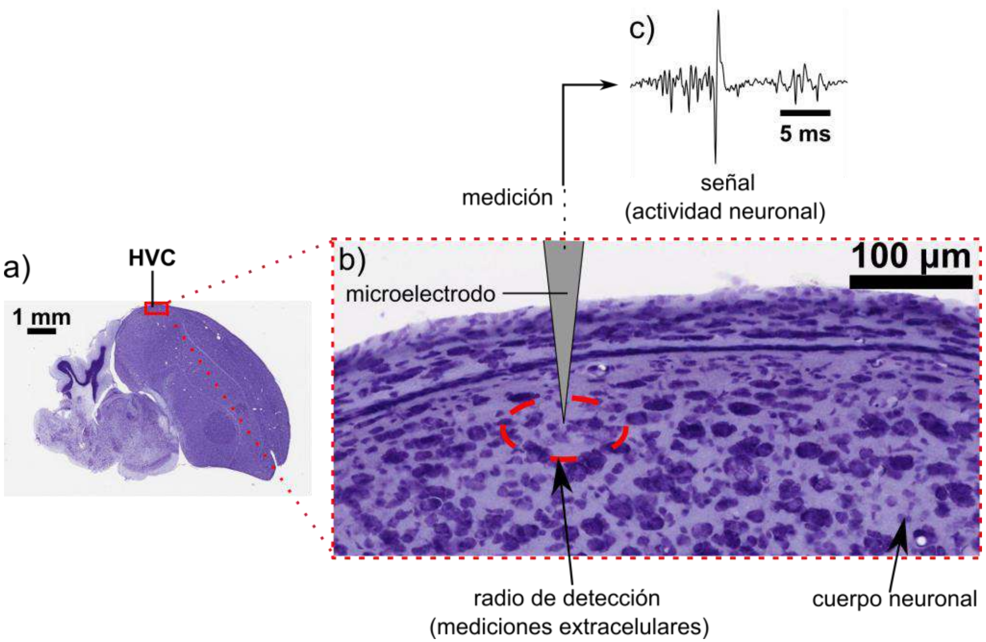
HVC
mMAN
Nif
LMAN
RA
DLM
X
YL medulla
AVT
P0
Pulsatile
P2
P1
HVC projection
RA projection
Initiating area activity
Expiratory related activity
Canary pressure patterns
Air sac pressure (arb. units)
pulsatile
period 1
period 2
P0

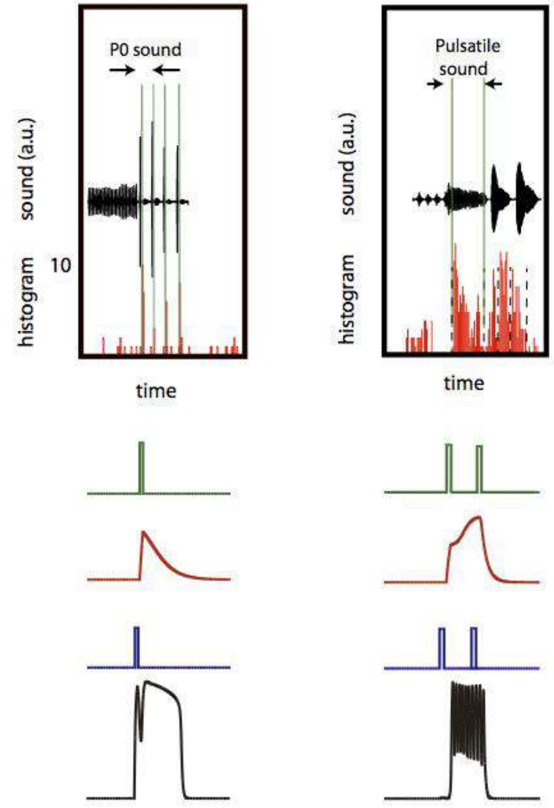

normal
-1.3ºC
-2.6ºC
-3.8ºC
-4.7ºC
-5.5ºC
-6.2ºC
-6.6ºC
-7.1ºC
-7.5ºC
0.5 sec
Normal Temperature
HVC activity
RA average activity
Pressure
0
0.5
1
1.5
2
Cooling
HVC activity
RA average activity
Pressure
0
0.5
1
1.5
2

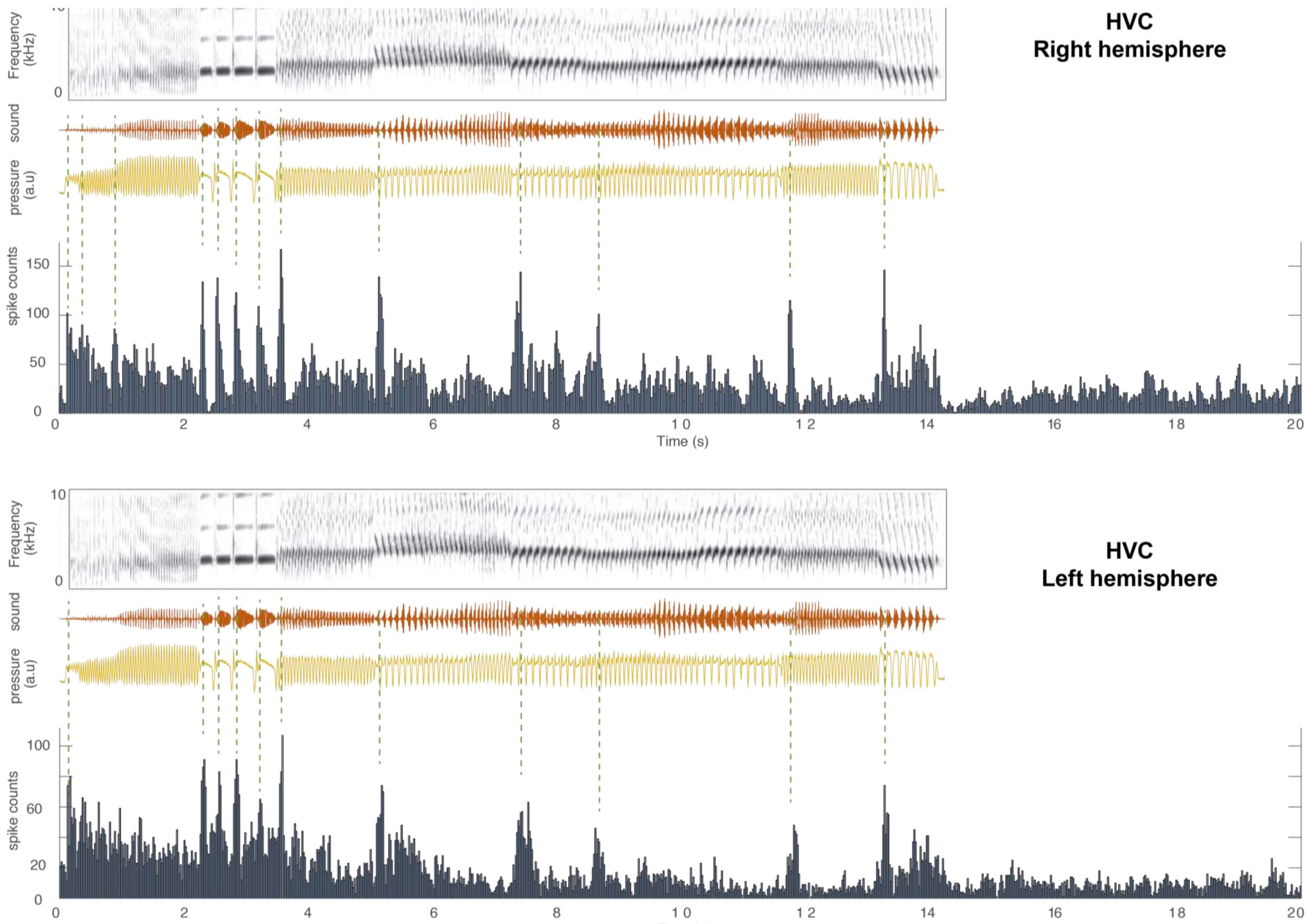
The experimental data



A good model is a predictive one






HVC
Right hemisphere
Frequency (kHz)
0
sound
pressure (a.u)
spike counts
150
100
50
0
0
2
4
6
8
10
12
14
16
18
20
Time (s)
HVC
Left hemisphere
10
Frequency (kHz)
0
sound
pressure (a.u)
spike counts
100
60
20
0
0
2
4
6
8
10
12
14
16
18
20
Time (s)

- Topics visited

Individual out of equilibrium units:

Isolated, coexisting stationary points - isolated periodic solutions.

Qualitative changes as the parameters are moved.

When coupled, there is a diversity of regimes:

Synchronicity among them, leading to a reduction of the dimensionality of the displayed dynamics.

Spatio-temporal complexity, chaos, turbulence.

- Supplemental material

Another situation where there
Is a snilc…

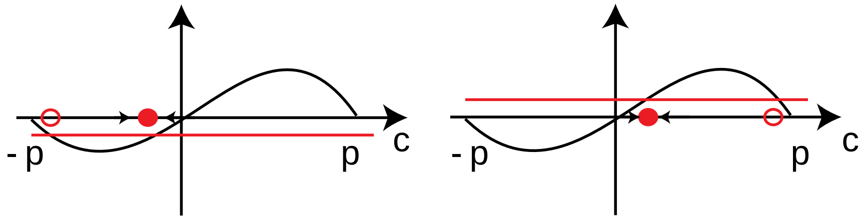
$$\frac{d\theta_1}{dt} = \omega_1$$

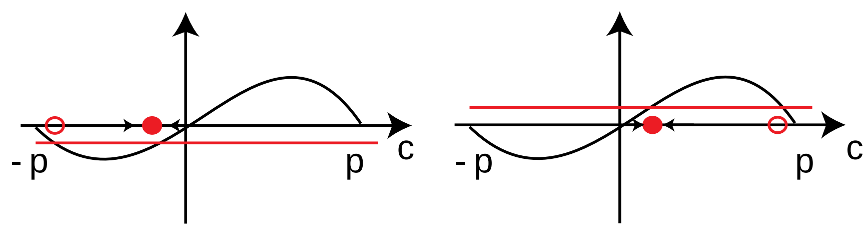
$$\frac{d\theta_2}{dt} = \omega_2 + k\sin(\theta_1 - \theta_2).$$

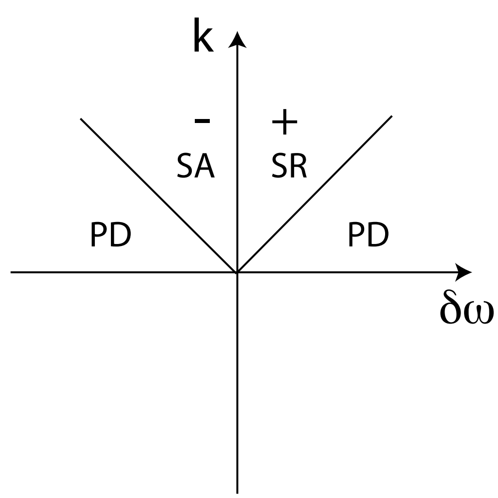
$\Gamma = \theta_1 - \theta_2$:

$$\frac{d\Gamma}{dt} = (\omega_1 - \omega_2) - k\sin(\Gamma)$$
$$= \delta\omega - k\sin(\Gamma)$$

$$\theta_1 - \theta_2 = \ sin^{-1}(\delta\omega/k) \Rightarrow$$

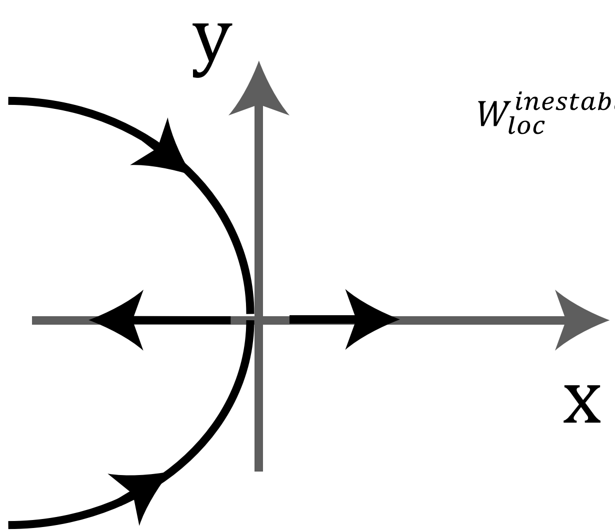



- p
p
c
- p
p
c


k
-
+
SA
SR
PD
PD
δω

Another element in the description

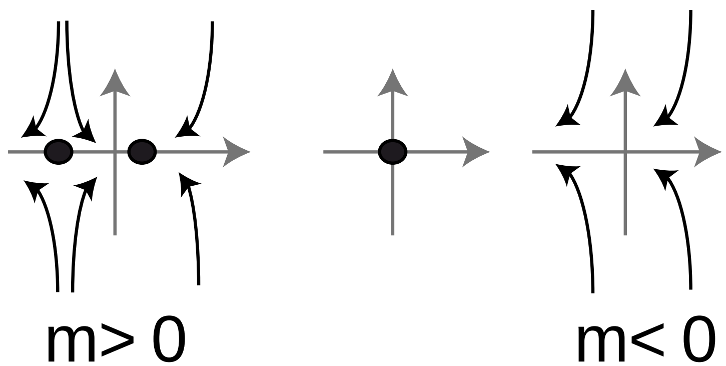
$$W_{loc}^{estable}(x^*) = \left\{x \in U | \phi_t(x) \xrightarrow[t\to\infty]{} x^*, con\ \phi_t(x) \in U\ para\ todo\ t\right\}$$

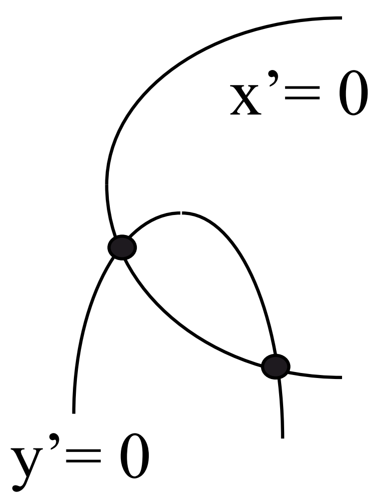


$$W_{loc}^{inestable}(x^*) = \left\{x \in U | \phi_t(x) \xrightarrow[t\to-\infty]{} x^*, con\ \phi_t(x) \in U\ para\ todo\ t\right\}$$

$$W^{estable} = \bigcup_{t\leq 0} \phi_t(W_{loc}^{estable}(x^*)),$$

$$W^{inestable} = \bigcup_{t\geq 0} \phi_t(W_{loc}^{inestable}(x^*)),$$


m> 0
m< 0


x'= 0
y'= 0

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Bifurcation diagrams

$$\frac{dx}{dt} = -x + S(\rho_x + ax - by)$$

$$\frac{dy}{dt} = -y + S(\rho_y + cx - dy)$$

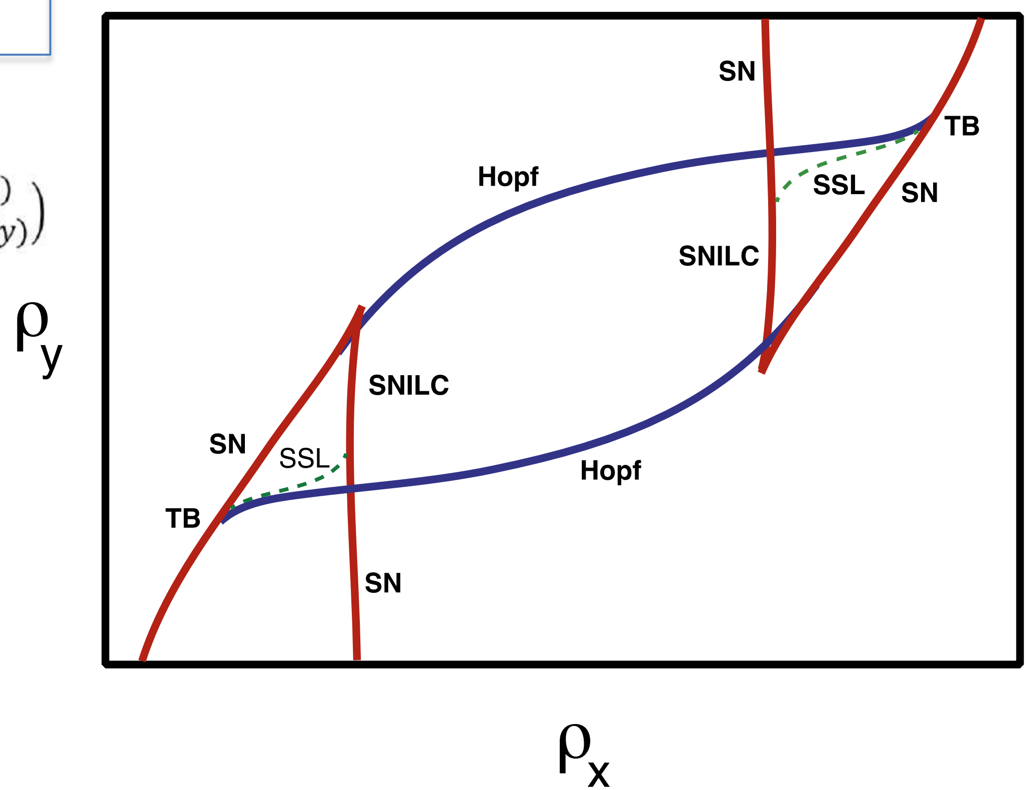
$$DF = \begin{pmatrix} -1 + aS'(\rho_x + ax - by) & -bS'(\rho_x + ax - by) \\ cS'(\rho_y + cx - dy) & 1 - dS'(\rho_y + cx - dy) \end{pmatrix}$$

Bifurcación de nodo silla SN

Takens Bogdanov TB

Bifurcación de nodo-silla en ciclo límite SNILC

Bifurcación homoclínica SSL



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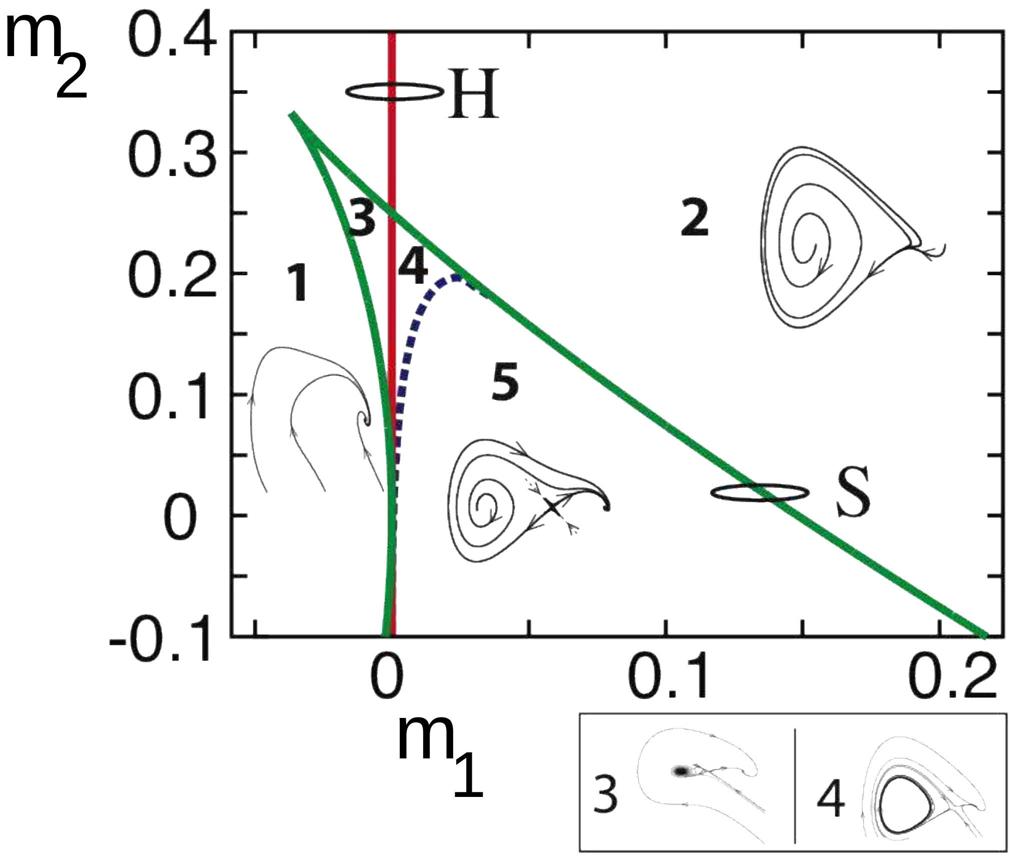
$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -\mu_1 - \mu_2 x - x^3 - x^2 y + x^2 - xy$$



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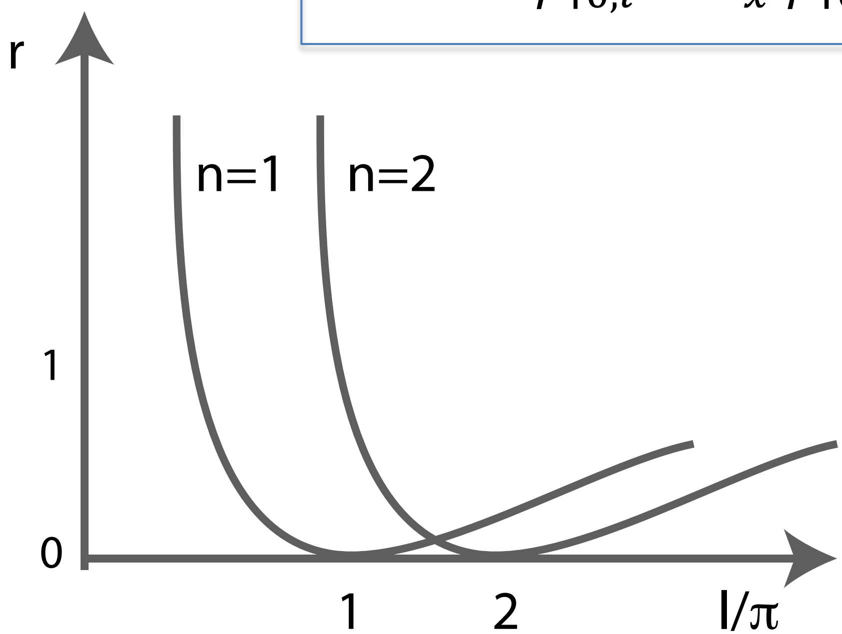
Center manifolds

$$\boxed{\begin{gathered}\partial_t\varphi + \varphi\partial_x\varphi = r\varphi - (\partial_x^2+1)^2\varphi,\\ \varphi|_{0,l} = \partial_x^2\varphi|_{0,l} = 0.\end{gathered}}$$



$$\partial_t\Psi = r\Psi - (\partial_x^2+1)^2\Psi,$$

$$\Psi = \sum\nolimits_n A_n(t)\sin(k_n x)$$

$$\frac{dA_n(t)}{dt}\sin(k_n x) = \big(r - (k_n^4 - 2k_n^2 + 1)\big)A_n(t)\sin(k_n x),$$

$$\boxed{s_n = r - (k_n^2 - 1)^2 = 0}$$

$$\boxed{\partial_t \varphi + w\partial_x \varphi = r\varphi - (\partial_x^2 + 1)^2 \varphi,}$$

$$\varphi = \sum_n A_n \sin(nx),$$

$$\varphi_x \varphi = \sum_m \sum_n m A_m A_n \sin(nx)\sin(mx) = \sum_m \sum_n m A_m A_n \left(\sin((m+n)x)\right.$$
$$\left. + \sin((n-m)x)\right)/2,$$

$$A_1\left(A_1 \frac{\sin(2x)}{2} + A_2 \frac{2}{2}(\sin(3x) + \sin(-x)) + A_3 \frac{3}{2}(\sin(4x) + \sin(-3x)) + \cdots\right) +$$

$$A_2\left(A_1 \frac{\sin(3x) + \sin(x)}{2} + A_2 \frac{2}{2}(\sin(4x)) + A_3 \frac{3}{2}(\sin(5x) + \sin(-x)) + \cdots\right) +$$

$$A_3\left(A_1 \frac{\sin(4x) + \sin(2x)}{2} + A_2 \frac{2}{2}(\sin(5x) + \sin(x)) + A_3 \frac{3}{2}(\sin(6x)) + \cdots\right) + \cdots$$

$$\frac{dA_1}{dt} = rA_1 + \frac{1}{2}(A_1A_2 + A_2A_3 + \cdots)$$

$$\frac{dA_2}{dt} = (r-9)A_2 - \frac{1}{2}({A_1}^2 + A_1A_3 + \cdots)$$

$$\frac{dA_3}{dt} = (r-64)A_3 - \frac{3}{2}(A_1A_2 - \cdots)$$

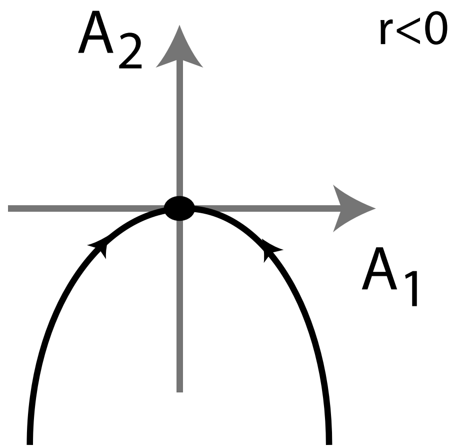
Restricted to two modes:

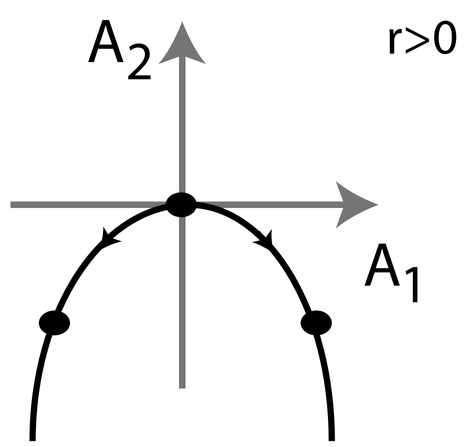
$$\frac{dA_1}{dt} = rA_1 + \frac{1}{2}(A_1A_2)$$

$$\frac{dA_2}{dt} = (r-9)A_2 - \frac{1}{2}({A_1}^2).$$

$$9A_2 + \frac{1}{2}\left({A_1}^2\right) = 0,$$

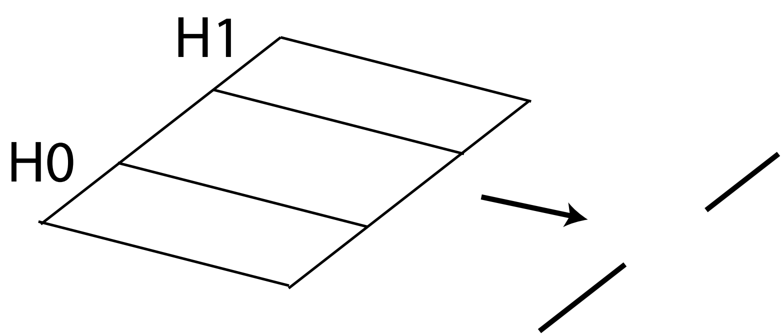
$$\frac{dA_1}{dt} = rA_1 - \frac{1}{36}{A_1}^3.$$

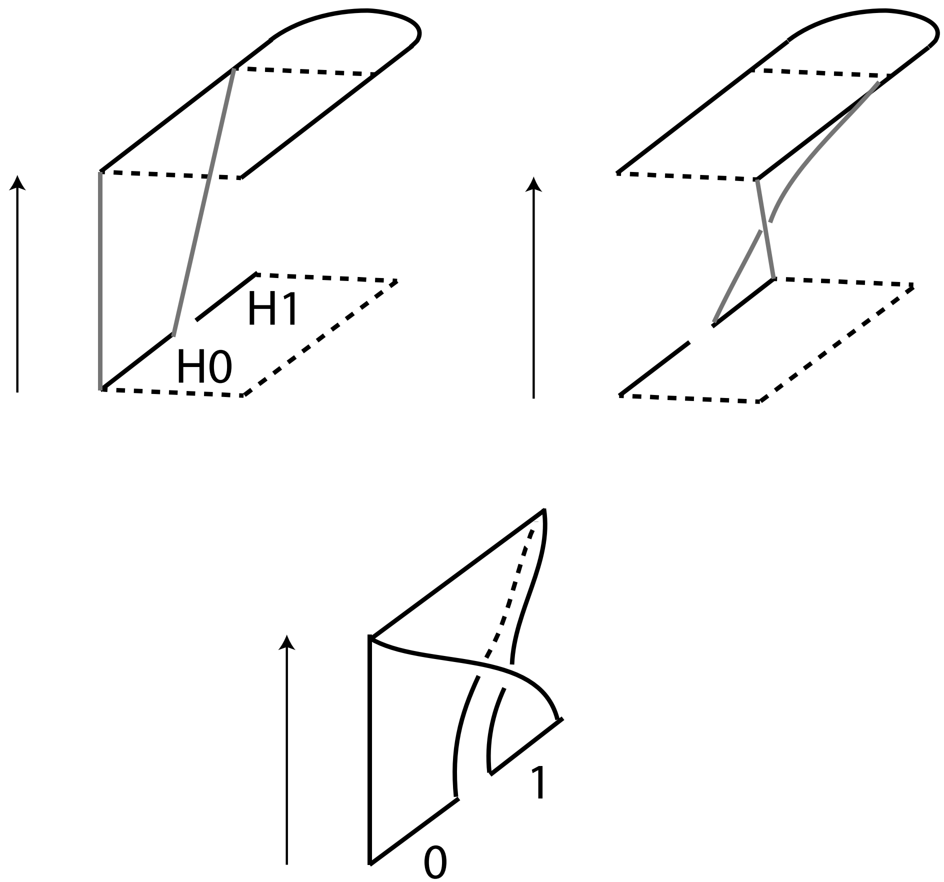


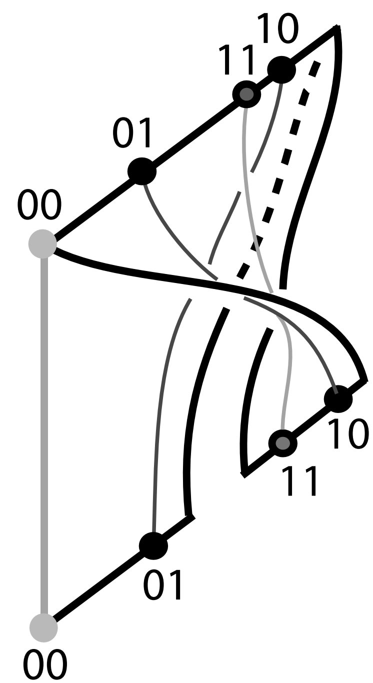


$$9A_2 + \frac{1}{2}\left({A_1}^2\right) = 0,$$

$$\boxed{\frac{dA_1}{dt} = rA_1 - \frac{1}{36}{A_1}^3.}$$


H1
H0


H1
H0
1
0


10
11
01
00
10
11
01
00

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