LECTURE 1

Beyond LCDM

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Plan for the lectures

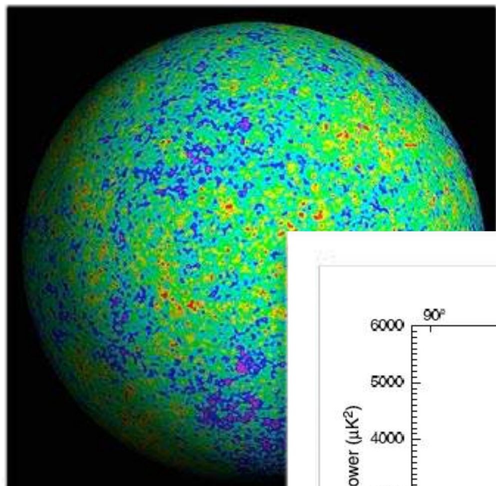
Lecture 1. Beyond LCDM - an overview of the gravitational landscape

Lecture 2. f(R) Gravity - what happens to background dynamics and growth of structure.

Lecture 3. Model Independent Approaches to Cosmological Tests of Gravity.

Lecture 4. Effective Field Theory (EFT) of Dark Energy - set up and applications to cosmological probes.

A success story



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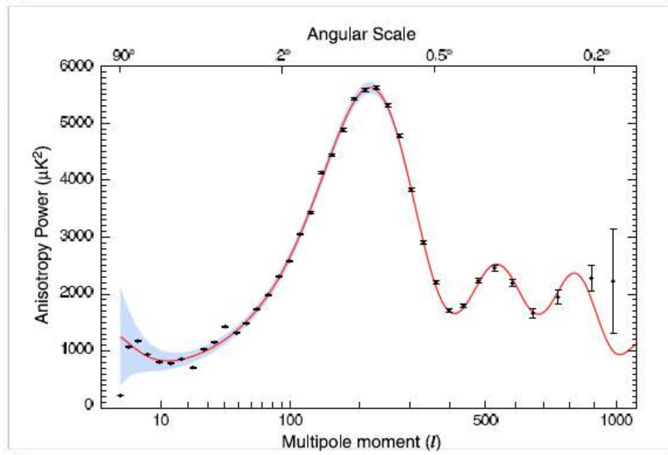

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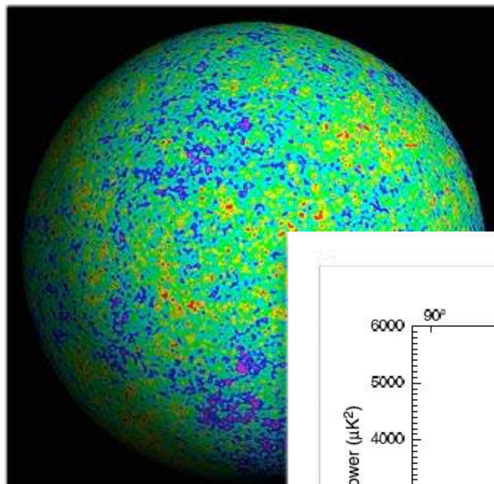

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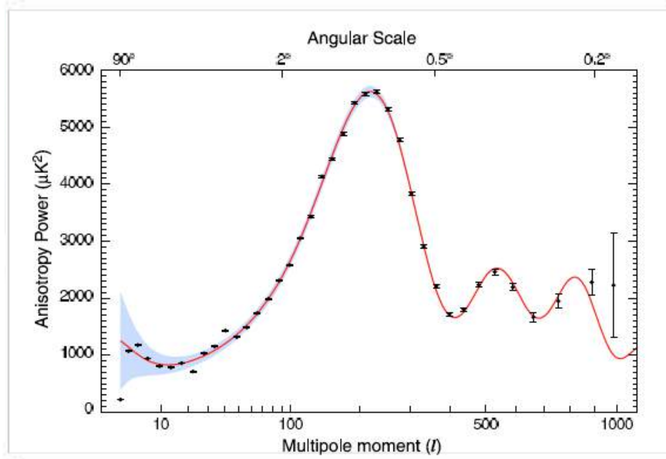
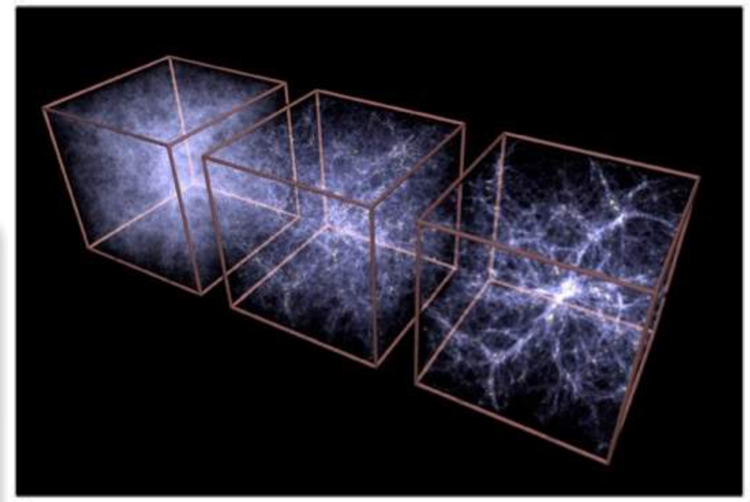
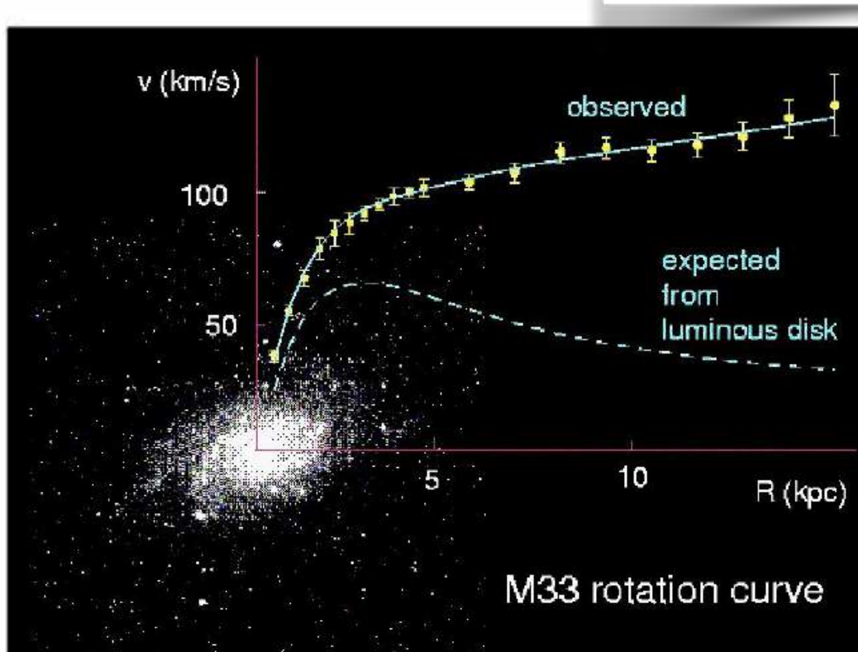


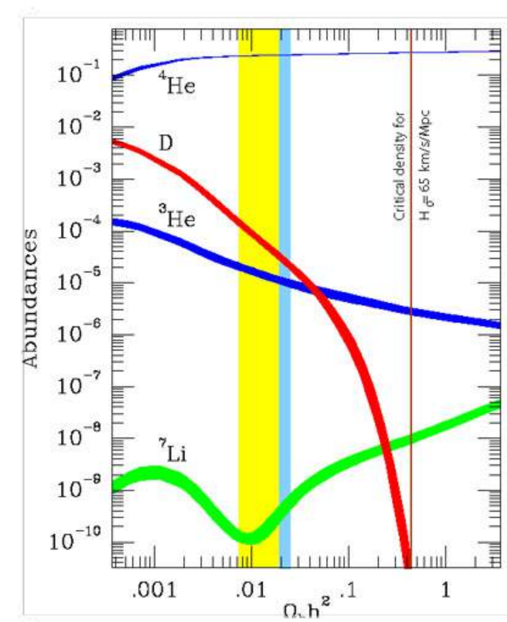


Large Scale Structure



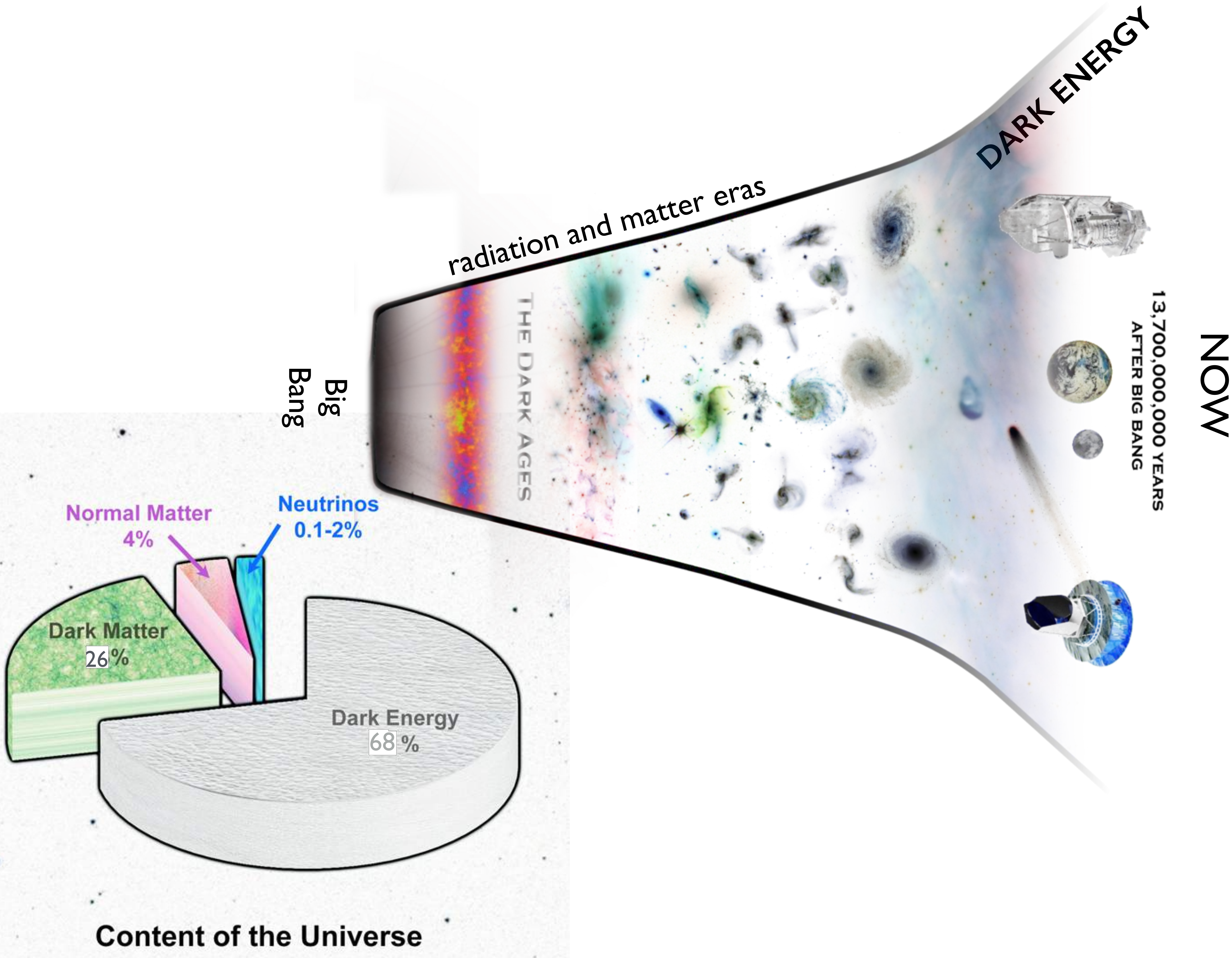
Galaxy rotation curves

Lensing



BBN Nucleosynthesis

Our Universe



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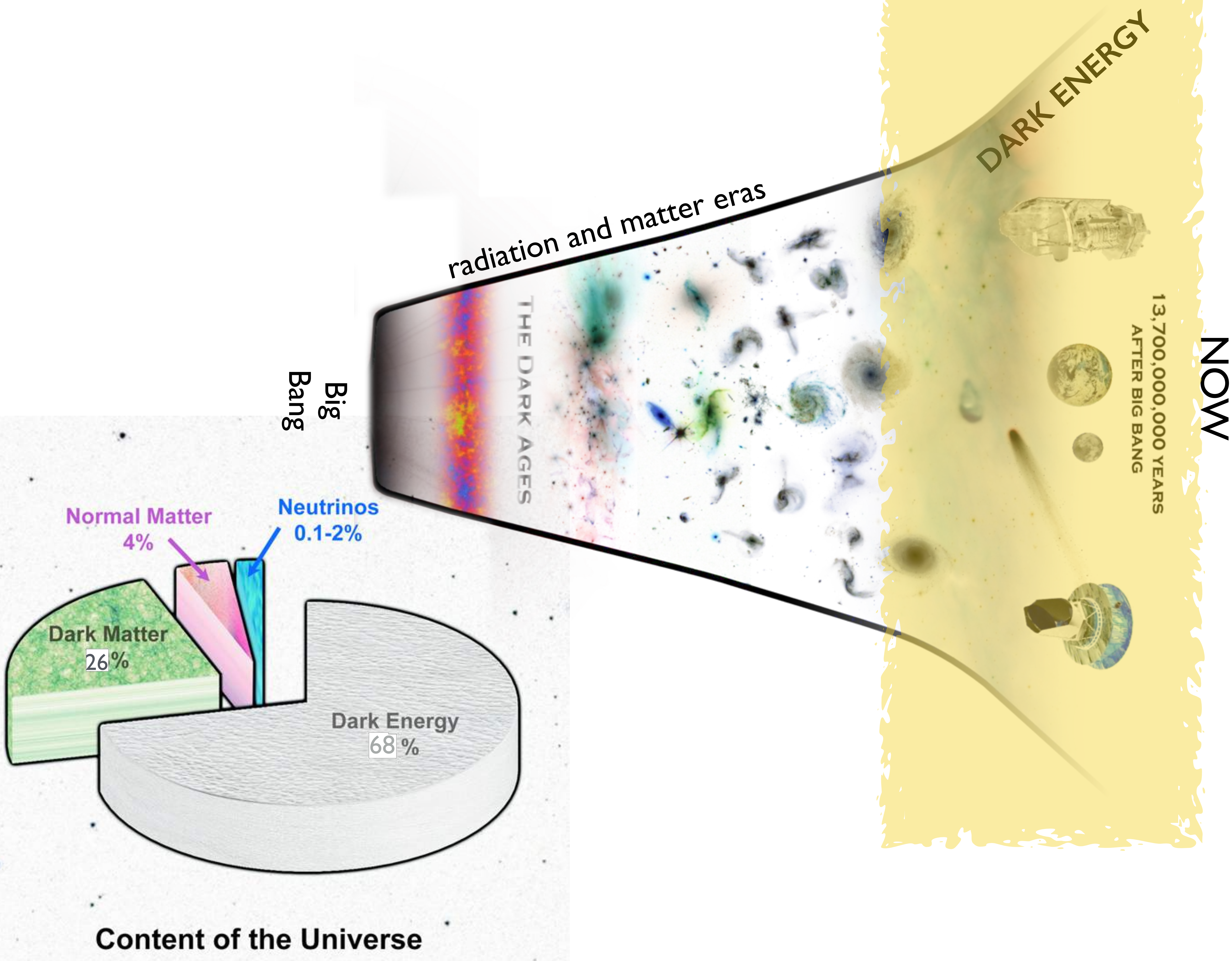

LCDM

That the expansion rate of the Universe is accelerating is now a firmly established aspect of cosmology and a testament to the breathtaking convergence of techniques that has emerged in observational cosmology. In turn, cosmic acceleration has introduced new wrinkles into almost every part of theoretical cosmology: what is sourcing it ??

In the standard model of cosmology: the cosmological constant Λ

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda) + \int d^4x\sqrt{-g}\mathcal{L}_m$$

it is equivalent to adding a component with:

$$T_{\mu\nu} = -\frac{\Lambda}{8\pi G}g_{\mu\nu}$$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}$$

$$p_\Lambda = -\rho_\Lambda \qquad (w = -1)$$

$$\frac{\ddot{a}}{a} \sim \frac{\Lambda}{3}$$

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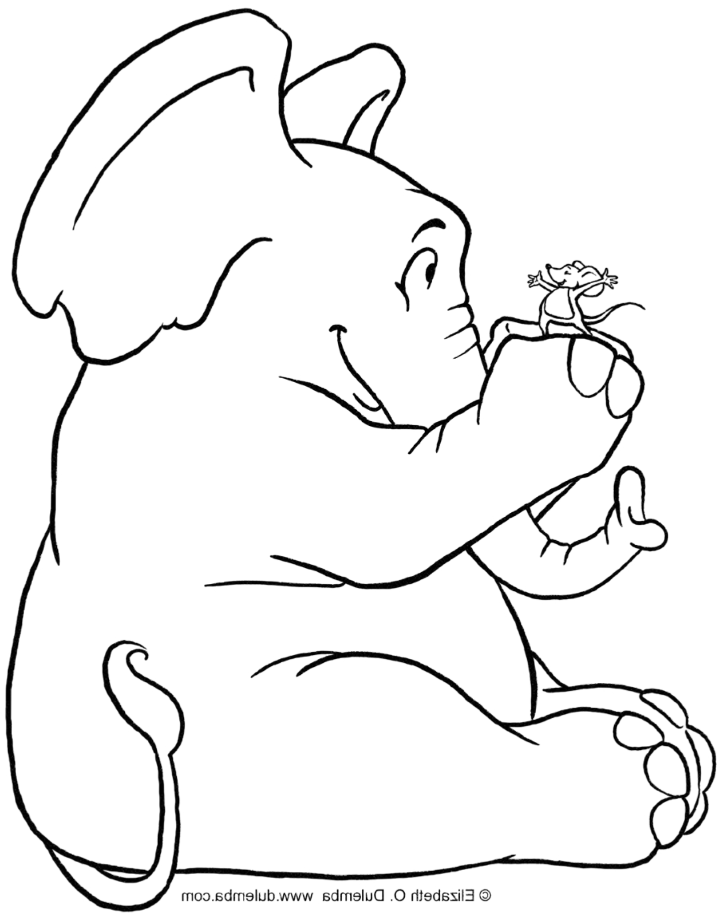
* what goes in here ?
* what do we measure it to be?

LCDM



* cosmological constant problem:

why is the vacuum energy so small (or zero)?

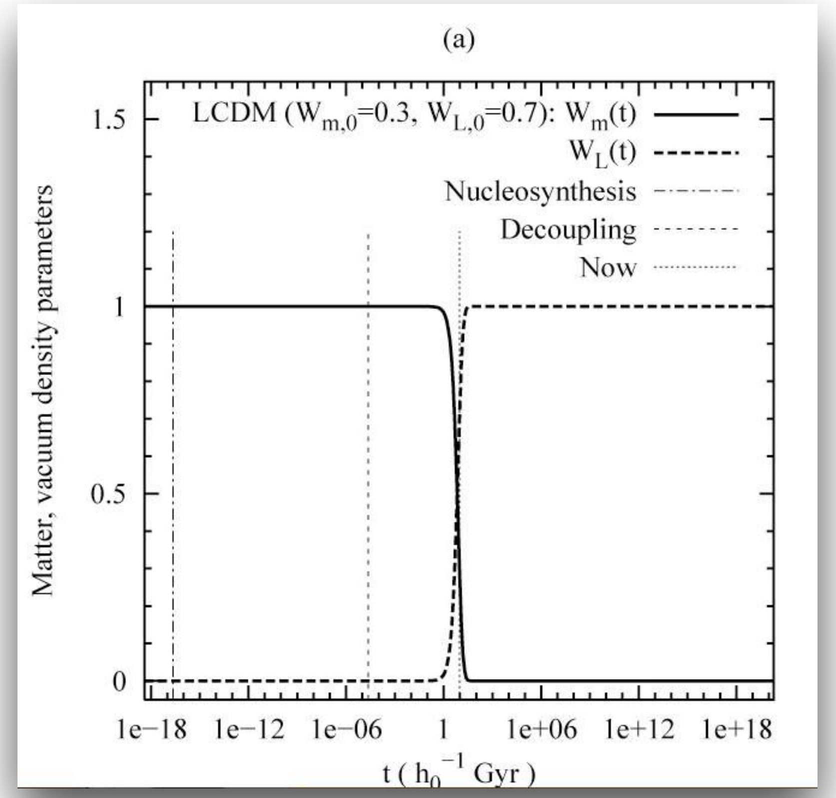
$$\rho_{\rm th} \sim ({\rm TeV})^4 - (M_P)^4$$

$$\rho_{\rm obs} \sim (10^{-3}{\rm eV})^4$$

* cosmic coincidence problem:

why are the matter and dark energy densities approximately equal today?

$$\frac{\Omega_\Lambda}{\Omega_m} \propto a^3$$



How special is GR ?

Weinberg-Deser theorem: A Lorentz invariant theory of a massless spin-2 particle must be GR at low energies.

$\downarrow$

So, to modify GR we can either give mass to the graviton, introduce new DOF or break Lorentz invariance

Any theory beyond ΛCDM does at least one of the above.

The new DOF will generally be Lorentz scalars.

The above scenarios can be achieved in different ways, e.g. through higher dimensional setup, higher derivatives (be aware of instabilities!!), explicit additional DOF, giving up locality, ...

In general, by modifying the original action, we change the equations of motion in such a way that some of those that were constraint equations become dynamical.

How special is GR ?

Lovelock's theorem: The only possible second-order, Euler-Lagrange equations obtainable in a 4D spacetime from an action containing solely the 4D metric and its derivatives are the Einstein field equations

(1971)

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More broadly, the most general metric theory of gravity yielding conserved 2nd order EOMs in D dimensions is:

Lovelock gravity: $\mathcal{L} = \sqrt{-g}\sum_{n=0}^{t} \alpha_n \mathcal{R}^n$

$$\mathcal{R}^n = \frac{1}{2^n}\delta^{\mu_1\nu_1\dots\mu_n\nu_n}_{\alpha_1\beta_1\dots\alpha_n\beta_n}\prod_{r=1}^{n} R^{\alpha_r\beta_r}_{\mu_r\nu_r}$$

$$D = 2(t+1) - (D \bmod 2)$$

$$\mathcal{L} = \sqrt{-g}\left(\alpha_0 + \alpha_1 R + \alpha_2 \underbrace{\left(R^2 + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu\nu}R^{\mu\nu}\right)}_{\mathcal{G}}\right) + \alpha_3 \mathcal{O}\left(R^3\right)$$

$\mathcal{G}$ Gauss-Bonnet term

Generalized Kronecker delta

$$\mathcal{R}^n = \frac{1}{2^n}\delta^{\mu_1\nu_1\ldots\mu_n\nu_n}_{\alpha_1\beta_1\ldots\alpha_n\beta_n}\prod_{r=1}^{n}R^{\alpha_r\beta_r}_{\mu_r\nu_r}$$

$$\delta^{\mu_1\ldots\mu_p}_{\nu_1\ldots\nu_p} = \begin{cases} +1 & \text{if } \nu_1\ldots\nu_p \text{ are distinct integers and are an even permutation of } \mu_1\ldots\mu_p \\ -1 & \text{if } \nu_1\ldots\nu_p \text{ are distinct integers and are an odd permutation of } \mu_1\ldots\mu_p \\ 0 & \text{in all other cases.} \end{cases}$$

$$\delta^{\mu_1\ldots\mu_p}_{\nu_1\ldots\nu_p} = p!\,\delta^{\mu_1}_{[\nu_1}\ldots\delta^{\mu_p}_{\nu_p]} = p!\,\delta^{[\mu_1}_{\nu_1}\ldots\delta^{\mu_p]}_{\nu_p}$$

$$\delta^{\mu_1\ldots\mu_p}_{\nu_1\ldots\nu_p} = \begin{vmatrix} \delta^{\mu_1}_{\nu_1} & \cdots & \delta^{\mu_1}_{\nu_p} \\ \vdots & \ddots & \vdots \\ \delta^{\mu_p}_{\nu_1} & \cdots & \delta^{\mu_p}_{\nu_p} \end{vmatrix}$$

How special is GR ?

A nice property of the Lovelock scalars is that any Lagrangian written in terms of Lovelock scalars, *including non linear functions of them*, will not contain extra tensorial DOF.

in 4D

$$\mathcal{L} = \sqrt{-g}\,(\alpha_0 + \alpha_1 R + \alpha_2 \mathcal{G})$$

1 massless spin-2 DOF

$$\mathcal{L} = \sqrt{-g}\,(\alpha_1 f(R) + \alpha_2 f(\mathcal{G}))$$

1 massless spin-2 DOF + 2 scalar DOF: $f_{\mathcal{G}}\,,\, f_R$

↓

$$\nabla_\mu \nabla_\nu f_R\,,\ \nabla_\mu \nabla_\nu f_{\mathcal{G}}$$

↗

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More generally, a theory of gravity which maintains second order EOMs for the tensor and has a <u>single</u> additional propagating scalar DOF in 4D is:

$$\mathcal{L} = \sqrt{-g} f(R, \mathcal{G}) \qquad \text{with} \quad f_{RR} f_{\mathcal{G}\mathcal{G}} - f_{R\mathcal{G}}^2 = 0$$

Scalar-tensor theories

And after Lovelock, came Horndeski !!!
but it took us some decades to notice it…

… so let us forget for a moment about the powerful framework provided by Horndeski gravity, and let us have a look at the gravitational landscape that emerged in the past decades.

Beyond LCDM

discrepancies in Uranus's observed orbit

↓

discovery of Neptune (1846)

(DARK ENERGY)

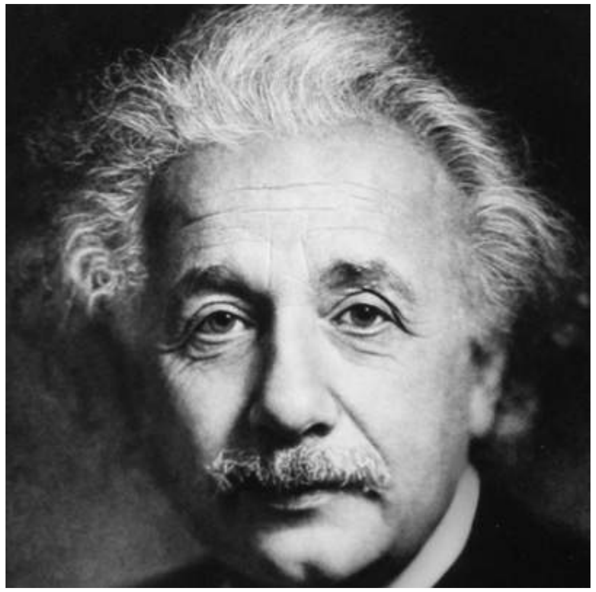
Le Verrier

precession of Mercury's orbit

↓

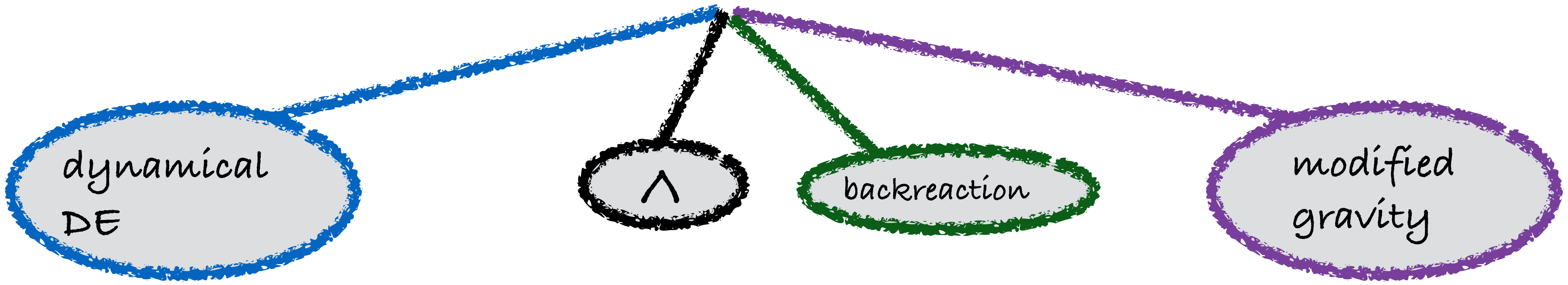
general relativity (1915)

(MODIFIED GRAVITY)



Einstein

Theoretical Landscape



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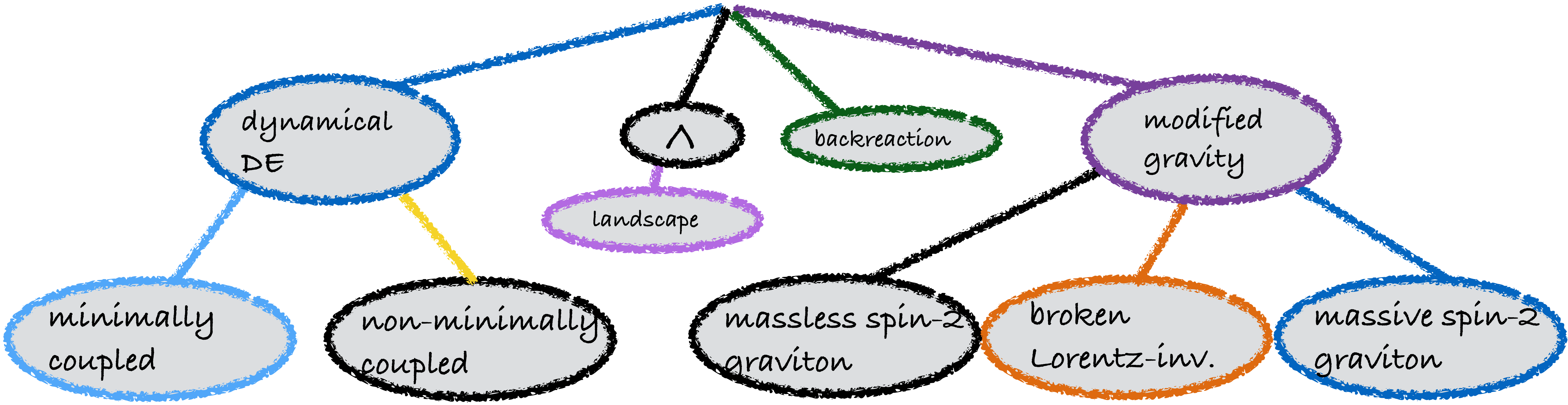

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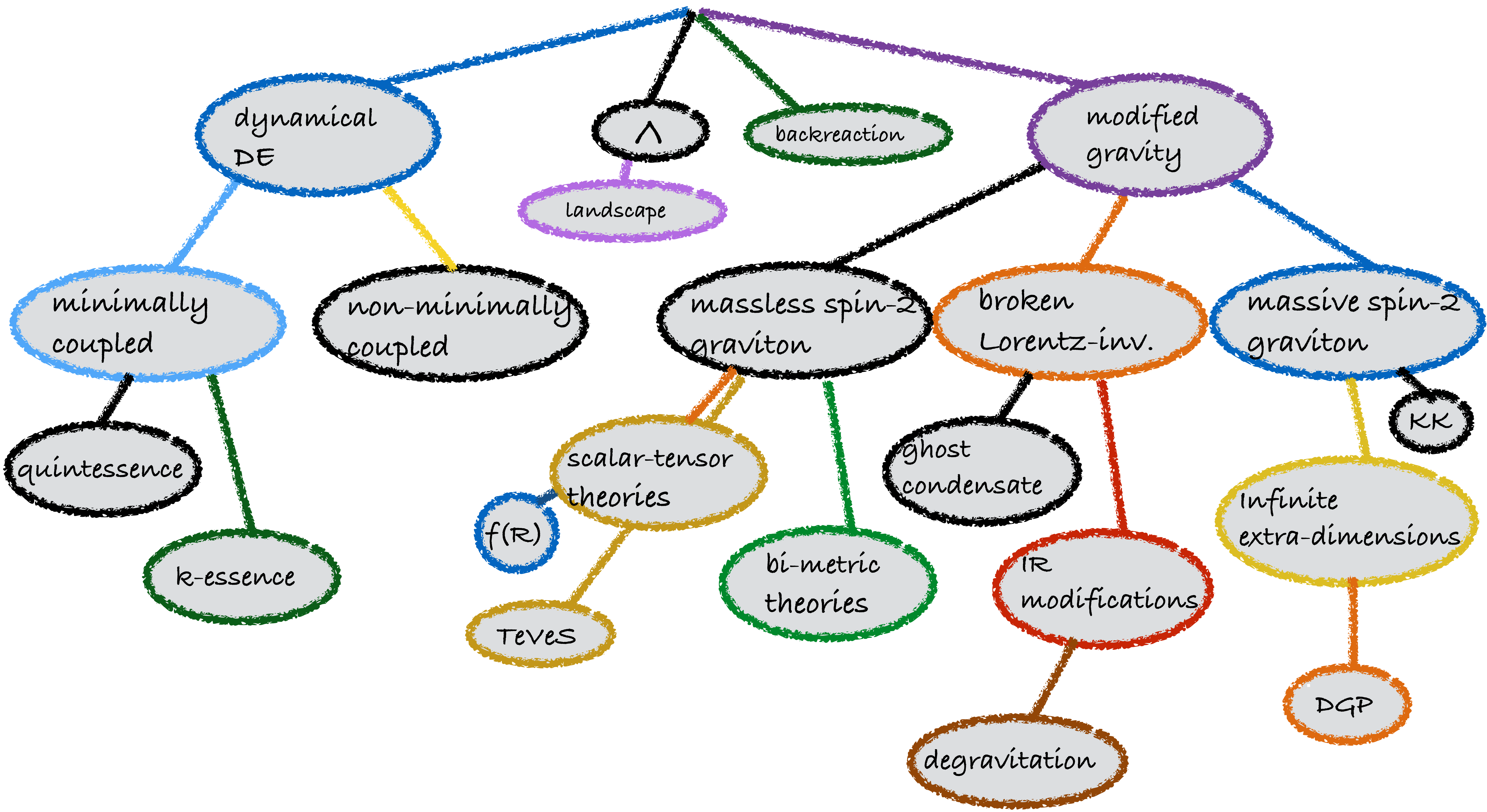

f(R) gravity

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\left[R + f(R)\right] + \int d^4x\sqrt{-g}\mathcal{L}_m\left[\chi_i, g_{\mu\nu}\right]$$

(S.Carroll, V.Duvvuri, M.Trodden & M.S.Turner, Phys.Rev.D70 043528 (2004), S.Capozziello, S.Carloni & A.Troisi, astro-ph/0303041)

$$\begin{cases} \left(1 + f_R\right) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\left(R + f\right) + \left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu\right) f_R = \frac{T_{\mu\nu}}{M_P^2} \\ \nabla_\mu T^{\mu\nu} = 0 \end{cases}$$

$$f_R \equiv \frac{df}{dR}$$

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The Einstein equations are fourth order !

f(R) g

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2004),

The following term:

$$g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\mu\nabla_\nu\left(-\delta g^{\mu\nu} + g^{\mu\nu}g_{\alpha\beta}\delta g^{\alpha\beta}\right)$$

in GR is a boundary term, but in this new action it comes multiplied by f_R and gives rise to:

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The Einstein equations are fourth order !

The trace-equation becomes:

dynamical !

$$(1-f_R)\,R + 2f - 3\Box f_R = \frac{T}{M_P^2}$$

Generalized Brans-Dicke

$$S = \int d^4x\sqrt{-g}\left[\frac{m_0^2}{2}F(\phi)R - \frac{1}{2}K(\phi)(\partial\phi)^2 - U(\phi)\right]$$

$$S_{\rm BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{2\omega}{\phi}\partial_\mu\phi\partial^\mu\phi - 2\Lambda(\phi)\right]$$ Brans-Dicke, 1961

resulting eoms:

$$FG_{\mu\nu} = \frac{1}{m_0^2}\left(T^{\rm m}_{\mu\nu} + T^{\phi}_{\mu\nu}\right) + \nabla_\mu\nabla_\nu F - g_{\mu\nu}\Box F$$

$$T^{\phi}_{\mu\nu} \equiv \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi + U(\phi)\right].$$

Quintessence:

$$F = 1\,,\quad K = 1$$

$$w^\phi = \frac{\frac{1}{2}\dot\phi^2 - U(\phi)}{\frac{1}{2}\dot\phi^2 + U(\phi)}$$

Galileons

Let us focus on the scalar DOF introduced by most, if not all, of these models. After all, that is the ingredient that could take care of cosmic acceleration.

We have discussed Lovelock gravity, which corresponds to the most general action for the metric leading to 2nd order EOMs in D dimensions. Let us now look at a scalar DOF. In Minkowski space there is something analogous to Lovelock argument that allows us to identify the most general Lagrangian for a scalar DOF with at most 2nd order EOMs in D dimensions.

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Studying the decoupling limit of the higher-dimensional DGP model, Nicolis *et al.* realized that the scalar field corresponding to the longitudinal mode of the massive graviton obeyed the galilean shift symmetry, inherited from the higher-dimensional Poincaré invariance :

$$\phi \to \phi + b + c^\mu x_\mu$$

Interestingly, requiring a theory for a scalar field to be galilean invariant and to have EOMs at most of 2nd order, identifies a finite number of terms !

In particular, we have D+1 galileon terms for a Lagrangian in D dimensions. All this, in Minkowski space. If we want to covariantize this, in order for it to be valid in curved spacetime, then we need to non-minimally couple the scalar to gravity in order to keep EOMs to 2nd order (... breaking the galilean invariance).

Galileons

Putting all this together, the most general 4D **scalar-tensor** theory having second-order field equations is described by the following Galileon Lagrangian:

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i \, ,$$

where

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}\, G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right. \\
&\quad \left. +2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)\right]
\end{aligned}$$

$$X \equiv -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$$

The above Lagrangian was first discovered by Horndeski in 1974 in a different, but equivalent form, in the context of Lovelock gravity. Notice that it is a higher-derivative theory which still gives 2nd order EOMs

A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D79, 064036 (2009).
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Horndeski

Second-order scalar-tensor field equations
in a four-dimensional space

(1974)

Starting from a generic action depending on the metric, a scalar field and their derivatives, in 4D:

$$S = \int \mathrm{d}^4 x \sqrt{-g} \mathcal{L}(g_{\mu\nu}, g_{\mu\nu,\lambda_1}, \cdots, g_{\mu\nu,\lambda_1,\cdots,\lambda_p}, \phi, \phi_{,\lambda_1}, \cdots, \phi_{,\lambda_1,\cdots,\lambda_q}),$$

$$p\,, q \geq 2$$

requesting diffeomorphism inv. and 2nd order eoms., leads to the following action:

$$\begin{aligned}\mathcal{L} = \delta^{\alpha\beta\gamma}_{\mu\nu\sigma} &\left[\kappa_1 \phi^\mu_\alpha R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X} \phi^\mu_\alpha \phi^\nu_\beta \phi^\sigma_\gamma + \kappa_3 \phi_\alpha \phi^\mu R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X} \phi_\alpha \phi^\mu \phi^\nu_\beta \phi^\sigma_\gamma\right] \\ &+ \delta^{\alpha\beta}_{\mu\nu} \left[(F + 2W) R_{\alpha\beta}{}^{\mu\nu} + 2F_X \phi^\mu_\alpha \phi^\nu_\beta + 2\kappa_8 \phi_\alpha \phi^\mu \phi^\nu_\beta\right] \\ &- 6\left(F_\phi + 2W_\phi - X\kappa_8\right)\Box\phi + \kappa_9.\end{aligned}$$

where κ_1 , κ_3 , κ_8 , κ_9 are arbitrary functions of ϕ and X

This is equivalent to the Generalized Galileon action.

Beyond Horndeski

Why avoiding higher-order derivatives in the Lagrangian ?

One of the guiding principles we follow is the *Ostrogradsky* theorem, which states that a system described by nondegenerate higher-derivative Lagrangian suffers from ghost-like instabilities.

An important postulate of the Ostrogradsky theorem is that the Lagrangian is **nondegenerate**. If this is not the case, one can reduce a set of higher-derivative field equations to a healthy second-order system.

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They have interesting cosmological phenomenology!

Massive Gravity

It is a vast and extremely challenging territory!

Writing a theory for a massive, spin-2 field is not too difficult. First attempt dates back to Fierz-Pauli in 1939, who wrote the action for a massive spin-2 on flat space.

But it gets very complicated as soon as we include interactions with other particles. The theory is always fully non-linear, and suffers very easily from instabilities.

The modern idea is that of linking the mass of the graviton to horizon scale, so that the scale of acceleration is technically natural (a symmetry, i.e. gauge invariance, is restored in the limit of $m \longrightarrow 0$)

I will not say much about modified gravity in these lectures. From the observational point of view, we mostly deal with the *massless limit of massive gravity, which corresponds to a massless graviton plus a scalar field* (i.e. the longitudinal mode of the massive graviton).

Massive Gravity

It is a vast and extremely challenging territory!

Writing a theory for a massive, spin-2 field is not too difficult. First attempt dates back to Fierz-Pauli in 1939, who wrote the action for a massive spin-2 on flat space.

But it gets very complicated as soon as we include interactions with other particles. The theory is always fully non-linear, and suffers very easily from instabilities.

The modern idea is that of linking the mass of the graviton to horizon scale, so that the scale of acceleration is technically natural (a symmetry, i.e. gauge invariance, is restored in the limit of $m \longrightarrow 0$)

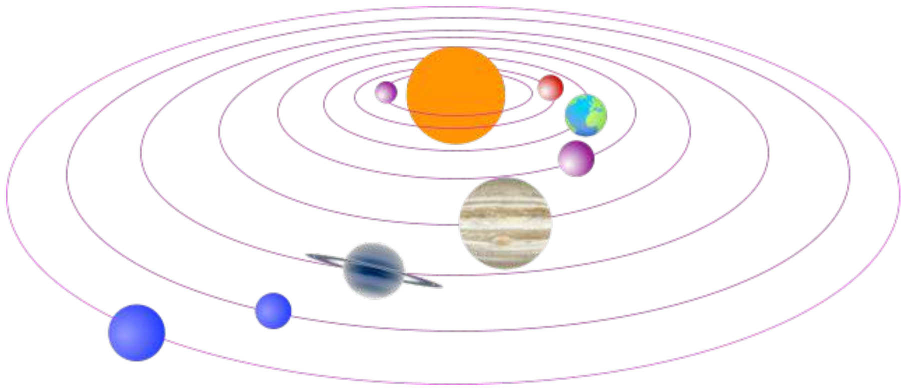
I will not say much about modified gravity in these lectures. From the observational point of view, we mostly deal with the *massless limit of massive gravity, which corresponds to a massless graviton plus a scalar field* (i.e. the longitudinal mode of the massive graviton).

There is a nice scenario that gives a technically small cosmological constant through the degravitation of the vacuum energy. It is realized by promoting Newton's constant to a high-pass filter (linked to the mass of the graviton).

DEGRAVITATION

When modifying gravity,
be aware of …

Local tests of gravity



Gravity has been tested to great accuracy in the solar system and in laboratory.
All alternative theories are bounded to resemble GR very closely on the corresponding scales.

Yet, as we have seen most of these models introduce a scalar field which is coupled to matter fields (there's no good symmetry argument to prevent it) and mediates an additional, fifth force among them with a long range $\sim\ m_\phi^{-1}$.
This interaction must be in some way suppressed in high density environments!

Over the past decade it has become clear that this can happen through non-linear screening mechanisms. Before this was realized, experiments such as the Cassini one were thought to rule out many Brans-Dicke models (all those with the coupling **ω** <40000), including f(R) which corresponds to **ω**=0 !

While there is a plethora of models of dark energy/modified gravity, there are only three general classes of screening. I will give a brief overview of this at the end of this lecture. There's an interesting phenomenology associated to these, that still needs to be fully explored and exploited.

Khoury and Weltman, Phys.Rev.Lett.93 (2004)

Stability of your theory

Extra DOFs are exactly what we needed to source cosmic acceleration in a dynamical way!
Yet extra dynamics might bring in instabilities in the system.
The latter have often a classical and a quantum facet.

For instance the DOFs might be ghost-like (~ wrong sign kinetic term), develop a gradient instability (~ wrong sign gradient term), be tachyonic, propagate superluminally etc...which would make the theory unhealthy for different reasons.

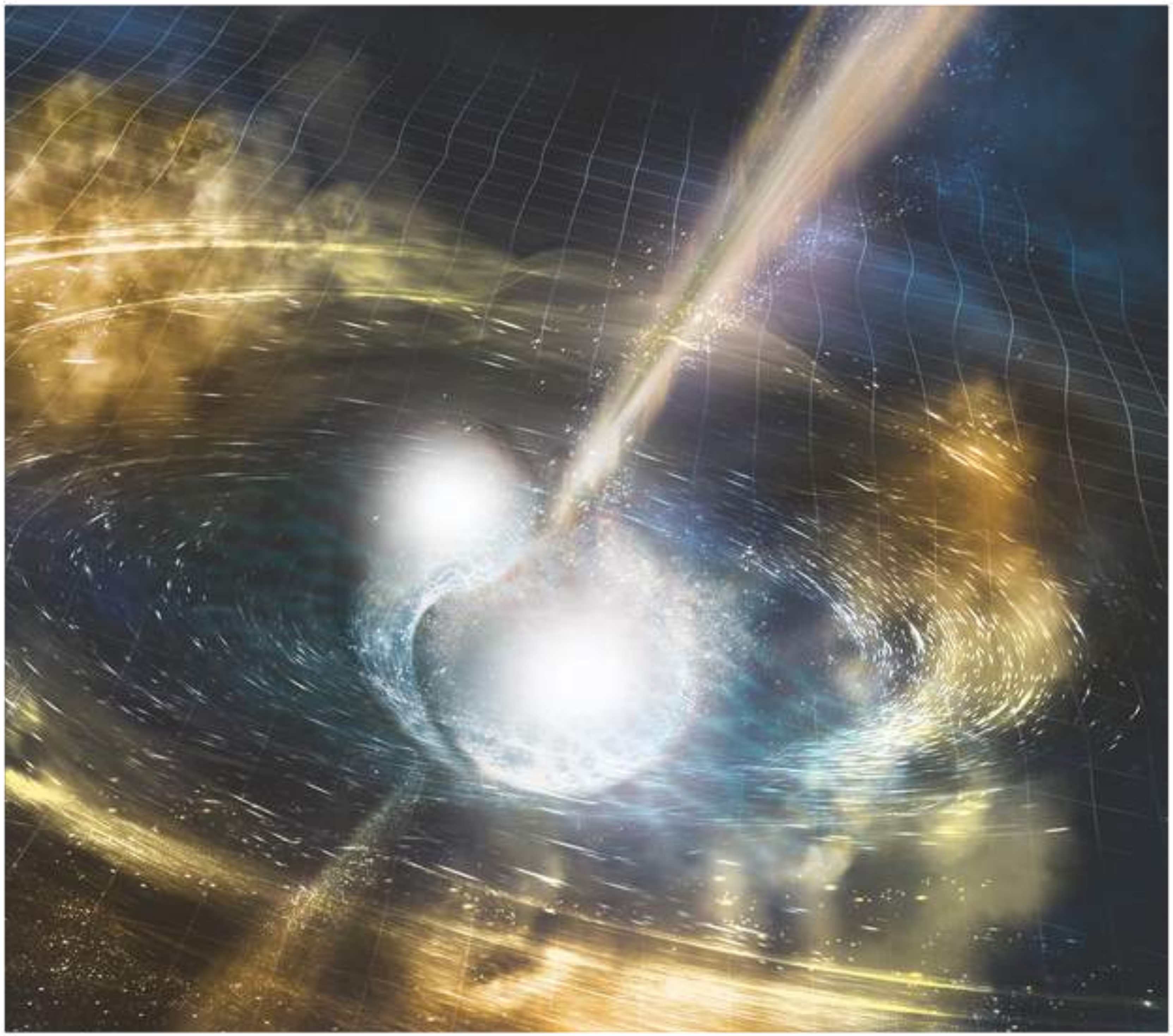
One can identify conditions to avoid these instabilities by performing a diagnostic of the dynamics of perturbations at the level of the action.

†

Gravitational Waves

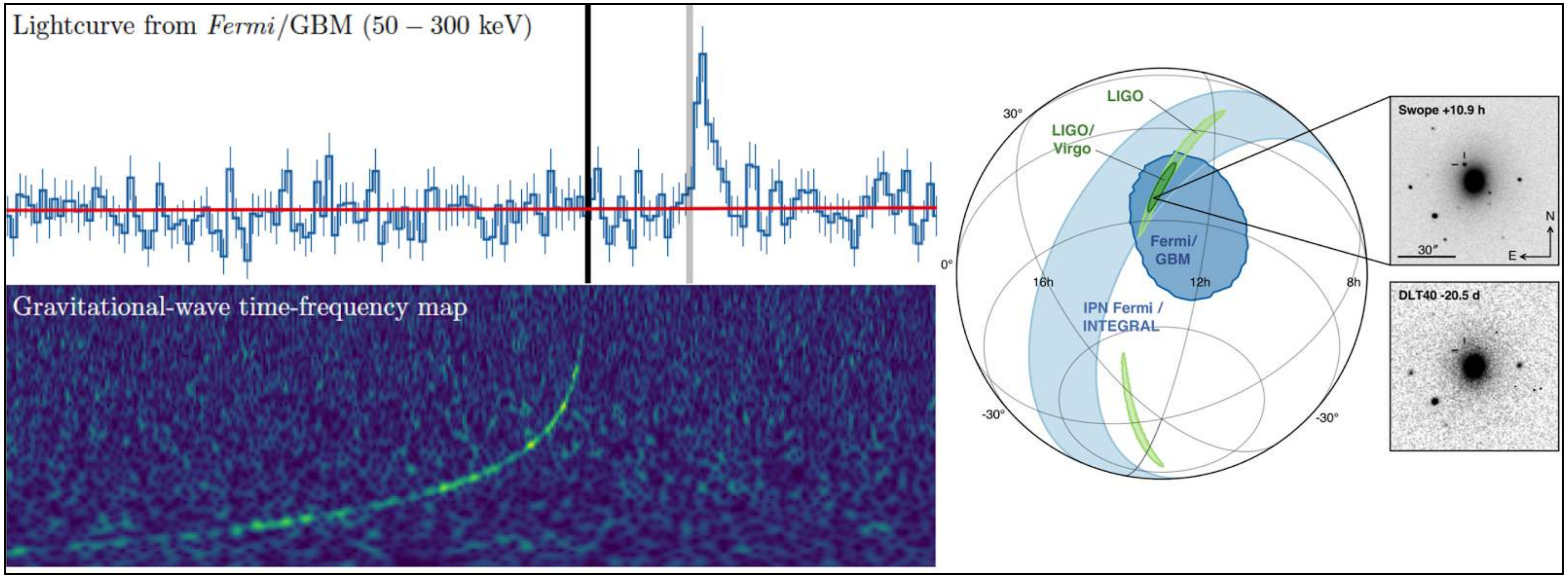
OCTOBER 2017

'After straining our eyes, it is now time to strain our ears !'



GW170817

& GRB170817A



LIGO, Virgo, Fermi,-GBM, INTEGRAL, Astrophys.J. 848 (2017)

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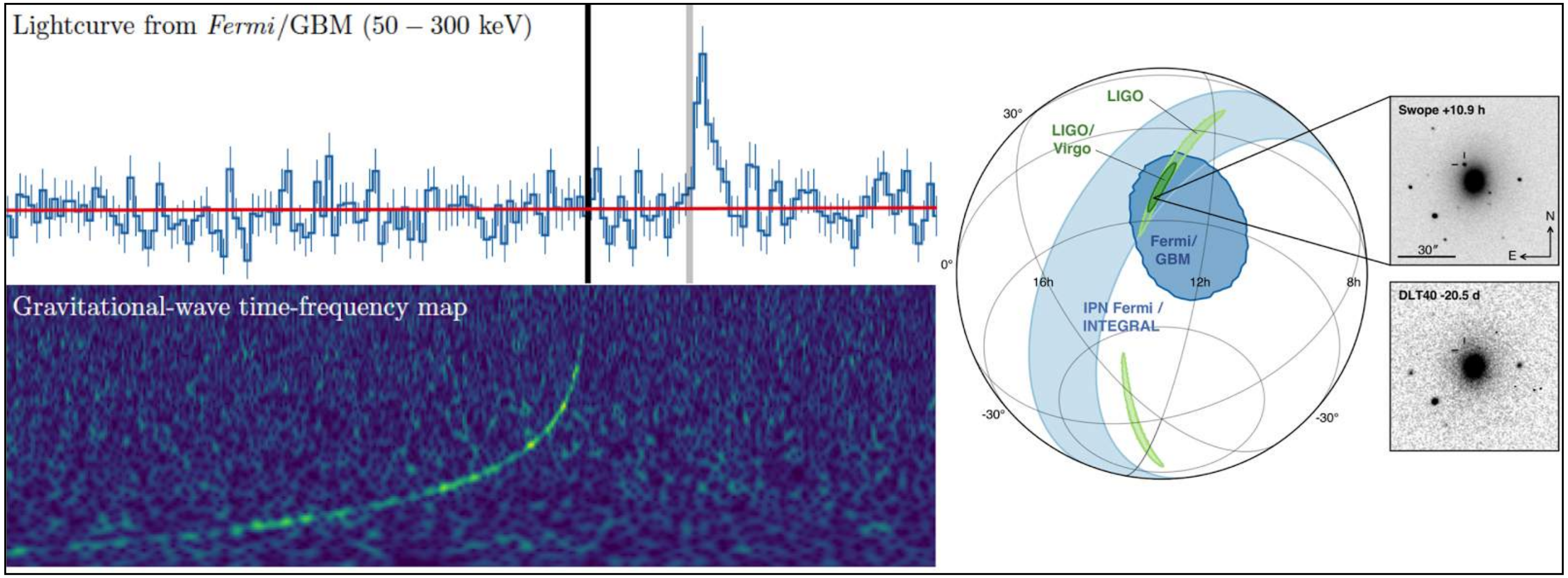

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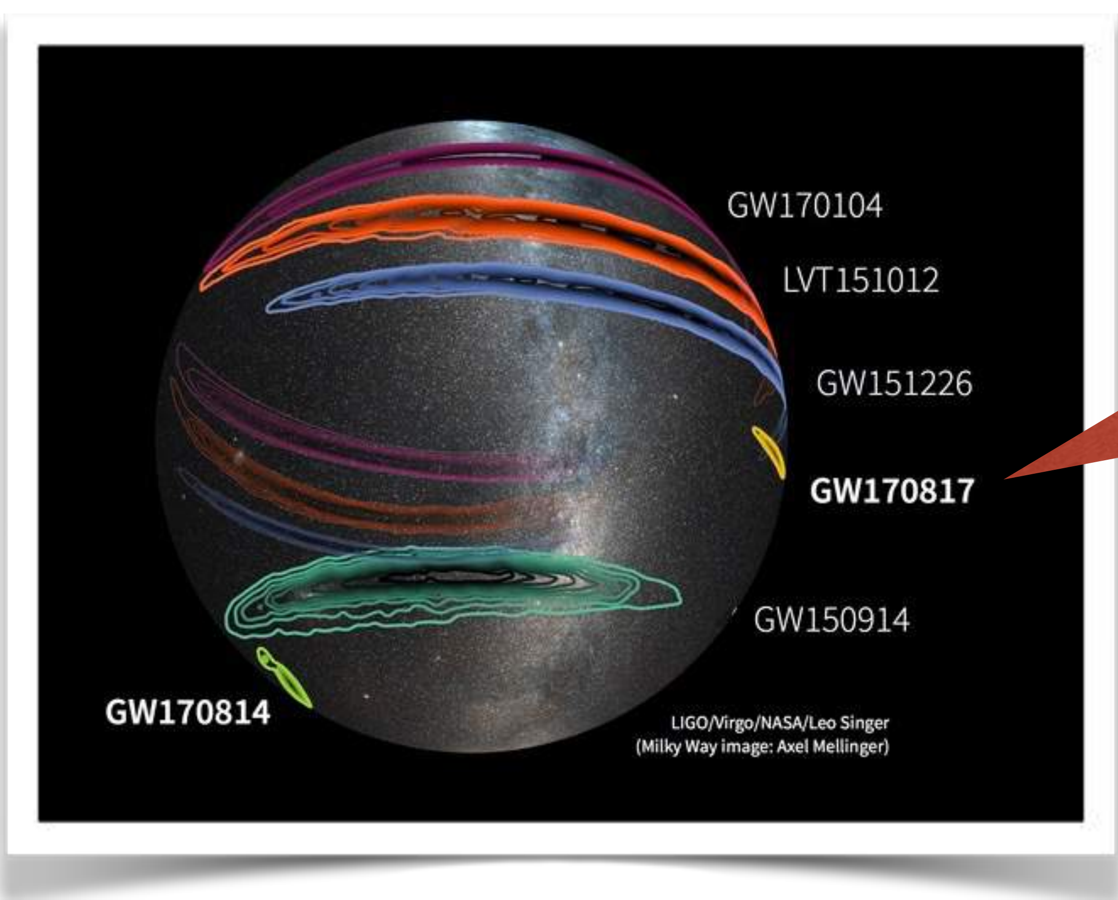
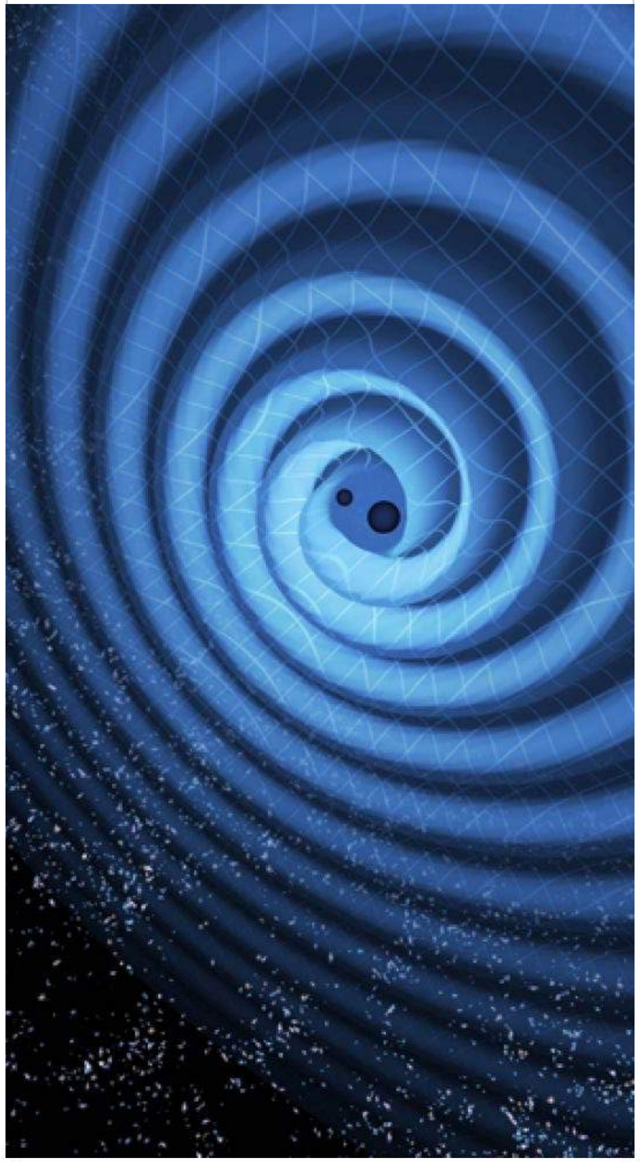


The electromagnetic counterpart allows us to determine the ***redshift*** of the source.

GW170817

& GRB170817A



Ripples in the fabric of spacetime described by the tensor h_{ij} .

In a cosmological background they propagate according to:

$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} + c_T^2 k^2 h_{ij} = 0$$

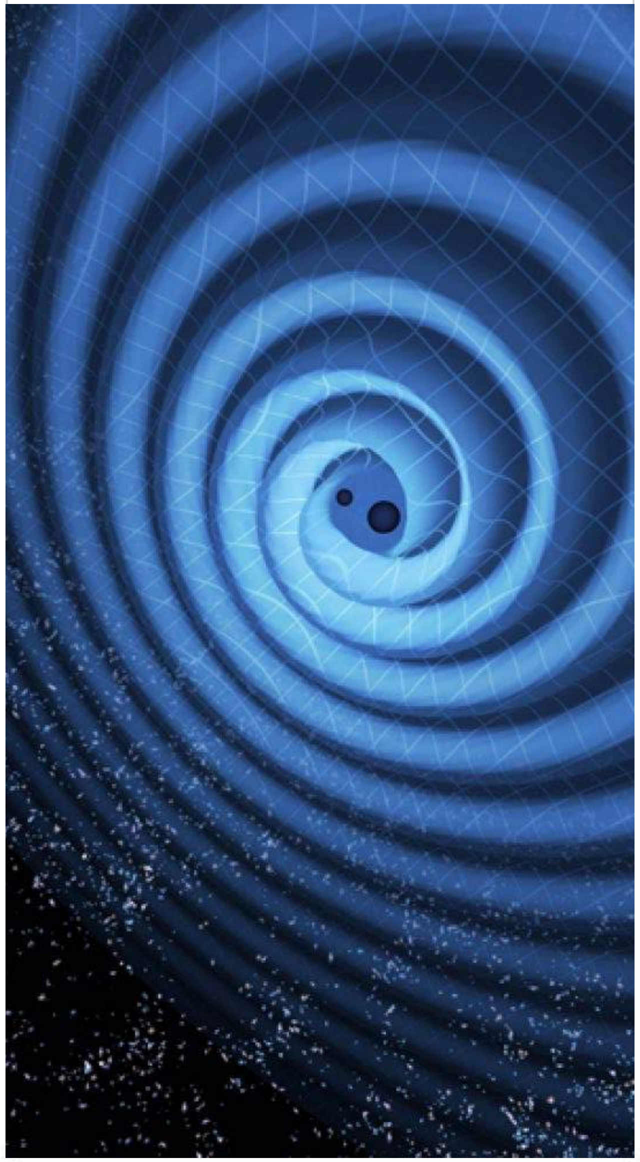
Using the observed **time-delay** btw GRB and GW,

$(1.74 \pm 0.05)\mathrm{s}$, they placed very stringent limits on the speed of gravity:

$$-3 \cdot 10^{-15} < \frac{c_T^2}{c^2} - 1 < 7 \cdot 10^{-16}$$

GW170817

& GRB170817A



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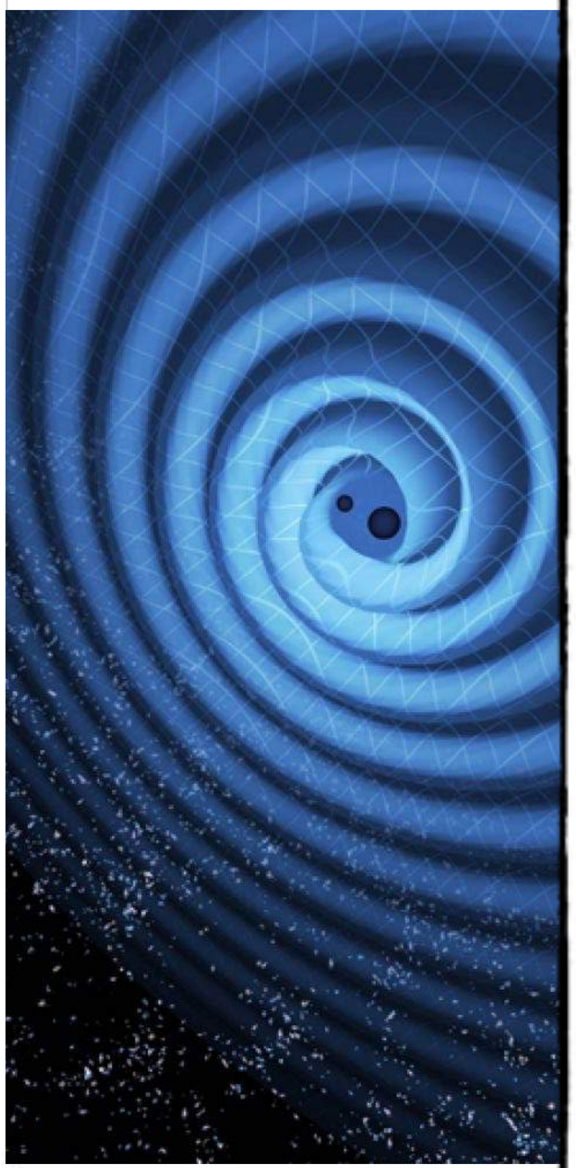
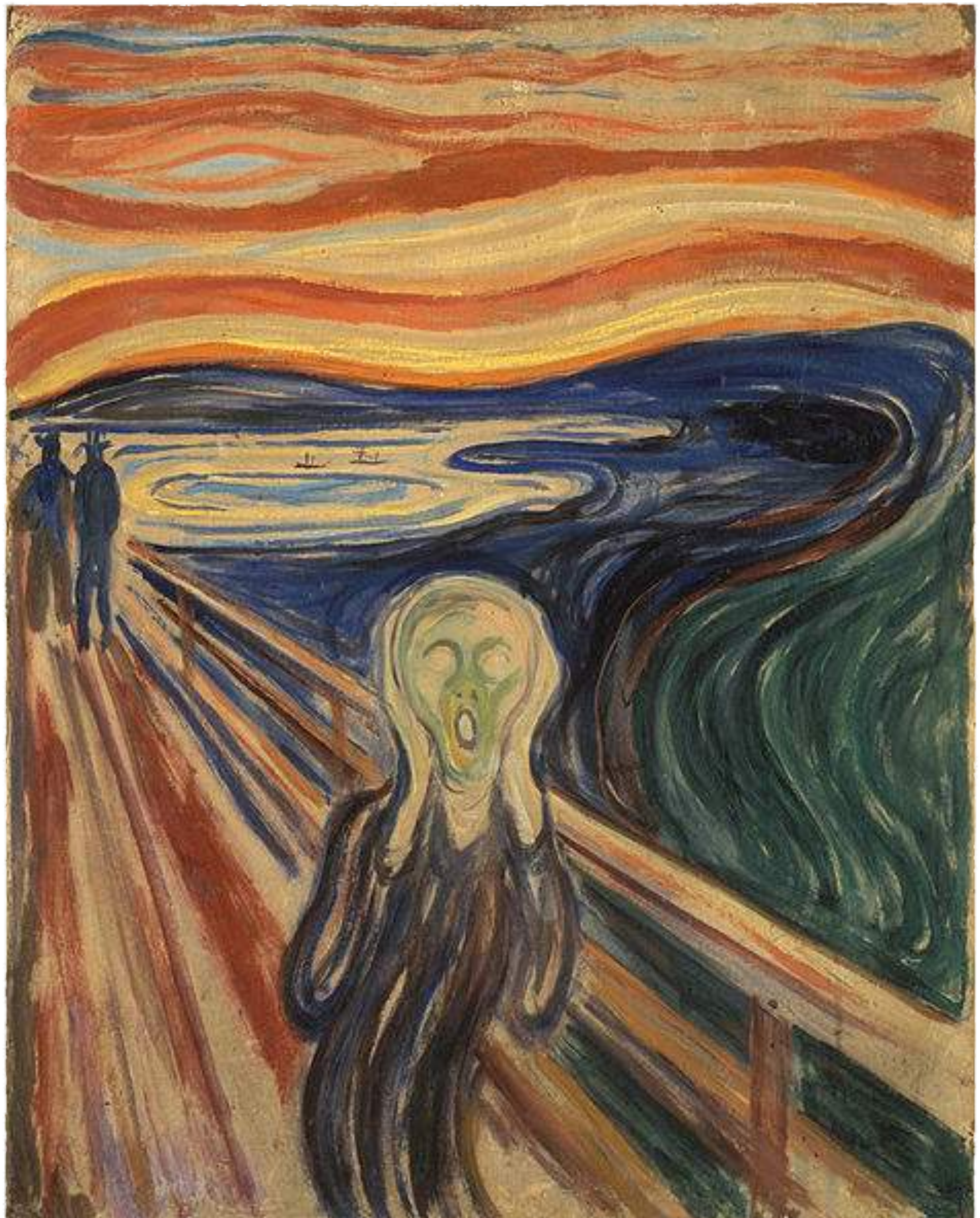
$$c_T^2 = 1$$

GW170817





The scream, E. Munch

170817A

tensor h_{ij} .

cording to:

0

,

the speed of gravity:

The aftermath of GW170817

& GRB170817A

e.g. general scalar-tensor theories give:

$$\ddot{h}_{ij} + (3 + \alpha_M)\, H\dot{h}_{ij} + (1 + \alpha_T)\, k^2 h_{ij} = 0$$

The aftermath of GW170817

& GRB170817A

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conformal coupling

non-trivial kinetic term

The aftermath of GW170817
& GRB170817A

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conformal coupling (α_M)

non-trivial kinetic term (α_T)

$$c_T^2 = 1 \quad \Longrightarrow \quad \alpha_T = 0$$

For Generalized Galileons:

$$\alpha_T \propto 2X[2G_{4X} - 2G_{5\phi} - (\ddot{\phi} - H\dot{\phi})G_{5X}],$$

and this implies that Covariant Galileons, as well as quartic and quintic Horndeski, are gone.
This cuts a big chunk of self-accelerating models.

Creminelli & Vernizzi, PRL 2017
Ezquiaga, Zumalacarregui, PRL 2017
Baker et al., PRL 2017

The aftermath of GW170817
& GRB170817A

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a caveat: see Rainbow paper by de Rham & Melville, June 2018

The aftermath of GW170817

& GRB170817A

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/$f(R)$ [47, 48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ [51], $f(\phi)\cdot$Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

Ezquiaga, Zumalacarregui, PRL 2017

Einstein-Aether, Generalized Proca, bimetric gravity, … are constrained but still there!

Cosmological Tests of Gravity

But this is a school on Observational Cosmology, so let me get to the more interesting part, i.e. how to test our theory of gravity on cosmological scales?

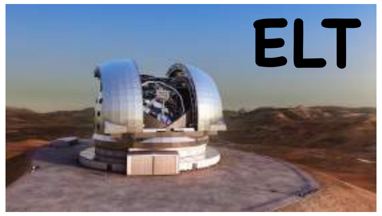
Generally theories beyond ΛCDM have enough freedom to reproduce 'any' desired expansion history, being higher order in nature.

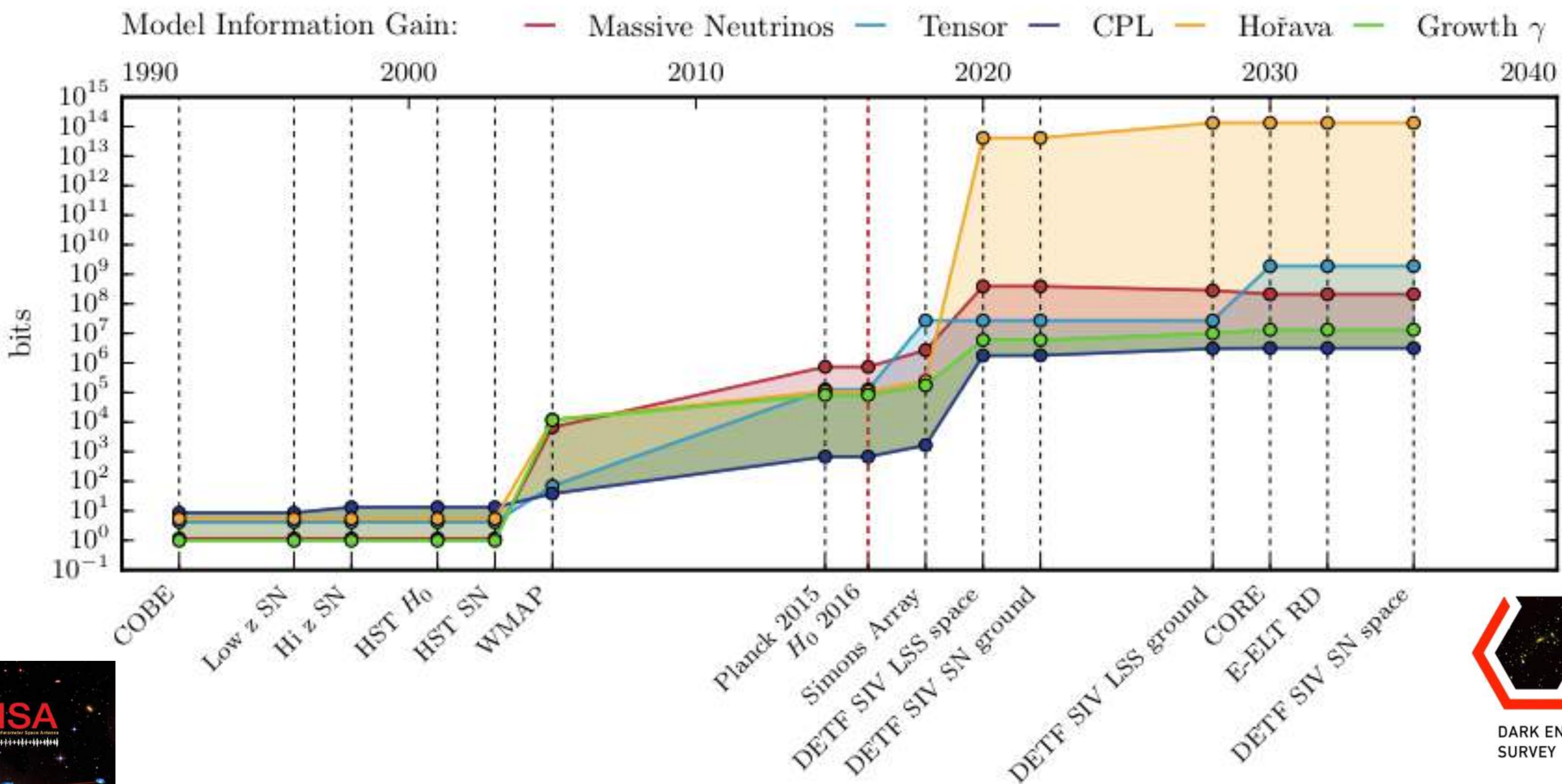
In other words, at the level of background expansion history there is a degeneracy among different approaches to the phenomenon of cosmic acceleration.

It has become increasingly clear that we need to go beyond geometrical probes in order to disentangle the theoretical landscape of cosmic acceleration.

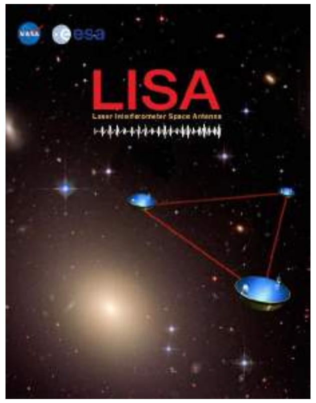
Large Scale Structure will provide a powerful testbed.

Information Gain in Cosmology





Model cumulative Information Gain



GIANT MAGELLAN TELESCOPE

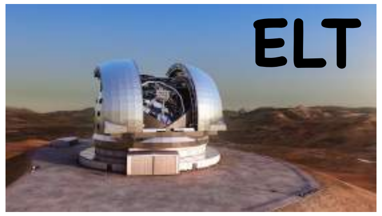
Large Synoptic Survey Telescope

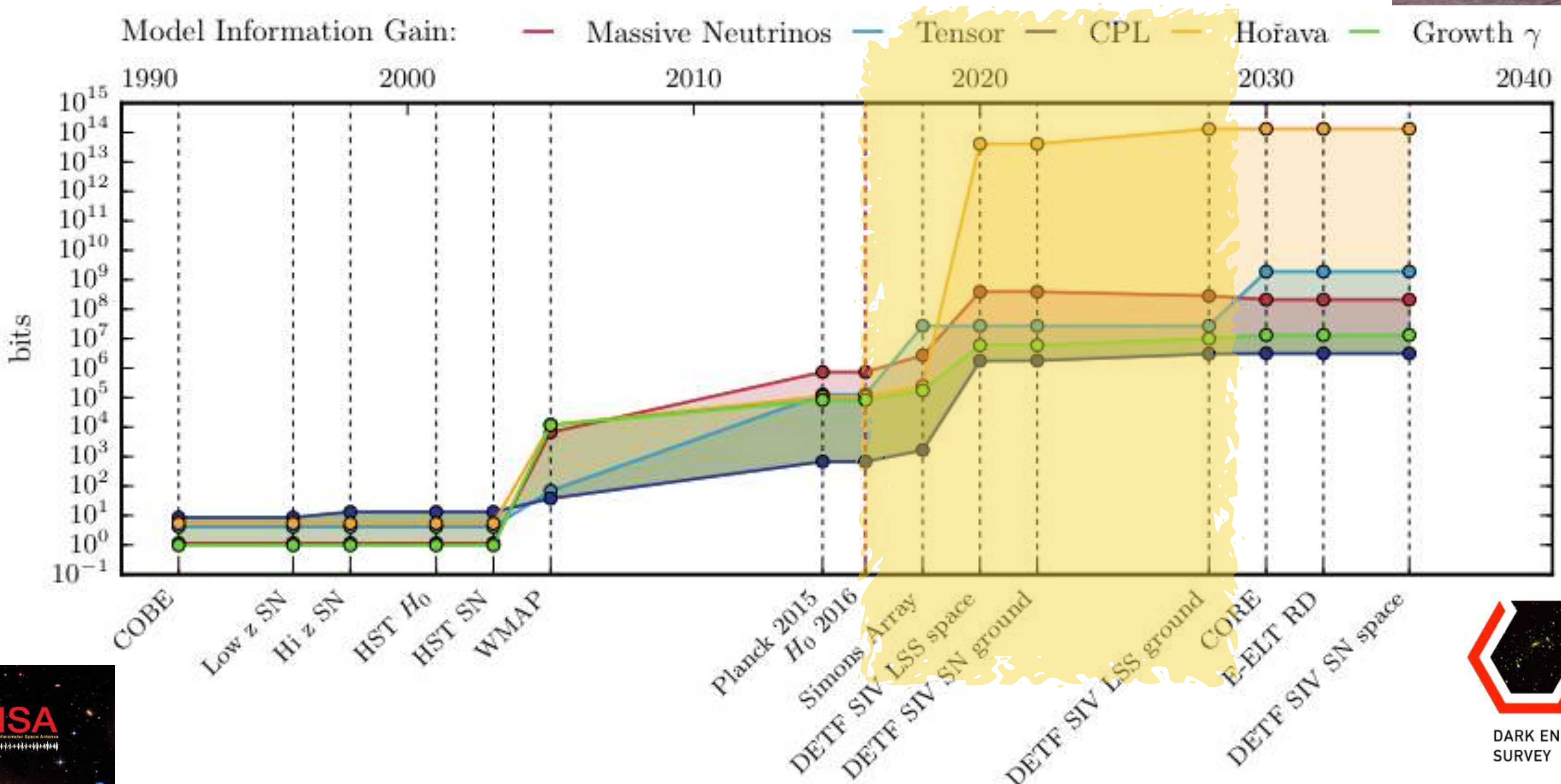
DARK ENERGY SURVEY

SKA

Raveri, Martinelli, Zhao, Wang, CosmicFish/arXiv:1606.06273

Information Gain in Cosmology





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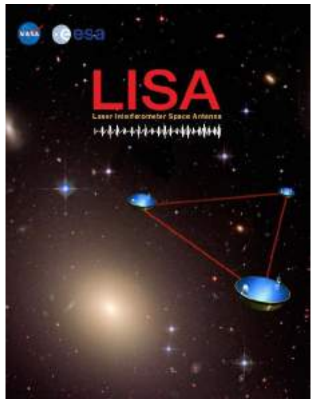

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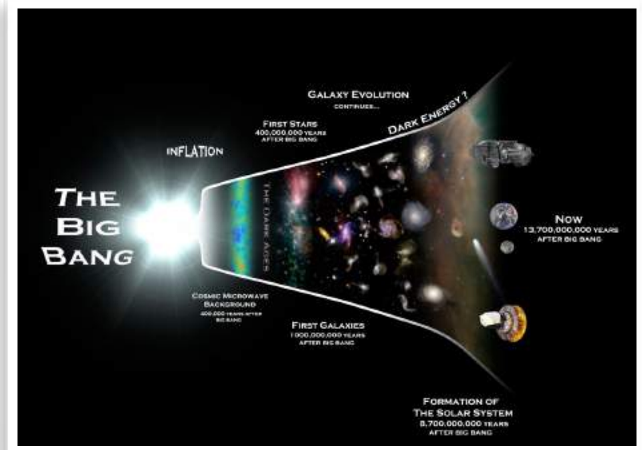
Going to the perturbed Universe

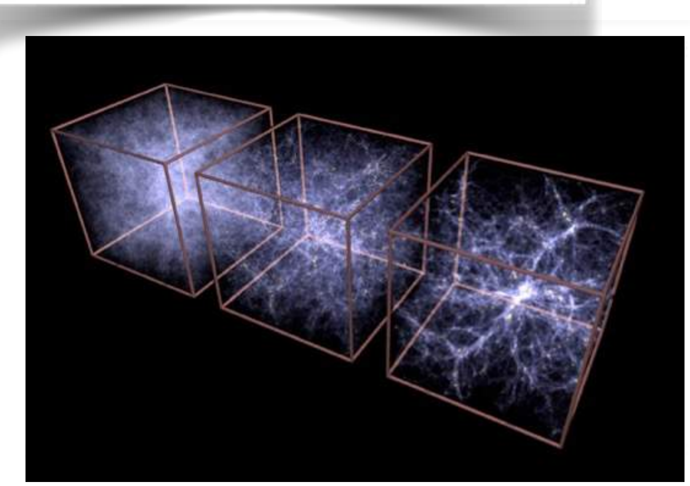
Perturbed metric and LSS

$$ds^2 = -a^2(\tau)\left[(1+2\Psi(\tau,\vec{x}))\,d\tau^2 - (1-2\Phi(\tau,\vec{x}))\right]$$

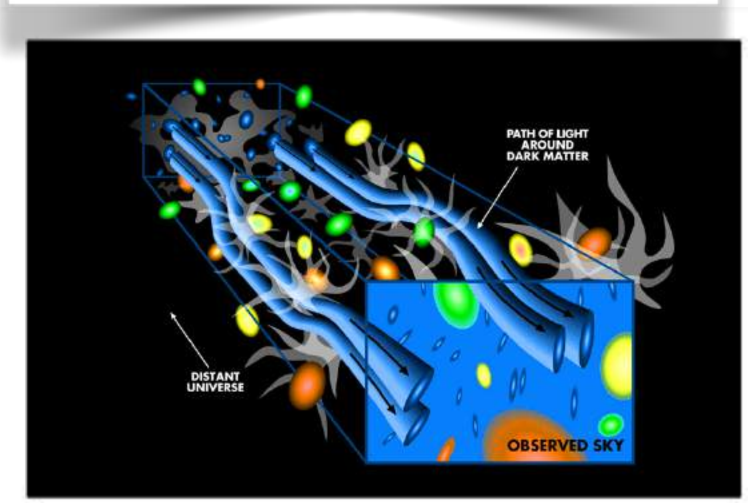
expansion history: $a(\tau)$



non-relativistic dynamics
(growth of structure, pec. vel.): $\Psi(\tau,\vec{x})$



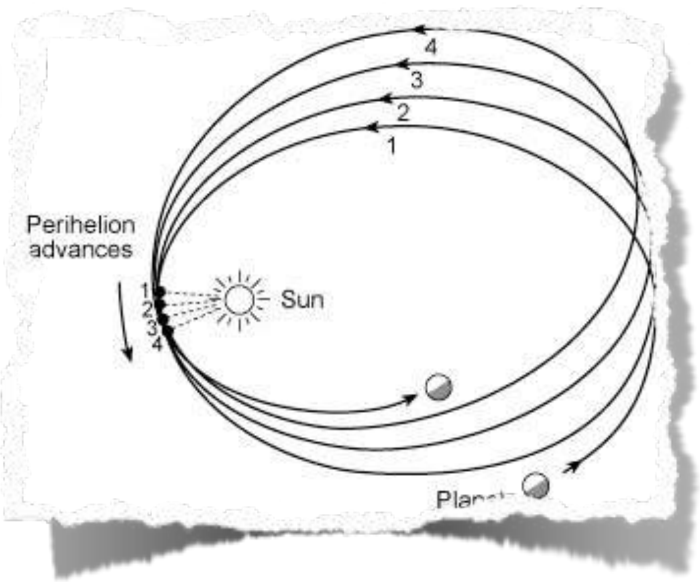
relativistic dynamics
(weak lensing, ISW): $(\Phi+\Psi)(\tau,\vec{x})$



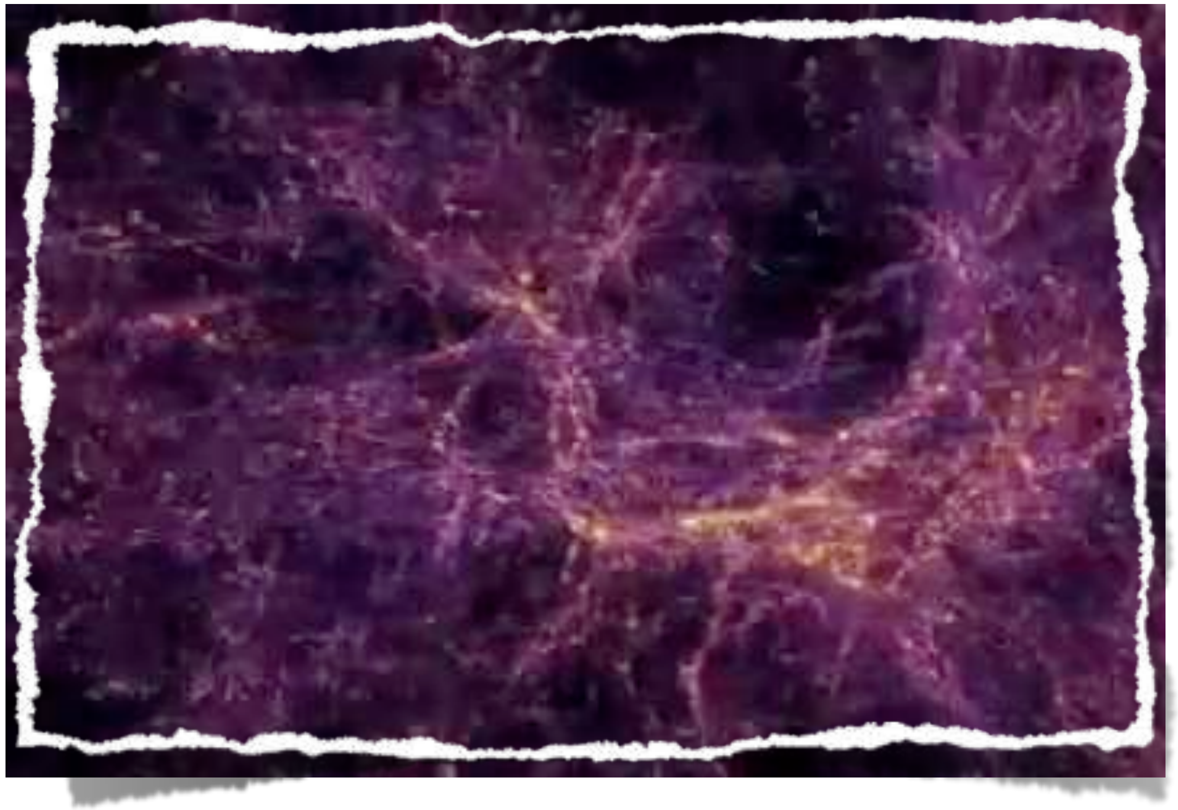
Cosmic Functions of Interest

Ψ : **non-relativistic tracers**

orbits of planets



galaxies: growth of structure and peculiar velocities



Newtonian potential

Cosmic Functions of Interest

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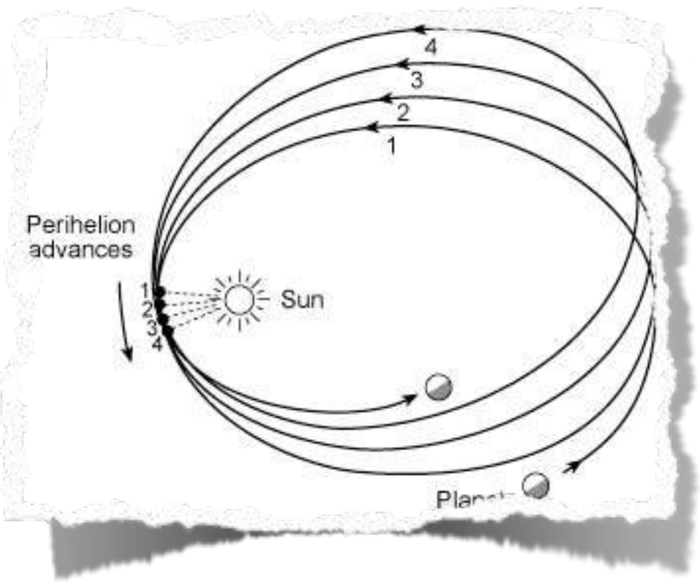

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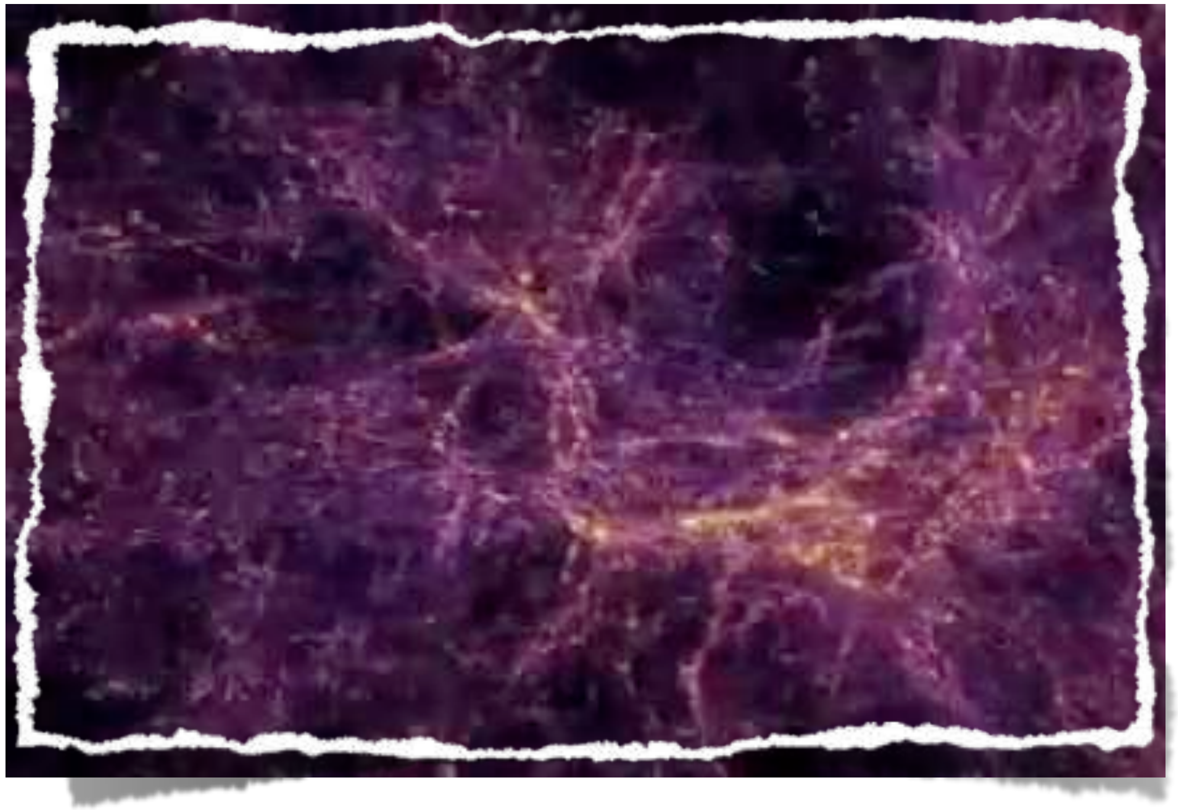
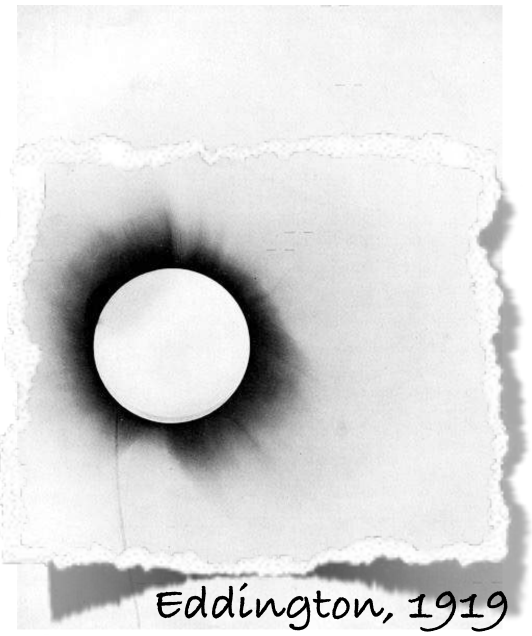


Newtonian potential

Cosmic Functions of Interest

$\Phi + \Psi$: **massless particles**

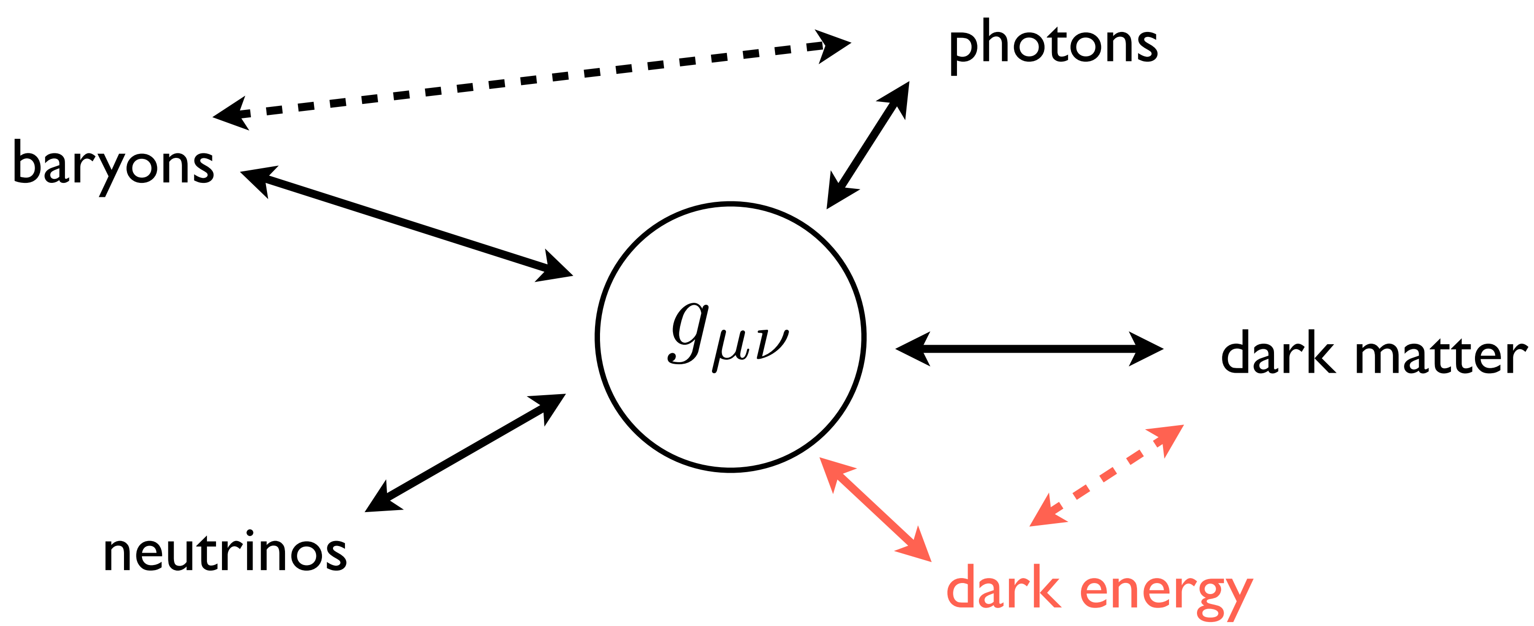
light deflection, time delay



weak lensing,
Integrated Sachs-Wolfe effect in CMB

lensing potential

Cosmic Functions of Interest



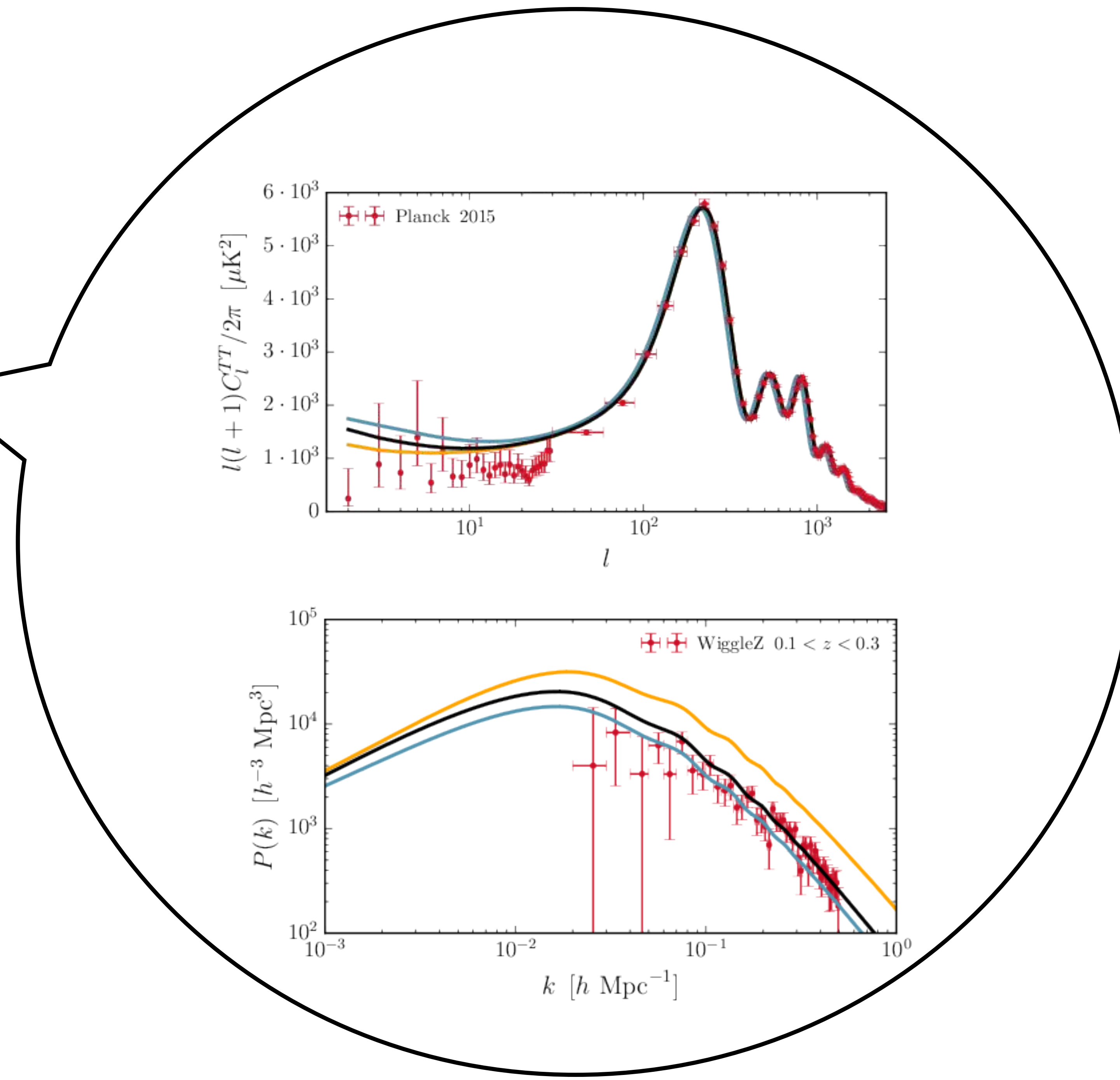
Boltzmann eqs.: $$\frac{df}{dt} = C[f]$$ where f is the phase-space distribution function of a given species

Einstein eqs.: $$G_{\mu\nu} \leftrightarrow T_{\mu\nu}$$

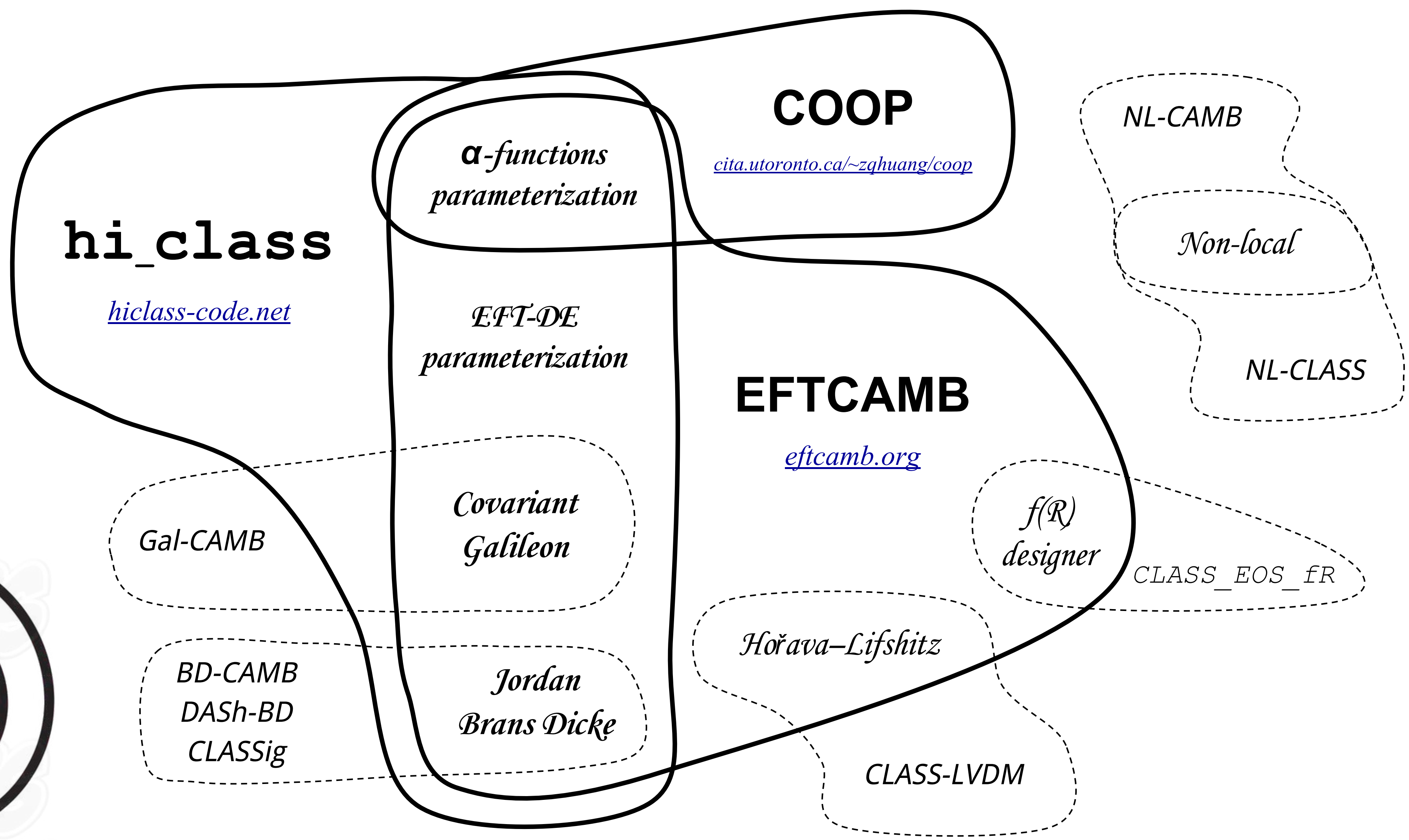
From Theory to Observables

matter and metric perturbations
+
theory

EINSTEIN-BOLTZMANN
SOLVER
e.g. CAMB



From Theory to Observables



Going to non-linear, dense regions

Screening Mechanisms

Screening Mechanisms

Essentially all attempts to explain cosmic acceleration introduce **new long range forces**, typically mediated by a scalar DOF.

Still, they have not been seen experimentally, therefore the **fifth force** that they mediate must be suppressed in local environments, where GR has been tested to high accuracy. This can happen by mean of the so-called **screening mechanism**.

Screening Mechanisms: types

PHENOMENOLOGICAL CLASSIFICATION

Chameleon the effective mass of the field depends on the local density of matter, so that it is light on cosmological scales (and can source acceleration) and heavy in local regions, effectively hiding from local tests of gravity

Symmetron the vev of the field depends on the local density of matter and the coupling of the field to matter is proportional to the vev, so that the scalar couples with gravitational strength in regions of low density, but is decoupled and screened in regions of high density

Vainshtein & k-mouflage it is a kinetic type of screening, in which either first (k-.) or second (V.) derivatives of the scalar field become large in dense regions, effectively weakening the interaction with matter

Small scale tests of gravity that rely on distinct signatures of screening are useful discriminants of cosmological models. This is a "recent" realization, destined to complement large scale tests of GR

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Symmetron

$$Z(\bar{\phi})\left(\ddot{\varphi} - c_s^2(\bar{\phi})\nabla^2\varphi\right) + m^2(\bar{\phi})\varphi = g(\bar{\phi})\mathcal{M}\delta^3(\vec{x})$$

and the coupling
he scalar couples
decoupled and
screened in regions of high density

Vainshtein & k-mouflage

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Chameleon Mechanism

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\left(\partial\phi\right)^2 - V\left(\phi\right)\right] + S_m\left[\psi_m^{(i)}, g_{\mu\nu}^{(i)}\right]$$

$$g_{\mu\nu}^{(i)} = e^{2\beta_i\phi/M_P}g_{\mu\nu}$$

Khoury and Weltman, Phys.Rev.Lett.93 (2004)

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$$\Box\phi = V_{,\phi} + e^{4\beta_i \phi / M_P} g_{(i)}^{\mu\nu} T_{\mu\nu}^{(i)}$$

Khoury and Weltman, Phys.Rev.Lett.93 (2004)

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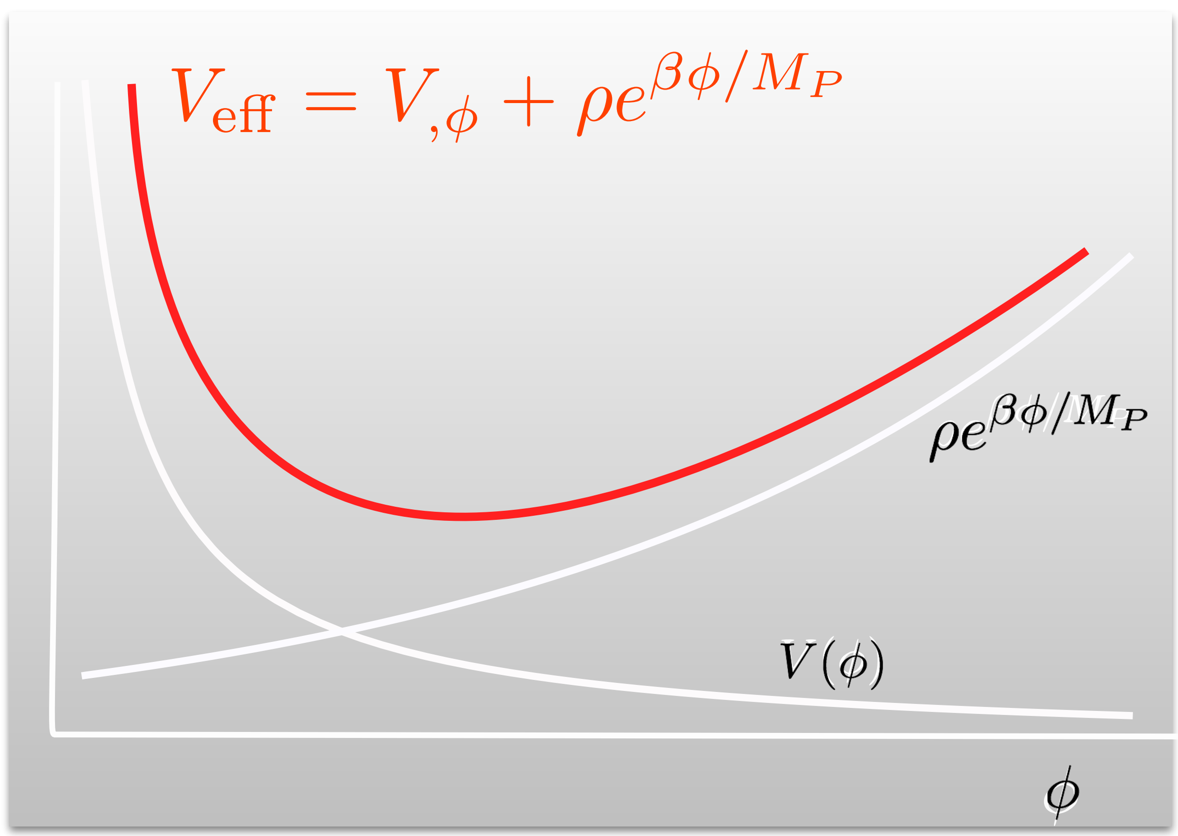
for a single, non-relativistic matter component



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