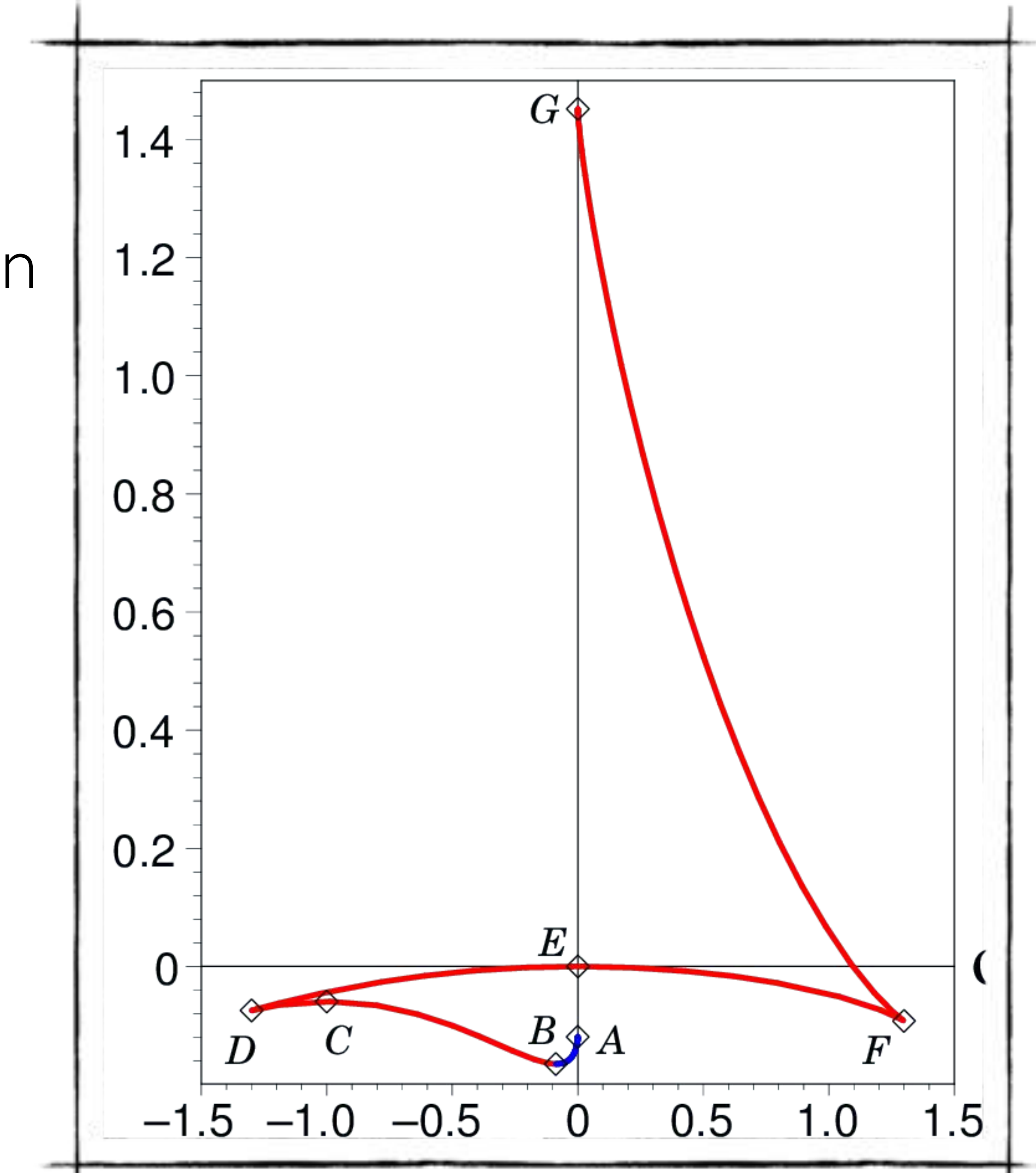
LECTURE 2

f(R) Gravity: background and structure formation



A. Frolov, Phys.Rev.Lett. 101 (2008)

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III Joint ICTP-Trieste/ICTP-SAIFR School on Observational Cosmology July 30, 2019

pre-LECTURE 2

finishing up Screening Mechanisms

Screening Mechanisms

Essentially all attempts to explain cosmic acceleration introduce **new long range forces**, typically mediated by a scalar DOF.

Still, they have not been seen experimentally, therefore the **fifth force** that they mediate must be suppressed in local environments, where GR has been tested to high accuracy. This can happen by mean of the so-called **screening mechanism**.

Screening Mechanisms: types

PHENOMENOLOGICAL CLASSIFICATION

Chameleon — the effective mass of the field depends on the local density of matter, so that it is light on cosmological scales (and can source acceleration) and heavy in local regions, effectively hiding from local tests of gravity

Symmetron — the vev of the field depends on the local density of matter and the coupling of the field to matter is proportional to the vev, so that the scalar couples with gravitational strength in regions of low density, but is decoupled and screened in regions of high density

Vainshtein & k-mouflage — it is a kinetic type of screening, in which either first (k-.) or second (V.) derivatives of the scalar field become large in dense regions, effectively weakening the interaction with matter

Small scale tests of gravity that rely on distinct signatures of screening are useful discriminants of cosmological models. This is a "recent" realization, destined to complement large scale tests of GR

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$$Z(\bar{\phi})\left(\ddot{\varphi} - c_s^2(\bar{\phi})\nabla^2\varphi\right) + m^2(\bar{\phi})\varphi = g(\bar{\phi})\mathcal{M}\delta^3(\vec{x})$$

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Vainshtein & k-mouflage

resulting static potential around point source $\rho = \mathcal{M}\delta^3(\vec{x})$

$$V(r) = -\frac{g^2(\bar{\phi})}{Z(\bar{\phi})c_s^2(\bar{\phi})}\frac{e^{-\frac{m(\bar{\phi})}{\sqrt{Z(\bar{\phi})c_s(\bar{\phi})}}r}}{4\pi r}\mathcal{M}$$

second (V.)
s, effectively

Small scale tests c ... l discriminants of
cosmological mo ... ge scale tests of

Chameleon Mechanism

$$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] + S_m\left[\psi_m^{(i)}, g_{\mu\nu}^{(i)}\right]$$

$$g_{\mu\nu}^{(i)} = e^{2\beta_i\phi/M_P} g_{\mu\nu}$$

$$\Box\phi = V_{,\phi} - \sum_i \frac{\beta_i}{M_P} e^{4\beta_i\phi/M_P} g_{(i)}^{\mu\nu} T_{\mu\nu}^{(i)}$$

Khoury and Weltman, Phys.Rev.Lett.93 (2004)

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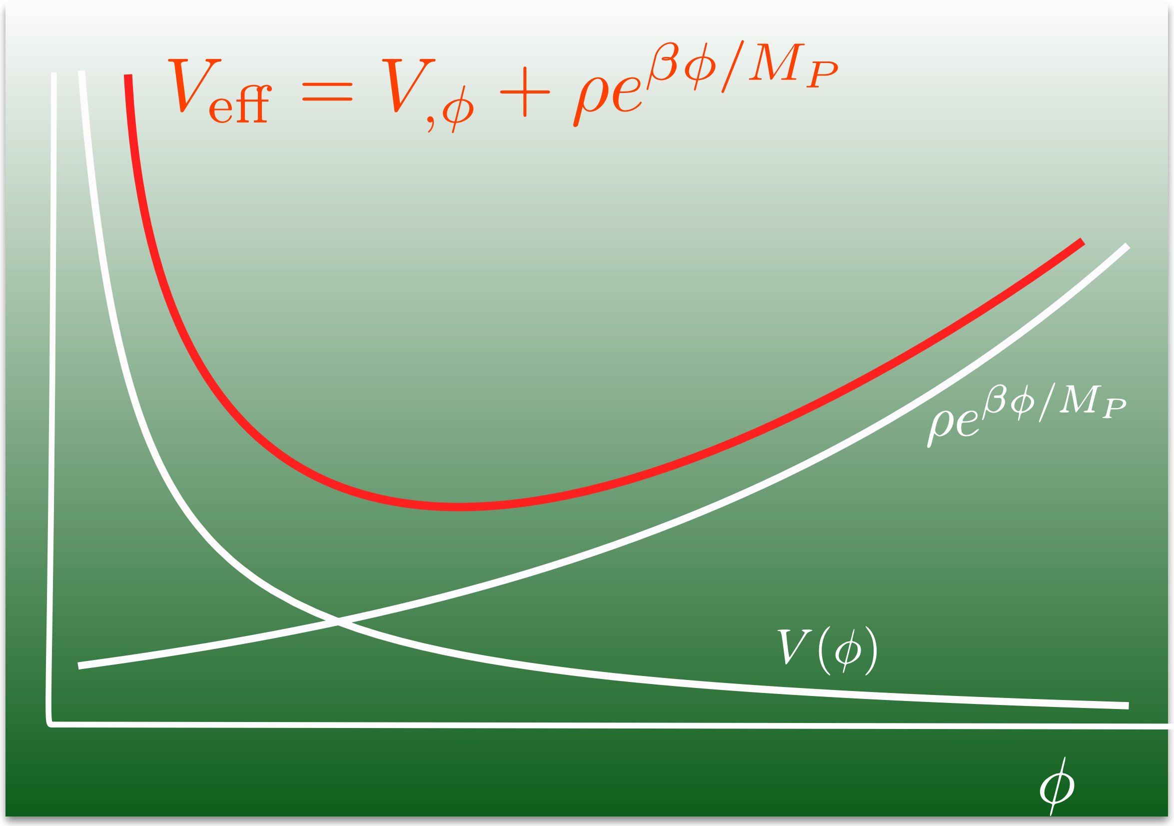
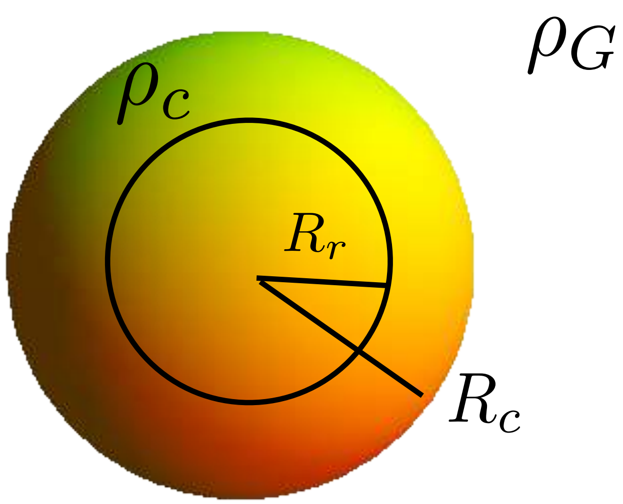
↓ *for a single, non-relativistic matter component*



Khoury and Weltman, Phys.Rev.Lett.93 (2004) calculation of environment dependent mass …

Around 'spherically symmetric cow'



$$\frac{d^2\phi}{2r^2} + \frac{2}{r}\frac{d\phi}{dr} = V_{,\phi} + \frac{\beta\rho}{M_P}$$

B.C.s:
$$\begin{cases} \dfrac{d\phi}{dr} = 0 \ \text{ at } r = 0 \\ \phi \to \phi_G \ \text{ as } r \gg R_c \end{cases}$$

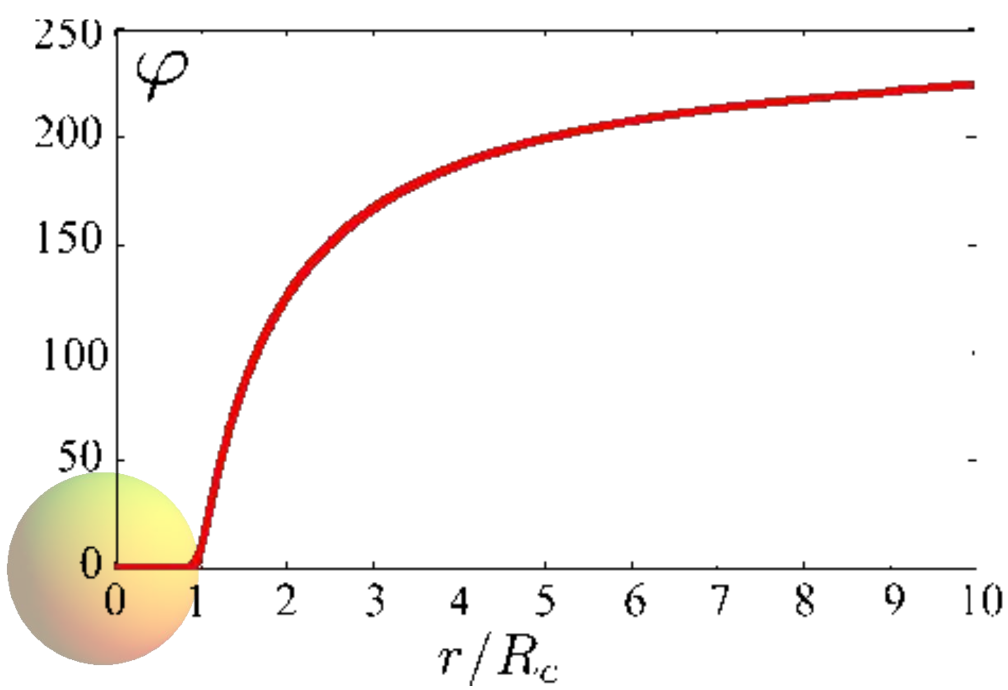
$$\phi = \begin{cases} \phi_c & r < R_r \\ A\phi_c + \frac{B}{r} + \frac{\beta\rho_c}{6M_P}r^2 & R_r < r < R_c \quad \left(\dfrac{d^2\phi}{dr^2} + \dfrac{2}{r}\dfrac{d\phi}{dr} \approx \dfrac{\beta\rho_c}{M_P}\right) \\ -C\dfrac{e^{-m_G(r-R_c)}}{r} + \phi_G & r > R_c \quad \left(\dfrac{d^2\phi}{dr^2} + \dfrac{2}{r}\dfrac{d\phi}{dr} \approx m_G^2(\phi - \phi_G)\right) \end{cases}$$

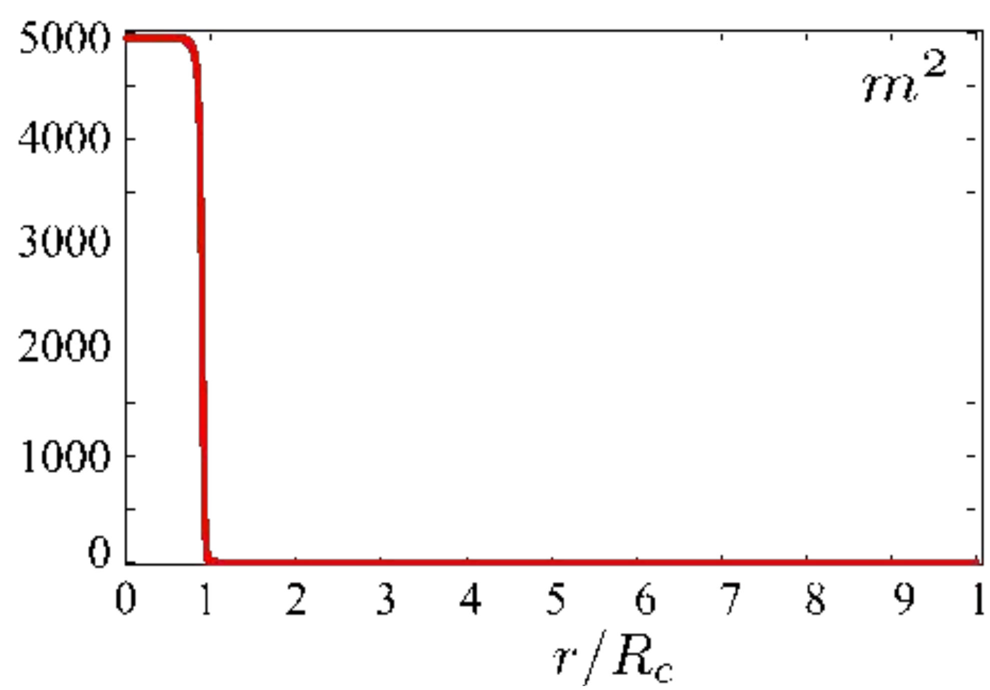
Around 'spherically symmetric cow'

If $\frac{\phi_G - \phi_c}{6\beta M_P \Phi_c} \ll 1$ we can achieve the following configuration: THIN-SHELL

$$\phi \approx -\frac{\beta}{4\pi M_P}\frac{3\Delta R_c}{R_c}\frac{M_c e^{-m_G(r-R_c)}}{r} + \phi_G$$

$$\frac{\Delta R_c}{R_c} = \frac{R_c - R_r}{R_c} \approx \frac{\phi_G - \phi_c}{6\beta M_P \Phi_c} \ll 1$$





Thin-shell configuration

Whether this configuration can be achieved, depends on the parameters of the theory and on how perturbing is the source.

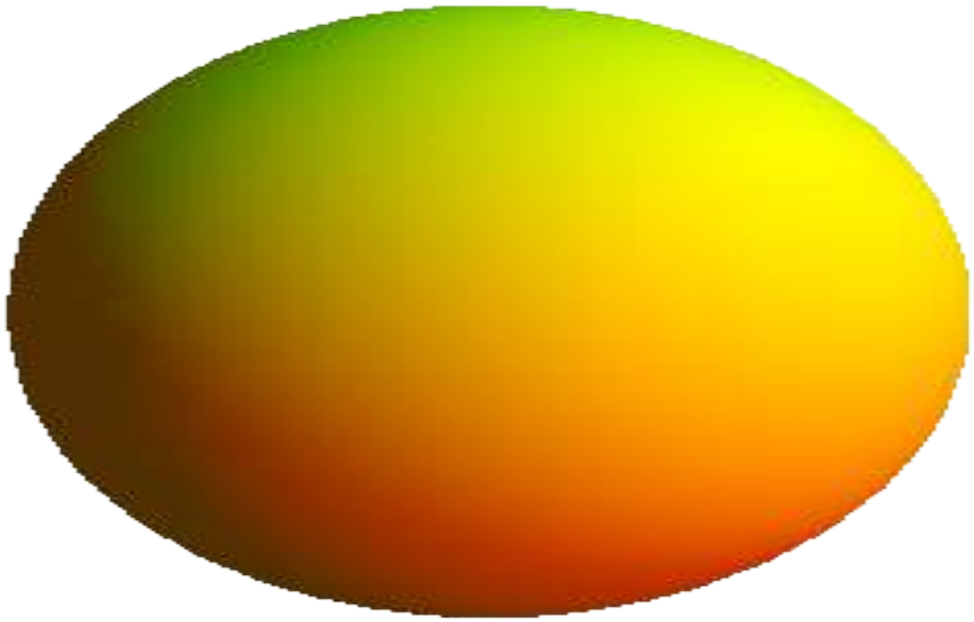
If it is strong enough, the chameleon field will settle to the minimum corresponding to the inner density, be massive over most of the source and the long range force it mediates will be suppressed. The interpolation between outer and inner values will happen over a thin-shell, further suppressing the intensity of the field outside the source. I.e. the source will be **screened**.

Otherwise the chameleon will change throughout the source, i.e. the shell will be thick, and the field won't be massive enough inside the source to hide from local tests, i.e. the source will be **unscreened**.

Something observable?

Viable screening theories make novel predictions for local gravitational experiments, offering a rich spectrum of testable predictions, from laboratory to extra-galactic scales.

And what happens if we go beyond spherical symmetry and/or static configurations?



We expect monopole, dipole, etc..radiation in dynamical configurations, e.g. different rates of energy loss in binary systems

… more on this at the end of the lecture !

f(R) Gravity

f(R) gravity

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[R + f(R) \right] + \int d^4x \sqrt{-g} \mathcal{L}_m \left[\chi_i, g_{\mu\nu} \right]$$

(S.Carroll, V.Duvvuri, M.Trodden & M.S.Turner, Phys.Rev.D70 043528 (2004),
S.Capozziello, S.Carloni & A.Troisi, astro-ph/0303041)

$$\begin{cases} \left(1 + f_R\right) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R + f\right) + \left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu\right) f_R = \frac{T_{\mu\nu}}{M_P^2} \\ \nabla_\mu T^{\mu\nu} = 0 \end{cases}$$

$$f_R \equiv \frac{df}{dR}$$

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The Einstein equations are fourth order !

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2004),

The following term:

$$g^{\mu\nu} \delta R_{\mu\nu} = \nabla_\mu \nabla_\nu \left(-\delta g^{\mu\nu} + g^{\mu\nu} g_{\alpha\beta} \delta g^{\alpha\beta} \right)$$

in GR is a boundary term, but in this new action it comes multiplied by f_R and gives rise to:

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The Einstein equations are fourth order !

The trace-equation becomes:

dynamical !

$$(1-f_R)\,R + 2f - 3\Box f_R = \frac{T}{M_P^2}$$

Background Cosmology

Background

Hence there is an additional dynamical DOF, dubbed the scalaron, which obey the following eom:

$$\Box f_R = \frac{1}{3}\left(R + 2f - Rf_R\right) - \frac{\kappa^2}{3}(\rho - 3P) \equiv \frac{\partial V_{\rm eff}}{\partial f_R}$$

By design, the f(R) theories we consider must have $f \ll R$ and $f_R \ll 1$ at high curvatures to be consistent with our knowledge of the high redshift universe.

In this limit, the extremum of the effective potential lies at the GR value $R = \kappa^2\left(\rho - 3P\right)$.

Whether this extremum is a minimum or a maximum is determined by the second derivative of the effective potential at the extremum:

$$m_{f_R}^2 \equiv \frac{\partial^2 V_{\rm eff}}{\partial f_R^2} = \frac{1}{3}\left[\frac{1+f_R}{f_{RR}} - R\right] \approx \frac{1+f_R}{3f_{RR}} \approx \frac{1}{3f_{RR}}$$

We will get back to this characteristic lengthscale of the model several times.

Einstein frame

Let's look at it from another angle! Introducing an auxiliary field we can write a dynamically equivalent action:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \, [(\Phi + f(\Phi)) + (1 + f_\Phi)(R - \Phi)] + \int d^4x \sqrt{-g} \, \mathcal{L}_{\rm m}[\chi_i, g_{\mu\nu}] \, , \qquad f_\Phi \equiv \frac{df}{d\Phi}$$

If $f_{\Phi\Phi} \neq 0$, the field equation for Φ gives R= Φ which reduces the above action to the original one.

T. Chiba, Phys. Lett. B 575 (2003) G. Magnano and L. M. Sokolowski, Phys. Rev. D 50, 5039 (1994)

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Now, let us perform a *conformal transformation*:

$$\tilde{g}_{\mu\nu} = e^{2\omega(x^\alpha)} g_{\mu\nu} \qquad \text{with} \qquad e^{-2\omega}(1+f_R) = 1$$

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Defining $\phi \equiv \frac{2\omega}{\beta\kappa}$, we get the following action:

$$\tilde{S} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\,\tilde{R} + \int d^4x\sqrt{-\tilde{g}}\,\left[-\frac{1}{2}\tilde{g}^{\mu\nu}(\tilde{\nabla}_\mu\phi)\tilde{\nabla}_\nu\phi - V(\phi)\right] + \int d^4x\sqrt{-\tilde{g}}\,e^{-2\beta\kappa\phi}\mathcal{L}_{\rm m}[\chi_i, e^{-\beta\kappa\phi}\tilde{g}_{\mu\nu}]$$

for a scalar field with the following potential:

$$V(\phi) = \frac{1}{2\kappa^2}\frac{Rf_R - f}{(1+f_R)^2}$$

$$\beta = \sqrt{\frac{2}{3}}$$

T. Chiba, Phys. Lett. B 575 (2003) G. Magnano and L. M. Sokolowski, Phys. Rev. D 50, 5039 (1994)

Einstein frame

This is the Einstein frame ! The gravitational action has the standard Einstein-Hilbert form,
there is an explicit additional scalar DOF which is coupled to matter.
This frame is physically equivalent to the one of the original action, i.e. the Jordan frame. The latter is defined by the fact that matter fields follow geodesics of the metric. f(R) has a universal coupling
and hence allows for a uniquely defined Jordan frame.

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Designer approach

From Einstein eqs.:

$$\frac{\ddot{a}}{a} - (1 + f_R)\mathcal{H}^2 + a^2 \frac{f}{6} + \frac{1}{2}\ddot{f}_R = -\frac{\kappa^2}{6} a^2 (\rho + 3P)$$

in conformal time!

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and fix the desired expansion history to that of a flat universe containing matter, radiation and dark energy:

$$E = \Omega_m a^{-3} + \Omega_r a^{-4} + \rho_{\rm eff}/\rho_c^0 \equiv E_m + E_r + E_{\rm eff}$$

$$E_{\rm eff} = (1 - \Omega_m - \Omega_r) \exp\left[-3 \ln a + 3 \int_a^1 w_{\rm eff}(a) d \ln a\right]$$

$$\rho_c^0 \equiv 3H_0^2/\kappa^2$$

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All standard background functions in the Friedmann equation can be expressed in terms of E and its derivatives. The resulting equation is a 2nd order ODE for f(R) in terms of derivatives wrt ln(a):

$$y'' - \left(1 + \frac{E'}{2E} + \frac{R''}{R'}\right)y' + \frac{R'}{6H_0^2 E}y = -\frac{R'}{H_0^2 E}E_{\rm eff}$$

Designer approach

In order to set the initial conditions, let us consider the general and the particular solutions at early times, when the effects of the effective dark energy on the expansion are negligible. At a certain early value a_i , the homogeneous part of the eq. is satisfied by a power law ansatz $y \propto a_i^p$. Substituting this ansatz in and solving the quadratic equation for p yields:

$$p_\pm = \frac{1}{2}\left(-b \pm \sqrt{b^2 - 4c}\right)$$

$$b = \frac{7 + 8r_i}{2(1 + r_i)} \ , \quad c = -\frac{3}{2(1 + r_i)} \ , \quad r_i = \frac{a_{eq}}{a_i}$$

The decaying mode solution corresponding to p_- leads to a large f(R) at early times which makes it unacceptable, and we set its amplitude to zero. The particular solution at a_i can be found by substituting $y_p = A_p E_{\rm eff}(a_i)$ in the ODE. One then finds:

$$A_p = \frac{-6c}{-3w'_{\rm eff} + 9w^2_{\rm eff} + (18 - 3b)w_{\rm eff} + 9 - 3b + c}$$

Put together, the initial conditions at a_i are

$$y_i = Ae^{p_+ \ln a_i} + y_p$$
$$y'_i = p_+ Ae^{p_+ \ln a_i} - 3[1 + w_{\rm eff}(a_i)]y_p$$

and A is the remaining arbitrary constant that can be used to parametrize different f(R) models with the same expansion history.

Designer approach

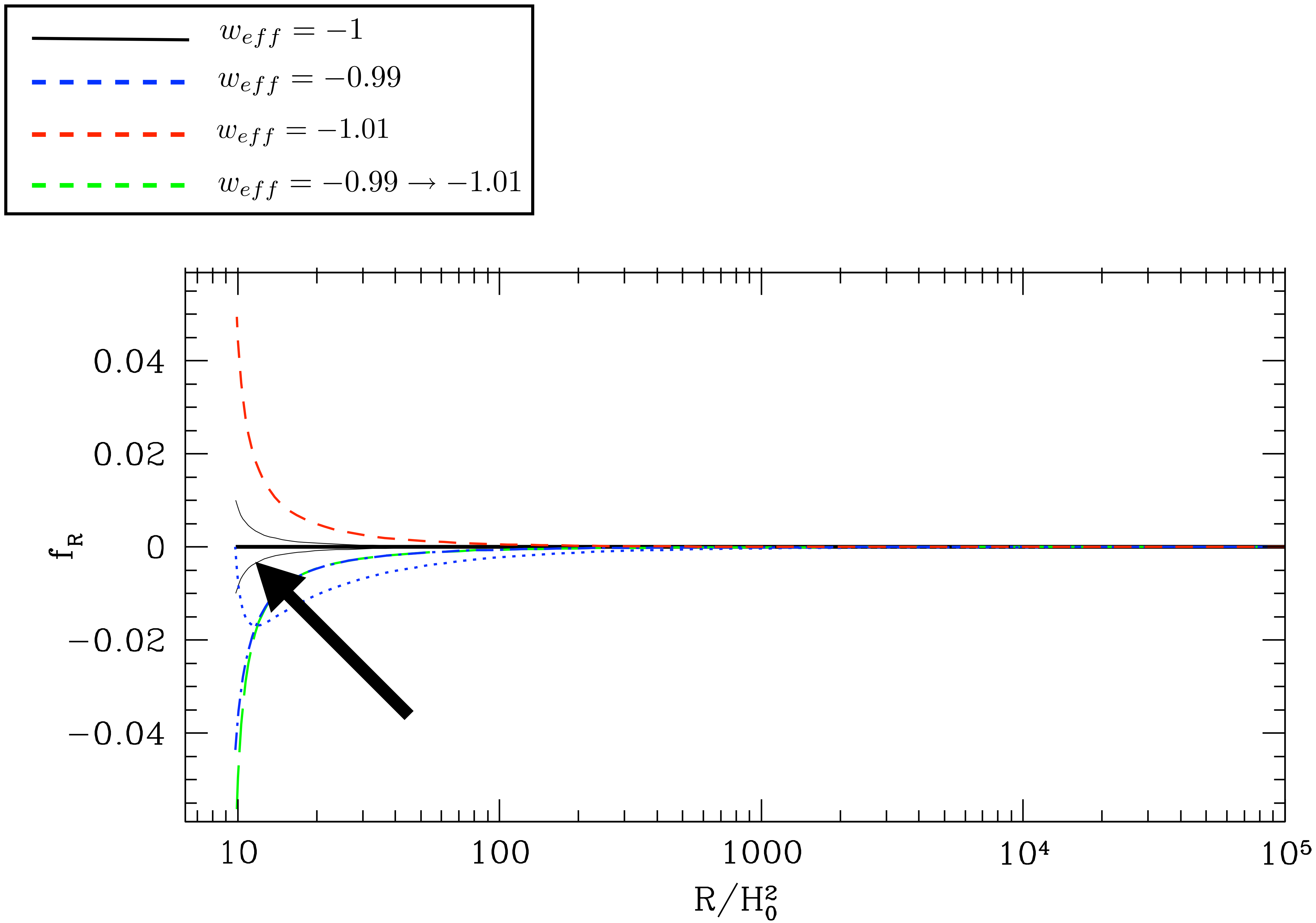
There is a family of f(R) models for each expansion history.

As a label for each family, it is common to use the boundary condition at a=0, B0, defined as the today value of:

$$B = \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

Which is the characteristic lengthscale of the scalaron in units of the horizon scale.

Designer approach



Viability conditions

L. Pogosian, A. Silvestri, Phys. Rev. D77 (2008) 023503

Conditions of viability:

* $f_{RR} > 0$ to have a stable high-curvature regime
* $1 + f_R > 0$ to have a positive effective Newton constant
* $f_R < 0$ negative, monotonically increasing function of R that asymptotes to zero from below

----

* $|f_R^0| \leq 10^{-6}$ must be small at recent epochs to pass local tests of gravity

Dolgov, Kawasaki, Phys.Lett.B 573 (2003), I.Navarro, K. van Acoyelen, JCAP 0702:022, 2007
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Take home message

Generally theories beyond ΛCDM have enough freedom to reproduce 'any' desired expansion history, being higher order in nature.

In other words, at the level of background expansion history there is a degeneracy among different approaches to the phenomenon of cosmic acceleration.

In the past decade it has become increasingly clear that we need to go beyond geometrical probes in order to disentangle the theoretical landscape of cosmic acceleration.

In other words, the growth of structure is expected to be a powerful testbed,
in particular through combinations of different observables.

Structure Formation

LSS in LCDM

Now let us close the system with the fluid equations for matter. Let us focus on <u>CDM</u> since we are interested in <u>late times clustering</u>:

$$\dot{\delta} = -\theta + 3\dot{\Phi}$$
$$\dot{\theta} = -\mathcal{H}\theta + k^2\Psi$$

$\xrightarrow{k \gg aH}$ $\ddot{\delta} + \mathcal{H}\delta + k^2\Psi = 0$ $\longrightarrow$ $\ddot{\delta} + \mathcal{H}\delta - \frac{a^2}{2M_P^2}\rho\delta = 0$

$$\ddot{\delta} + \mathcal{H}\delta - \frac{3}{2}\Omega_m(a)\delta = 0$$

scale independent !

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the growing mode goes like:

$$D_1(a) = \frac{5\Omega_m}{2}\frac{H(a)}{H_0}\int_0^a \frac{da'}{\left[a'\frac{H(a')}{H_0}\right]^3}$$

which gives a <u>consistency relation btw expansion history and growth of structure</u>

The growth rate:

$$f \equiv \frac{d\ln\delta}{d\ln a} \sim \Omega_m(a)^{6/11}$$

LSS in a nutshell

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LCDM:

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- relativistic and non-relativistic probes respond to the same metric potential
- the growth of structure is scale-independent

LSS in f(R)

Let us start from the Einstein equations in f(R) gravity:

$$(1 + f_R)\, R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\, (R + f) + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu)\, f_R = \frac{T_{\mu\nu}}{M_P^2}$$

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One does the usual expansion to linear order in scalar perturbations. The tricky term is the last one on the l.h.s.. In the 00 case it gives:

$$\begin{aligned}
&\delta\left[\left(\delta^0_0\Box-\nabla^0\nabla_0\right)f_R\right]\\
&=\delta\left(\nabla^i\nabla_i f_R\right)\\
&=\delta\left(g^{ij}\nabla_j\nabla_i f_R\right)\\
&=\delta\left[\left(g^{ij}\partial_i\partial_j-g^{ij}\Gamma^\alpha_{ij}\partial_\alpha\right)f_R\right]\\
&=\delta\left[\left(g^{ij}\partial_i\partial_j-g^{ij}\Gamma^0_{ij}\partial_0-g^{ij}\Gamma^k_{ij}\partial_k\right)f_R\right]\\
&=g^{(0)ij}\partial_i\partial_j\delta f_R-\delta g^{ij}\Gamma^{(0)0}_{ij}\partial_0 f_R-g^{(0)ij}\Gamma^{(0)0}_{ij}\partial_0\delta f_R\\
&=-\frac{k^2}{a^2}\delta f_R-\frac{3}{2}\mathcal{H}\dot{\delta f}_R+\frac{\dot{f}_R}{a^2}\left(6\mathcal{H}\Psi+3\dot{\Phi}\right)
\end{aligned}$$

$F \equiv 1 + f_R$

$\mathbf{E_{00}}$:

$$\left[k^2\Phi + 3\mathcal{H}\left(\dot{\Phi} + \mathcal{H}\Psi\right)\right] + \frac{3}{2}\dot{\mathcal{H}}\frac{\delta f_R}{F} - \frac{3}{2}\mathcal{H}\frac{\delta \dot{f}_R}{F}$$
$$-\frac{k^2}{2}\frac{\delta f_R}{F} + \left(3\dot{\Phi} + 6\mathcal{H}\Psi\right)\frac{\dot{F}}{2F} = -\frac{a^2}{2M_P^2}\frac{\rho}{F}\delta$$

$\mathbf{E_{0i}}$:

$$\left[\dot{\Phi} + \mathcal{H}\Psi\right] - \frac{1}{2}\frac{\delta \dot{f}_R}{F} + \frac{1}{2}\mathcal{H}\frac{\delta f_R}{F} + \frac{1}{2}\frac{\dot{F}}{F}\Psi = \frac{a^2}{2M_P^2}\frac{(\rho + P)}{F}\theta$$

$\mathbf{E_{ii}}$:

$$\left[2\ddot{\Phi} + 2\mathcal{H}\left(\dot{\Psi} + 2\dot{\Phi}\right) + 2\left(2\dot{\mathcal{H}} + \mathcal{H}^2\right)\Psi + \frac{2k^2}{3}(\Phi - \Psi)\right]$$
$$+\left(2\mathcal{H}b^2 + \dot{\mathcal{H}}\right)\frac{\delta f_R}{F} - \frac{\delta \ddot{f}_R}{F} - \mathcal{H}\frac{\delta \dot{f}_R}{F} - \frac{2k^2}{3}\frac{\delta f_R}{F} + 2\frac{\ddot{F}}{F}\Psi$$
$$+\frac{\dot{F}}{F}\left(2\dot{\Phi} + 2\mathcal{H}\Psi + \dot{\Psi}\right) = \frac{1}{F}\frac{a^2}{M_P^2}\delta P$$

$\mathbf{E_{ij}}$:

$$k^2(\Phi - \Psi) - k^2\frac{\delta f_R}{F} = \frac{3a^2}{2M_P^2}\frac{(\rho + P)}{F}\sigma\,.$$

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$\mathbf{E_{00}}$:

$$\left[k^2\Phi + 3\mathcal{H}\left(\dot{\Phi} + \mathcal{H}\Psi\right)\right] + \frac{3}{2}\dot{\mathcal{H}}\frac{\delta f_R}{F} - \frac{3}{2}\mathcal{H}\frac{\delta \dot{f}_R}{F}$$

$$-\frac{k^2}{2}\frac{\delta f_R}{F} + \left(3\dot{\Phi} + 6\mathcal{H}\Psi\right)\frac{\dot{F}}{2F} = -\frac{a^2}{2M_P^2}\frac{\rho}{F}\delta$$

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Poisson eq.:

$$k^2\Phi - k^2\frac{\delta f_R}{F} + \frac{3}{2}\left[\left(\dot{\mathcal{H}} - \mathcal{H}^2\right)\frac{\delta f_R}{F} + \left(\dot{\Phi} + \mathcal{H}\Psi\right)\frac{\dot{f}_R}{F}\right] = -\frac{a^2}{2M_P^2}\rho\Delta$$

$\mathbf{E_{ij}}$:

$$k^2\left(\Phi - \Psi\right) - k^2\frac{\delta f_R}{F} = \frac{3a^2}{2M_P^2}\frac{(\rho + P)}{F}\sigma\,.$$

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entering the QS regime!

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On Quasi-Static approximation

Often employed on sub-horizon scales. It significantly simplifies the work because it reduces the Einstein equations, and any equation for additional scalar d.o.f., to algebraic relations in Fourier space. What does it effectively correspond to?
Is it always a good approximation?

in LCDM

- sub-horizon scales: $k \gg aH$

↓

- time derivatives of metric potentials negligible w.r.t. space derivatives

in DE/MG

- sub-horizon scales: $k \gg aH$

and

- time derivatives negligible w.r.t. space derivatives for both metric potentials and additional scalars, i.e.

$$\delta\ddot{\phi} \ll c_s^2 k^2 \delta\phi$$

LSS in f(R)

$$k^2\Psi = -\frac{a^2}{2M_P^2}\frac{1}{F}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{F}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{F}}\rho\Delta$$

$$\frac{\Phi}{\Psi} = \frac{1+2\frac{k^2}{a^2}\frac{f_{RR}}{F}}{1+4\frac{k^2}{a^2}\frac{f_{RR}}{F}}$$

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time and scale dependent
rescaling of Newton constant

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time and scale dependent rescaling of Newton constant

$$\frac{k^2}{a^2}\frac{f_{RR}}{F} = \frac{k^2}{a^2}\frac{1}{m^2}$$

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β

$$\frac{G_{\rm eff}}{G} = \frac{1+\frac{4}{3}\frac{k^2}{a^2m^2}}{1+\frac{k^2}{a^2m^2}} \qquad \frac{\Phi}{\Psi} = \frac{1+\frac{2}{3}\frac{k^2}{a^2m^2}}{1+\frac{4}{3}\frac{k^2}{a^2m^2}}$$