EXERCISE SET Lecture 1

1. **Λ-matter equality and acceleration**
   By assuming a flat universe, with only matter (non-relativistic) and dark energy (in the form of a cosmological constant, Λ), find the redshift $z_\Lambda = 1/a_\Lambda - 1$ of equality between matter and dark energy, i.e. the redshift at which the two fractional energy densities are equal. [Use the Friedmann equation expressed in terms of the fractional energy densities]

   Still assuming these are the two dominant components, calculate the redshift $z_a = 1/a_a - 1$, at which acceleration kicks in.

2. **Meszaros equation**
   Writing the Friedmann equation as follows

$$\mathcal{H}^2 = \frac{H_0^2 \Omega_m^2}{\Omega_r} \left(\frac{1}{y} + \frac{1}{y^2} \right) , \tag{1}$$

   where $\mathcal{H} = aH$ is the Hubble parameter in conformal time, show that the equation for the evolution of matter density perturbation on sub-horizon scales

$$\delta_m'' + \mathcal{H}\delta_m' - 4\pi G a^2 \rho_m \delta_m = 0 \, , \tag{2}$$

   (where a prime denotes derivation with respect to conformal time), can be written as

$$\frac{d^2\delta_m}{dy^2} + \frac{2+3y}{2y(1+y)} \frac{d\delta_m}{dy} - \frac{3}{2y(1+y)} \delta_m = 0 \, , \tag{3}$$

   known as the Meszarós equation. Verify that the solutions are

$$\begin{aligned} \delta_m^1 &\propto 2 + 3y \\ \delta_m^2 &\propto (2+3y) \ln \frac{\sqrt{1+y}+1}{\sqrt{1+y}-1} - 6\sqrt{1+y} \, . \end{aligned} \tag{4}$$

   Determine how δ_m grows with y for $y \ll 1$ (RD) and $y \gg 1$ (MD).