EXERCISE SET Lecture 3

1. **Growth factor**
   Focus on a Universe with only cold dark matter and dark energy, with a varying equation of state $w(a)$

$$\frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3}\left[1 + \frac{\Omega_w(a)}{\Omega_m(a)}\right] . \tag{1}$$

   Let us define the growth factor as $f \equiv \frac{d\ln\delta}{d\ln a}$. Starting from the equation for the growth of structure in cosmic time

$$\frac{d^2\delta}{dt^2} + 2H\frac{d\delta}{dt} - 4\pi\rho_m\delta = 0 \tag{2}$$

   derive an equation for the growth factor f. Neglect terms of order f^2 and solve the equation by quadratures. Now substitute Ω_m in favor of Ω_w.
   Finally, assume dark energy comes in the form of the cosmological constant and show that $f \sim \Omega_m(a)^{6/11}$.

2. **Growth index and rescaled Newton constant**
   Assuming

$$f \equiv \frac{d\ln\delta}{d\ln a} = (\ln\delta)' = \Omega_m(a)^\gamma \text{ and } k^2\Phi = -\frac{a^2}{2M_P^2}\mu\,\rho\,\delta \tag{3}$$

   use the equation for the growth of structure

$$\delta'' + \left(2 + \frac{H'}{H}\right)\delta' + k^2\Phi = 0 \tag{4}$$

   to obtain an expression for the rescaling of the Newton constant μ in terms of the growth index γ.