EXERCISE SET Lecture 4

1. **Background in EFT of DE**
   Varying the EFT action for DE (in a slightly different convention than the one showed in the lecture) w.r.t. the metric, the Einstein equations in terms of the background EFT operators only, read:

$$\left(\Omega G_{\mu\nu} - \nabla_\mu \nabla_\nu \Omega + g_{\mu\nu}\right) M_P^2 + \left(c g^{00} + \Lambda\right) g_{\mu\nu} - 2c\delta^0_\mu \delta^0_\nu = T_{\mu\nu}\,. \tag{1}$$

   Using the trace and the 00 component of these equations, find algebraic equations for the EFT functions c, Λ. Consider that on the background (for a flat universe)

$$G_{00} = 3H^2\,, \;\; R = 12H^2 + 6\dot{H}\,, \;\; T^\mu_\nu = \mathrm{diag}(-\rho, P, P, P)\,. \tag{2}$$

2. **Mapping into EFT of DE**
   Using the following:

$$n_\mu = -\gamma \phi_{;\mu}\,, \;\; \gamma = \frac{1}{\sqrt{-X}}\,, \;\; K_{\mu\nu} = h^\sigma_\mu n_{\nu;\sigma}\,, \;\; \dot{n}_\mu = n^\nu n_{\mu;\nu}\,, \tag{3}$$

   where $X \equiv \phi^{;\mu}\phi_{;\mu}$ and a semicolon indicates a covariant derivative, derive the following expression for the second derivative of the scalar field:

$$\phi_{;\mu\nu} = -\gamma^{-1}\left(K_{\mu\nu} - n_\mu \dot{n}_\nu - n_\nu \dot{n}\mu\right) + \frac{\gamma^2}{2}\phi^{;\lambda} X_{;\lambda} n_\mu n_\nu\,. \tag{4}$$

   By taking the trace of the expression above, show that:

$$\Box\phi = -\gamma^{-1} K + \frac{1}{2}\phi^{;\mu} X_{;\mu}/X\,. \tag{5}$$