Exercises for the Course on Inflation

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1. Solve for $a(t)$ in the closed radiation-dominated cosmology as a function of maximum size $a_{\rm max}$. How much Λ should we add to have static universe?

2. Derive the required field range to get $N_e = 60$ when inflaton potential is linear $V \propto \phi$.

3. The normalization of the primordial scalar power is $\Delta_\zeta^2 = \frac{k^3}{2\pi^2}\left\langle \zeta_{\vec k}\zeta_{-\vec k}\right\rangle' \simeq 2\times 10^{-9}$. For what value of m the $m^2\phi^2$ model would give this?

4. Derive equations (2.12) and (2.27) of Maldacena's paper 0210603.

5. What are the predictions of the $V \propto \phi$ model for r and n_s? Is it a viable model of the observed universe?

6. a) Calculate $\langle x^4\rangle$ for a harmonic equation. b) Calculate the same for an anharmonic oscillator with interaction $-V = -\lambda X^4$, to linear order in λ. Hint: Use perturbation theory:

$$\langle X^4\rangle = \langle 0|\bar T\left[e^{i\int_{-\infty(1+i\epsilon)}^0 dtH_I}\right] X_I^4(0) T\left[e^{-i\int_{-\infty(1-i\epsilon)}^0 dtH_I}\right]|0\rangle \tag{1}$$

   where T and $\bar T$ indicate time-ordering and anti-time-ordering, respectively, and $H_I = \lambda X_I^4$. The interaction-picture operator is:

$$X_I(t) = \frac{1}{\sqrt{2\omega}}(ae^{-i\omega t} + a^\dagger e^{i\omega t}). \tag{2}$$

7. $\varphi_L(t)$ is the following average of the inflation fluctuations in the $\zeta = 0$ gauge

$$\varphi_L(t) = \int_0^{a(t)/L} \frac{d^3\vec k}{(2\pi)^3}\varphi_{\vec k}, \qquad L \gg 1/H. \tag{3}$$

   (This is almost, but not quite, the average of $\varphi(t,\vec x)$ over a region of fixed physical radius L.) For a scale invariant spectrum $P_\varphi(k) = H^3/2k^3$ we get

$$\left\langle(\varphi_L(t_1) - \varphi_L(t_2))^2\right\rangle = D(t_1 - t_2). \tag{4}$$

   Calculate the diffusion constant D. If we discretize time as $t_n = n/H$, the above linearly growing correlation can be thought of as a random-walk with step size $\Delta\varphi = \sqrt{D/H}$. For what value of $\zeta_{\rm rms}$ do these quantum jumps become comparable with the classical rolling? This is the transition point to the eternal inflation phase, in which there is a finite probability for the inflation to never end. See Susskind's lectures in PiTP 2011.

8. Consider the potential $V(\phi) = ce^{\alpha\phi}$, with $\alpha \ll 1$. What is the maximum ϕ for which inflation is not eternal? What is the maximum N_e in the non-eternal phase?

9. Calculate the 3-point function of ζ if inflaton has a cubic interaction $\dot\varphi^3$. See astro-ph/0306122. Show that this 3-point function is suppressed in the squeezed limit. See 1503.08043 [hep-th] for another application of non-Gaussianities.

10. Derive eq. (14) of 1506.03447 [hep-th].

11. Some works on the initial patch problem: 1511.05143 [hep-th],1712.07352 [hep-th],1602.03520 [hep-th].