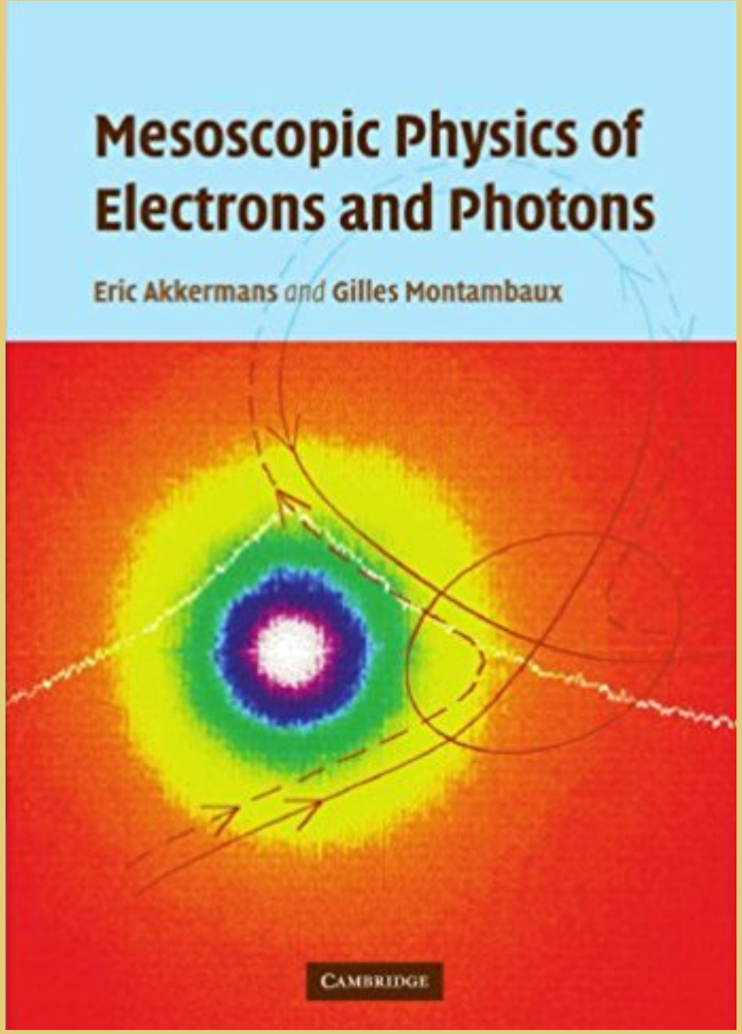
# Mesoscopic Physics of Photons

Eric Akkermans

School on Interaction of Light with Cold Atoms, Sept. 16-27, 2019, Sao Paulo, Brazil, ICTP-SAIFR/IFT-UNESP.

Based on *Mesoscopic physics of electrons and photons*, by Eric Akkermans and Gilles Montambaux, Cambridge University Press, 2007

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- Phase coherence and self-averaging: universal fluctuations.

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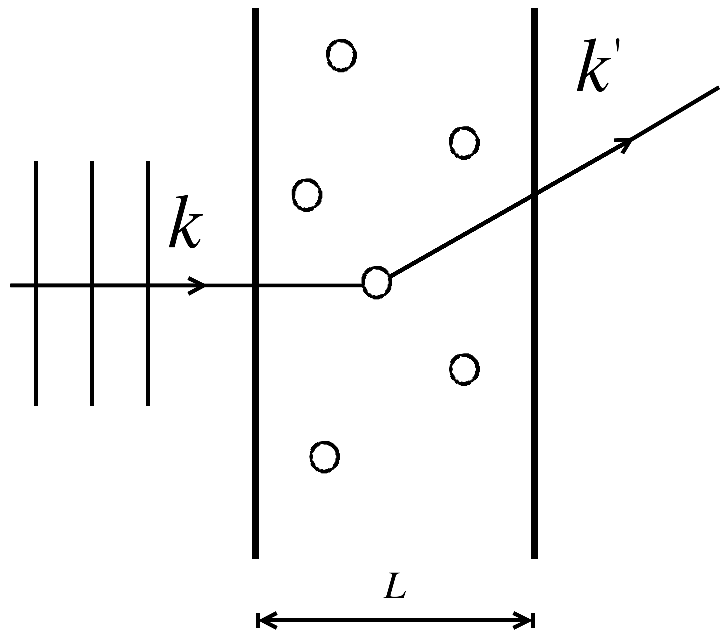
- The Aharonov-Bohm effect in disordered conductors.
- Phase coherence and effect of disorder.
- Average coherence: Sharvin$^2$ effect and coherent backscattering.
- Phase coherence and self-averaging: universal fluctuations.
- Classical probability and quantum crossings.

The framework :

*Multiple scattering of waves*

# *Multiple scattering of waves*



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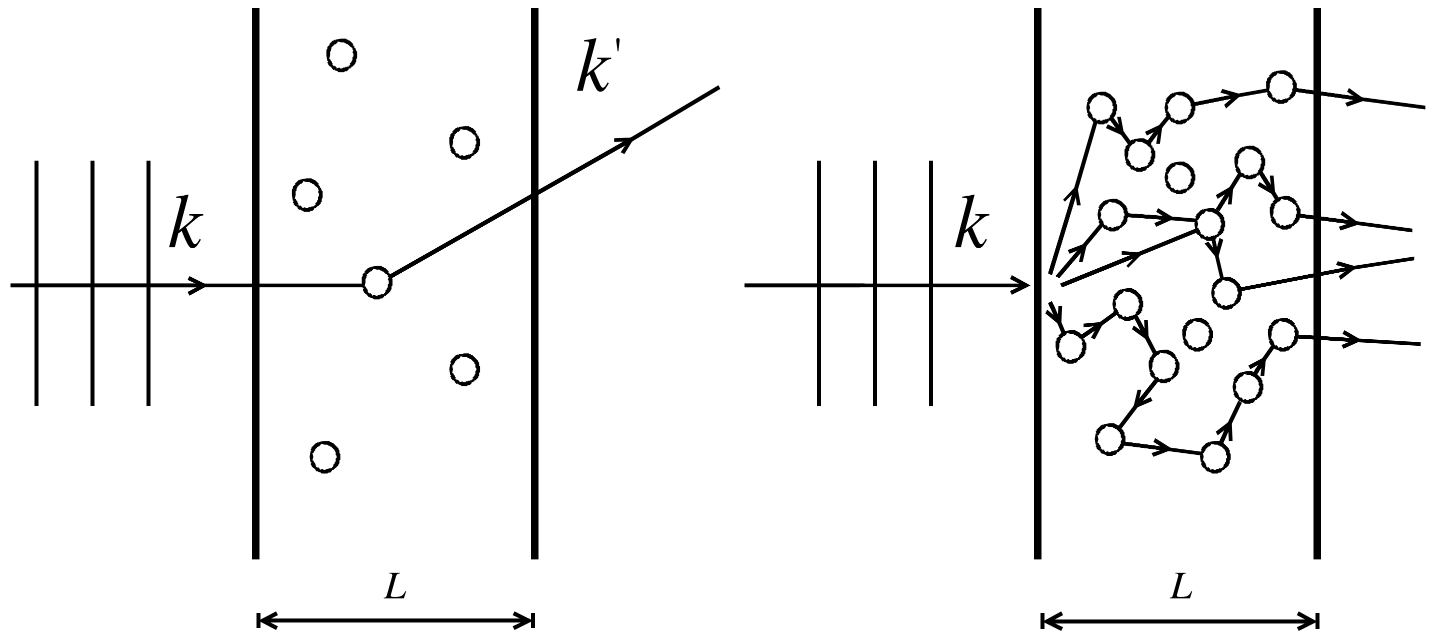

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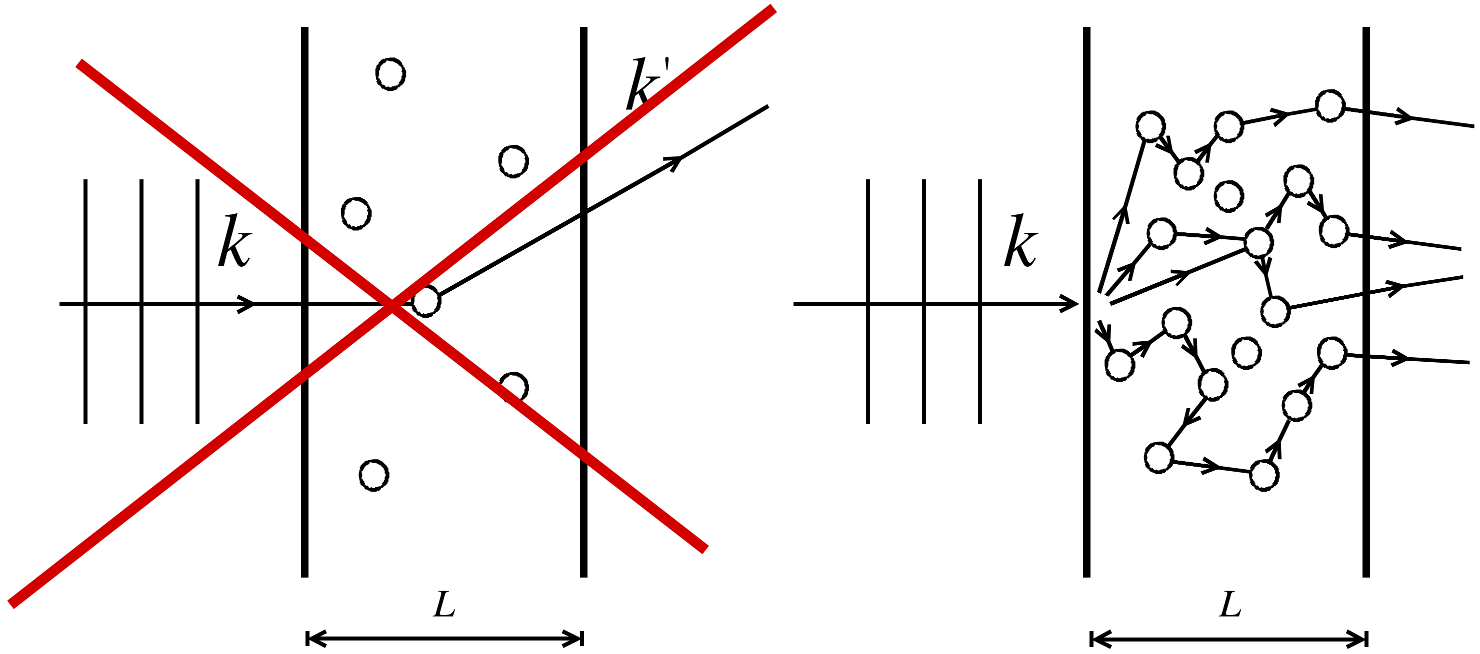

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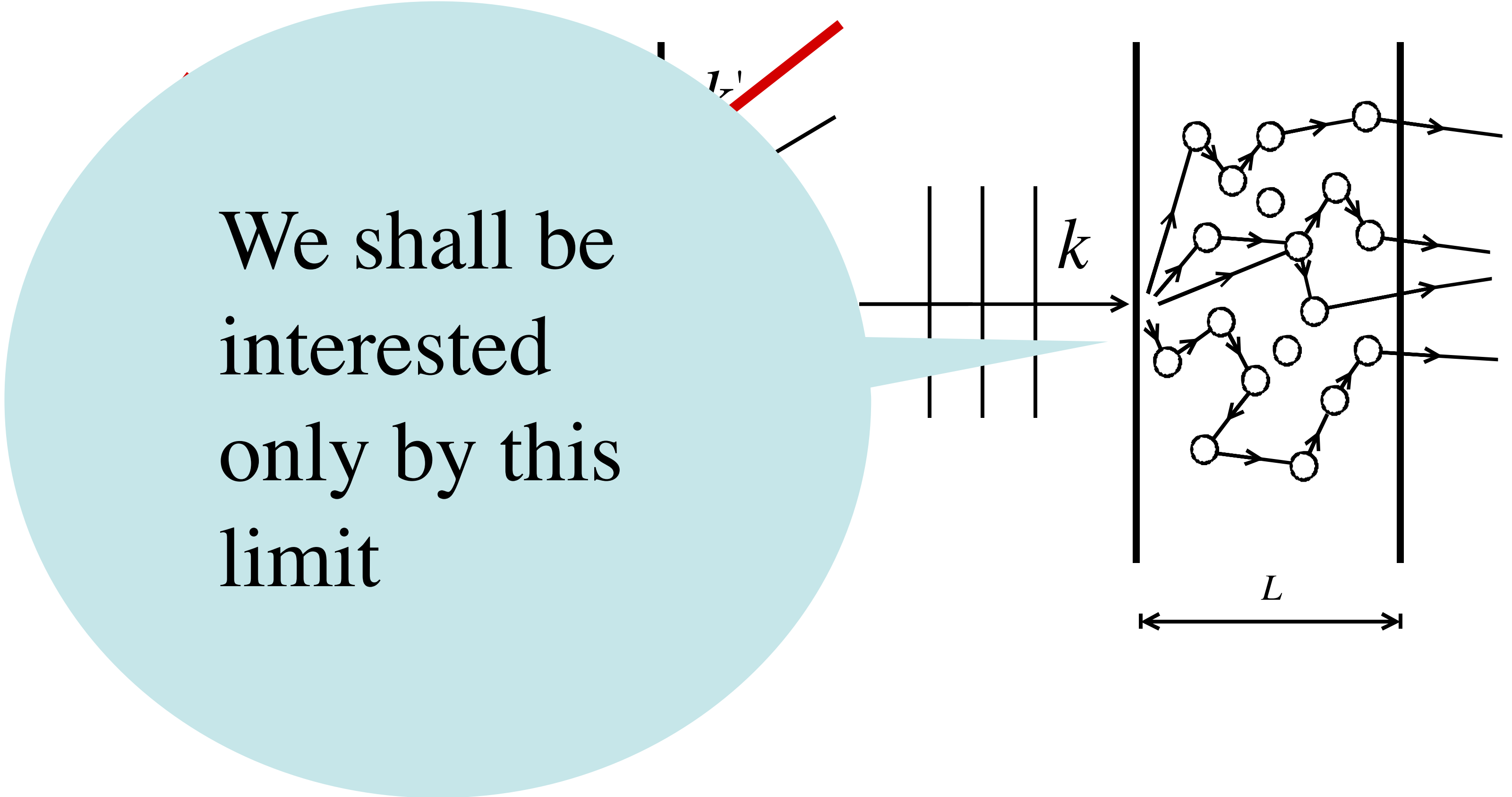


We shall be interested only by this limit

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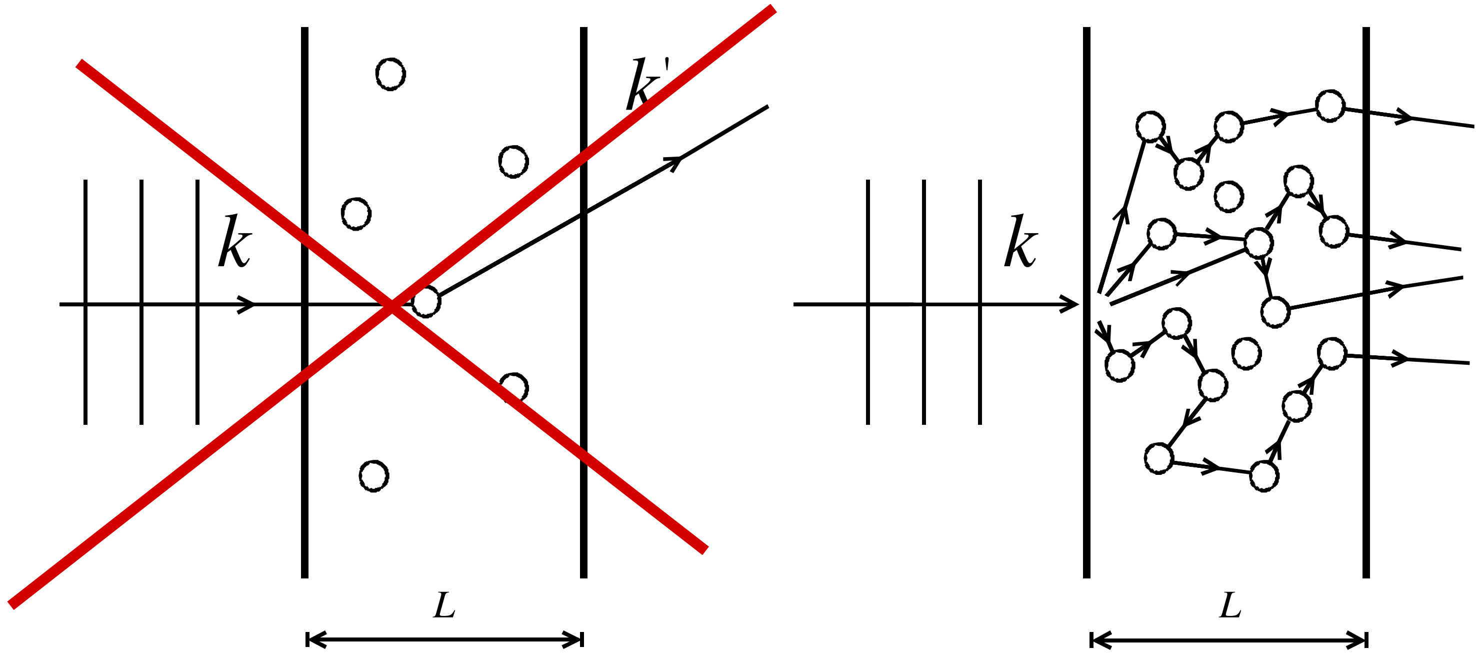


2 characteristic lengths:

Wavelength: $\lambda_F = k_F^{-1}$

Elastic mean free path: $l$

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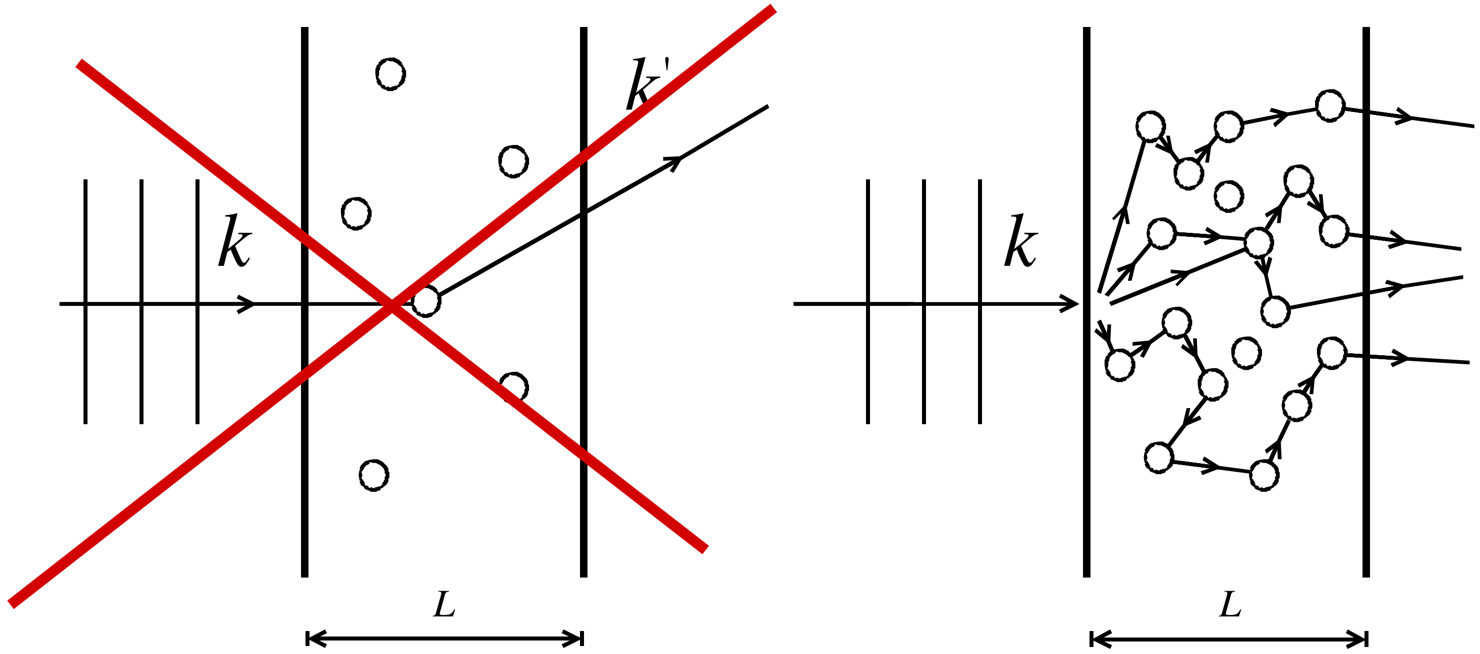


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Elastic mean free path: $l$

Weak disorder $\lambda_F \ll l$ : independent scattering events

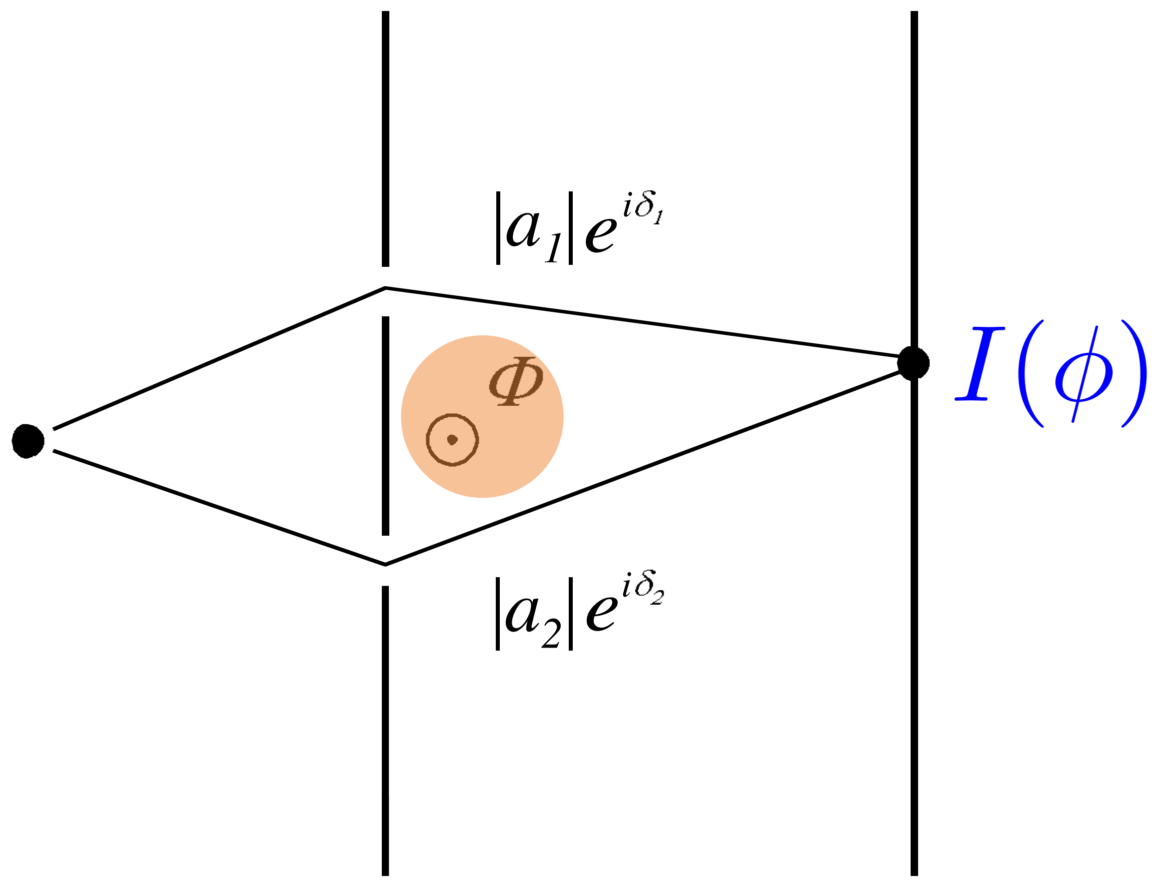
# A “canonical” mesoscopic effect

## The Aharonov-Bohm effect

Aharonov-Bohm (1959)

# Aharonov-Bohm effect

No magnetic field on the electrons : no Lorentz force and no orbital motion.



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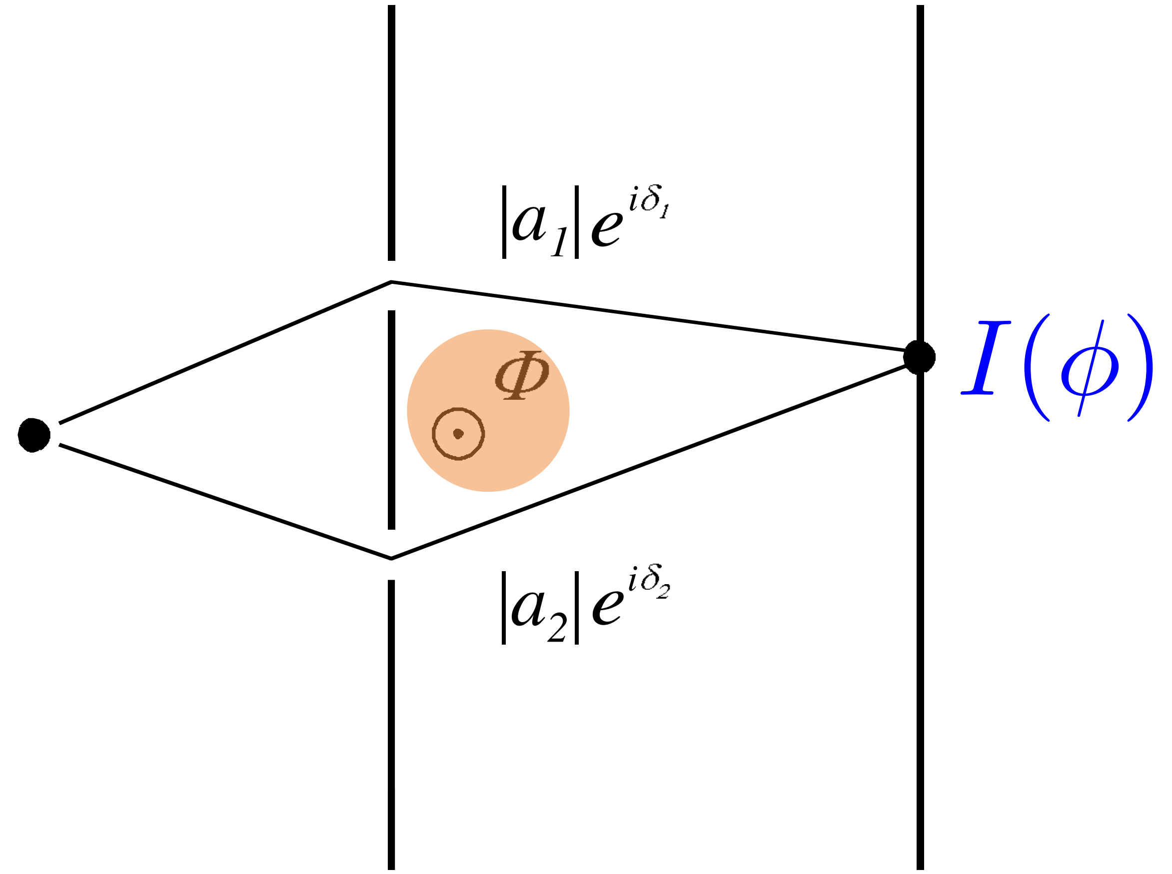


The quantum amplitudes $a_{1,2} = |a_{1,2}|e^{i\delta_{1,2}}$ have phases:

$$\delta_1 = \delta_1^{(0)} - \frac{e}{\hbar}\int_1 \mathbf{A}.d\mathbf{l} \quad \text{and} \quad \delta_2 = \delta_2^{(0)} - \frac{e}{\hbar}\int_2 \mathbf{A}.d\mathbf{l}$$

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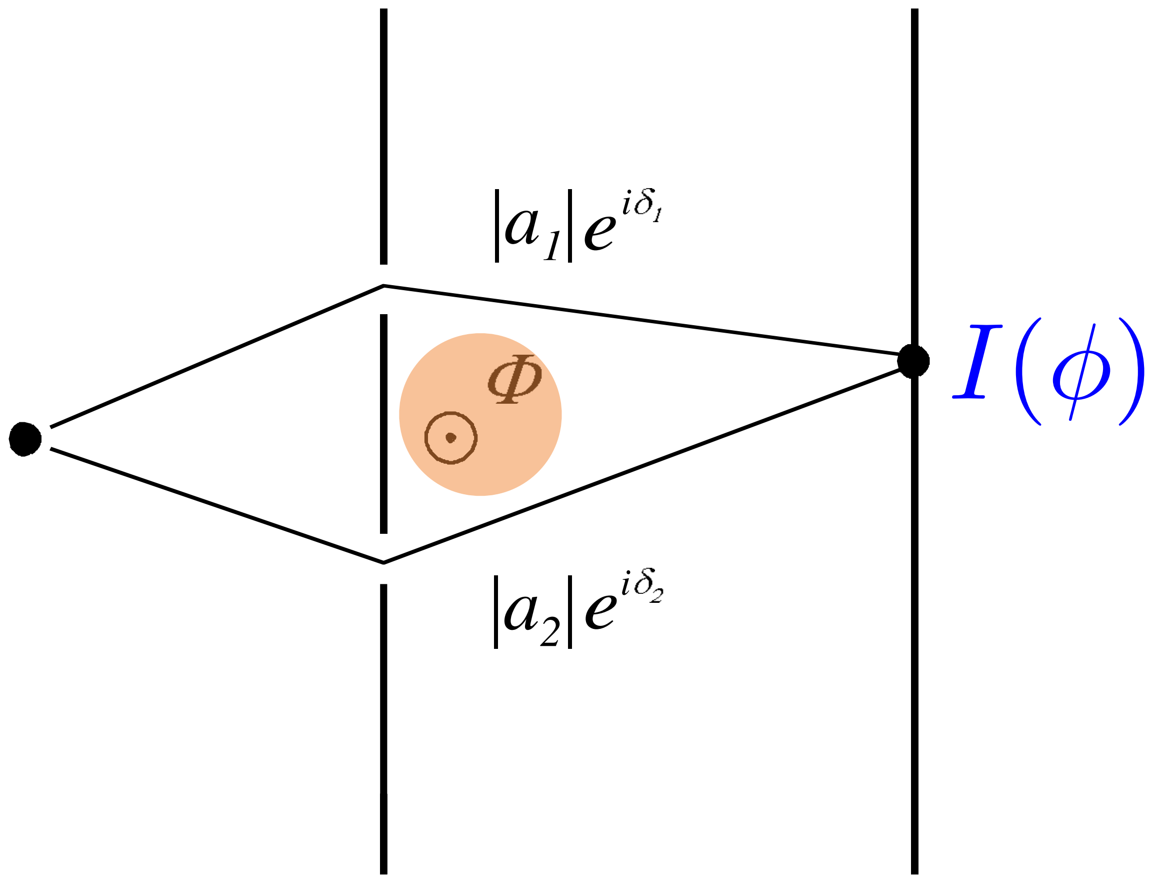
There is a continuous change of the state of interference:

Aharonov-Bohm effect (1959).

***Implementation in metals*** : the conductance $G(\phi)$ is the analog of the intensity.



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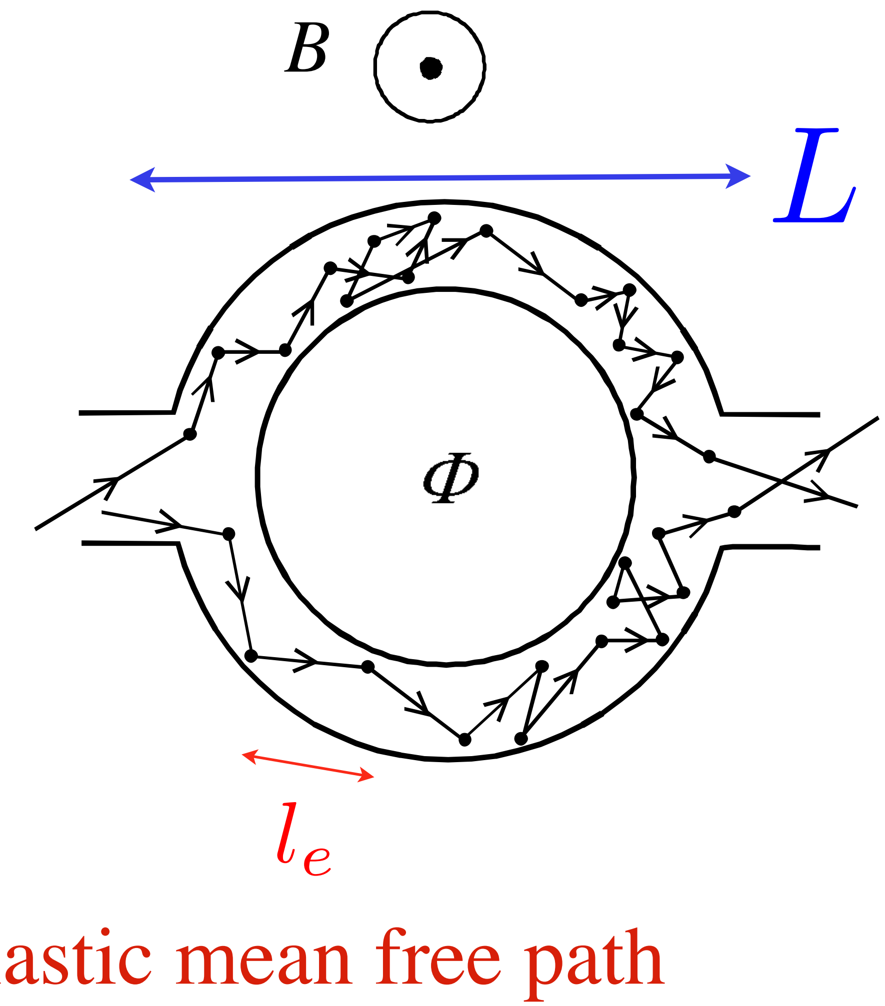


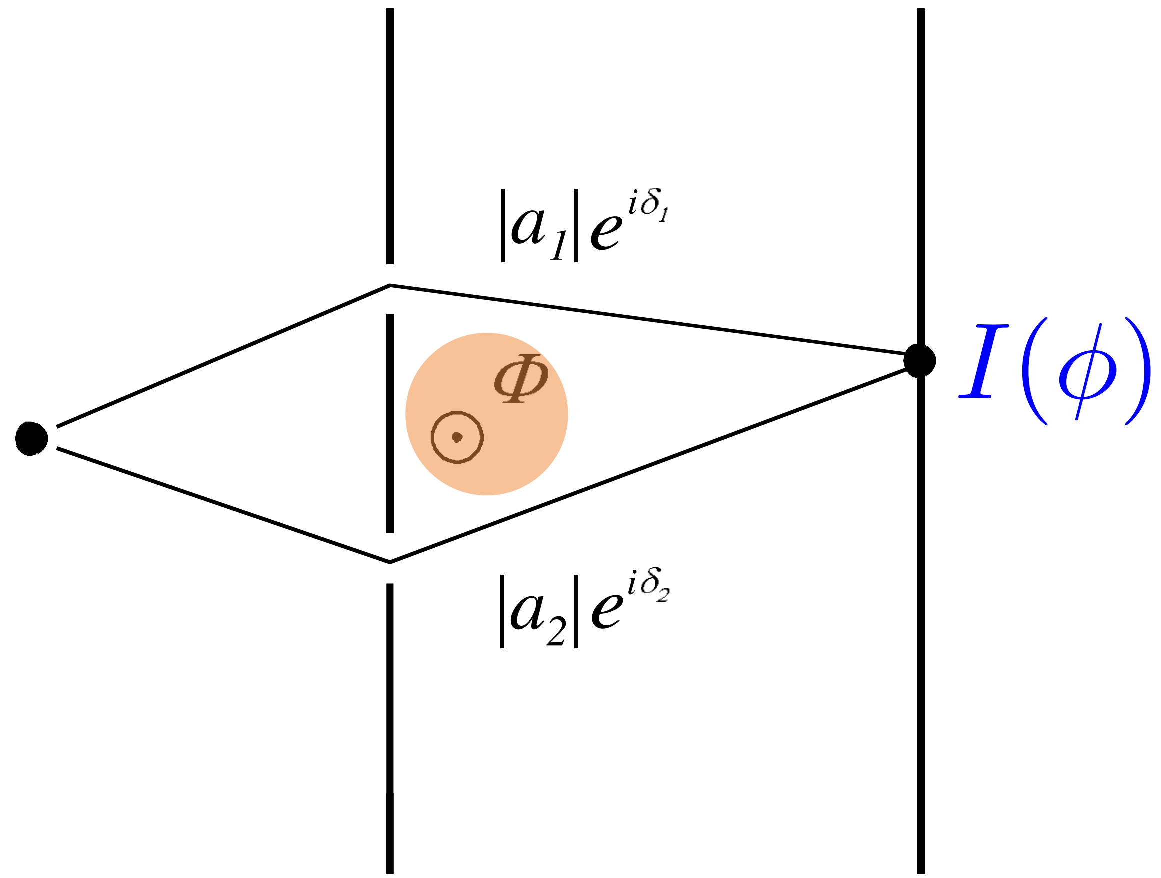

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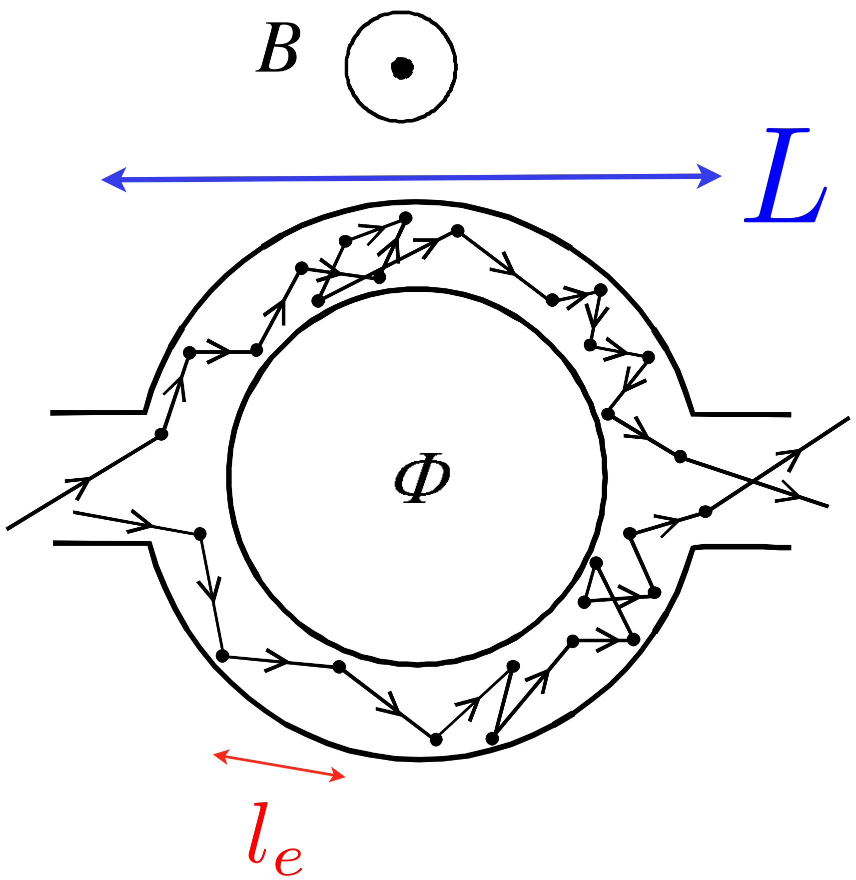


elastic mean free path

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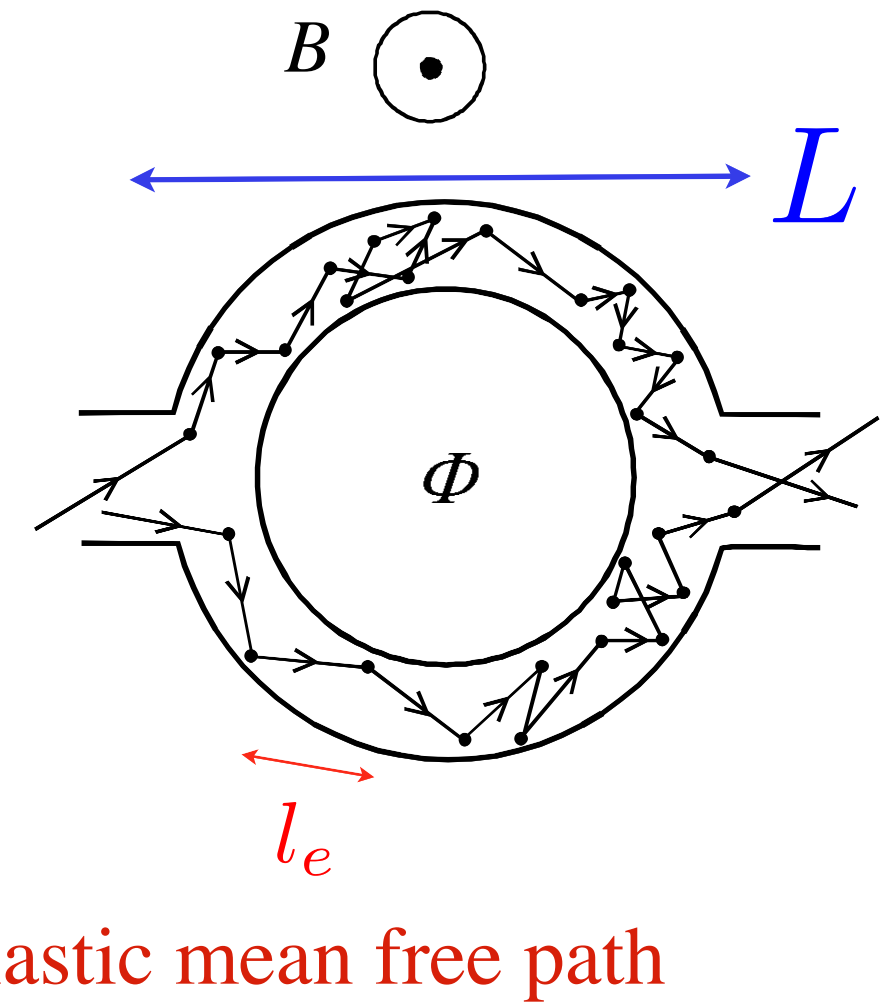


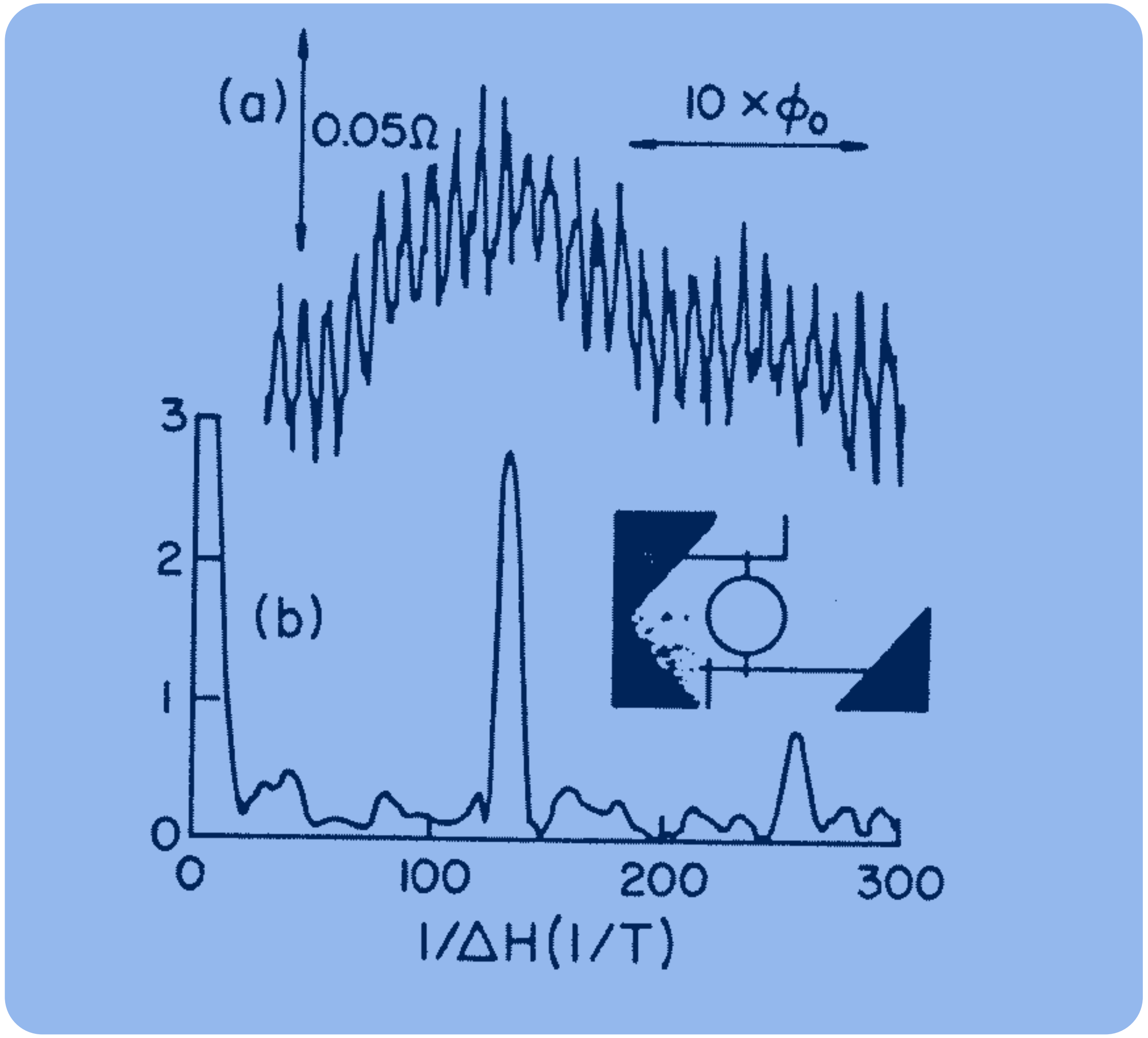

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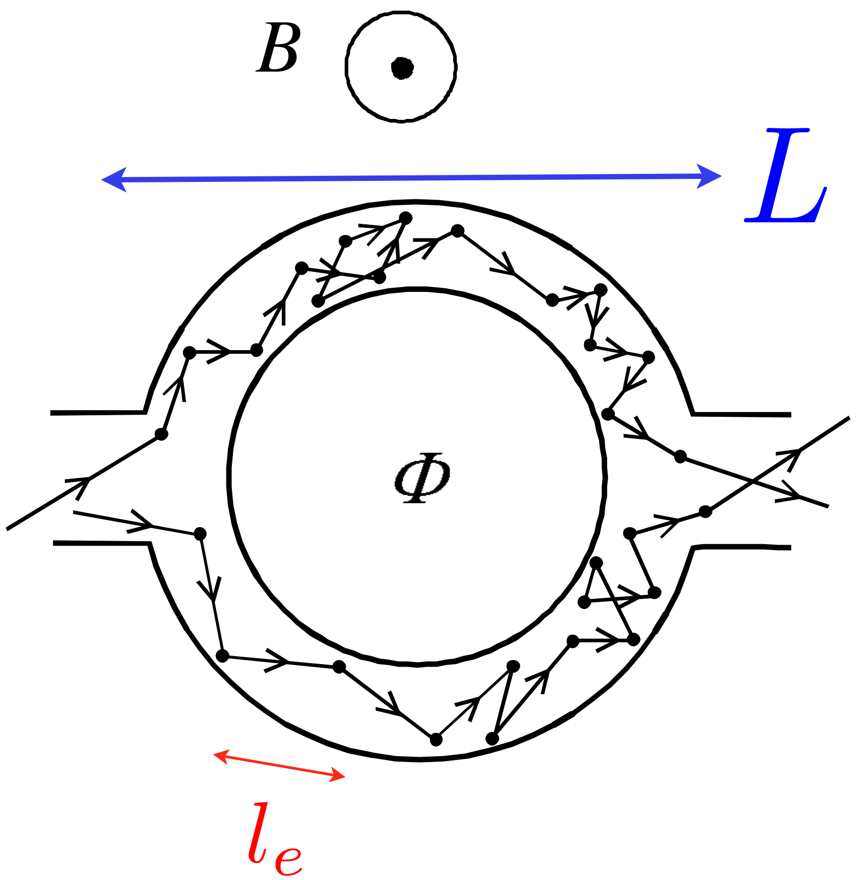


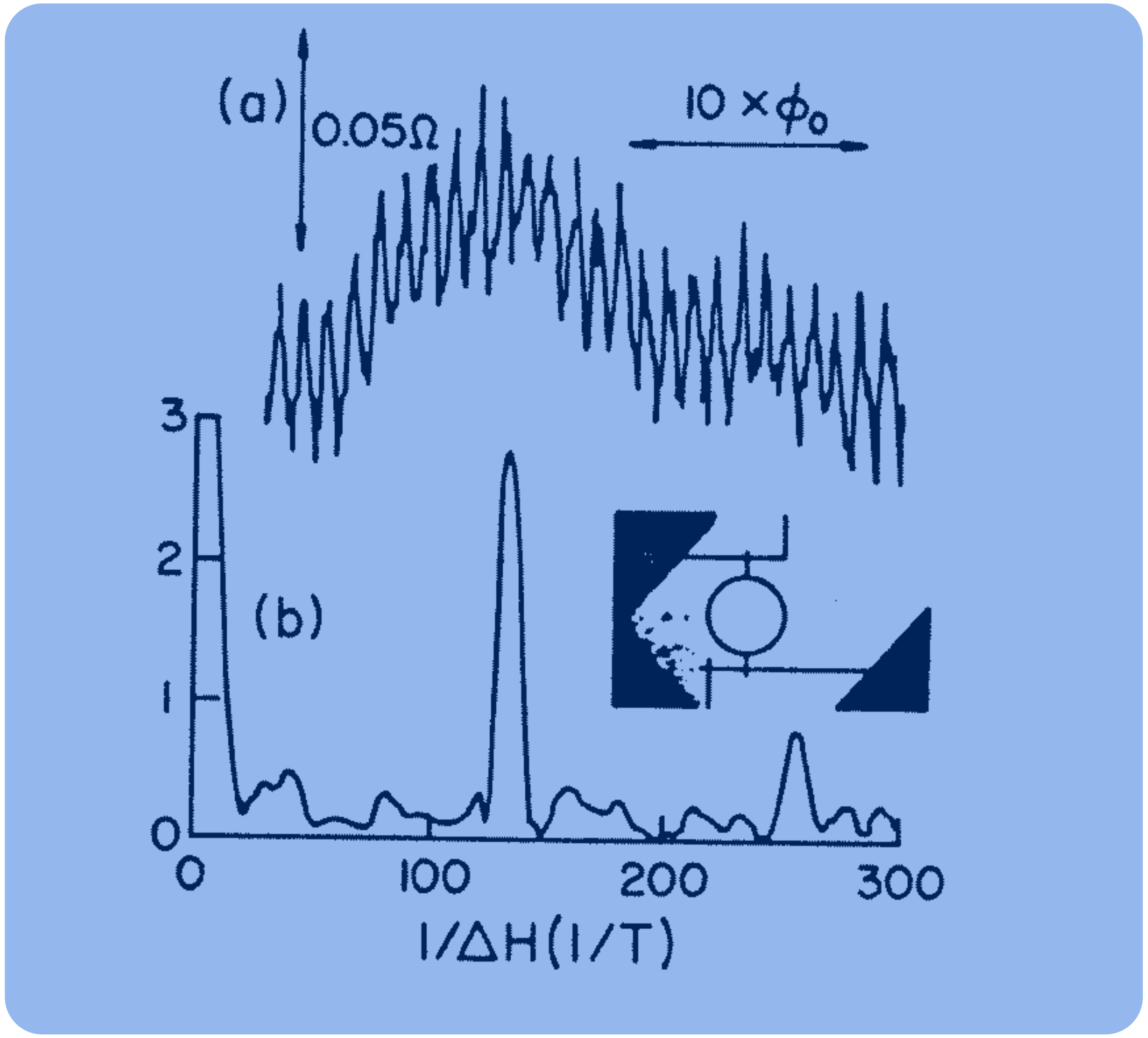


*Webb et al. 1985*

$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

Phase coherent effects subsist in disordered metals.

Reconsider the Drude theory.

# Phase coherence and effect of disorder

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For a macroscopic normal metal, coherent effects are washed out.

⇒ It must exist a characteristic length $L_\phi$ called phase coherence length beyond which all coherent effects disappear.

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The observation of coherent effects requires

$$L << L_\phi$$

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Averaging over disorder ⇒ vanishing of the Aharonov-Bohm effect

$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

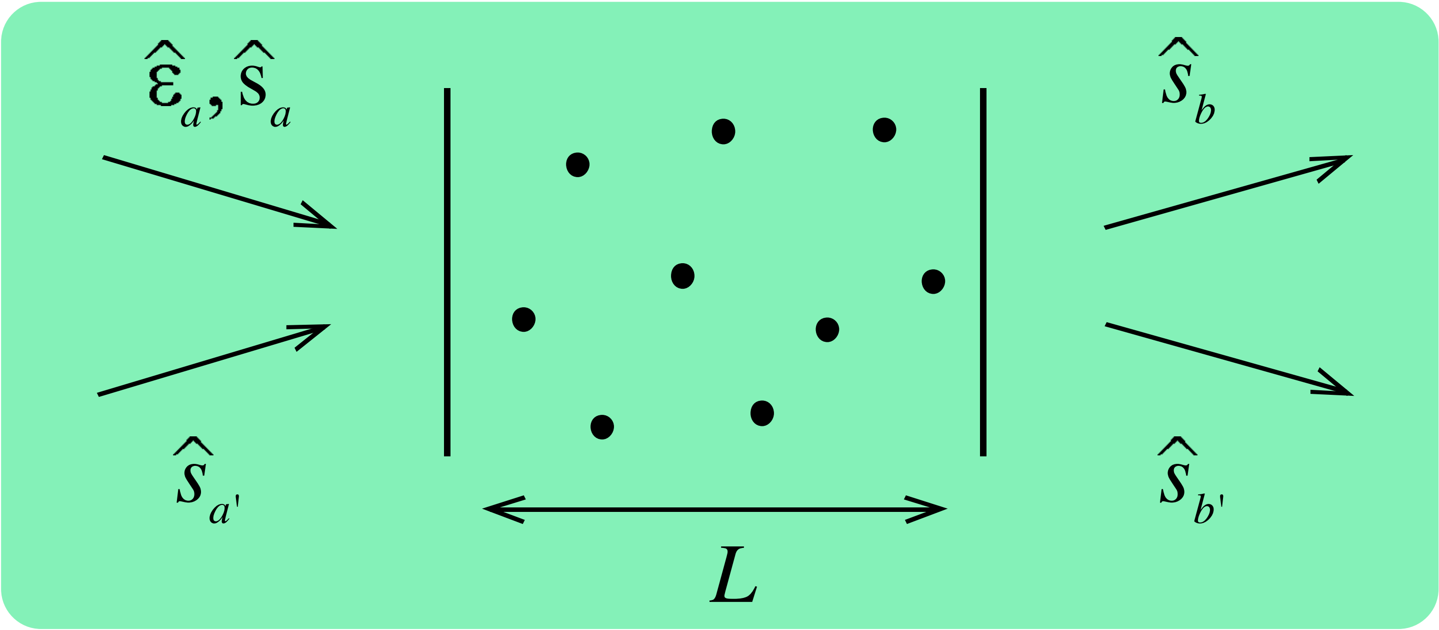
⇒ $\langle G(\phi) \rangle = G_0$

Disorder seems to erase coherent effects....

*Formulate the same question :*
*disorder vs. coherent effects in optics*

# An analogous problem: *Speckle patterns in optics*

Consider the elastic multiple scattering of light transmitted through a fixed disorder configuration.



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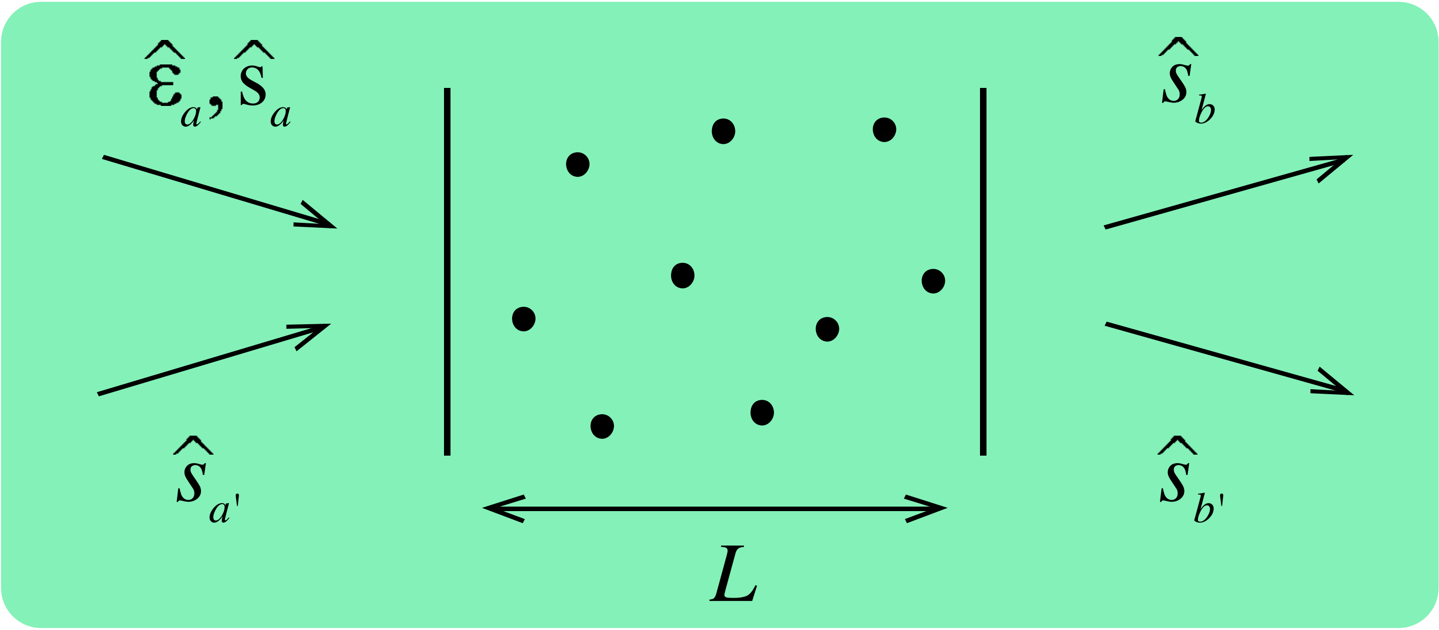
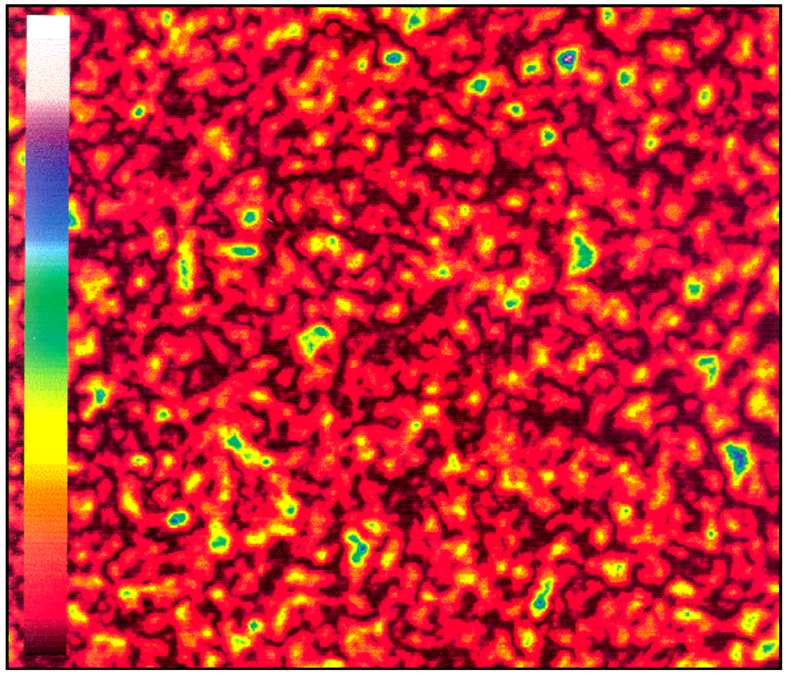
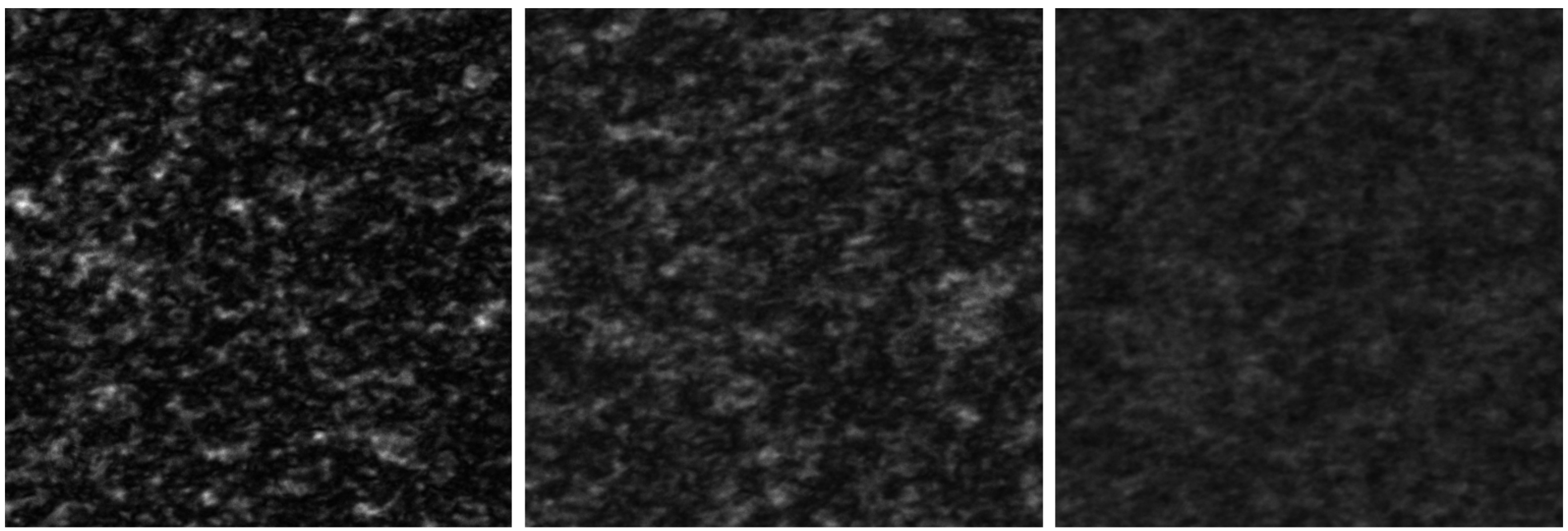



Outgoing light builds a speckle pattern *i.e.*, an interference picture:



# Averaging over disorder erases the speckle pattern:

Integration over the motion of the scatterers leads to self-averaging

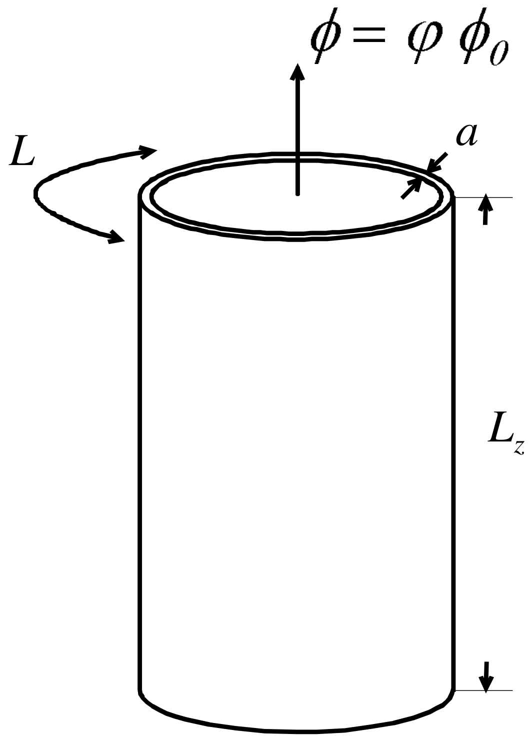


(a) (b) (c)

Time averaging

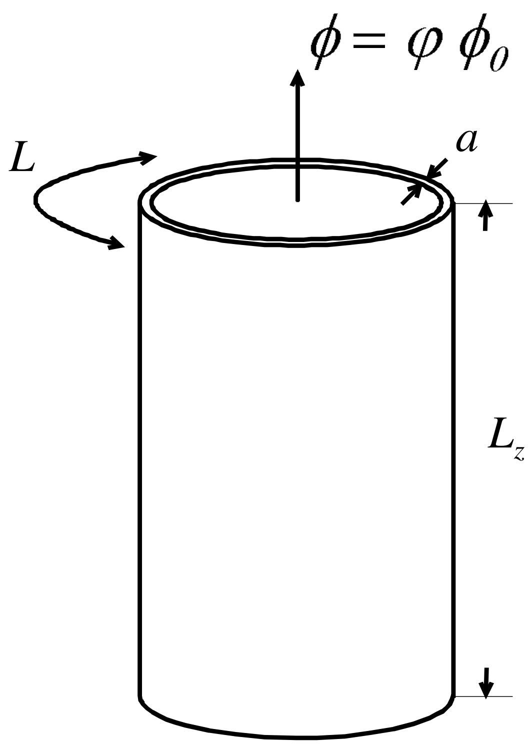
*There is an equivalent for the Aharonov-Bohm effect*

# The Sharvin[2] experiment




Experiment analogous to that of *Webb* but performed on a hollow cylinder of height larger than $L_\phi$ pierced by a Aharonov-Bohm flux.
Ensemble of rings identical to those of *Webb* but incoherent between themselves.

# The Sharvin$^2$ experiment

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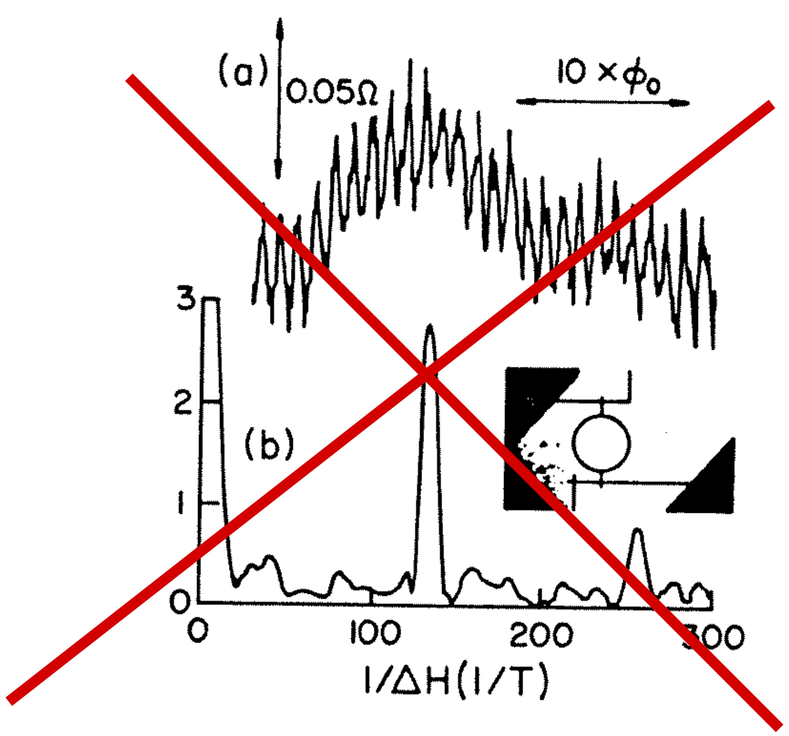


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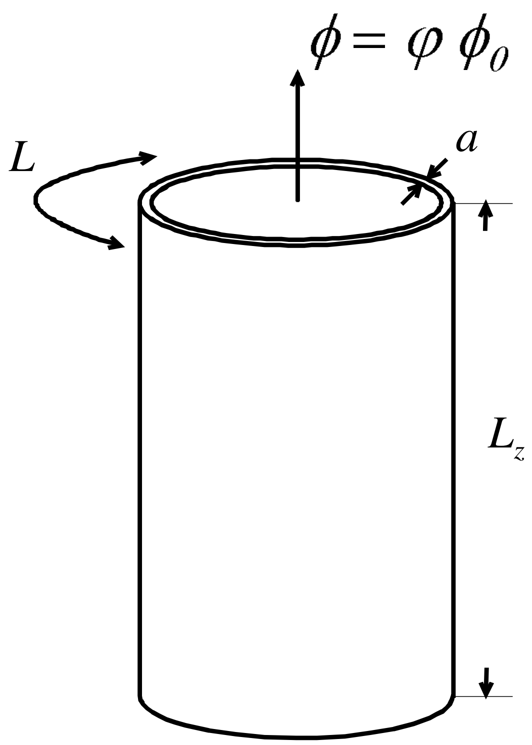


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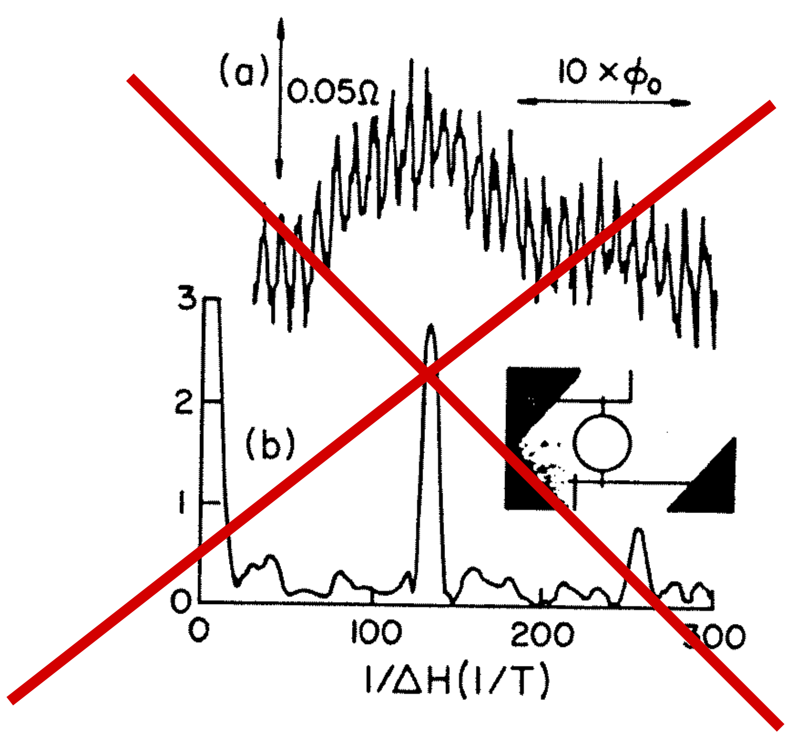


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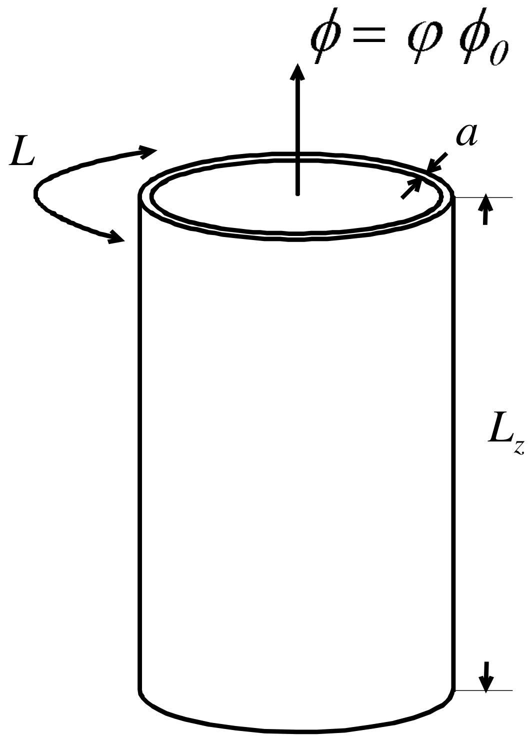


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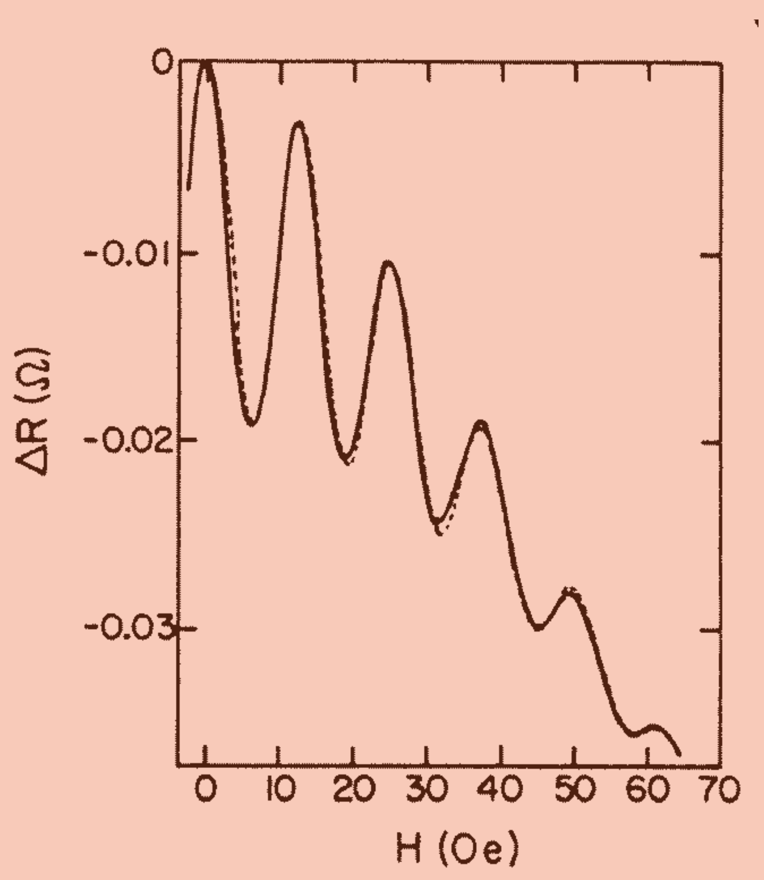


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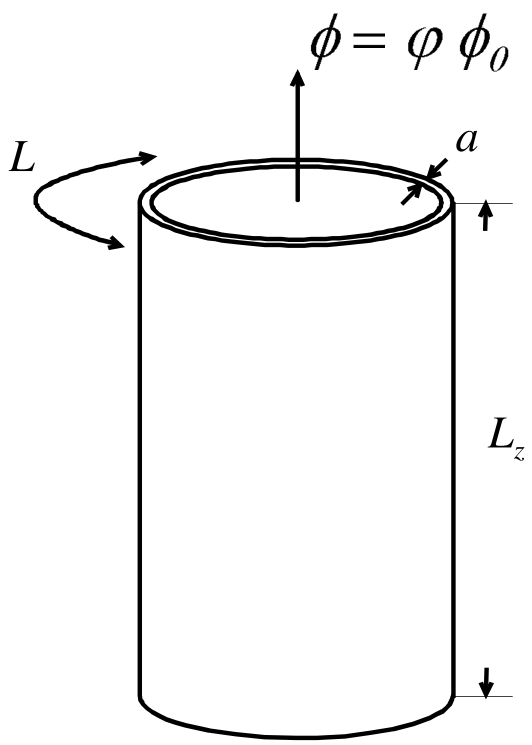


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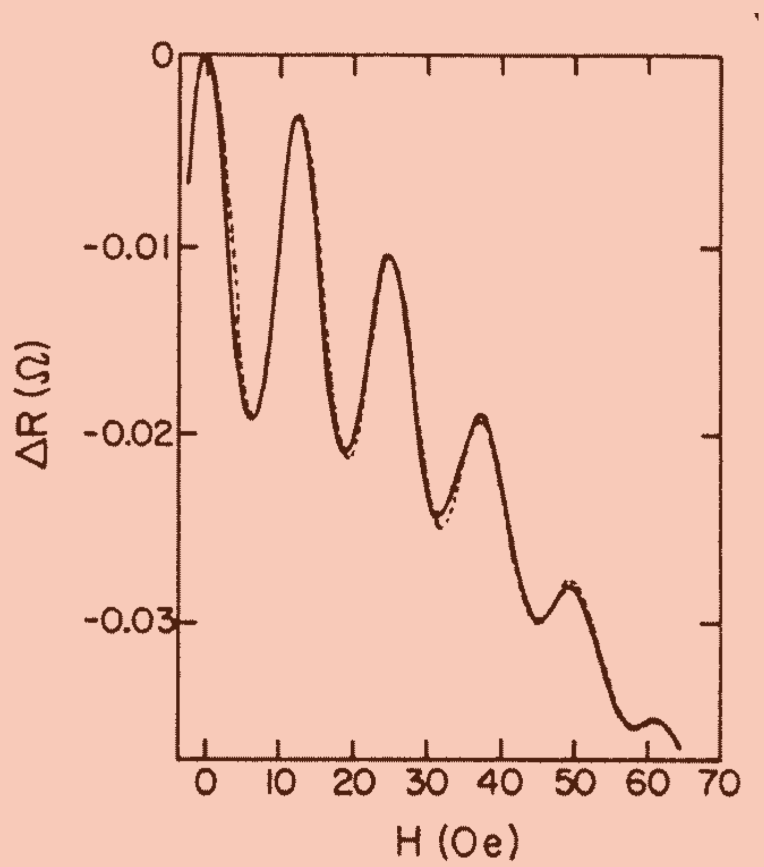


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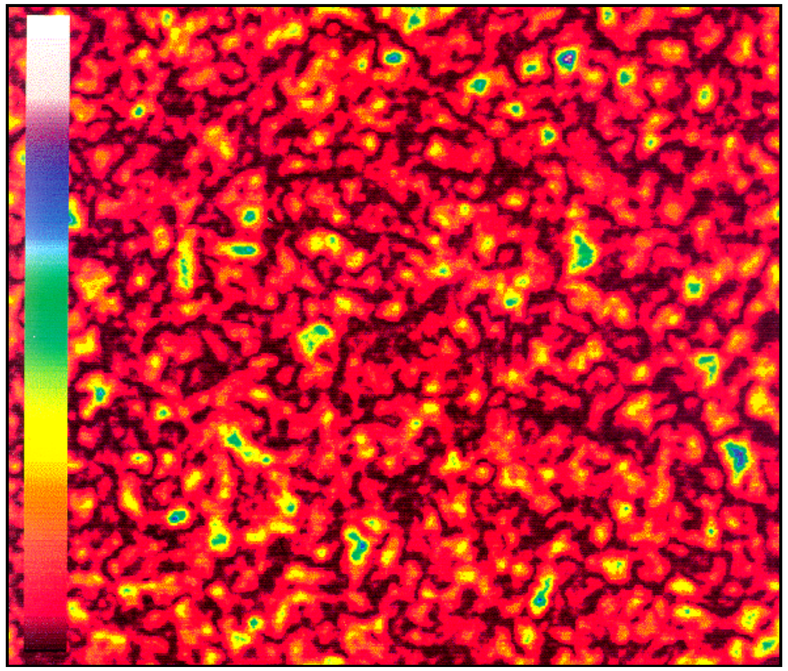
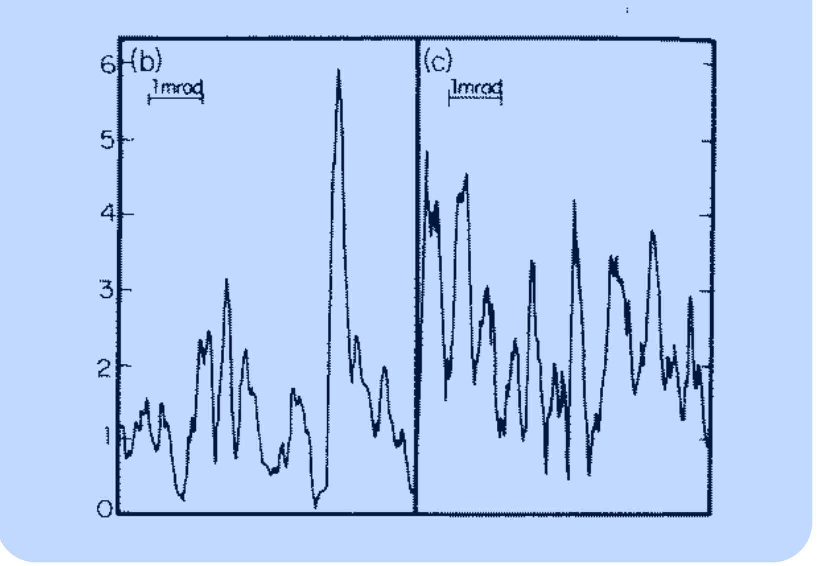
After all, disorder does not seem to erase coherent effects, but to modify them....

# What about speckle patterns ?





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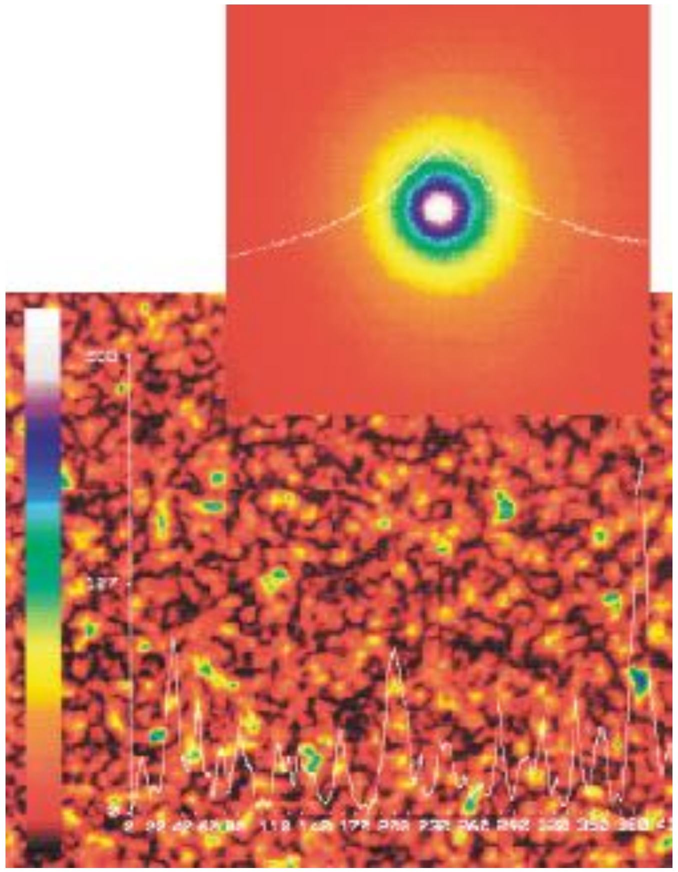

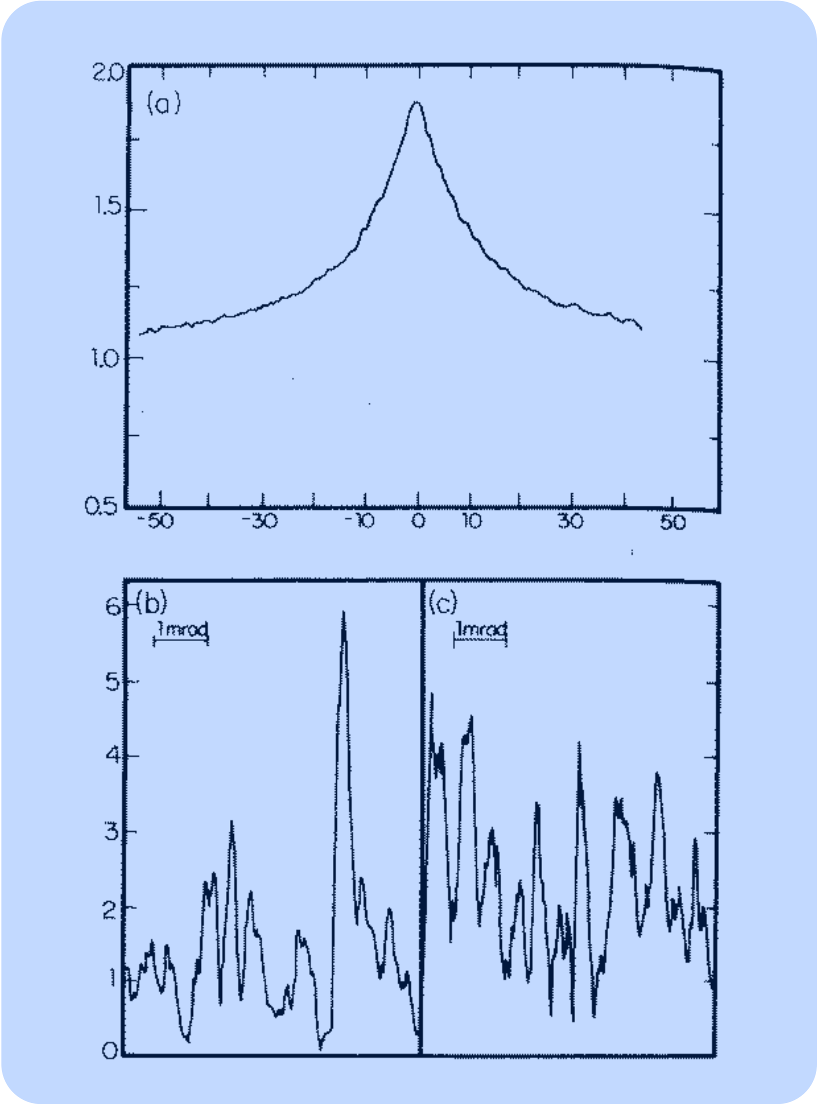


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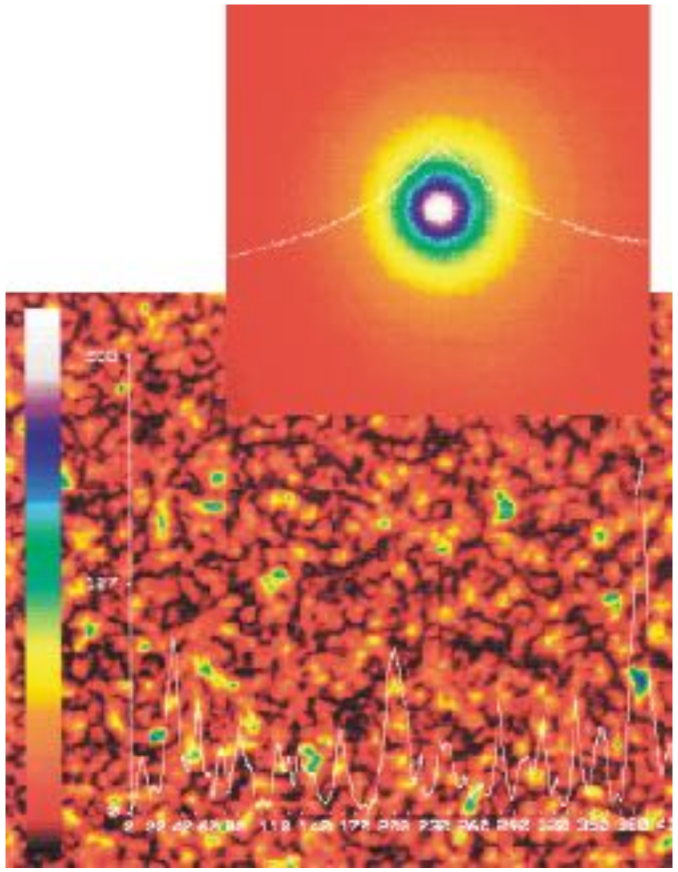

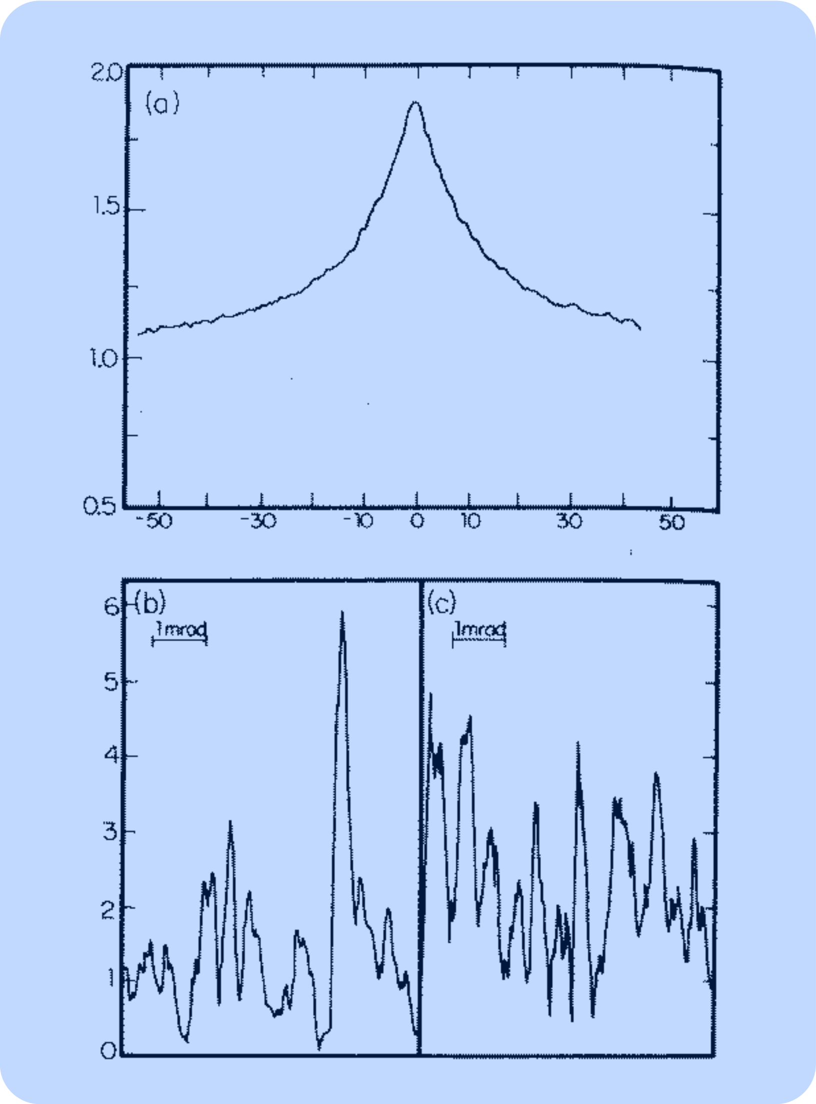


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Elastic disorder is not related to decoherence : disorder does not destroy phase coherence and does not introduce irreversibility.

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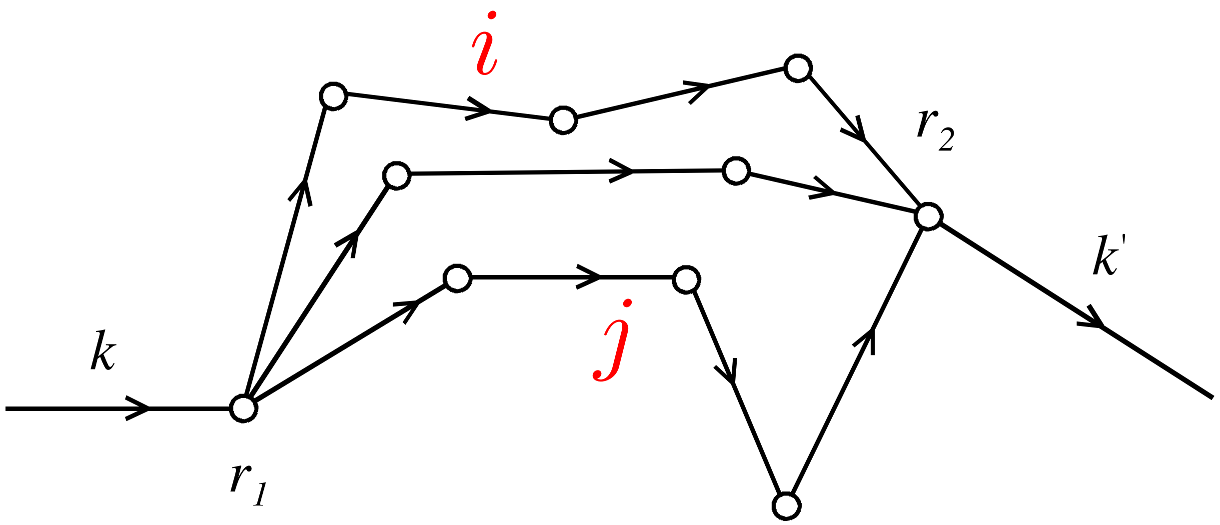


Complex amplitude $A(\mathbf{k},\mathbf{k}')$ associated to the multiple scattering of a wave (electron or photon) incident with a wave vector $\mathbf{k}$ and outgoing with $\mathbf{k}'$

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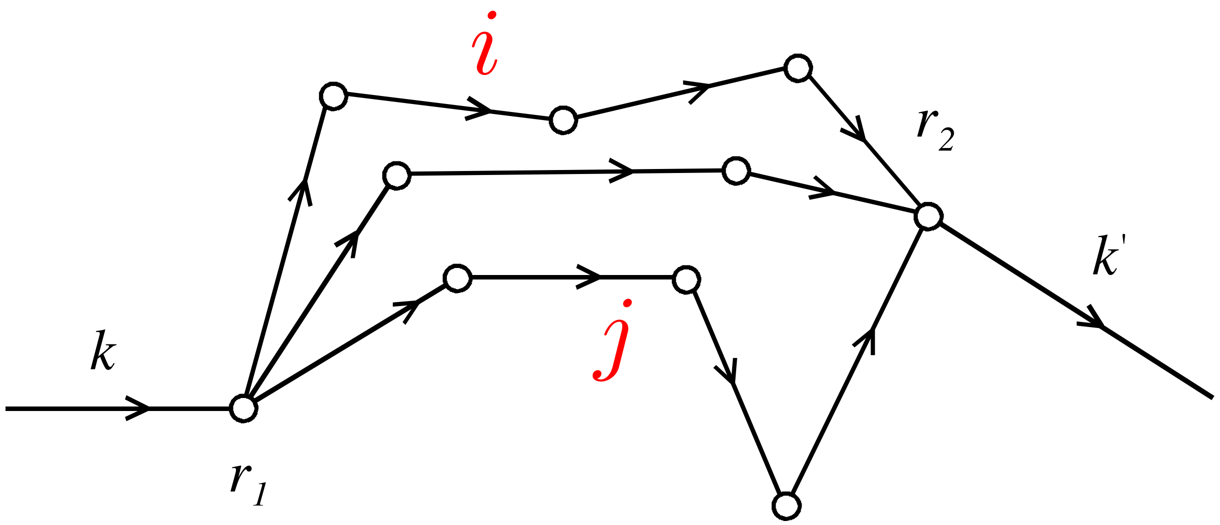


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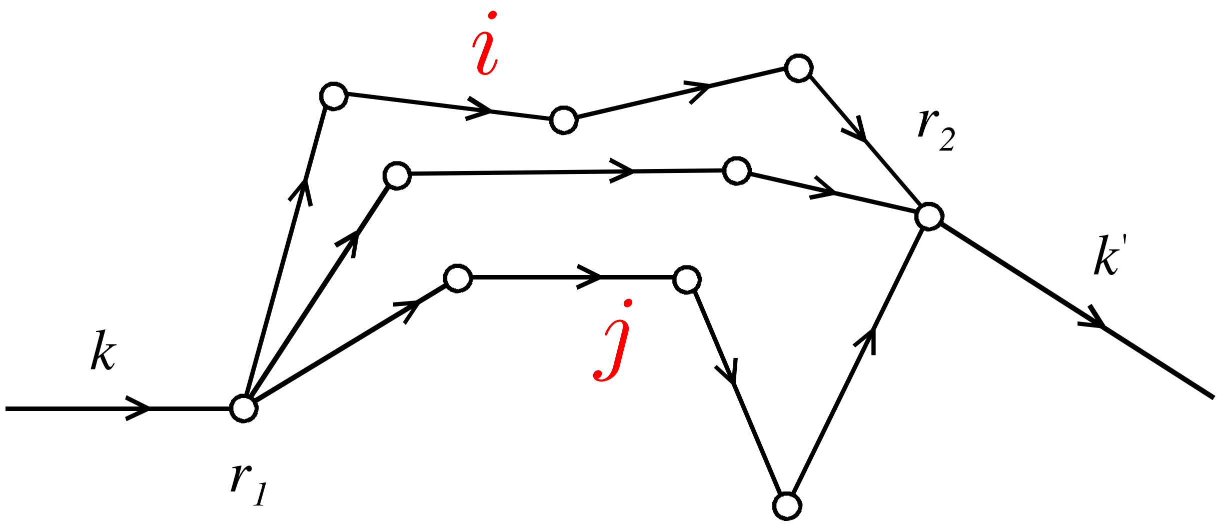


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the complex amplitude $f(\mathbf{r_1},\mathbf{r_2}) = \sum_j |a_j| e^{i\delta_j}$ describes the propagation of the wave between $\mathbf{r_1}$ and $\mathbf{r_2}$ .

The corresponding intensity is

$$|A(\mathbf{k},\mathbf{k}')|^2 = \sum_{\mathbf{r_1},\mathbf{r_2}} \sum_{\mathbf{r_3},\mathbf{r_4}} f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) e^{i(\mathbf{k}.\mathbf{r_1}-\mathbf{k}'.\mathbf{r_2})} e^{-i(\mathbf{k}.\mathbf{r_3}-\mathbf{k}'.\mathbf{r_4})}$$

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with

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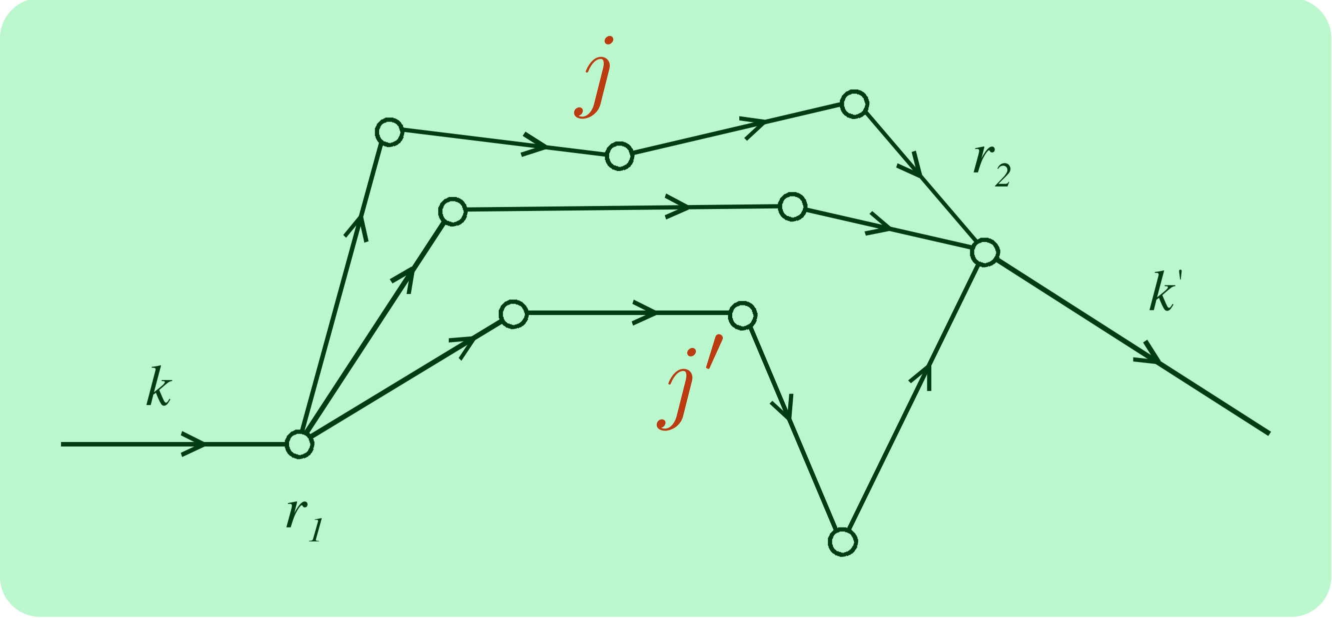


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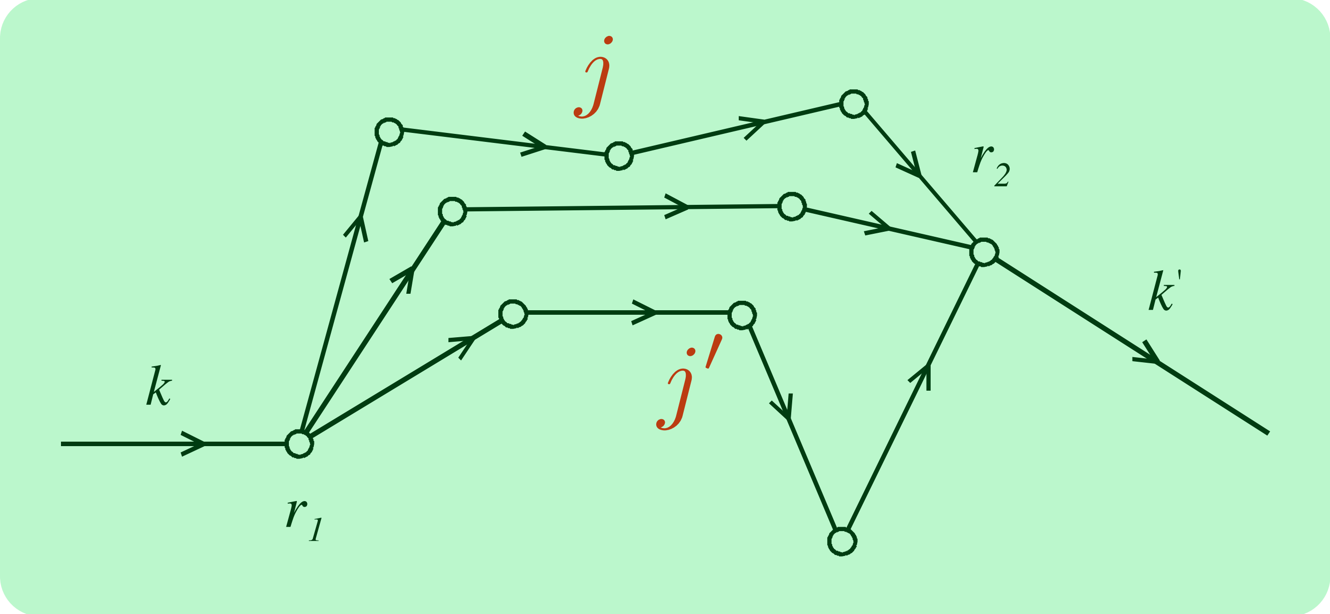


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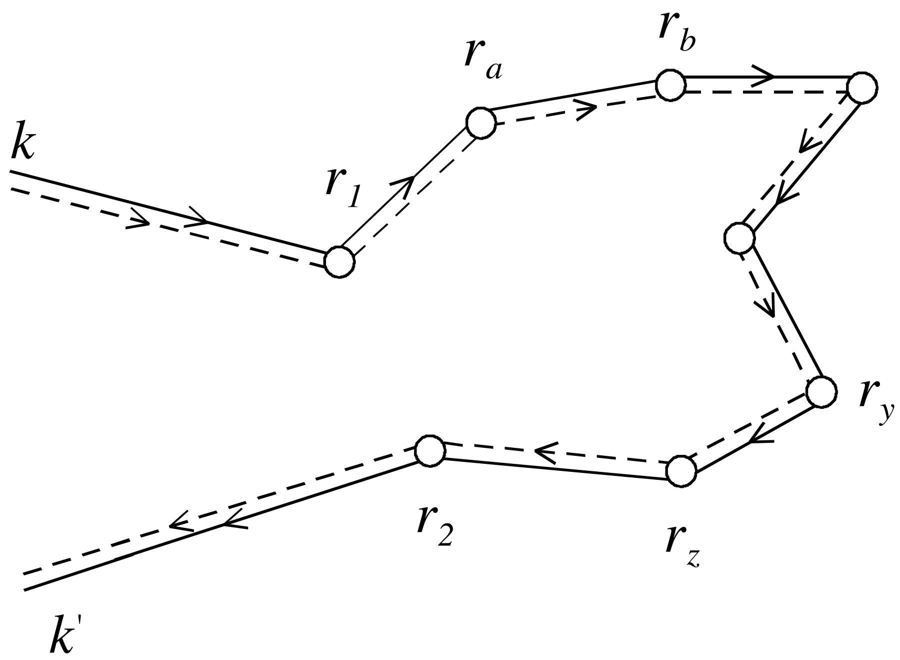
The only remaining contributions to the intensity correspond to terms with zero dephasing, *i.e.*, to identical trajectories.

(a)

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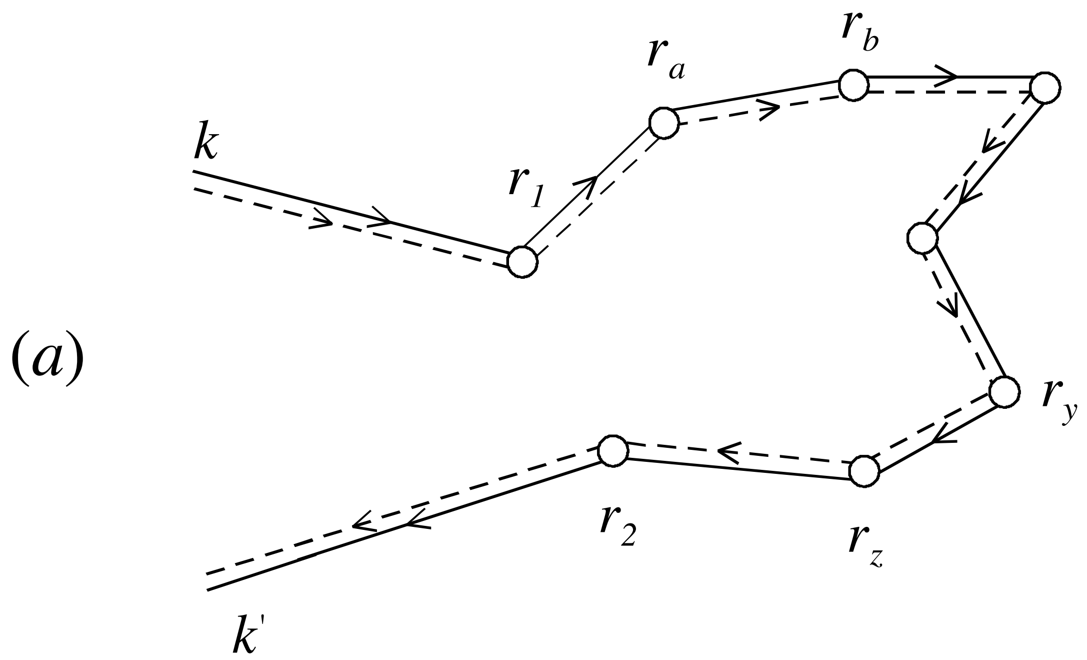


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Reciprocity theorem:

If I see you, then you see me.

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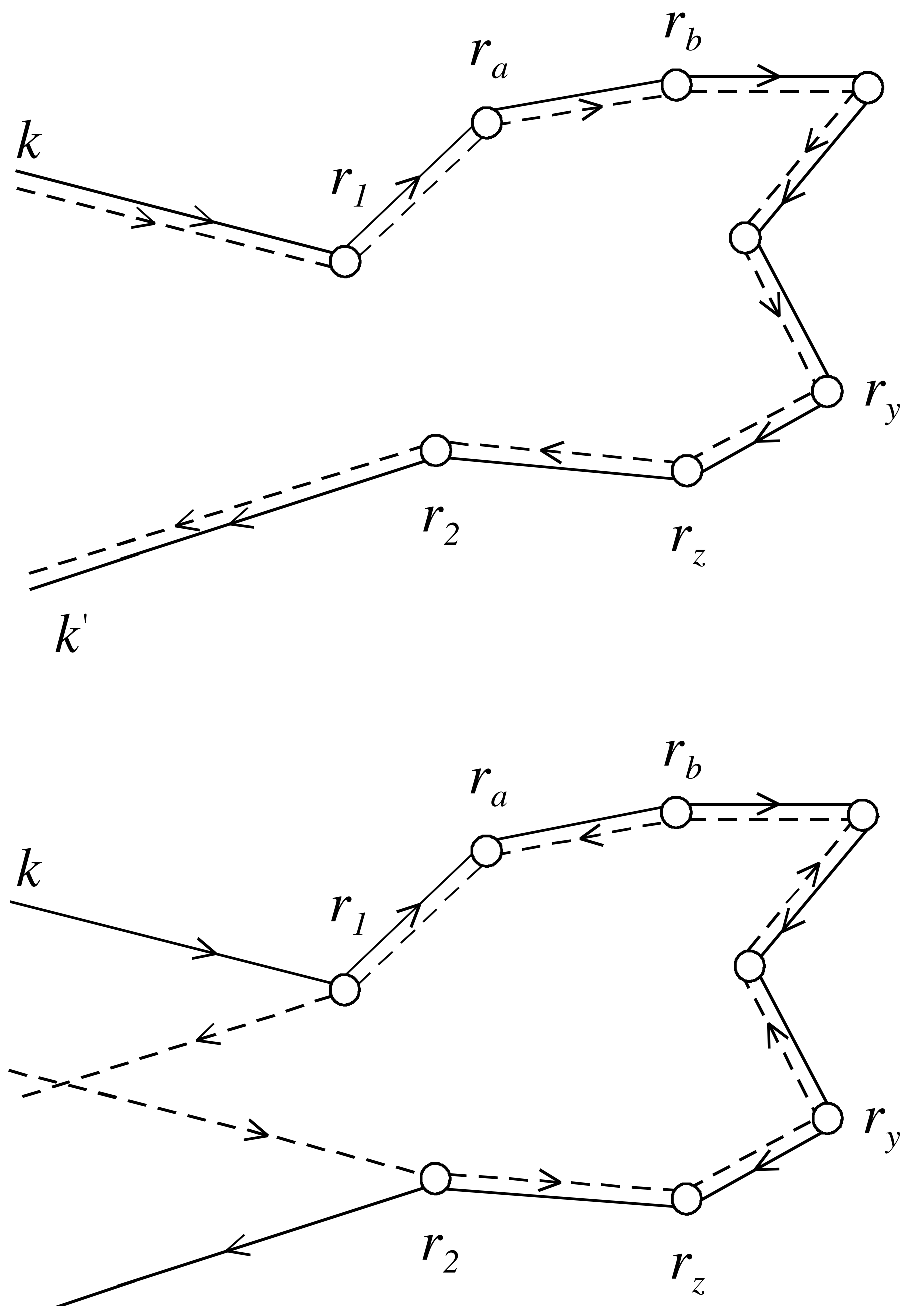


(b)

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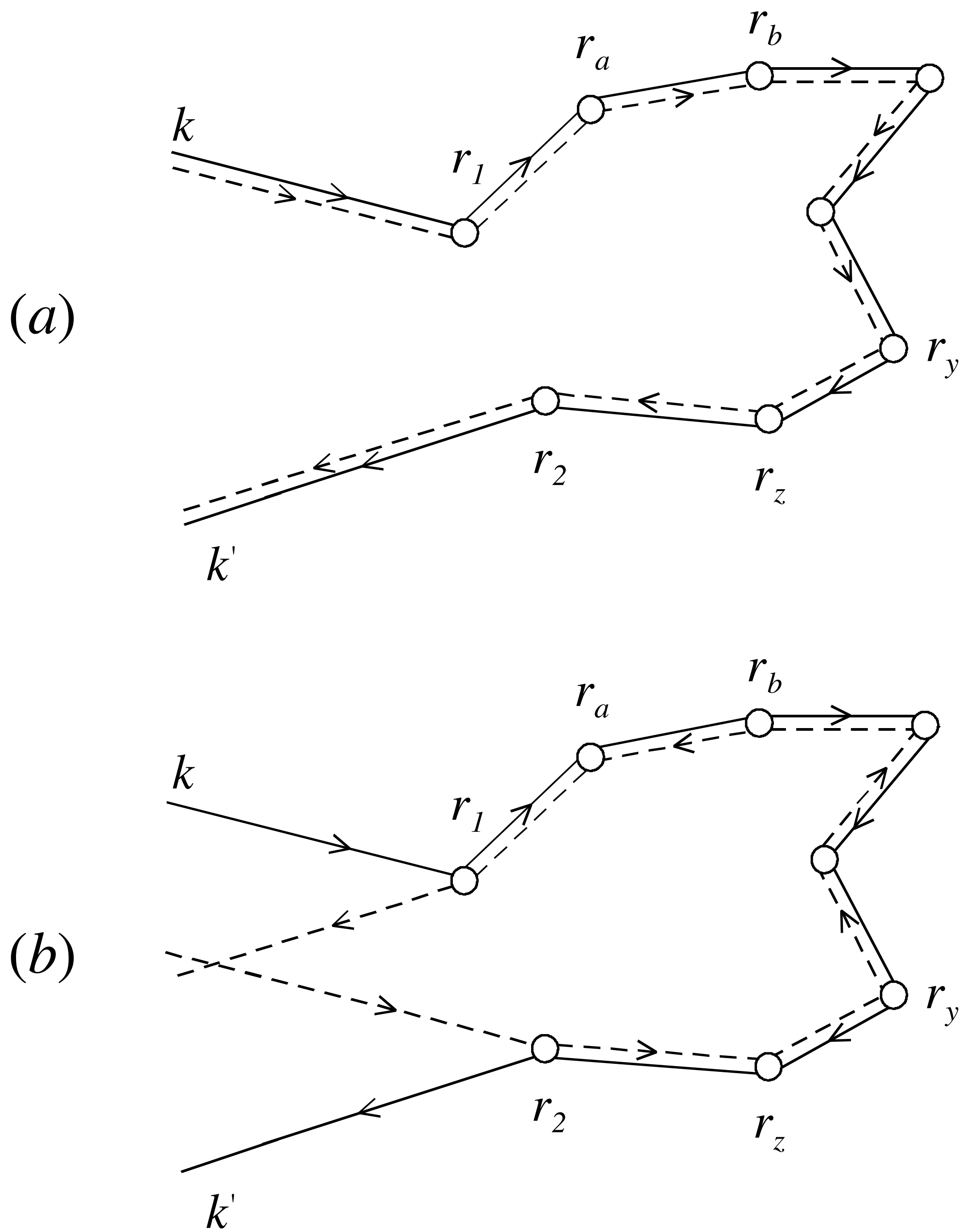


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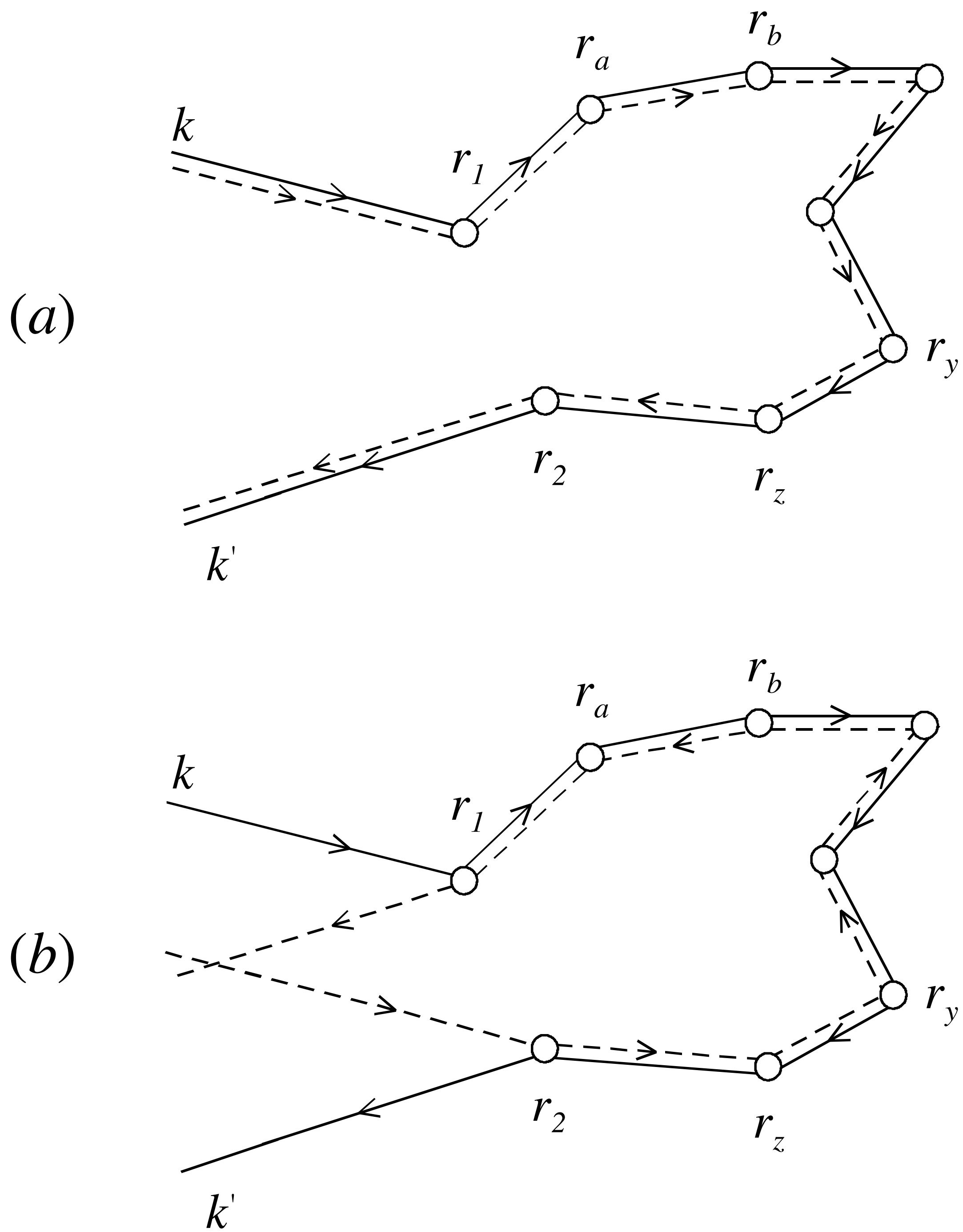


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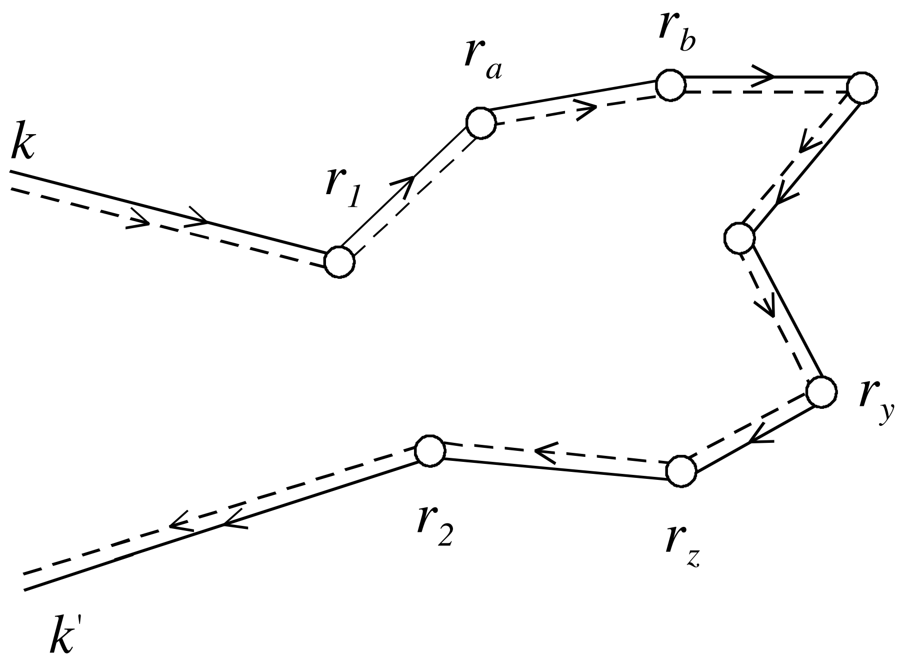


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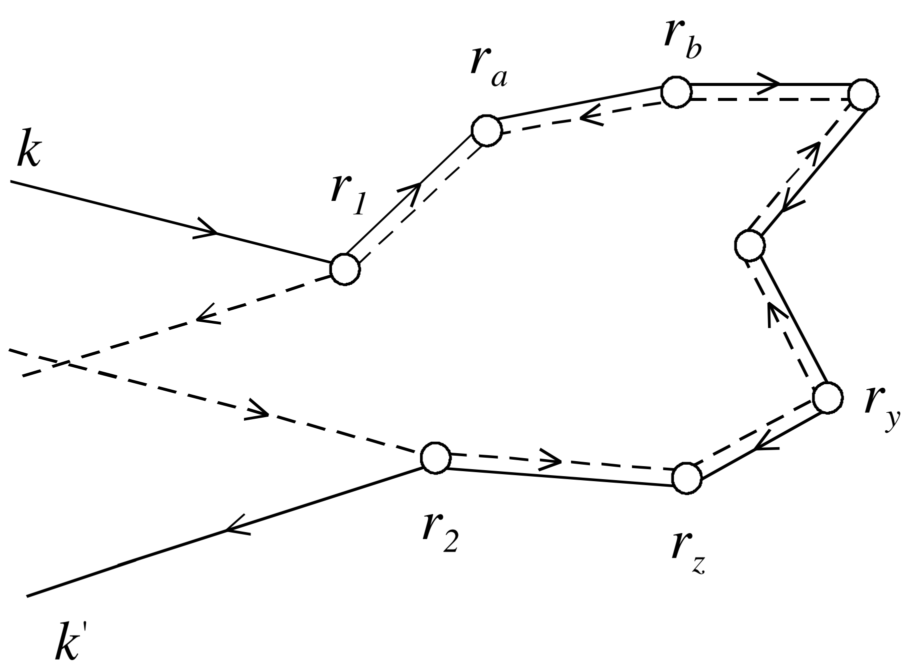


$$\mathbf{r_2} \longrightarrow \mathbf{r_z} \longrightarrow \mathbf{r_y} \cdots \longrightarrow \mathbf{r_b} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_1}$$

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interference term

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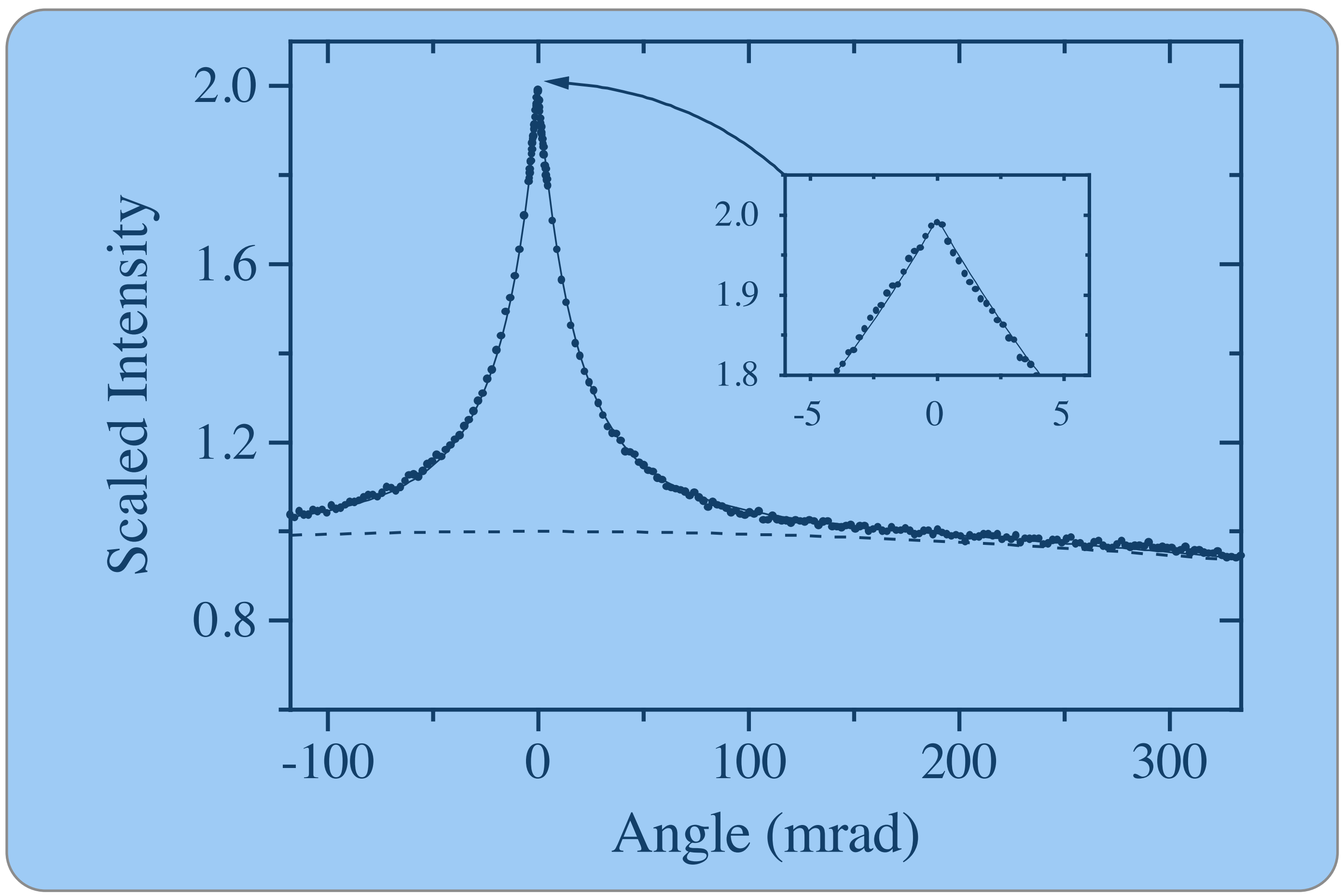
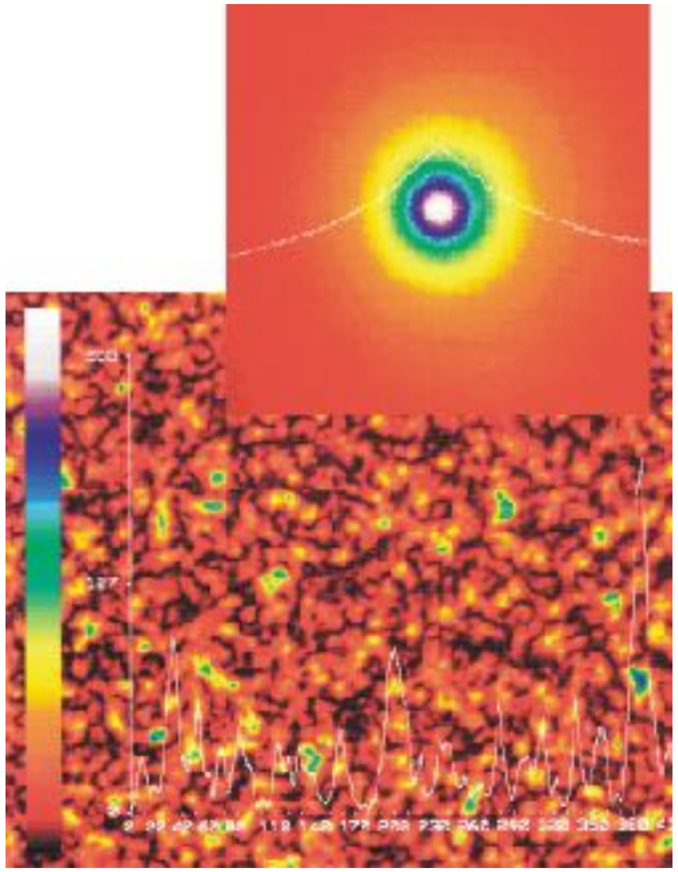
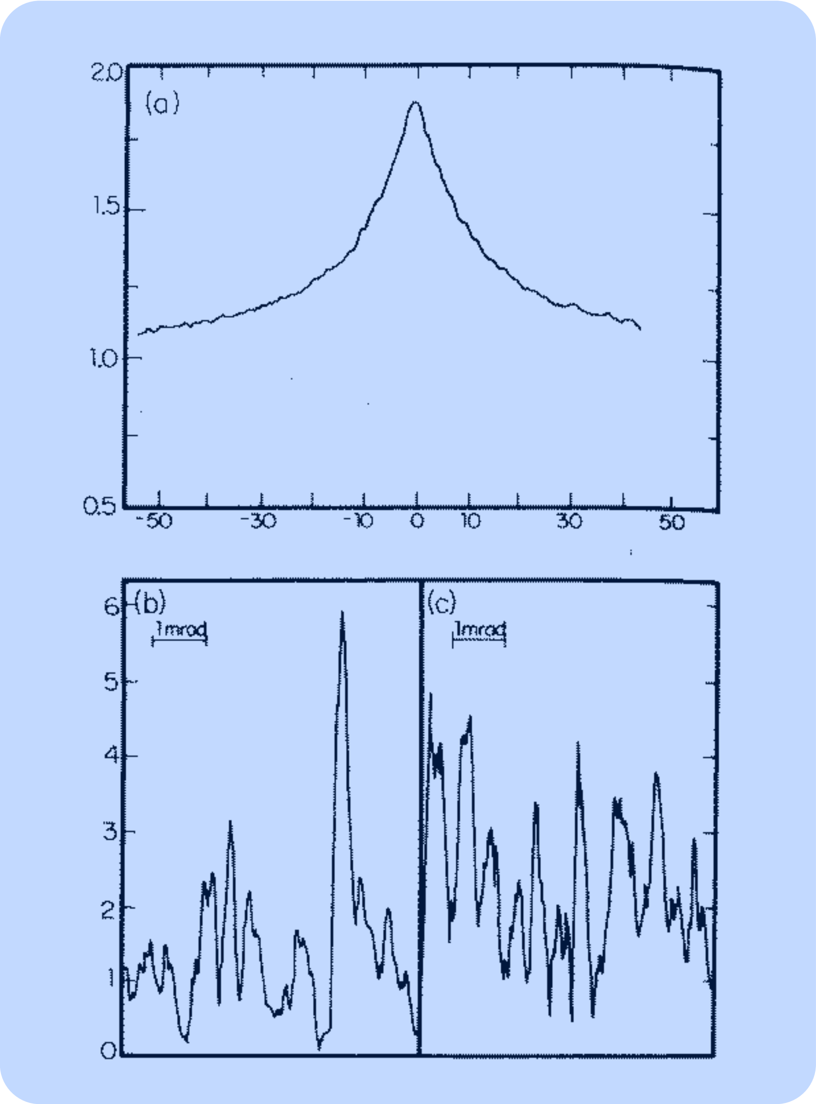



Coherent backscattering

# What about speckle patterns ?






Averaging over disorder does not produce incoherent intensity only, but also an angular dependent part, the coherent backscattering, which is a coherence effect.

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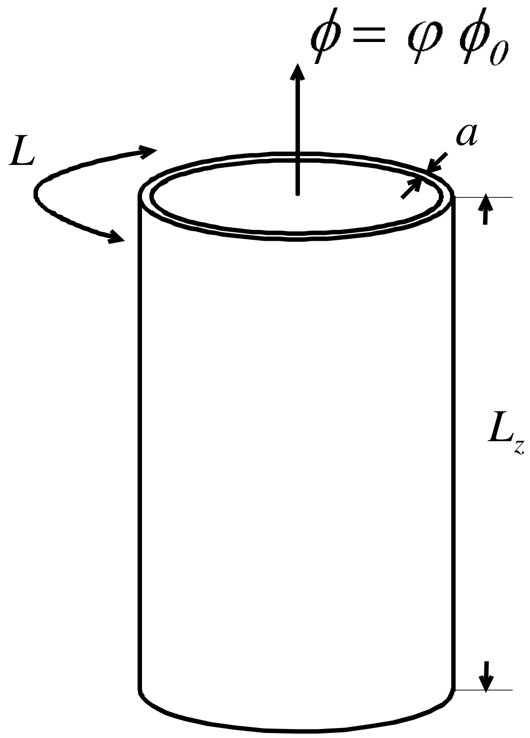
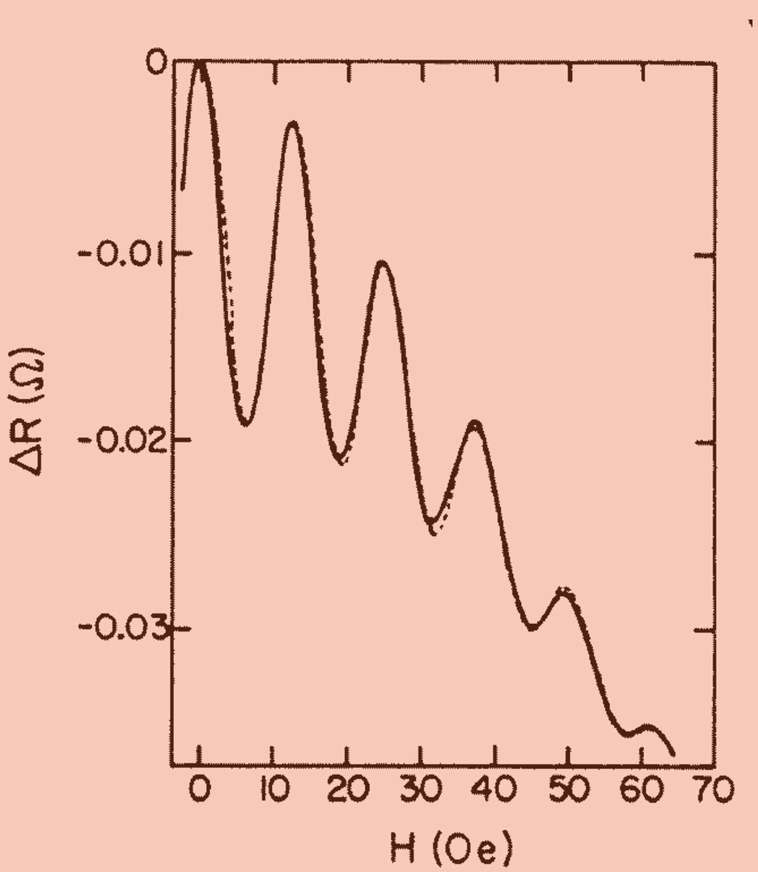
$\mathbf{r_1}-\mathbf{r_2} \simeq 0$ : closed loops, weak localization and $\phi_0/2$ periodicity of the Sharvin effect.

# The Sharvin$^2$ experiment




Experiment analogous to that of *Webb* but performed on a hollow cylinder of height larger than $L_\phi$ pierced by a Aharonov-Bohm flux.
Ensemble of rings identical to those of *Webb* but incoherent between themselves.




The signal modulated at $\phi_0$ *disappears* but, instead, it appears a new contribution modulated at $\phi_0/2$

# *Quantum complexity*

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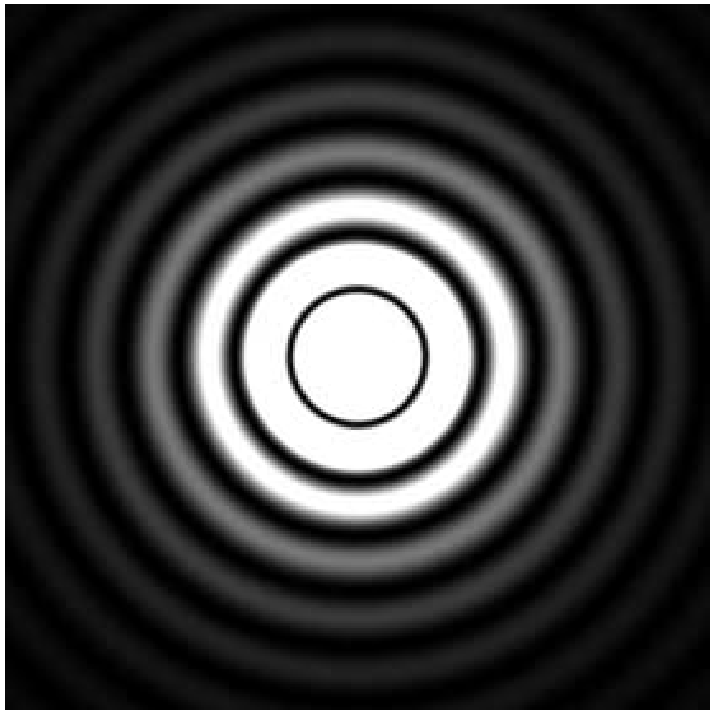
It introduces randomness and complexity: all symmetries are lost, there are no good quantum numbers.

# Exemple: speckle patterns in optics

Diffraction through a circular aperture: order in interference



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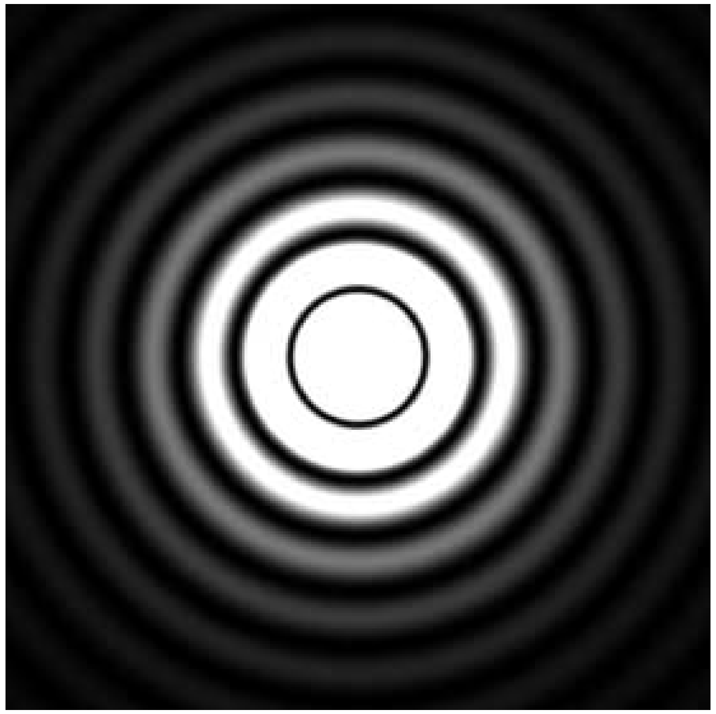

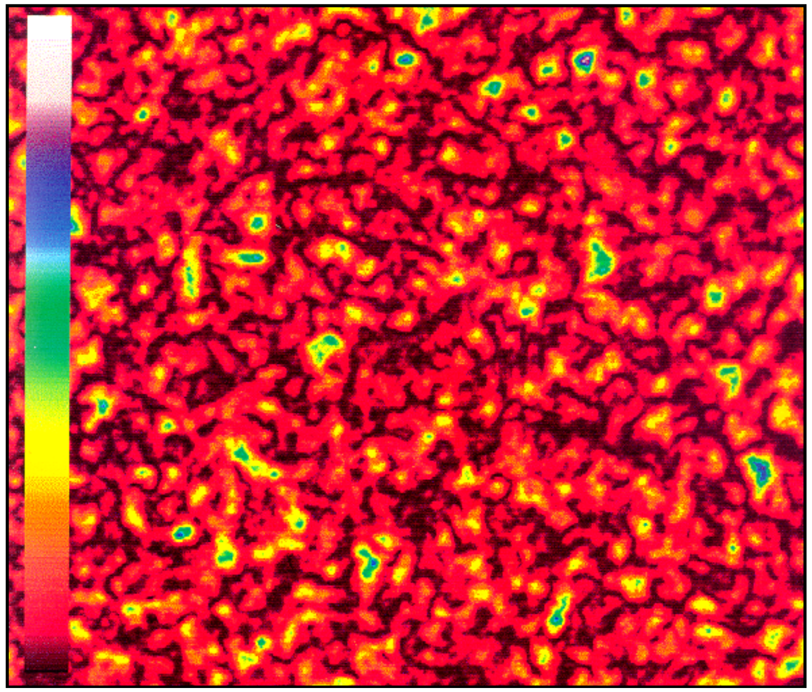

Transmission of light through a disordered suspension: complex system

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A mesoscopic quantum system is a coherent complex quantum system with $L \leq L_\varphi$

# An Exemple

# Phase coherence and self-averaging: universal fluctuations.

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*The system performs an average over realizations of the disorder.*

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- a description of fluctuations and coherence in a quantum complex system.

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Classically, the conductance of a cubic sample of volume $L^d$ is given by Ohm's law: $G = \sigma L^{d-2}$ where $\sigma$ is the conductivity.

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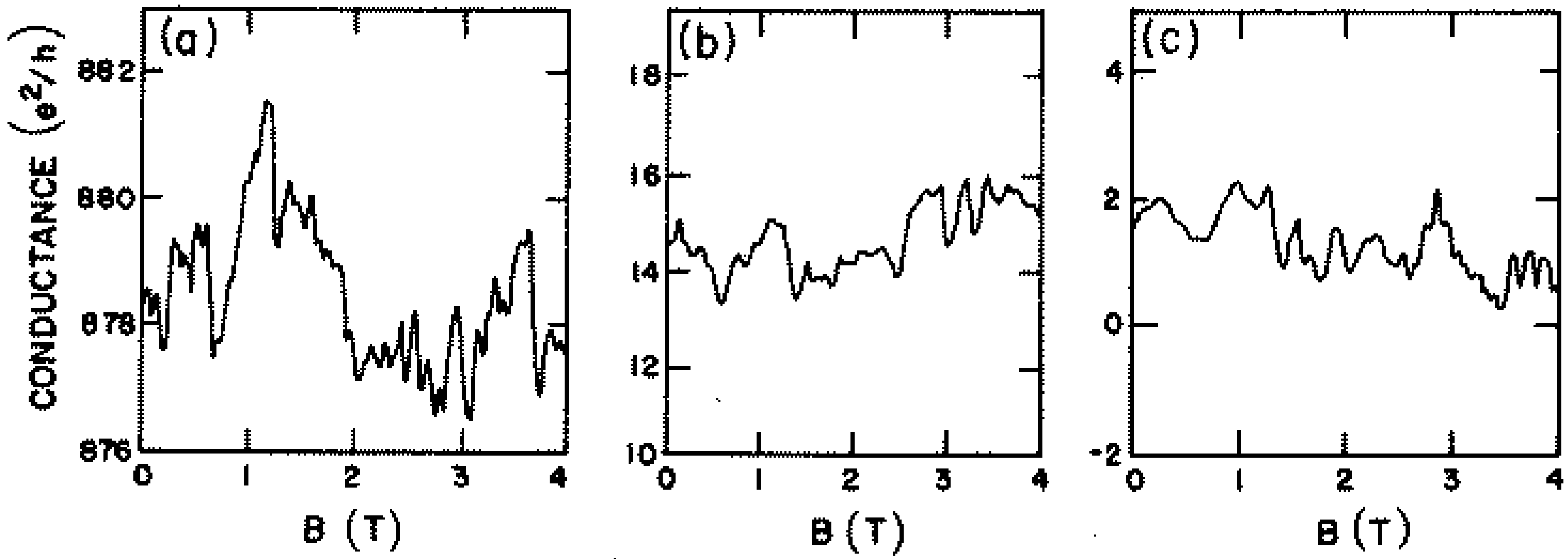
*In the mesoscopic limit, the electrical conductance is not self-averaging.*




Gold ring

Si-MOSFET

NUMERICS ON
THE ANDERSON MODEL