Mesoscopic Physics of Photons

Based on *Mesoscopic physics of electrons and photons*, by Eric Akkermans and Gilles Montambaux, Cambridge University Press, 2007

Eric Akkermans

Technion Israel Institute of Technology

PAZY EXCELLING IN SCIENCE

ISRAEL SCIENCE FOUNDATION

School on Interaction of Light with Cold Atoms, Sept. 16-27, 2019, Sao Paulo, Brazil, ICTP-SAIFR/IFT-UNESP.

Part 2

Introduction to mesoscopic physics

The tools (some of them)

Do coherent effects survive disorder average?

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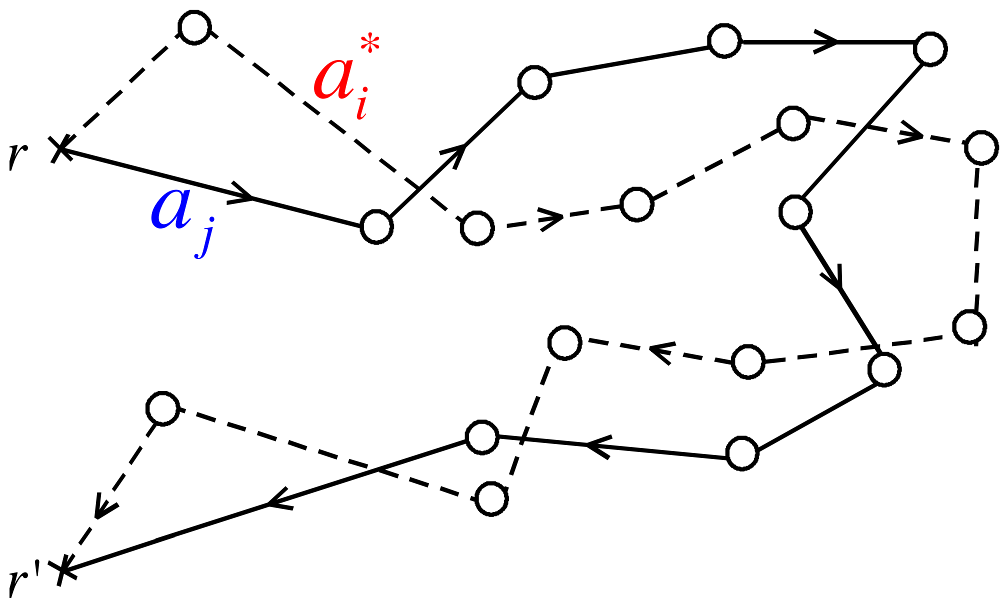
Quantum probability for electron diffusion between two points



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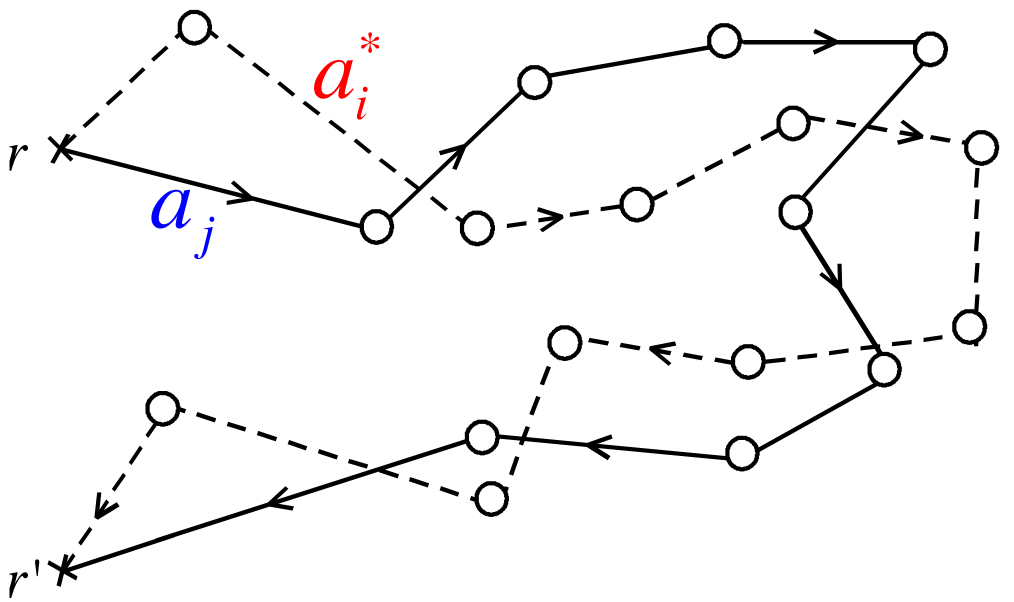

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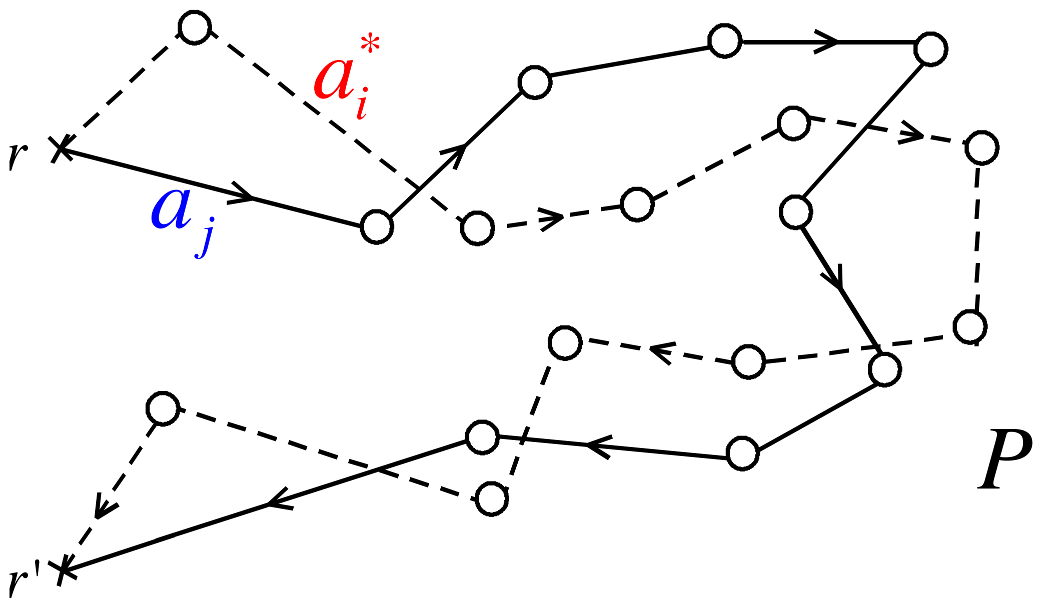

$$P(r,r') = \sum_{i,j} \overline{a_i^*(r,r')\, a_j(r,r')}$$

$$P(r,r') = \sum_{j} \overline{\left|a_j(r,r')\right|^2} + \sum_{i\neq j} \overline{a_i^*(r,r') a_j(r,r')}$$

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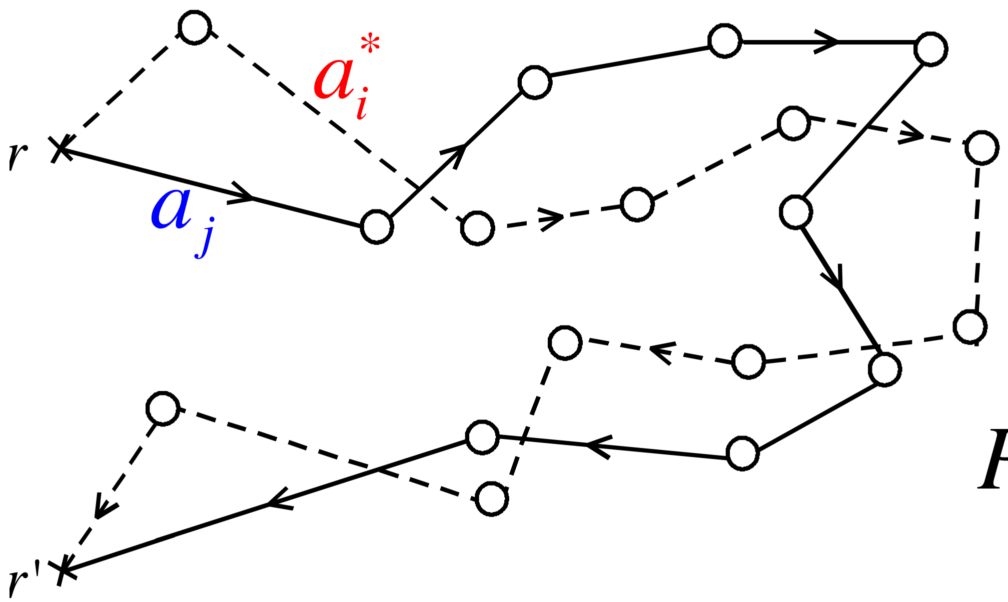

$$a_i^* \, a_j = |a_i|\,|a_j|\, e^{i(\delta_i - \delta_j)}$$

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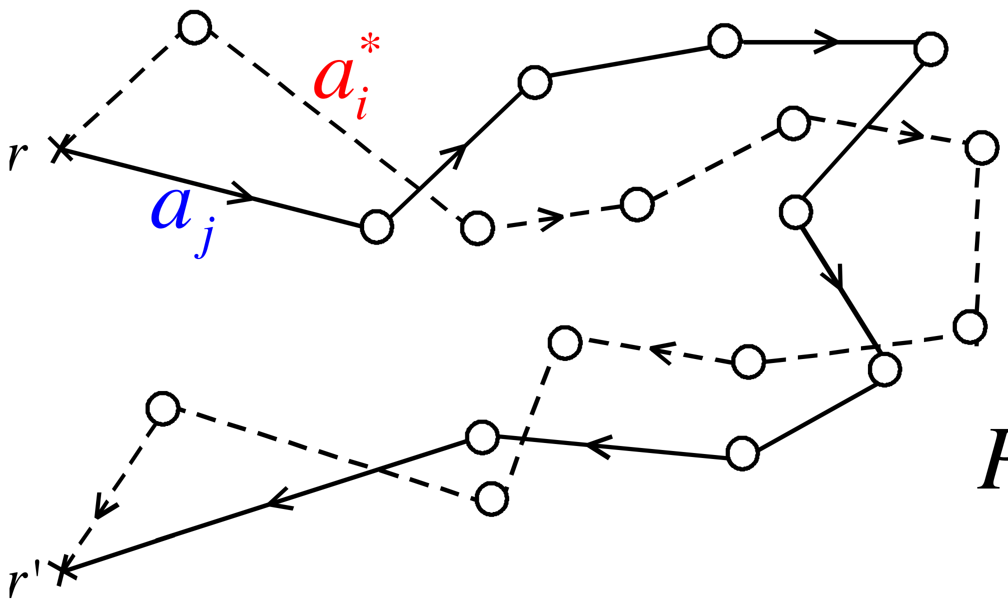

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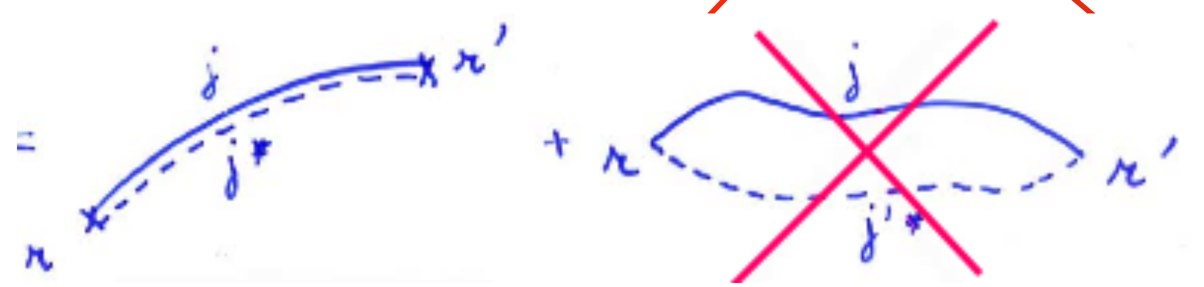
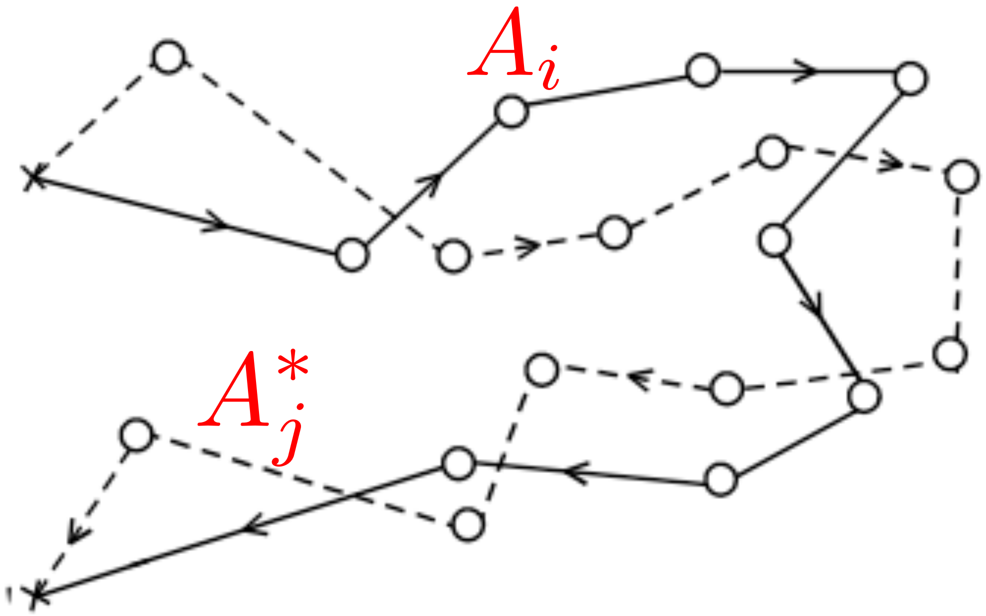
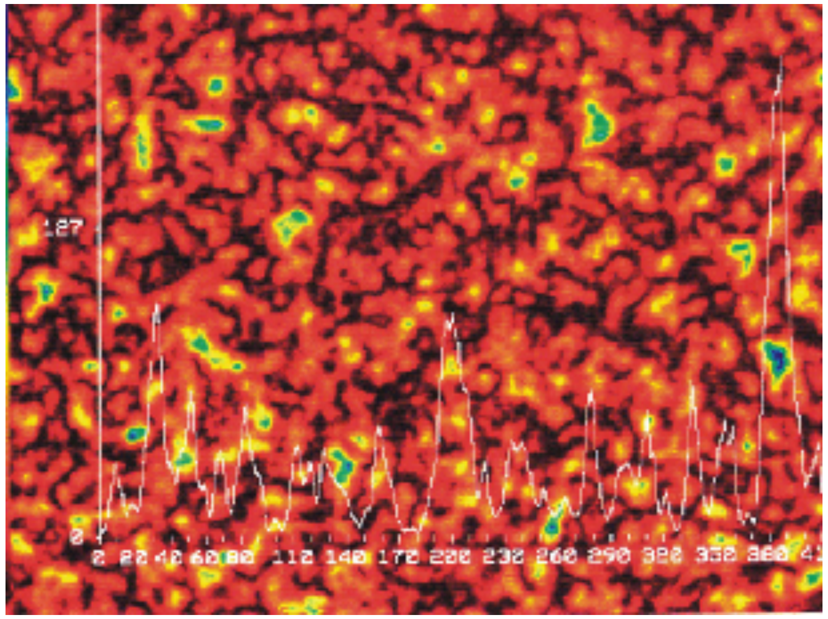


$$\delta_i - \delta_j \gg 1$$

Incoherent propagation !

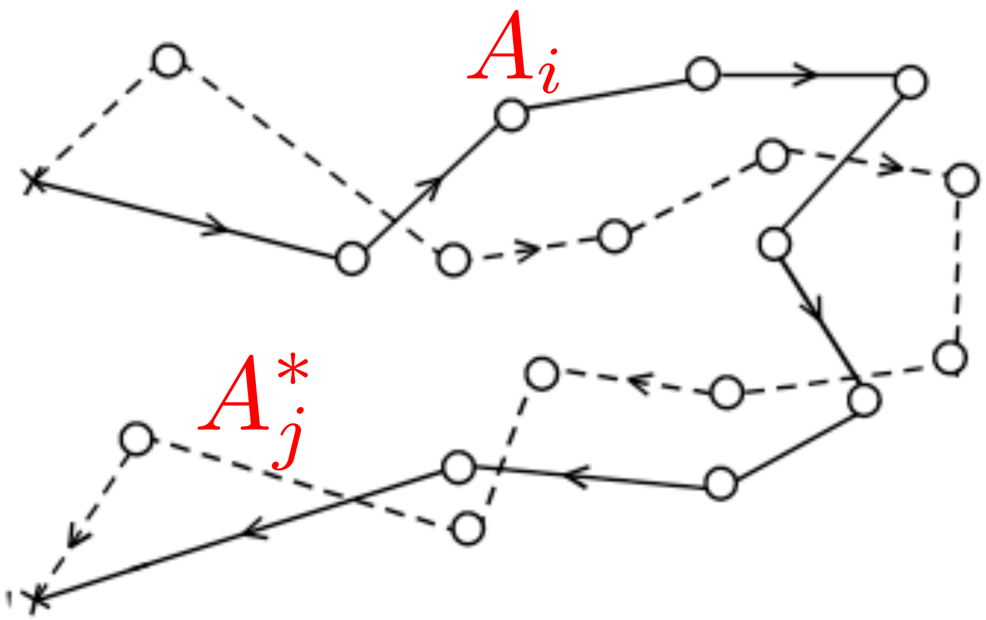
vanishes on average

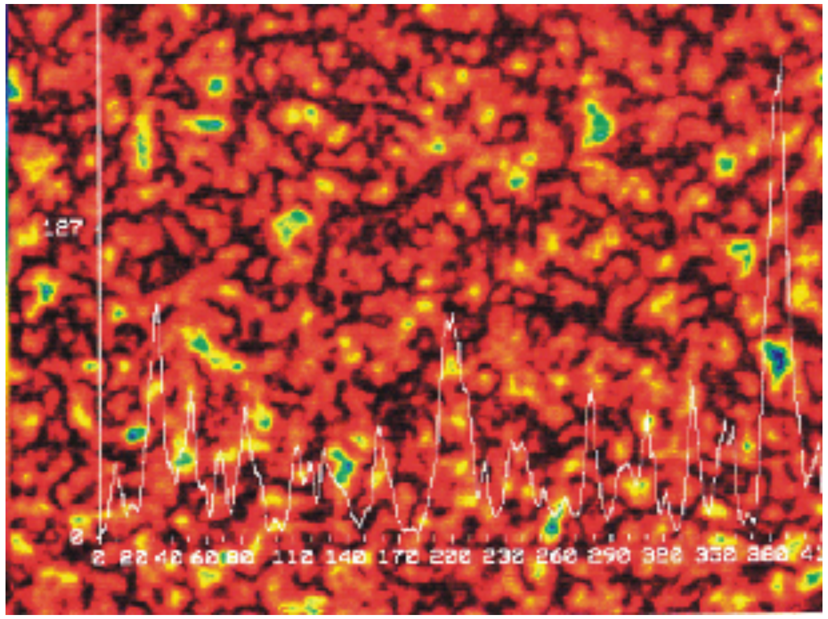




Before averaging : speckle pattern (full coherence)

Configuration average: most of the contributions vanish because of large phase differences.





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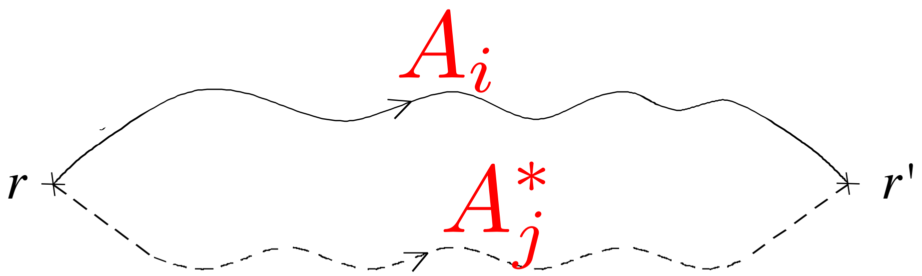
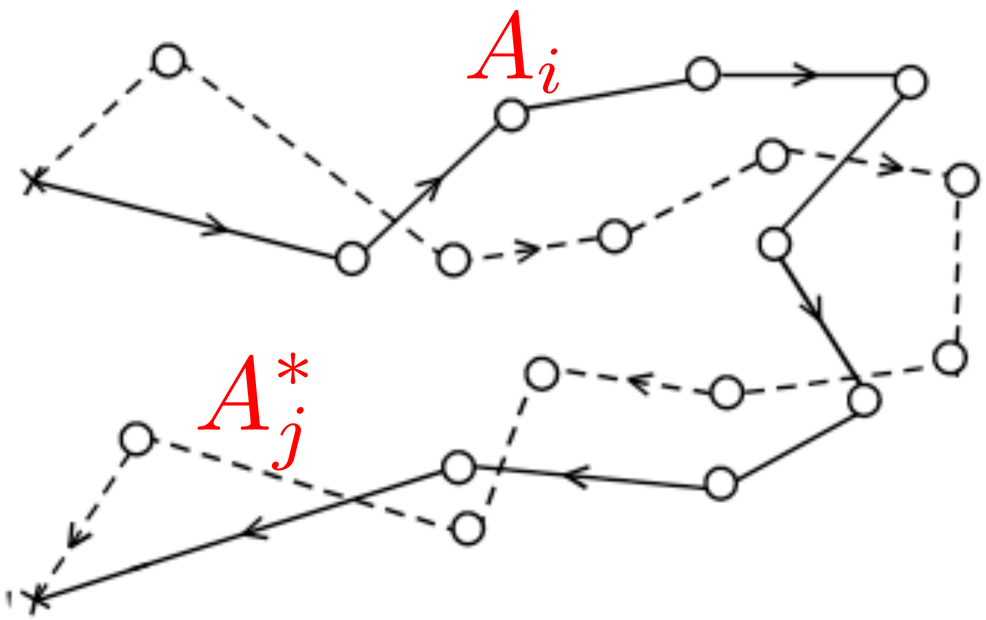
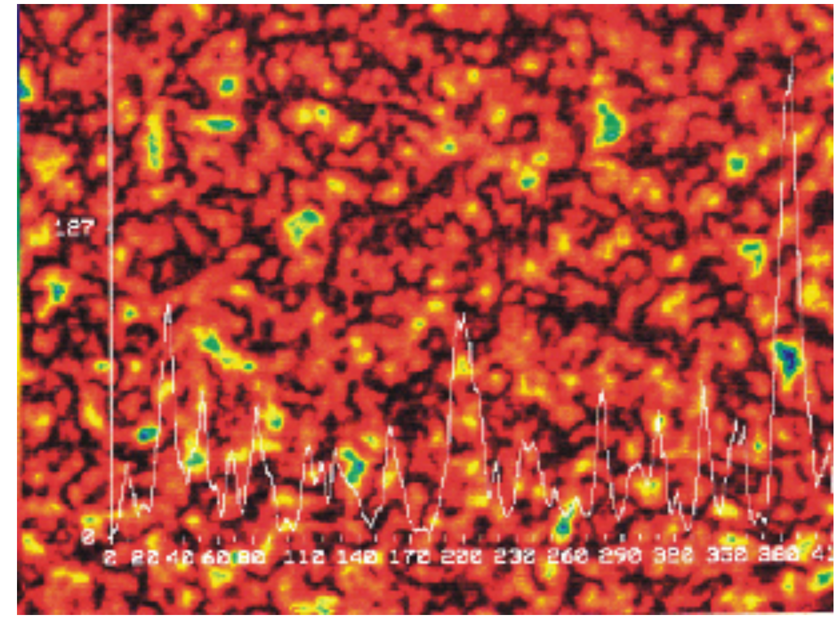


Vanishes upon averaging





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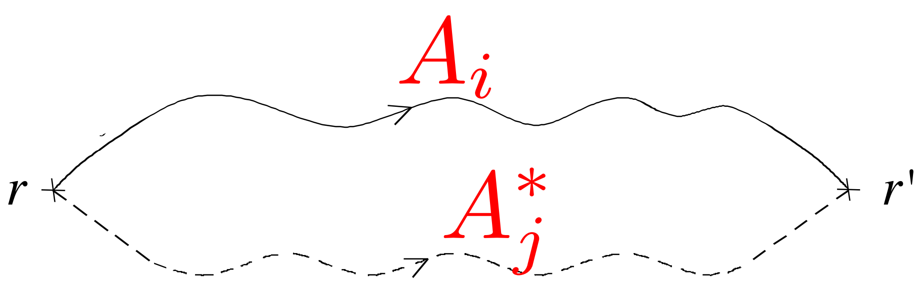

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Diffuson

To a good approximation, the incoherent contribution obeys a classical diffusion equation

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Incoherent electrons diffuse in the conductor with a *diffusion coefficient D*

with $D = \dfrac{v_g l}{3}$

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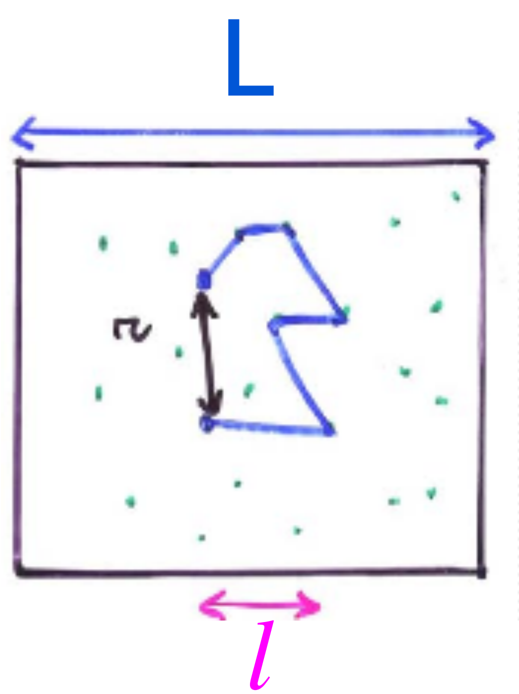

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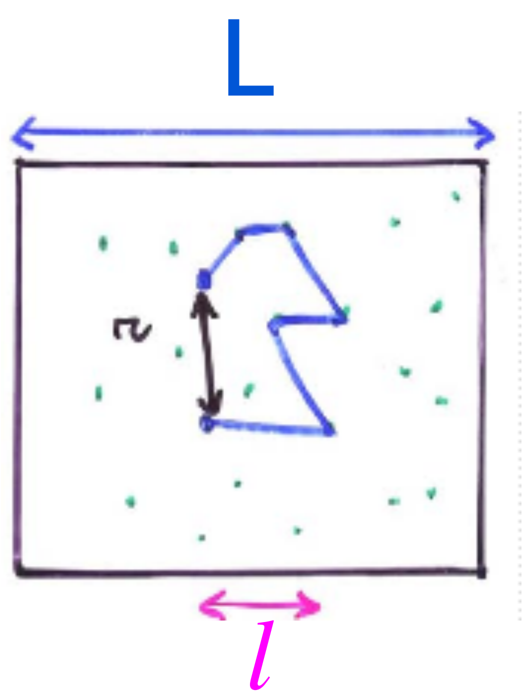

$$\left\langle r^2 \right\rangle = 2d\ Dt$$

space dimensionality

Thouless time

$$L^2 = D\tau_D$$

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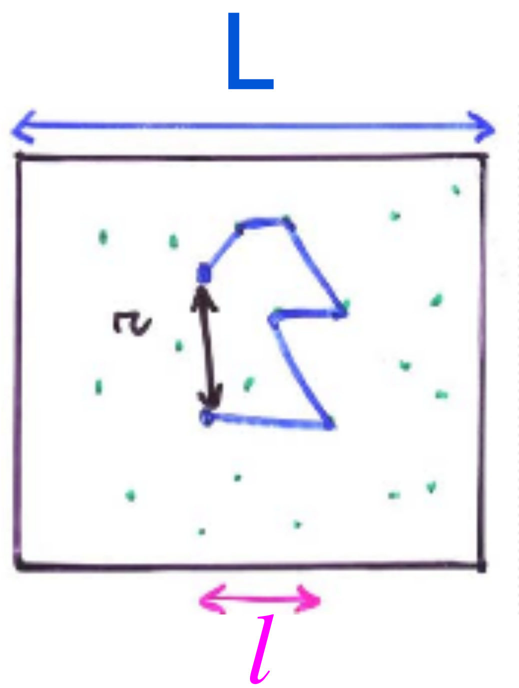

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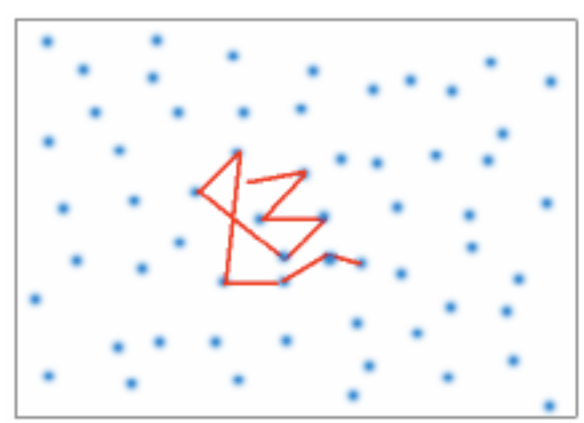

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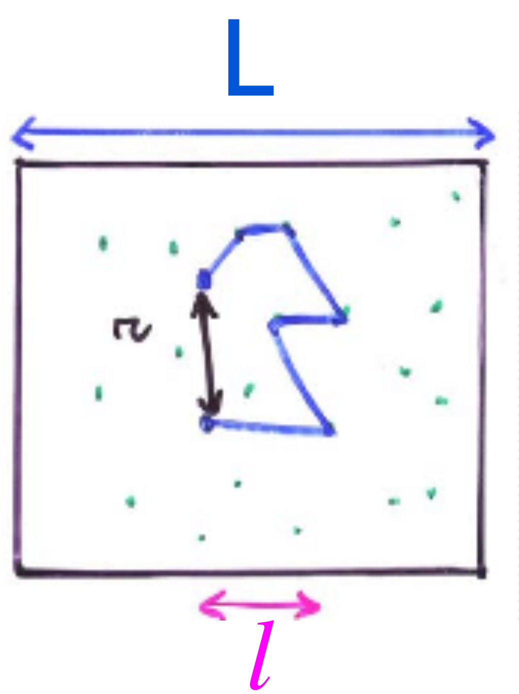

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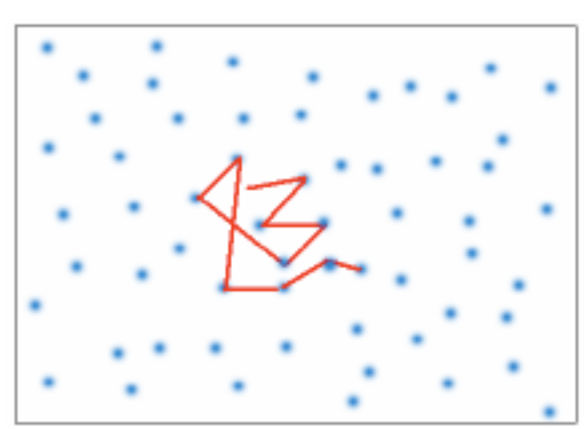

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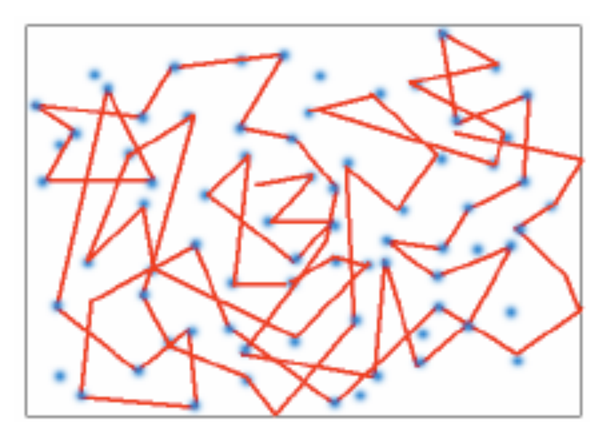

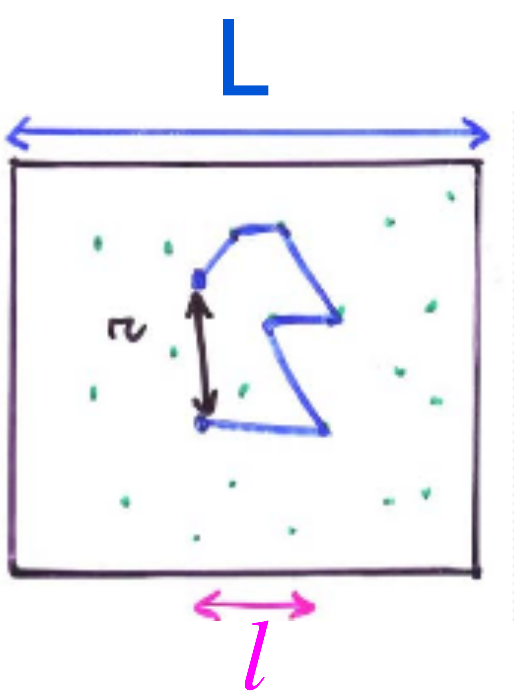

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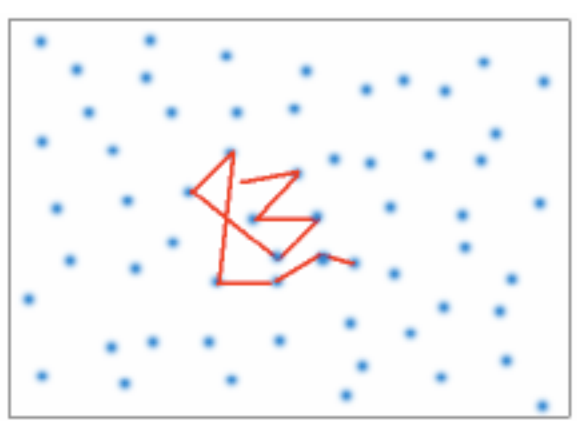

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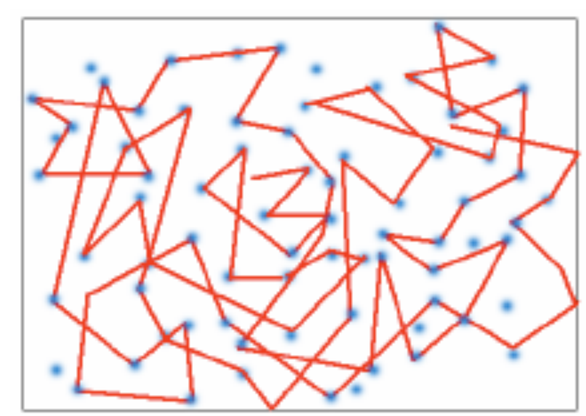

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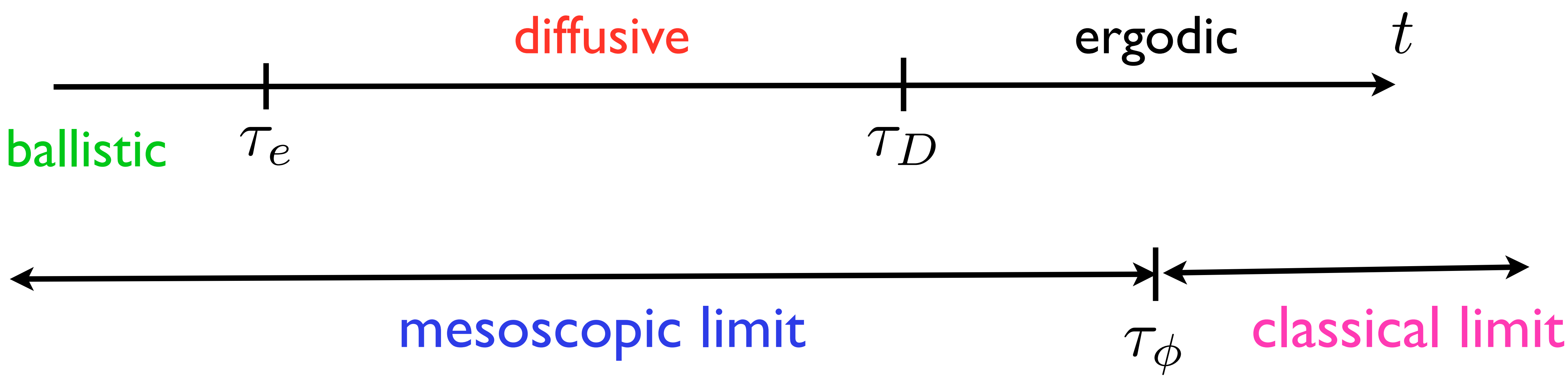

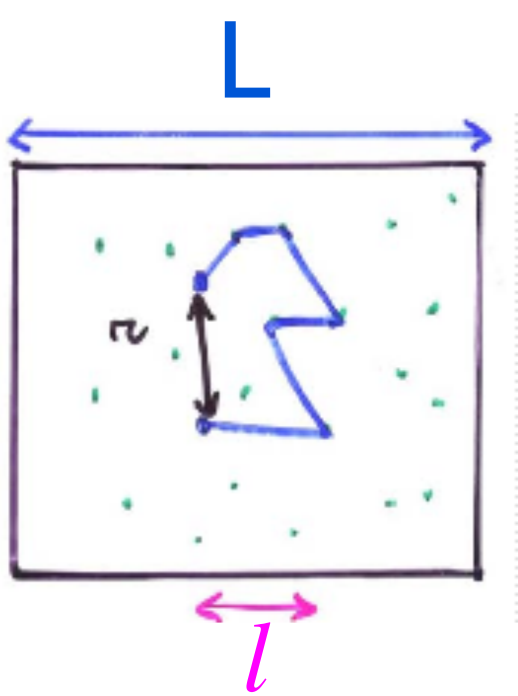

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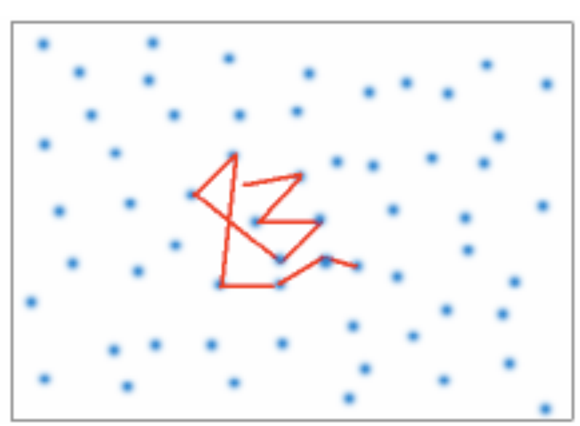

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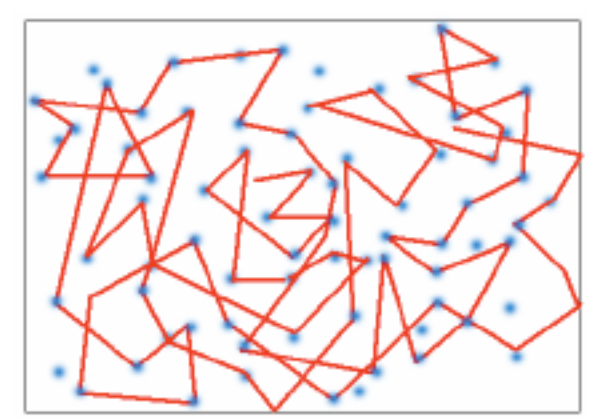
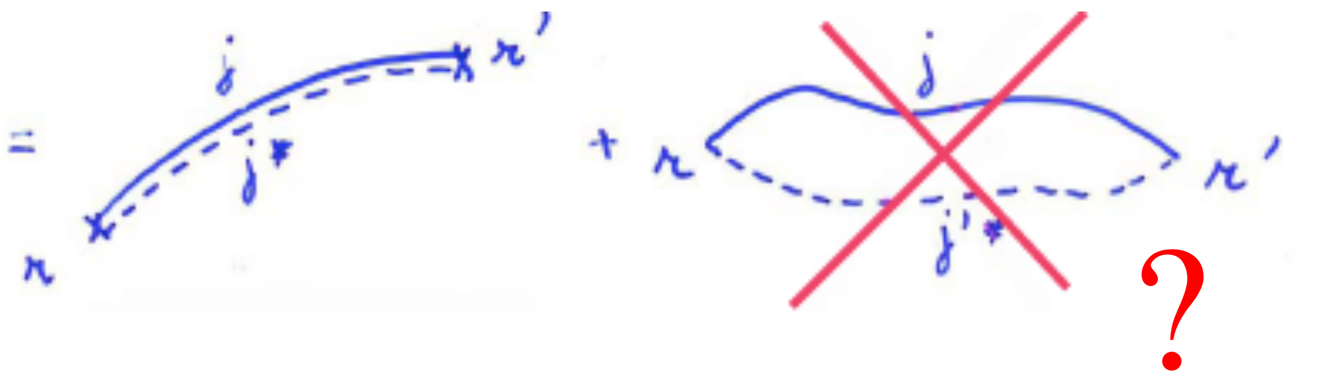


Coherent effects



What is the first correction *i.e.*, with the *smallest phase shift* ?

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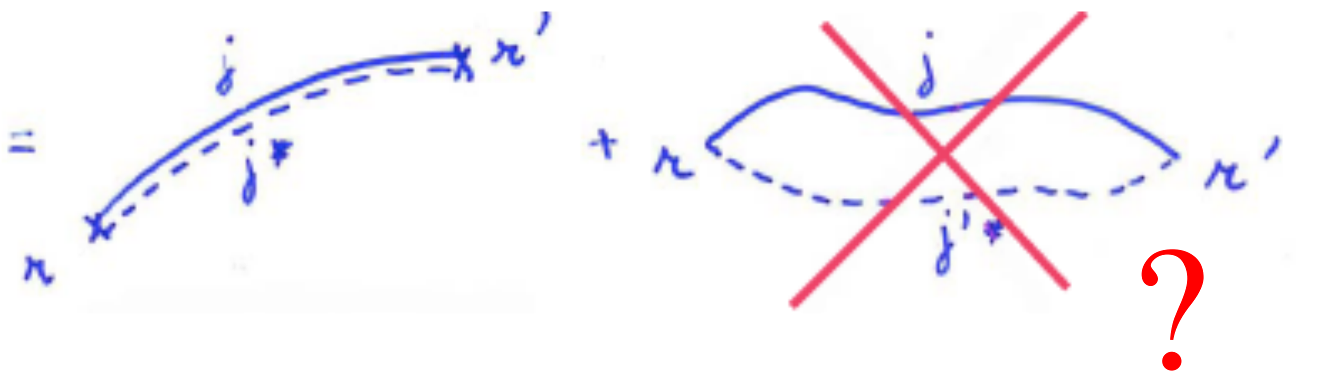

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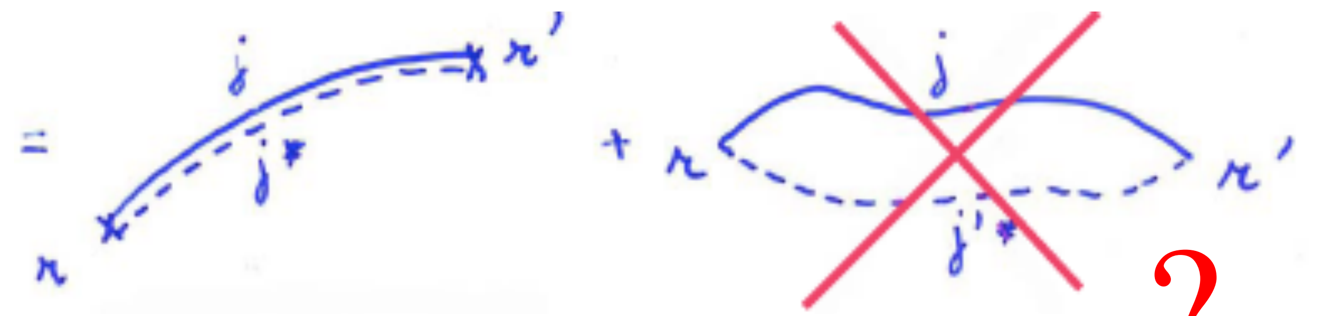

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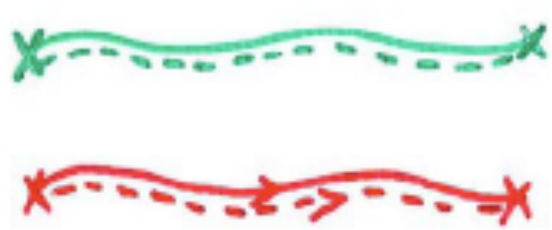
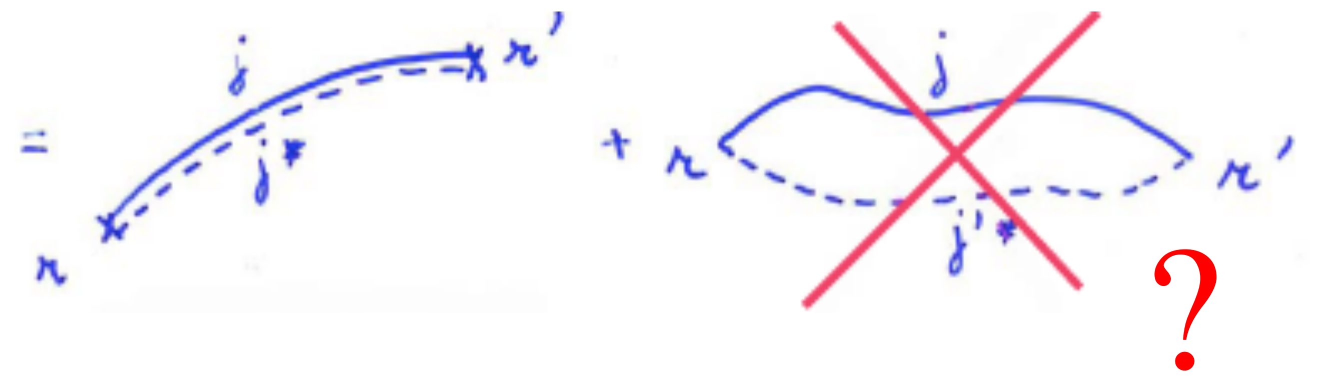
Example :



Classical diffusion

Coherent effects



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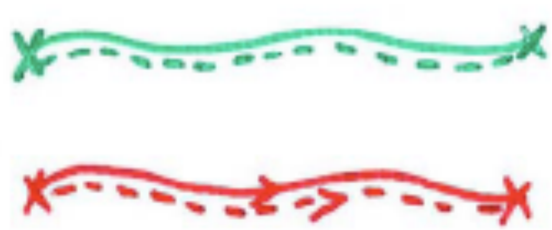

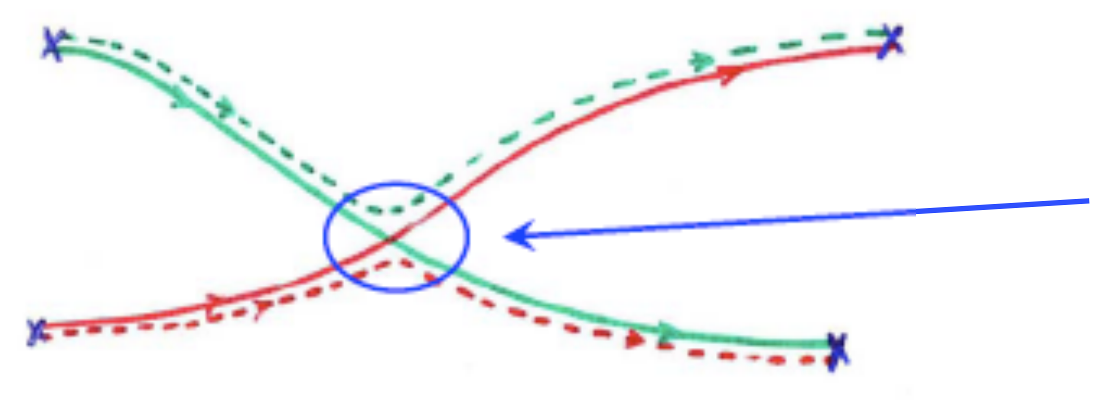

quantum crossing

Classical diffusion

Exchange of amplitudes

Occurrence of a *quantum crossing* after a time t for a photon diffusing in a volume L^d

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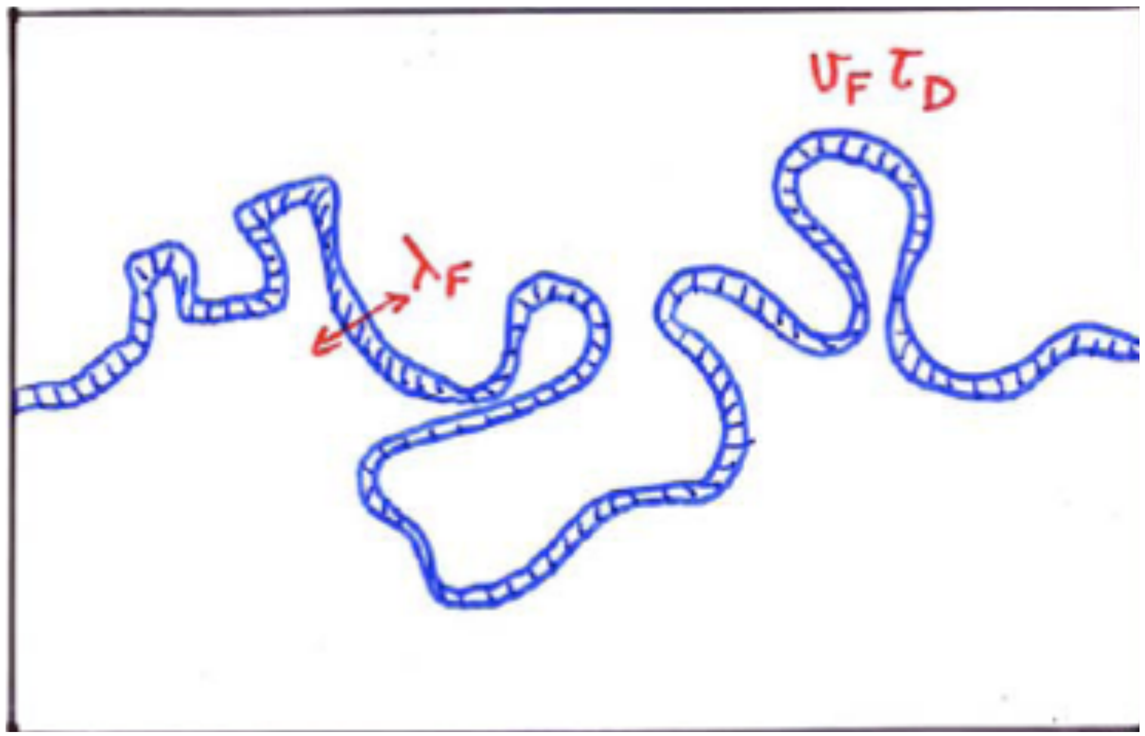

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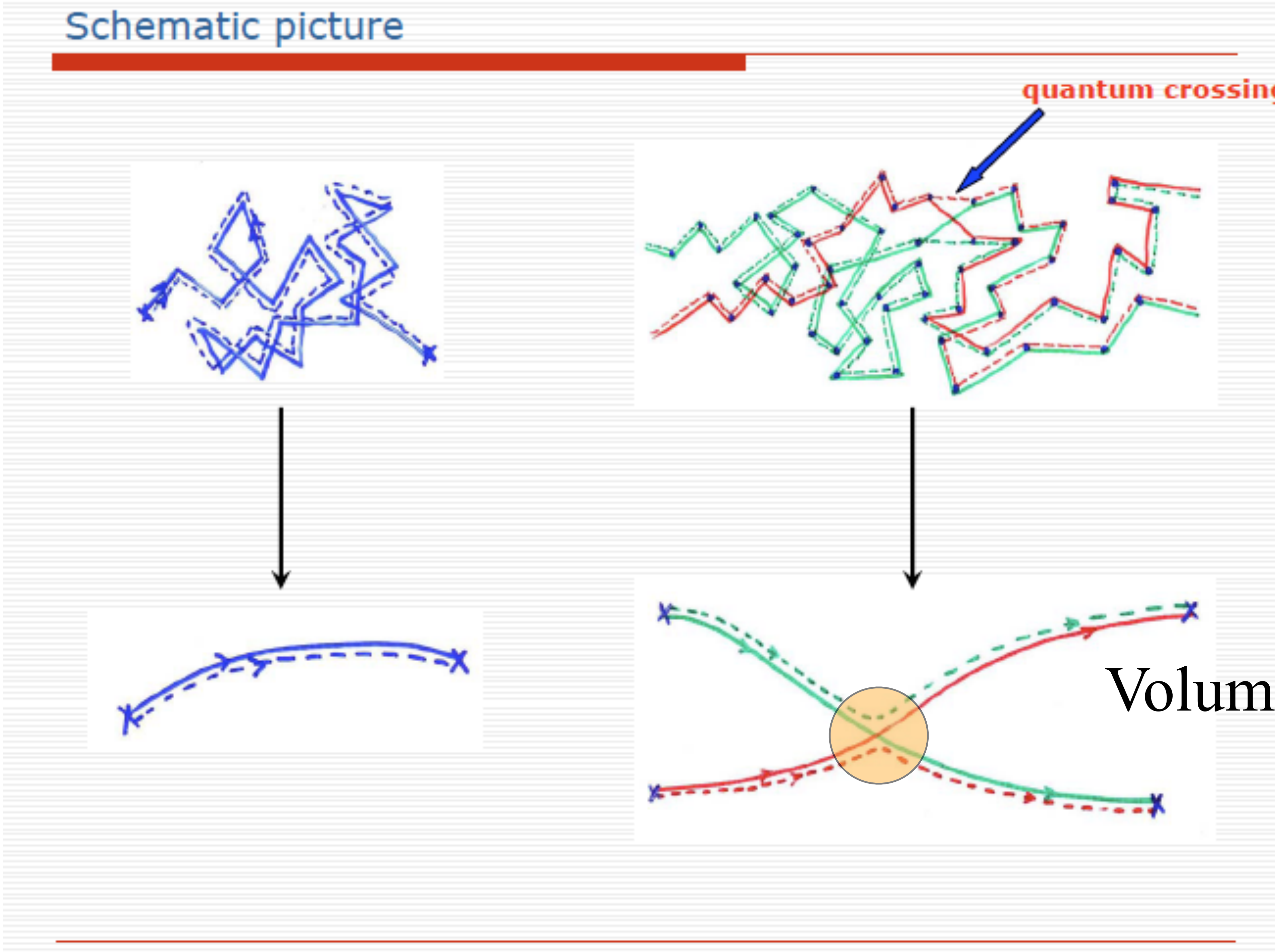
$$g = \frac{D}{c\lambda^{d-1}} L^{d-2}$$



Volume $\boxed{\lambda^{d-1}\, l}$

Quantum crossings decrease the diffusion coefficient D :

weak localization

λ : wavelength

$$g = \frac{D}{c\lambda^{d-1}} L^{d-2}$$

Physical meaning of this parameter ?

Electrical conductance of a metal

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$$g = \frac{l_e}{3\lambda^{d-1}} L^{d-2} = G_{cl}/(e^2/h)$$

G_{cl} is the classical electrical conductance so that

$$G_{cl}/(e^2/h) \gg 1$$

A direct consequence: quantum corrections to electrical transport

Classical transport : $G_{cl} = g \times \frac{e^2}{h} \,\mathrm{with}\, g \gg 1$

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Quantum corrections: $\Delta G = G_{cl} \times \frac{1}{g}$

so that $\boxed{\Delta G \simeq \frac{e^2}{h}}$

A direct consequence: quantum corrections to electrical transport

Classical transport : $G_{cl} = g \times \frac{e^2}{h}$ with $g \gg 1$

Independent of the microscopic (and often unknown) disorder - Depends only on the geometry

Quantun

so that $\boxed{\Delta G \simeq \frac{e^2}{h}}$ is universal

A direct consequence: quantum corrections to electrical transport

Cla[...] : $G_{cl} = g \times \frac{e^2}{h}$ with $g \gg 1$

Not that simple ! We wish to obtain precise numbers... Need to sum up Feynman diagrams.

Quantum correc[...]

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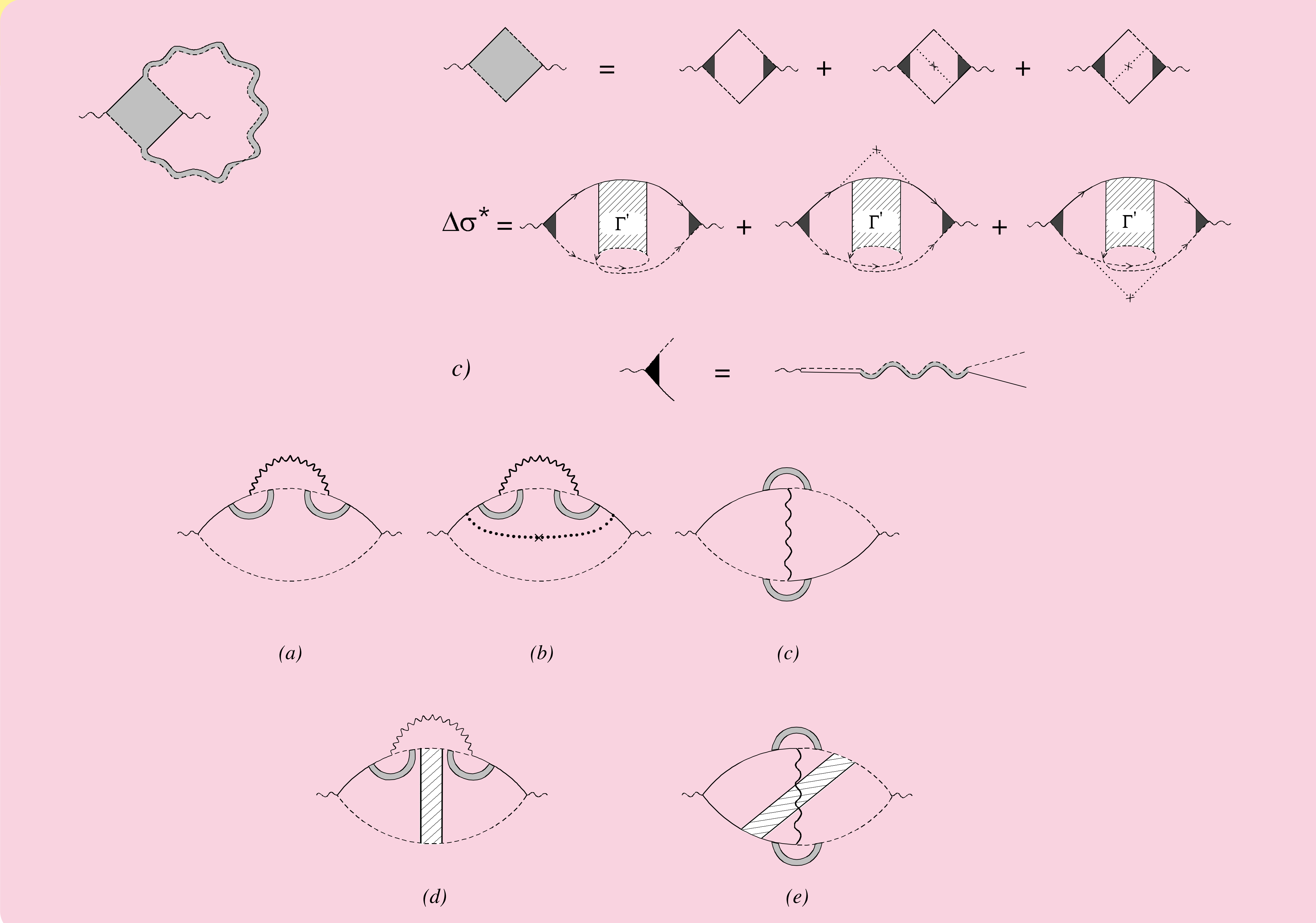

An intermezzo based on our understanding of coherent effects

A quantum phase transition: Anderson localization

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The diffusion coefficient D is reduced (weak localization) and becomes size dependent :

$$D(L) = D\left(1 - \frac{1}{\pi g}\ln\left(L/l\right) + \cancel{\left(\frac{1}{\pi g}\ln\left(L/l\right)\right)^2} + \dots\right) \qquad (d=2)$$

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A renormalization of *D(L)* changes also *g(L)*:

$$g(L) = \frac{D(L)}{c\,\lambda^{d-1}} L^{d-2}$$

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Scaling behavior :

$$\boxed{g(L,W) = f\left(L/\xi(W) \right)}$$

$\xi(W)$ is the localization length

Numerical calculations on the (universal) Anderson Hamiltonian

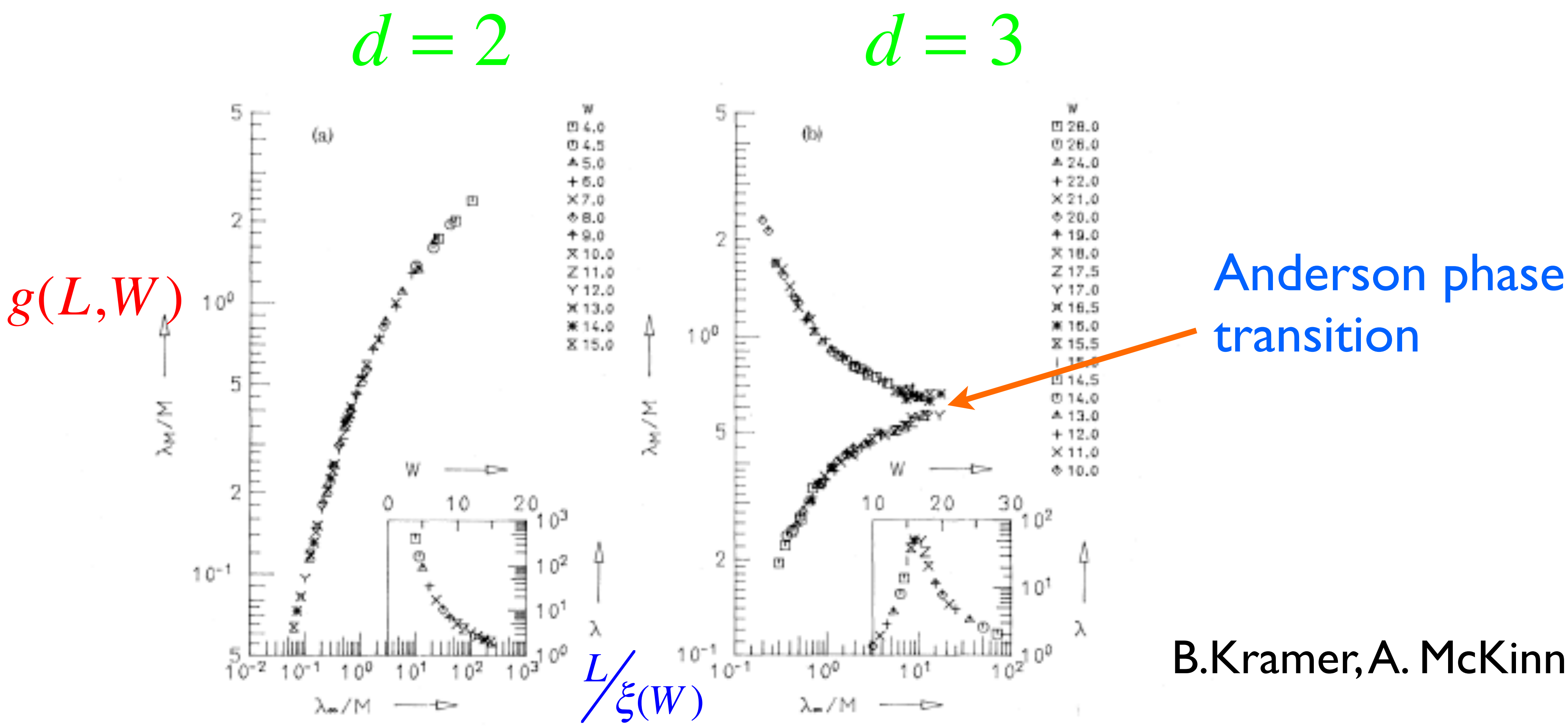

FIG. 1. Scaling function λ_M/M vs λ_∞/M for the localization length λ_M of a system of thickness M for (a) $d=2$ ($M \geq 4$) and (b) $d=3$ ($M \geq 3$). Insets show the scaling parameter λ_∞ as a function of the disorder W.

B.Kramer, A. McKinnon, 1981

Anderson localization phase transition occurs in $d > 2$

End of the intermezzo based on our understanding of coherent effects

Weak disorder physics

Weak disorder limit: $\lambda << l \Rightarrow g >> 1$

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Probability of a crossing $\left(\propto 1/g\right)$ is small: phase coherent corrections to the classical limit are small.

Weak disorder physics

Weak disorder limit: $\lambda << l \Rightarrow g >> 1$

Probability of a crossing $\left(\propto 1/g\right)$ is small: phase coherent corrections to the classical limit are small.

Quantum crossings modify the classical probability (*i.e*. the Diffuson).

Due to its long range behavior, the Diffuson propagates (localized) coherent effects over large distances.

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Quantum crossings are independently distributed :

We can generate higher order corrections to the Diffuson as an expansion in powers of $1/g$

Weak localization- Electronic transport

To the classical probability corresponds the Drude conductance G_{cl}

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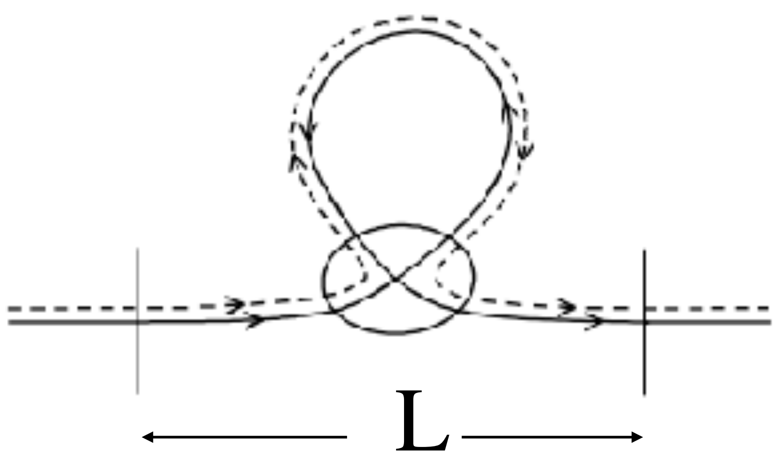
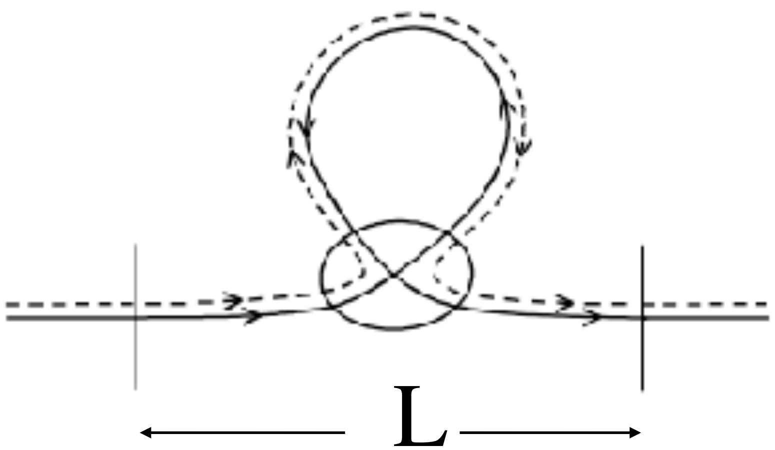


First correction $(\propto 1/g)$ involves one quantum crossing and the probability $p_o(\tau_D)$ to have a closed loop:

Weak localization- Electronic transport

$\Longrightarrow$ To the classical probability corresponds the Drude conductance G_{cl}



First correction $(\propto 1/g)$ involves one quantum crossing and the probability $p_o(\tau_D)$ to have a closed loop:

$$\boxed{\frac{\Delta G}{G_{cl}} = -p_o(\tau_D)}$$

$$\tau_D = L^2/D$$

quantum correction decreases the conductance: weak localization

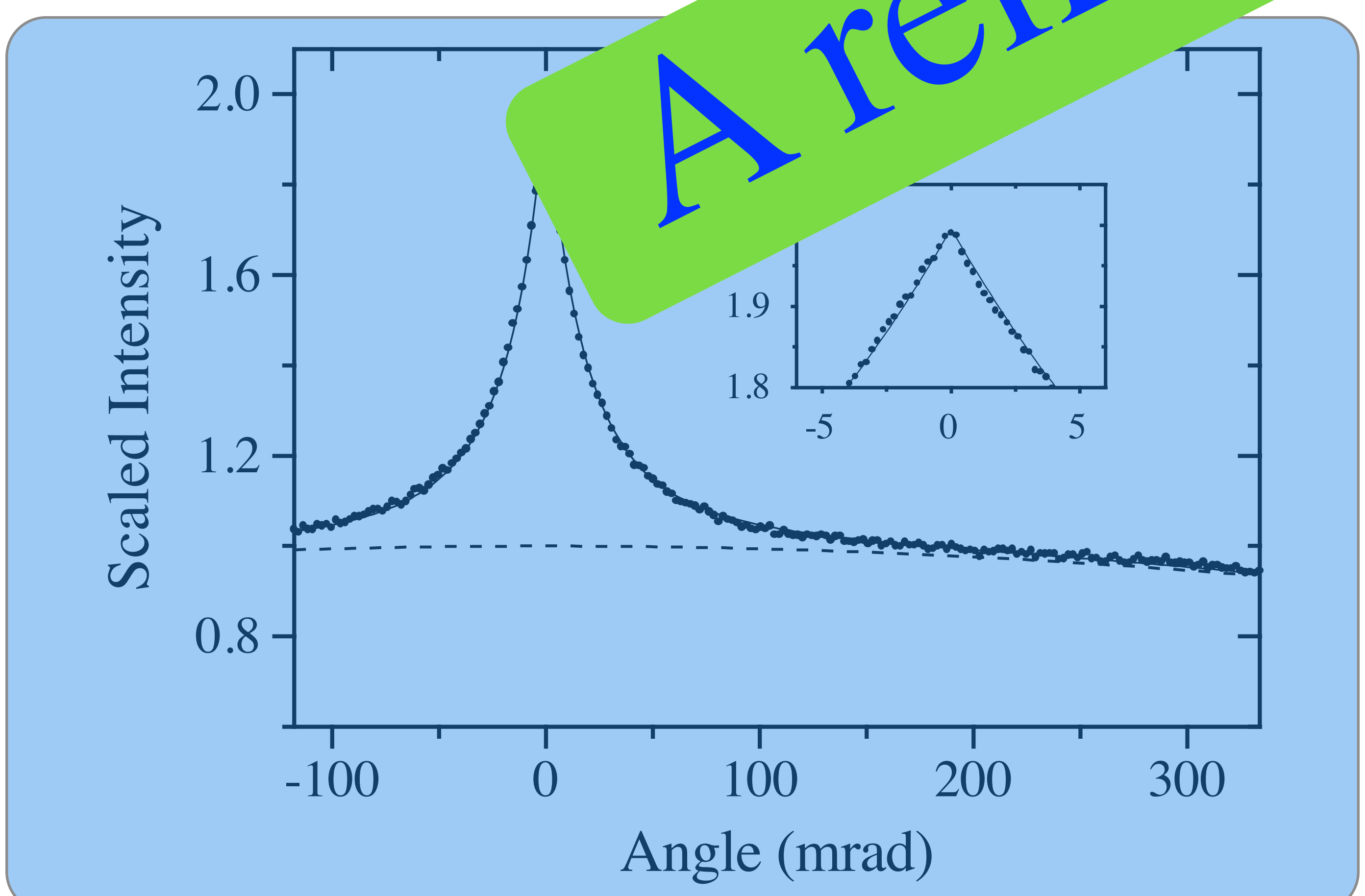
$$\overline{|A(\mathbf{k},\mathbf{k}')|^2} = \overline{\sum_{\mathbf{r_1},\mathbf{r_2}} |f(\mathbf{r_1},\mathbf{r_2})|^2 \left[1 + e^{i(\mathbf{k}+\mathbf{k}').(\mathbf{r_1}-\mathbf{r_2})}\right]}$$

Generally, the interference term vanishes due to the sum over $\mathbf{r_1}$ and $\mathbf{r_2}$, except for two notable cases:

$\mathbf{k} + \mathbf{k}' \simeq 0$: Coherent backscatterin

$\mathbf{r_1} - \mathbf{r_2} \simeq 0$: closed loops, w n and $\phi_0/2$ periodicity of the Sha

A reminder !



Coherent backscattering

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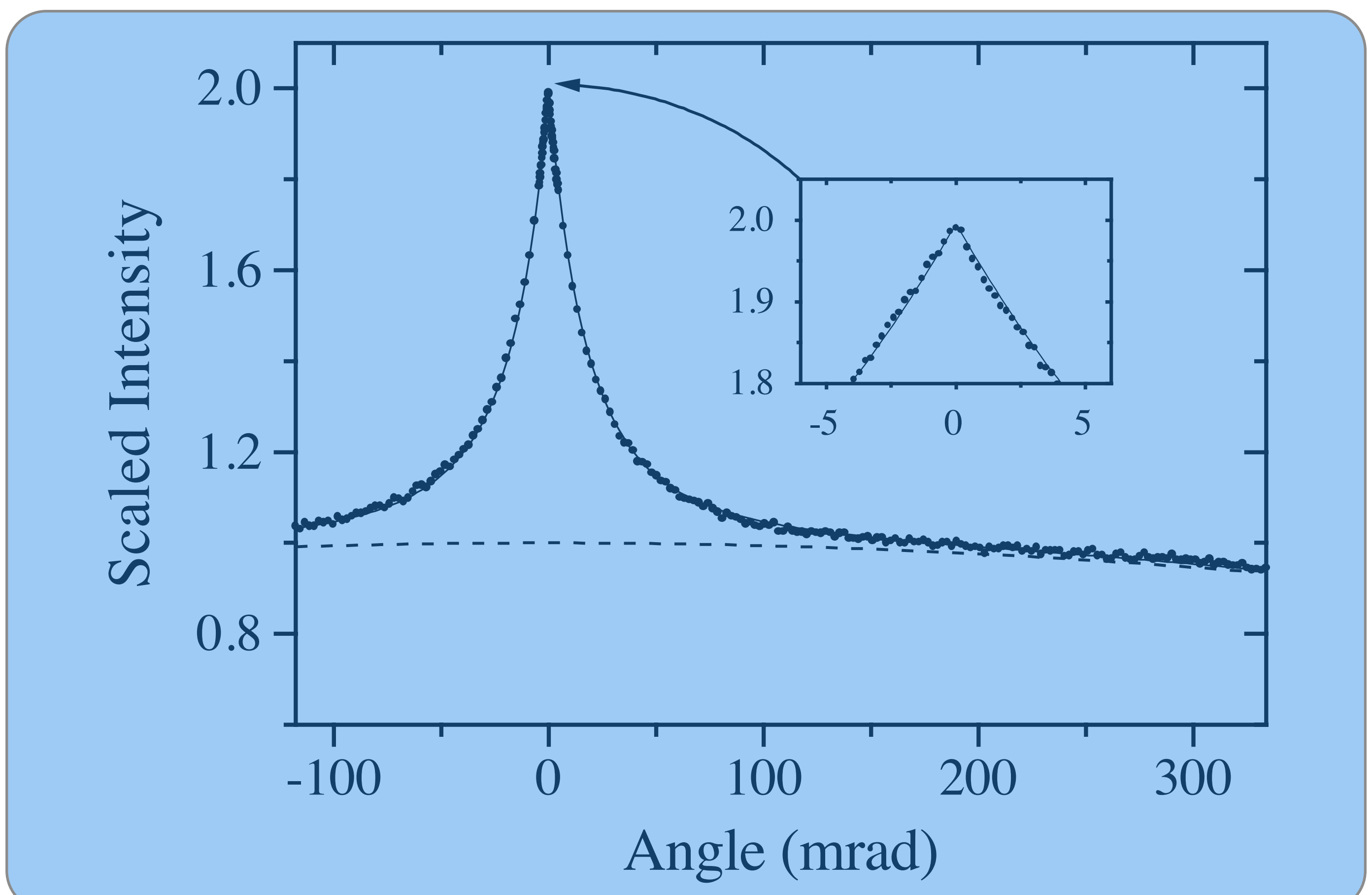

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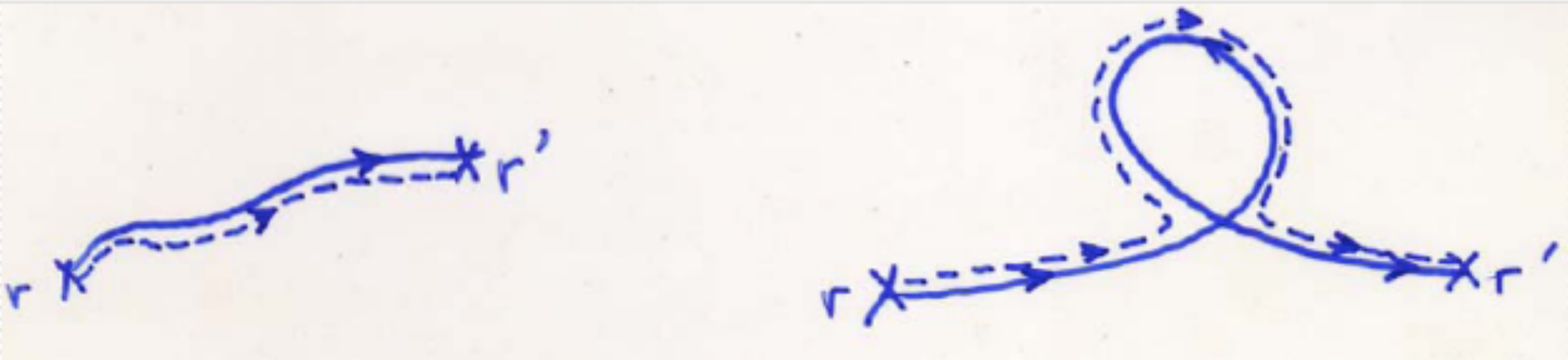
this case

How to calculate $P_{\text{int}}(t)$?

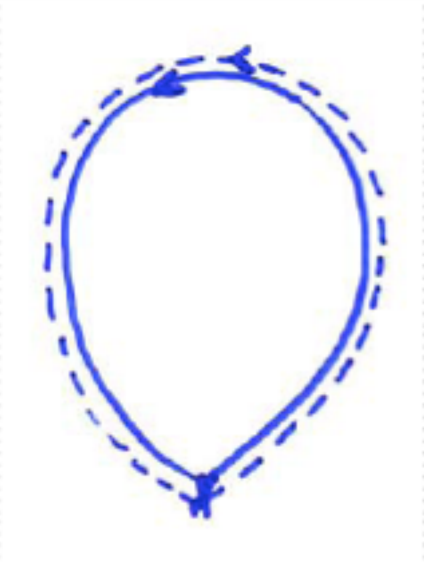
$$P_{\text{int}}(t) = \int P_{\text{int}}(r,r,t)d^d r$$



$P_{cl}(r,r',t)$ $\qquad$ $P_{\text{int}}(r,r',t) \sim \dfrac{1}{g}$ if $r \neq r'$

If $r = r'$

Diffuson



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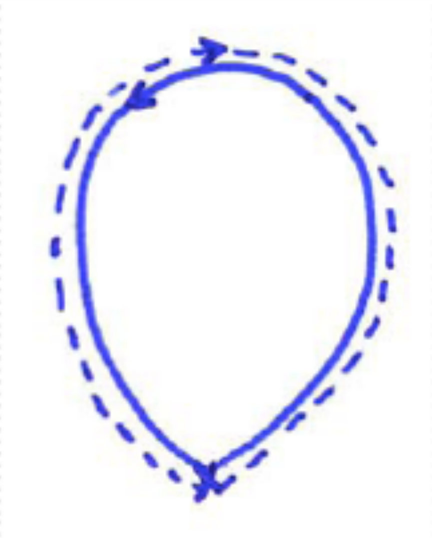

Cooperon

Classical return probability $\qquad$ Interference term

$$P_{cl}(r,r,t) = P_{\text{int}}(r,r,t)$$

Return probability is doubled !! $\qquad$ If time reversal invariance

In the presence of a dephasing mechanism that breaks time coherence, only trajectories with $t < \tau_\phi$ contribute.

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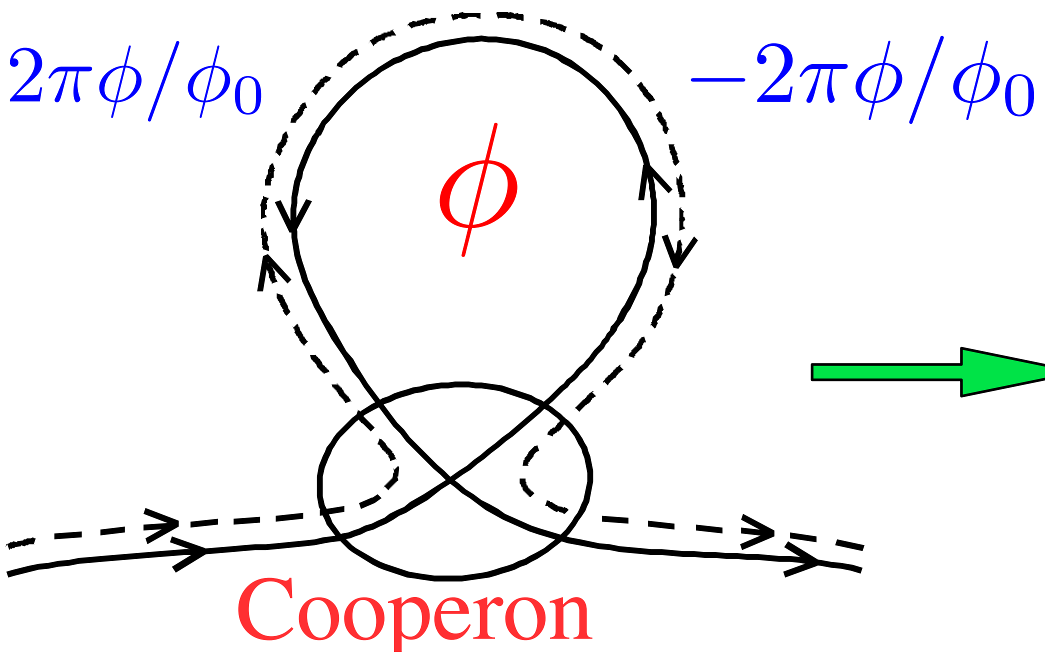

the phase difference becomes: $4\pi\phi/\phi_0$

$\phi_0/2$ periodicity of the Sharvin effect

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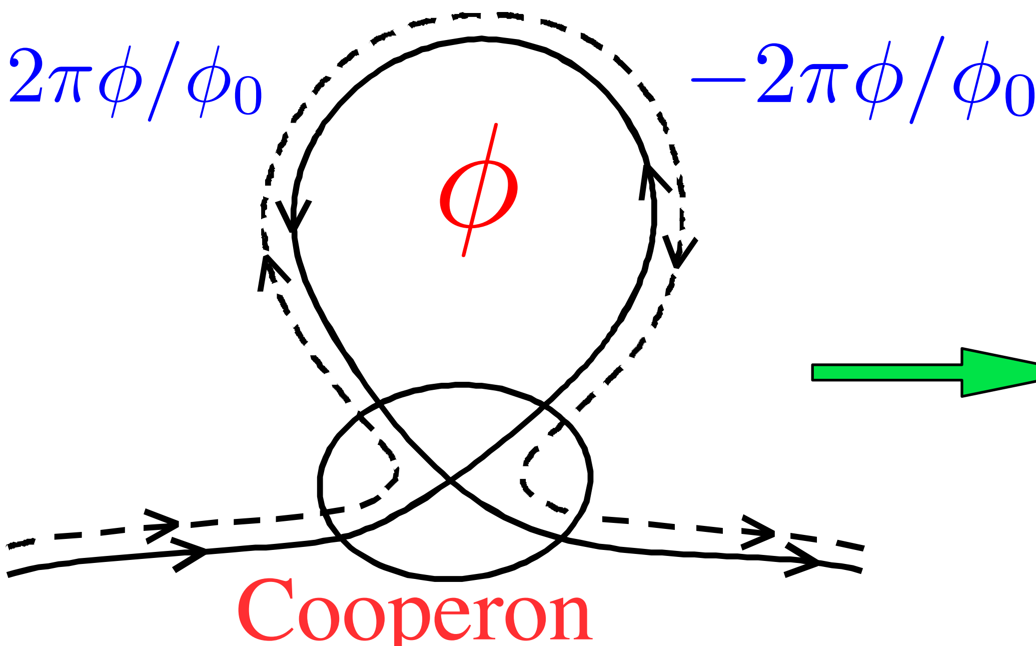

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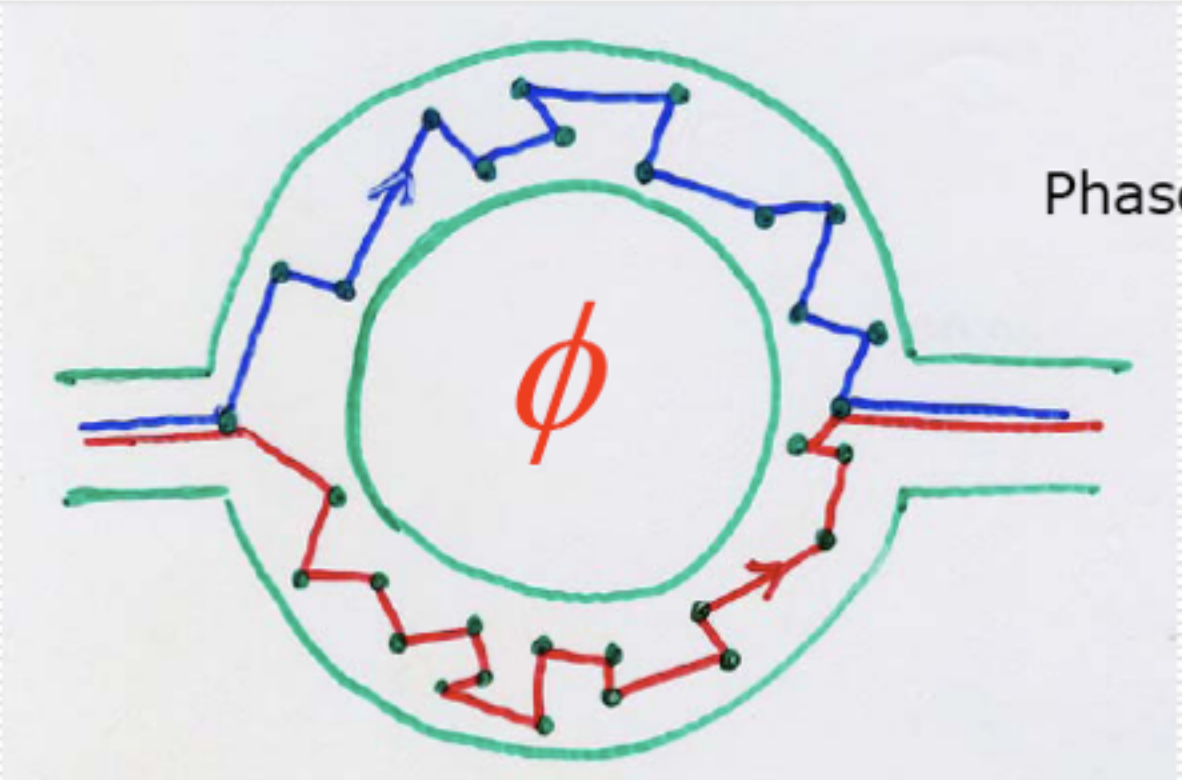
$\phi_0/2$ periodicity of the Sharvin effect

$P_{int}(r, r', t)$ is obtained from the *covariant* diffusion equation

effective charge 2e

$$\left(\frac{1}{\tau_\phi} + \frac{\partial}{\partial t} - D\left[\nabla_{r'} + i\frac{2e}{\hbar}\mathbf{A}(r')\right]^2\right) P_{int}(r, r', t) = \delta(r - r')\delta(t)$$

Webb

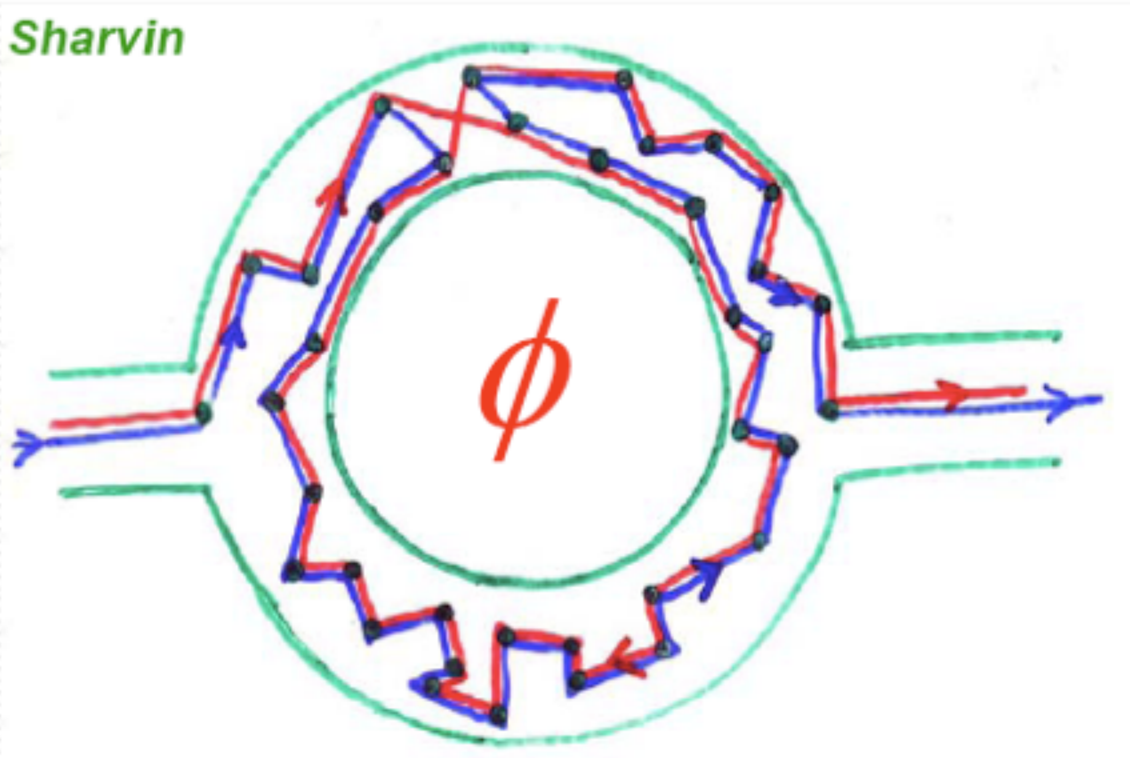


Sample specific interference

Phase difference $2\pi \frac{\phi}{\phi_0}$

Oscillates with period h/e

Sharvin, Sharvin



Survives disorder average

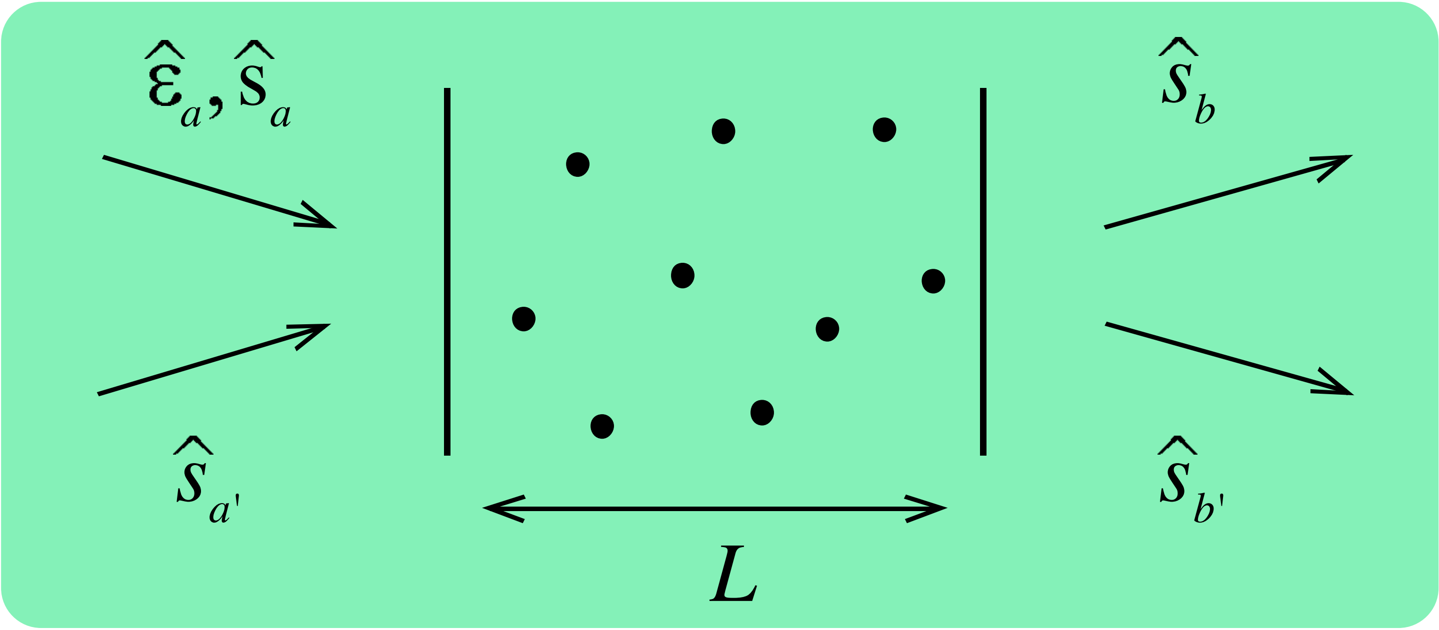
Phase difference $4\pi \frac{\phi}{\phi_0}$

Oscillates with period $h/2e$

Back to coherent effects for light

An analogous problem: *Speckle patterns in optics*

Consider the elastic multiple scattering of light transmitted through a fixed disorder configuration.



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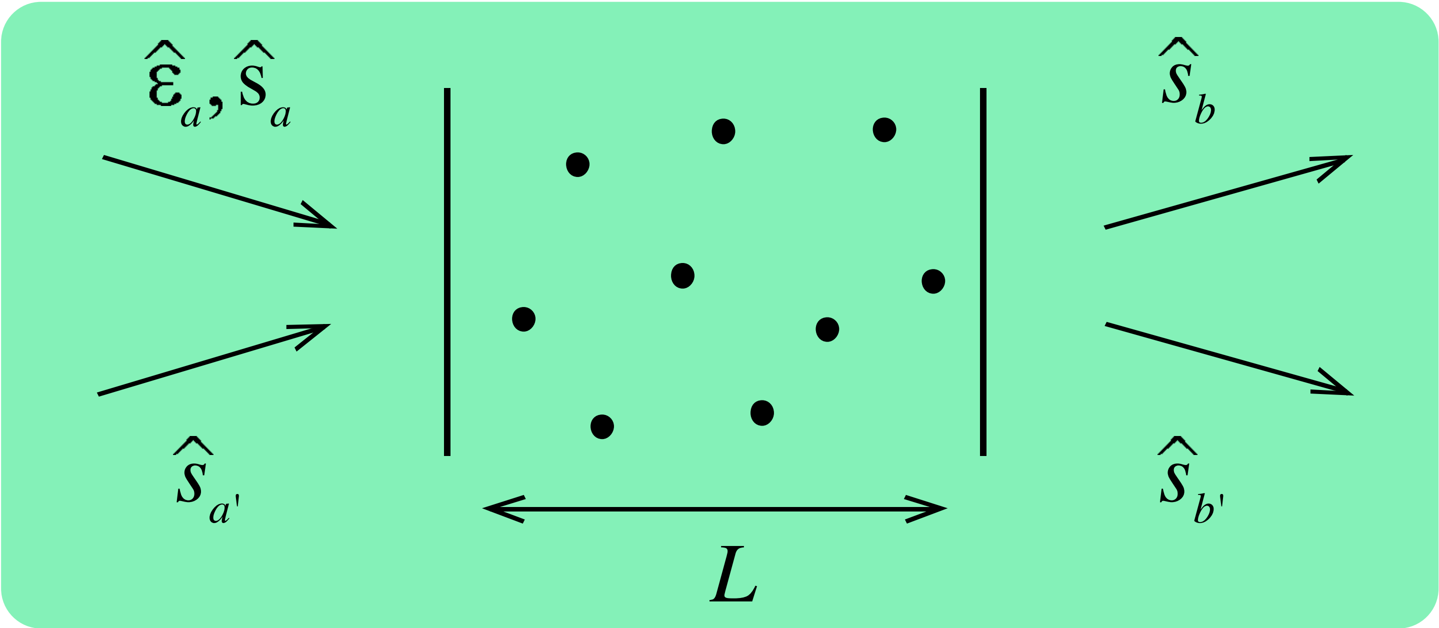
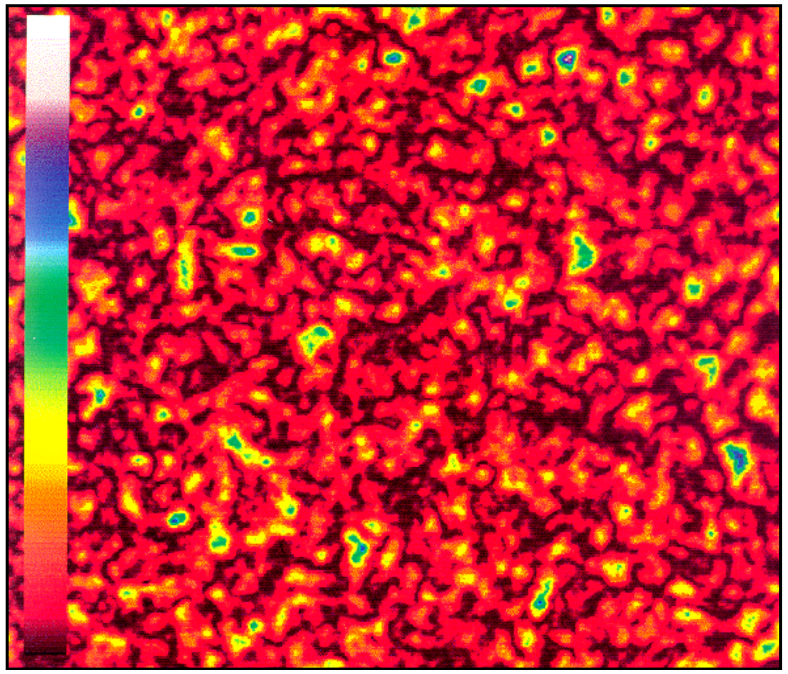
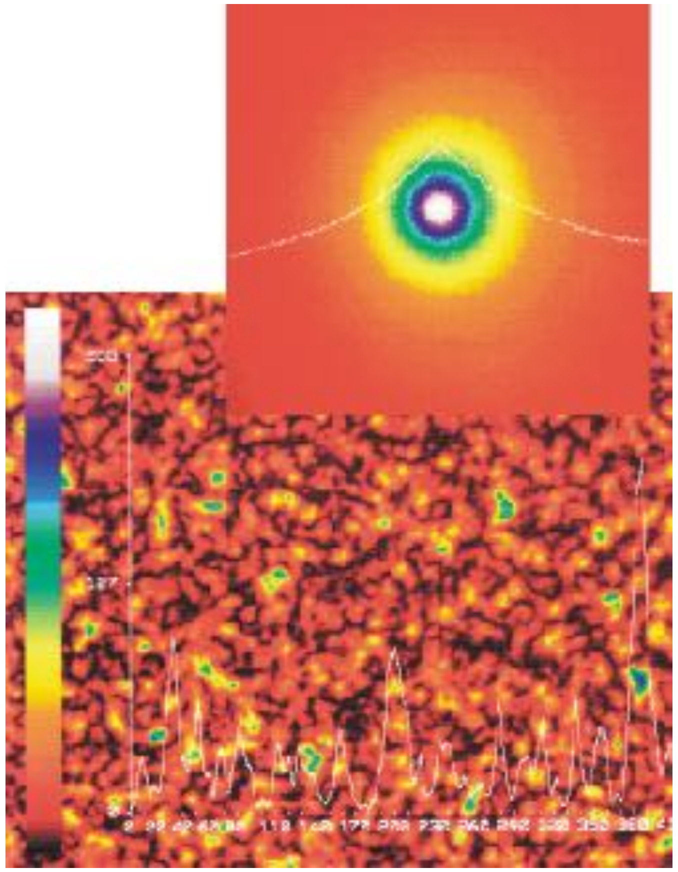
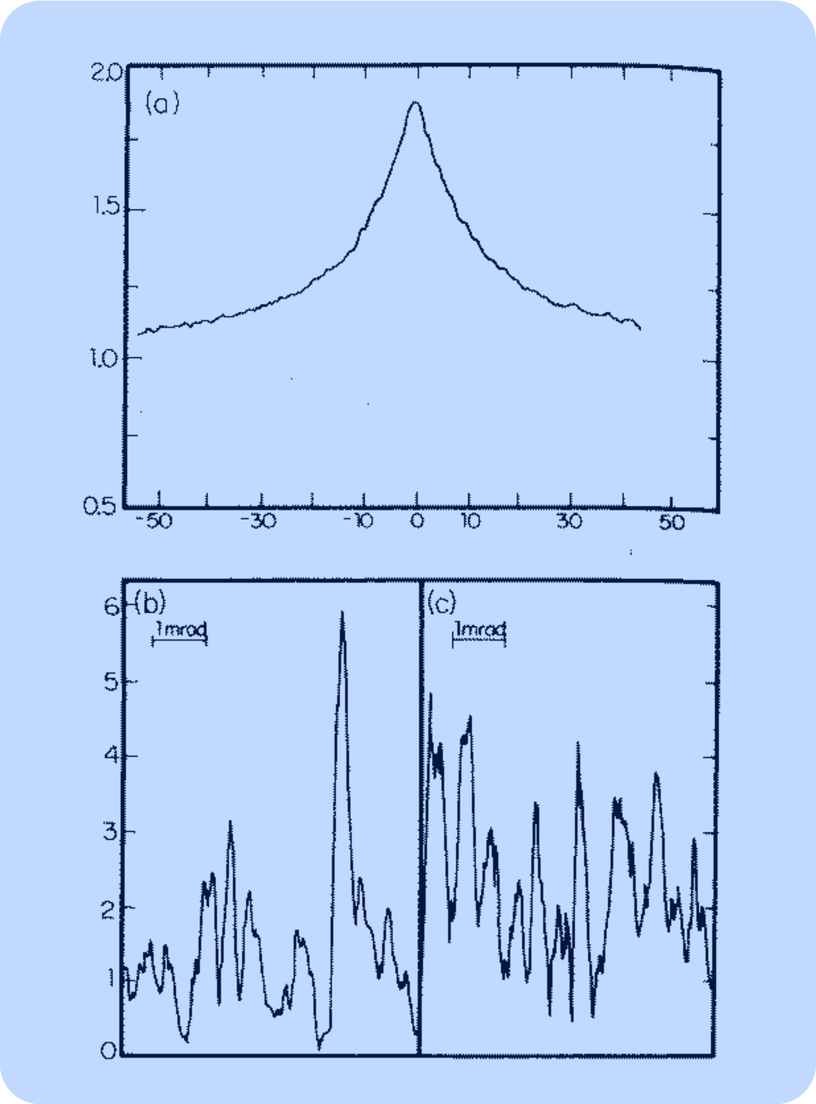


Outgoing light builds a speckle pattern *i.e.*, an interference picture:



What about speckle patterns ?



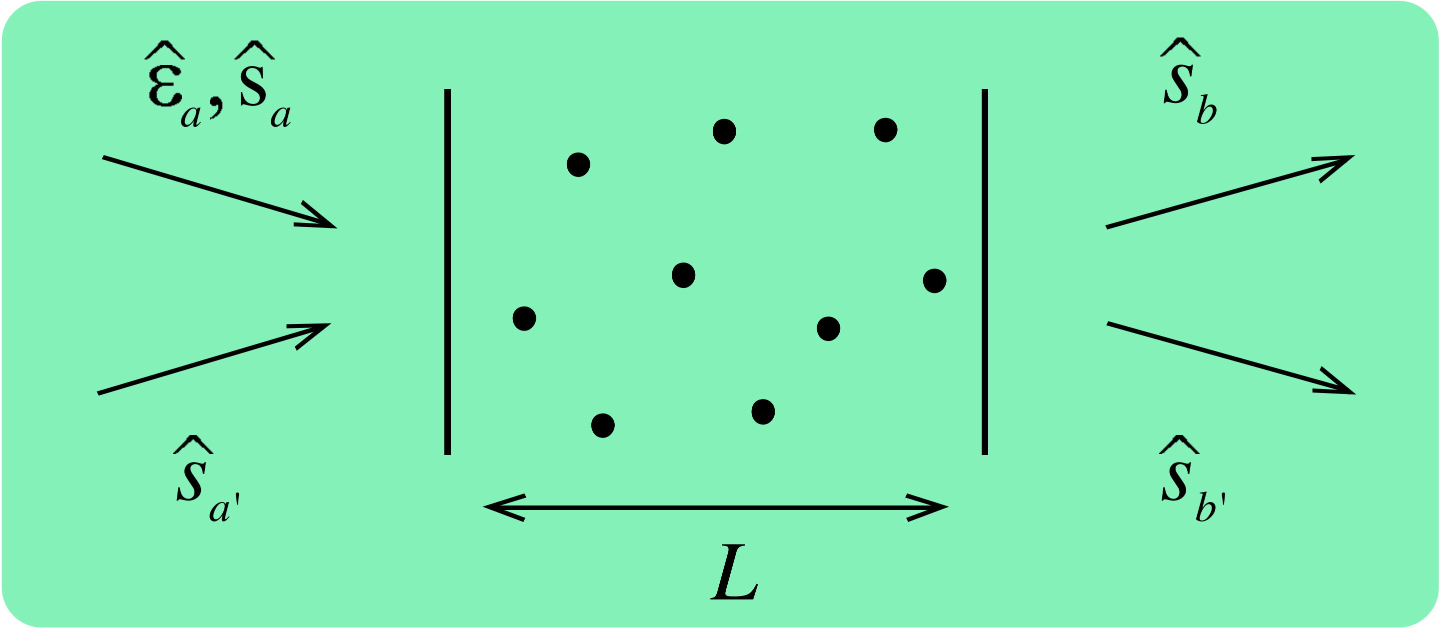


Averaging over disorder does not produce incoherent intensity only, but also an angular dependent part, the <u>coherent backscattering</u>, which is a coherence effect. We may conclude:

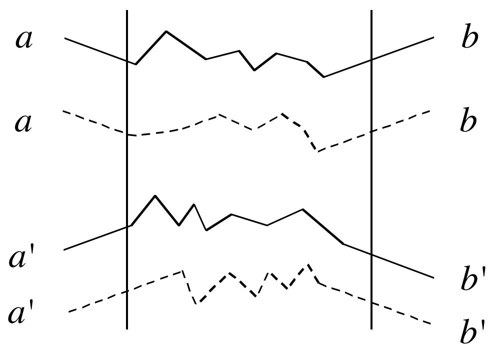
Elastic disorder is not related to decoherence : disorder does not destroy phase coherence and does not introduce irreversibility.

Fluctuations and correlations

Slab geometry



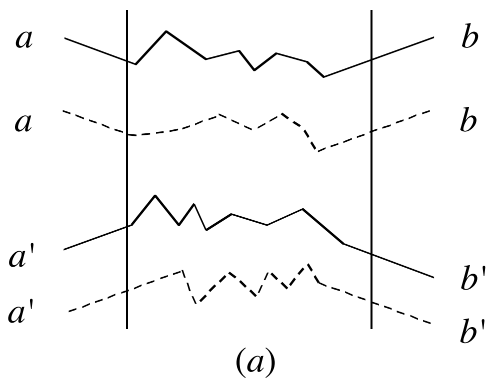
Fluctuations and correlations



transmission coefficient

$$T_{ab} = \left| t_{ab} \right|^2$$

Fluctuations and correlations



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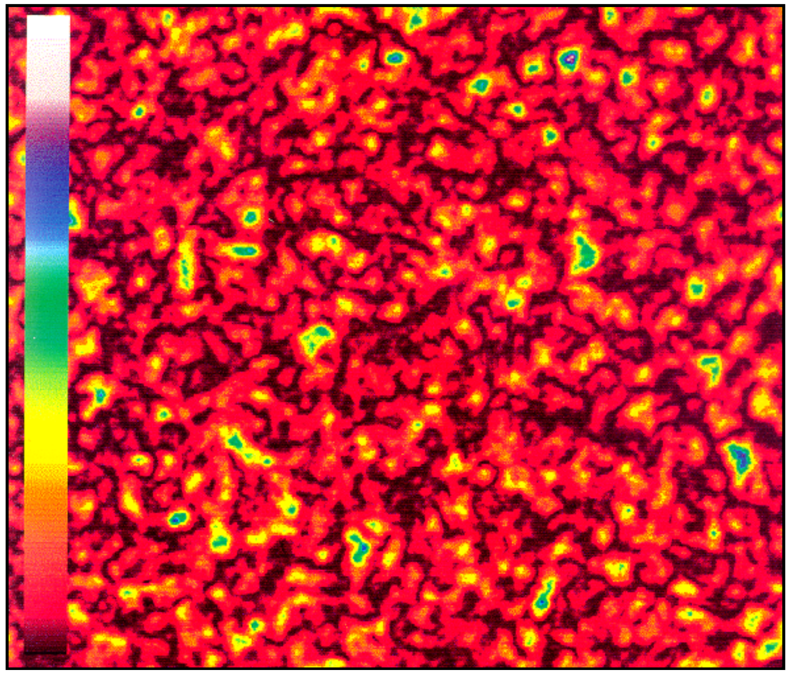
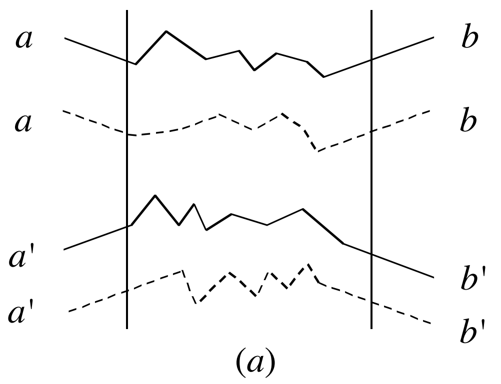
correlations involve the product of 4 complex amplitudes with or without quantum crossings



Correlation function of the transmission coefficient :

$$C_{aba'b'} = \frac{\overline{\delta T_{ab} \delta T_{a'b'}}}{\overline{T}_{ab} \overline{T}_{a'b'}}$$

Fluctuations and correlations



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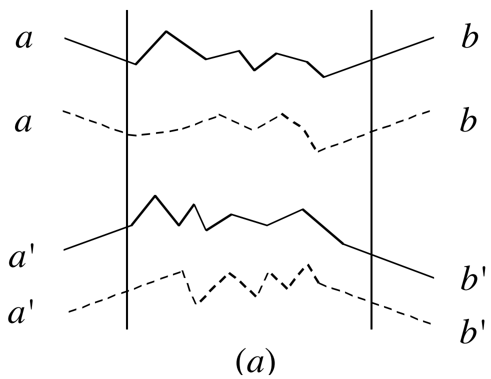

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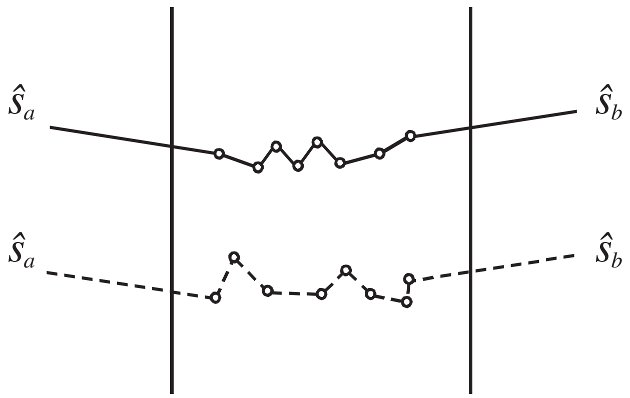

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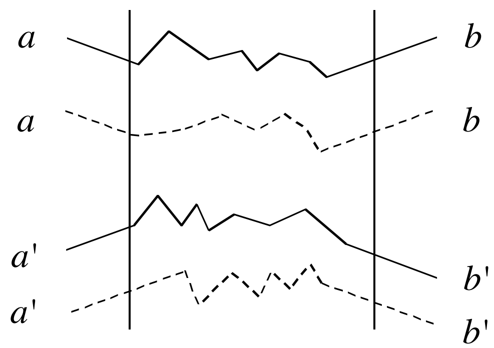

(a)

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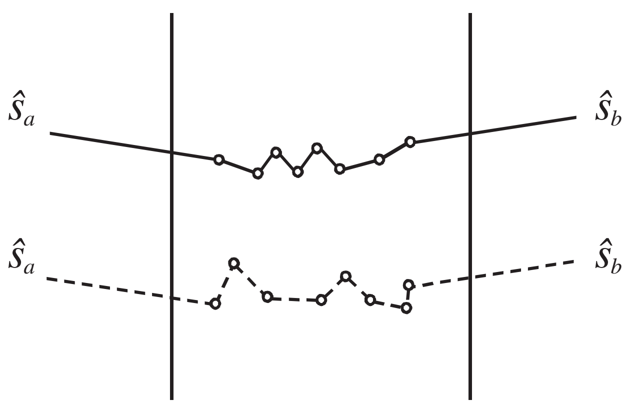

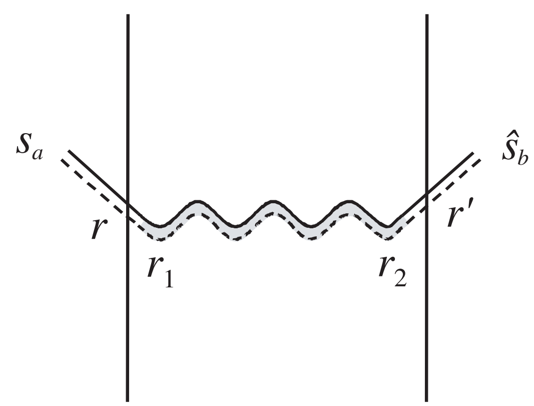

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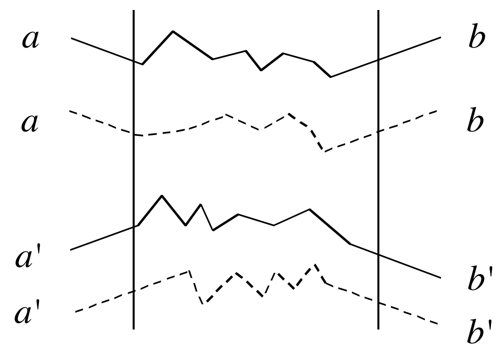

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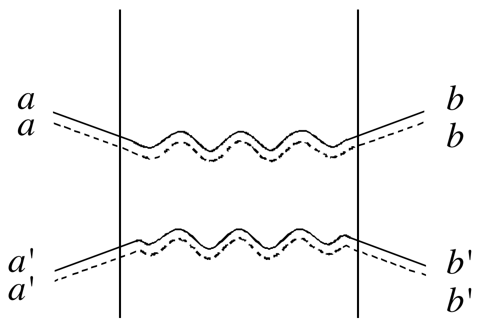

(b)

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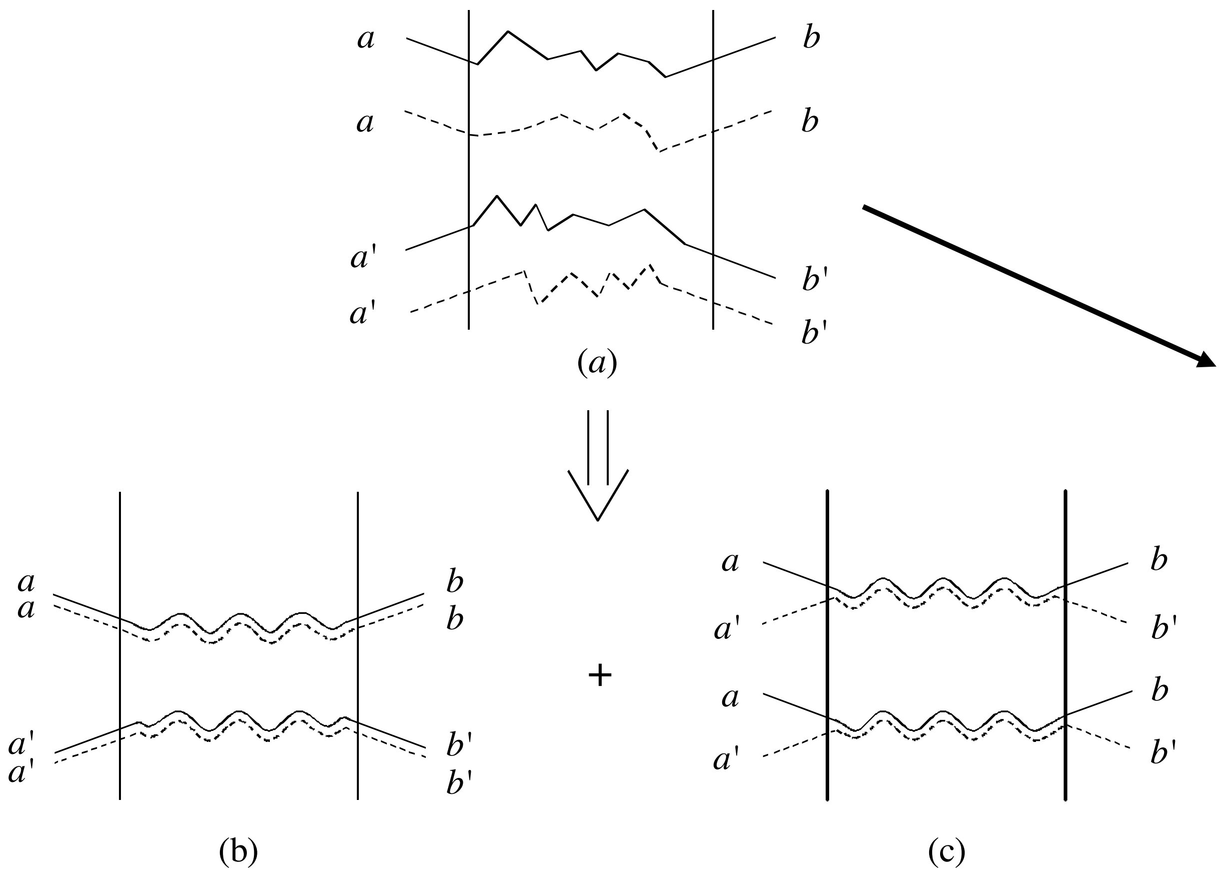

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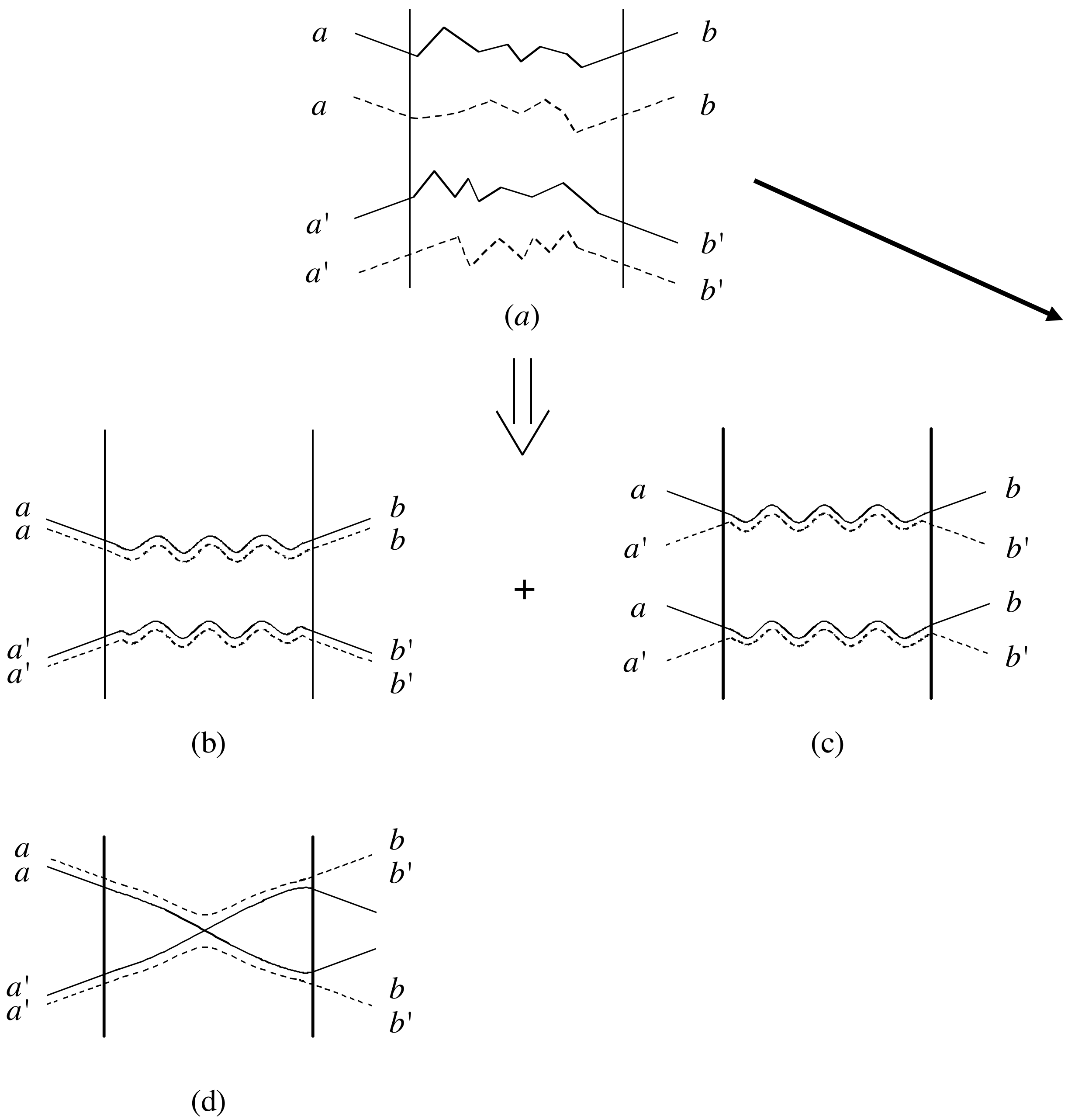

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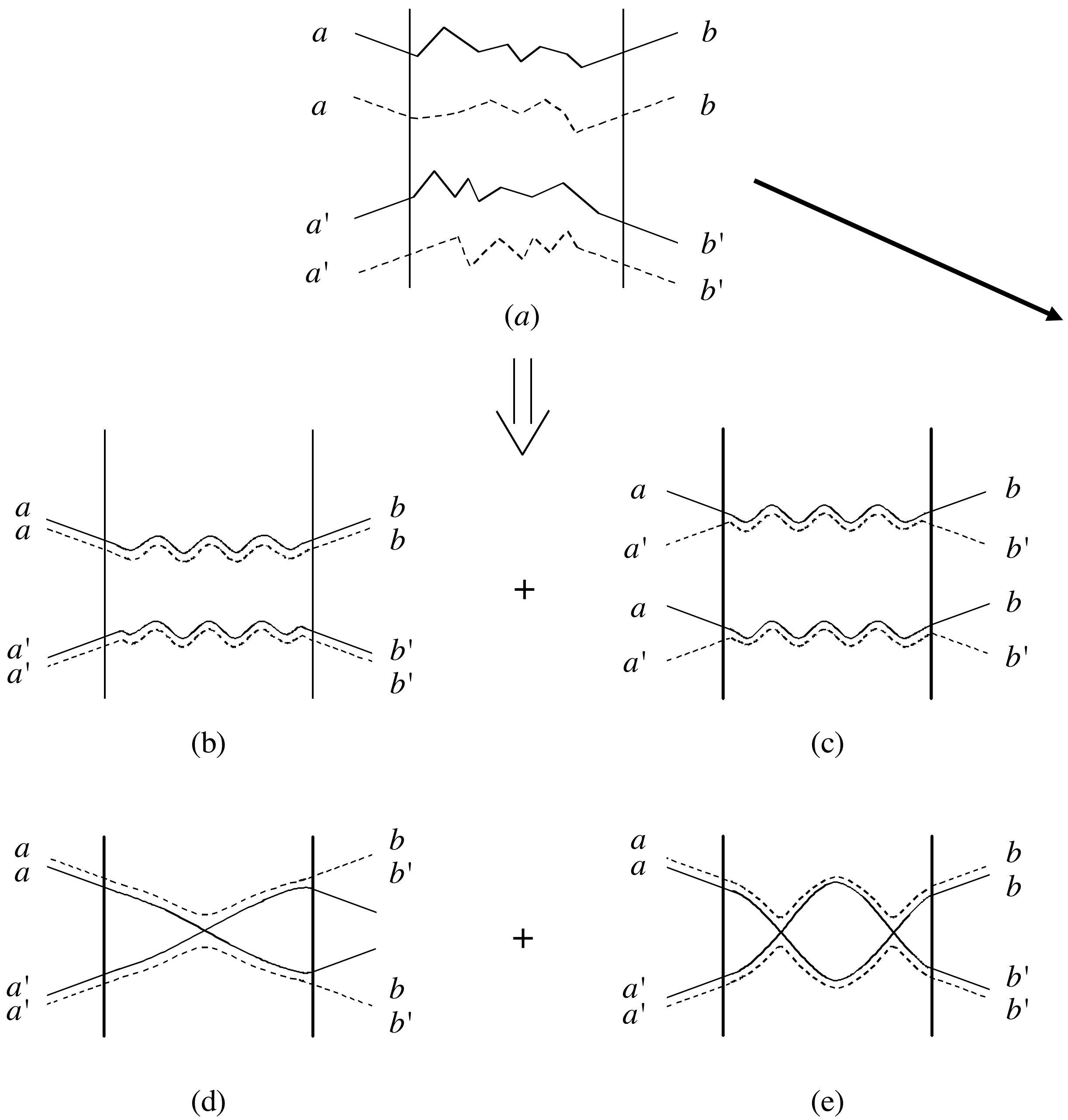

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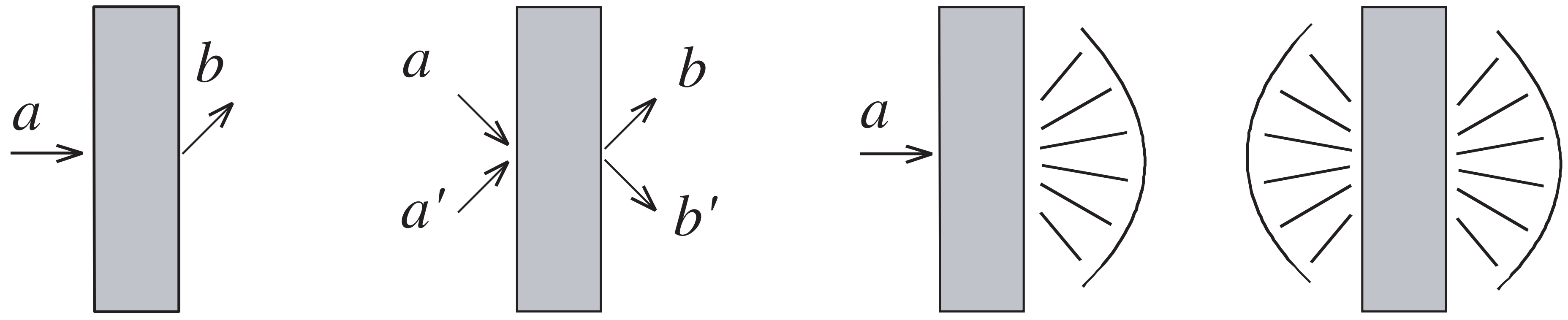

Measure different configurations

Fluctuations and correlations

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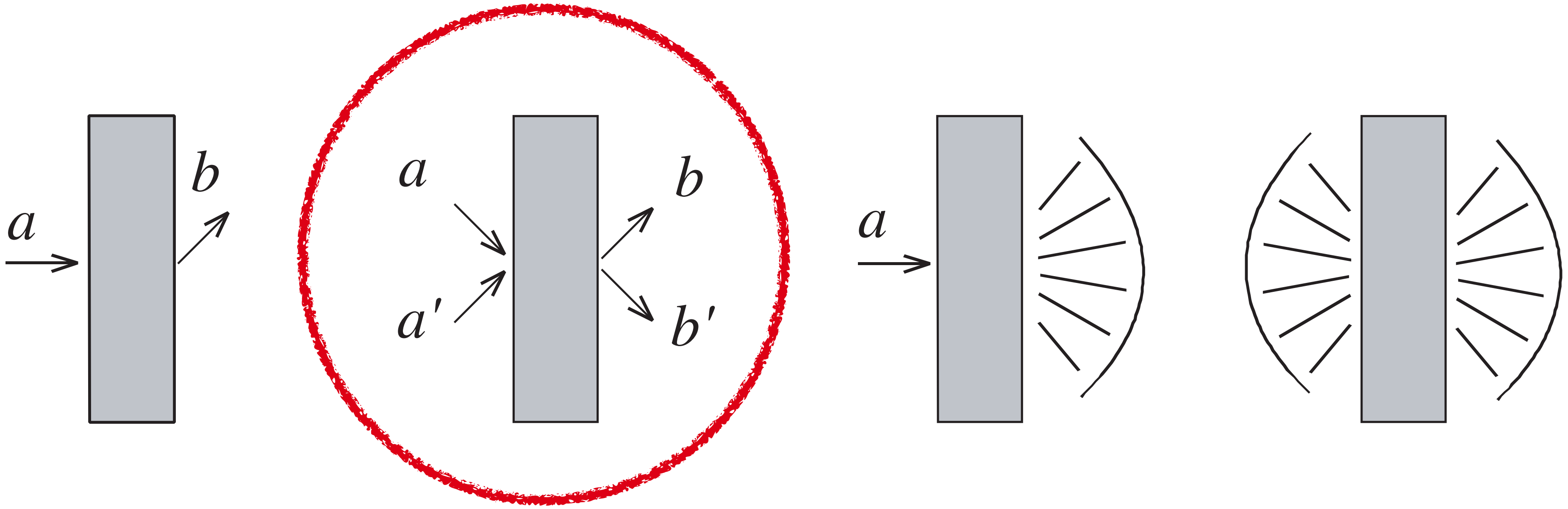
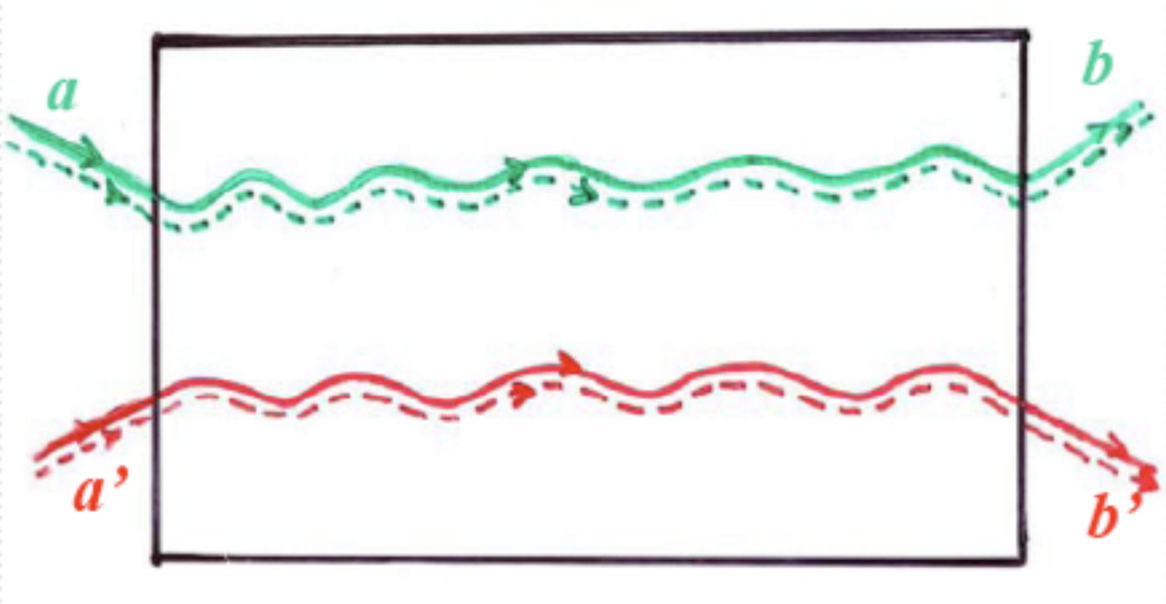


Measure different configurations

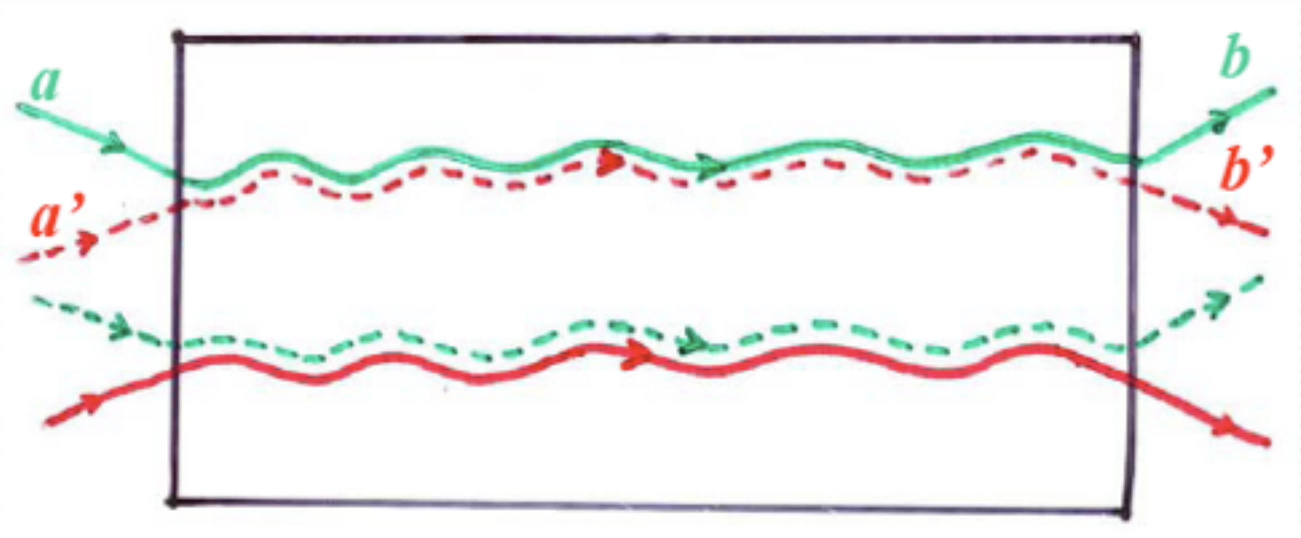
Speckle and conductance fluctuations

$$\overline{T_{ab}T_{a'b'}} = \overline{T_{ab}}\ \overline{T_{a'b'}} + \overline{\delta T_{ab}\delta T_{a'b'}}$$

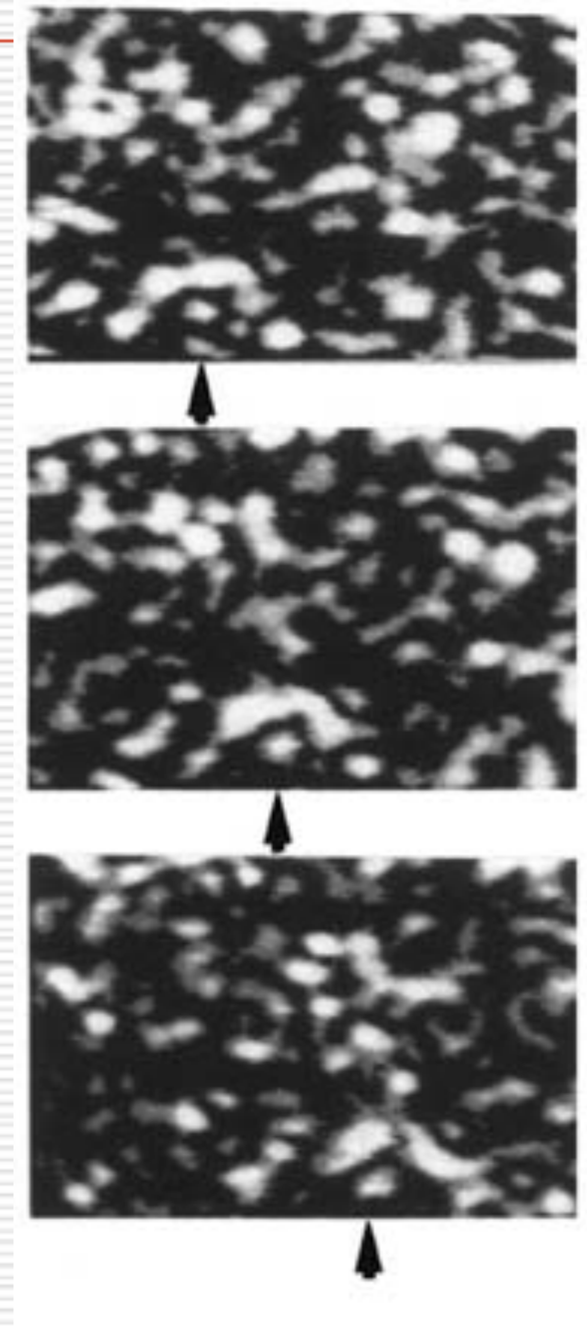
$$\overline{T_{ab}}\ \overline{T_{a'b'}}$$



$$\overline{\delta T_{ab}\delta T_{a'b'}}$$



$$= \overline{T_{ab}}\ \overline{T_{a'b'}}\ f(a,a',b,b')$$



Memory effect

$$f(a,a',b,b') = g(\Delta a)\delta(\Delta a - \Delta b)$$

Fluctuations and correlations

transmission coefficient

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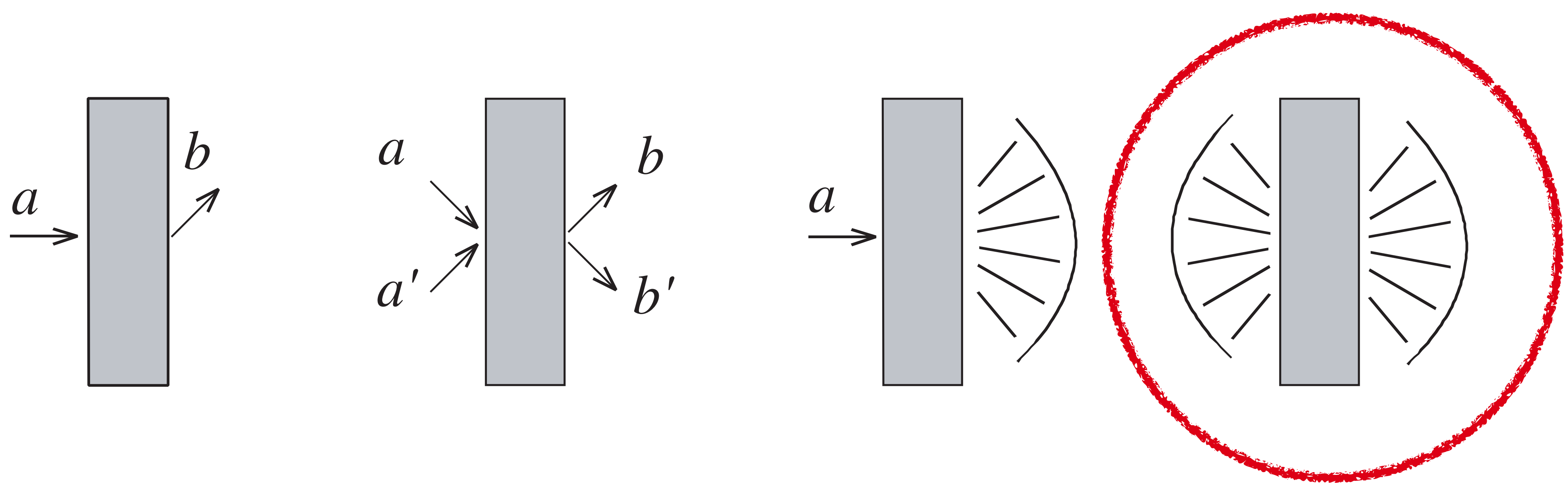
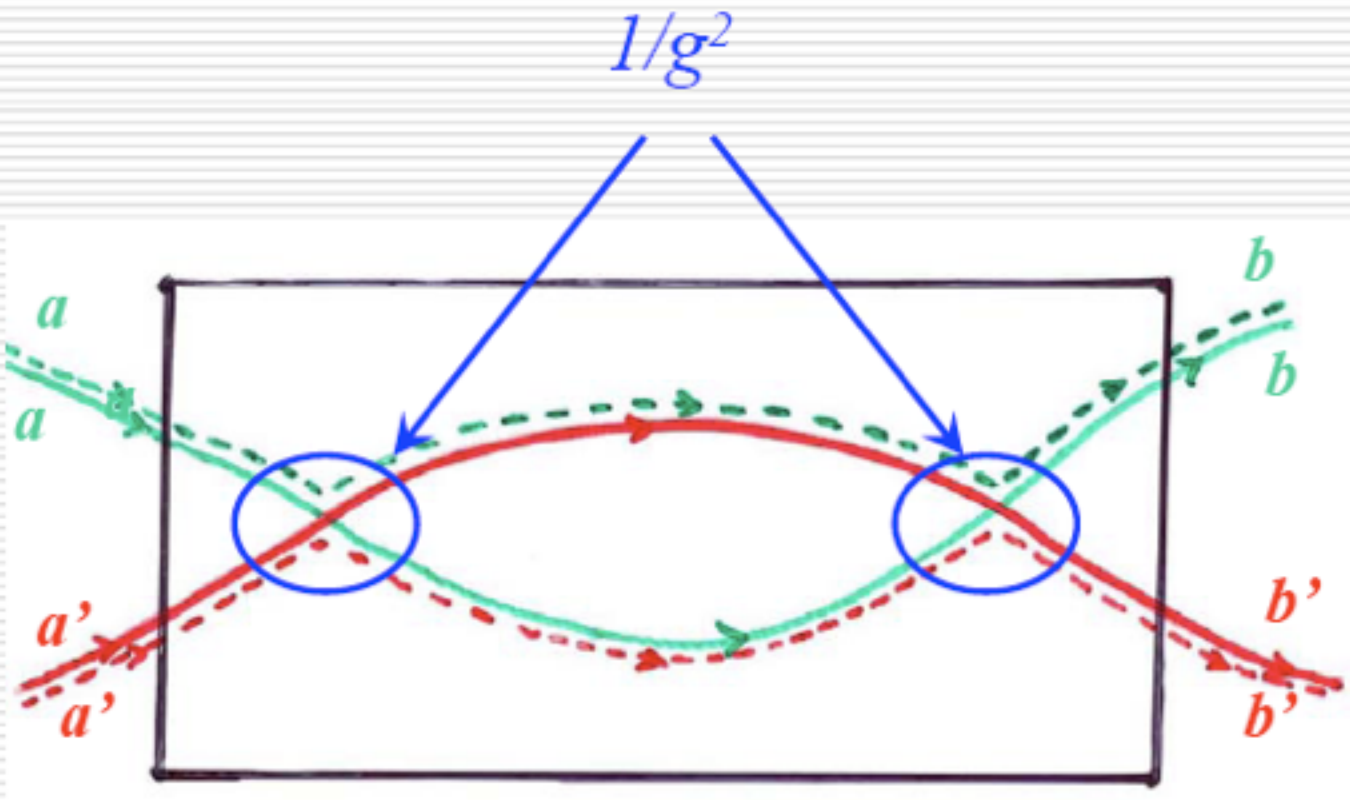


Measure different configurations

Speckle fluctuations *vs* conductance fluctuations C_3



$$\overline{\delta T_{ab}\delta T_{a'b'}} = \frac{2}{15g^2}\overline{T_{ab}}\ \overline{T_{a'b'}}$$

Long-range angular correlations, with very weak amplitude

$$\overline{\delta g^2} = \frac{2}{15g^2}\sum_{a,a',b,b'}\overline{T_{ab}}\ \overline{T_{a'b'}} = \frac{2}{15}$$

Universal conductance fluctuations

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Landauer description :

$$G = \frac{e^2}{h} \sum_{ab} T_{ab}$$

Universal conductance fluctuations

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0 crossing: $\overline{G}^2 = G_{cl}^2 = \left(e^2/h\right)^2 \; g^2$

Universal conductance fluctuations

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2 crossings: correction $\overline{\delta G^2} \propto \overline{G}^2 / g^2 = (e^2/h)^2$ universal

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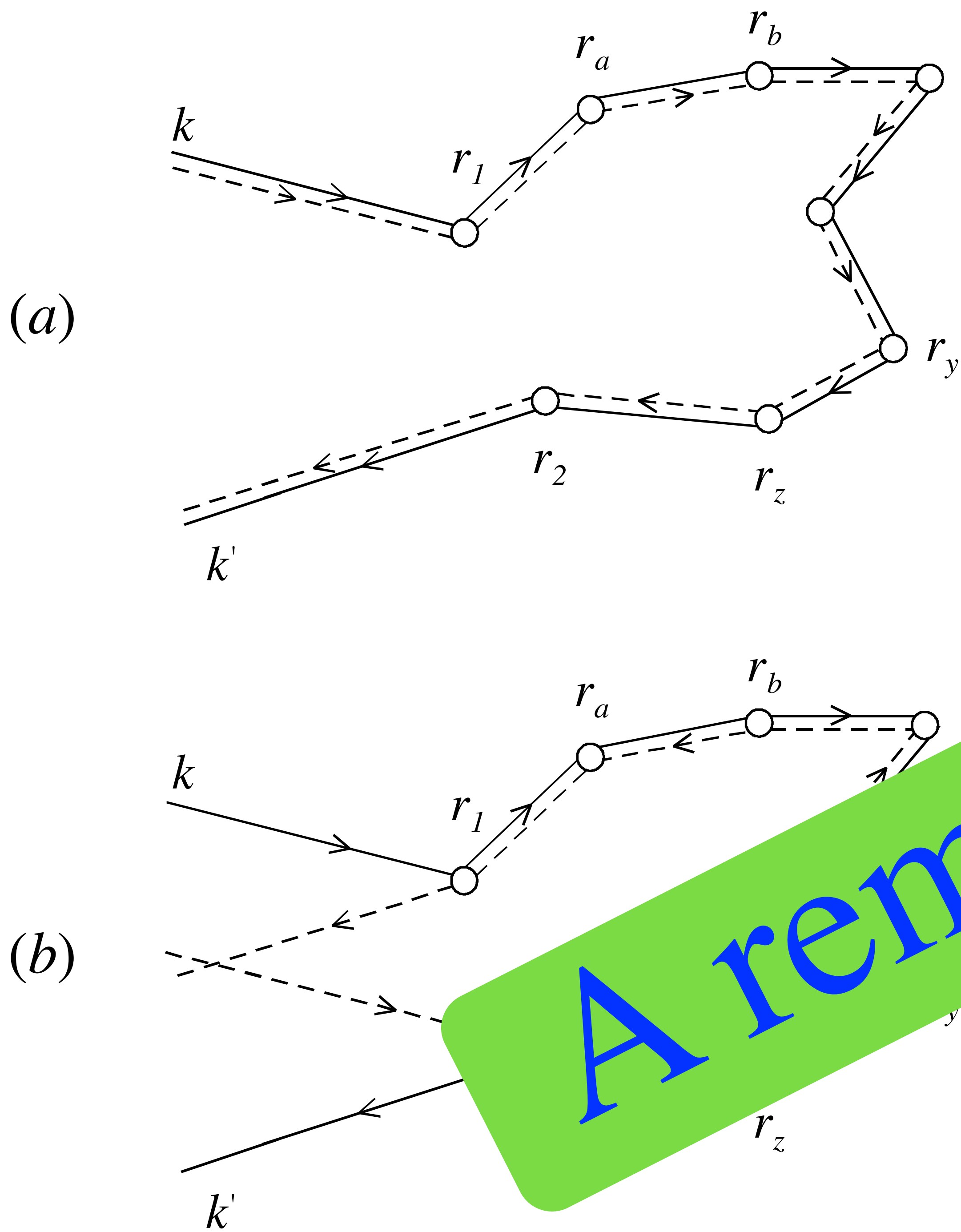
2 crossings: correction $\overline{\delta G^2} \propto \overline{G}^2 / g^2 = (e^2 / h)^2$ universal

(very different from the classical self-averaging limit $\overline{\delta G^2} \propto L^{d-4}$)

Dephasing and decoherence

Universal conductance fluctuations

Different contributions either sensitive or not to dephasing



$\mathbf{r_1} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_b} \cdots \longrightarrow \mathbf{r_y} \longrightarrow \mathbf{r_z} \longrightarrow \mathbf{r_2}$

Recipro … heorem:

I … then you see me.

$\mathbf{z} \longrightarrow \mathbf{r_y} \cdots \longrightarrow \mathbf{r_b} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_1}$

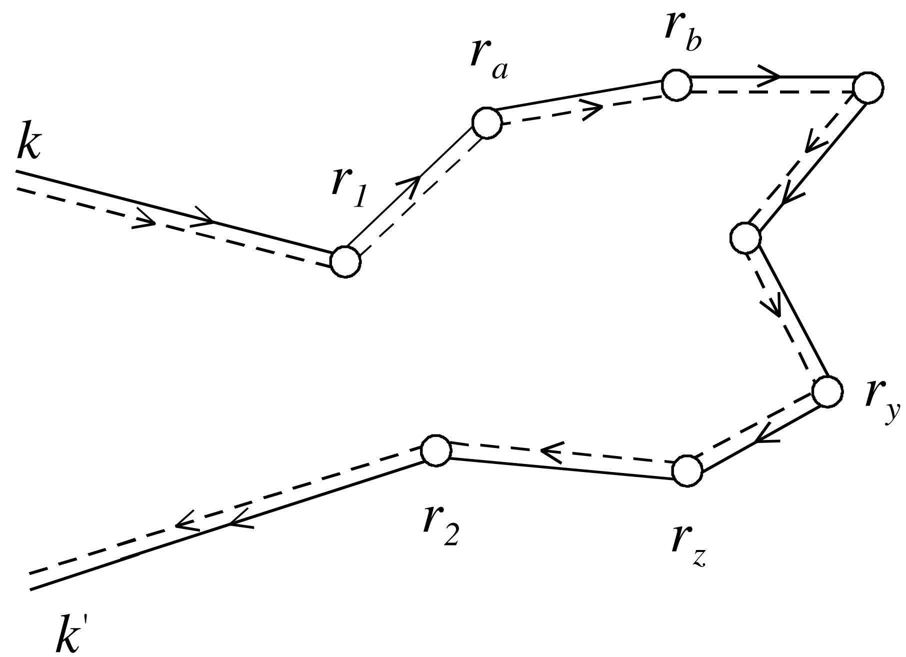
A reminder !

incoherent classical term

interference term

The total average intensity is:

$$\overline{|A(\mathbf{k},\mathbf{k}')|^2} = \overline{\sum_{\mathbf{r_1},\mathbf{r_2}} |f(\mathbf{r_1},\mathbf{r_2})|^2 \left[1 + e^{i(\mathbf{k}+\mathbf{k}').(\mathbf{r_1}-\mathbf{r_2})}\right]}$$

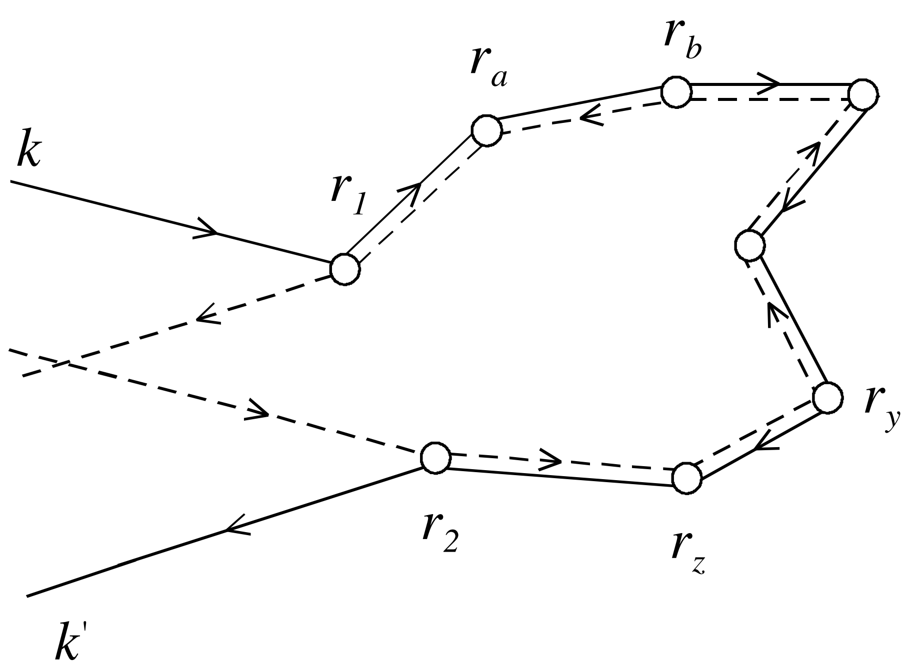


(a)

$$\mathbf{r_1} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_b} \cdots \longrightarrow \mathbf{r_y} \longrightarrow \mathbf{r_z} \longrightarrow \mathbf{r_2}$$

Reciprocity theorem:

If I see you, then you see me.



(b)

$$\mathbf{r_2} \longrightarrow \mathbf{r_z} \longrightarrow \mathbf{r_y} \cdots \longrightarrow \mathbf{r_b} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_1}$$

incoherent classical term

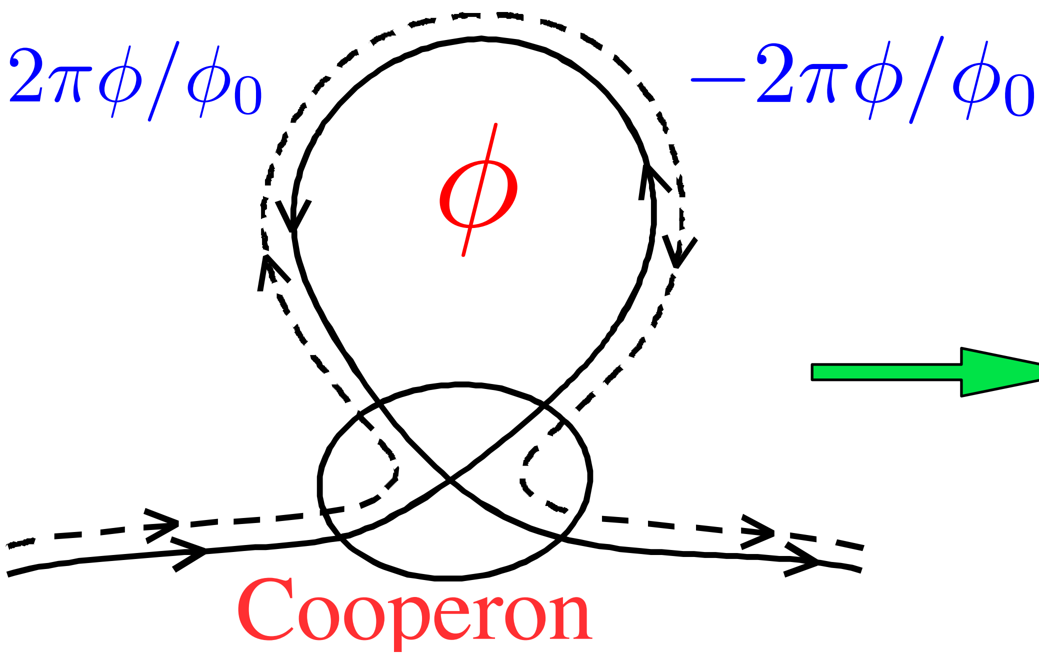
interference term

The total average intensity is:

$$\overline{|A(\mathbf{k},\mathbf{k}')|^2} = \overline{\sum_{\mathbf{r_1},\mathbf{r_2}} |f(\mathbf{r_1},\mathbf{r_2})|^2 \left[1 + e^{i(\mathbf{k}+\mathbf{k}').(\mathbf{r_1}-\mathbf{r_2})}\right]}$$

In the presence of a dephasing mechanism that breaks time coherence, only trajectories with $t < \tau_\phi$ contribute.

In the presence of an Aharonov-Bohm flux, paired amplitudes in the Cooperon acquire opposite phases:



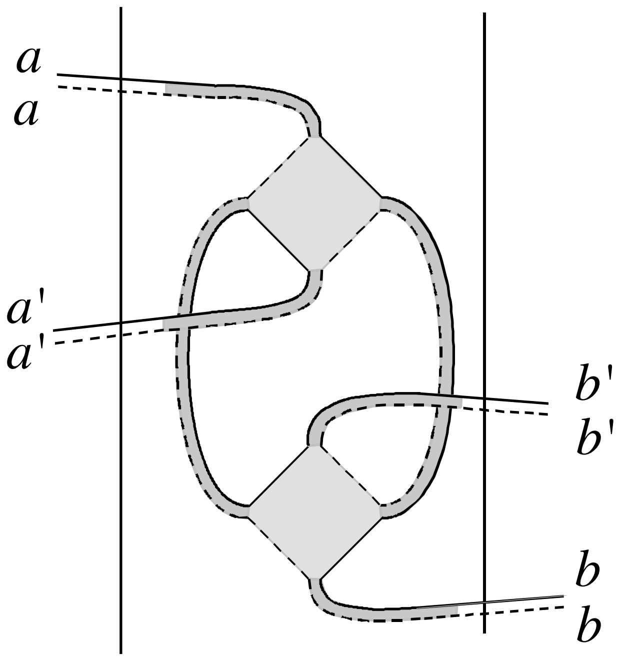
the phase difference becomes: $4\pi\phi/\phi_0$

$\phi_0/2$ periodicity o[illegible]harvin effect

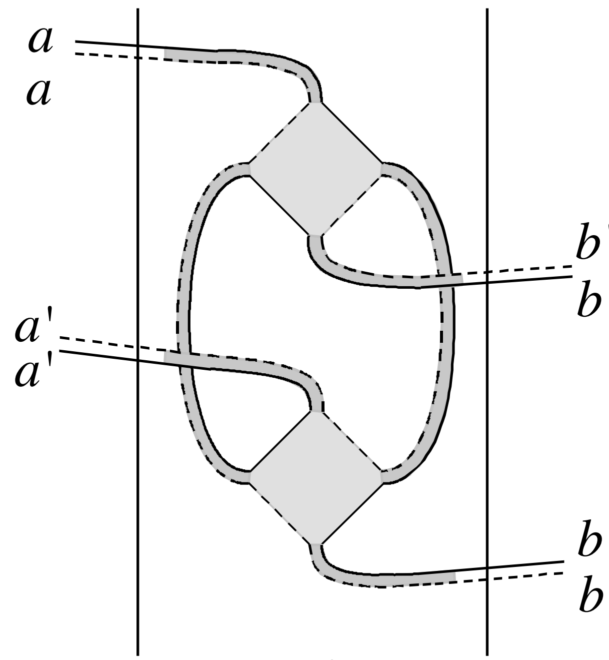
A reminder !

Dephasing and decoherence

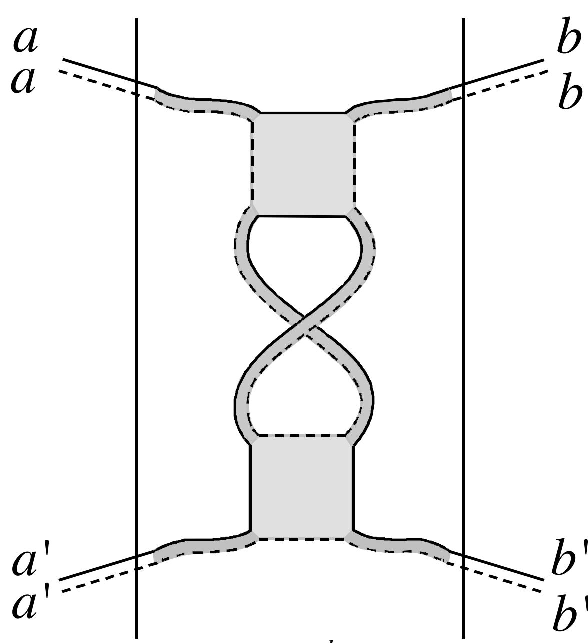
Universal conductance fluctuations



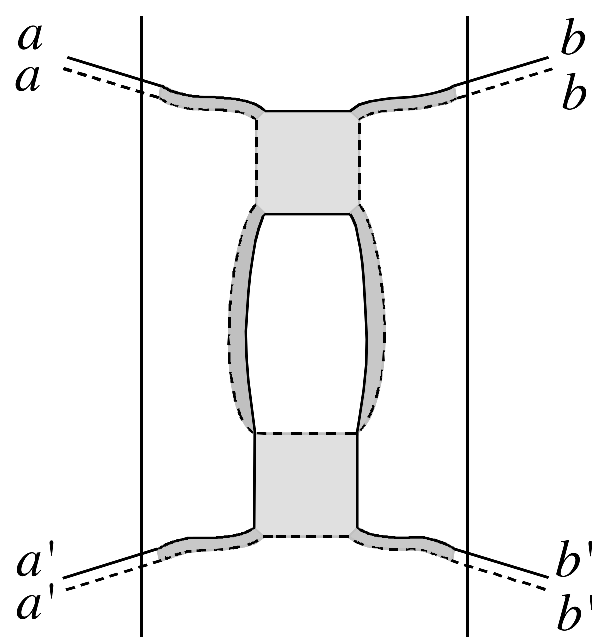
(K_1^d)



(K_2^c)



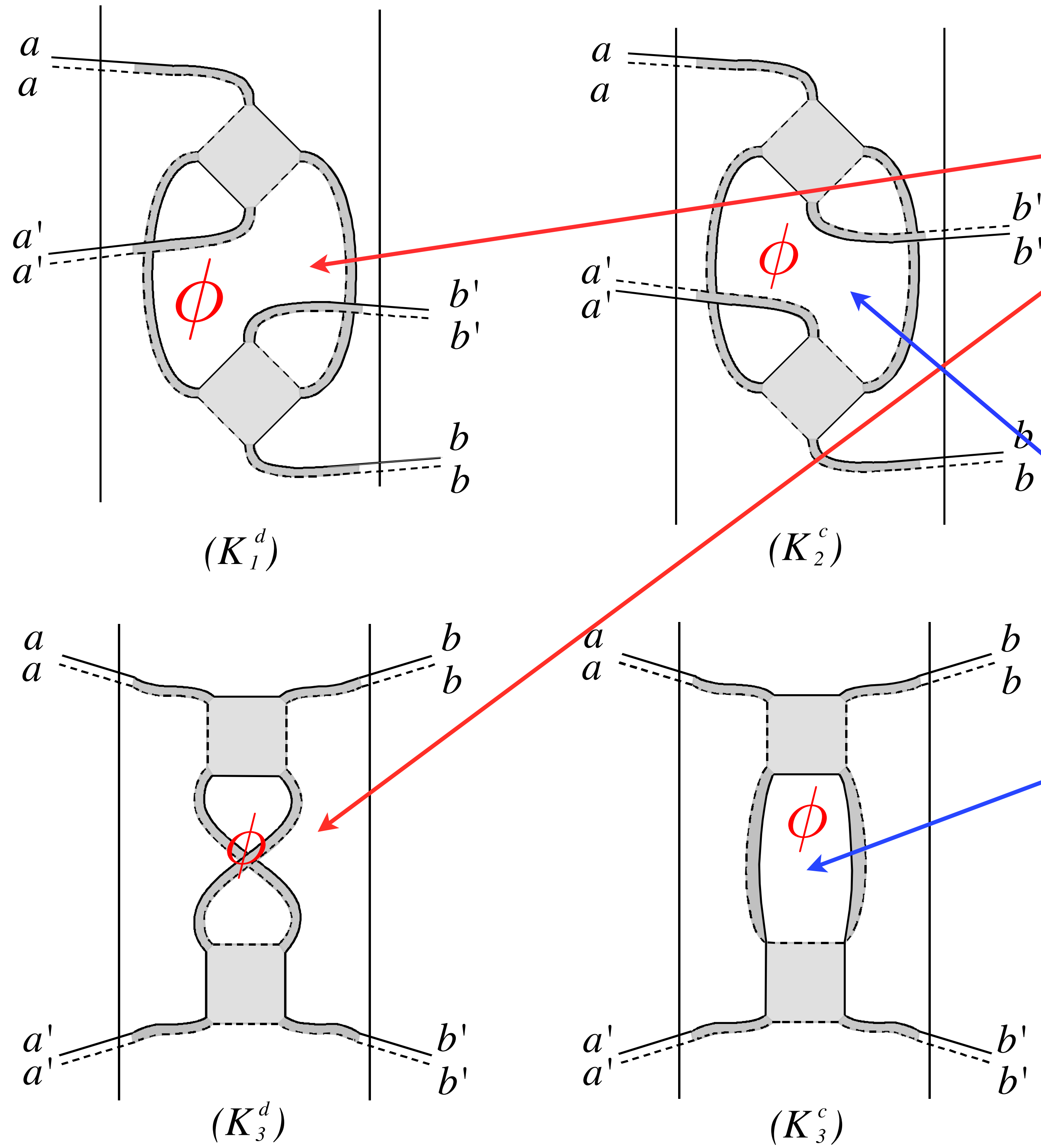
(K_3^d)



(K_3^c)

Dephasing and decoherence

Universal conductance fluctuations



2 Diffusons

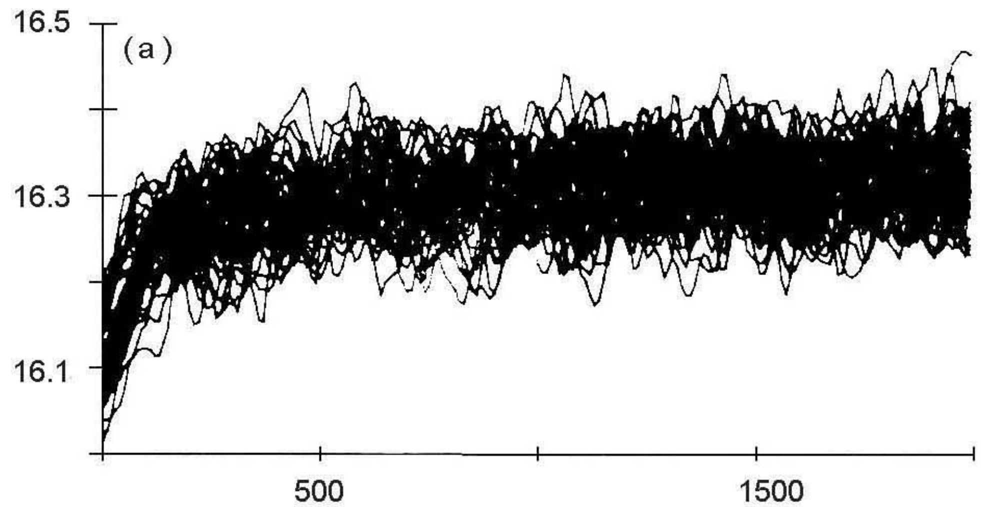
2 Cooperons

sensitive to an applied Aharonov-Bohm magnetic flux

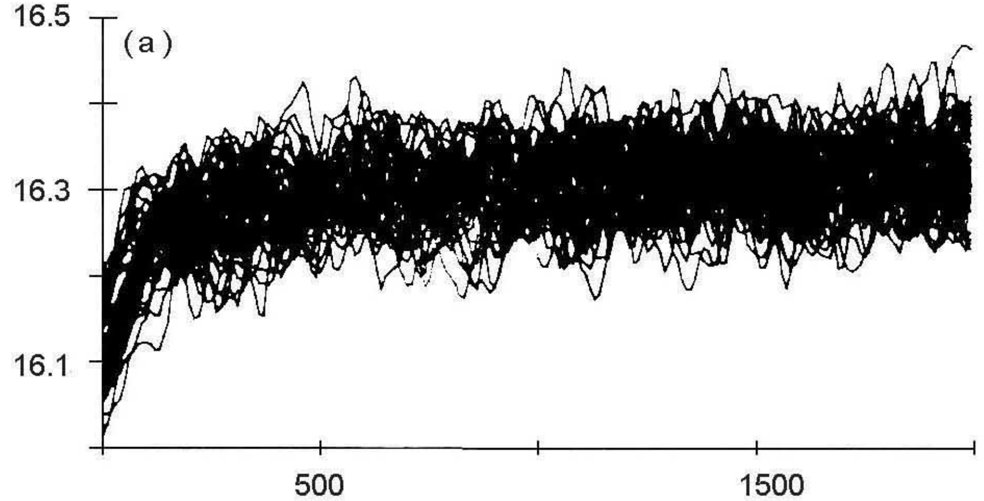
ϕ

46 *Si*-doped *GaAs* samples at 45 mK

(Mailly-Sanquer)



46 *Si*-doped *GaAs* samples at 45 mK



(Mailly-Sanquer)

We expect the conductance fluctuations to be reduced by a factor 2

$$\overline{\delta G^2} \xrightarrow{\phi} \frac{\overline{\delta G^2}}{2}$$

46 *Si*-doped *GaAs* samples at 45 mK

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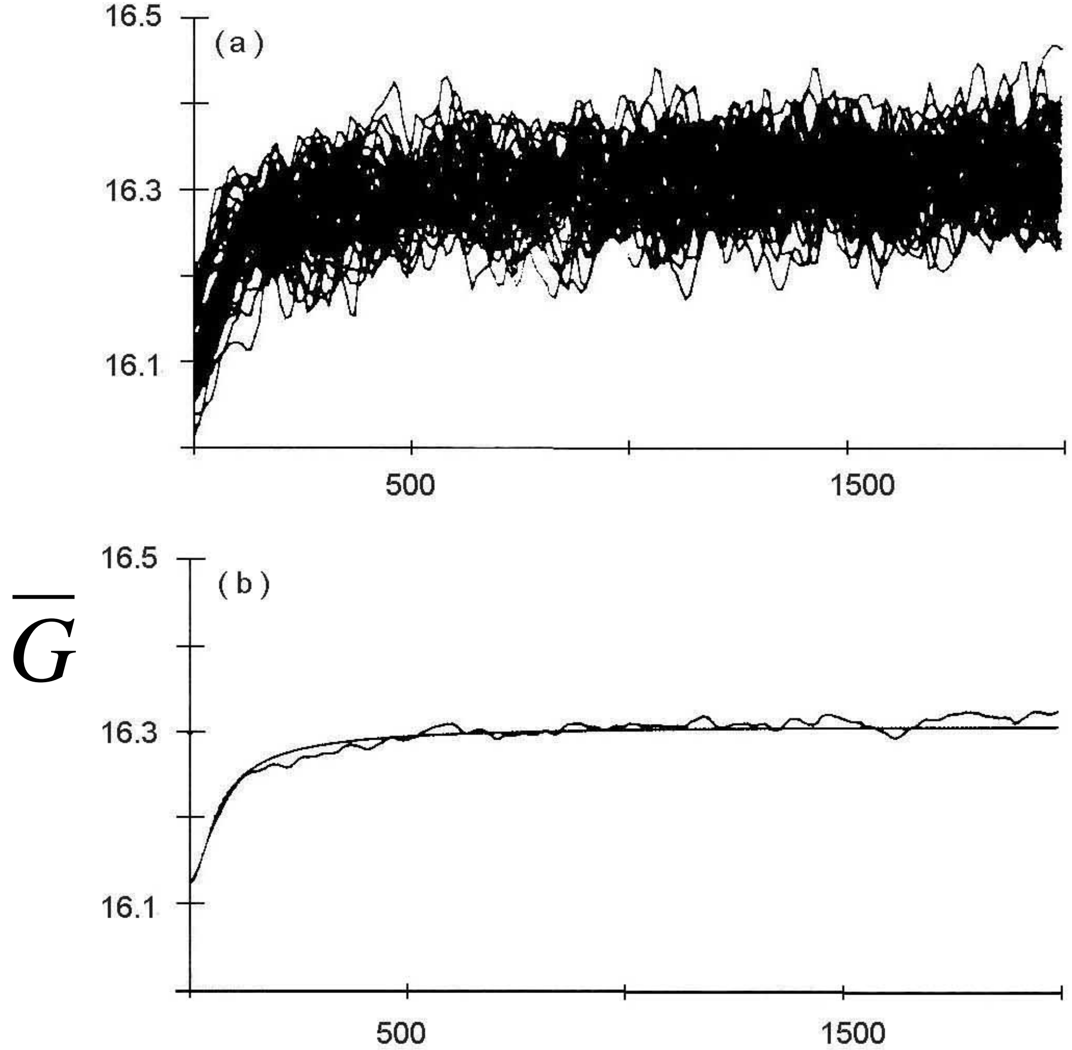

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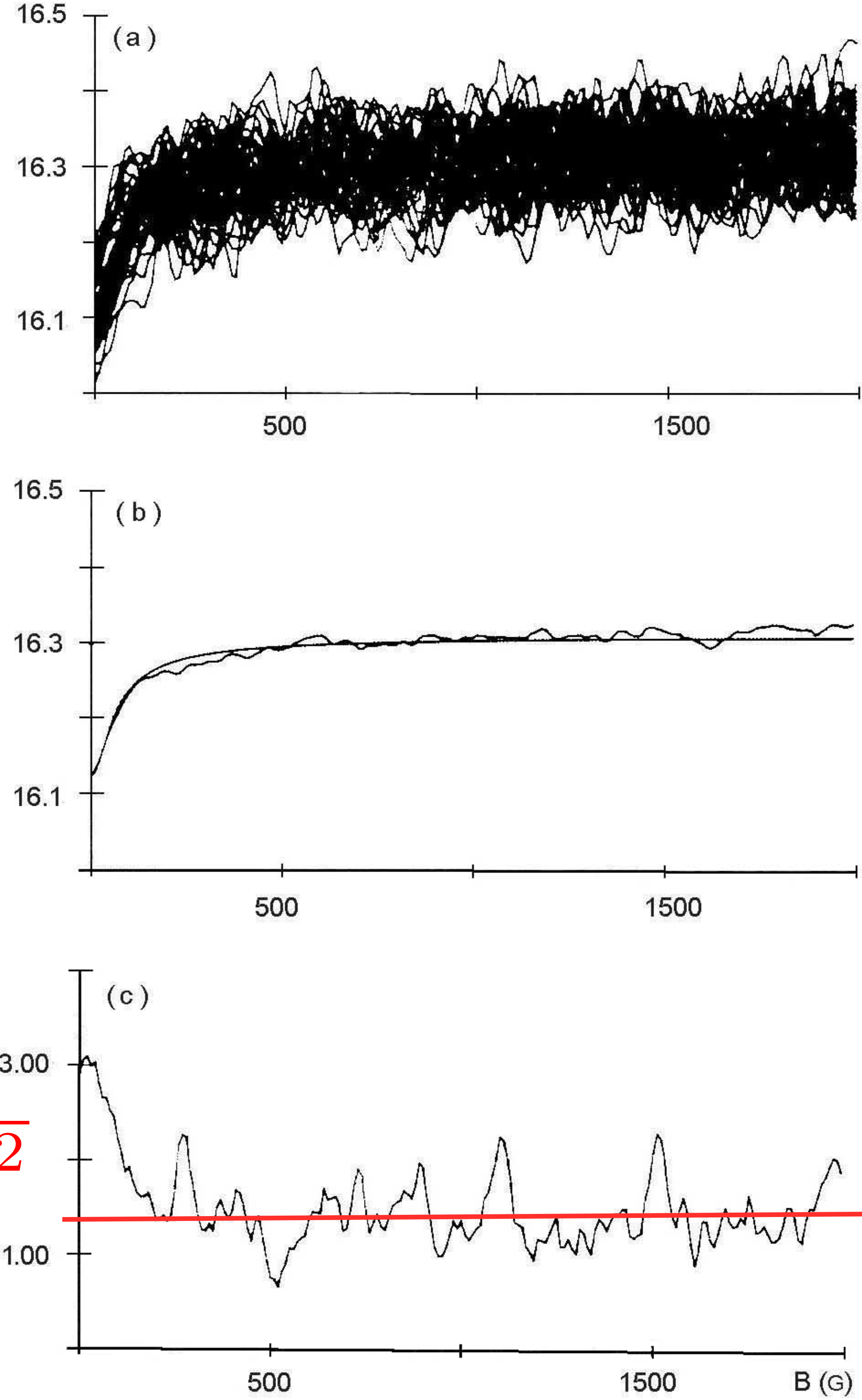
46 *Si*-doped *GaAs* samples at 45 mK

(Mailly-Sanquer)

$\overline{G}$



$\overline{\delta G^2}$

1.5

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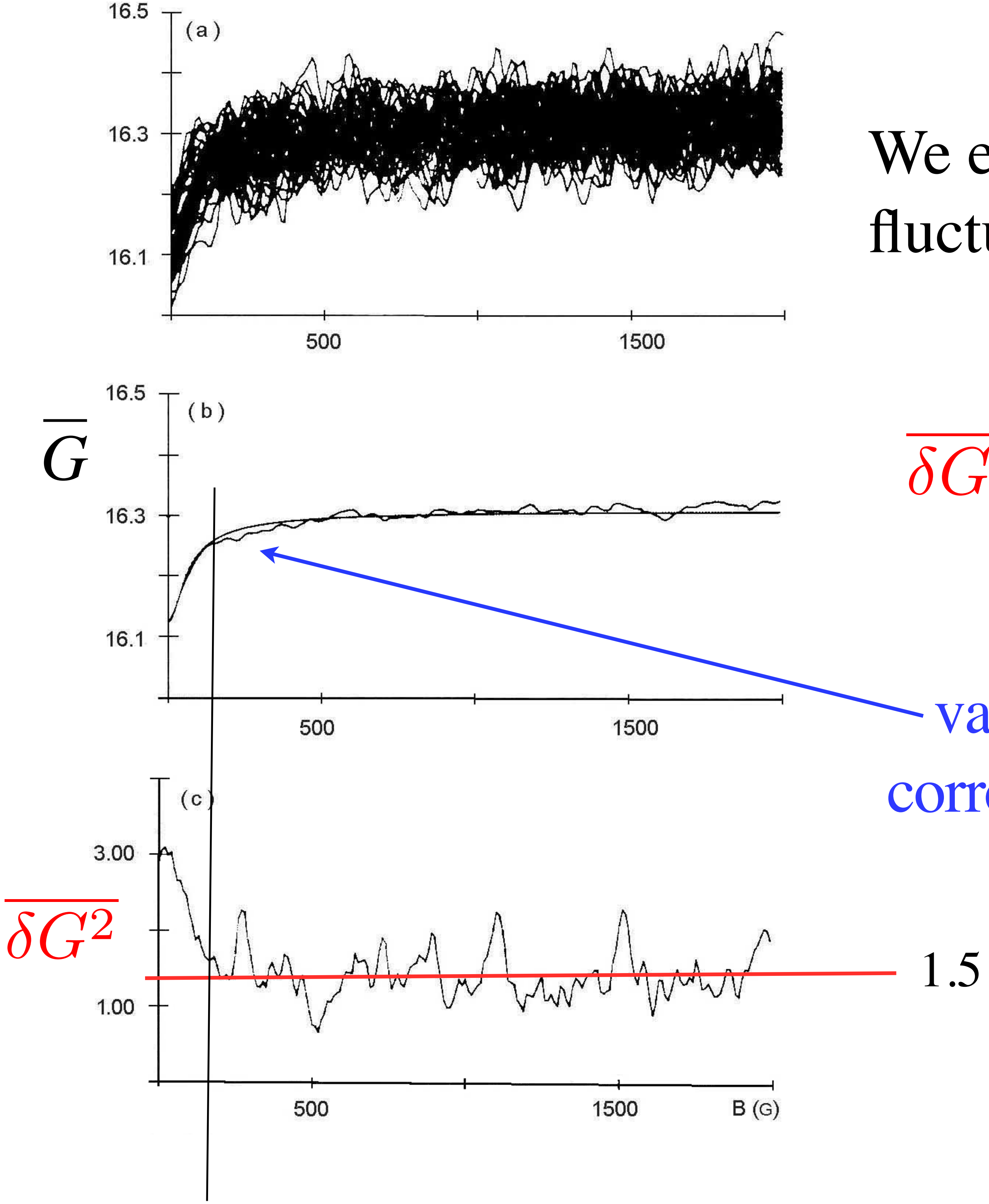

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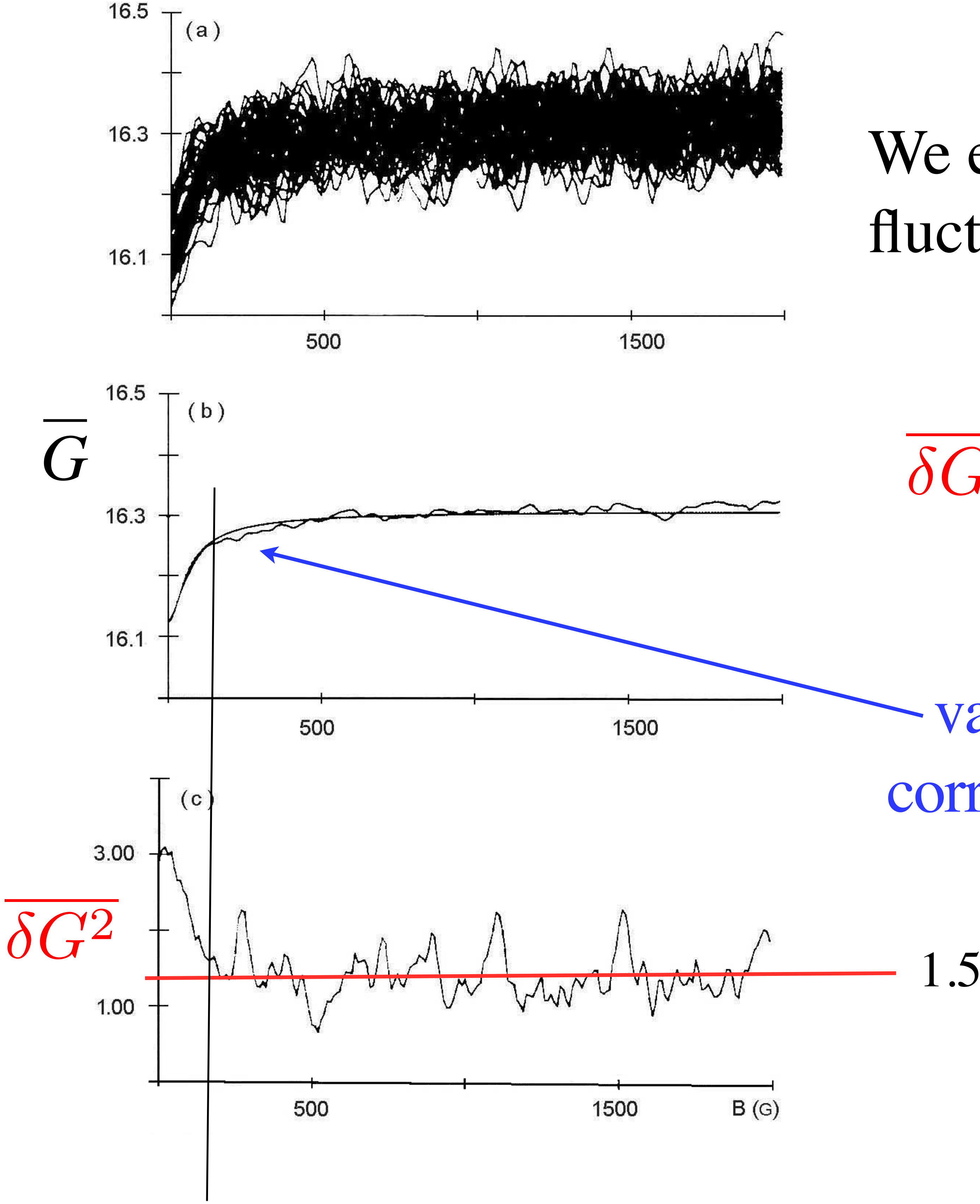
vanishing of the weak localization correction for the same magnetic field

46 *Si*-doped *GaAs* samples at 45 mK

(Mailly-Sanquer)



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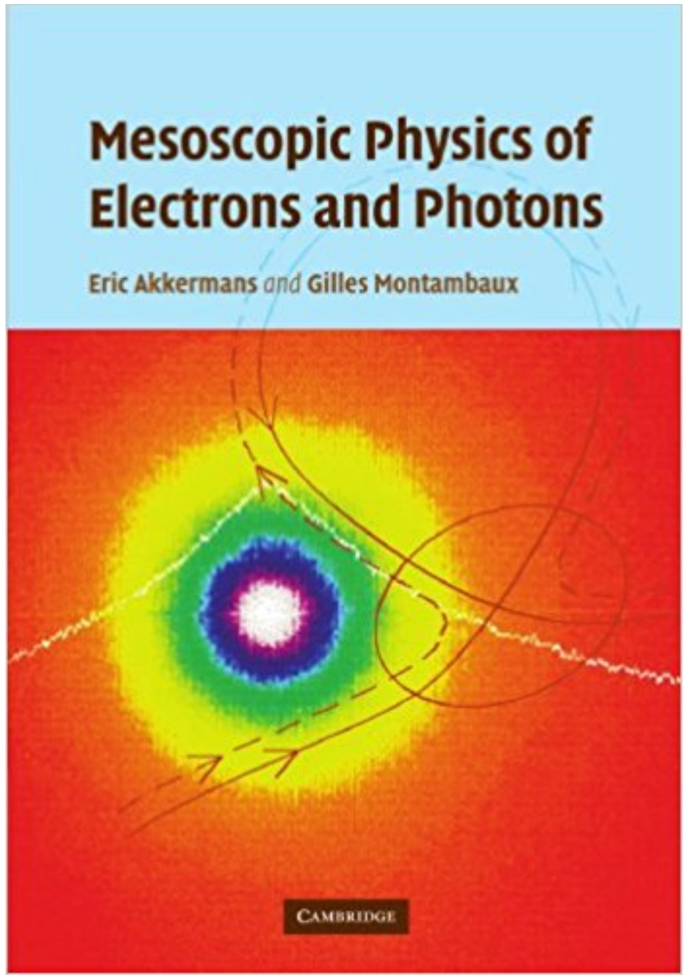
vanishing of the weak localization correction for the same magnetic field

In the presence of incoherent processes $L > L_\phi$:

$$\overline{\delta G^2} \rightarrow 0$$

Thank you for your attention.



Based on *Mesoscopic physics of electrons and photons*,
by Eric Akkermans and Gilles Montambaux, Cambridge University Press, 2007