Lectures on quantum gases

Lecture 5

Feshbach resonances

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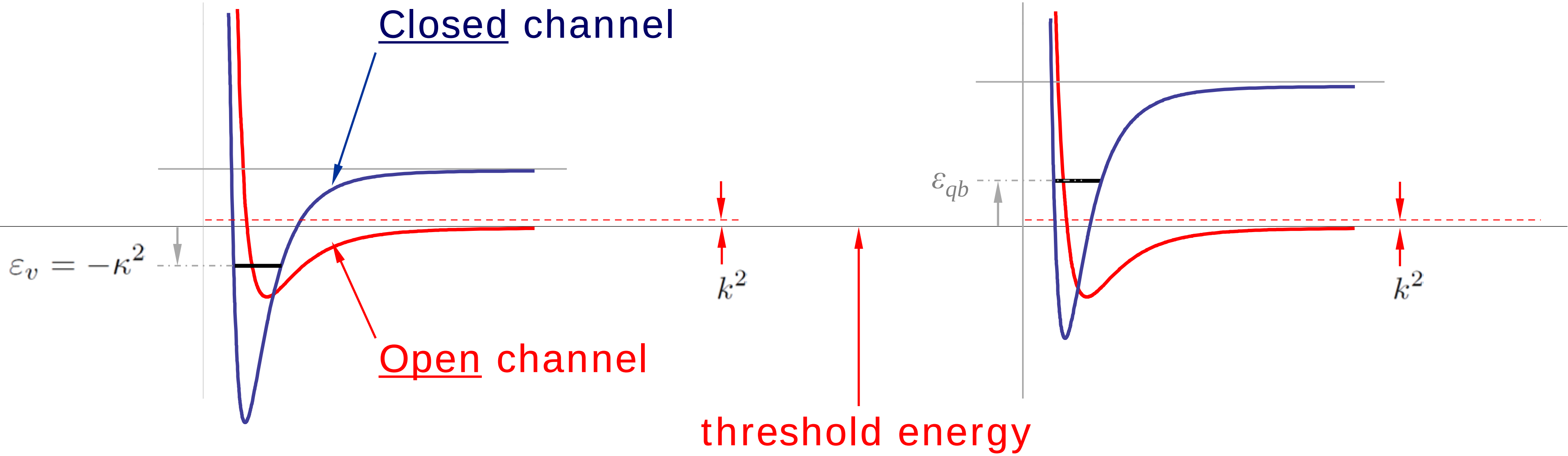
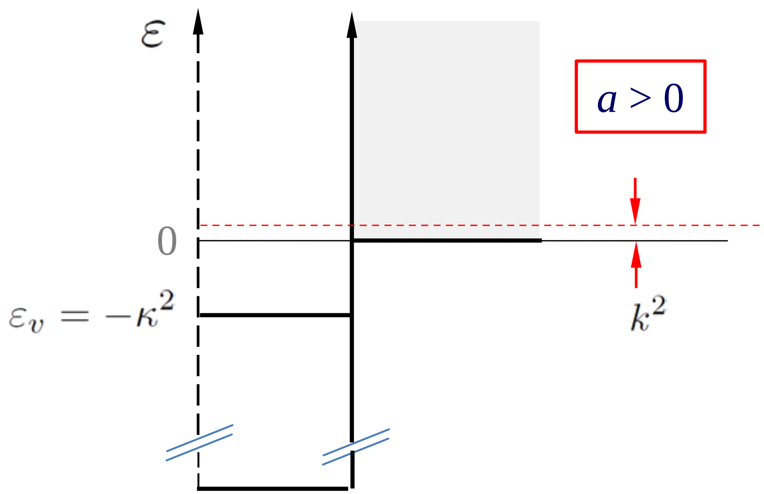
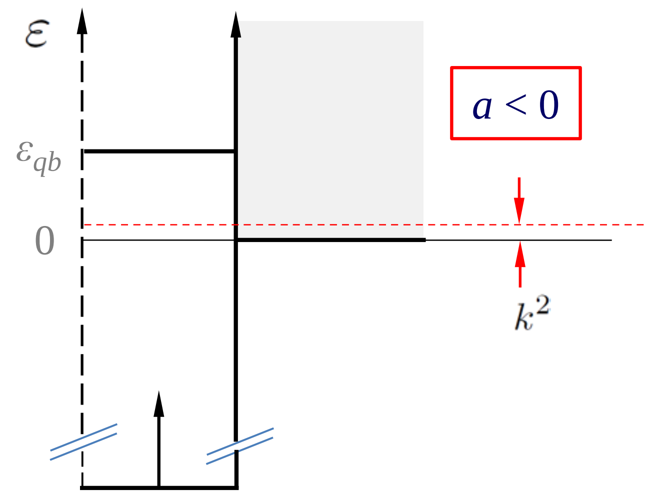
Feshbach resonance



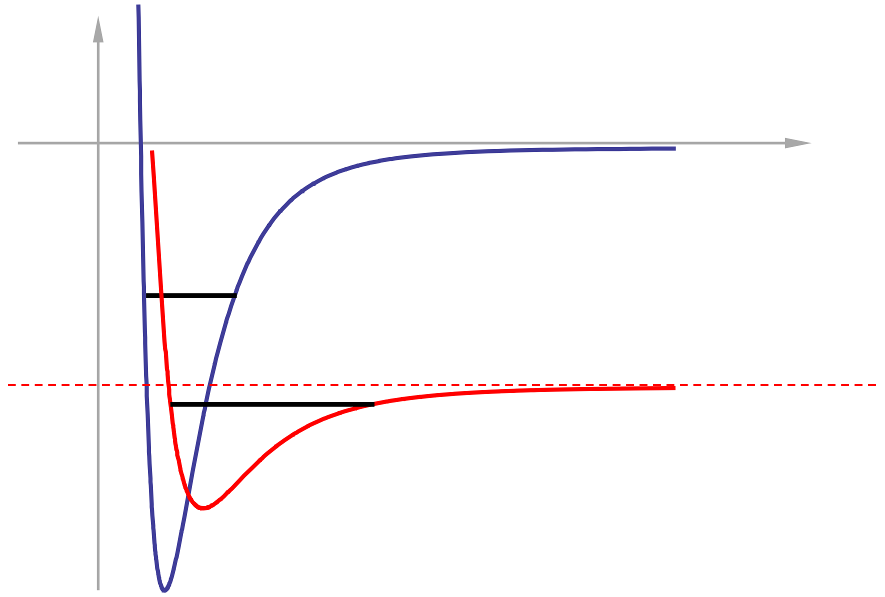
Closed channel: below threshold



Closed channel: above threshold



Feshbach resonance



With cold alkali atoms we can tune to a Fesbach resonance at arbitrary, <u>fixed</u> (low) collisional energy by varying the magnetic field: Zeeman tuning

Interactions between two alkali atoms

Hamiltonian including spin Zeeman term:

$$\mathcal{V}(r) = V_D(r) + J(r)\mathbf{s}_1 \cdot \mathbf{s}_2$$

$$\mathcal{H} = \frac{1}{2\mu}\left(p_r^2 + \frac{\mathbf{L}^2}{r^2}\right) + \mathcal{V}(r) + \gamma_e B\, S_z \underbrace{- (\gamma_1 i_{z1} + \gamma_2 i_{z2})\, B}_{(-\,\gamma_n B\, I_z\,)} + \mathcal{H}_{\rm hf}^{+} + \mathcal{H}_{\rm hf}^{-}$$

all terms conserve M_F

only term

not singlet/triplet conserving

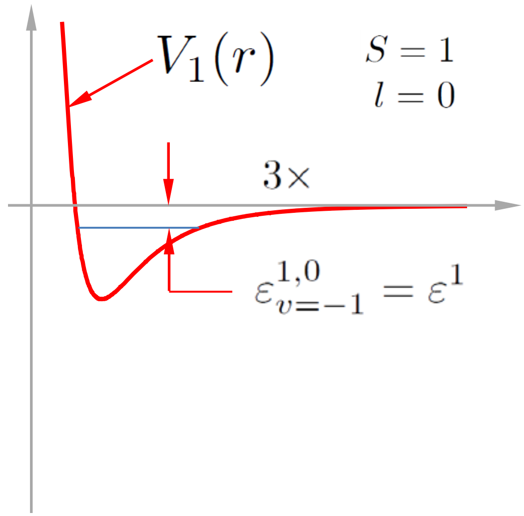
Good basis states: $|\psi\rangle = |R_l^S\rangle |l, m_l\rangle |S, M_S\rangle |i_1, m_1\rangle |i_2, m_2\rangle$

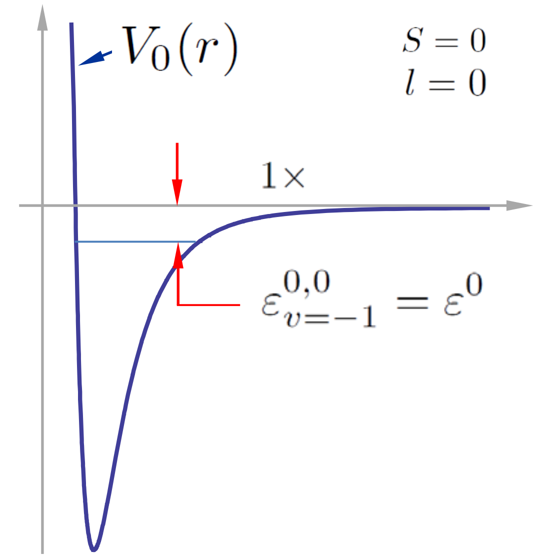
$$|\psi\rangle = |R_l^{S,I}\rangle |l, m_l\rangle |S, M_S\rangle |I, M_I\rangle$$

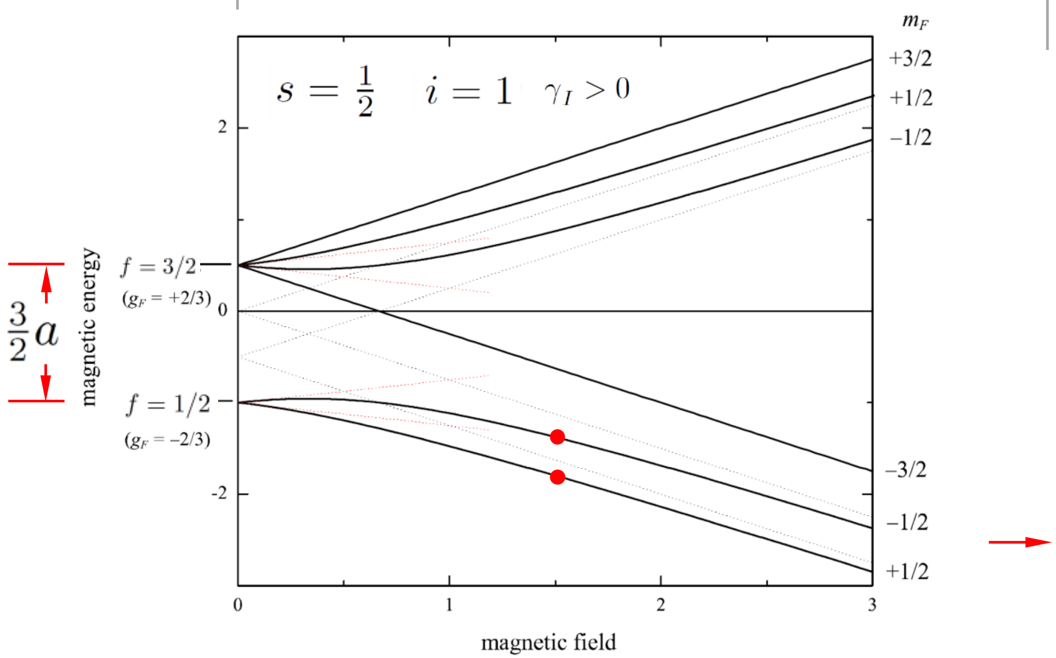
Effective potential:

$$U_{S,l}(r) = U_S(r) + \frac{l\,(l+1)}{r^2} + \frac{2\mu}{\hbar^2}\left[\gamma_e B\, M_S \underbrace{- (\gamma_1 m_1 + \gamma_2 m_2)\, B}_{(-\gamma_n B M_I\,)}\right]$$

Example: two ^{6}Li atoms







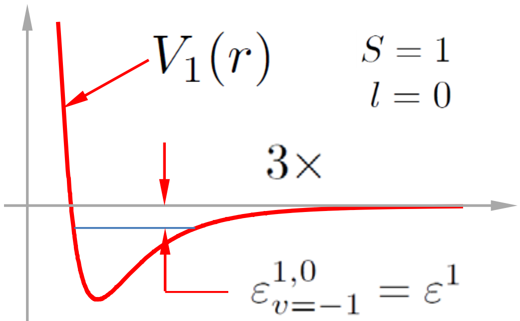
Find all s-wave molecules with $M_F = 0$

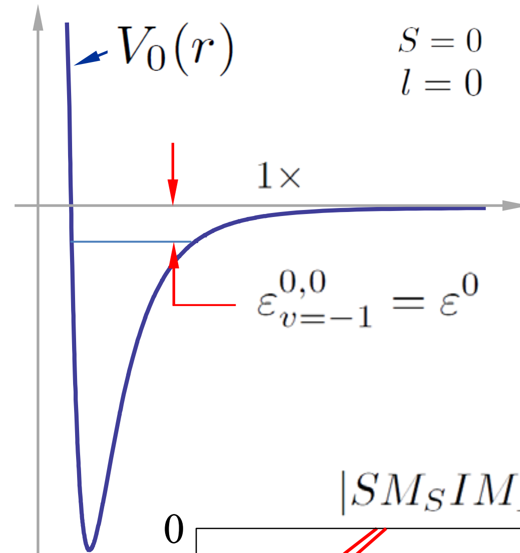
$S = 0$ $\begin{cases} I = 0, 2 \\ I = 1 \end{cases}$ $X^1\Sigma_g^+$ $\begin{array}{l} l = 0, 2, 4, \cdots \\ l = 1, 3, 5, \cdots \end{array}$

$S = 1$ $\begin{cases} I = 0, 2 \\ I = 1 \end{cases}$ $a^3\Sigma_u^+$ $\begin{array}{l} l = 1, 3, 5, \cdots \\ l = 0, 2, 4, \cdots \end{array}$

free atom pair with $M_F = 0$

Example: two ^{6}Li atoms





$M_F = 0$

$S=0$ $\begin{cases} I = 0,2 \\ I = 1 \end{cases}$ $X^1\Sigma_g^+$ $\begin{matrix} l = 0,2,4,\cdots \\ l = 1,3,5,\cdots \end{matrix}$

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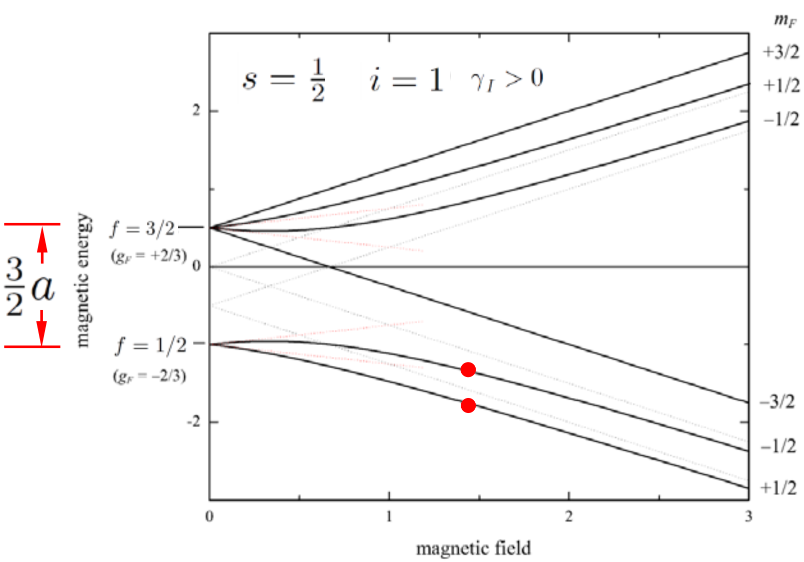
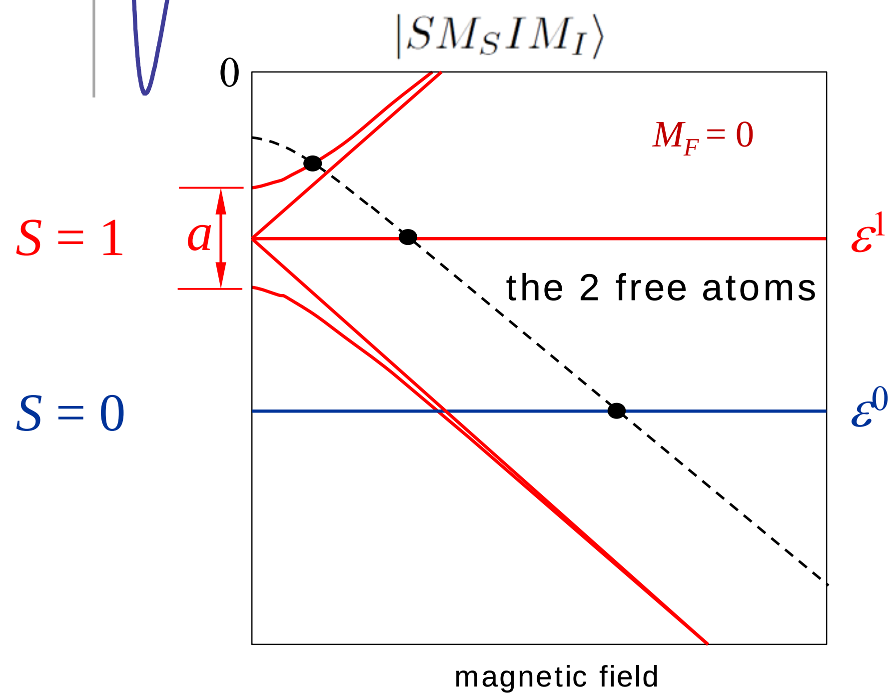
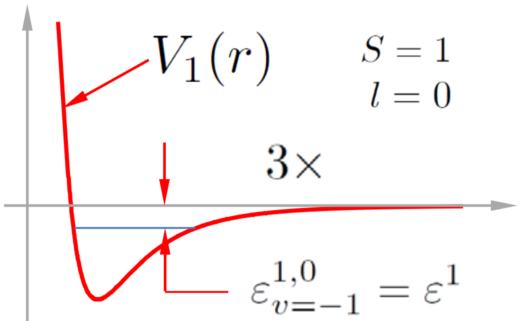
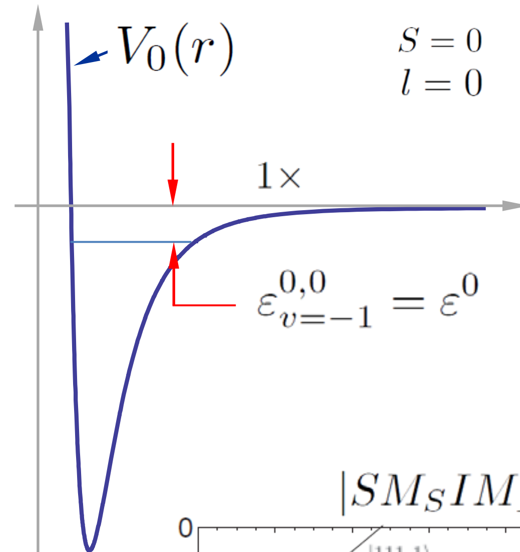




Example: two ^{6}Li atoms

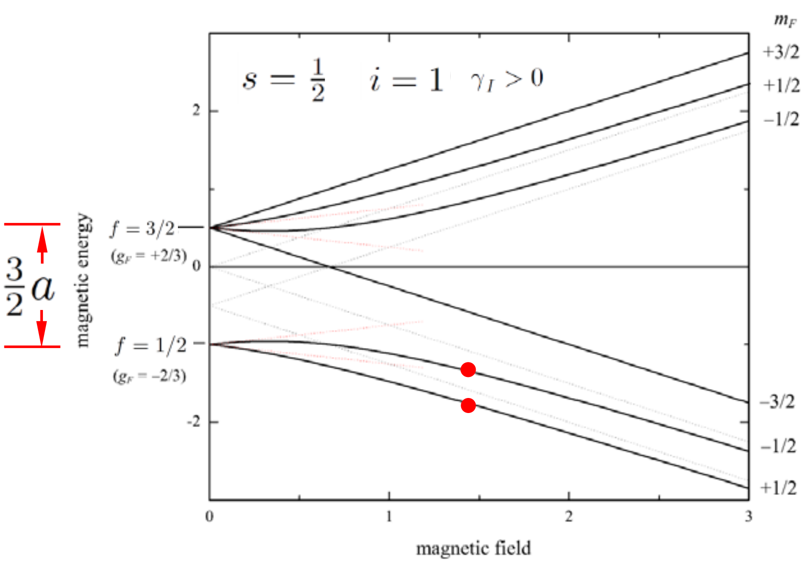




$M_F = 0$

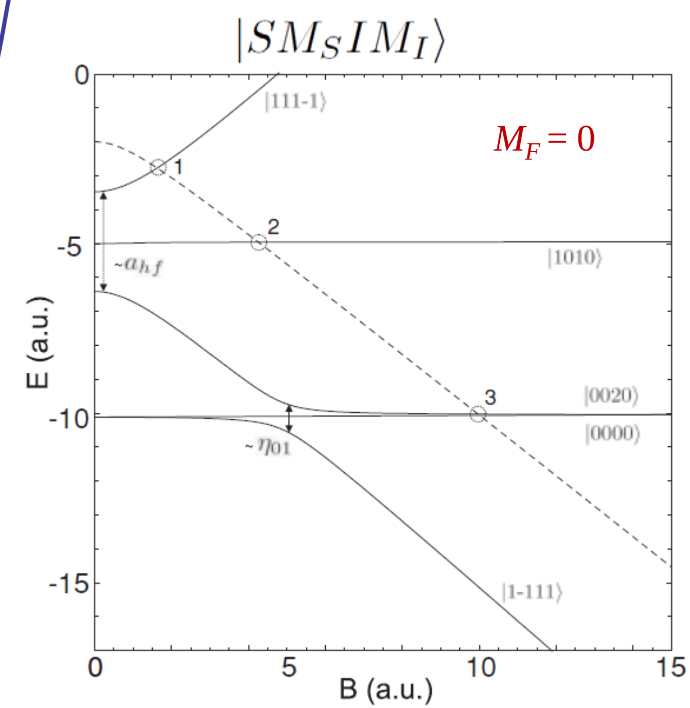
$$S = 0 \left\{ \begin{array}{l} I = 0, 2 \\ I = 1 \end{array} \right. \quad X^1\Sigma_g^+ \quad \begin{array}{l} l = 0, 2, 4, \cdots \\ l = 1, 3, 5, \cdots \end{array}$$

$$S = 1 \left\{ \begin{array}{l} I = 0, 2 \\ I = 1 \end{array} \right. \quad a^3\Sigma_u^+ \quad \begin{array}{l} l = 1, 3, 5, \cdots \\ l = 0, 2, 4, \cdots \end{array}$$



$S = 1$

$S = 0$



Diagonalization of Hamiltonian

$\mathcal{V}(r) = V_D(r) + J(r)\mathbf{s}_1 \cdot \mathbf{s}_2$

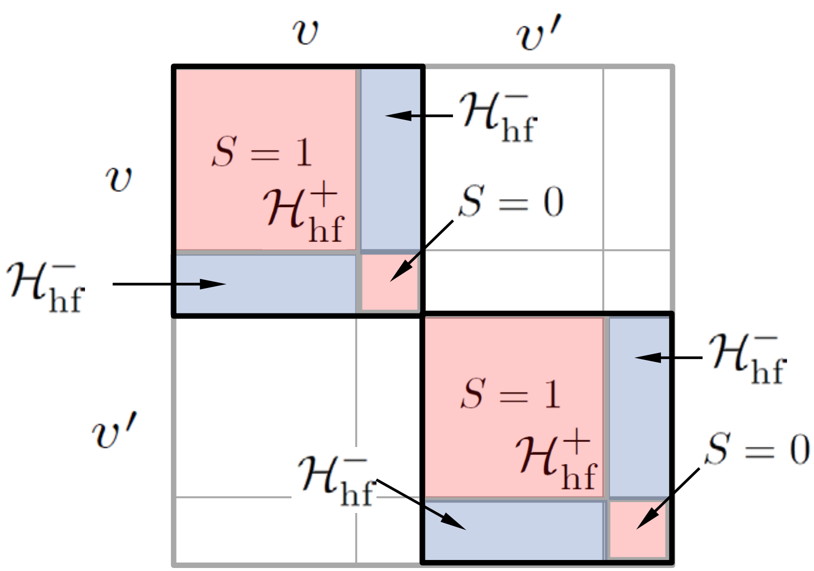
$$\mathcal{H} = \underbrace{\frac{1}{2\mu}\left(p_r^2 + \frac{\mathbf{L}^2}{r^2}\right) + \mathcal{V}(r)}_{\mathcal{H}_0} + \gamma_e B\, S_z - \gamma_n B I_z + \mathcal{H}^+_{\mathrm{hf}} + \mathcal{H}^-_{\mathrm{hf}}$$

$$|\alpha\rangle = |R^S_{v,l}\rangle |l, m_l\rangle \underbrace{|S, M_S; I, M_I\rangle}_{|\sigma\rangle}$$

$$\langle\beta|\mathcal{H}|\alpha\rangle = \left(\varepsilon_v^{S,l} + \gamma_e B M_S - \gamma_n B M_I\right)\delta_{\alpha\beta} + \langle\beta|\mathcal{H}^+_{\mathrm{hf}} + \mathcal{H}^-_{\mathrm{hf}}|\alpha\rangle$$

$$= [\varepsilon_v^{S,l} + E_\sigma(B)]\delta_{\sigma,\sigma'} + \underbrace{\langle\sigma'|\mathcal{H}^+_{\mathrm{hf}}|\sigma\rangle}_{=0}\delta_{v,v'} + \underbrace{\langle\sigma'|\mathcal{H}^-_{\mathrm{hf}}|\sigma\rangle}_{=0}\underbrace{\langle R^{S',l}_{v'}|R^{S,l}_v\rangle}$$

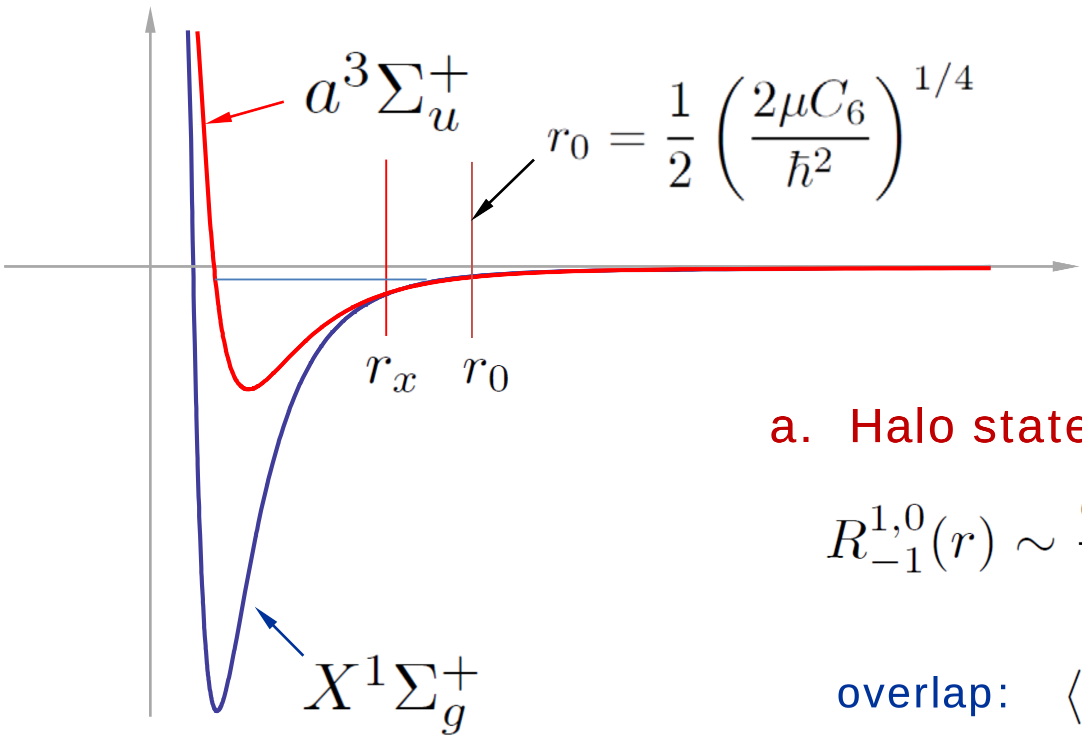
for $S \neq S'$ for $S = S'$



Franck-Condon factor
(only part that depends on radial wavefunction)

3 fit parameters: ε^0, ε^1, $\langle R^{S',l}_{v'}|R^{S,l}_v\rangle$

Franck-Condon factor



a. Halo states (universal regime)

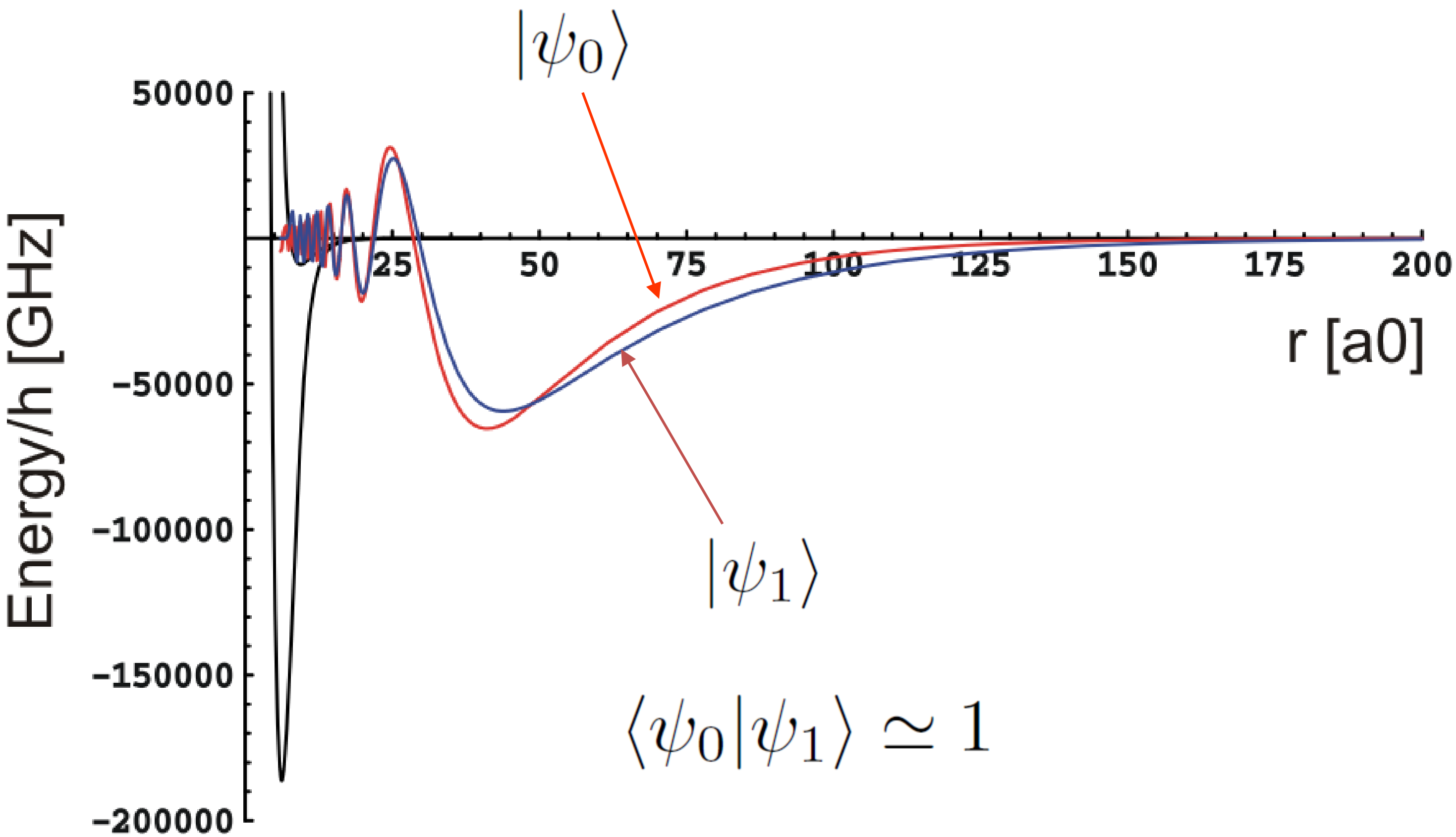
$$R_{-1}^{1,0}(r) \sim \frac{e^{-\kappa_1 r}}{r} \qquad R_{-1}^{0,0}(r) \sim \frac{e^{-\kappa_0 r}}{r}$$

overlap: $\langle R_{-1}^{0,0} | R_{-1}^{1,0} \rangle = 2\frac{\sqrt{\kappa_0 \kappa_1}}{\kappa_0 + \kappa_1}$

b. Asymptotic bound states ($r_{cl} > r_x$)

can be calculated numerically
starting from Van der Waals tail

singlet/triplet overlap

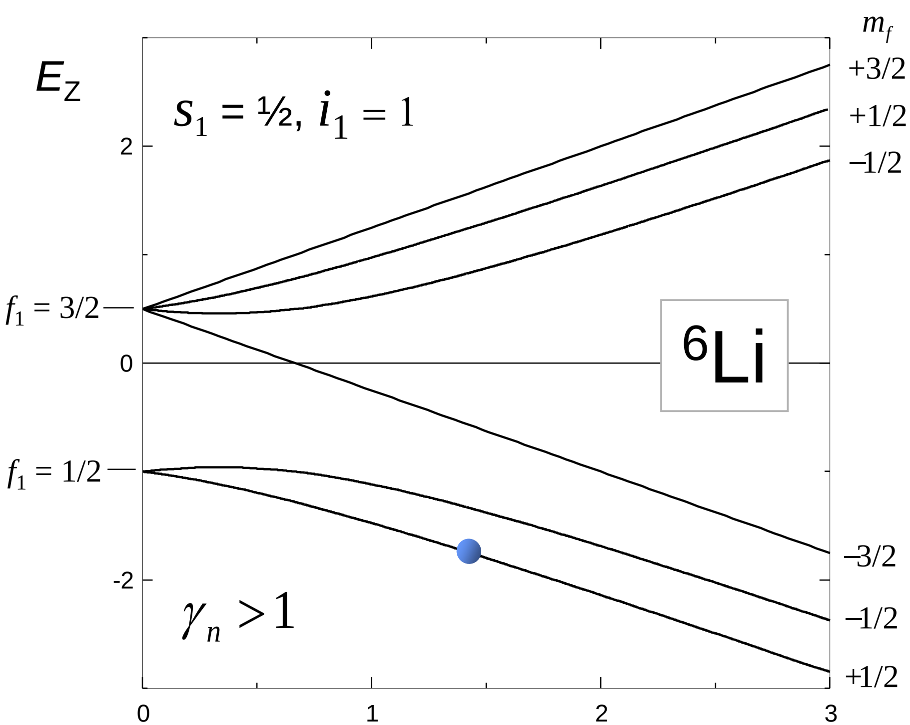


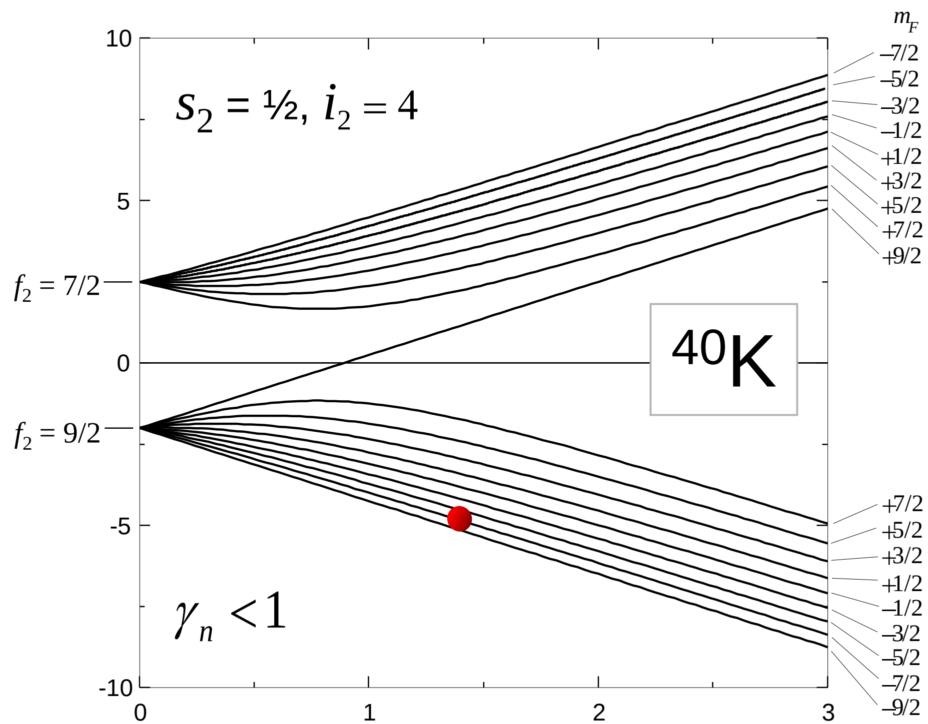
So we only need 2 fit parameters: $\varepsilon^0, \varepsilon^1$

So we can fit the resonances without knowing the radial wavefunctions!

^{6}Li-^{40}K pairs

$$\mathcal{H}_{\mathrm{hf+Z}} = \frac{a_1}{\hbar^2}\mathbf{i}_1 \cdot \mathbf{s}_1 + \frac{a_2}{\hbar^2}\mathbf{i}_2 \cdot \mathbf{s}_2 + \gamma_s\left(\mathbf{s}_1 + \mathbf{s}_2\right) \cdot \mathbf{B} - \left(\gamma_1 \mathbf{i}_1 + \gamma_2 \mathbf{i}_2\right) \cdot \mathbf{B}$$





magnetic field

magnetic field

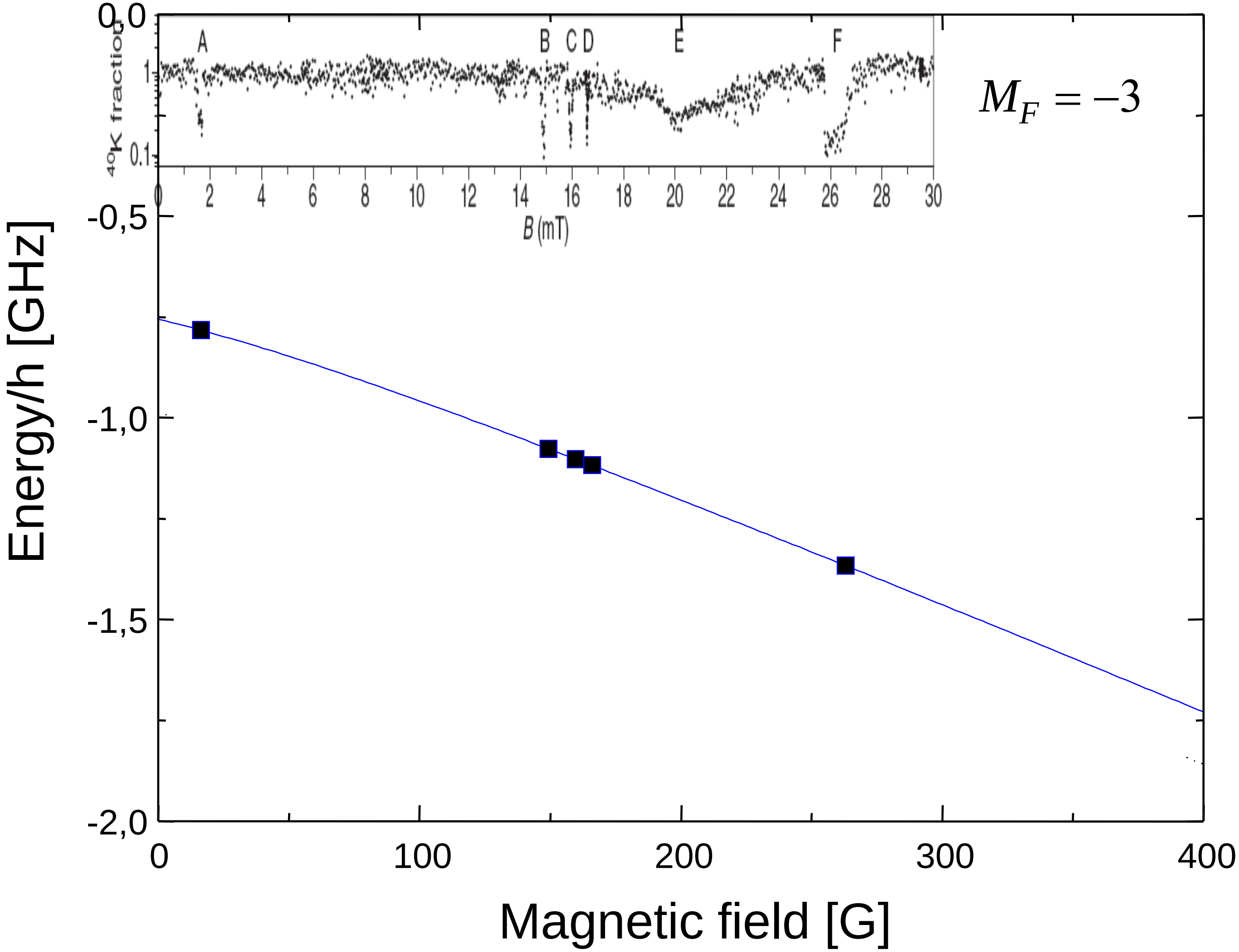
81 Gauss

$M_F = -3$

459 Gauss

^{6}Li-^{40}K loss features with $M_F = -3$

$$|\tfrac{9}{2}, -\tfrac{7}{2}\rangle_{\mathrm{K}} + |\tfrac{1}{2}, \tfrac{1}{2}\rangle_{\mathrm{Li}}$$



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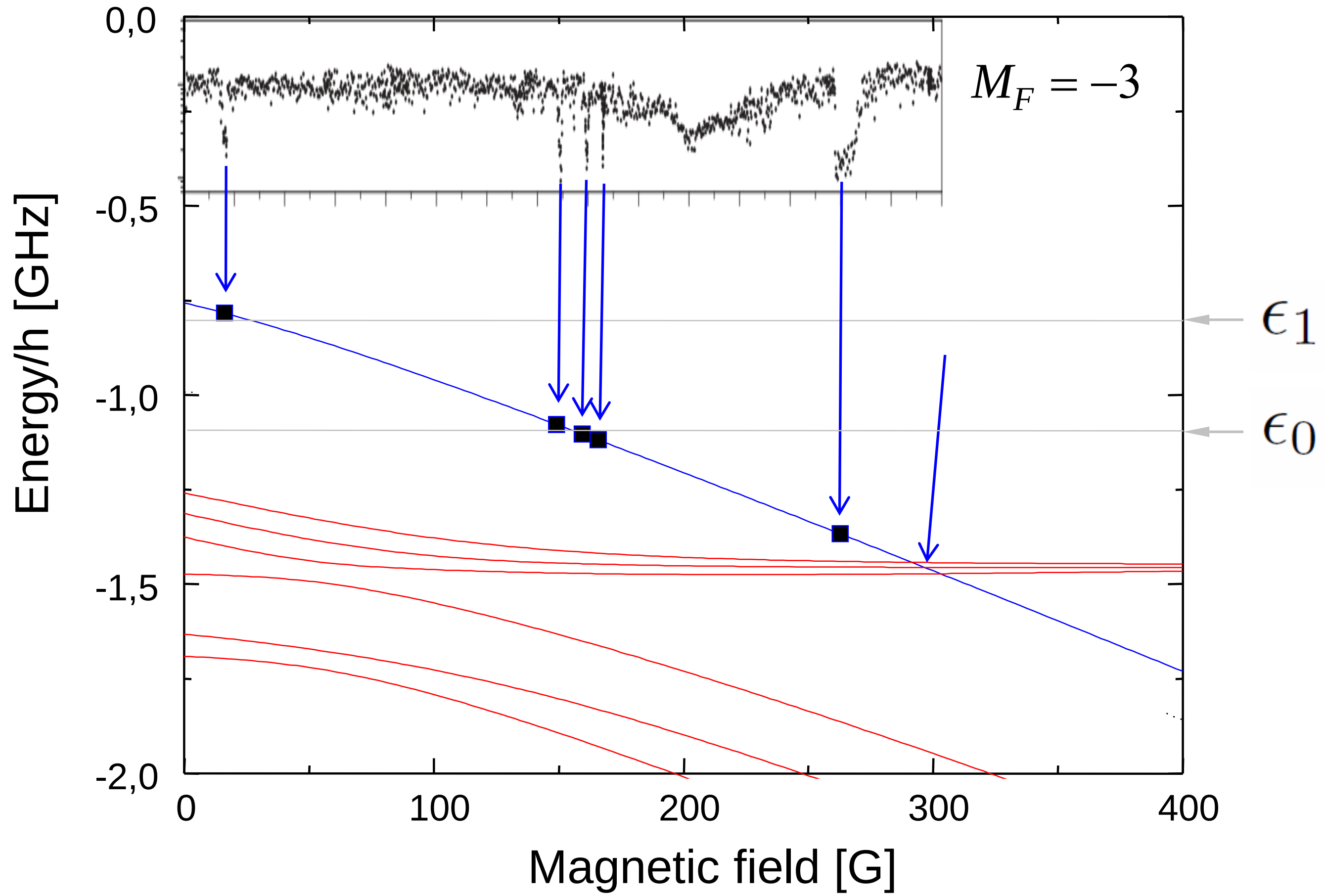
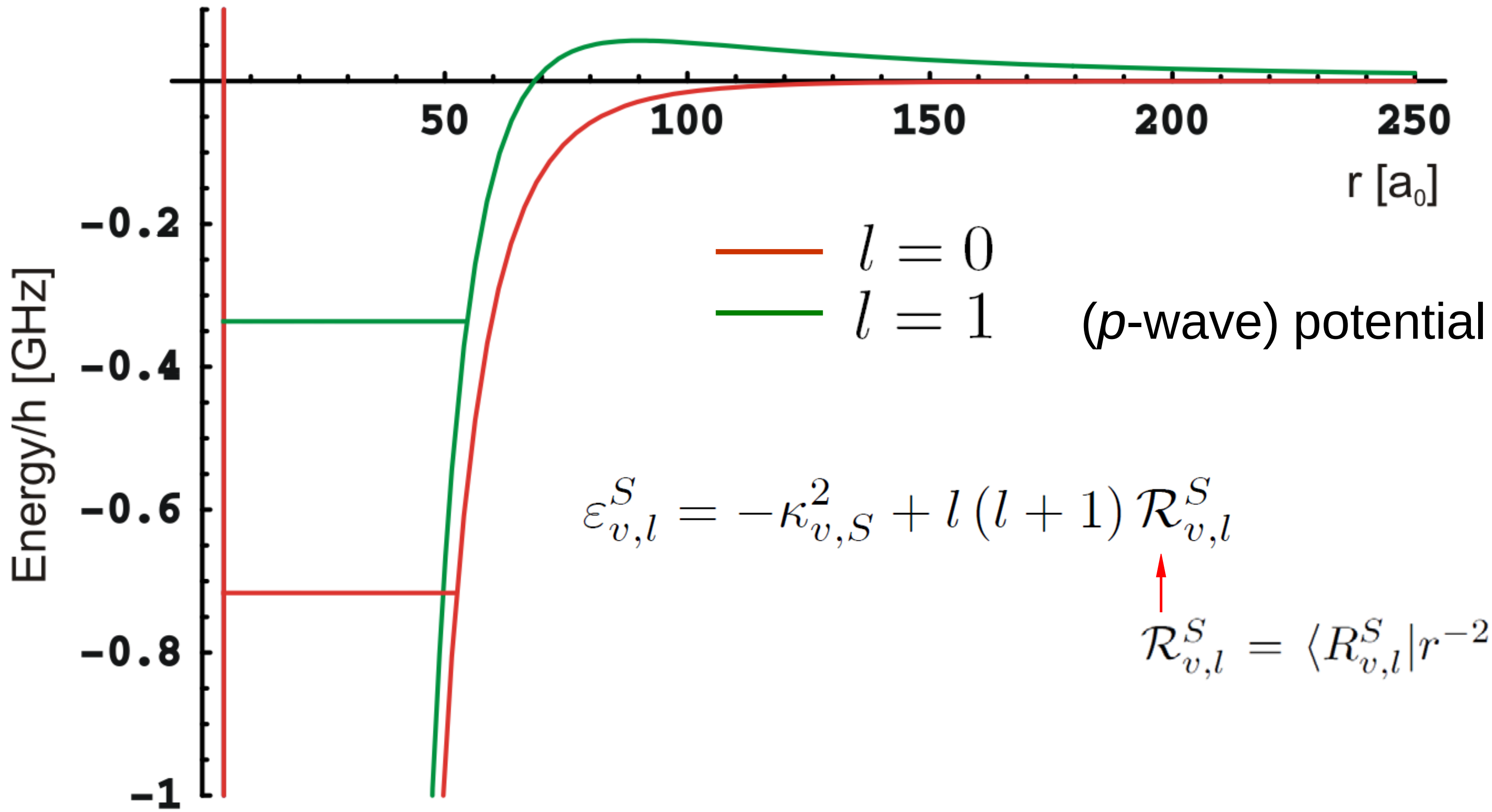


Include $l = 1$ molecular levels



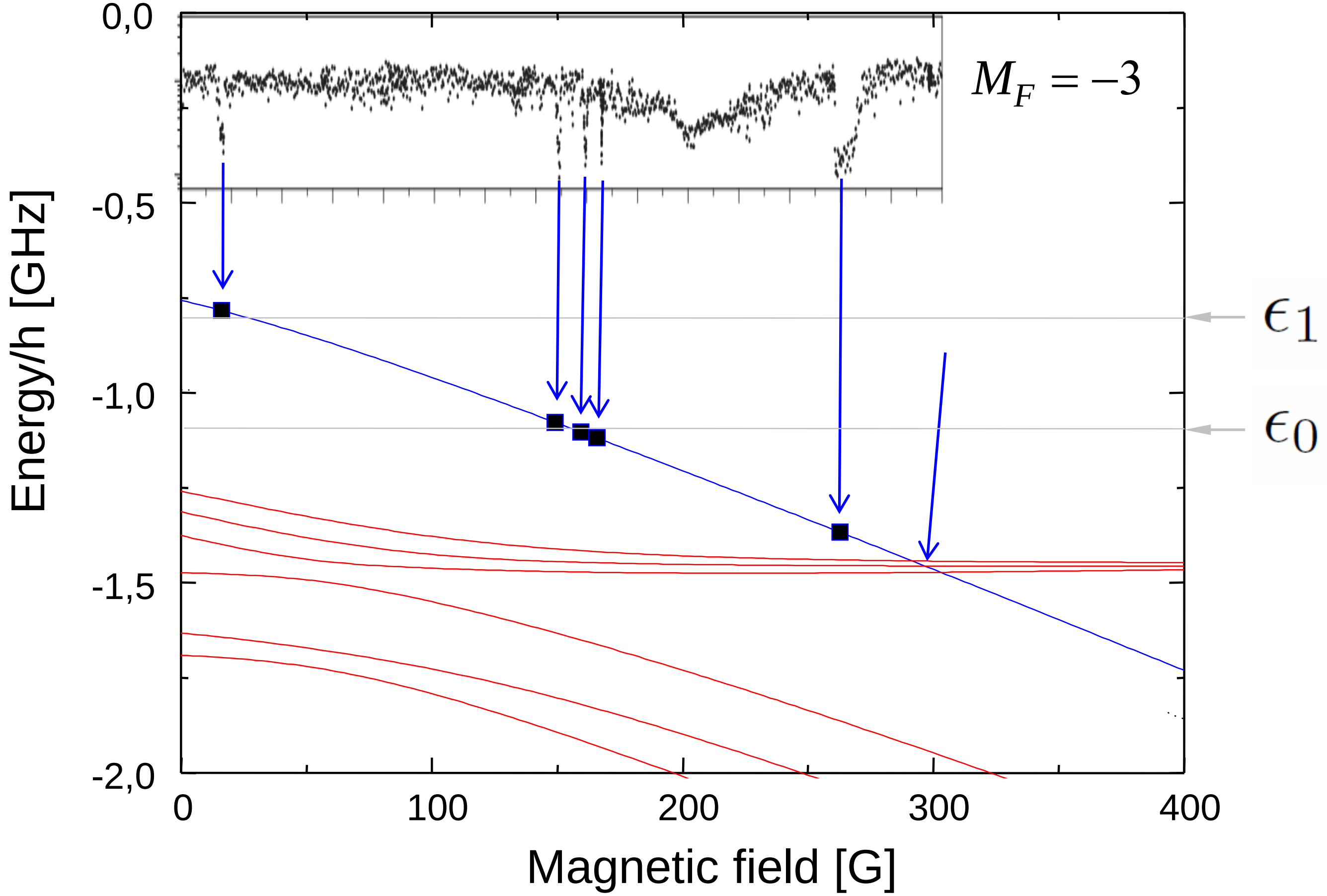
$$\varepsilon_{v,l}^{S} = -\kappa_{v,S}^{2} + l\,(l+1)\,\mathcal{R}_{v,l}^{S}$$

$$\mathcal{R}_{v,l}^{S} = \langle R_{v,l}^{S} | r^{-2} | R_{v,l}^{S} \rangle$$

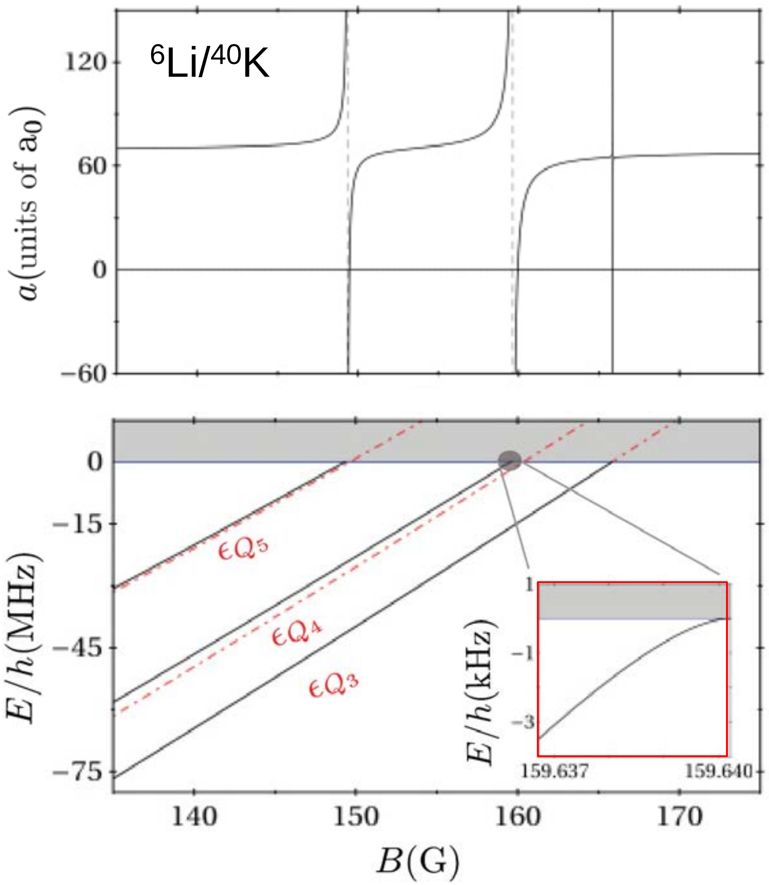
Conclusion: $l = 0$ and $l = 1$ levels can be related through C_6 coefficient

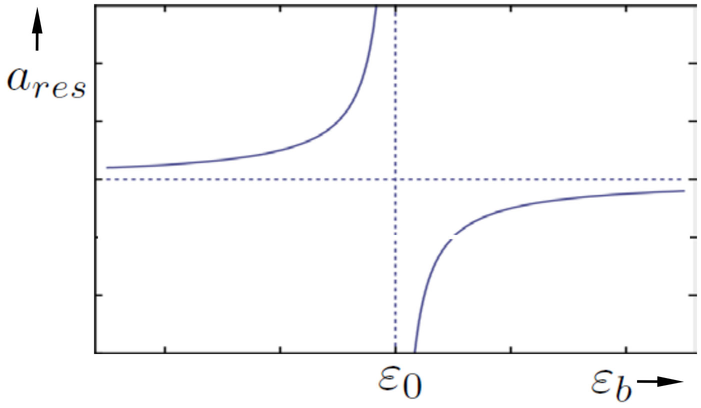
^{6}Li-^{40}K loss features with $M_F = -3$

$$|\tfrac{9}{2}, -\tfrac{7}{2}\rangle_{\mathrm{K}} + |\tfrac{1}{2}, \tfrac{1}{2}\rangle_{\mathrm{Li}}$$

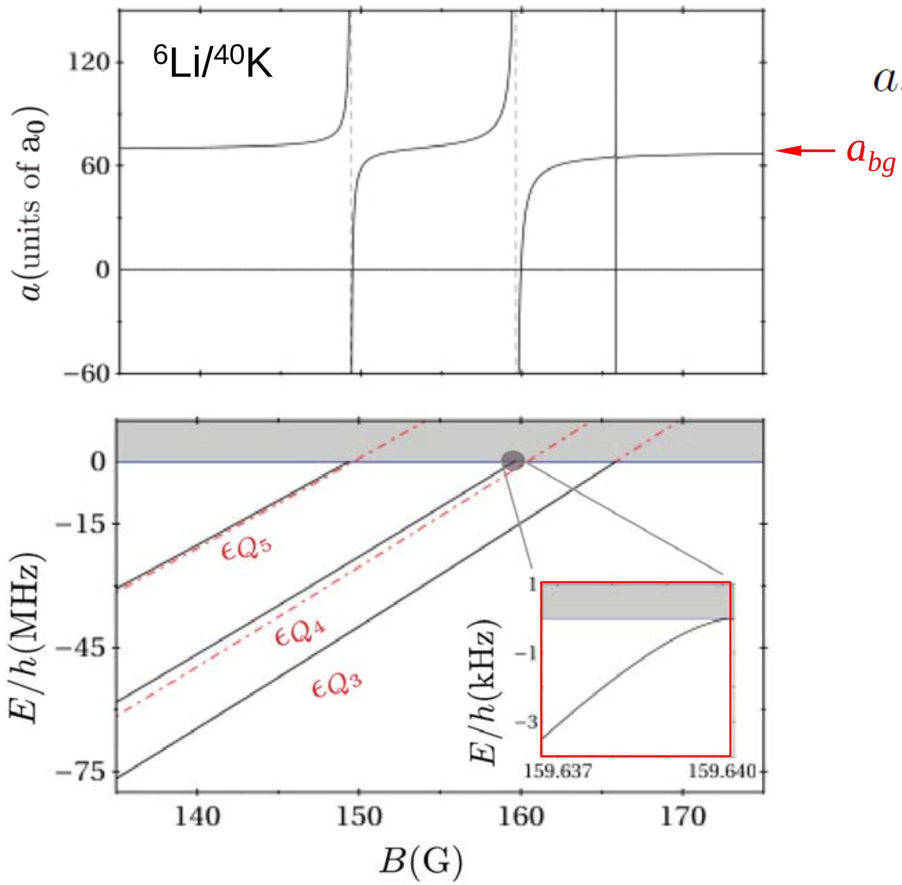


Relation to scattering length





Relation to scattering length



$$a_{res} = -\frac{1/R^*}{(\varepsilon_b - \varepsilon_0)} = -\frac{(\hbar^2/m_r R^*)}{\mu_{\rm eff}(B - B_0)}$$

$$a(B) = a_{bg} - \frac{(\hbar^2/m_r R^*)}{\mu_{\rm eff}(B - B_0)}$$

$$= a_{bg}\left[1 - \frac{(\hbar^2/m_r a_{bg} R^*)}{\mu_{\rm eff}(B - B_0)}\right]$$

$$a(B) = a_{bg}\left[1 - \frac{\Delta}{B - B_0}\right]$$

$$\Delta = \hbar^2/(m_r a_{bg} \mu_{\rm eff} R^*)$$

Review paper

C. Chin et al., R.M.P., 82 1225 (2010)

Relation to scattering length

strength parameter:

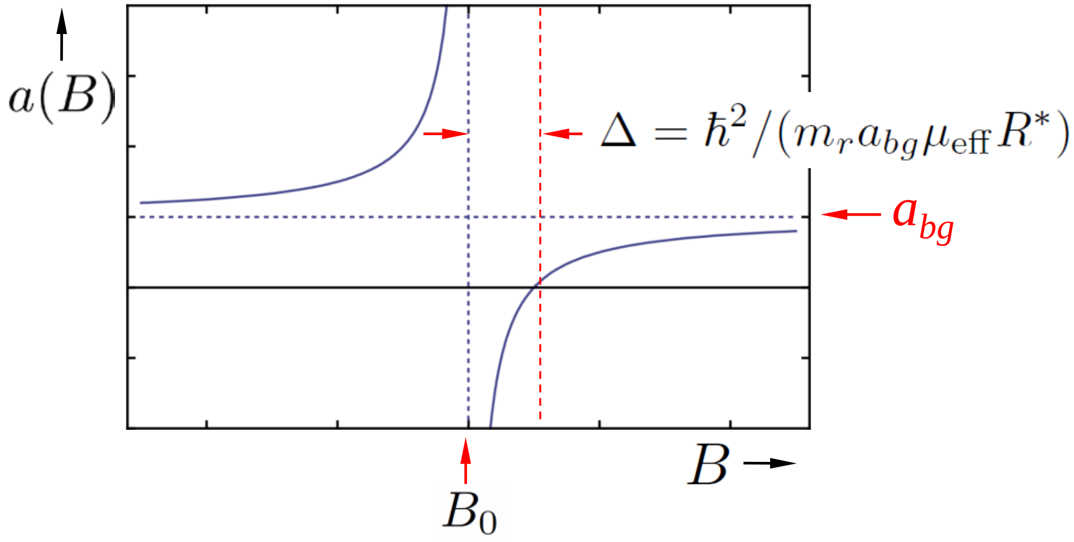
$$s \equiv \frac{r_0}{R^*} = \frac{a_{bg}}{r_0}\frac{\mu_{\rm eff}}{E_0}\Delta$$

$$E_0 = \frac{\hbar^2}{mr_0^2}$$

$$a_{res} = -\frac{1/R^*}{(\varepsilon_b - \varepsilon_0)} = -\frac{(\hbar^2/m_r R^*)}{\mu_{\rm eff}(B-B_0)}$$

$$a(B) = a_{bg} - \frac{(\hbar^2/m_r R^*)}{\mu_{\rm eff}(B-B_0)}$$

$$= a_{bg}\left[1 - \frac{(\hbar^2/m_r a_{bg} R^*)}{\mu_{\rm eff}(B-B_0)}\right]$$

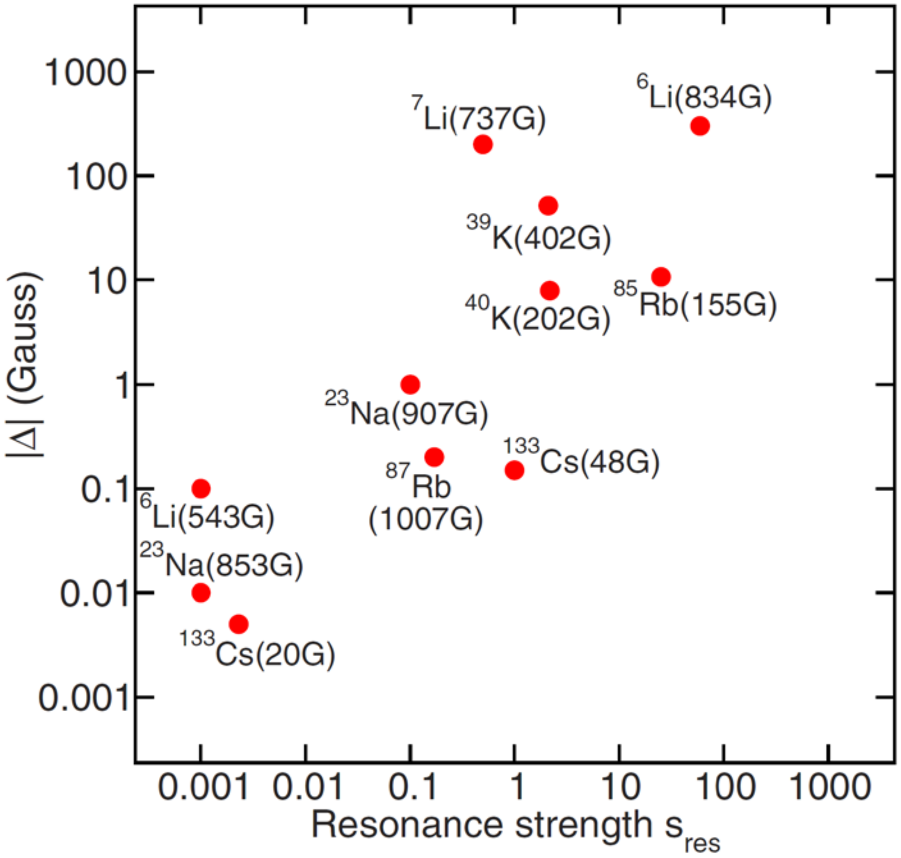


$$a(B) = a_{bg}\left[1 - \frac{\Delta}{B-B_0}\right]$$

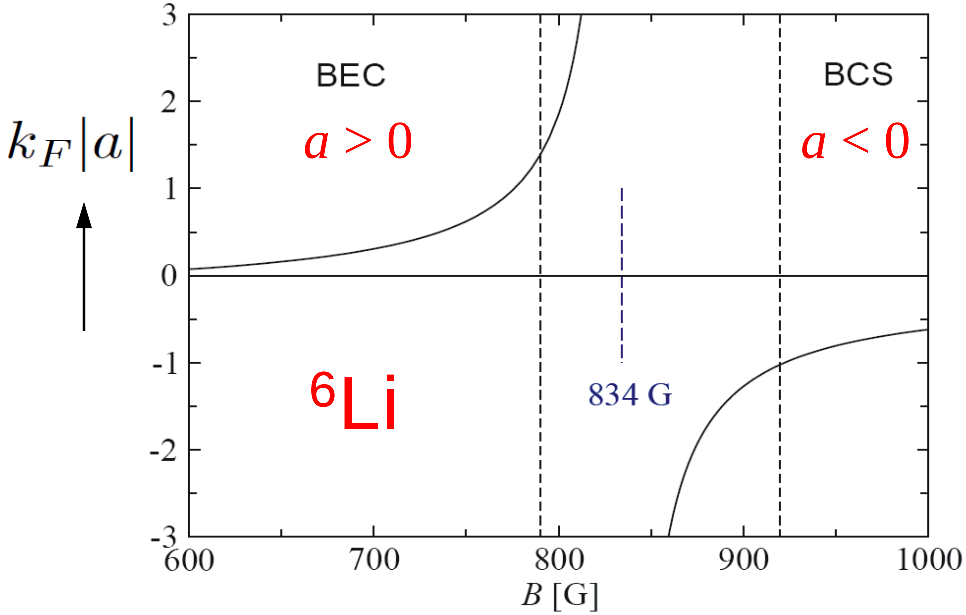
Relation to scattering length

strength parameter:

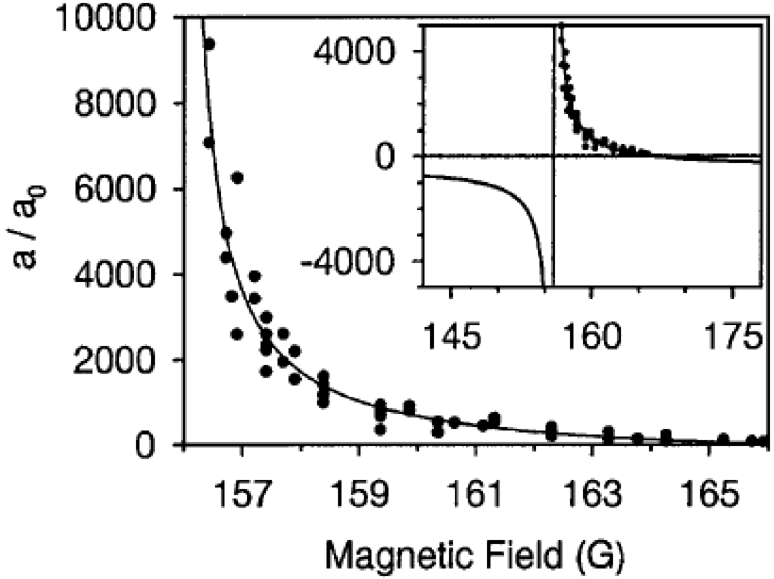
$$s \equiv \frac{r_0}{R^*} = \frac{a_{bg}}{r_0}\frac{\mu_{\rm eff}}{E_0}\Delta$$

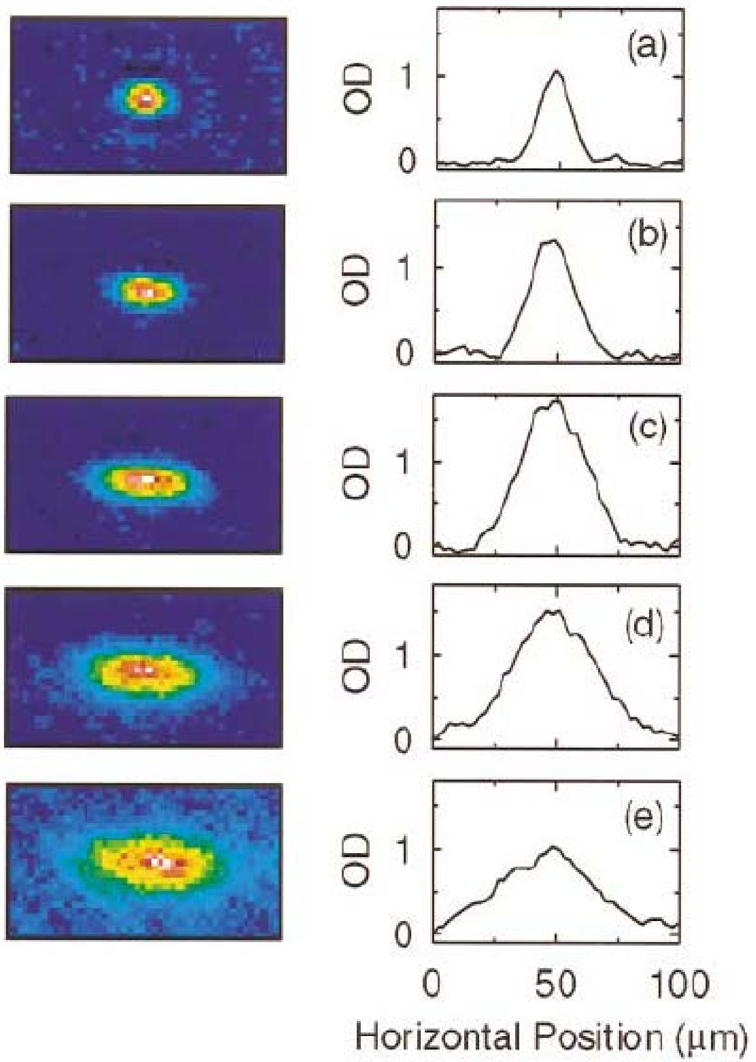


Li-6



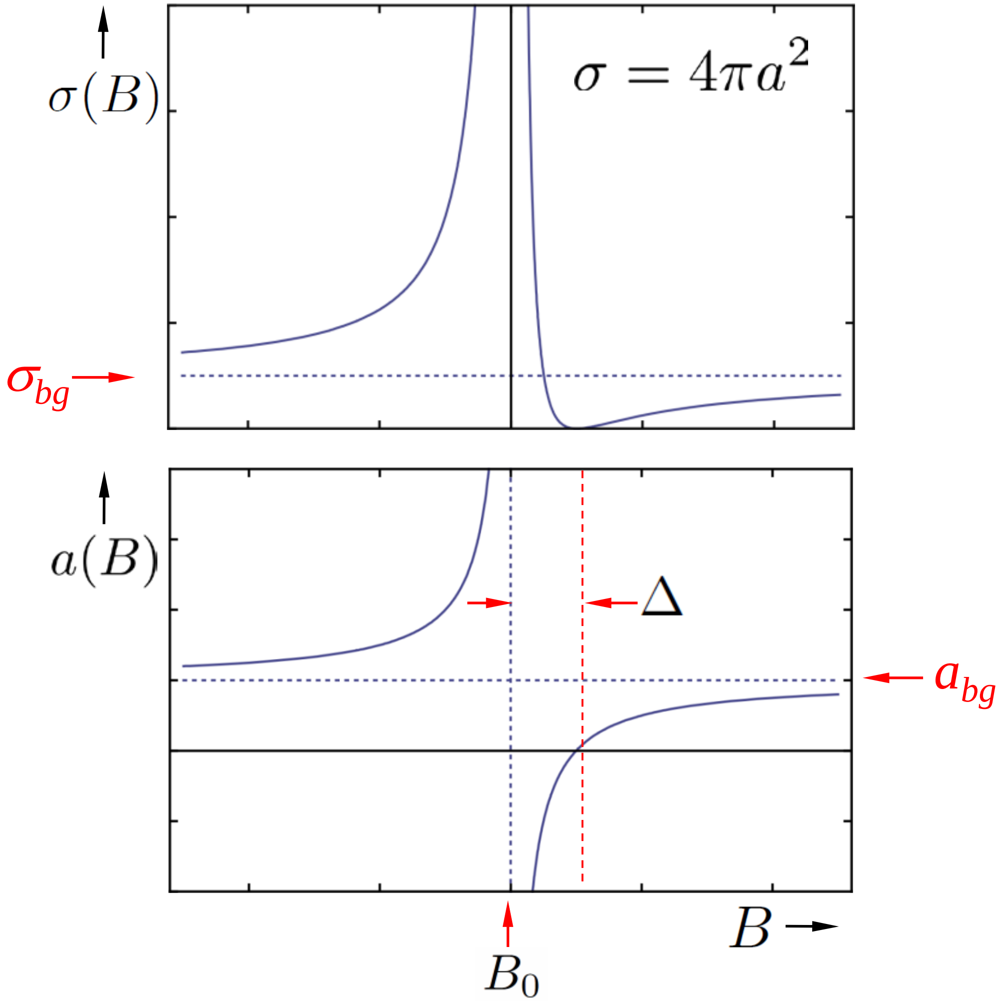
Rb-85



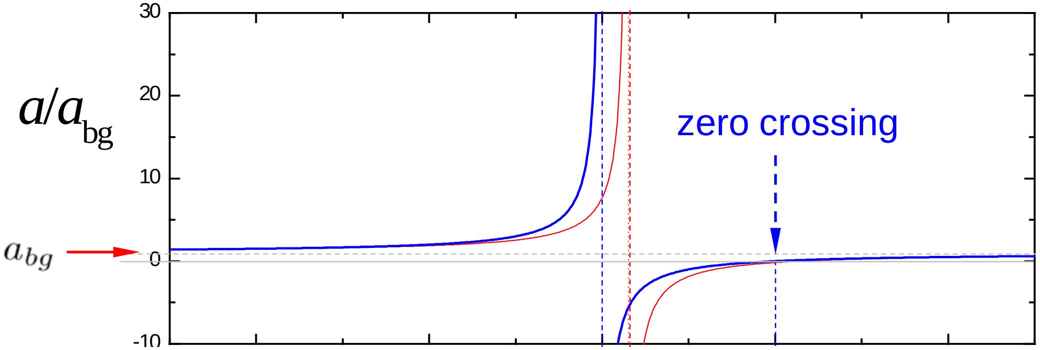


S.L. Cornish et al., PRL 85 (2000) 1995

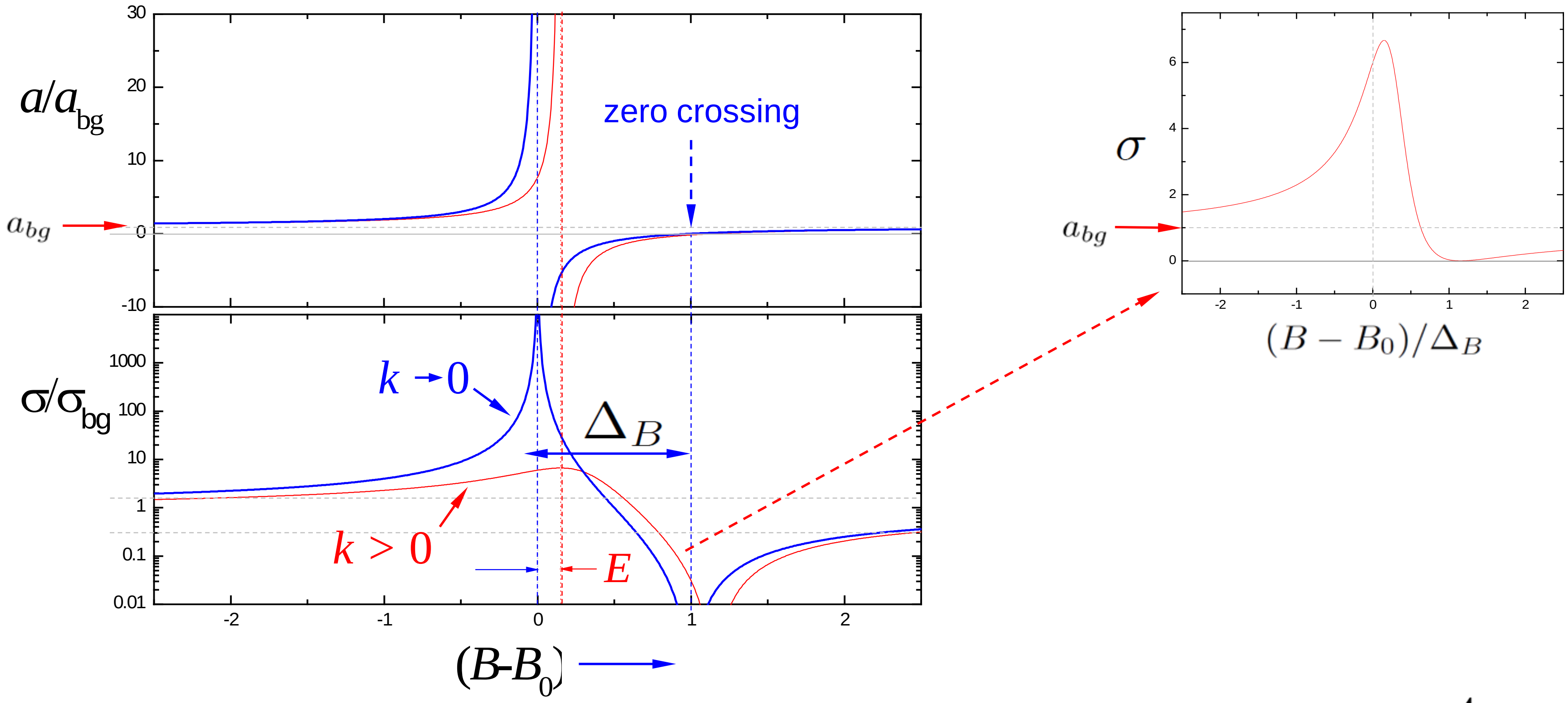
Fano lineshape



width from: *elastic cross section*



width from: *elastic cross section*



on resonance ($a \to \infty$): *unitarity limited scattering* ($\sigma = \dfrac{4\pi}{k^2}$)

for $a_{bg} \neq 0$ asymmetric line shape: *Fano profile*

Narrow versus broad resonances

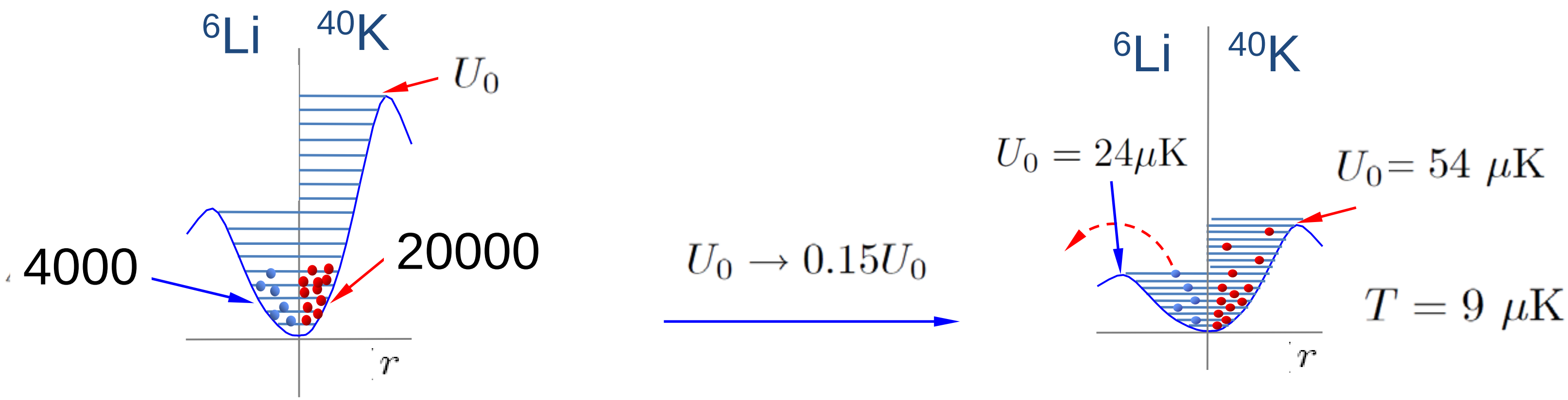
Importantly:

The meaning of broad and narrow depends on the context

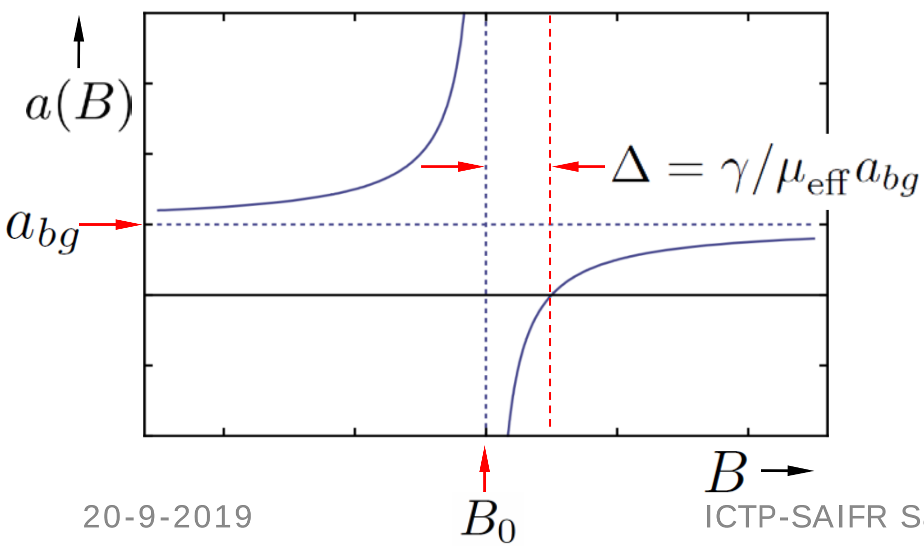
- Broad as measured in magnetic field
- Broad as compared to the collisional band of energies
- Broad due to strong Feshbach coupling (large R*)

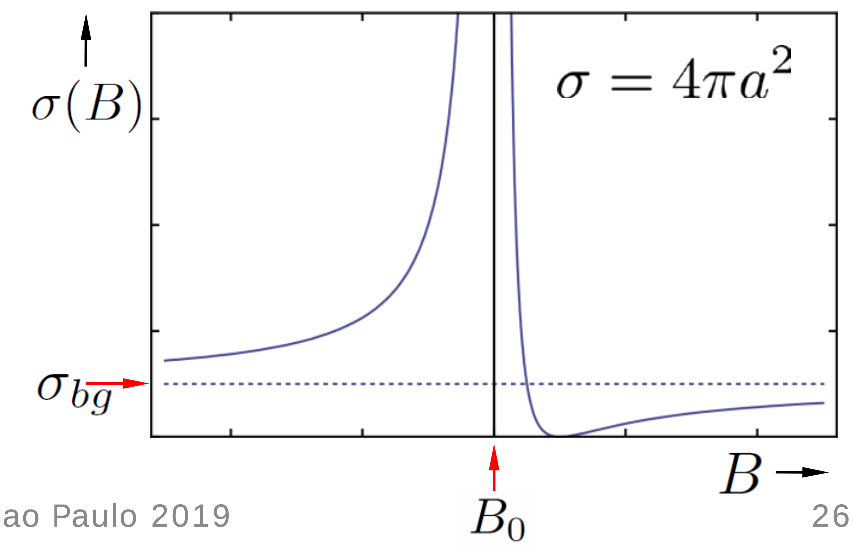
experimental

adiabatic expansion of Gaussian trap induces evaporation:

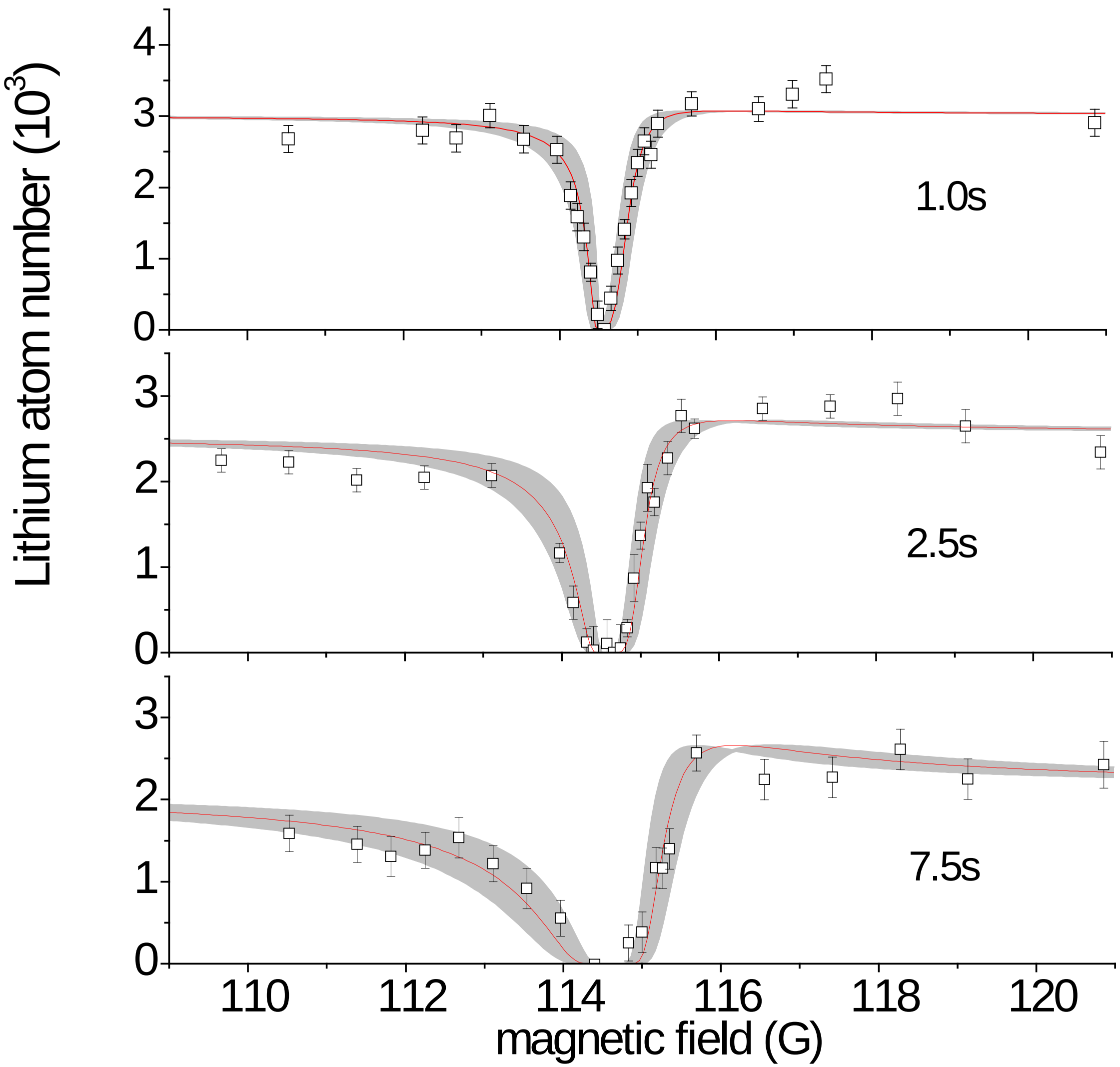


Measure evaporation of ^{6}Li due to collisions with ^{40}K





characterization of a Feshbach resonance



position: $B_0 = 114.47(5)$ G

width: $\Delta B = 1.5(5)$ G

T.G. Tiecke, et al.
PRL 104, 053202 (2010)

End of ICTP-SAIFR Lectures

Thank you