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ZOQ Zentrum für Optische Quantentechnologien

ILP INSTITUT FÜR LASERPHYSIK

Optical Lattices and Artificial Gauge Potentials

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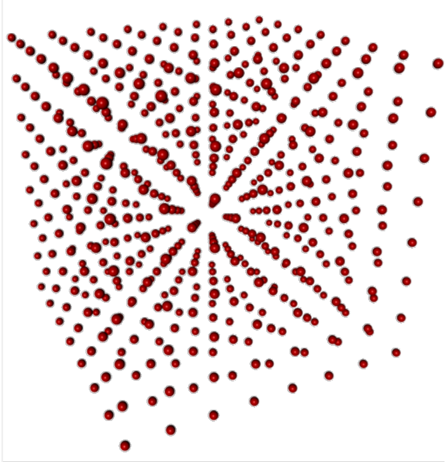
Optical Lattices and Artificial Gauge Potentials

Part 1: Build up the Hamiltonian
Optical Lattices
Non-interacting properties (band structure, wave functions)
Hubbard models

Cold atoms simulator



Part 2: Read out the quantum state
Probing quantum gases in optical lattice
Mapping phase diagrams of Hubbard models

Part 3: Beyond Hubbard models in optical lattices
Topological properties and transport
Magnetic phenomena for neutral atoms

Optical Lattices and Artificial Gauge Potentials

Part 2: Read Out the Quantum State

Part 2

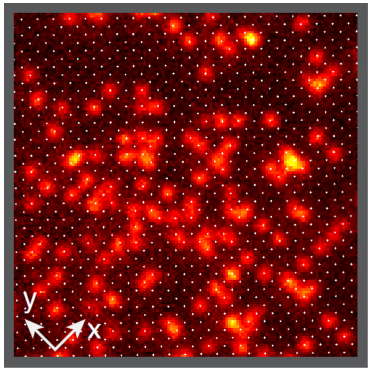
2.1 Probing quantum gases in optical lattices

2.2 Bose-Hubbard Model

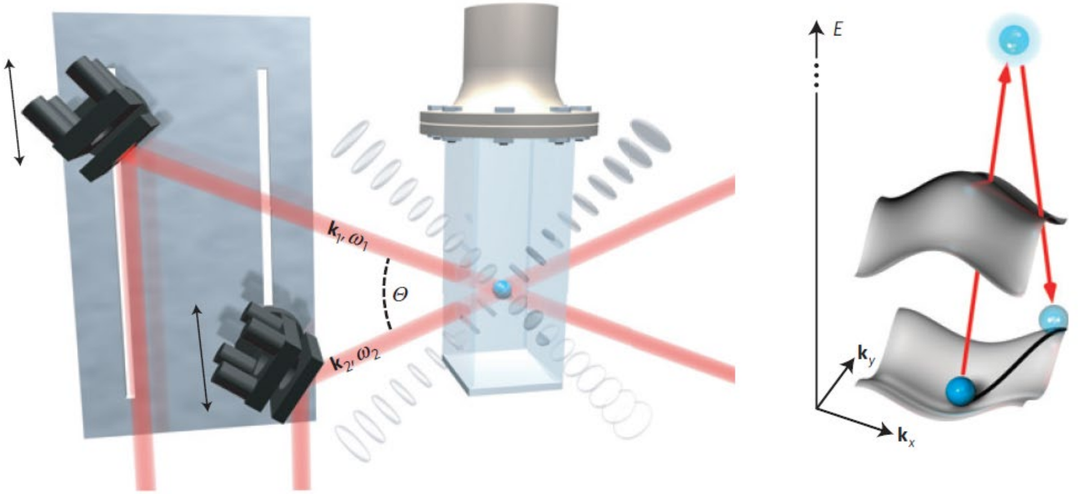
2.3 Fermi-Hubbard Model

2.1 Probing quantum gases in optical lattices

- Momentum space
  - » Time-of-flight measurement (TOF)
  - » Band mapping
- Real space: Quantum Gas Microscopes
  - » Single site / single atom detection
  - » Occupation number at each lattice site

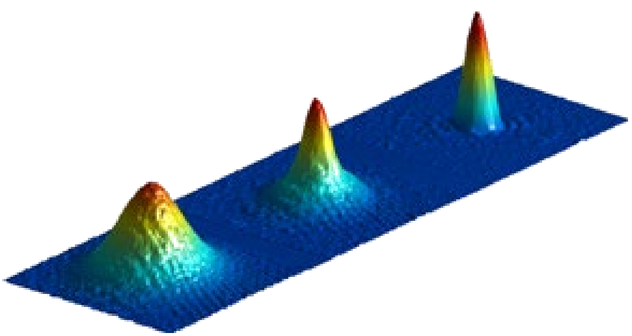


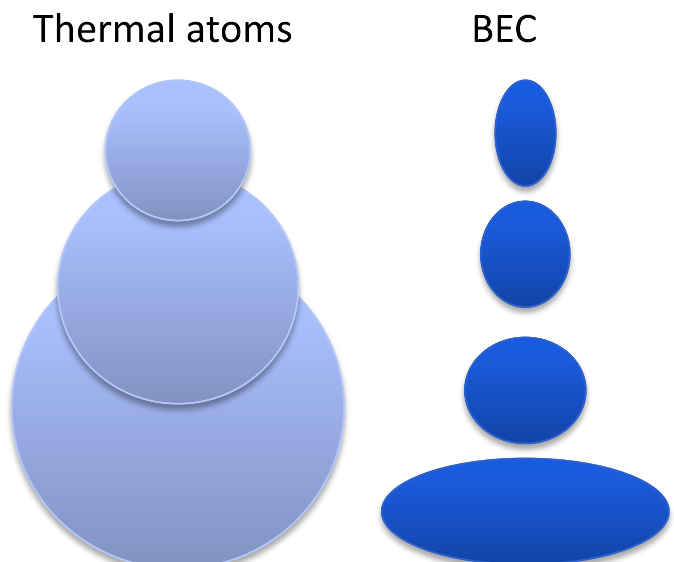
- Excitation spectrum
  - » Bragg spectroscopy
  - » Amplitude modulation

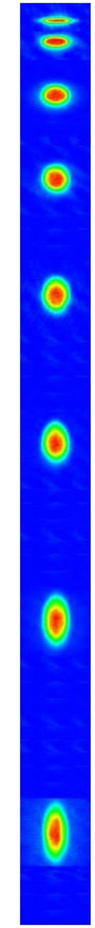


Momentum Space / Time-of-flight measurement

- Time-of-flight measurement (TOF)
  - » Sudden extinction of lattice and trap potentials (Projection onto free momentum states)
  - » Free expansion of the atomic cloud under gravity (interactions neglected due to low density)
  - » Absorption imaging with resonant laser light



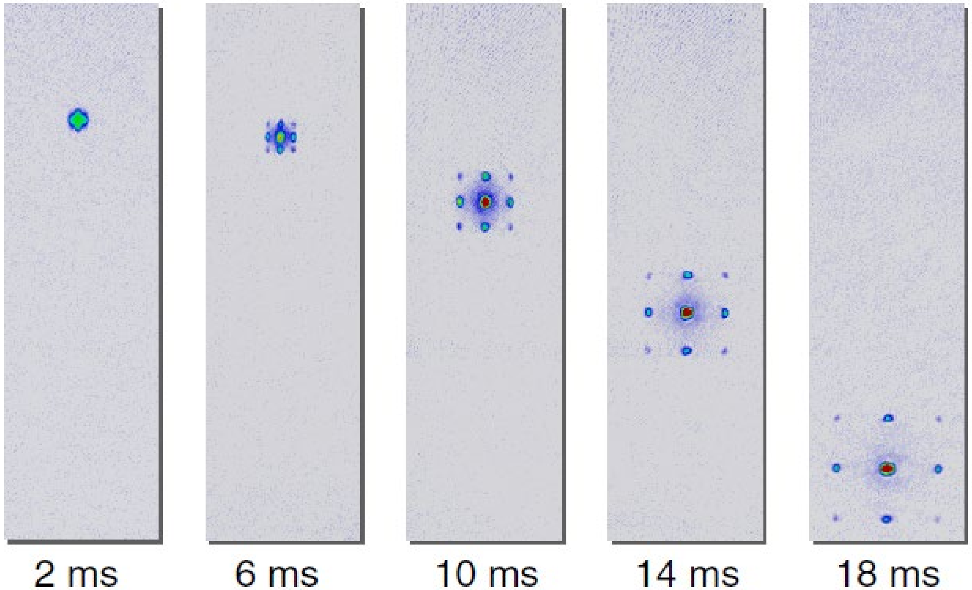




Momentum Space / Time-of-flight measurement

- Time-of-flight measurement (TOF)
  - » Sudden extinction of lattice and trap potentials (Projection onto free momentum states)
  - » Free expansion of the atomic cloud under gravity (interactions neglected due to low density)
  - » Absorption imaging with resonant laser light
- Free momentum states expand ballistically $r = \mathbf{p}t/m$
- Time-of-flight-distribution corresponds to the momentum distribution in the lattice

$$\rho^p(\mathbf{k}) \approx \left(\frac{\hbar t}{m}\right)^3 \rho^{\mathrm{TOF}}\left(\mathbf{r} = \frac{\hbar \mathbf{k} t}{m}\right)$$

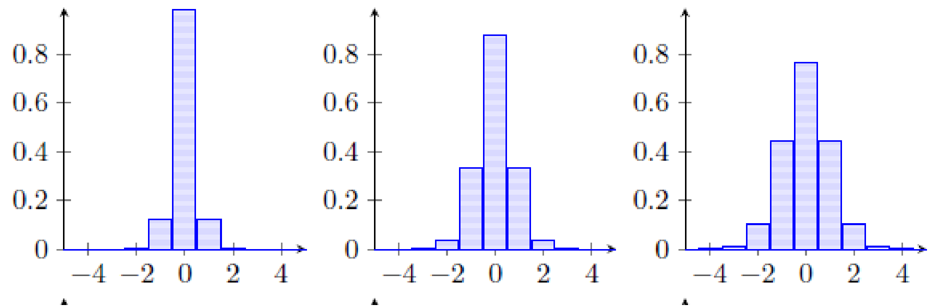


Figures: Greiner, PhD thesis, Munich (2003)

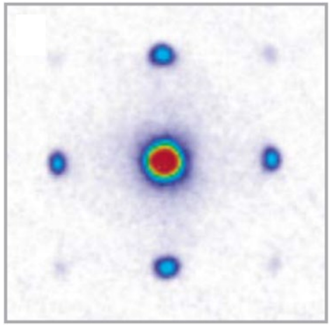
Momentum Space / Time-of-flight measurement

- Bosons condense into Bloch state $\boldsymbol{q} = 0$
  - » Superposition of plane waves $\boldsymbol{k} = 0 + n\boldsymbol{G}$

    → Bragg peaks in ToF
  - » Envelope according to Bloch coefficients



Bloch coefficients for n=0, q=0, V_0/E_r=2,8,20

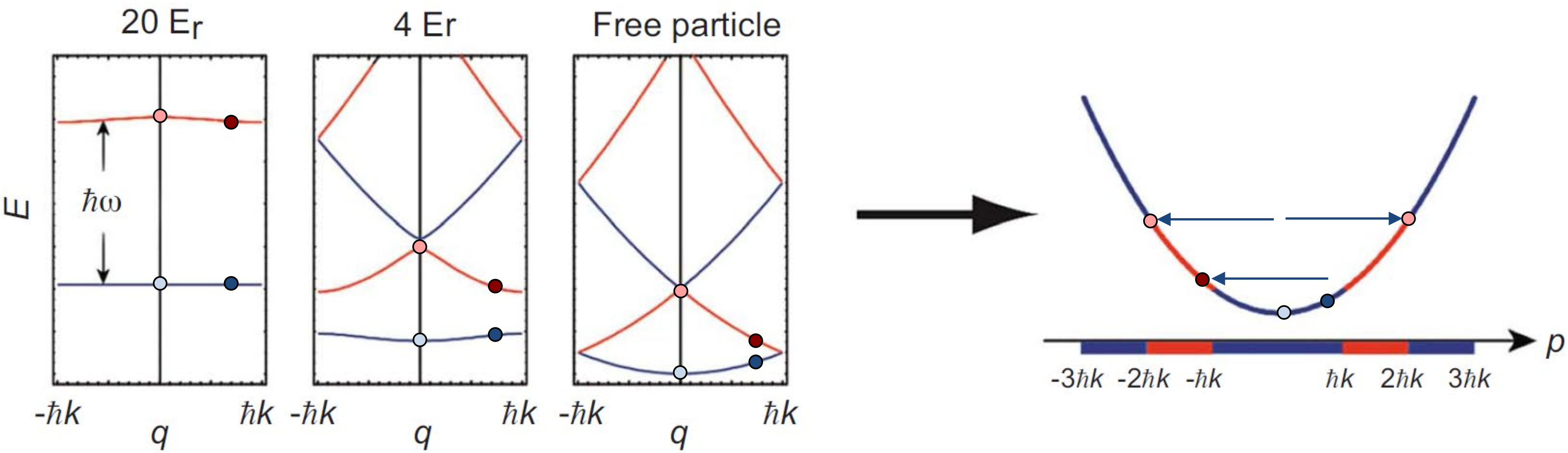


Absorption imaging of a BEC after ToF

- Alternative description
  - » Interference of the coherent matter waves from the lattice sites
  - » Contrast is a measure for the coherence in the lattice
  - » Envelope from the finite size of the Wannier function

Figures: Greiner, Bloch, et al. Nature 415, 39 (2002), Greiner PhD thesis, Munich (2003)

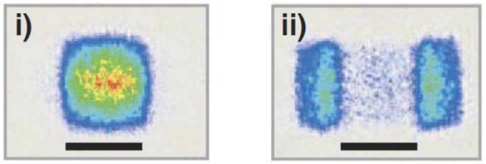
Momentum Space / Adiabatic band mapping



- Adiabatic ramping down of the lattice
  - » Mapping onto the dispersion of the free particle
  - » Quasi momentum **q** is conserved
  - » Timescale matters: avoid excitations to higher bands (not too fast)

- Imaging momentum distribution after ToF

- Often used for fermions, which fill up the bands



Band (000) filled

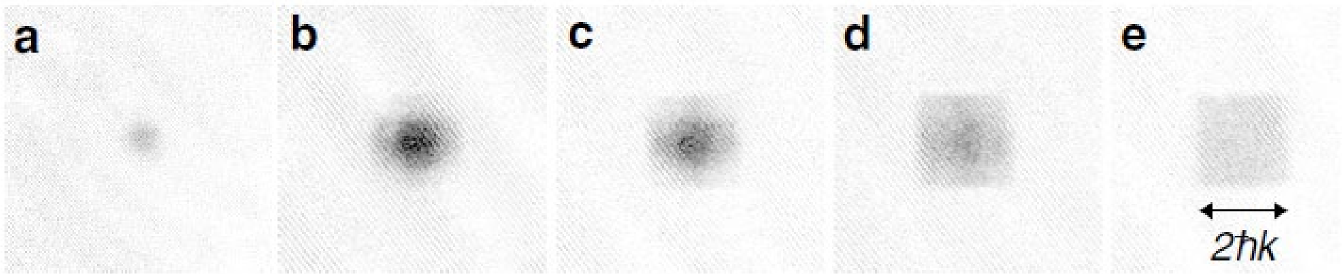
Band (100) filled

Figures: Bloch et al., Rev. Mod. Phys. (2007); Müller, Bloch et al. PRL 99, 200405 (2007)

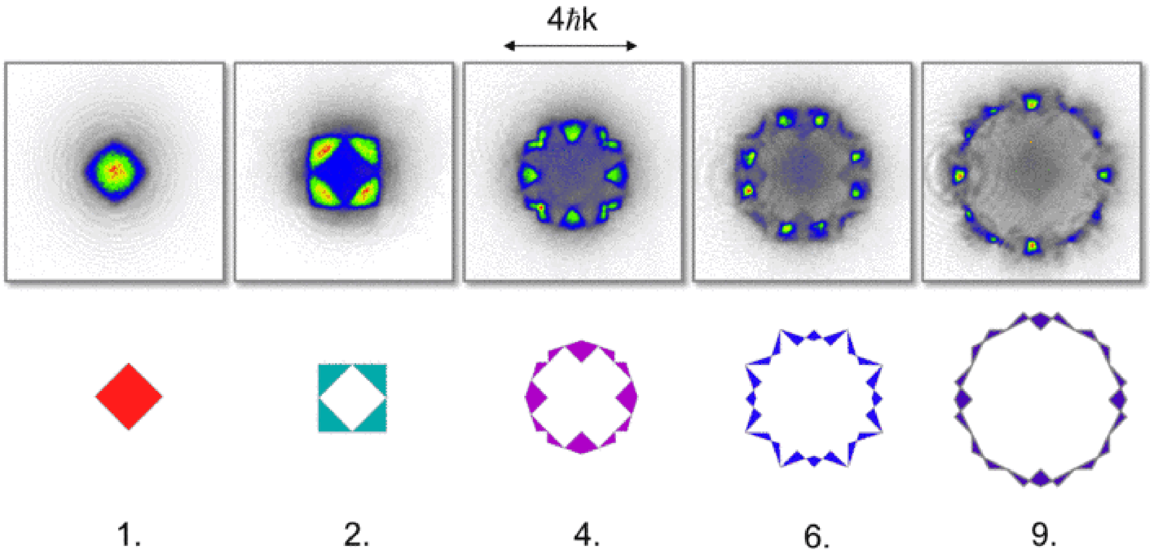
Momentum Space / Adiabatic band mapping

- Imaging Fermi surfaces

Non-interacting fermions in a cubic lattice filling up the lowest band



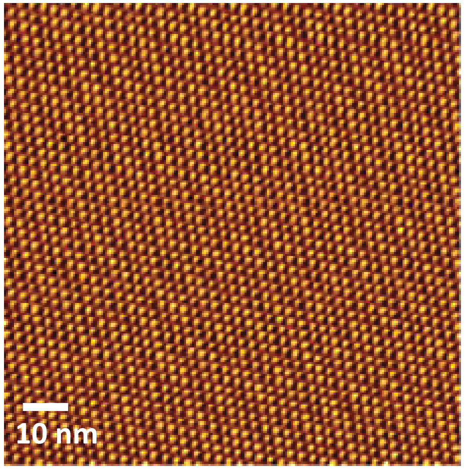
- Superfluids in higher bands of a bipartite square lattice



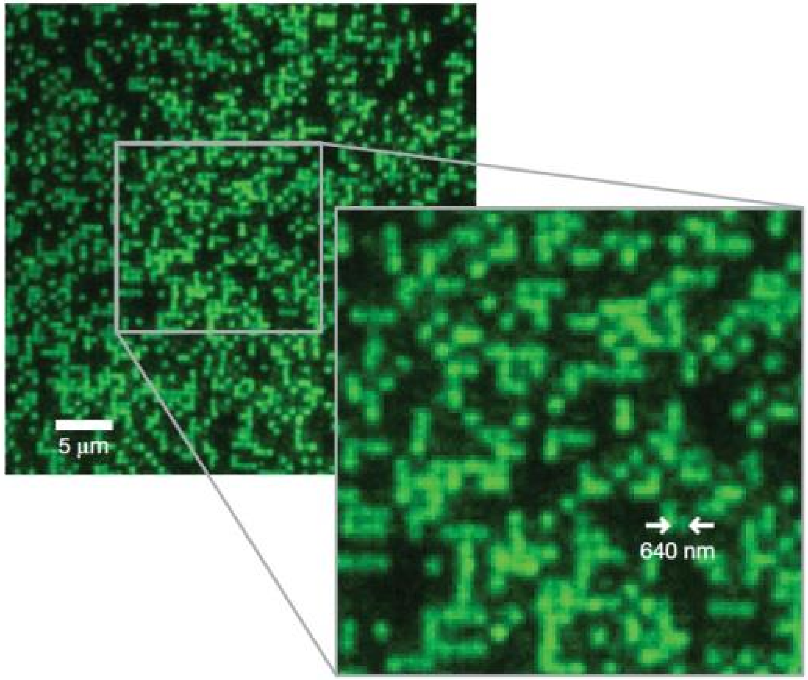
Real Space / Quantum Gas Microscopes

- Optical lattices are model systems for solid state physics
  but 1000 times larger lattice spacing!
- Resolve lattice sites with visible light (no STM needed)
- Advantage: freeze distribution and image single snapshots
- Only recent development due to technical challenges

Scanning tunneling microscope



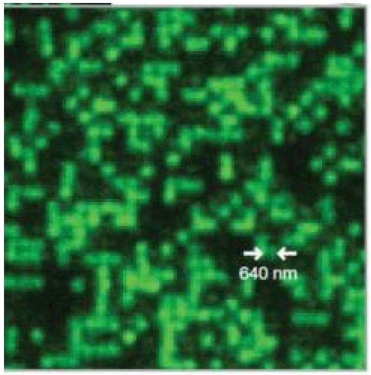
Quantum gas microscope



Bakr/Greiner et al. Nature 462, 74 (2010).
Figure: http://afmuniversity.em.keysight.com/index.php/component/content/article/14-sample-data-articles/197-scanning-tunneling-microscopy-stm (STM image of HOPG, scan size 100 nm)

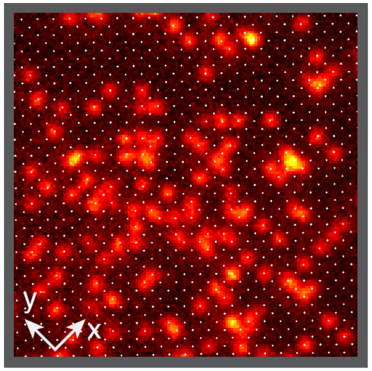
Gallery: Bosonic quantum gas microscopes

Group of Markus Greiner
(Harvard University)



W. Bakr et al.:
Science **329**, 547 (2010)

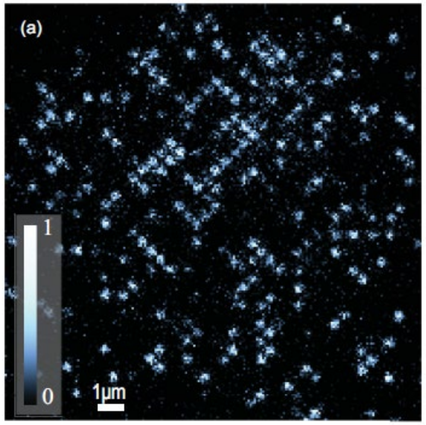
Group of Stefan Kuhr,
Immanuel Bloch (MPQ)



J. F. Sherson et al.:
PRL **114**, 193001 (2011)

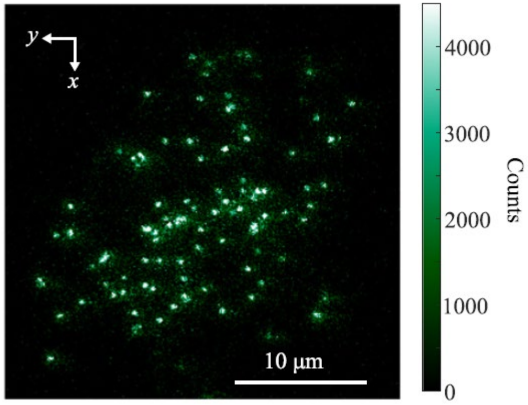
^{87}Rb

Group of Mikio Kozuma
(Tokyo)



M. Miranda et al.
PRA 91, 063414 (2015)

Group of Yoshiro Takahashi
(Kyoto)

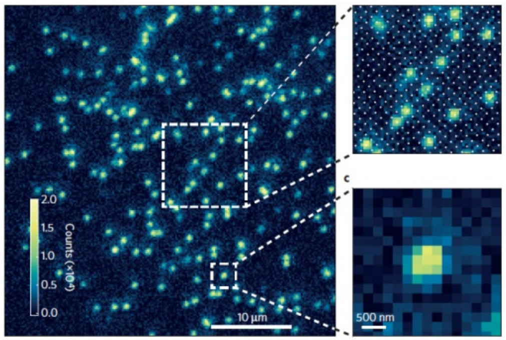


Yamamoto et al.
New J. Phys. 18, 023016 (2016)

^{174}Yb

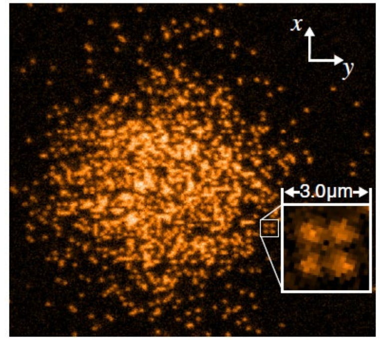
Gallery: Fermionic quantum gas microscopes

Group of Stefan Kuhr
(University of Strathclyde, Glasgow)



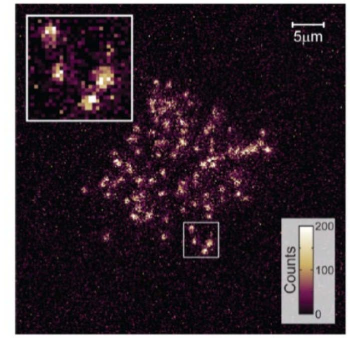
E. Haller et al.:
Nature Physics **11**, 738 -742 (2015)

Group of Martin Zwierlein
(MIT)



L. W. Cheuk et al.:
PRL **114**, 193001 (2015)

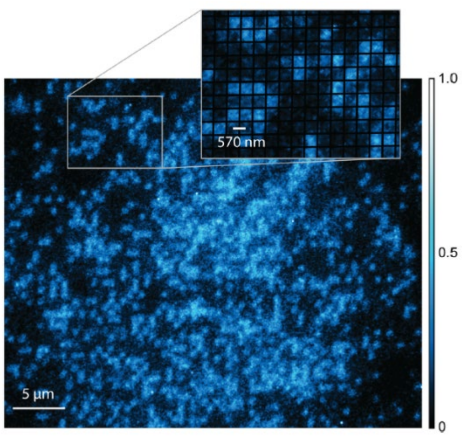
Group of Joseph Thywissen
(University of Toronto)



G. J. A. Edge et al.:
PRA **92**, 063406 (2015)

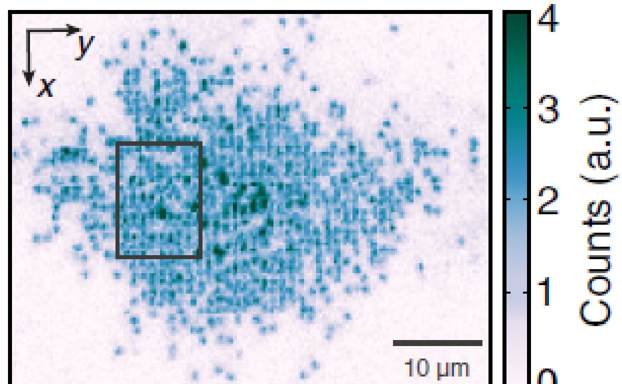
^{40}K

Group of Markus Greiner
(Harvard University)



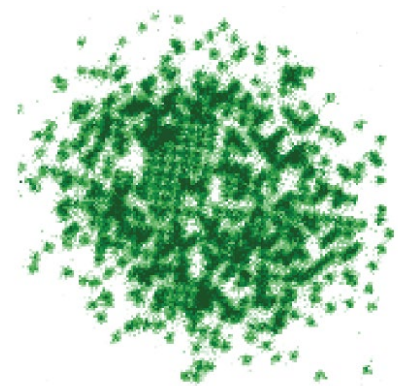
M. F. Parsons et al.:
PRL **114**, 213002 (2015)

Group of Christian Gross
/ Immanuel Bloch (MPQ)



A. Omran et al.:
PRL **115**, 263001 (2015)

Group of Wasseem Bakr
(Princeton)



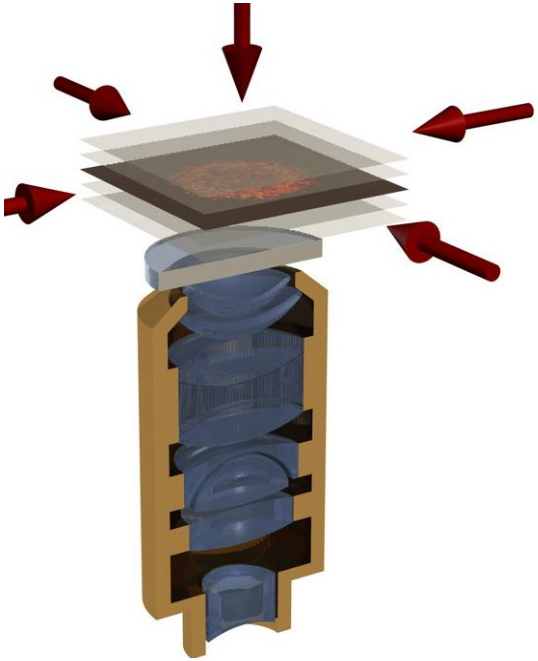
P. T. Brown et al.:
Science **357**, 1385 (2017)

^{6}Li

Real Space / Quantum Gas Microscopes

Experimental procedure

- Freezing the fluorescing atoms
  - » Pin atoms in a very deep optical lattice (0.3…1 mK)
  - » Induce fluorescence while laser cooling the atoms



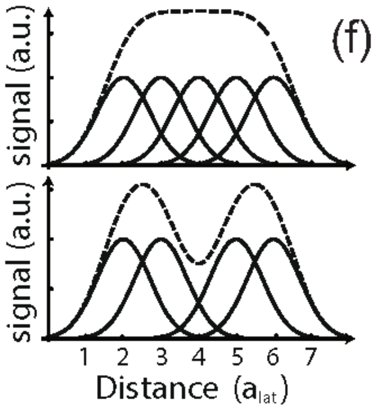
- Fluorescence Imaging
  - » High resolution setups (NA = 0.7)

Rayleigh resolution

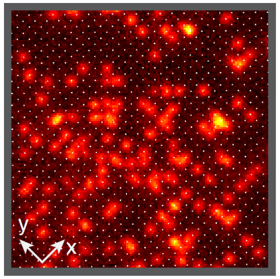
$$r_0 = \frac{1.22\,\lambda}{2\mathrm{NA}} \sim 700\ \mathrm{nm}$$

Distance between lattice sites

$$a_{lat} = \frac{\lambda_{lat}}{2} \sim 500\ \mathrm{nm}$$



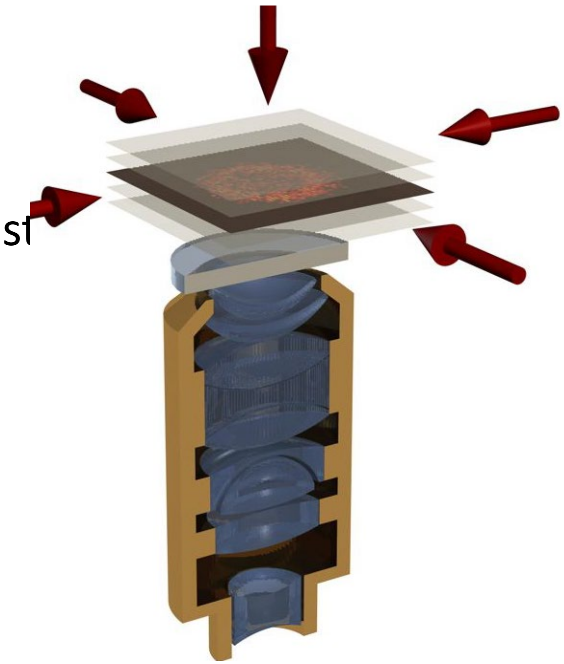
  - » Reconstruction algorithm



Figures: Sherson, Weitenberg, Bloch, Kuhr et al., Nature 467, 68 (2010)

Real Space / Quantum Gas Microscopes

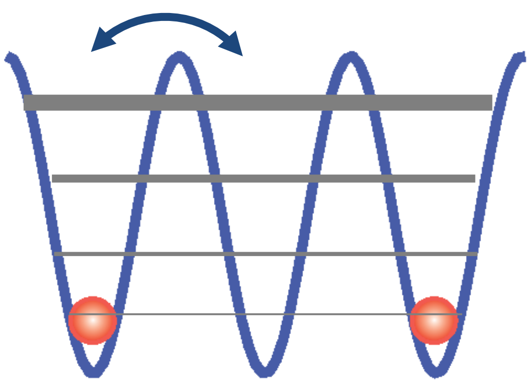
Freezing the fluorescing atoms



- Pinning with an optical lattice
  - » Very deep lattices (e.g. 3000 E_r) to freeze the quantum st
  - » Tunneling time longer than experimental time scales
  - » Pin atoms to their lattice sites during imaging

- Fluorescence imaging
  - » Resonant light (maximize scattering)
  - » Typical time scales: few seconds required for 50 photons per atoms
  - » Photon recoil: excitation to higher bands having larger tunneling $J_n \propto V_0^{n/2}$
  - » Cooling techniques required

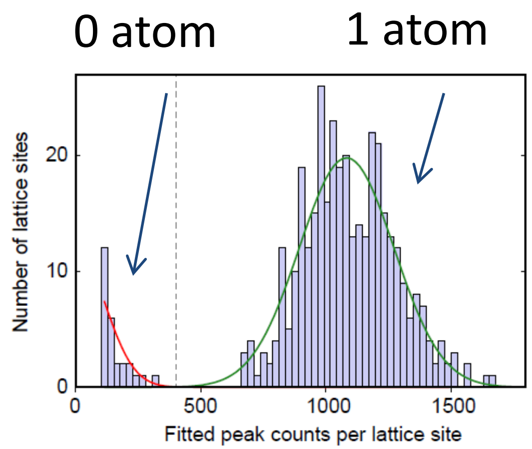
  (optical molasses, Raman sideband cooling, EIT cooling)

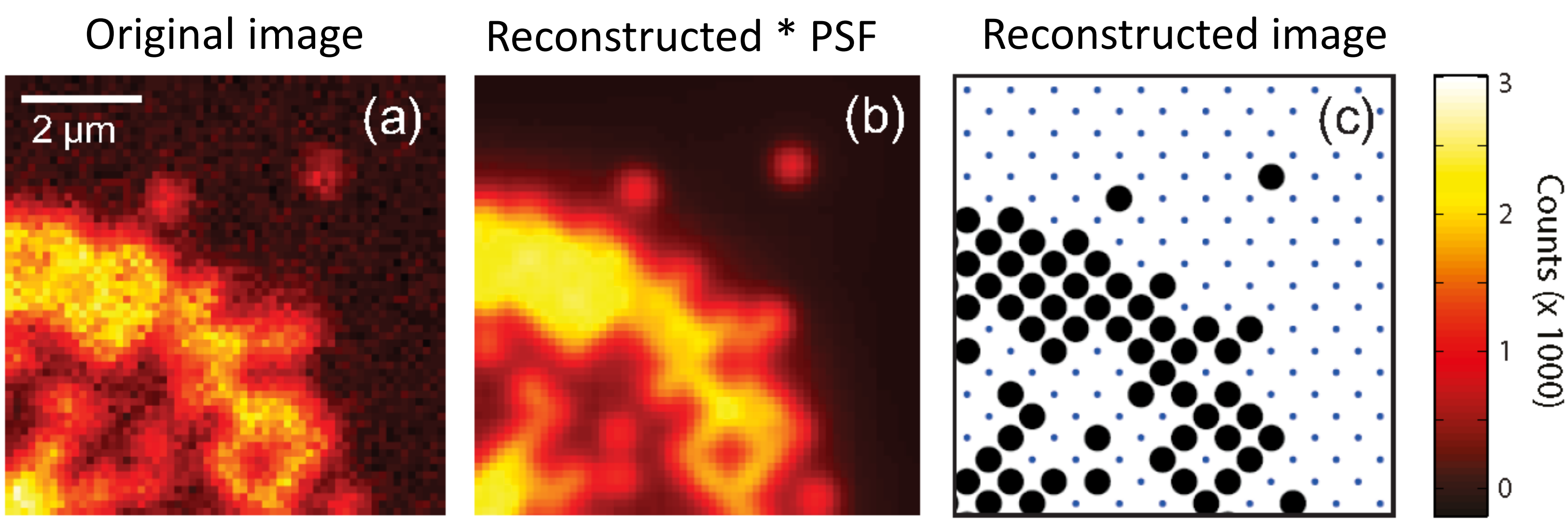


Real Space / Quantum Gas Microscopes

Reconstruction of lattice occupation

- Lattice structure known (calibrated from sparse images)
- Lattice phase fixed using isolated atoms
- Only occupation on lattice sites needed (0 or 1)
- Histogram of counts on lattice sites
- Use iterative reconstruction algorithm





Minimize difference
Original – reconstructed * PSF

try different occupations

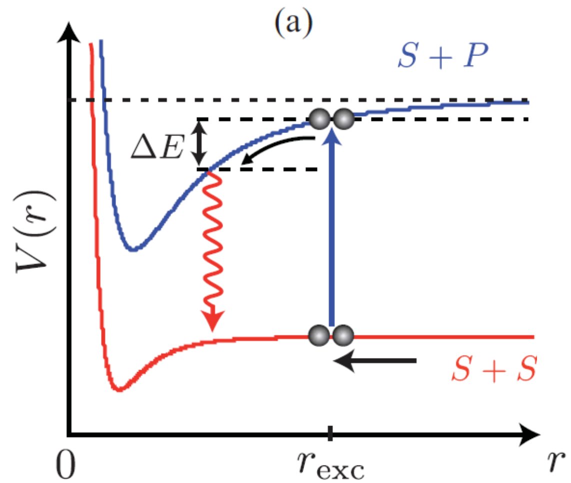
Sherson/Bloch et al. Nature 467, 68-72 (2010)

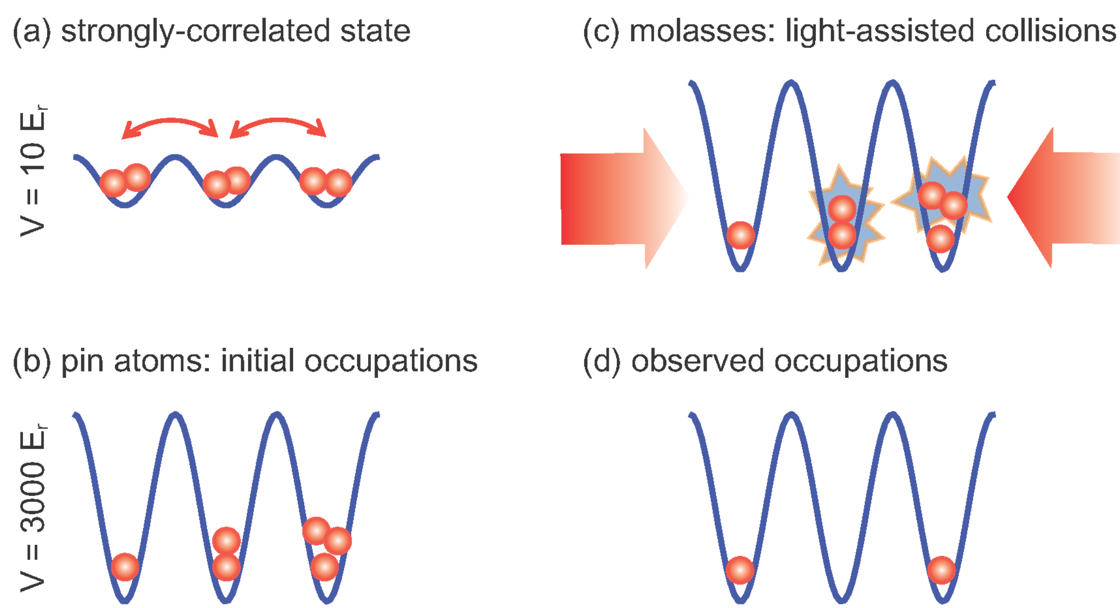
Real Space / Quantum Gas Microscopes

Light-assisted collisions: Parity projection

- Light assisted collision: large kinetic energies.
- Short time-scale: no contribution to the fluorescence signal
- Pairwise loss: measurement of the parity

  of the initial occupation of a lattice site



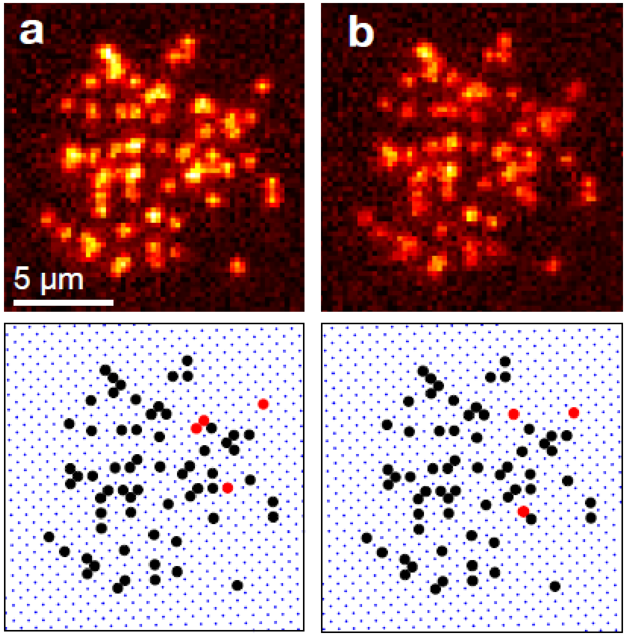


Fuhrmanek/Browaeys et al. Phys. Rev. A 85, 062708 (2012)
Omran et al. PRL 115, 263001 (2015). Preiss et al. PRA 91, 041602(R) (2015)

Real Space / Quantum Gas Microscopes

Detection fidelity of quantum gas microscopes

- Detection fidelity limited by two processes
  - » Loss rate (vacuum lifetime)
  - » Hopping rate (re-cooling to other lattice site)
- Characterization of detection fidelity
  - » Compare two consecutive images of the same cloud
  - » Overall fidelity typically around 98%
- Detection time
  - » Compromise between signal and hopping/loss rate
  - » Typical value: 1 s

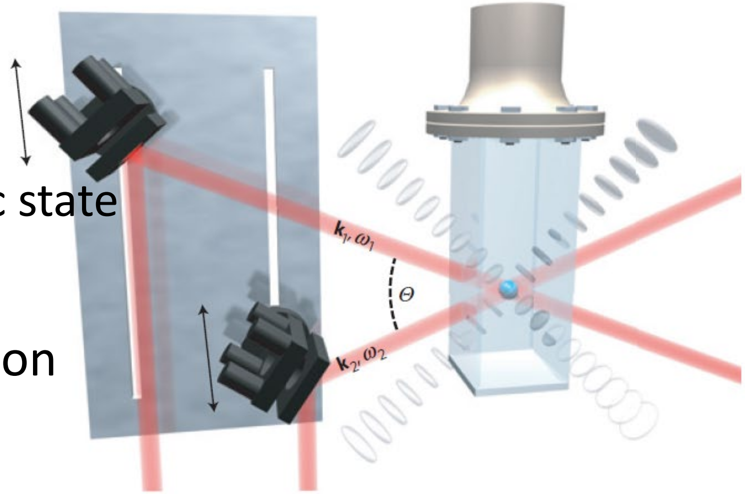


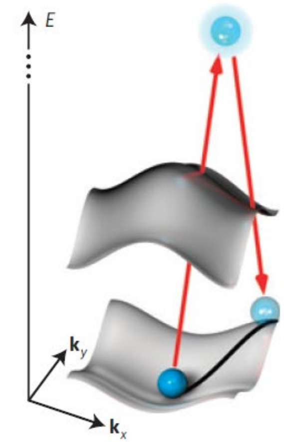
Two consecutive images of a cloud of ^{87}Rb atoms (differences marked in red)

Weitenberg, PhD thesis, Munich (2011)

Spectroscopy Techniques / Bragg spectroscopy

- Bragg spectroscopy
  - » Coherent two-photon transition
  - » No change of the internal atomic state





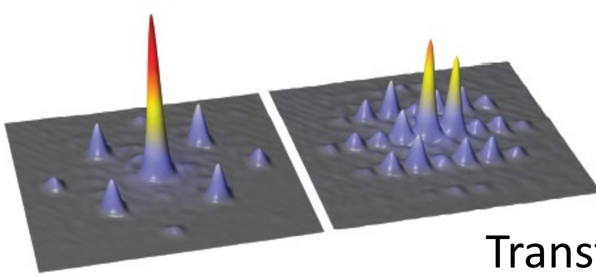
- Energy and momentum conservation

$$\hbar\,\Delta E = \hbar(\omega_1 - \omega_2)$$

$$\hbar\mathbf{k}_{\mathrm{Bragg}} = \hbar(\mathbf{k}_1 - \mathbf{k}_2)$$

$$\approx 2\hbar k \sin(\theta/2)\,\frac{\mathbf{e}_{k_1} - \mathbf{e}_{k_2}}{|\mathbf{e}_{k_1} - \mathbf{e}_{k_2}|}$$

- Resonances at excitations matching values of $\boldsymbol{k}_{\mathrm{Bragg}}$ and ΔE
  fix $\boldsymbol{k}_{\mathrm{Bragg}}$, vary ΔE and find maximal transfer



Transfer visible in ToF

Ernst, Sengstock et al. Nature Phys. 6, 56 (2010)

Spectroscopy Techniques / Bragg spectroscopy

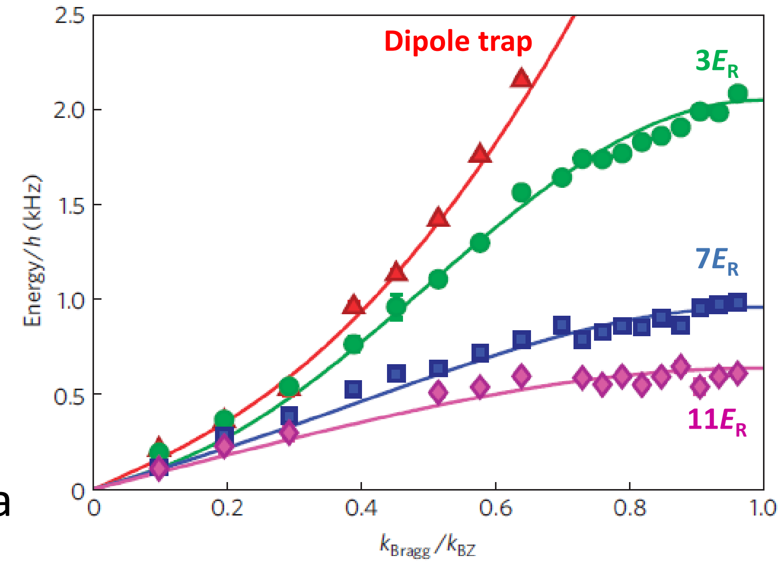
- Bragg spectroscopy for a BEC
  - » Dipole trap: free particle dispersion
  - » Optical lattices: band structure
- Interactions between the atoms
  - » Modification of the single particle energies
  - » Effective band structure Bogoliubov approxima

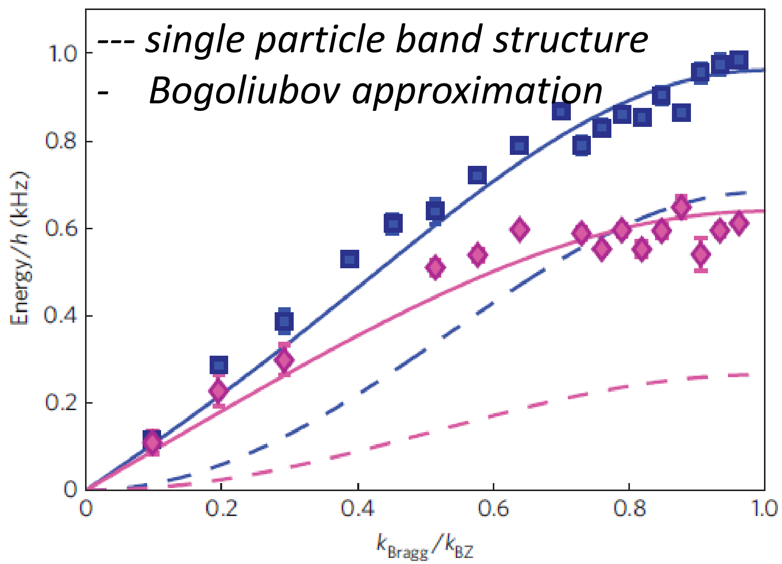
$$E_q^{\mathrm{eff}} = \sqrt{E_q(E_q + 2\,n\,U)}$$

n : filling per lattice site

U : Interaction strength

E_q

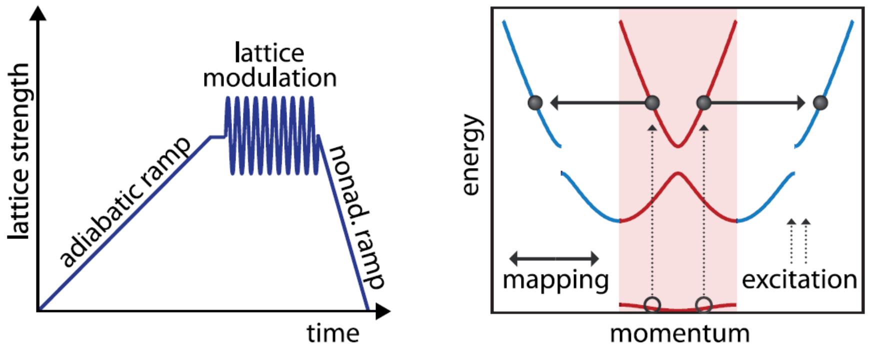




Ernst, Sengstock et al. Nature Phys. 6, 56 (2010)

Spectroscopy Techniques / Amplitude modulation

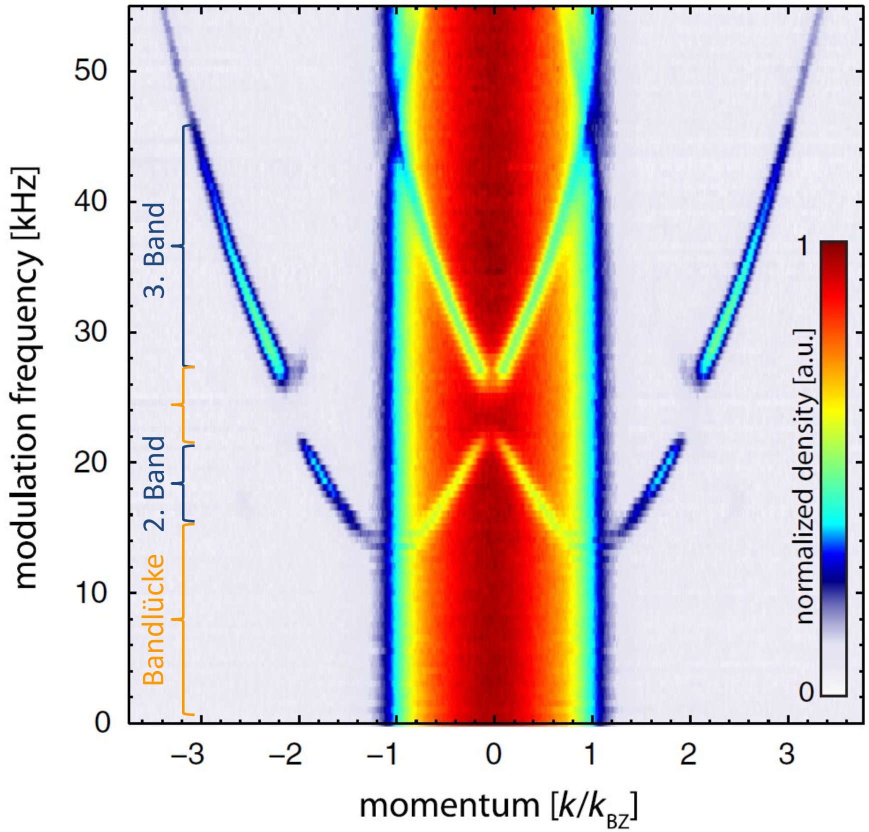
- Amplitude modulation
  - » Excitation with $\Delta q = 0$ and energy transfer
  - » Population of higher bands
  - » Subsequent band mapping and ToF



- Band-structure measurement
  - » Non-interacting fermions filling the lowest
  - » Depopulation in the lowest band shows the reduced zone scheme
  - » If the lowest band is approximately flat

    corresponds to the band energy

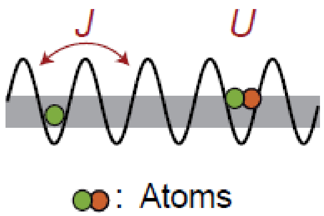
- Measurement of excitation spectrum



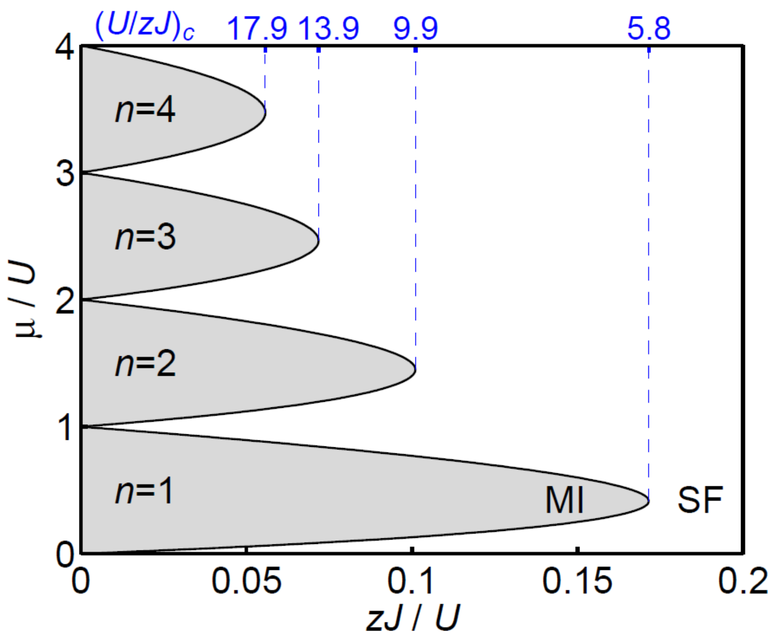
Heinze, Sengstock, et al. PRL 107, 135303 (2011)

2.2 Bose-Hubbard Model

$$\hat{H} = -J\sum_j \hat{b}_{j+1}^\dagger \hat{b}_j + h.c. + \frac{U}{2}\sum_j \hat{n}_j(\hat{n}_j - 1)$$



Potential created by standing light wave



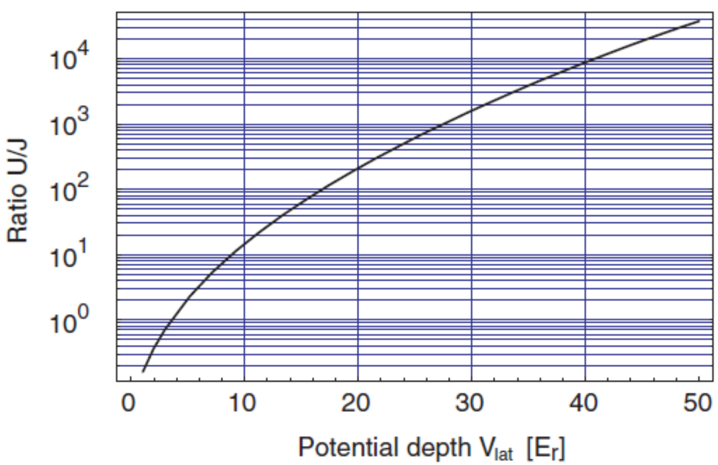


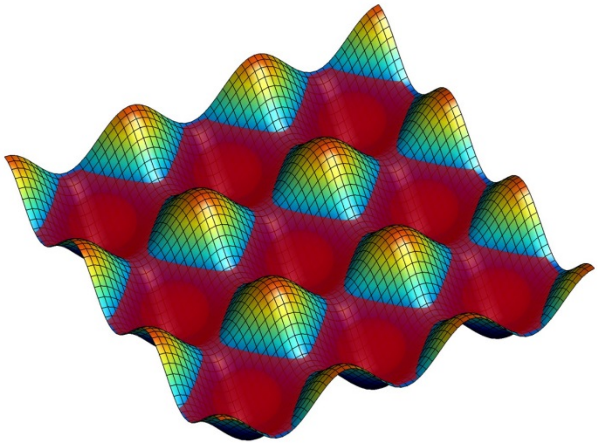
Figure: Fischer et al. PRB (1989)

Bose-Hubbard Model

- Superfluid phase
  - » Tunneling dominates

    Minimization of the kinetic energy
  - » Delocalized in space, localized in momentum space

    "BEC in the lattice"
  - » Coherent state on each lattice site (number fluctuations)



- Mott insulating phase
  - » Interactions dominate

    Minimization of $\frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$
  - » Localized in real space, delocalized in momentum space
  - » Number state on each lattice site
  - » Gapped excitation spectrum
  - » Incompressible $\frac{\partial N}{\partial \mu} = 0$

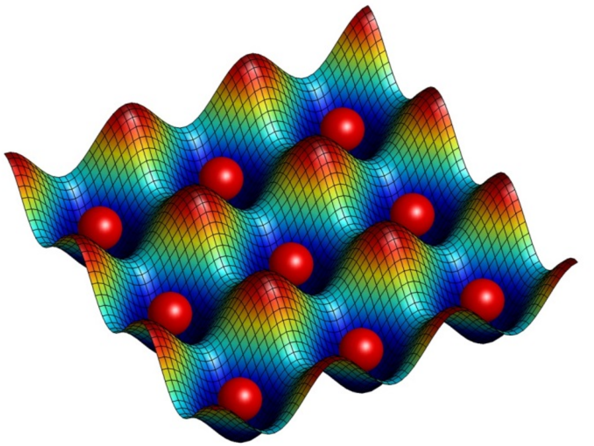
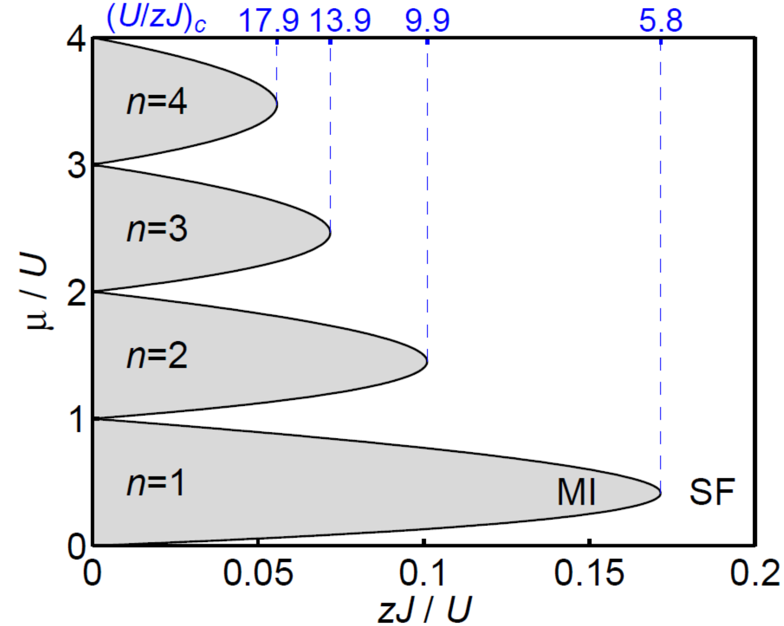


Figure: Lühmann, PhD thesis, Hamburg (2009)

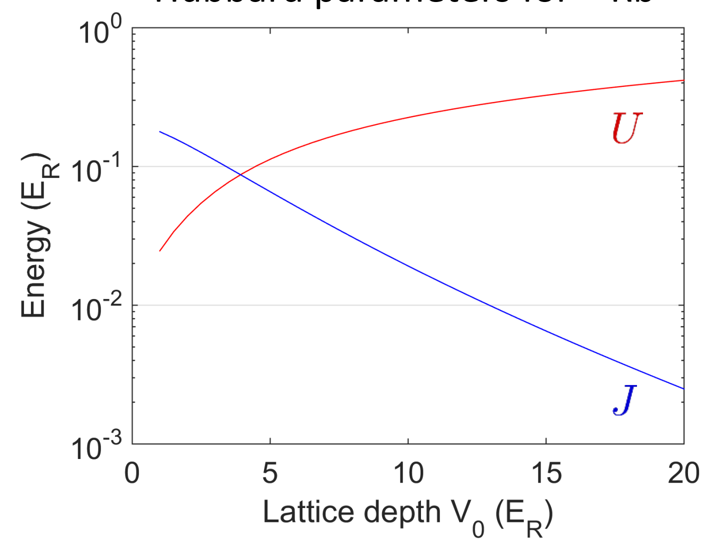
Bose-Hubbard Model / Mapping Phase Diagram

- Experimental procedure
  - » BEC in a dipole trap
  - » Adiabatically ramping of the lattice
  - » Adjust the ratio U/J via the lattice depth V_0
  - » Measure the obtained quantum phase



- Experimental observables
  - » Long range coherence
  - » Number squeezing
  - » Gap in the excitation spectrum
  - » Shell structure

Hubbard parameters for ^{87}Rb

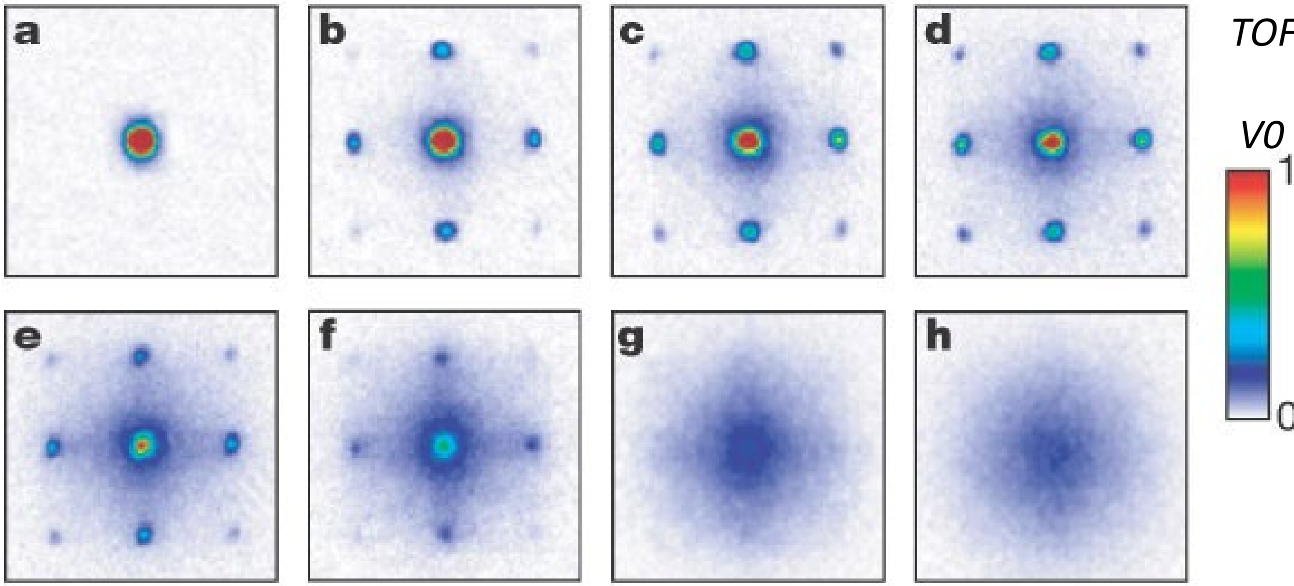


Figures: Fischer et al. PRB (1989); Lühmann, PhD thesis, Hamburg (2009)

Bose-Hubbard Model / Long-range Coherence

- Superfluid
  - » Condensate in lowest Bloch state **q**=0 (a)
  - » Bragg peaks in TOF image, long-range coherence (b-e)

- Mott Insulator
  - » Atoms localize, long-range coherence vanishes (g-h)
  - » Observe only the envelope (approx. Gaussian)



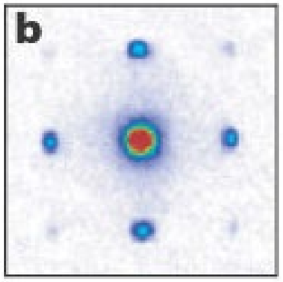
TOF measurement for increasing lattice depth
V0 = 0, 3, 7, 10; 13, 14, 16, 20 Er.

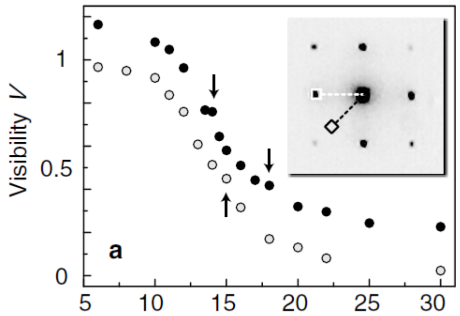
Greiner, Bloch et al. Nature (2002)

Bose-Hubbard Model / Number squeezing

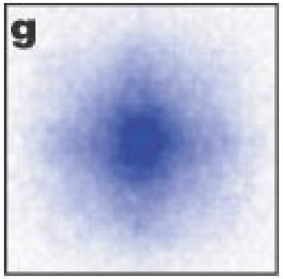
Superfluid

Mott insulator

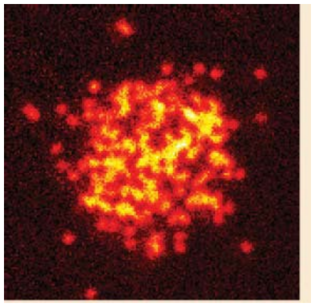


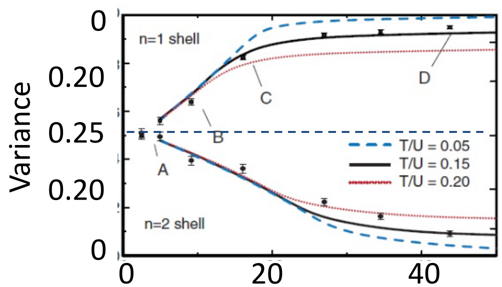


Coherence loss

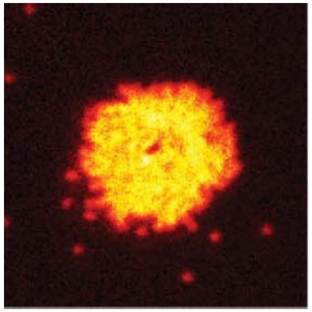


ToF images





Number squeezing



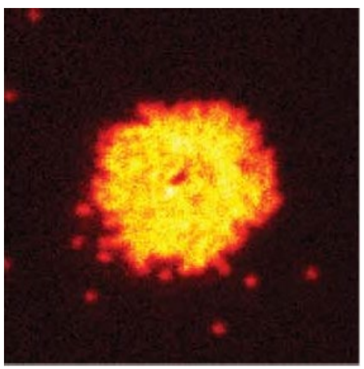
In situ images

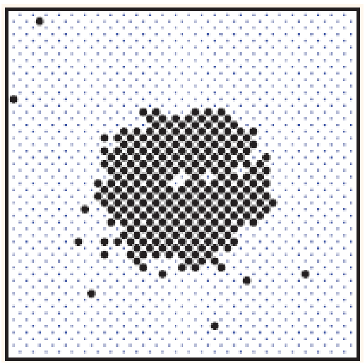
Interaction strength *U/J*

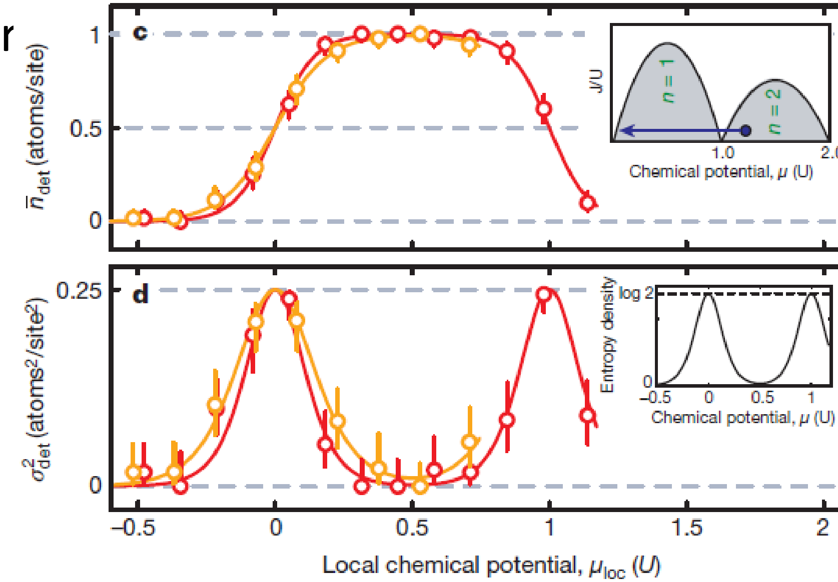
Figures: Greiner (2002), Gerbier (2005), Sherson (2010), Bakr (2010), Endres (2011)

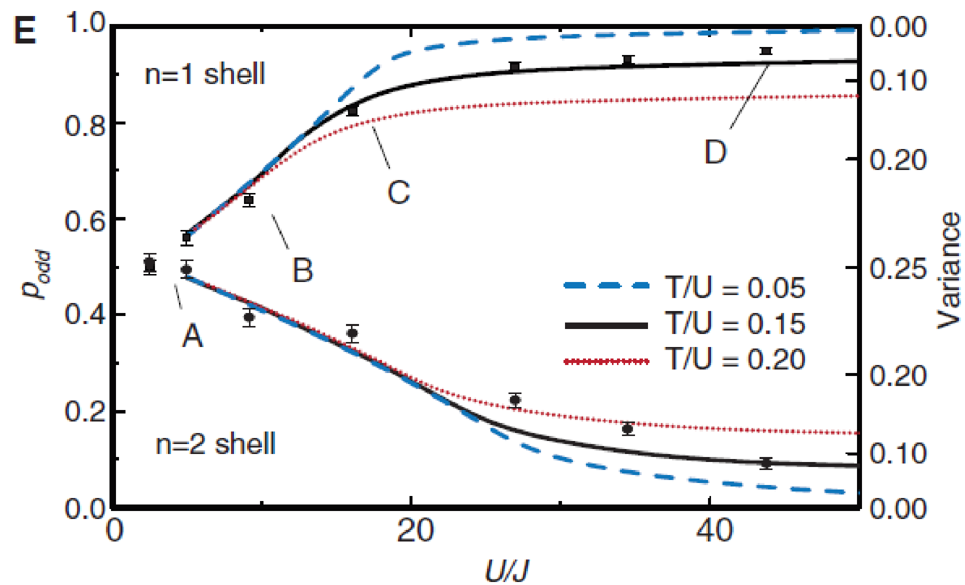
Bose-Hubbard Model / Number squeezing

- Number squeezing for the Mott insulator
  - » Measurement of the parity of the occupation number (odd 0, even 1)
  - » Suppression of density fluctuations in the Mott shells is directly visible
  - » Density fluctuations change over the superfluid to Mott insulator transition







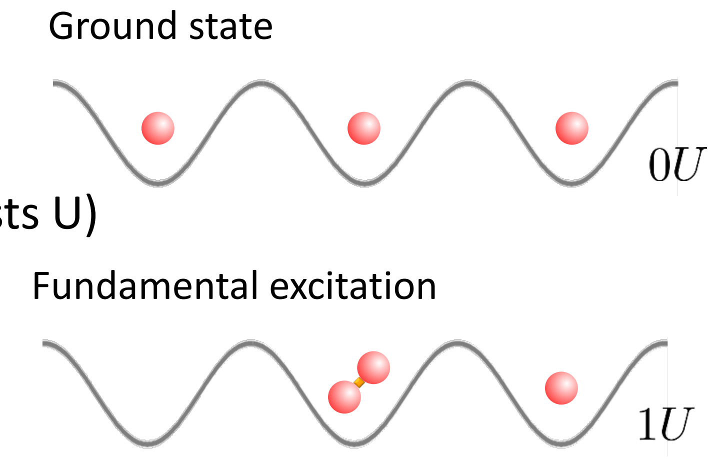


- Precise thermometry
  - » >99% probability of n=1
  - » T/U=0.15

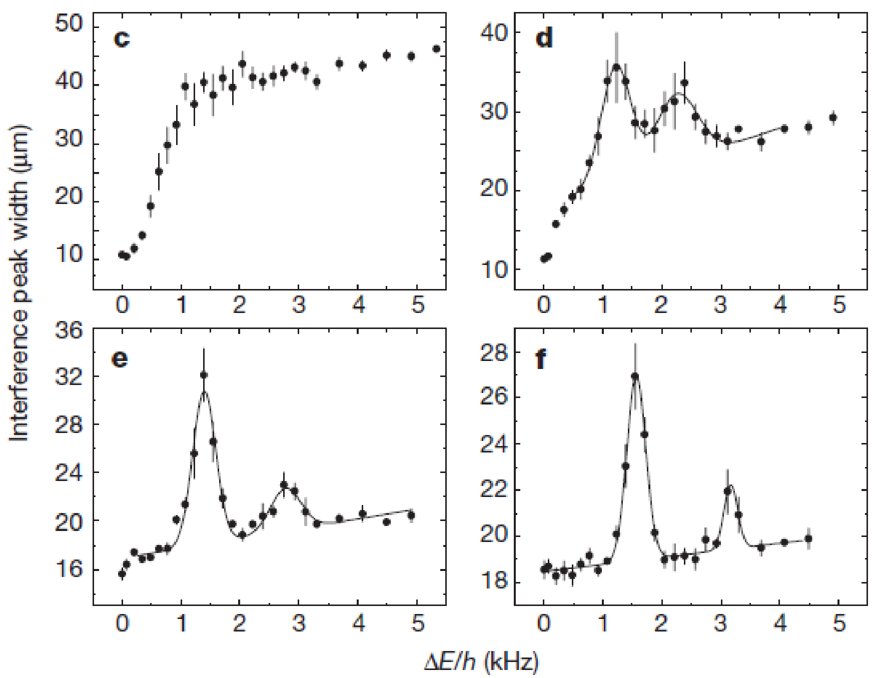
Sherson, Weitenberg, Bloch, Kuhr et al., Nature 467, 68 (2010), Bakr, Greiner et al. Science 329, 547 (2010)

Bose-Hubbard Model / Gap in the excitation spectrum

- Mott insulator
  - » Number state on each lattice site
  - » Gapped excitation spectrum (lowest excitation costs U)



- Measurements
  - » Amplitude modulation at the right frequency
  - » Tilted lattice using magnetic field gradient

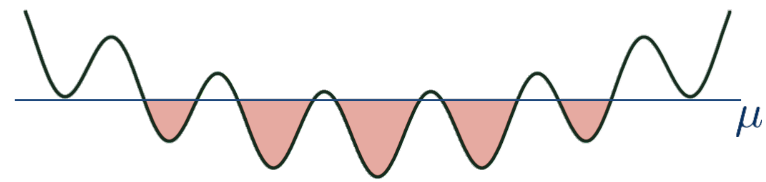


Lattice depths V_0 = 10, 13, 16, 20 Er

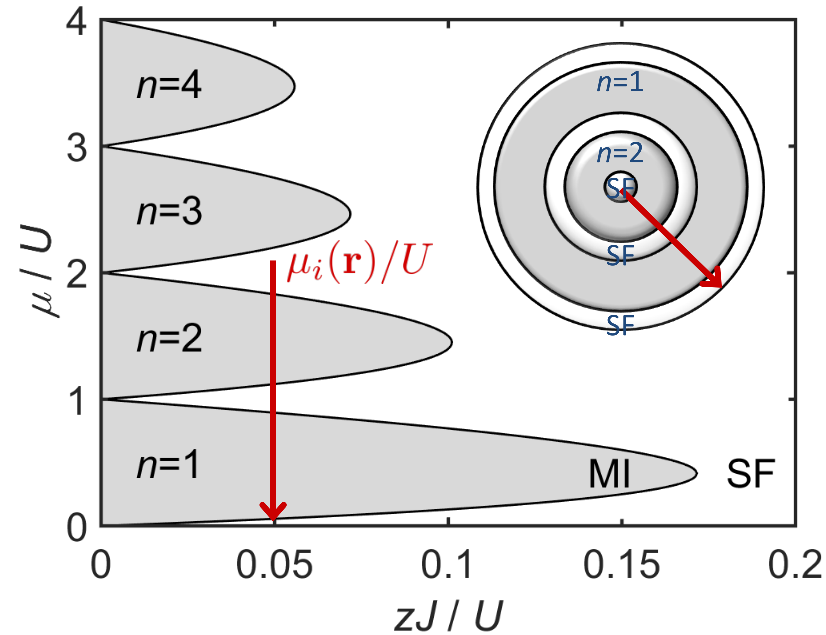
Greiner, Bloch et al. Nature (2002)

Bose-Hubbard Model / Mott shells in inhomogeneous systems

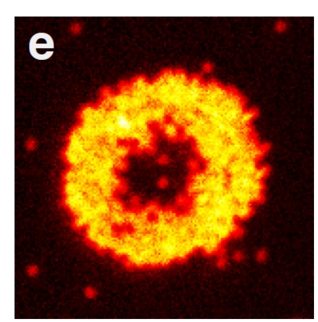
- Inhomogeneous system
  - » Gaussian beams induces external confinement
  - » Additional term in the Bose-Hubbard model
    $\hat{H}_C = \sum_i \epsilon_i \hat{n}_i$ with $\epsilon_i = V_C(R_i)$

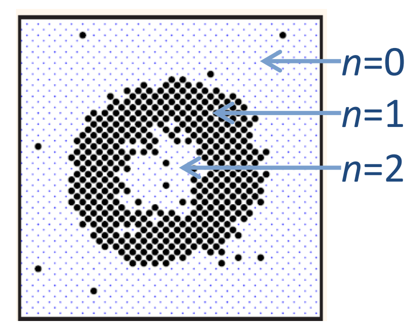


- Local density approximation (LDA)
  - » Introduce effective chemical potential
    $$\hat{H}_C - \hat{N}\mu = \sum_i \epsilon_i \hat{n}_i - \sum_i \mu \hat{n}_i = -\sum_i \mu_i \hat{n}_i$$
    with $\mu_i = \mu - \epsilon_i$
  - » Regions exchange particles: no exact matching of particle number necessary!



- „Wedding cake structure“
  - » Cut through phase diagram (red arrow)
  - » Formation of Mott shells
  - » Observable in 2D in situ image





Sherson, Weitenberg, Bloch, Kuhr et al., Nature 467, 68 (2010), Bakr, Greiner et al. Science 329, 547 (2010)

2.3 Fermi Hubbard Model

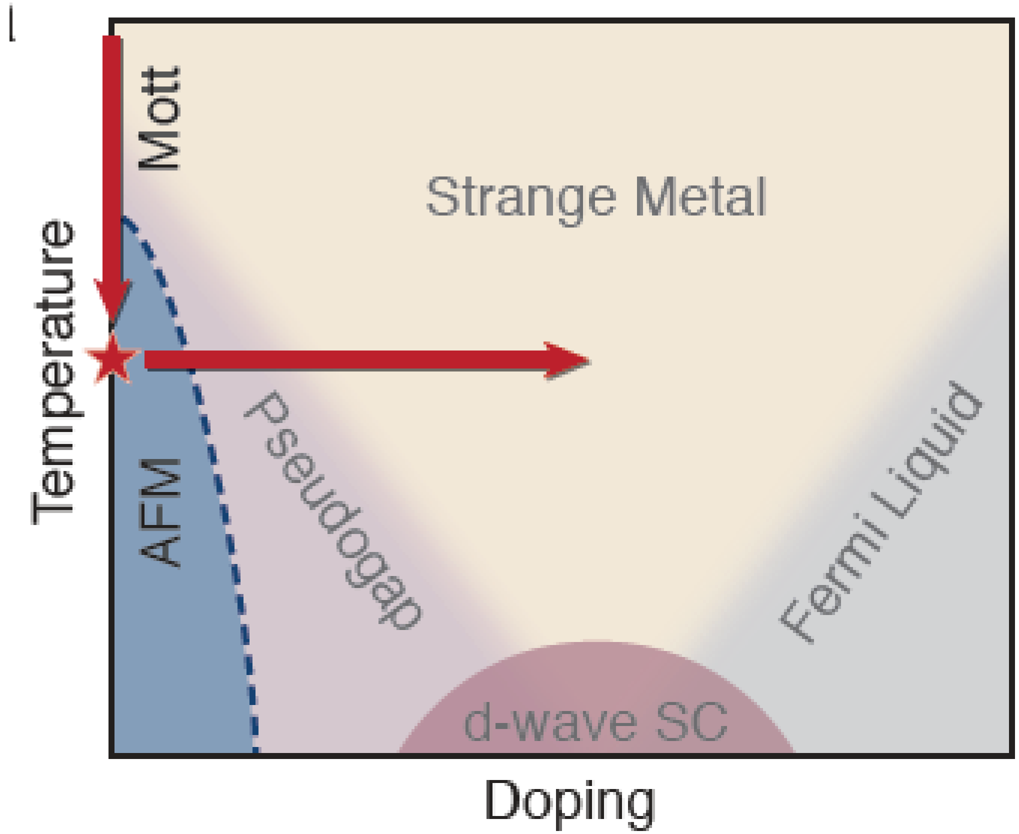
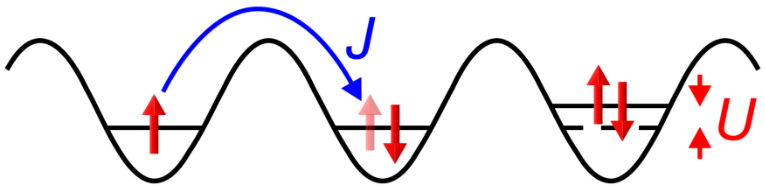


Figure: Mazurenko, Greiner et al. Nature 545, 462 (2017)

Fermi-Hubbard Model



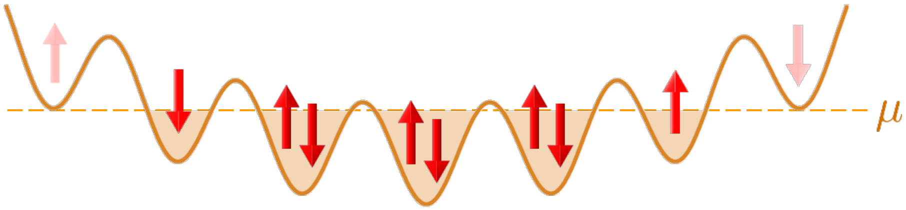
- Fermi Hubbard model
  - » Fermions with spin s=1/2 ($\uparrow, \downarrow$)
  - » Contact interactions on a lattice site $U_{\uparrow,\downarrow} = U$

$$\hat{H} = -J \sum_{\substack{\langle i,j \rangle \\ \sigma \in \{\uparrow,\downarrow\}}} \hat{c}^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

- For medium particle numbers and confinement occupation of only the lowest band

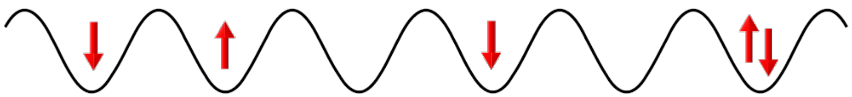
- Hubbard model with local chemical potential

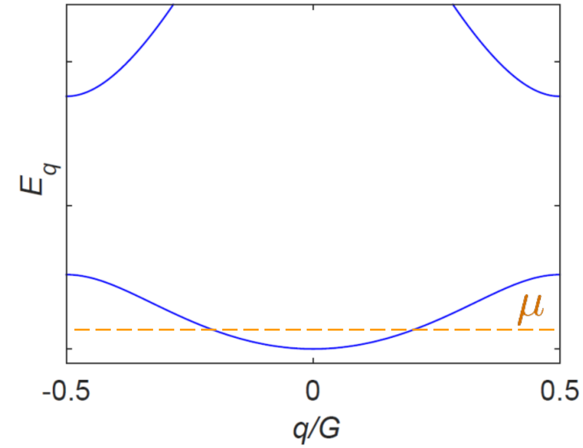
$$\hat{H} = -J \sum_{\langle i,j \rangle, \sigma} \hat{c}^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \sum_i (\mu - \epsilon_i)(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow})$$



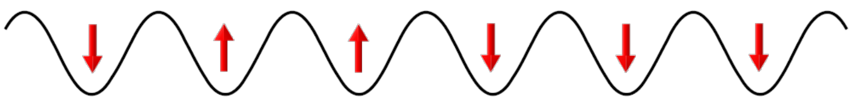
Fermi-Hubbard Model / Quantum Phases

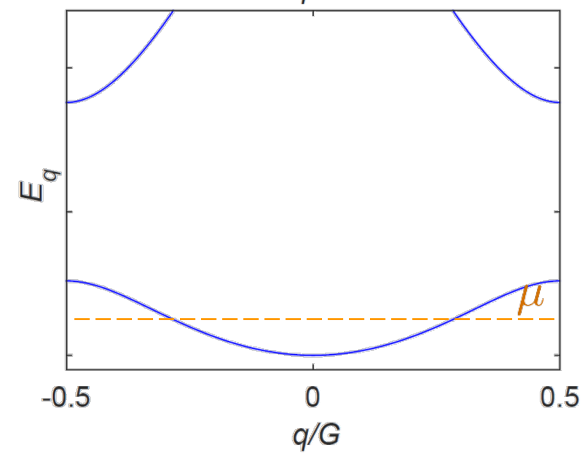
- Metallic phase
  - » No excitation gap
  - » Delocalized Boch states, fluctuations



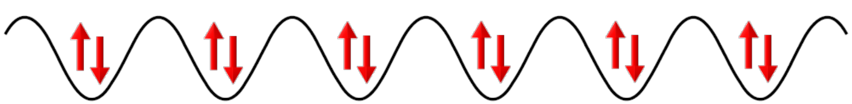


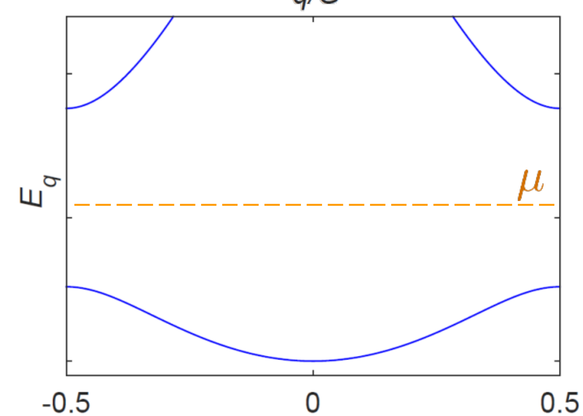
- Mott Insulating phase
  - » Excitation gap *U*, half filling
  - » Interaction *U* suppresses the double occupancy





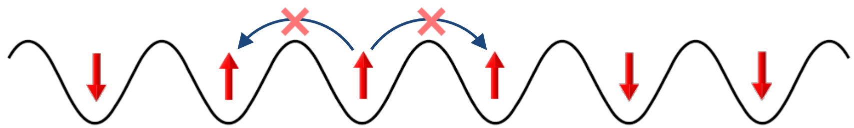
- Band Insulating phase
  - » All Bloch states of the band are occupied
  - » Excitation gap (into the next band)
  - » No hopping possible: Insulator



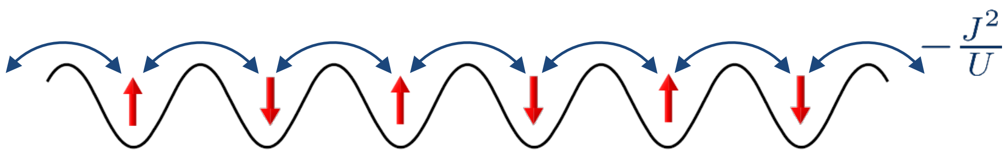


Fermi-Hubbard Model / Quantum magnetism

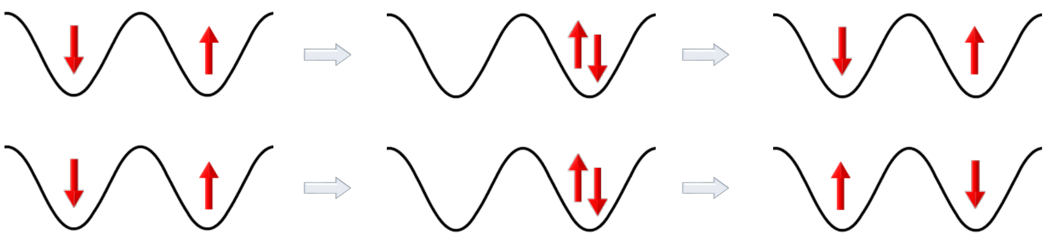
- Mott Insulator
  - » Double occupancy suppressed ($U \gg J$)
  - » No fixed spin order



  - » Fluctuations are suppressed by unfavorable ordering of the spins (Pauli principle)
- Ground state has antiferromagnetic order



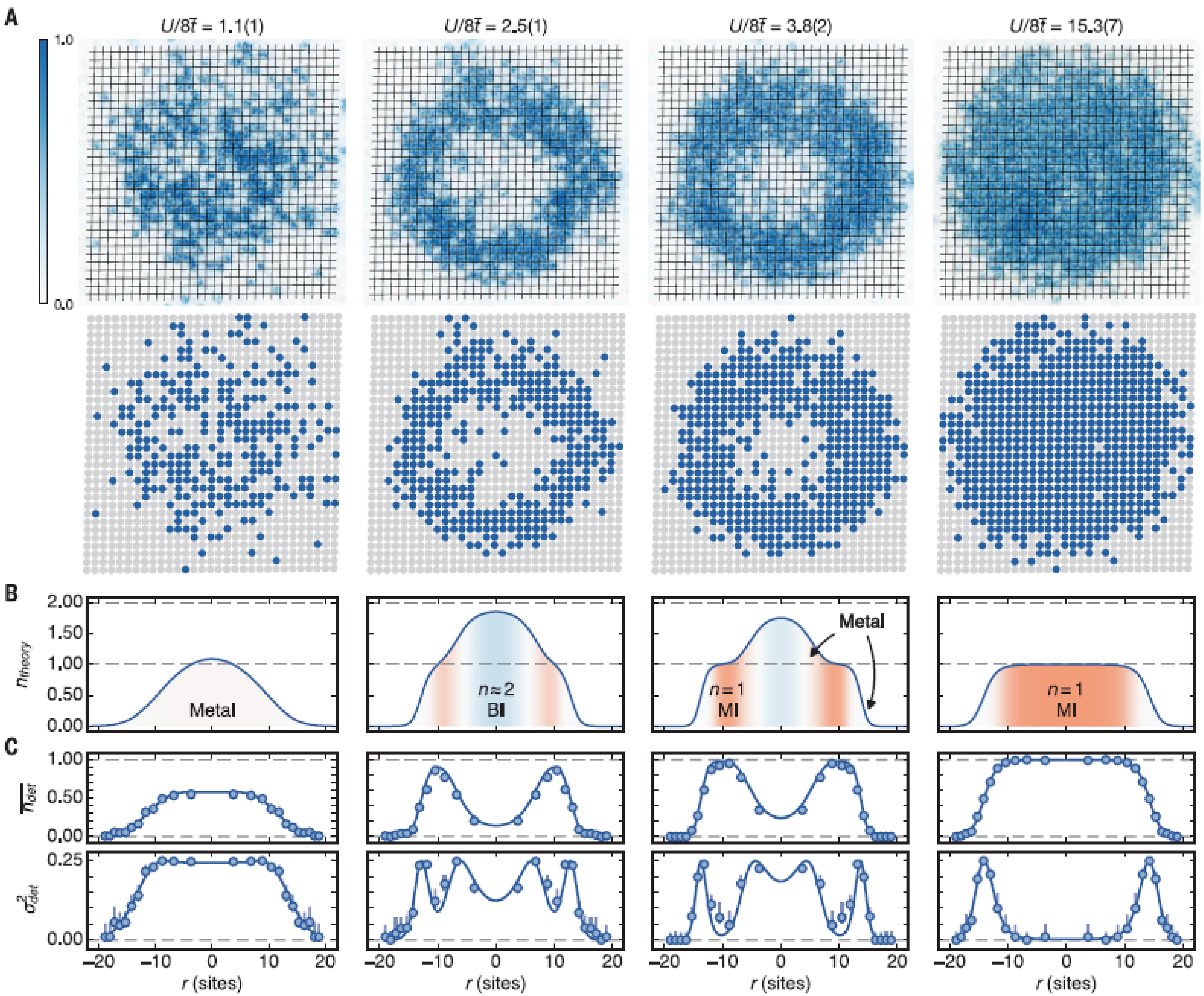
  - » Super exchange: Tunnel processes of second order



$$\hat{H}_{\text{eff}} = -\frac{J^2}{U}\sum_{\langle i,j\rangle,\sigma}\hat{n}_{i,\sigma}\hat{n}_{j,-\sigma} - \frac{J^2}{U}\sum_{\langle i,j\rangle,\sigma}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,-\sigma}\hat{c}^{\dagger}_{j,-\sigma}\hat{c}_{j,\sigma}$$

  - » Energy scale J^2/U is small (on the order of currently reached thermal energy)

Fermi-Hubbard Model / Mott insulator state

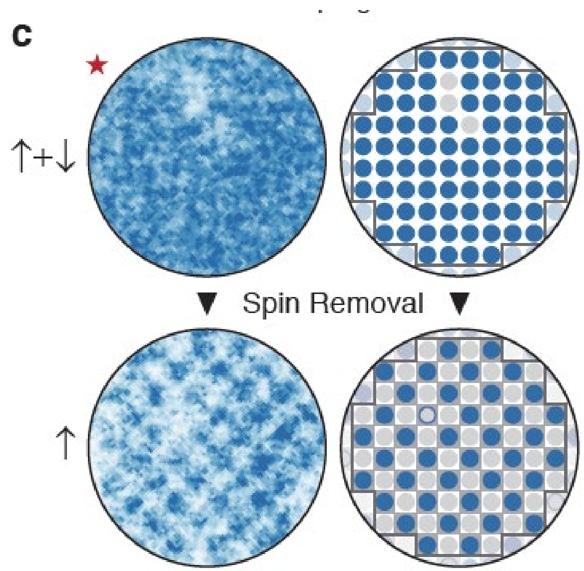


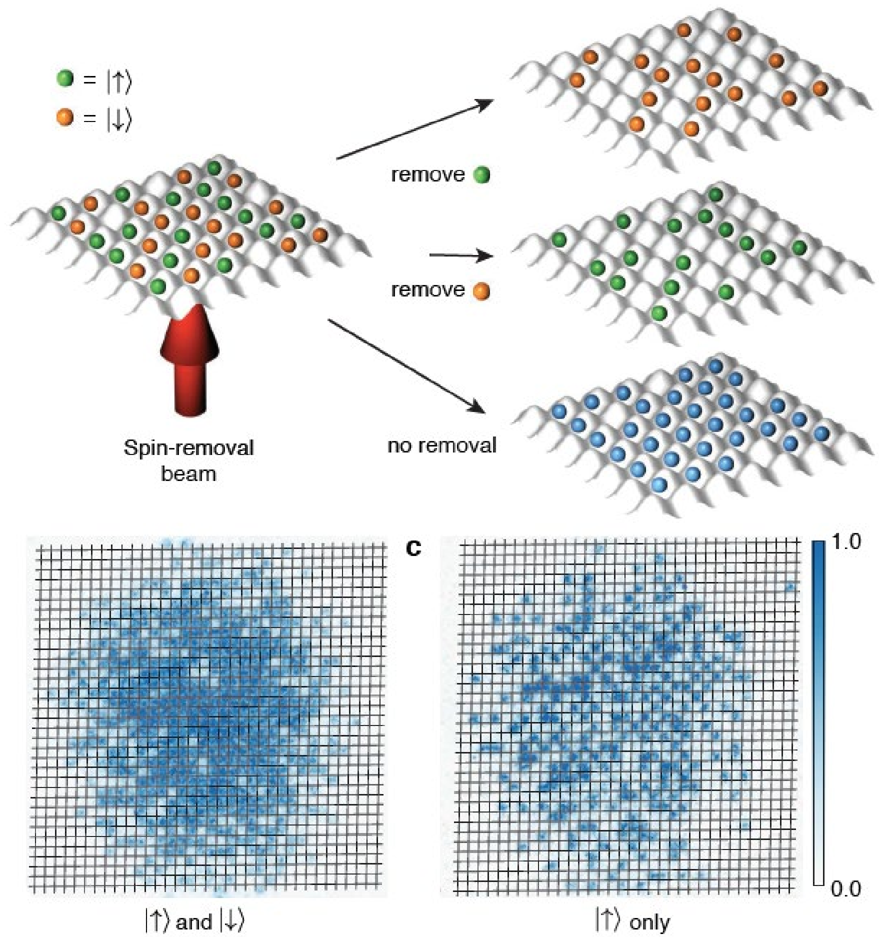
But random distribution of the two spin states? (not resolved here)

Greif, Greiner, et al. Science 351, 953 (2016)

Fermi-Hubbard Model / Spin correlations

- Spin-dependent detection under a quantum gas microscope
  - » Selectively remove one spin state before imaging and extract spin correlations
  - » Spatially separate spin states via magnetic field gradient (1D system)





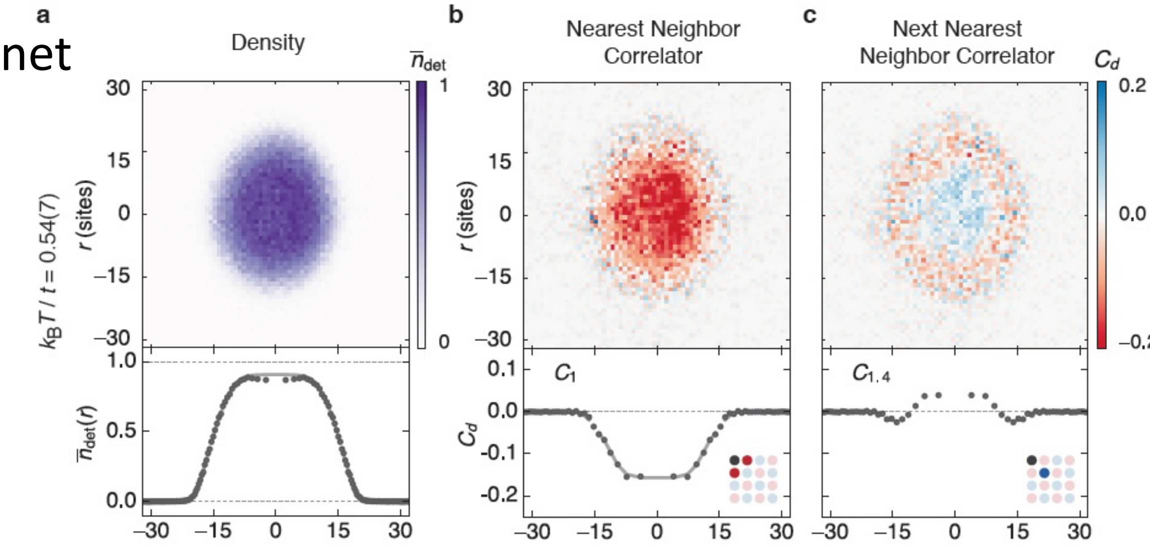
Parsons, Greiner et al., Science 353, 1253 (2016).
Boll, Bloch et al., Science 353, 1257 (2016)
Cheuk, Zwierlein et al. Science 353, 1260 (2016)
Mazurenko, Greiner et al. Nature 545, 462 (2017)

Fermi-Hubbard Model / Spin correlations in 2D

- First signatures of antiferromagnet
  - » Density

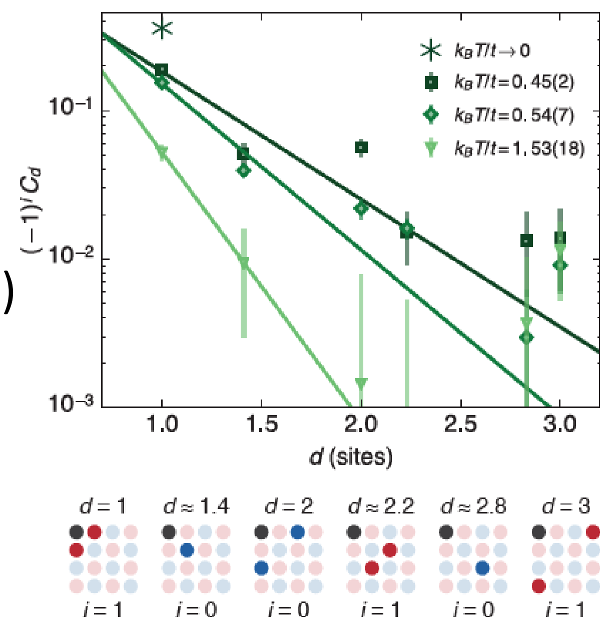
    n=1 in the center
  - » Correlators

    $C_1 < 0$ in the center



- Correlation length
  - » Exponential decay
  - » Correlation length increases

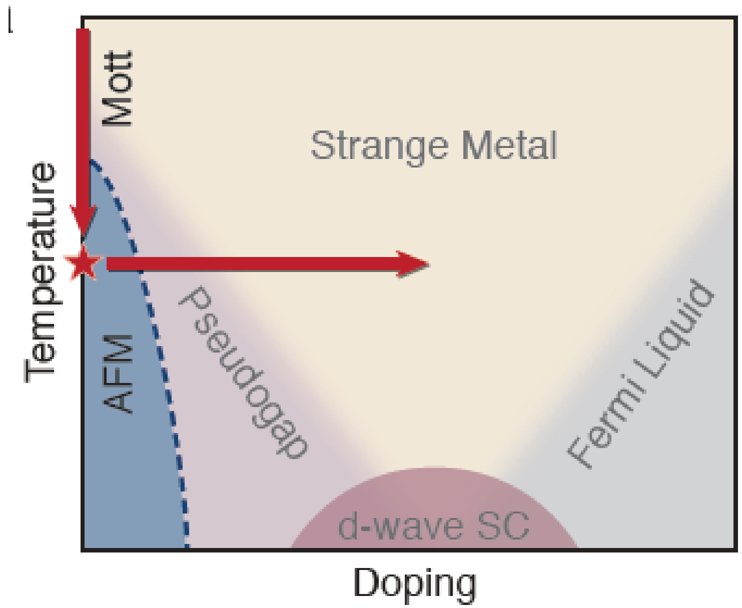
  with decreasing temperature (0.2, 0.4, 0.5 lattice sites)



Parsons, Greiner et al., Science 353, 1253 (2016).
Boll, Bloch et al., Science 353, 1257 (2016)
Cheuk, Zwierlein et al. Science 353, 1260 (2016)
Mazurenko, Greiner et al. Nature 545, 462 (2017).

Outlook: Hubbard models and quantum magnetism

- Open question: is there a transition to a d-wave superconducting state in the Fermi Hubbard model?
- Quantum simulation with cold atoms need even lower temperatures
  - » Advances entropy redistribution schemes
  - » Cooling in lattices
- Long term goal of quantum simulation: reach regimes experimentally, which cannot be computed efficiently



= away from half filling

AFM = antiferromagnet
SC = superconductor

Lee, Nagaosa & Wen, Rev. Mod. Phys. 78, 17 (2006). Figure: Mazurenko, Greiner et al. Nature 545, 462 (2017)