Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

ZOQ
Zentrum für Optische Quantentechnologien

ILP
INSTITUT FÜR LASERPHYSIK

Optical Lattices and Artificial Gauge Potentials

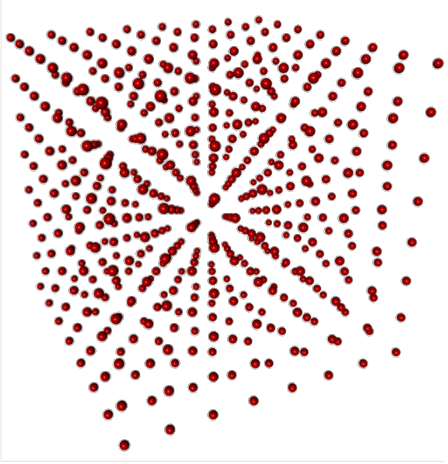
Juliette Simonet

Institute of Laser Physics, Hamburg University

Optical Lattices and Artificial Gauge Potentials

Part 1: Build up the Hamiltonian
Optical Lattices
Non-interacting properties (band structure, wave functions)
Hubbard models

Cold atoms simulator



Part 2: Read out the quantum state
Probing quantum gases in optical lattice
Mapping phase diagrams of Hubbard models

Part 3: Beyond Hubbard models in optical lattices
Topological properties and transport
Magnetic phenomena for neutral atoms

Optical Lattices and Artificial Gauge Potentials

Part 3: Artificial Gauge Potentials

Part 3

3.1 Magnetic phenomena in presence of a lattice

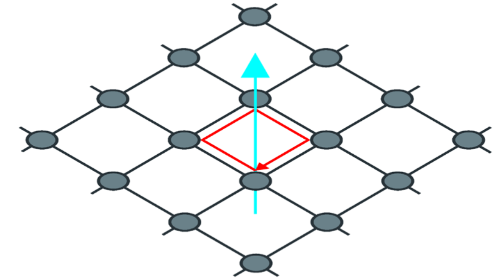
3.2 Generating *artificial* gauge potentials on a lattice

3.3 Engineering and probing topological band structures

Dalibard, Introduction to the physics of artificial gauge fields, Cours du Collège de France

3.1 Magnetic phenomena in presence of a lattice

3.1 Magnetic phenomena in presence of a lattice



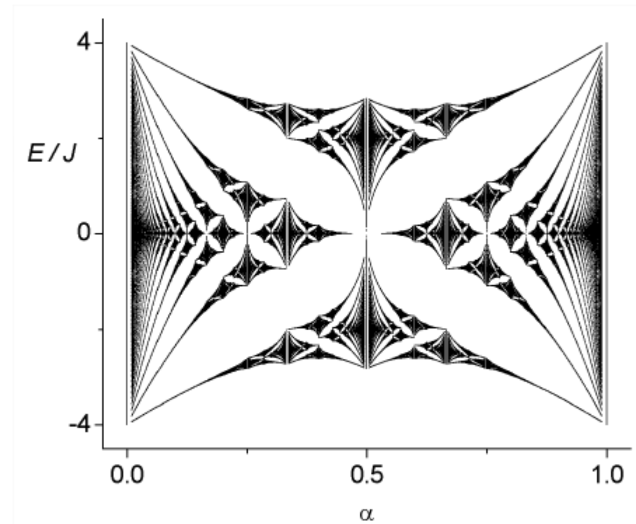
- Competition between two length scales
  - » Lattice spacing: *a*
  - » Magnetic length: $l_{\text{mag}} = \sqrt{\frac{\hbar}{eB}}$

- New phenomena when $l_{\text{mag}} \approx a$

$$\frac{a^2}{l_{\text{mag}}^2} = \frac{e\,Ba^2}{\hbar} = 2\pi\frac{\Phi}{\Phi_0} \qquad \Phi_0 = h/e$$

Fractal structure for the energy spectrum

"Hofstadter butterfly"

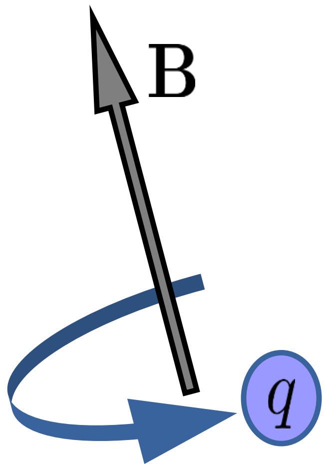


- Quantum Hall effect

Simulation with quantum gases in optical lattices?

Artificial magnetic field

- Coupling between electromagnetic fields and **charged** particles central for many phenomena:
  - » Integer and fractional Quantum Hall effect
  - » Spin-orbit coupling
  - » Topological insulators
  - » ….



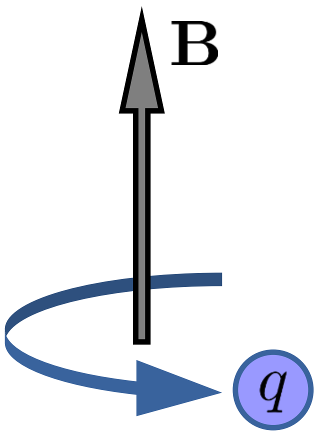
- Quantum simulation with quantum gases
  - » Well controlled systems to study solid-state models
  - » **Neutral** atoms (q = 0)

Simulating magnetic effects with quantum gases is a challenge:

Requires the creation of “substitutes” to real electromagnetic fields: “Artificial gauge potentials”

Quantum mechanics / Gauge transformation

- Classical physics
  - » Equation of motion $m\ddot{\boldsymbol{r}} = q\dot{\boldsymbol{r}} \times \boldsymbol{B}$
  - » Gauge transformation $\boldsymbol{A}(\boldsymbol{r}) \rightarrow \mathbf{A}'(\mathbf{r}) = \mathbf{A}(\mathbf{r}) + \boldsymbol{\nabla}\chi(\boldsymbol{r})$



- Quantum mechanics
  - » Schrödinger equation

$$\imath\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \frac{(\hat{\mathbf{p}} - q\mathbf{A}(\hat{\mathbf{r}}))^2}{2M}\psi(\mathbf{r},t)$$

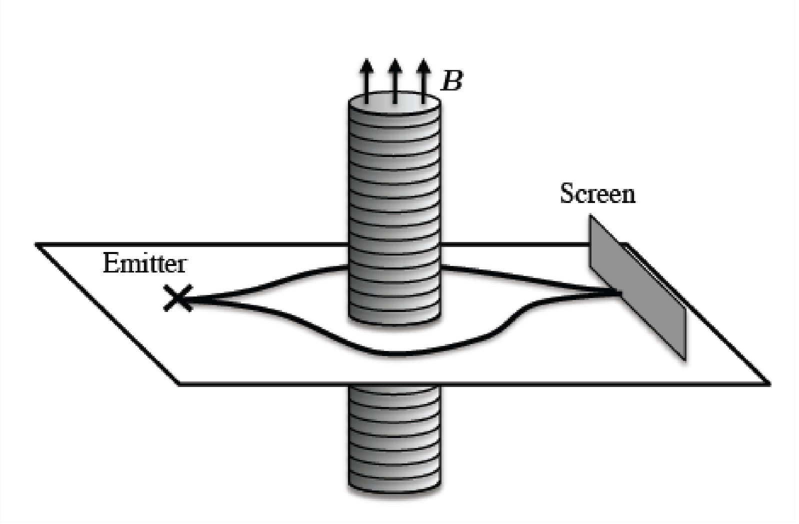
  - » Gauge transformation (imposed by the Schrödinger equation)

$$\mathbf{A} \rightarrow \mathbf{A}'(\mathbf{r}) = \mathbf{A}(\mathbf{r}) + \nabla\chi(\mathbf{r})$$

$$\psi(\mathbf{r},t) \rightarrow \psi'(\mathbf{r},t) = \exp[\imath q\chi(\hat{\mathbf{r}})/h]\psi(\mathbf{r},t)$$

The wave-function is modified by a gauge transformation: it acquires a phase!

Quantum mechanics / The Aharonov-Bohm effect

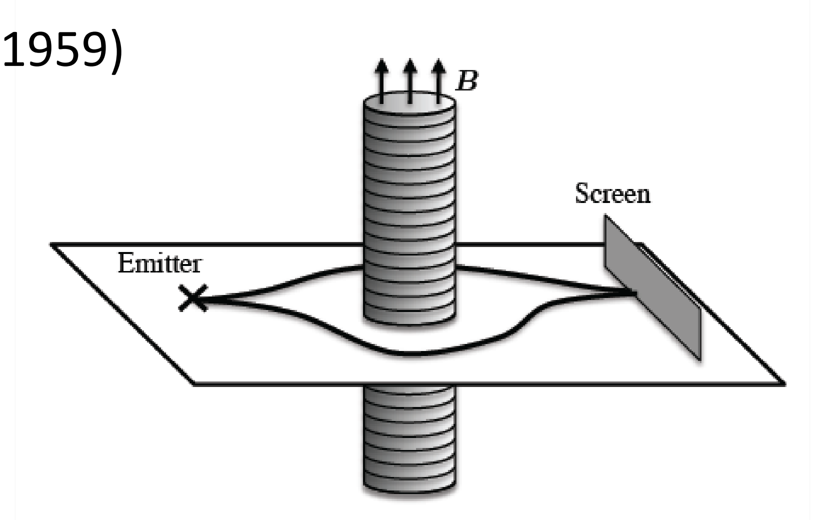


- Gedanken experiment of Aharonov and Bohm (1
  - » Two path interferometer for single electrons
  - » Infinite solenoid: $\mathbf{B}_{\mathbf{in}}$ = **B** and $\mathbf{B}_{\mathbf{out}}$ = **0**
- Probing a magnetic field without seeing it
  - » Zero Lorentz force outside the solenoid
  - » **BUT:** Shift of the interference pattern
- One of the "seven wonders of the quantum world" [*New Scientist* magazine]
  - » Several experimental demonstrations
  - » Questions the locality of electromagnetic fields
    - Local electromagnetic fields (**B**, **E**) and delocalized particle in the solenoid,
    - Gauge potentials (**A**, V) and particle localized around the solenoid.
  - » Global action versus local forces: Lagrangian formalism (based on energies) is not just a computational aid to the Newtonian formalism (based on forces).

Aharonov and Bohm, Phys. Rev. 115, 485 (1959)

Quantum mechanics / The Aharonov-Bohm effect

- Gedanken experiment of Aharonov and Bohm (1959)
  - » Two path interferometer for single electrons
  - » Infinite solenoid: $\mathbf{B}_{\mathbf{in}}$ = **B** and $\mathbf{B}_{\mathbf{out}}$ = **0**



- Probing a magnetic field without seeing it
  - » Zero Lorentz force outside the solenoid
  - » **BUT:** Shift of the interference pattern

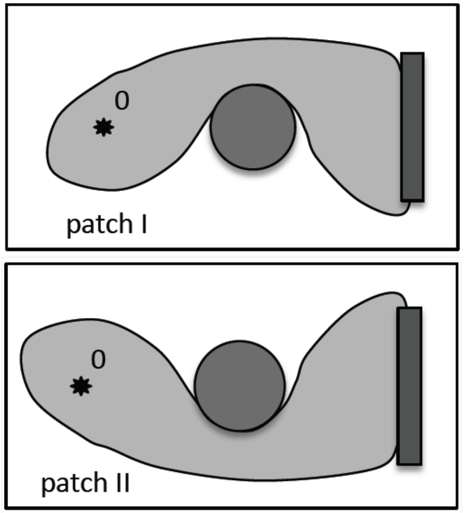
- Aharonov-Bohm phase
  - » Switching the current corresponds to a gauge change:

$$\mathbf{A}(\mathbf{r}) = \mathbf{0} \rightarrow \mathbf{A}(\mathbf{r}) = \nabla \chi_{I,II}(\mathbf{r})$$

  - » The matter-wave interference at ***r*** is related to:

$$\psi_l^*(\mathbf{r})\psi_r(\mathbf{r}) = \exp[\imath q(\chi_{II}(\mathbf{r}) - \chi_I(\mathbf{r}))]\psi_l^{(0)*}\psi_r^{(0)}$$

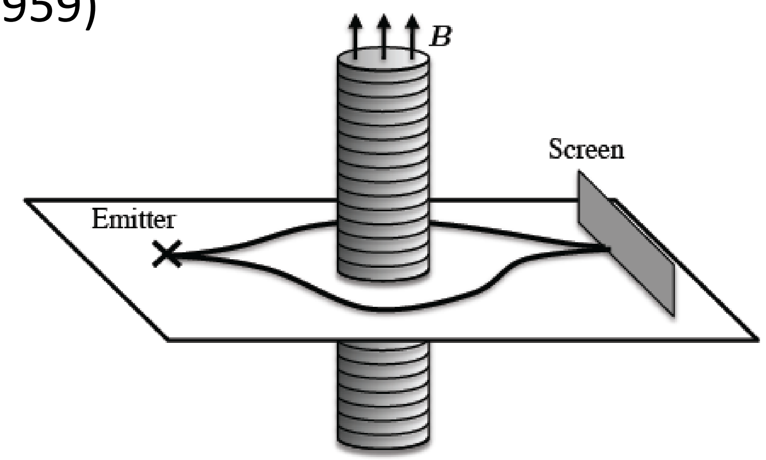
$$\chi_I(\mathbf{r}) - \chi_{II}(\mathbf{r}) = \int_{\mathbf{0},\mathbf{CI}}^{\mathbf{r}} \mathbf{A}(\mathbf{r}')d\mathbf{r}' - \int_{\mathbf{0},\mathbf{CII}}^{\mathbf{r}} \mathbf{A}(\mathbf{r}')d\mathbf{r}'$$



Aharonov and Bohm, Phys. Rev. 115, 485 (1959)

Quantum mechanics / The Aharonov-Bohm effect

- Gedanken experiment of Aharonov and Bohm (1959)
  - » Two path interferometer for single electrons
  - » Infinite solenoid: $\mathbf{B}_{in} = \mathbf{B}$ and $\mathbf{B}_{out} = \mathbf{0}$



- Probing a magnetic field without seeing it
  - » Zero Lorentz force outside the solenoid
  - » **BUT:** Shift of the interference pattern

- Aharonov-Bohm phase

$$\Delta\varphi = \frac{1}{\hbar}\oint q\mathbf{A}(\mathbf{r}).d\mathbf{r} = \frac{2\pi}{\Phi_0}\int\int B_z(x,y)dydx = 2\pi\frac{\Phi}{\Phi_0}$$

Flux quantum $\Phi_0 = \dfrac{h}{q}$

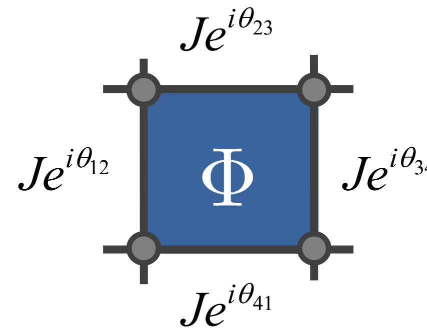
  - » Gauge invariant
  - » Geometric phase (no dependency on velocity)
  - » Even topological (constant under path deformation)

Aharonov and Bohm, Phys. Rev. 115, 485 (1959)

Magnetic field on a lattice / Peierls phase

- Peierls substitution
  - » Presence of a gauge potential

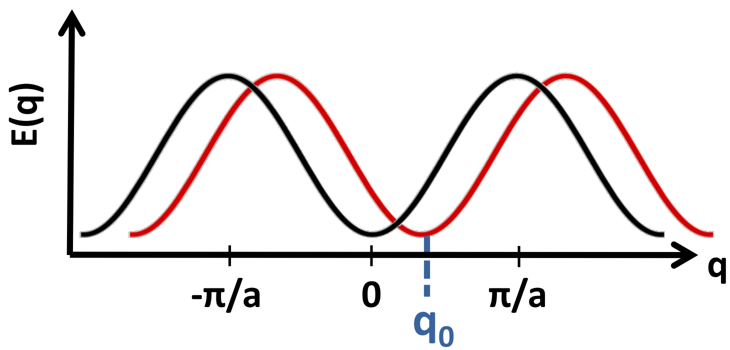
    $\leftrightarrow$ Complex tunneling matrix element

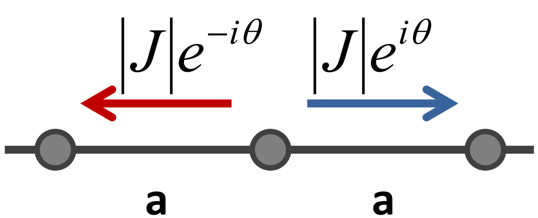


  - » Peierls phase $\theta_{i,j} = \frac{e}{\hbar}\int_{R_i}^{R_j} \boldsymbol{A}(\boldsymbol{r}) \cdot d\boldsymbol{r}$
  - » Magnetic flux through a plaquette - Aharonov-Bohm phase

$$\sum \theta_{ij} = \frac{e}{\hbar}\oint \boldsymbol{A}(\boldsymbol{r}) \cdot d\boldsymbol{r} = \frac{e}{\hbar}\iint \boldsymbol{B}(\boldsymbol{r}) \cdot d\boldsymbol{S} = 2\pi \frac{\Phi}{\Phi_0}$$

- Gauge potential in momentum space

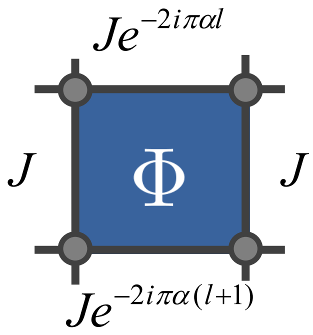




$$|\psi(q_0)\rangle = \sum_j e^{iajq_0}|j\rangle \Rightarrow q_0 a = \theta$$

Harper Hamiltonian / Hofstadter butterfly

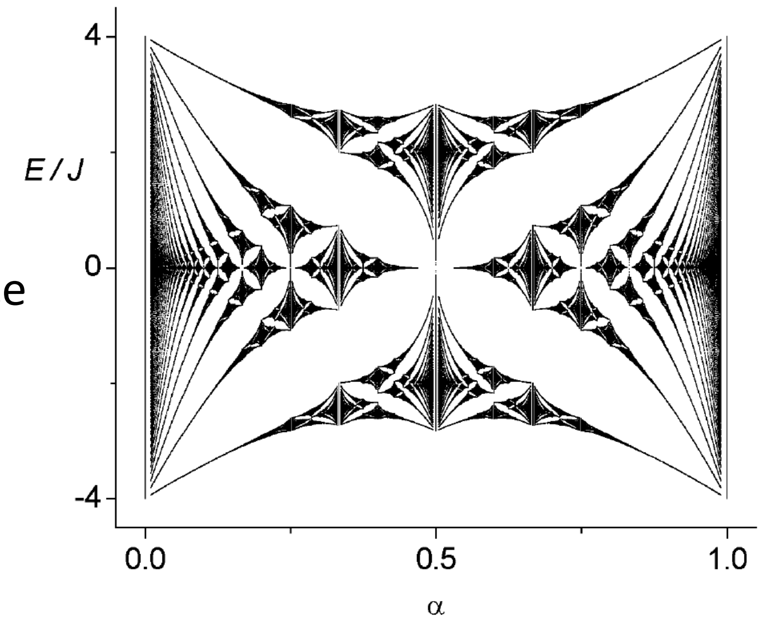
- Particle moving on a square lattice in presence of a magnetic field
  - » Same flux through each plaquette $\Phi = \alpha\Phi_0$
  - » Landau gauge $\boldsymbol{A} = -By\,\boldsymbol{e}_x$
  - » Peierls phase $\theta(|j,l\rangle \to |j,l+1\rangle) = 0$
    $\theta(|j,l\rangle \to |j+1,l\rangle) = -2\pi\alpha l$



- Harper Hamiltonian

$$\hat{H}_{\text{Harper}} = -J\sum_{j,l}\left(e^{-i2\pi\alpha l}|j+1,l\rangle\langle j,l| + |j,l+1\rangle\langle j,l|\right) + hc$$

- Energy spectrum: Hofstadter butterfly
  - » Invariant under $\alpha \to \alpha + 1$
    → study of the spectrum for $0 \le \alpha < 1$
  - » Magnetic field breaks the translational invariance along *y*
  - » Fractal structure

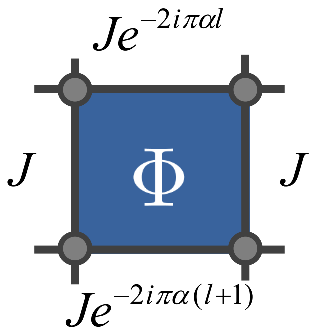


Harper Hamiltonian / Hofstadter butterfly

- Rational values of the flux $\alpha = p'/p$
  - » Translational symmetry restored along y

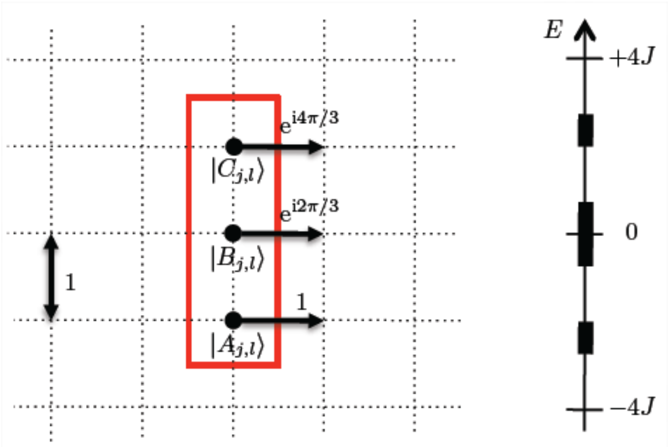
$$\theta(|j, l+p\rangle \to |j+1, l+p\rangle) = -2\pi\alpha(l+p)$$
$$= \theta(|j, l\rangle \to |j+1, l\rangle) \quad \text{modulo } 2\pi$$

  - » Increased spatial period *pa*: magnetic unit cell



- Case α=1/3
  - » Magnetic unit cell: length of *a* along *x* and *3a* along *y*
  - » Each unit cell contains 3 sites

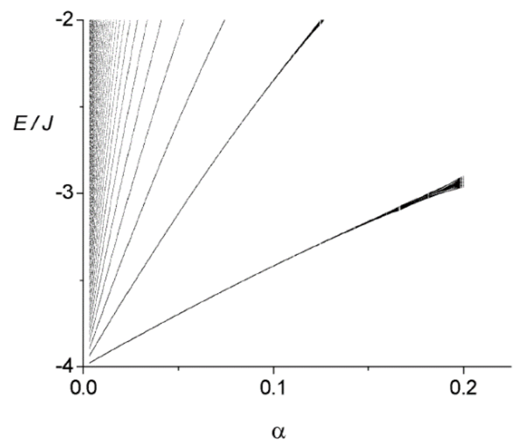
    → Splitting of the energy spectrum in 3 sub-bands

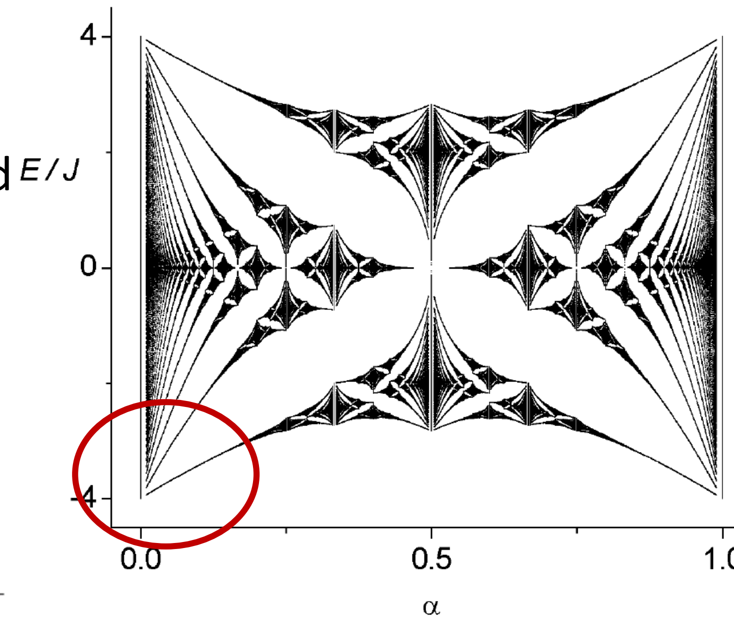


- Origin of the fractal structure
  - » α=1/3 and α=10/31: very close values of α
  - » But very different results as 3 or 31 sub-bands!

Harper Hamiltonian - Hofstadter butterfly

- Recovering the Landau levels
  - » For low magnetic fluxes: $l_{\mathrm{mag}} \gg a$
  - » Analog to a free particle in a static magnetic field
  - » Landau levels?

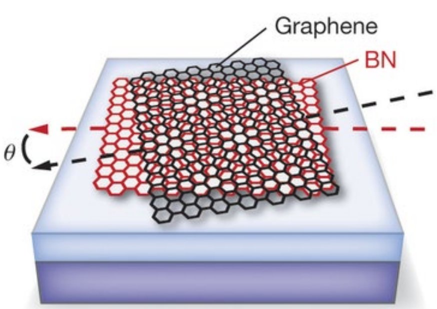




- Measurement of the Hofstadter butterfly

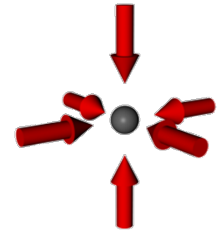
$$\Phi = \Phi_0 \Leftrightarrow B \approx \Phi_0 / a^2$$

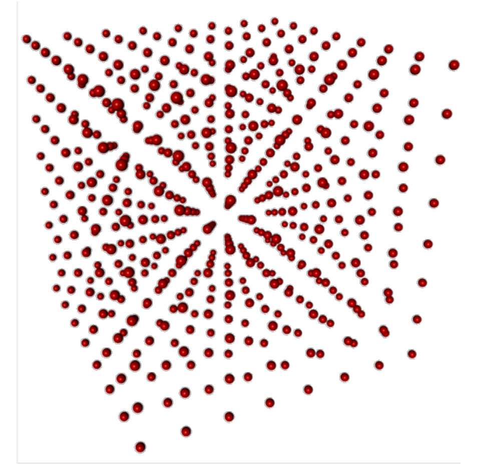
  - » Solid state systems $a \approx 1\,A \Rightarrow B \approx 4\;10^5 T$
  - » Realized using the Moiré pattern in monolayer graphene
  - » Quantum gases?



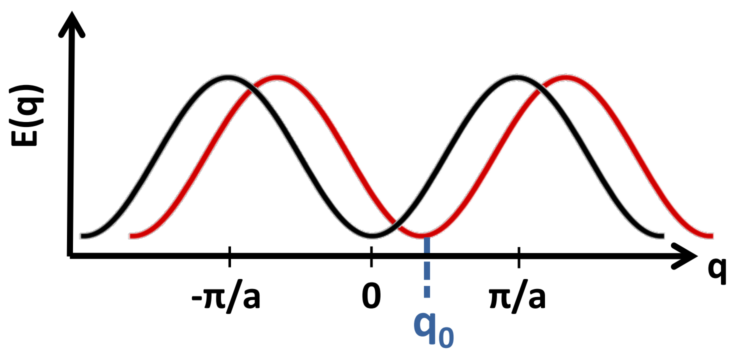
3.2 Generating *artificial* gauge potentials on a lattice

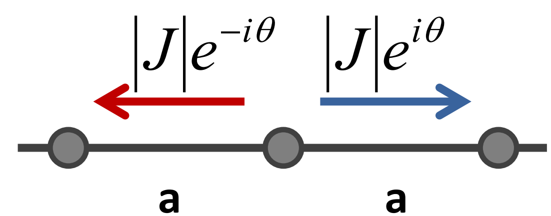
3.2 Generating *artificial* gauge potentials on a lattice





- Natural tunneling in an optical lattice
  - » Well controlled with the lattice depth
  - » Tunneling = hopping probability $J \geq 0$
- Getting complex tunneling
  - » Shift the dispersion relation





$$|\psi(q_0)\rangle = \sum_j e^{iajq_0}|j\rangle \Rightarrow q_0 a = \theta$$

- » Strong field regime reachable as one simulates directly the Peierls phase

$$0 \leq \theta < 2\pi$$

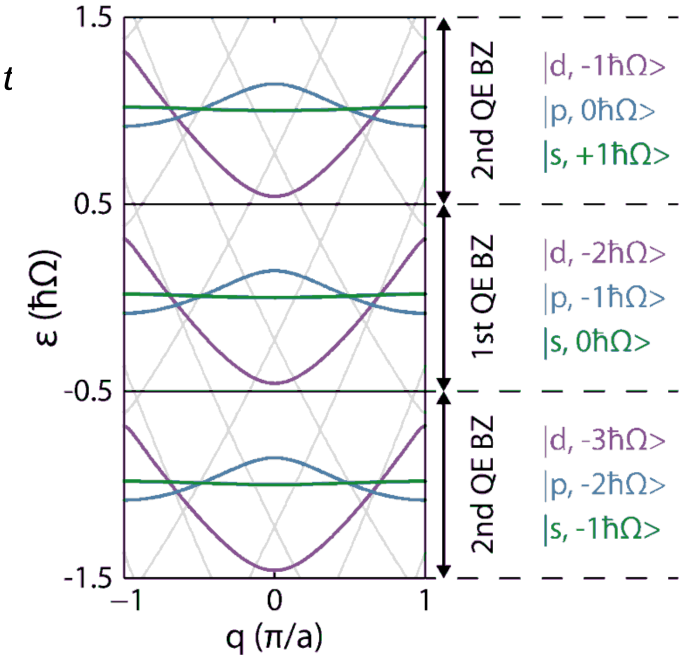
Floquet engineering

- Band engineering via periodic driving: "Floquet engineering"
  - » Periodic driving of the quantum system $\hat{H}(t+T) = \hat{H}(t)$
  - » Analog to the Bloch theorem in time

    Eigenstate: Floquet states $|\psi_n(t)\rangle = |u_n(t)\rangle e^{-i\epsilon_n t}$

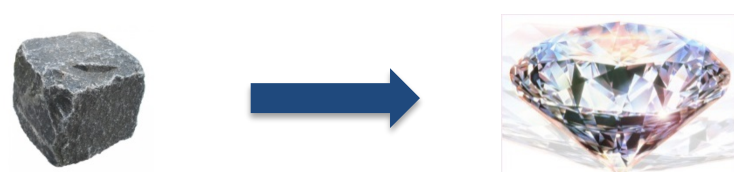
    $$|u_n(t+T)\rangle = |u_n(t)\rangle$$
  - » Floquet theorem

    $$|U(t_1,t_2)\rangle = P(t_2)e^{iH_{\text{eff}}(t_1-t_2)}P^+(-t_1)$$



- High frequency limit
  - » Faster than all other timescales in the system
  - » Effective Hamiltonian is time independent

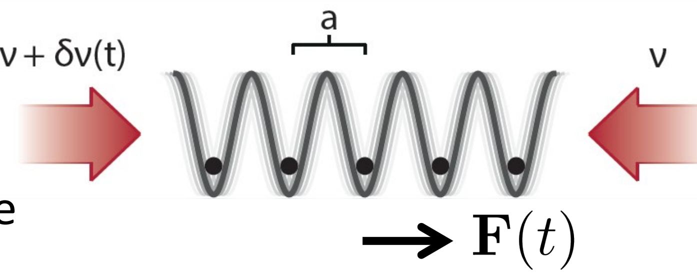
    $$\hat{H}_{\text{eff}} = \langle \hat{H}(t) \rangle_T$$
  - » New properties can emerge in the effective Hamiltonian, especially gauge fields



Band engineering via lattice shaking

- Lattice shaking
  - » Modification of the frequency of one lattice beam
  - » Acceleration of the lattice in space → inertial force

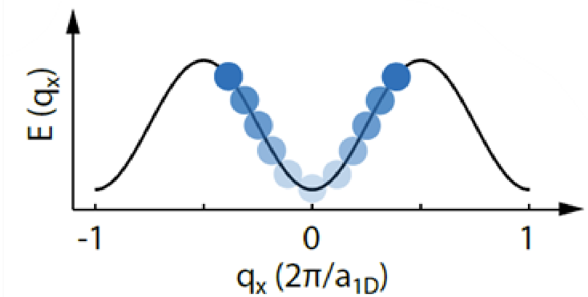
$$\boldsymbol{F}(t) = -m\ddot{\boldsymbol{r}}(t)$$



  - » Semi-classical equation for the quasi-momentum

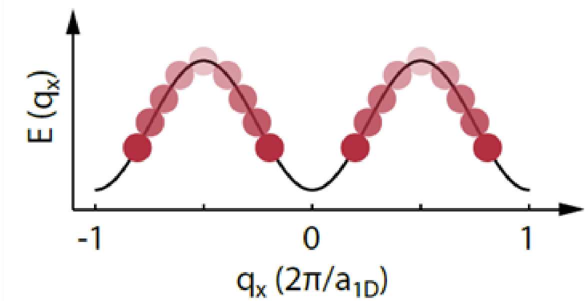
$$\hbar\dot{\boldsymbol{q}}_k(t) = \boldsymbol{F}(t)$$

  - » Time-periodic force with zero mean value $\langle \boldsymbol{F}(t)\rangle_T = 0$



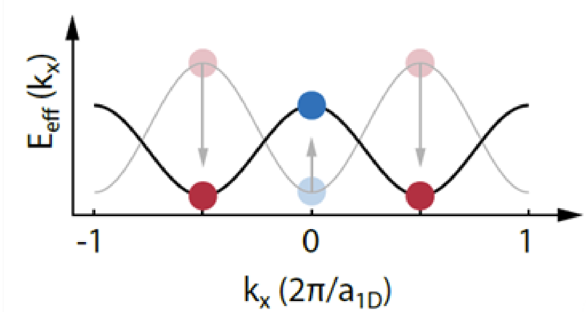
- Renormalization of the band structure in 1D
  - » Sinusoidal shaking $F(t) = F_0 \sin(\omega t)$

$$\Rightarrow q_k(t) = k + \frac{F_0}{\hbar\omega}\cos(\omega t)$$

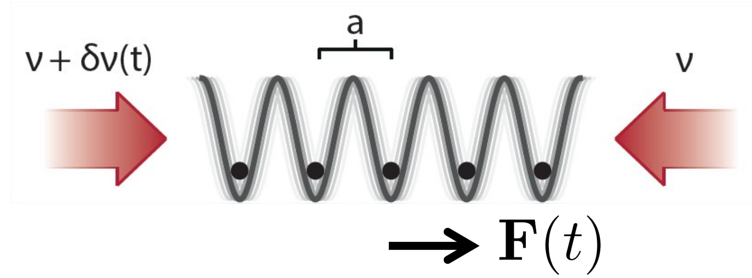


  - » Effective band-structure

$$\mathrm{E}_{\mathrm{eff}}(k) = \frac{1}{T}\int_0^T E\big(q_k(\tau)\big)d\tau$$



Band engineering via lattice shaking



- Effective tunneling
  - » Band structure and tunneling

    $$E(q) = -2J_{\text{bare}}\cos(qa)$$

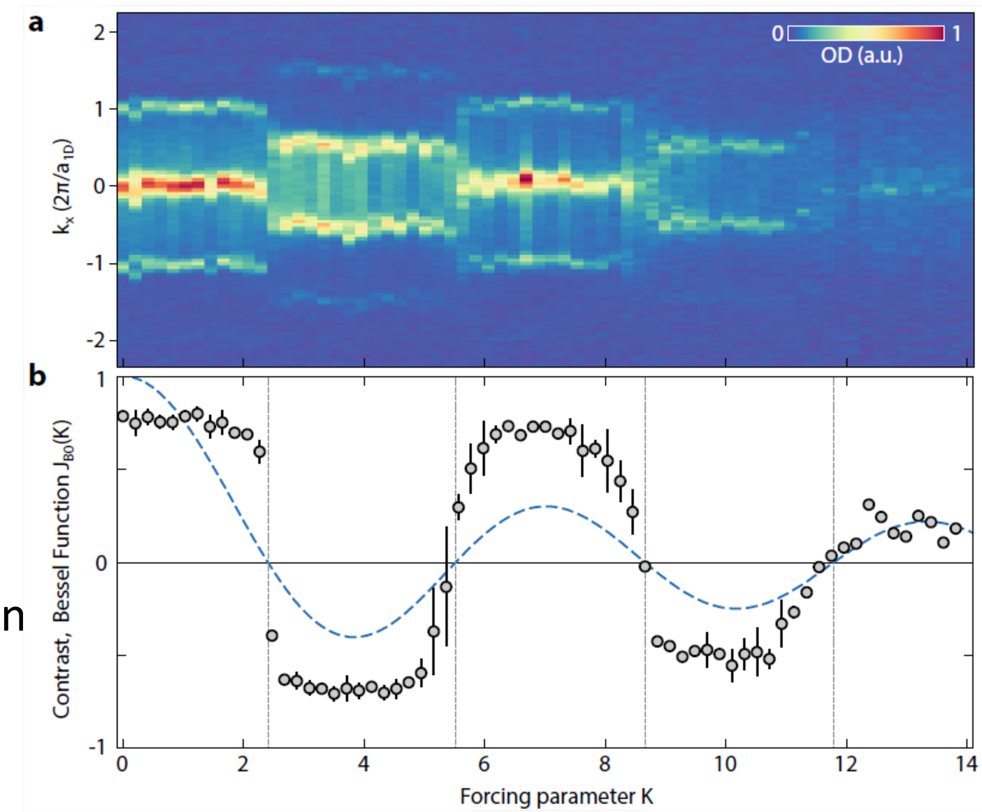
  - » Effective tunneling

    $$\mathrm{E}_{\text{eff}}(k) = \frac{1}{T}\int_0^T E\big(q_k(\tau)\big)d\tau = -2J_{\text{eff}}\cos(ka)$$

    $$J_{\text{eff}} = J_{\text{bare}}J_0(K)$$

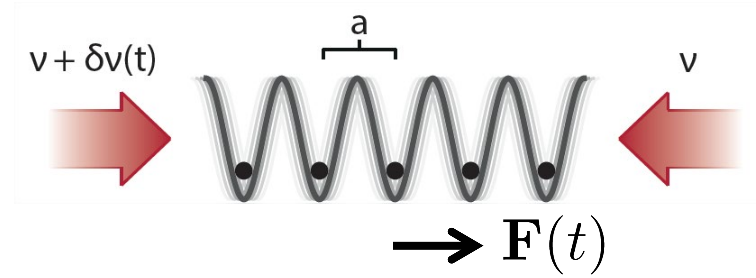
    $$K = \frac{F_0 a}{\hbar\omega}$$

- Measurement with a condensate
  - » BEC: occupies the minimal energy quasi-momentum *k*
  - » Quasi-momentum retrieved after time-of-flight expansion for different forcing amplitude K

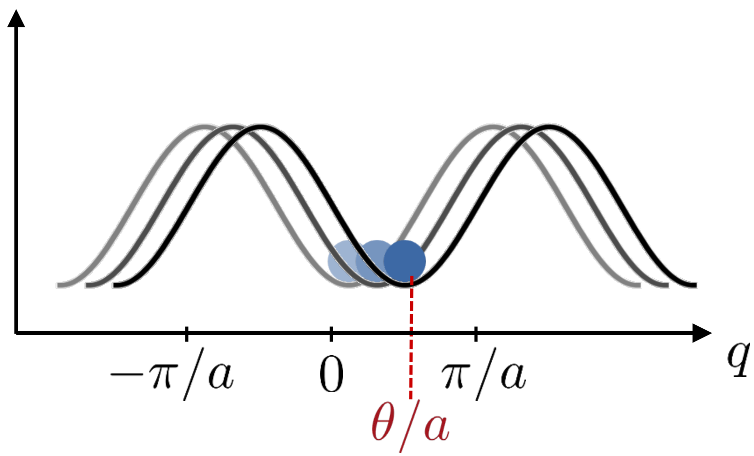


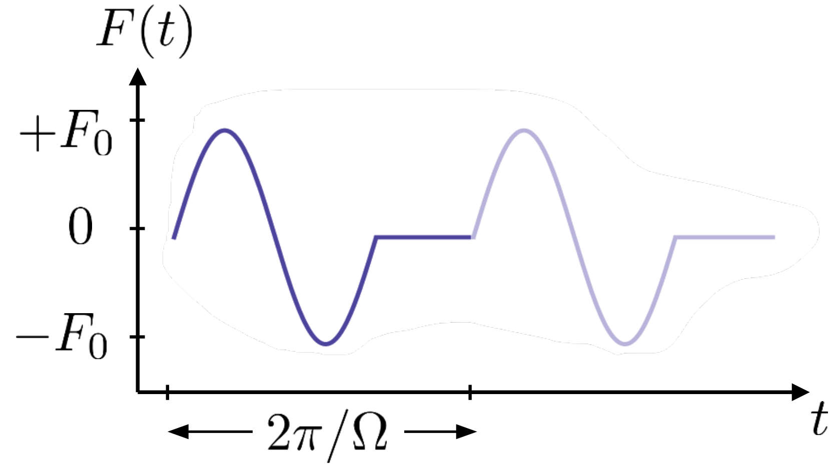
Band engineering via lattice shaking

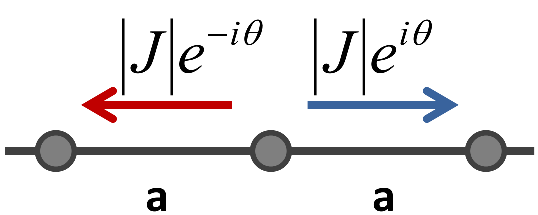
- Realization of complex tunneling
  - » Inertial force asymmetric around q=0

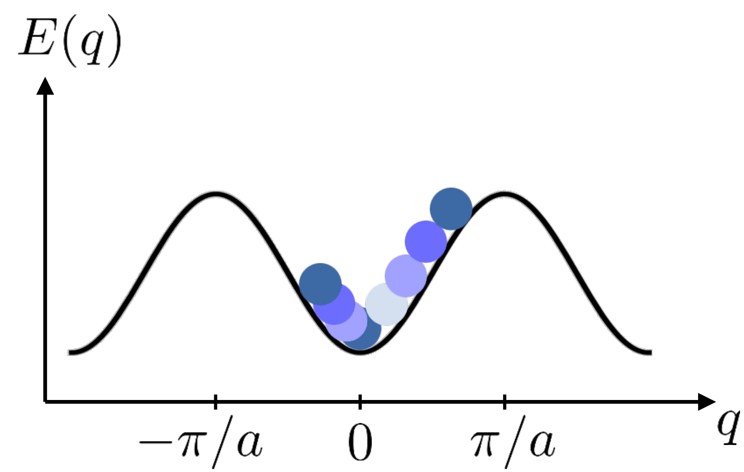


$$E_{\mathrm{eff}}(q) = E(q + \theta/a)$$





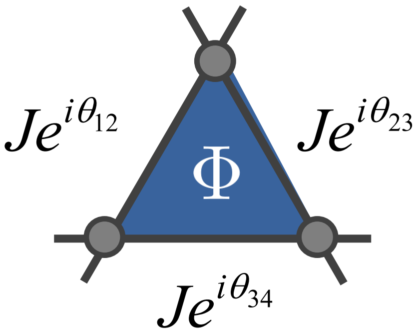


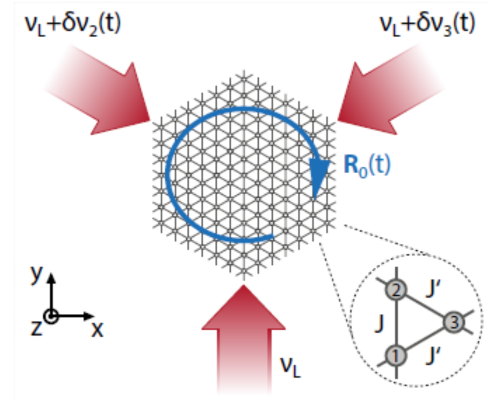


Sengstock, Simonet, Phys. Rev. Lett. **108**, 225304 (2012)

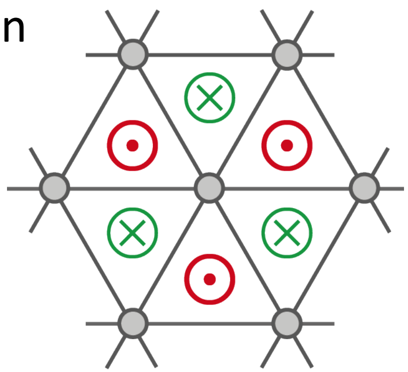
Band engineering via lattice shaking

- Realization of artificial magnetic fluxes
  - » Shaking of a triangular lattice → complex tunneling

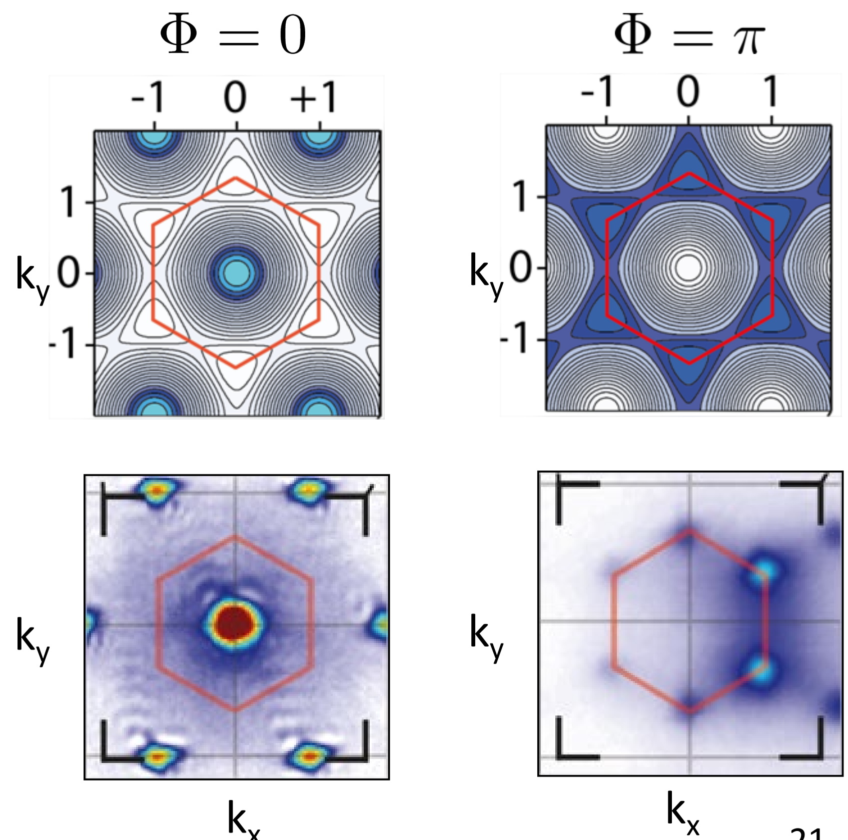




  - » Alternating flux pattern



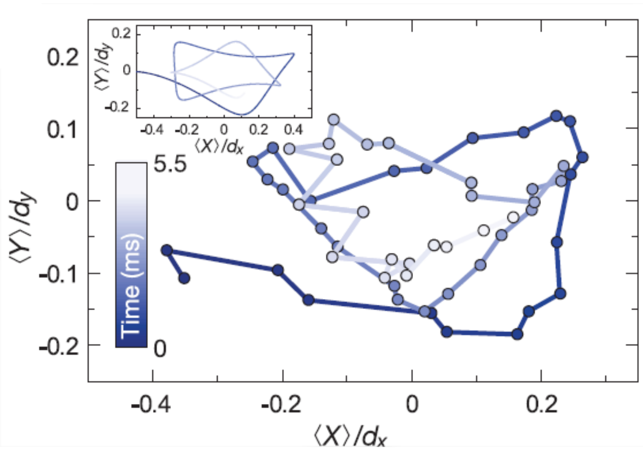
  - » Modification of the band structure can be retrieved after time-of-flight

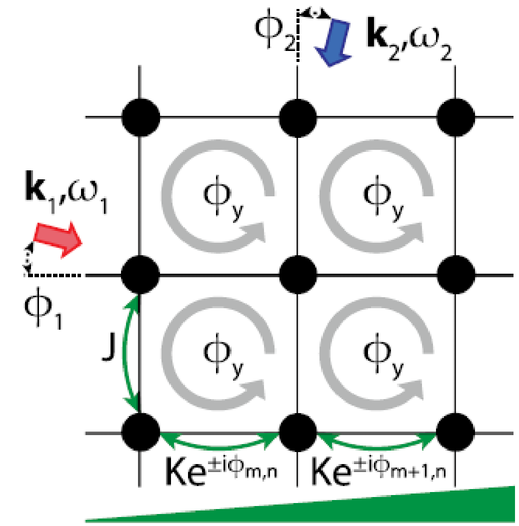


Sengstock, Simonet, Nature Physics **9**, 738 (2013)

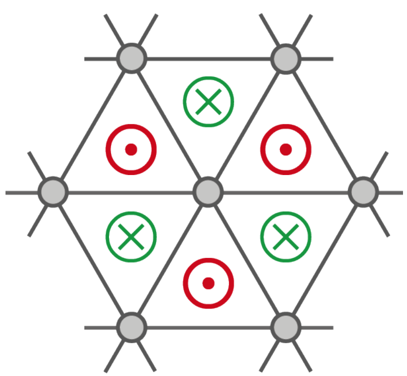
Band engineering via amplitude modulation

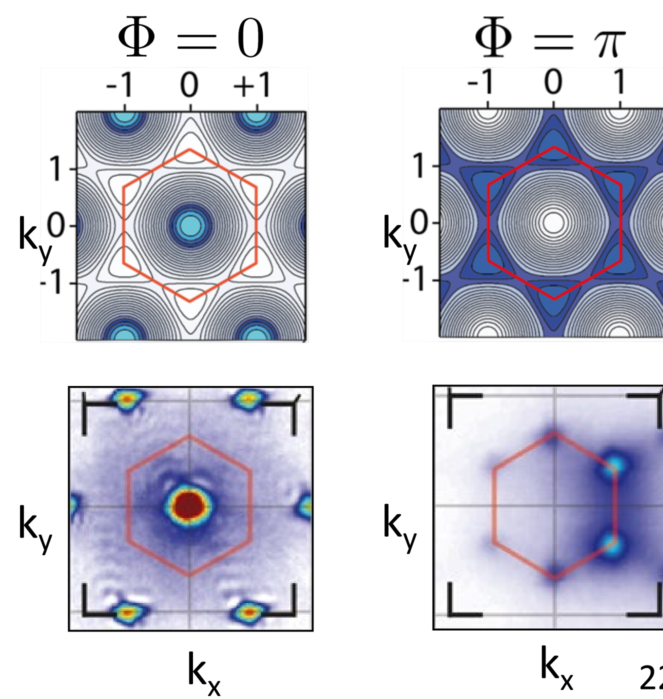
- Cyclotron motion of mass currents
  - » Cyclotron orbit around a square plaquette





  - » Finite mass current ↔ finite quasi-momentum





📄 Bloch / Ketterle, Phys. Rev. Lett. **111** (2013)

3.3 Engineering and probing topological band structures

3.3 Engineering and probing topological band structures

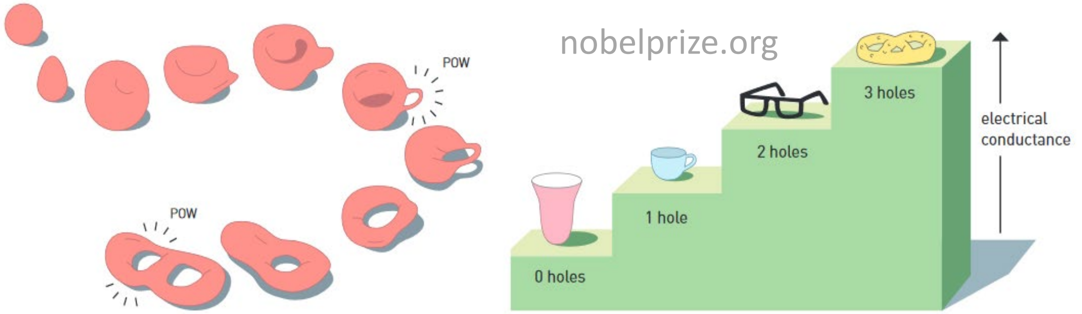
Topology and material properties

» Energy bands of a solid can be topologically non-trivial

- Berry curvature, Chern number

» Topologically non trivial bands give rise to new properties

- Anomalous velocity
- Quantized conductance
- Topological insulators
- Edge states



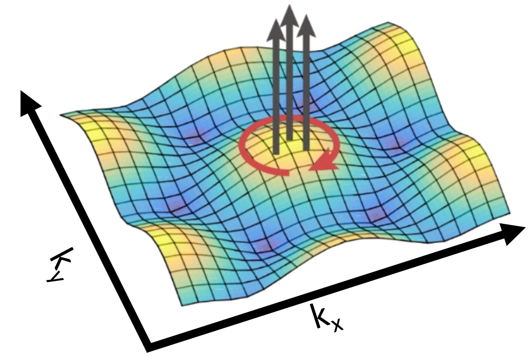
Topological properties of Bloch bands

- Berry curvature in Bloch bands
  - » Eigenstates $|u_{n,\boldsymbol{k}}\rangle$ (band index *n*, quasi-momentum ***k***)
  - » Berry connection $\mathbf{A}_n(\boldsymbol{k}) = i\hbar \left\langle u_{n,\boldsymbol{k}} \middle| \boldsymbol{\nabla}_{\boldsymbol{k}} u_{n,\boldsymbol{k}} \right\rangle$
  - » Berry curvature

$$\mathbf{B}_n(\boldsymbol{k}) = \boldsymbol{\nabla}_{\mathbf{k}} \times \boldsymbol{A}_n(\boldsymbol{k})$$

$$\mathbf{B}_n(\boldsymbol{k}) = \boldsymbol{\nabla}_{\mathbf{k}} \times i\hbar \left\langle u_{n,\boldsymbol{k}} \middle| \frac{\partial}{\partial \boldsymbol{k}} \middle| u_{n,\boldsymbol{k}} \right\rangle$$

  - » Magnetic field in momentum space $\boldsymbol{F} \propto -\dot{\boldsymbol{k}} \times \boldsymbol{B}_\lambda(\boldsymbol{k})$



$$|\psi\rangle \rightarrow \exp(i\Phi^{\text{geom}})\,|\psi\rangle$$

„The remarkable and rather mysterious result of this paper…“ Berry 1984

- Berry phase: Geometrical phase accumulated around a closed path

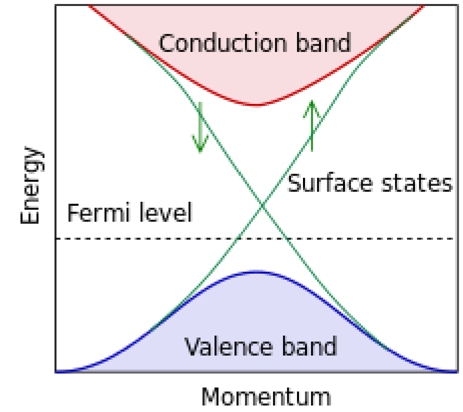
$$\Phi_n^{\text{geom}} = \frac{e}{\hbar}\oint \boldsymbol{A}_n(\boldsymbol{k}) \cdot d\boldsymbol{k} = \frac{e}{\hbar}\iint \boldsymbol{B}_n(\boldsymbol{k}) \cdot d\boldsymbol{S}$$

- Chern number
  - » Integral of the Berry curvature over the Brillouin zone

$$\mathrm{C}_n = \frac{1}{2\pi}\iint_{FBZ} \boldsymbol{B}_n(\boldsymbol{k}) \cdot d\boldsymbol{k}$$

  - » Gauge invariant

Non-trivial topological bands and material properties



- Topological insulator
  - » Electronic band structure: band insulator with Fermi level falling between valence and conduction band
  - » Insulating in the bulk
  - » Metallic at the surface: edge/surface-states (bulk energy gap)
- Chern number and transport properties
  - » Influences the transport properties
    - Anomalous velocity
    - Quantized conductance (Quantum Hall effect)
  - » Determines the number of edge states (Bulk/Edge equivalence)
- Models leading to non-trivial topological bands
  - » Spin-orbit coupling
  - » Harper model
  - » Haldane model

Chern number and transport measurements

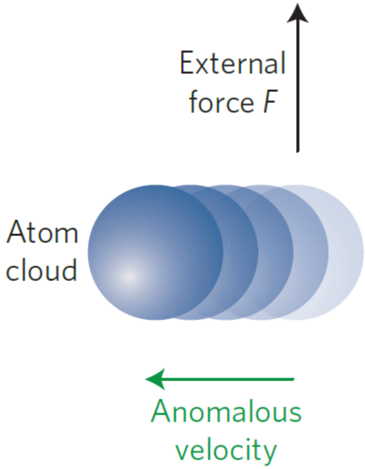
- Band velocity in a 1D lattice
  - » Atom cloud submitted to a constant force F along y
  - » Average velocity of the eigenstate $|u_{n,\boldsymbol{k}}\rangle$

$$\mathrm{v}_n(k) = \left\langle u_{n,k} \,|\hat{v}|u_{n,k} \right\rangle = \frac{1}{\hbar}\frac{\partial E_n(k)}{\partial k}$$

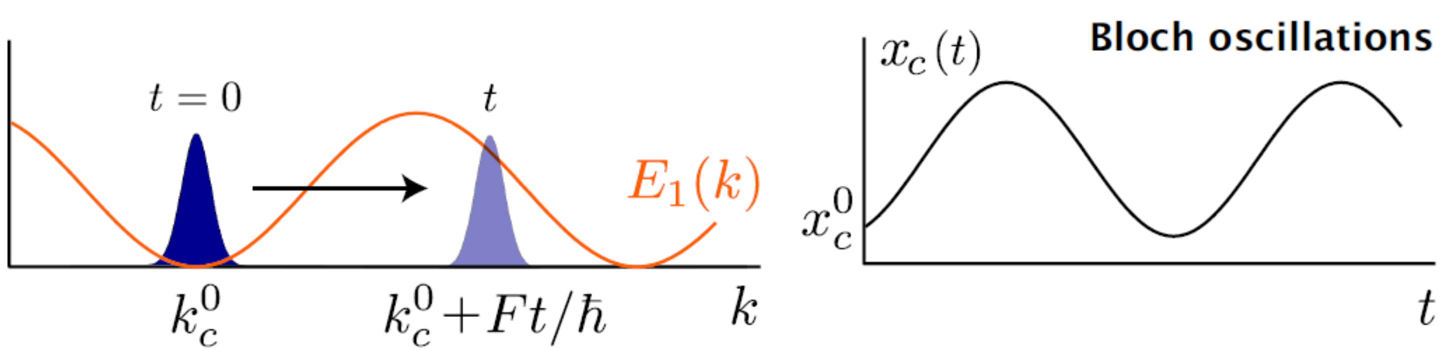
  - » Semi-classical equations of motion for a wave-packet

$$\hbar\dot{x}_c = \hbar \mathrm{v}_n(k_c) = \frac{1}{\hbar}\frac{\partial E_n(k)}{\partial k}$$

$$\hbar\dot{k}_c = F$$



- Bloch oscillations



Chern number and transport measurements

- Anomalous velocity in 2D lattices
  - » Modification of the velocity along the transverse direction *x*

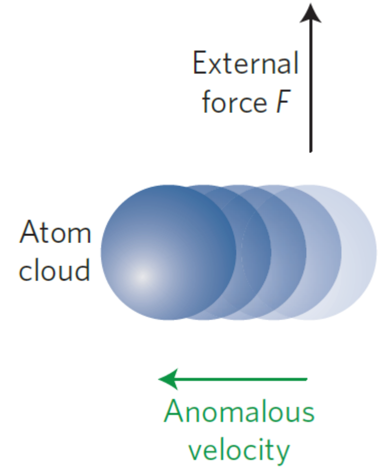
$$\mathrm{v}_n^x(\boldsymbol{k}) = \left\langle u_{n,\boldsymbol{k}} \middle| \widehat{v^x} \middle| u_{n,\boldsymbol{k}} \right\rangle = \frac{1}{\hbar}\frac{\partial E_n(k)}{\partial k_x} - \frac{F_y}{\hbar} B_n(\boldsymbol{k})$$

$$B_n(\boldsymbol{k}) = i\left(\left\langle \partial_{k_x} u_{n,\boldsymbol{k}} \middle| \partial_{k_y} u_{n,\boldsymbol{k}} \right\rangle - \left\langle \partial_{k_y} u_{n,\boldsymbol{k}} \middle| \partial_{k_x} u_{n,\boldsymbol{k}} \right\rangle\right)$$

  1st term: Usual band velocity responsible for Bloch oscillations

  2nd term: Anomalous velocity due to the Berry curvature

  - » Net drift transverse to the applied force

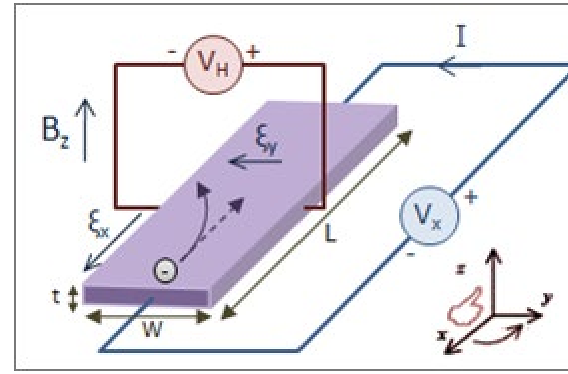


- Transverse velocity for uniformly populated bands
  - » Number of states per band $N_{\mathrm{states}} = A_{\mathrm{sys}}/A_{\mathrm{cell}}$
  - » Average particle number uniform over the Brillouin zone $\rho^n(\boldsymbol{k}) = \rho^n = N^n/N_{\mathrm{states}}$
  - » Mean transverse velocity

$$v_{tot}^x = \sum_n \rho^n \sum_{\boldsymbol{k}} v_n^x(\boldsymbol{k}) \rightarrow -\frac{F_y A_{\mathrm{cell}}}{\hbar} \sum_n C_n \qquad \left(\int_{BZ} \frac{\partial E_n(k)}{\partial k_x} d^2k = 0\right)$$

Quantum Hall effect

- Hall effect
  - » 2D electrons gas in presence of a magnetic field
  - » Electrons are deviated by the Lorentz force
  - » Separation of charges induces an electric field
  - » Hall voltage non zero

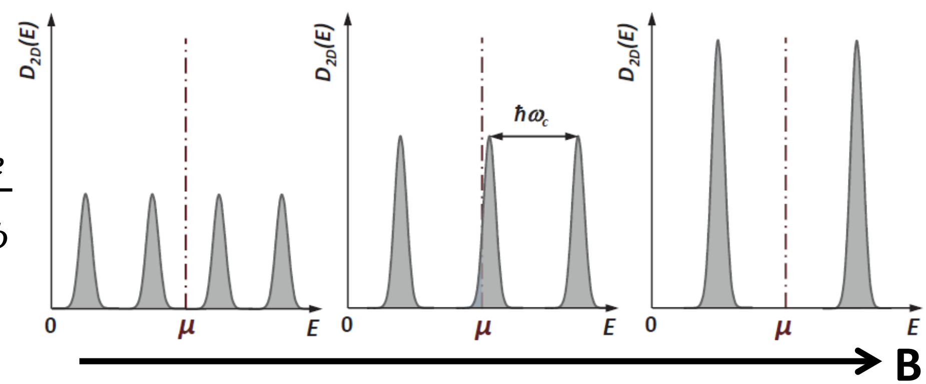


- Quantum Hall effect – macroscopic occupation of Landau levels
  - » Macroscopic degeneracy of each level (sample area A)

$$p = \frac{A}{2\pi l_{mag}^2} = \frac{eAB}{2\pi\hbar} = \frac{\Phi}{\Phi_0} = N_\phi$$

  - » Filling factor –

number of Landau levels involved $\nu = \frac{N_e}{N_\phi}$



  - » Quantum Hall effect reached for $\nu = 1$

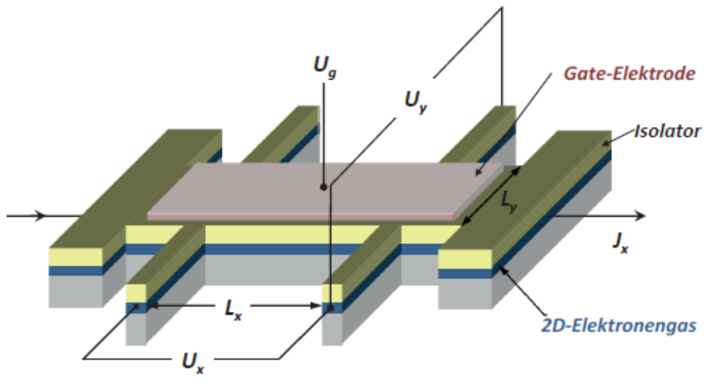
  - » Effect of the chemical potential

Insulating material when chemical potential between a filled and an empty band

Quantum Hall effect

- Integer Quantum Hall effect
  - » Current along x fixed $J_x = n_{2D} e\, v_X$
  - » Gate voltage U_g varied in order to vary μ
  - » Measurement of U_x and U_y $U_x = \rho_{xx} L_x J_x$

    $U_y = \rho_{xy} L_y J_x$



- Observations
  - » U_x vanishes periodically :

  Insulating when μ between 2 Landau levels

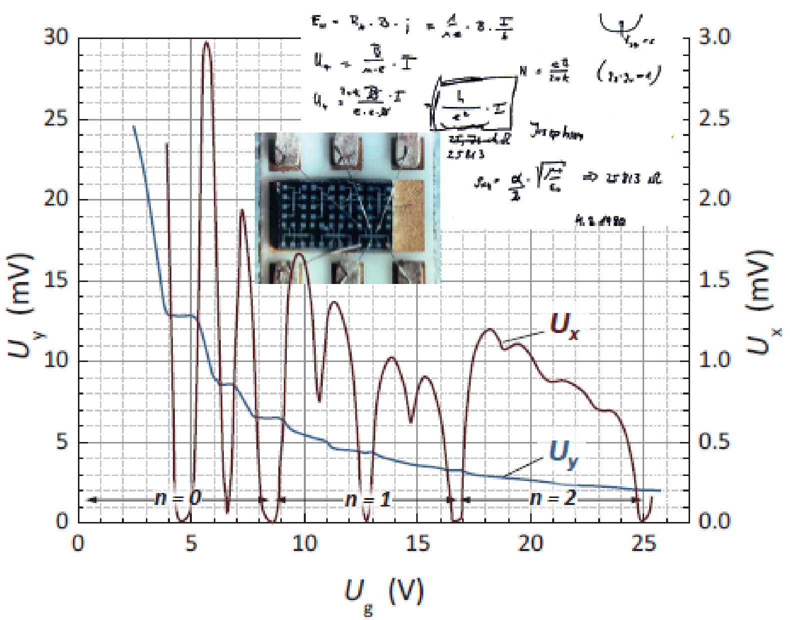
  $$\rho_{xx} = \rho_{yy} = 0$$

  - » U_y has plateaus for the same values of U_g

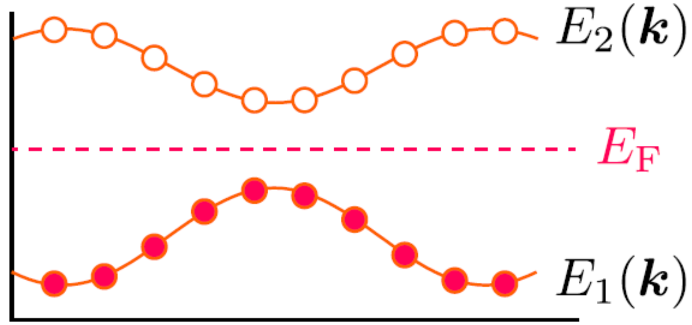
Completely unexpected…

Von Klitzing constant: $R_K = \frac{e}{h^2}$

Effect of non-trivial topological bands!



Quantum Hall effect



- 2D polarized Fermi gas at *T=0K*
  - » Fermi energy within a spectral gap
  - » Perfect filling of the bands below the gap

$$\rho = \frac{N_{tot}}{N_{states}} = 1 \text{ for } E_n < E_F$$

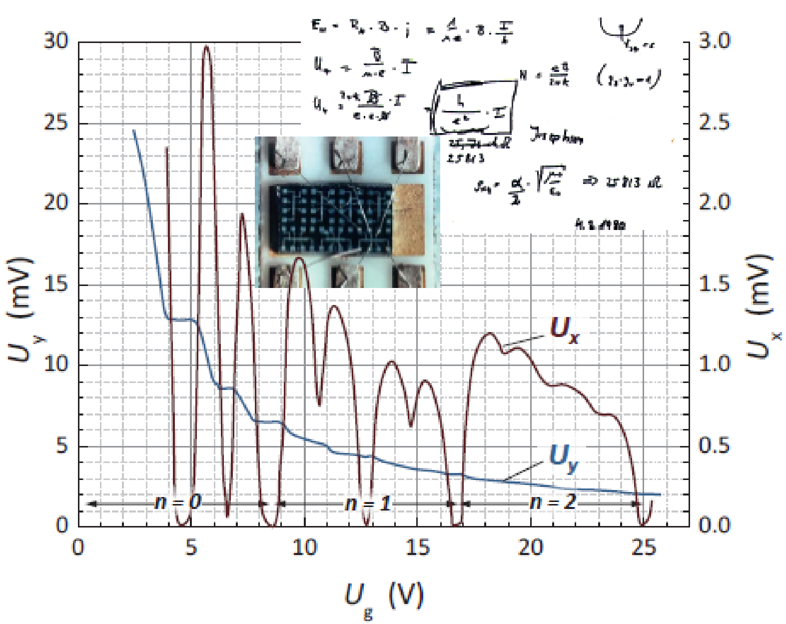
- Quantum Hall effect
  - » Total transverse velocity

$$v_{tot}^x = -\frac{F_y A_{\mathrm{sys}}}{\hbar} \sum_{E_n < E_F} C_n$$

  - » Electric Hall conductivity $\sigma_{x,y}$

$$j_x = \sigma_{x,y} E_y$$

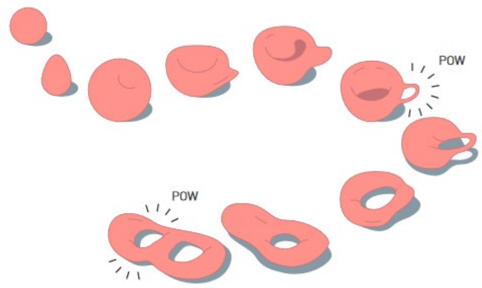
$$j_x = \frac{e v_{tot}^x}{A_{\mathrm{sys}}} \quad \Rightarrow \sigma_{x,y} = \frac{e^2}{\hbar} \sum_{E_n < E_F} C_n$$



  - » Transport measurements reveal the Chern numbers

Quantum Hall effect

- Integer Quantum Hall effect
  - » Quantized conductivity
  - » Transport measurements reveal topological properties



nobelprize.org

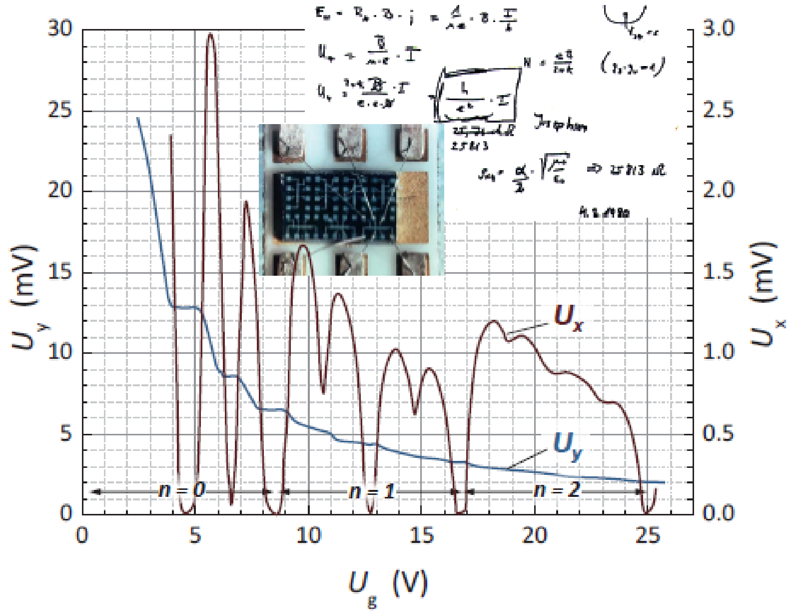
- Fractional Quantum Hall effect
  - » Plateaus at fractional values of the Hall resistance
  - » Collective behavior:

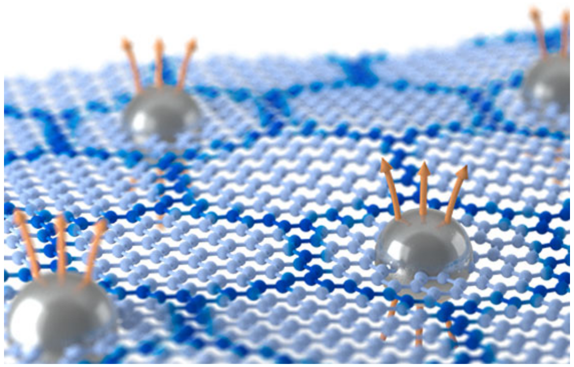
  condensation of the electron gas

  - » Microscopic origin unknown

  Induced by e-e repulsion?

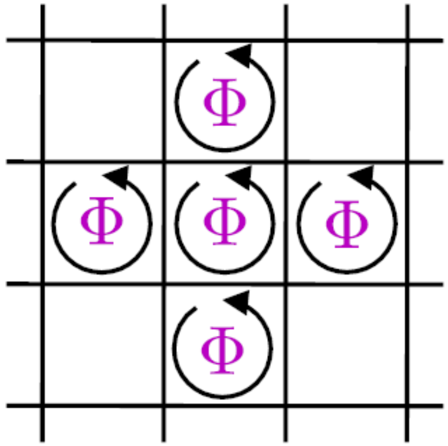
  Quantum simulation with model systems!





Engineering topologically non-trivial models - Harper Model

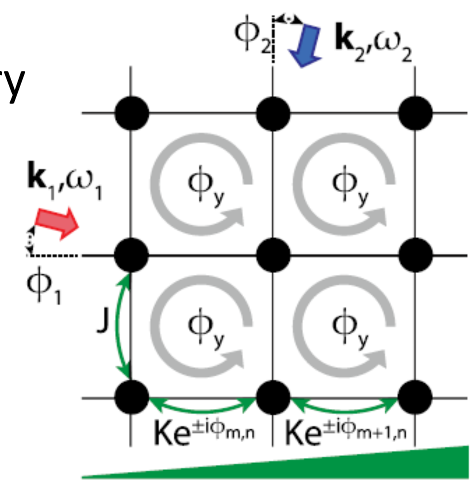
- Uniform magnetic field on a lattice
  - » Energy spectrum - Hofstadter butterfly
  - » For rational values of the flux $\alpha = \Phi/\Phi_0 = p'/p$
    - Increased spatial periodicity pa (magnetic cell)
    - The energy band for $\Phi = 0$ splits into p sub-bands
    - Each sub-band has a non-zero Chern number



- Consequence of the non-trivial topology
  - » Quantized conductance: quantum Hall effect
  - » Edge states: macroscopic consequence of the cyclotron orbits induced by a magnetic field truncated at the sample's boundary

- Realized for quantum gases

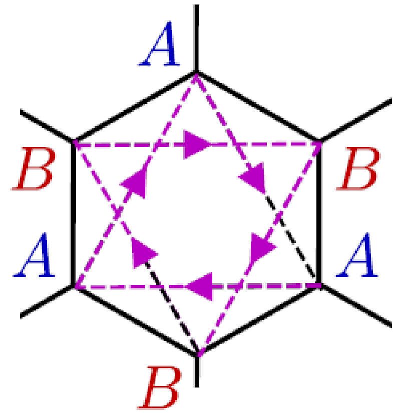
  Periodic amplitude modulation in a square lattice

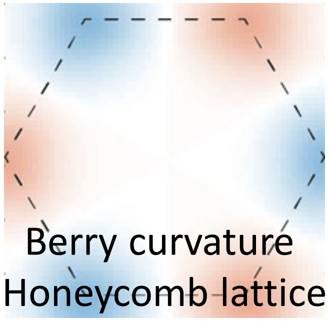


Bloch / Ketterle, Phys. Rev. Lett. **111** (2013)

Engineering topologically non-trivial models - Haldane model

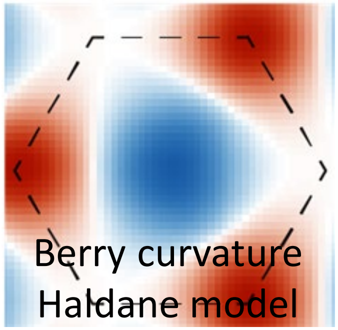
- Graphene-like honeycomb lattice
  - » Unit cell contains two equivalent sites A and B
  - » Nearest neighbor tunneling of amplitude J $(A \leftrightarrow B)$
  - » Band structure: two bands touching at the Dirac points
  - » Berry curvature non zero
    - Berry phase around the Dirac point
    - $\mathbf{B}_1(\boldsymbol{k}) = -\mathbf{B}_1(-\boldsymbol{k}) \Rightarrow C_1 = 0$





- Breaking time-reversal symmetry
  - » Addition of complex next-neighbor tunneling $(A \leftrightarrow A \text{ or } B \leftrightarrow B)$
  - » Lift the degeneracy at the Dirac points
  - » 2 sub-bands separated by a gap with Chern numbers +1 and -1

$$\mathbf{B}_1(\boldsymbol{k}) \neq -\mathbf{B}_1(-\boldsymbol{k}) \Rightarrow C_1 \neq 0$$



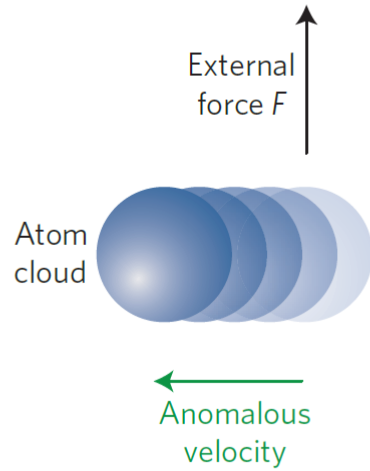
- Realized for quantum gases

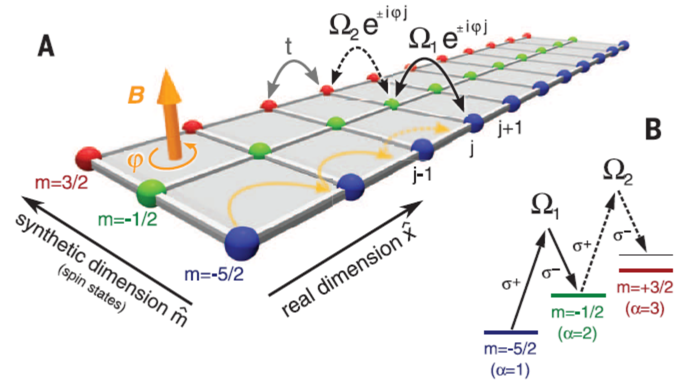
  Circular acceleration of an honeycomb lattice

Esslinger, Nature **515**, 238 (2014)

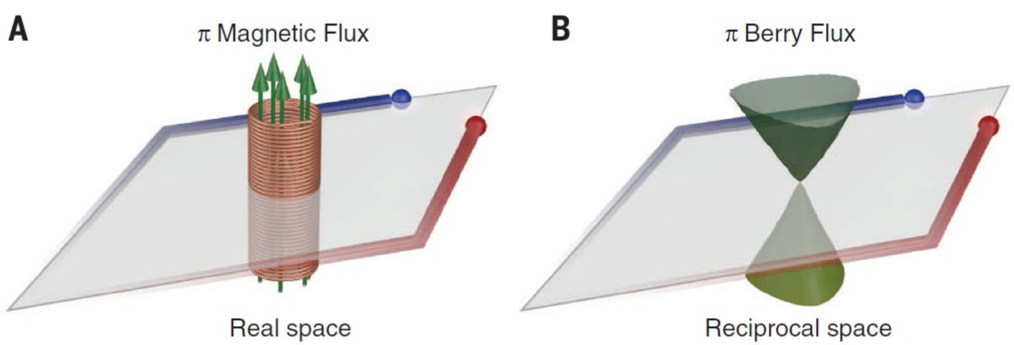
Evidencing topological properties with quantum gases

- Chern number
  - » Transport measurements
    - Anomalous velocity
    - Quantized conductance
  - » Counting the edge states



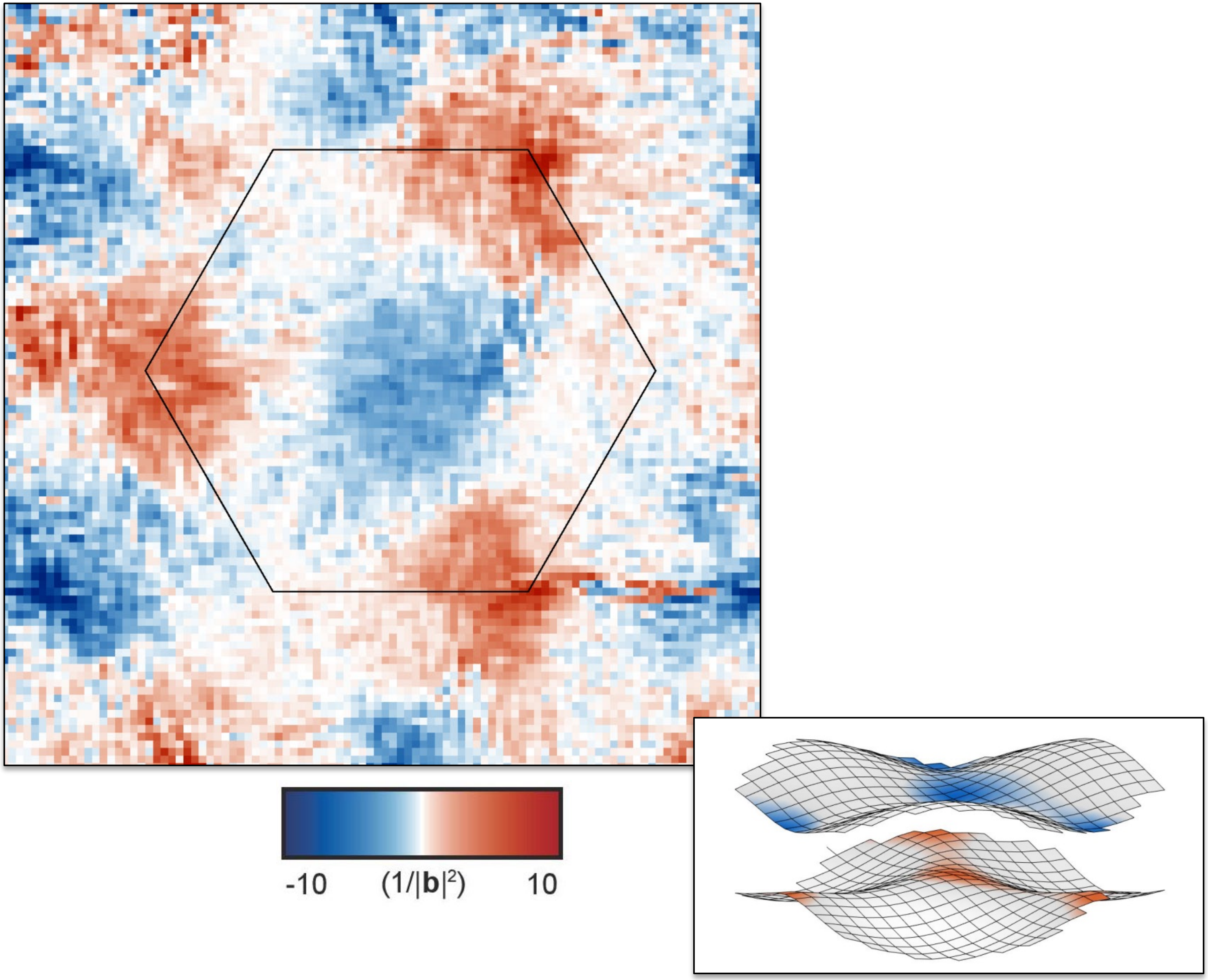


- Measuring the Berry phase with a momentum space interferometer



- Mapping the Berry curvature

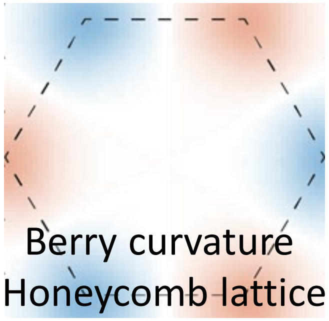
Mapping the Berry curvature



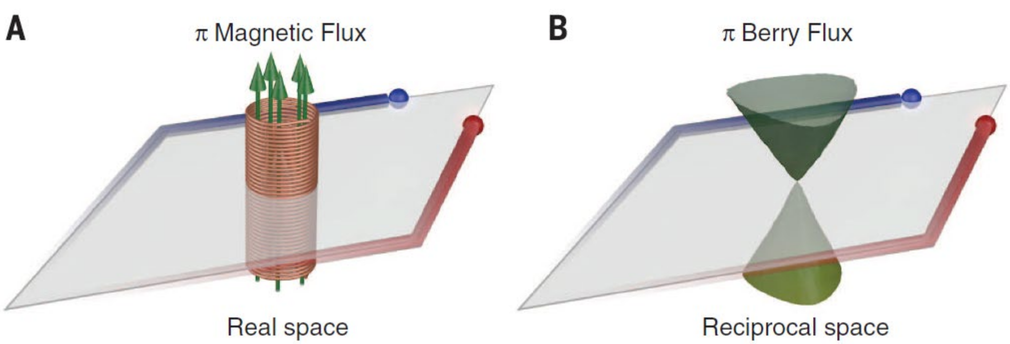
Sengstock, Weitenberg, Science **352**, 1091 (2016)

Measuring the Berry phase

- Honeycomb lattice
  - » Berry curvature non zero
  - » Berry phase of π accumulated around the Dirac points
  - » Opposite signs for the 2 Dirac points

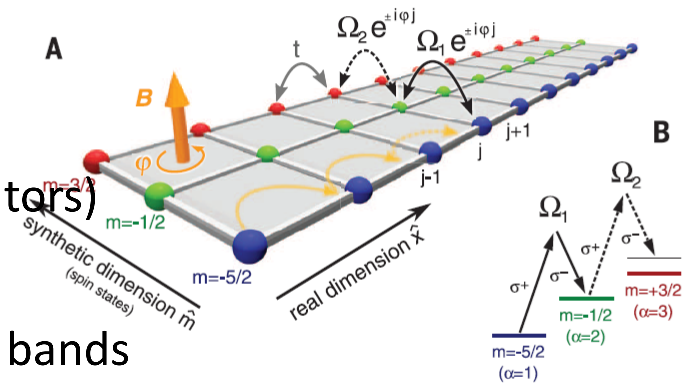


- Measurement of the Berry phase
  - » Berry flux analog to a magnetic flux
  - » Aharonov-Bohm interferometer: observation of the phase accumulated proportional to the magnetic flux
  - » Berry flux interferometer: closed path in reciprocal space



Bloch, Science **347**, 288 (2015)

Direct observation of edge states



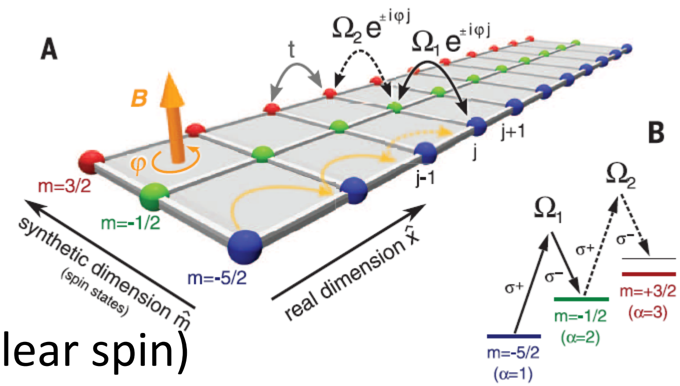
- Edge states
  - » Metallic states located at the edge of the sample
  - » Reveal non-trivial bulk properties (topological insulators)
  - » For non-interacting fermionic system:

    Number of edge states = Chern number of the filled bands

- Edge states for quantum gases
  - » Requires a non-zero Chern number for the lowest band
  - » Harper model: macroscopic consequence of the cyclotron orbits induced by a magnetic field truncated at the physical boundary of the sample
  - » Direct observation challenging
    - Corresponding to mass current (Time-of-Flight imaging)
    - Large imbalance between population bulk and edge states
    - Difficult to observe in harmonic traps (no sharp edges): box potentials required

Direct observation of edge states

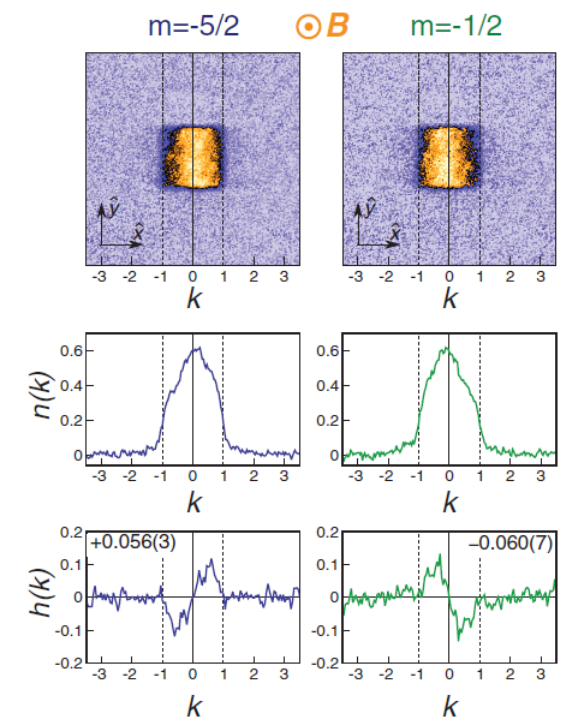
- Synthetic magnetic fields in synthetic dimensions
  - » Magnetic fields are two-dimensional objects
  - » Synthetic magnetic field
    - One dimensional lattice with tunneling *J*
    - Extra dimension: internal degree of freedom (nuclear spin)
    - Two-photon Raman transition couples the spins and induces a complex tunneling amplitude along the extra dimension
  - » Realization of the Harper model



- Direct observation of edge states
  - » Two-legs ladder with fermions
  - » Opposite mass currents along the two legs
  - » Chiral dynamics revealed

  by spin resolved time-of-flight measurement

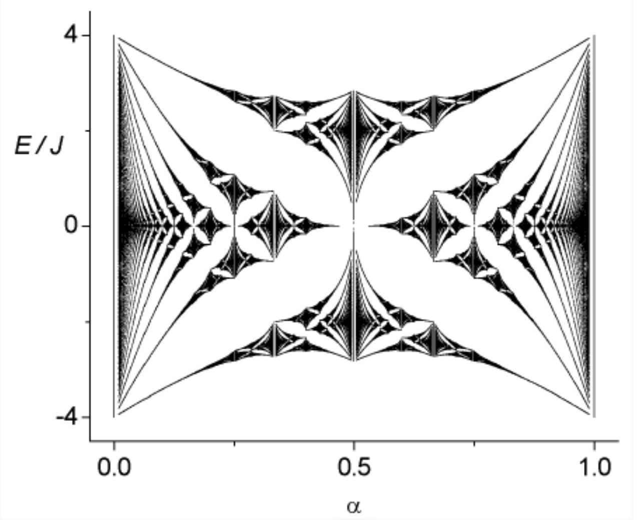
$$h(k) = n(k) - n(-k)$$



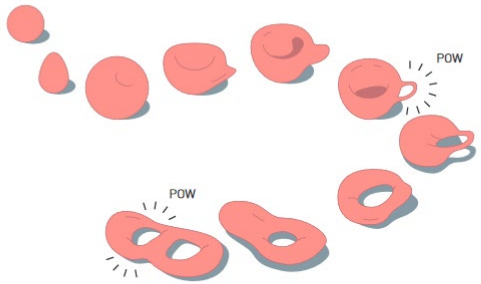
Lewenstein, Phys. Rev. Lett. **112**, 043001 (2014)

Spielman & Inguscio, Science **349** (2015)

Artificial gauge fields / Summary



- Artificial gauge fields
  - » Electromagnetic fields in free space (Raman coupling)
  - » Magnetic fields on a lattice (Floquet engineering)
  - » Spin-orbit coupling
- Topological non-trivial bands
  - » Harper model
  - » Haldane model
  - » ..many more as of today
- Evidencing topological properties with quantum gases
  - » Measurement of the Chern number via transport properties
  - » Direct observation of edge states
  - » Measuring the Berry phase
  - » Mapping the Berry curvature



nobelprize.org

Artificial Gauge Fields / Challenges and Outlook

- Static electromagnetic fields
  - » No feedback of the matter onto the artificial fields (neutral)
    → Maxwell equation not valid for artificial gauge fields

- Topological materials and edge states
  - » For non-interacting system: number of edge states = Chern number
    Effect of interactions?
  - » Edge states observed only for small systems so far (two/three legs ladder)
    Difficult to realize in a trap (no sharp edges): box potentials required

- Interactions
  - » So far no effect of interaction – single particle physics
  - » Realization of strongly correlated phases still not achieved