QCD DETERMINATION OF QUARK MASSES

Cesareo A. Dominguez

Centre for Theoretical & Mathematical Physics

&

Department of Physics

University of Cape Town, South Africa

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QUARK MASS

A FRACTAL

FRACTAL: object of non-integer dimension

$$\alpha_s(Q^2) = \frac{-2/\beta_1}{\ln(Q^2/\mu^2)} \qquad m_q(Q^2) = \frac{\widehat{m}_q}{[\ln(Q^2/\mu^2)]^{\frac{-\gamma_1}{\beta_1}}}$$

SHORT HISTORY OF QUARKS **(before QCD)**

1960's

SU(2) (Heisenberg) SU(3) (Gell-Mann)

FUNDAMENTAL REPRESENTATION: VOID (QUARKS)

CURRENT ALGEBRA: SU(2) $\otimes$ SU(2) , SU(3) $\otimes$ SU(3)

CHIRAL SYMMETRIES

V_μ & A_μ :

- **CHIRAL-SYMMETRIES**

- **$SU(3)_V \otimes SU(3)_A$:** $\partial_\mu V_\mu \propto (m_s - m_{u,d})$ $\partial_\mu A_\mu \propto (m_s + m_{u,d})$
- **$m_s = m_u = m_d = 0$**

- **1^{st} SB: $m_s \neq 0$ $m_u = m_d = 0$**

- $\partial_\mu V_\mu \propto (m_d - m_u)$ $\partial_\mu A_\mu \propto (m_u + m_d)$

- **$SU(2)_V \otimes SU(2)_A$ & $SU(2)_V$**

- **2^{nd} SB: $m_s \neq 0$ $m_u = m_d \neq 0$**

- **$SU(2)_V$**

- **3^{rd} SB: $m_u \neq m_d \neq 0$**

- **$U(1)$**

IN THE ABSENCE OF A THEORY (LAGRANGIAN)

ONLY QUARK MASS RATIOS **(chiral perturbation theory)**

related to hadron masses & some weak-interaction processes

$$H(x) = H_0(x) + \epsilon_0 u_0(x) + \epsilon_3 u_3(x) + \epsilon_8 u_8(x)\,.$$

$$R \equiv \frac{m_s - m_{ud}}{m_d - m_u} = \frac{\sqrt{3}}{2}\frac{\epsilon_8}{\epsilon_3}\,,$$

$$\frac{m_u}{m_d} = \frac{M_{K^+}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} = 0.56\,,$$

$$\frac{m_s}{m_d} = \frac{M_{K^+}^2 + M_{K^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} = 20.2\,,$$

K_{l3} decay (CAD Phys. Lett. B 86 (1979) 171) FLAG (Eur. Phys. J. C77,112 (2017)

$m_u / m_d = 0.55 \pm 0.21$ (0.50 ± 0.04)

$2\, m_s / (m_u + m_d) = 29 \pm 7$ 27.3 ± 0.9

$$Q^2 \equiv \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2} = \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2} \frac{M_K^2}{M_\pi^2} .$$

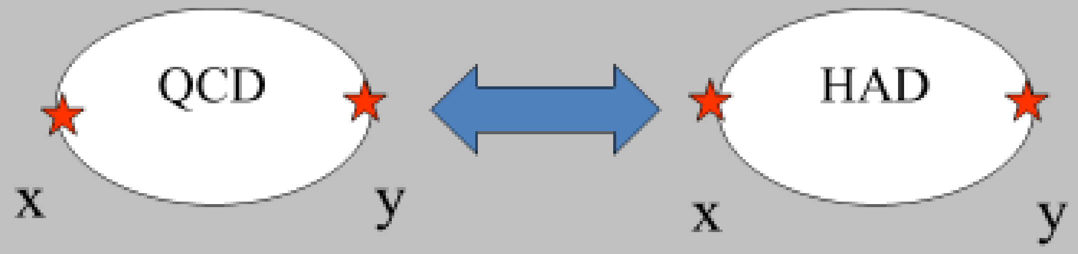
$\eta \to 3\pi$ (CAD & A. Zepeda, Phys. Rev. D18, 884 (1978))

$$r_s = 29 \pm 7$$

$$r_s \equiv \frac{m_s}{m_{ud}} = \frac{2m_s/m_d}{1 + m_u/m_d} = 28.1 \pm 1.3$$

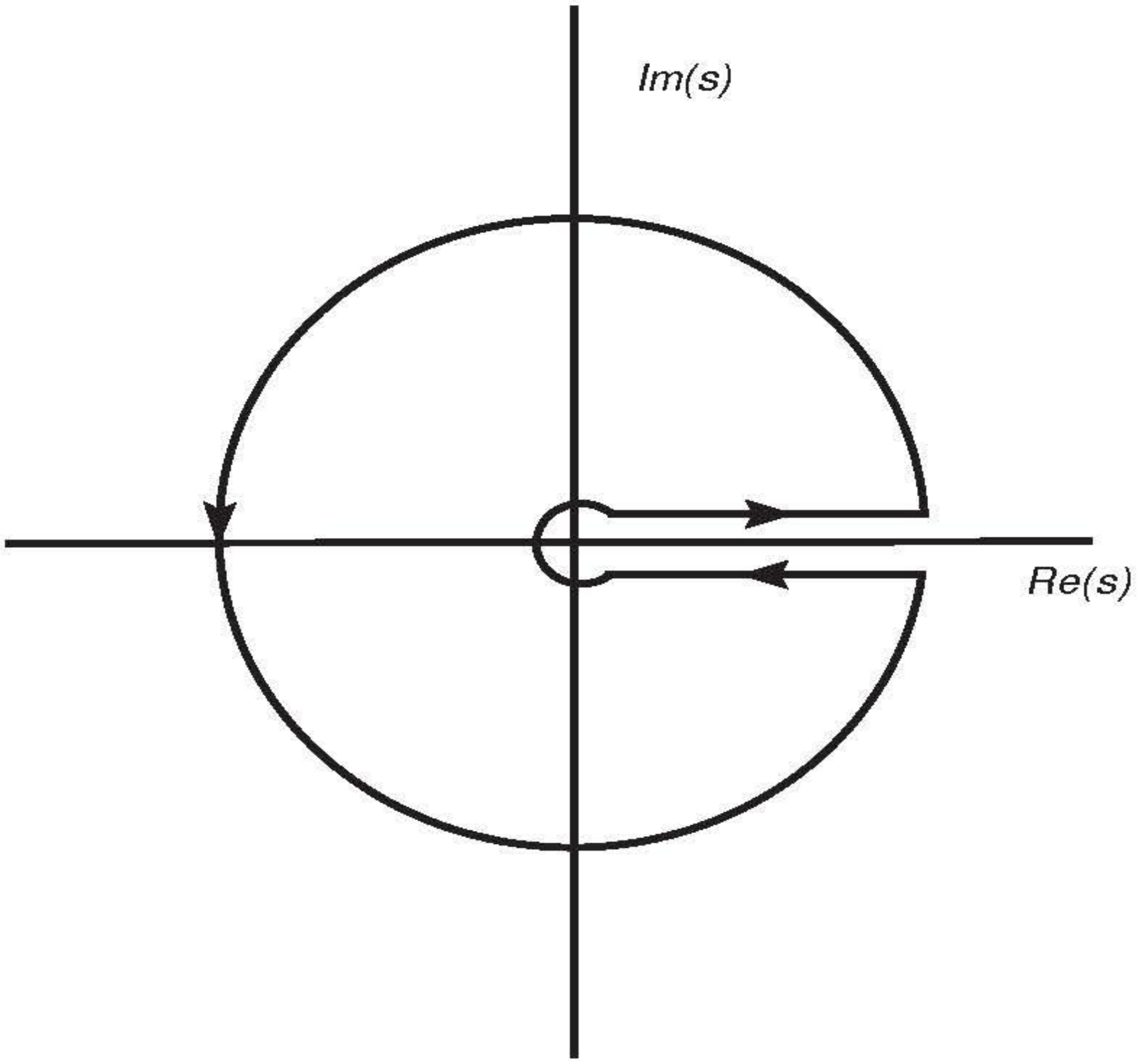
FLAG

NLO CHPT



CAUCHY'S THEOREM IN THE COMPLEX ENERGY PLANE

SQUARED ENERGY: $s \equiv E^2$


Im(s)
Re(s)

QCD SUM RULES

QCD

$$\Pi(q^2) = i\int d^4x \; e^{iqx} < 0|T(J(x)\,J^+(0))|0 >$$

$$J(x) \Rightarrow \partial^\mu A_\mu(x)|^i_j = (m_i + m_j)\,\overline{\psi}^i(x)\, i\gamma_5\, \psi_j(x)$$

$$\Pi(q^2) = \int d^4x\, e^{iqx} < 0 | T\,(J(x)\, J^+(0) | 0 >$$

$$\Pi(q^2)|_{QCD} = I + \sum_{N=0} C_{2N+2}(q^2, \mu^2) < 0 | \hat{O}_{2N+2}(\mu^2) | 0 >$$

$$I \Rightarrow O(\alpha_s^4)$$

$$C_{2N+2} \Rightarrow \frac{1}{(-q)^{2N+2}}$$

$$m_q < 0 | \overline{q}\; q | 0 >, \quad < 0 | \alpha_s\, G_{\mu\nu} G^{\mu\nu} | 0 >, \quad etc.$$

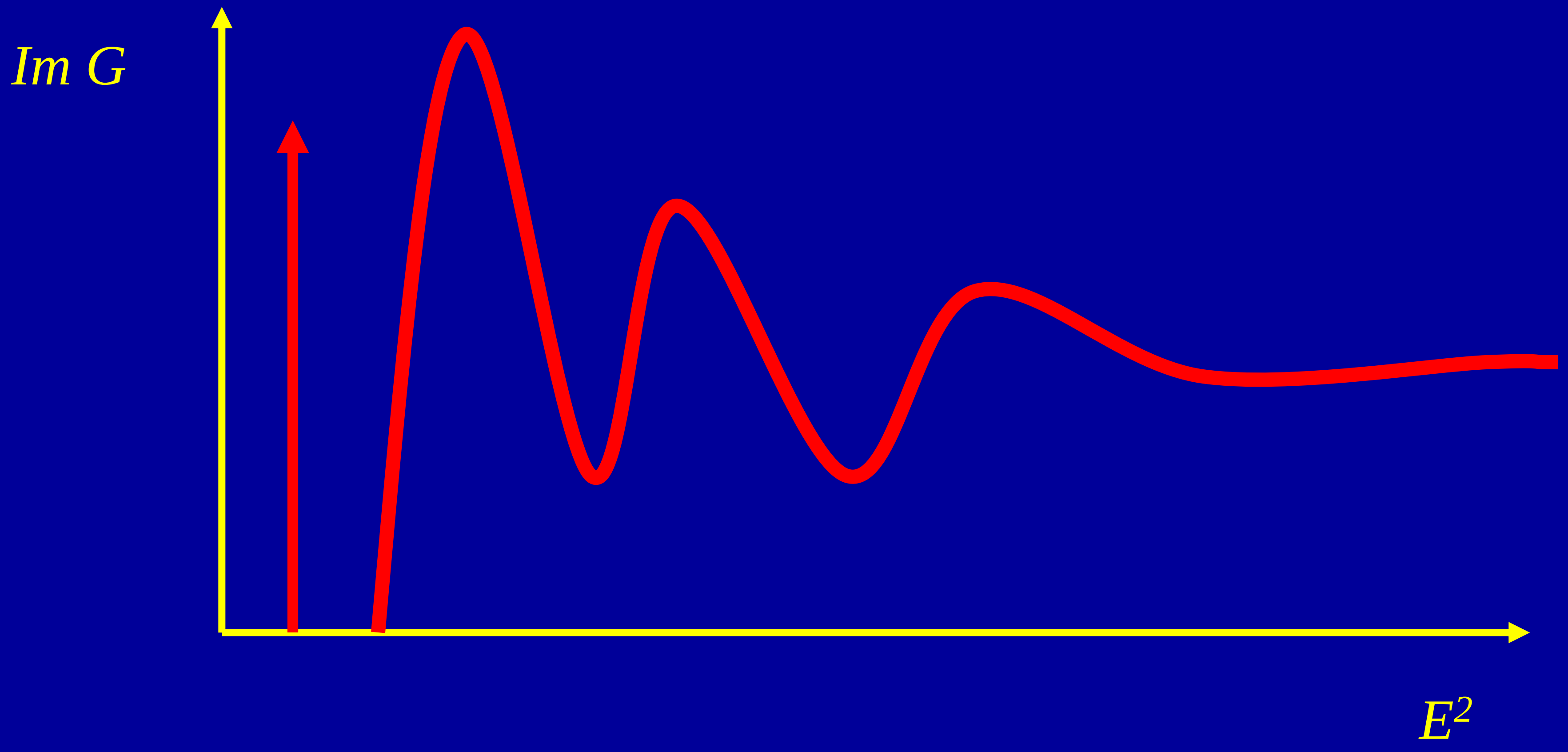
$$
\begin{aligned}
\bar{m}(s) = \bar{m}(s^*)\Big\{ & 1 - a(s^*)\,\gamma_0\,\eta + \frac{1}{2}\,a^2(s^*)\,\eta\,\Big[-2\,\gamma_1 + \gamma_0\,(\beta_0 + \gamma_0)\,\eta\Big] \\
- & \frac{1}{6}\,a^3(s^*)\,\eta\,\Big[6\,\gamma_2 - 3\,\Big(\beta_1\,\gamma_0 + 2\,(\beta_0 + \gamma_0)\,\gamma_1\Big)\,\eta + \gamma_0\,(2\,\beta_0^2 + 3\,\beta_0\,\gamma_0 + \gamma_0^2)\,\eta^2\Big] \\
+ & \frac{1}{24}\,a^4(s^*)\,\eta\,\Big[-24\,\gamma_3 + 12(\beta_2\,\gamma_0 + 2\beta_1\,\gamma_1 + \gamma_1^2 + 3\,\beta_0\,\gamma_2 + 2\,\gamma_0\,\gamma_2)\,\eta \\
- & 4\,\Big(6\,\beta_0^2\,\gamma_1 + 3\,\gamma_0^2\,(\beta_1 + \gamma_1) + \beta_0\,\gamma_0\,(5\,\beta_1 + 9\,\gamma_1)\Big)\,\eta^2 + \gamma_0\,(6\,\beta_0^3 + 11\,\beta_0^2\,\gamma_0 \\
+ & 6\,\beta_0\,\gamma_0^2 + \gamma_0^3)\,\eta^3\Big] \\
+ & \frac{1}{120}\,a^5(s^*)\,\eta\,\Big[-120\,\gamma_4 + \frac{1}{\beta_0}60\,\Big(-7\,\beta_1\,\beta_2\,\gamma_0 + 4\,\beta_0^2\,\gamma_3 + \beta_0\,(7\,\beta_1\,\gamma_0 + \beta_3\,\gamma_0 \\
+ & 2\,\beta_2\,\gamma_1 + 3\,\beta_1\,\gamma_2 + 2\gamma_1\,\gamma_2 + 2\,\gamma_0\,\gamma_3)\Big)\,\eta - 20\,\Big(3\beta_1^2\,\gamma_0 + \beta_1\,(14\,\beta_0 + 9\,\gamma_0)\,\gamma_1 \\
+ & 3\,(2\beta_0 + \gamma_0)(\beta_2\,\gamma_0 + \gamma_1^2 + 2\,\beta_0\,\gamma_2 + \gamma_0\,\gamma_2)\Big)\,\eta^2 + 10\Big(12\,\beta_0^3\,\gamma_1 + \gamma_0^3(3\,\beta_1 + 2\,\gamma_1) \\
+ & \beta_0\,\gamma_0^2\,(13\,\beta_1 + 12\,\gamma_1) + \beta_0^2\,\gamma_0\,(13\,\beta_1 + 22\,\gamma_1)\Big)\,\eta^3 - \gamma_0\,\Big(24\,\beta_0^4 + 50\,\beta_0^3\,\gamma_0 \\
+ & 35\,\beta_0^2\,\gamma_0^2 + 10\,\beta_0\,\gamma_0^3 + \gamma_0^4\Big)\,\eta^4\Big] + \mathcal{O}(a^6(s^*))\Big\}\,, \qquad (20)
\end{aligned}
$$

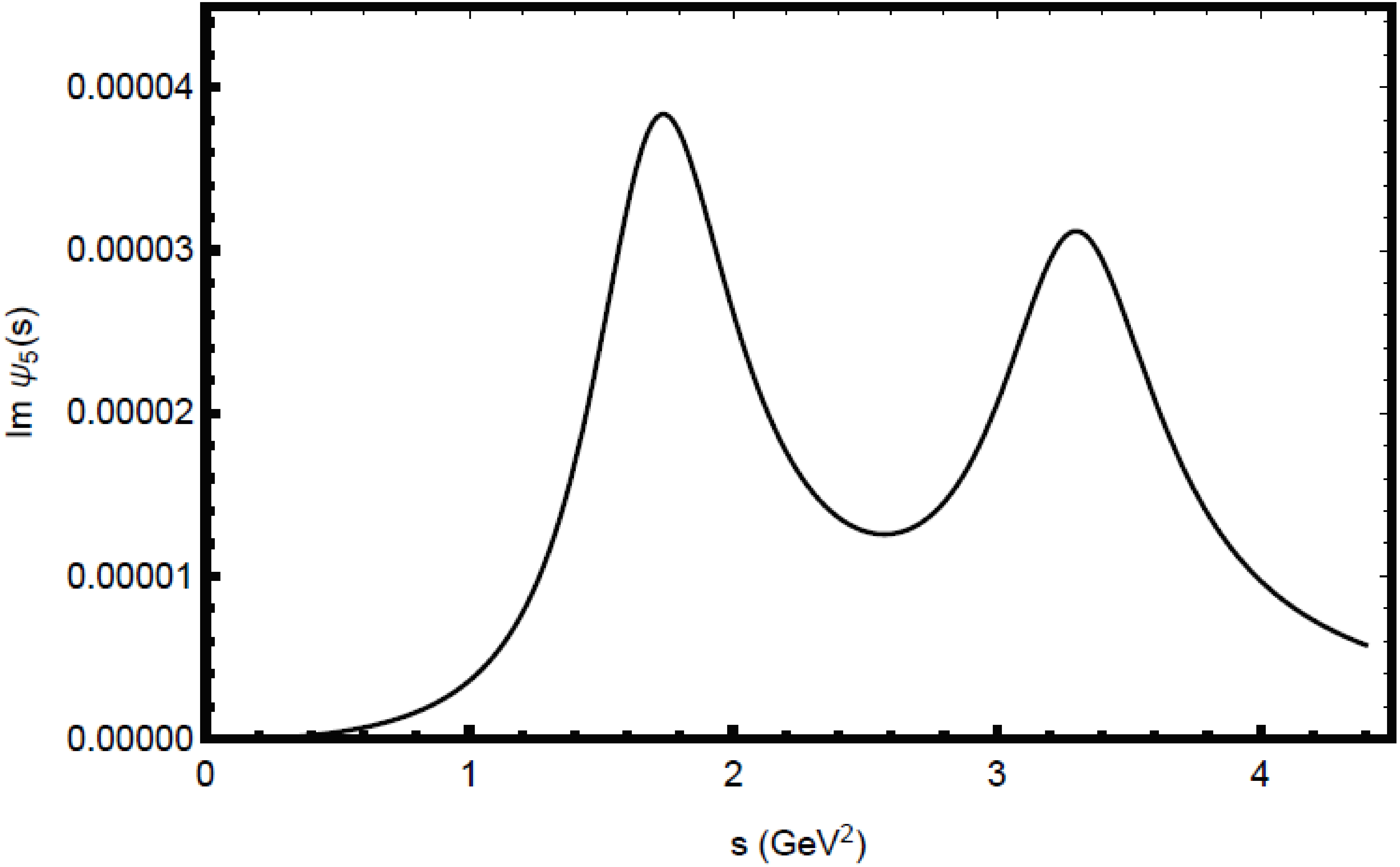
HADRONIC

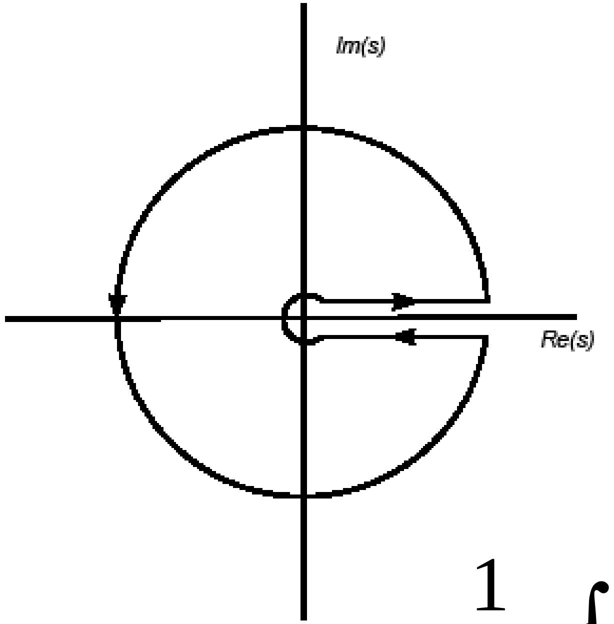
$$\Pi(q^2) = i\int d^4x\, e^{iqx} < 0|T(J(x)\, J^+(0))|0 >$$

$$J(x) \Rightarrow \partial^\mu A_\mu(x)|^i_j \propto f_{\pi/K} M^2_{\pi/K}$$

Real Spectral Function




0.00004
0.00003
0.00002
0.00001
0.00000
Im $\psi_5(s)$
0
1
2
3
4
s (GeV^2)



$$\oint_C \Pi(s)\, ds = 0$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\, \Pi(s) = \int_{s_{th}}^{s_0} ds\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s)$$

LIGHT QUARK MASSES (up, down, strange)

QCD sector: Perturbative QCD known to $O(\alpha_s^4)$ + $<\bar{q}\,q>$ & $<\alpha_s G^2>$

Hadronic sector: Pole (π, K) + (π' , K', π'' , K'') + χ-perturbation theory

CAD (1984), CAD & de Rafael (1987), Bijnens, Prades, de Rafael (1995), Maltman, Kambor (2002)

$$m_q^2 = \frac{\text{HADRONIC}}{\text{QCD}}$$ ≫ bad PQCD convergence

PADE APPROXIMANTS ↑

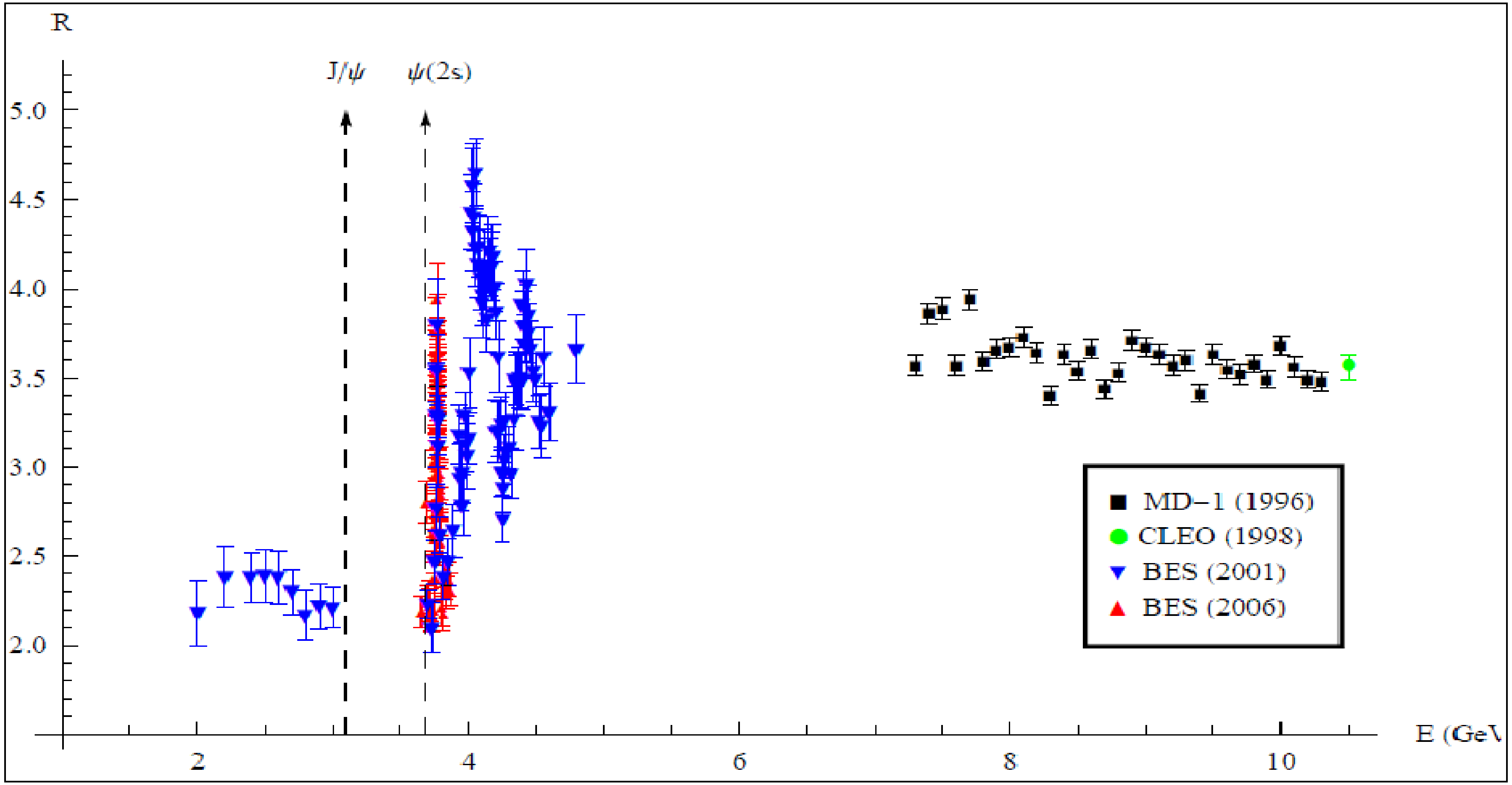
HEAVY QUARK MASSES (charm, bottom)

Vector Current Correlator

- QCD sector: Perturbative QCD known to $O(\alpha_s^3) + <\alpha_s G^2>$
- Hadronic sector: **(good)** Experimental e^+e^- data in J/ψ & ϒ regions

$e^+ e^- \to$ hadrons

charm-quark region



RESULTS

Quark	Mass (QCDSR) (MeV)	Mass (PDG/LQCD) (MeV)
up (2 GeV)	$\mathbf{2.6 \pm 0.4}$	$\mathbf{2.2 \pm 0.5}$ (PDG)
down (2 GeV)	$\mathbf{5.3 \pm 0.4}$	$\mathbf{4.7 \pm 0.5}$ (PDG)
strange	$\mathbf{94.0 \pm 9.0}$	$\mathbf{101.0 \pm 3.0}$ (PDG)
charm	$\mathbf{987 \pm 9.0}$	$\mathbf{986 \pm 6.0}$ (LQCD)
bottom (10 GeV)	$\mathbf{3623 \pm 9.0}$	$\mathbf{3617 \pm 25}$ (LQCD)

References to other recent quark-mass determinations will appear in the written version of this talk

SPRINGER BRIEFS IN PHYSICS

Cesareo A. Dominguez

Quantum Chromodynamics Sum Rules

Springer

CAPETOWN (ZA)