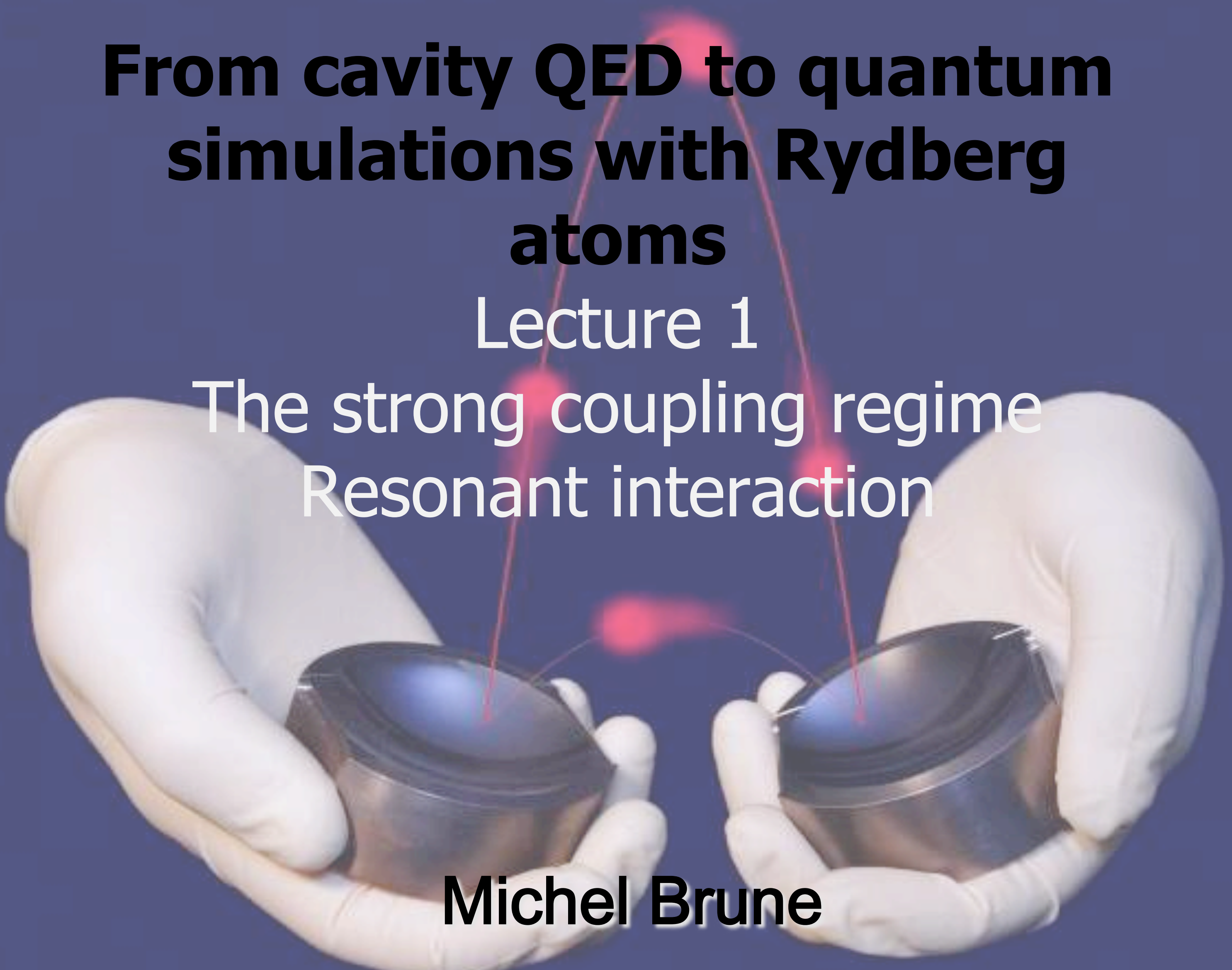
S Paulo Spt 2019



From cavity QED to quantum simulations with Rydberg atoms

Lecture 1

The strong coupling regime

Resonant interaction

Michel Brune

Laboratoire Kastler Brossel
Physique quantique et applications

École Normale Supérieure, CNRS,
Université Pierre et Marie Curie,
Collège de France, Paris

« Ridiculous » quantum phenomena

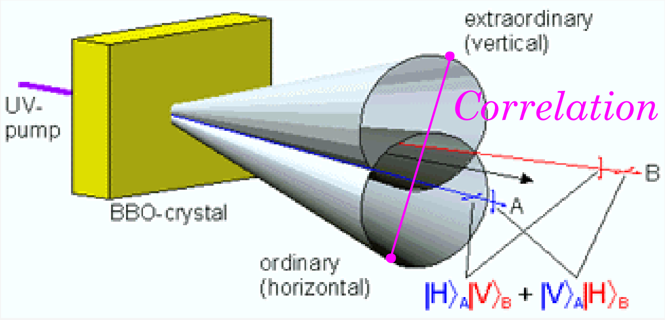
Schrödinger 1952 :

« one never experiments with just one electron, one atom or one molecule. In thought experiments we sometimes assume that we do, this invariably entails **ridiculous consequences**… »

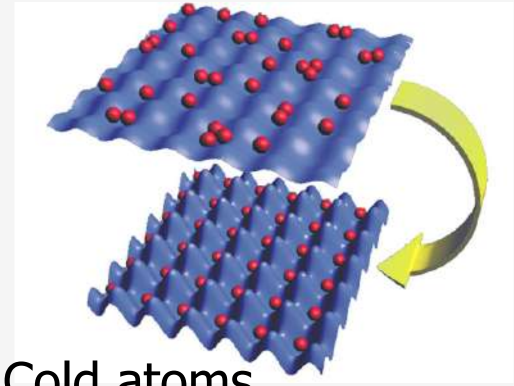
(British Journal of the Philosophy of Sciences, vol 3, 1952)

Experiment with individual quantum systems

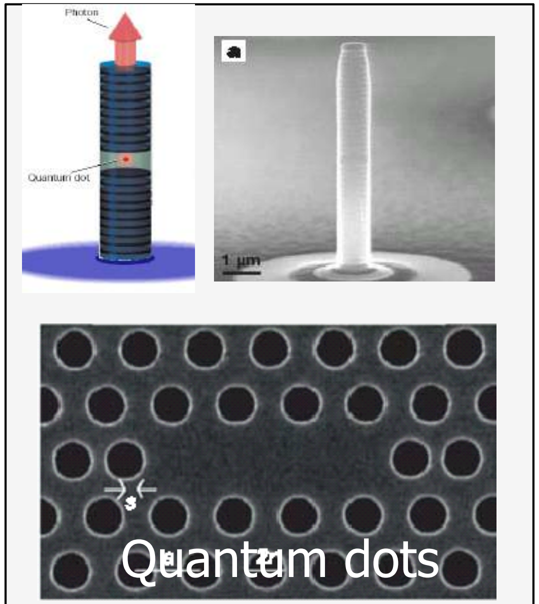
- A thriving field worldwide



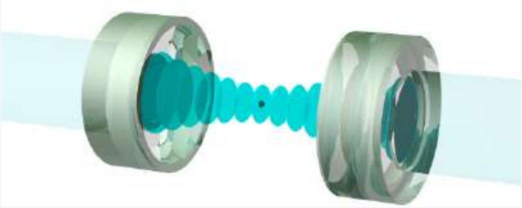
Twin photons



Cold atoms

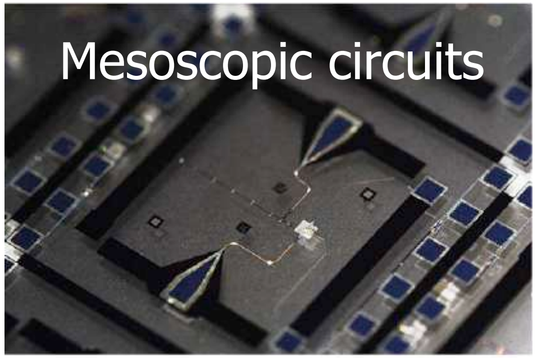


Quantum dots



Atoms and cavities

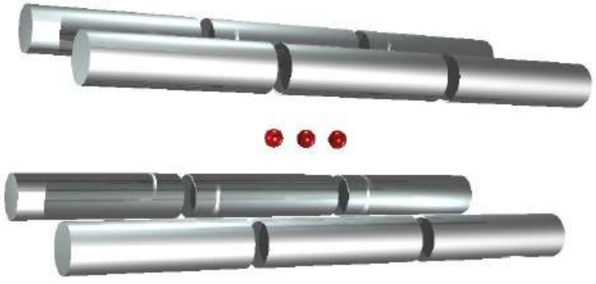
Ion traps



Mesoscopic circuits

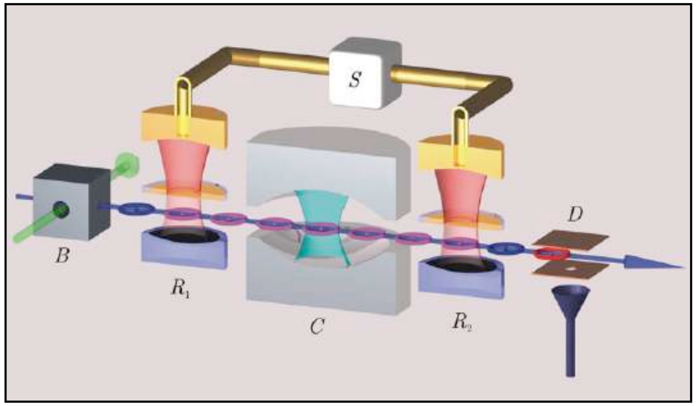
Trapped ions and photons

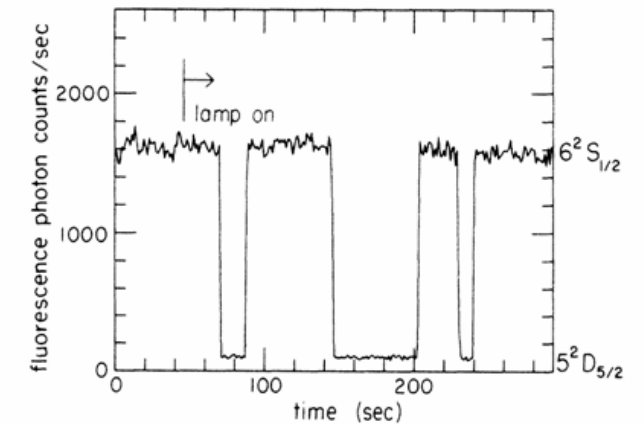
- Trapped ions



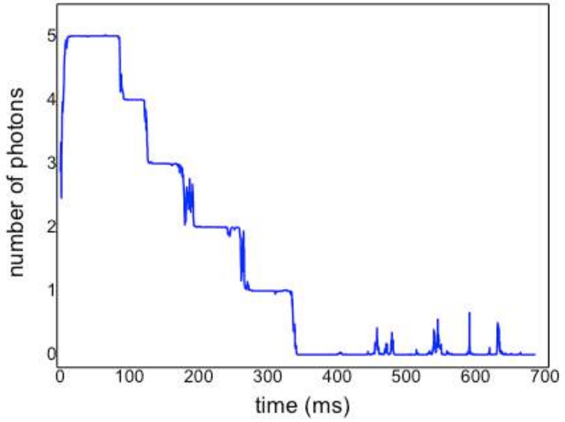
- Trapped photons

D.J. Wineland and S. Haroche





H. Dehmelt, JOSAB 1986



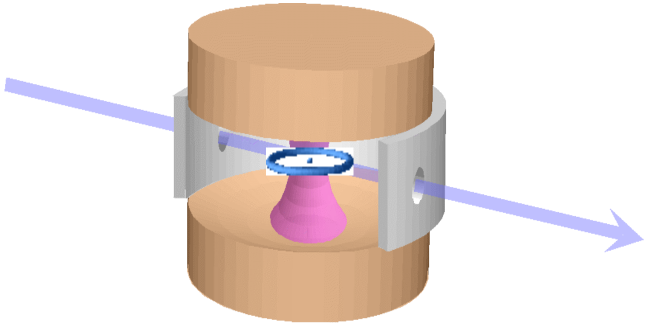
C. Gerlin et al. Nature 2007

→ Observation of quantum jumps in a single realization of an experiment

Main topic of the course

Exploring the quantum with Rydberg atoms

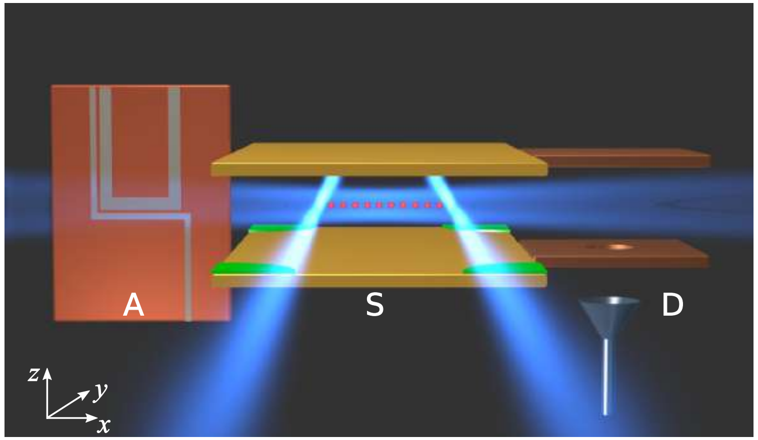
With photons and cavities



→cavity QED exploration of the fundamental aspects of quantum measurement:
- QND photon counting:
- Schrödinger cat and decoherence

→Topic of lectures 1-4

With trapped Rydberg atoms



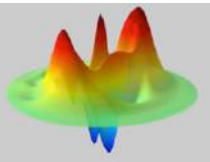
→High potential for performing quantum simulation of XXZ spin Hamiltonian

→Topic of lectures 5

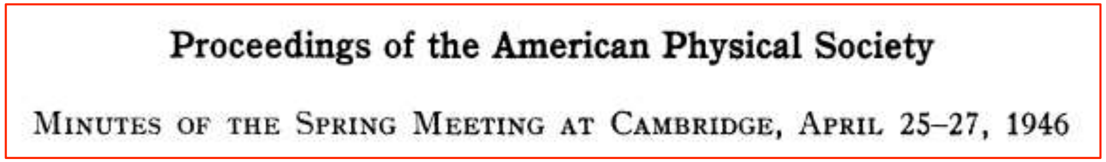
0. Cavity QED: a brief historical introduction

CQED, brief history

- 1870-1920: Effect of boundary conditions on dipole radiation (Maxwell, Hertz, Sommerfeld: Dipole radiating over conducting earth…).
- 1930: atom-metal surface interaction (London, Lennard Jones).
- **1946: Spins coupled with a tuned resonator (Purcel).**
- 1947: Vacuum fluctuations between two mirrors, Casimir effect.
- 1949: Boundary effects on Synchrotron radiation (Schwinger).
- 1954-60: Masers and Lasers: collective radiation of atoms in a cavity (Townes, Schalow…)
- 1974: Modification of molecular fluorescence near surfaces (Drexhage).
- 1983-87: Modification of spontaneous emission, experiments: ENS, MIT, Seattle, Yale, Rome…



A history of CQED: the origin

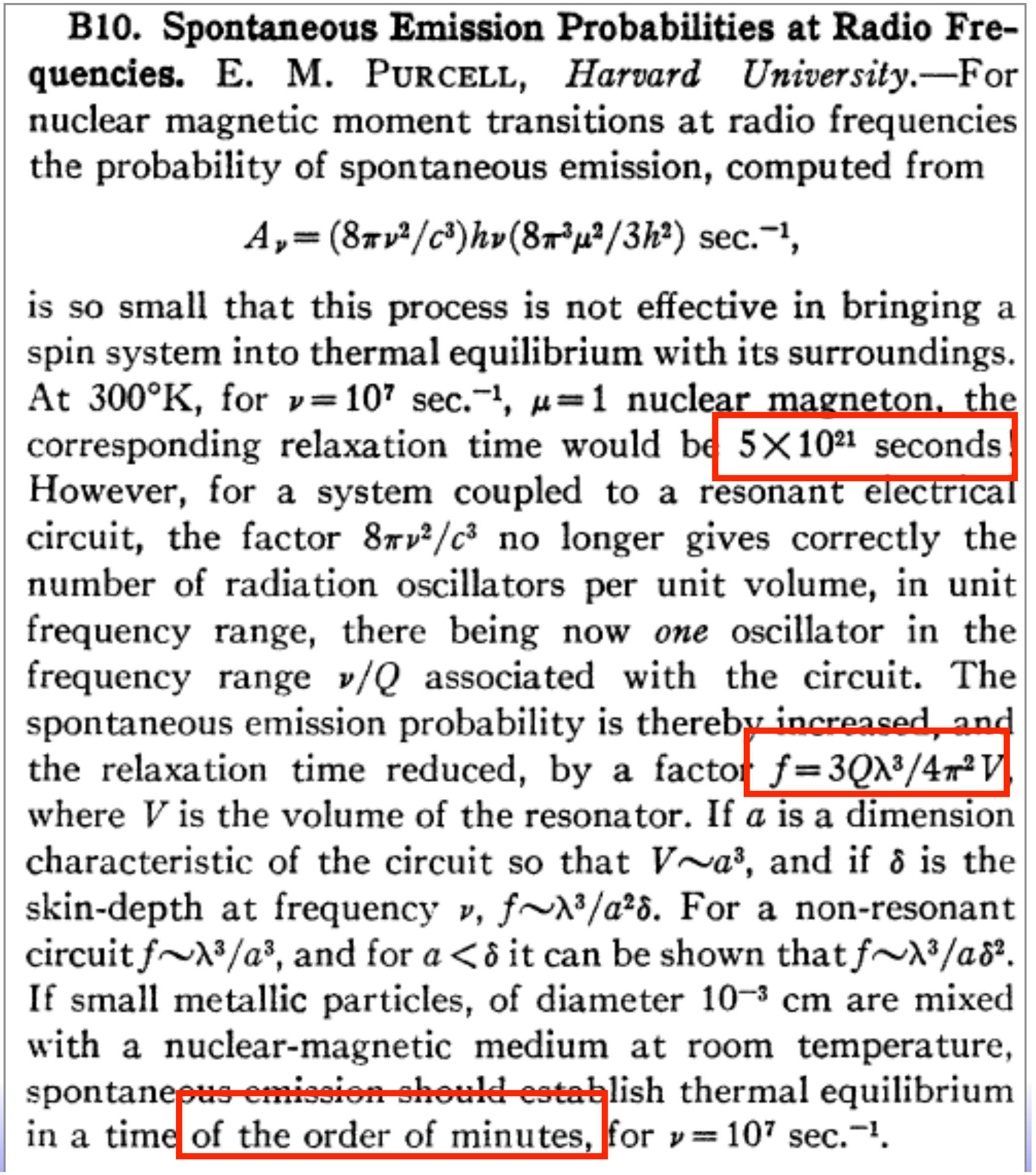


Proceedings of the American Physical Society

MINUTES OF THE SPRING MEETING AT CAMBRIDGE, APRIL 25–27, 1946

Physical Review, 69, 681,(1946)

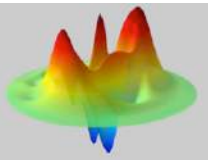
- Purcell 1946
  - ❑ spontaneous emission rate modification for a spin in a resonant circuit
  - ❑ Definition of the ‘Purcell factor’
  - ❑ Brief but seminal paper



B10. Spontaneous Emission Probabilities at Radio Frequencies. E. M. PURCELL, *Harvard University.*—For nuclear magnetic moment transitions at radio frequencies the probability of spontaneous emission, computed from

$$A_\nu = (8\pi\nu^2/c^3)h\nu(8\pi^3\mu^2/3h^2)\ \text{sec.}^{-1},$$

is so small that this process is not effective in bringing a spin system into thermal equilibrium with its surroundings. At 300°K, for $\nu = 10^7$ sec.$^{-1}$, $\mu = 1$ nuclear magneton, the corresponding relaxation time would be 5×10^{21} seconds! However, for a system coupled to a resonant electrical circuit, the factor $8\pi\nu^2/c^3$ no longer gives correctly the number of radiation oscillators per unit volume, in unit frequency range, there being now *one* oscillator in the frequency range ν/Q associated with the circuit. The spontaneous emission probability is thereby increased, and the relaxation time reduced, by a factor $f = 3Q\lambda^3/4\pi^2V$, where V is the volume of the resonator. If a is a dimension characteristic of the circuit so that $V\sim a^3$, and if δ is the skin-depth at frequency ν, $f\sim\lambda^3/a^2\delta$. For a non-resonant circuit $f\sim\lambda^3/a^3$, and for $a<\delta$ it can be shown that $f\sim\lambda^3/a\delta^2$. If small metallic particles, of diameter 10^{-3} cm are mixed with a nuclear-magnetic medium at room temperature, spontaneous emission should establish thermal equilibrium in a time of the order of minutes, for $\nu = 10^7$ sec.$^{-1}$.

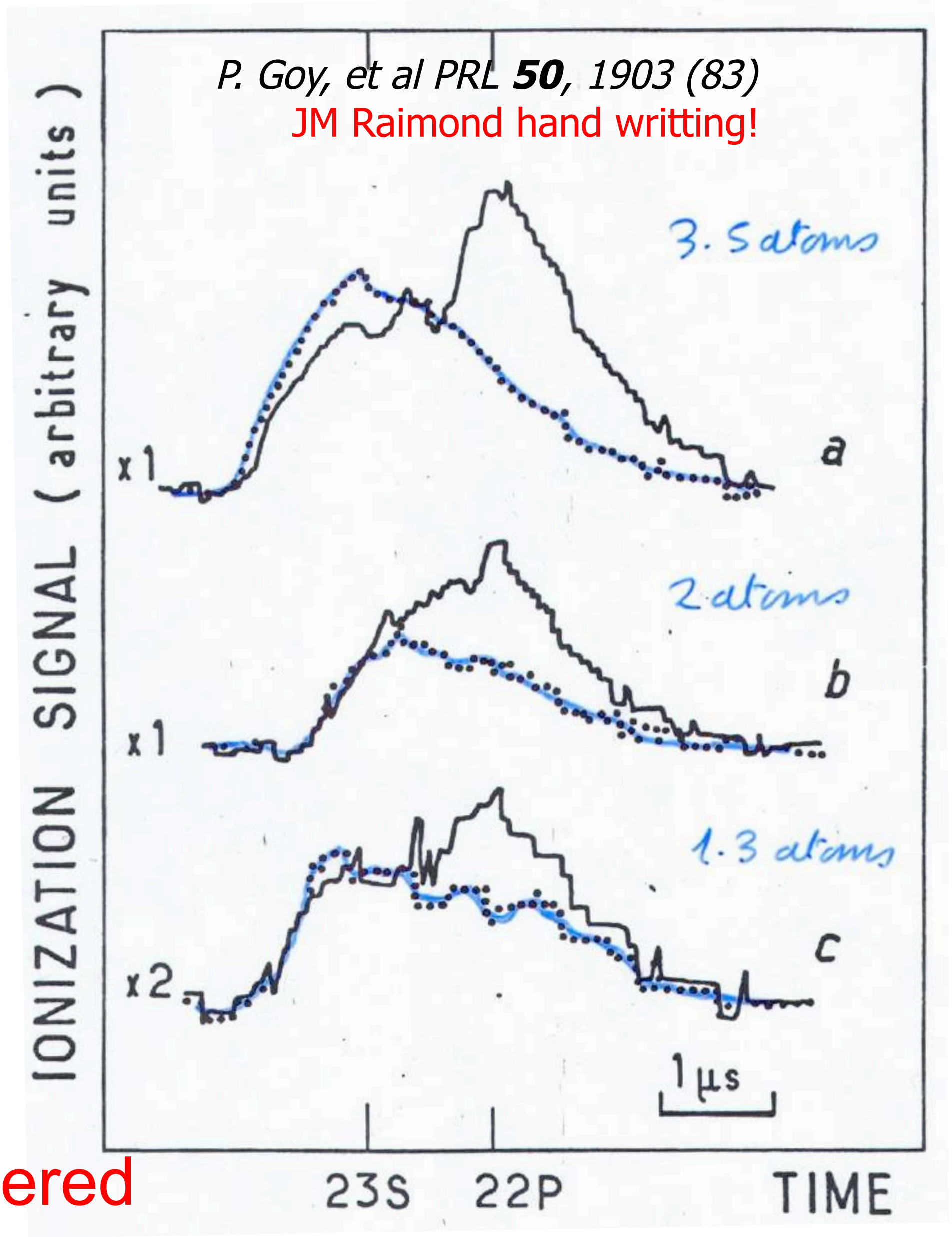


CQED, brief history

- 1870-1920: Effect of boundary conditions on dipole radiation (Maxwell, Hertz, Sommerfeld).
- 1930: atom-metal surface interaction (London, Lennard Jones).
- 1946: Spins coupled with a tuned resonator (Purcel).
- 1947: Vacuum fluctuations between two mirrors, Casimir effect.
- 1949: Boundary effects on Synchrotron radiation (Schwinger).
- 1954-60: Masers and Lasers: collective radiation of atoms in a cavity (Townes, Schalow…)
- 1974: Modification of molecular fluorescence near surfaces (Drexhage).
- **1983-87: Modification of spontaneous emission of one atom. Experiments at ENS, MIT, Seattle, Yale, Rome…**

First single-atom experiments

- Spontaneous emission enhancement (83)
  - ❑ Superconducting FP cavity
    - ➙ Q α 10^6
    - ➙ 340 GHz transition
  - ❑ Acceleration x 530
  - ❑ First experimental evidence of Purcell effect



P. Goy, et al PRL ***50****, 1903 (83)*

JM Raimond hand writting!

- Spontaneous emission inhibition
  - ❑ Gabrielse and Dehmelt (85)
  - ❑ Hulet, Hilfer and Kleppner (85)

- Spontaneous emission can be altered at will by imposing limiting conditions to the field

CQED, brief history

- 1870-1920: Effect of boundary conditions on dipole radiation (Maxwell, Hertz, Sommerfeld).
- 1930: atom-metal surface interaction (London, Lennard Jones).
- 1946: Spins coupled with a tuned resonator (Purcel).
- 1947: Vacuum fluctuations between two mirrors, Casimir effect.
- 1949: Boundary effects on Synchrotron radiation (Schwinger).
- 1954-60: Masers and Lasers: collective radiation of atoms in a cavity (Townes, Schalow…)
- 1974: Modification of molecular fluorescence near surfaces (Drexhage).
- 1983-87: Modification of spontaneous emission, experiments: ENS, MIT, Seattle, Yale, Rome…

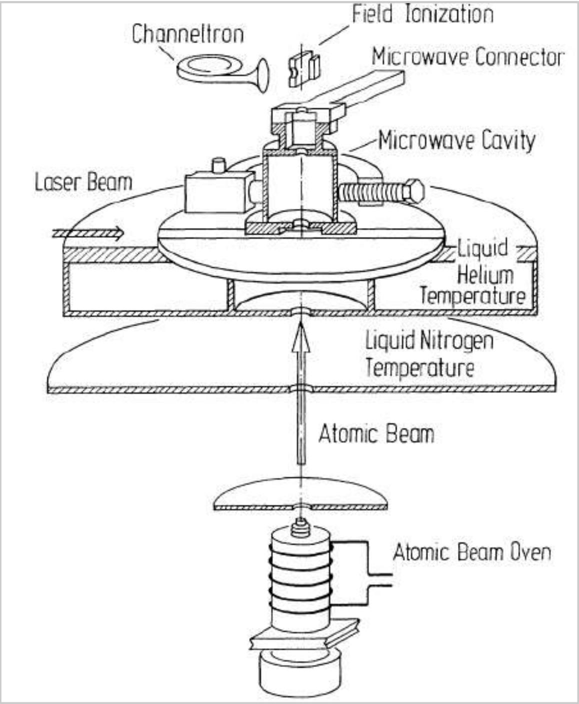
➙CQED in the PERTUBATIVE regime: Low Q cavity or effect of a single mirror: coupling strength << dissipation rates

Perturbation of atomic radiative properties which remains qualitatively the same as in free space.

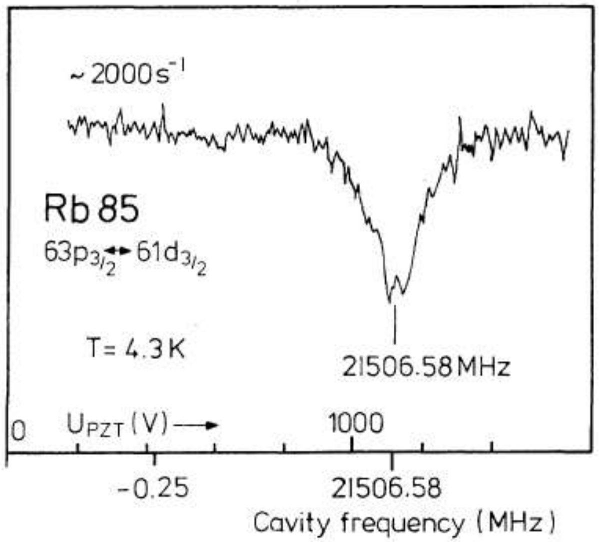
- **1985➙.. : Rydberg atoms microMaser (ENS, Munich).**

The Micromaser

- H. Walther and D. Meschede, 85
  - ❑ Cumulative emissions in the cavity in the strong coupling regime



One-Atom Maser
D. Meschede, H. Walther, and G. Müller
Phys. Rev. Lett. 54, 551 1985



- ❑ A maser with less than one atom at a time in the cavity
- ❑ Strong coupling regime
  - ➙ Single-Atom-cavity coupling overwhelms dissipation

The two regimes of cavity QED

- Weak coupling regime
  - ➙ Atom-field coupling small compared to dissipation
    - No qualitative modifications of the atomic radiative properties
      - Modification of the spontaneous emission rate
      - Modification of the atomic energies

- Strong coupling regime
  - ❑ Atom-cavity interaction overwhelms dissipative processes
    - ➙ The simplest matter-field coupling situation
      - Radical modification of the atomic radiative properties: Rabi oscillation replaces exponential decay
      - Creates and manipulates atom/field entangled state

The four time scales of CQED

- Atomic levels lifetime

$$\boxed{T_{at} = 1/\Gamma}$$

- Cavity: photon lifetime

$$\boxed{T_c = 1/\kappa}$$

- Atom-cavity coupling

$$\boxed{\Omega_0 = 2g = 1/T_{vac}}$$

- Atom-cavity interaction time

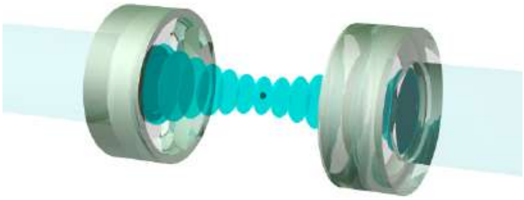
$$\boxed{T_{\text{int}}}$$

- Strong coupling conditions

$$\boxed{T_{\text{int}}\Omega_0 \approx 1; \quad T_{vac}, T_{\text{int}} << T_{at}, T_c}$$

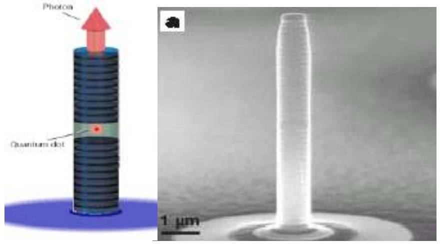
The four flavors of modern CQED

- Optical CQED
  - ❑ Ordinary atomic transitions and high finesse FP cavities



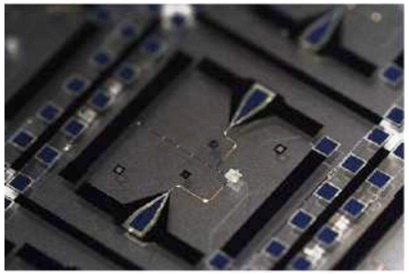
$$g \approx 50\ \text{MHz};\ \kappa \approx 100\ \text{kHz};\ \Gamma \approx 10\ \text{MHz};\ T_{\text{int}} \approx 1\text{s}$$

- Solid-state CQED
  - ❑ Quantum dots coupled to bragg mirrors/PBG



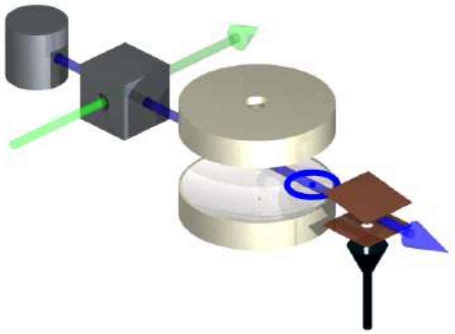
$$g \approx 10\ \text{GHz};\ \kappa \approx 1\ \text{GHz};\ \Gamma \approx\ 1\text{GHz};\ T_{\text{int}} = \infty$$

- Circuit QED
  - ❑ Solid-state qubits and stripline cavities



$$g \approx 100\ \text{MHz};\ \Gamma \ll \kappa \approx 0.1\ \text{MHz};\ T_{\text{int}} = \infty$$

- Microwave CQED
  - ❑ (Circular) Rydberg atoms and superconducting cavities



$$g \approx 10\ \text{kHz};\ \kappa \approx 1\ \text{Hz};\ \Gamma \approx 30\ \text{Hz};\ T_{\text{int}} \approx 400\ \mu s$$

Outline of the course

Topic of lectures 1-4:

CQED with Rydberg atoms

- Lecture 1: the strong coupling regime
- Lecture 2-3: Quantum Non Demolition photon counting

Quantum jumps, quantum feedback, past quantum state approach

- Lecture 4: Quantum measurement Schrödinger cat and decoherence

State reconstruction

Lecture 5:

Toward a circular Rydberg atom quantum simulator
of XXZ spin Hamiltonian

Outline of Course 1

1. One atom, one mode, the Jaynes-Cummings model

2. Rydberg atoms in a cavity: the tools achieving the strong coupling regime

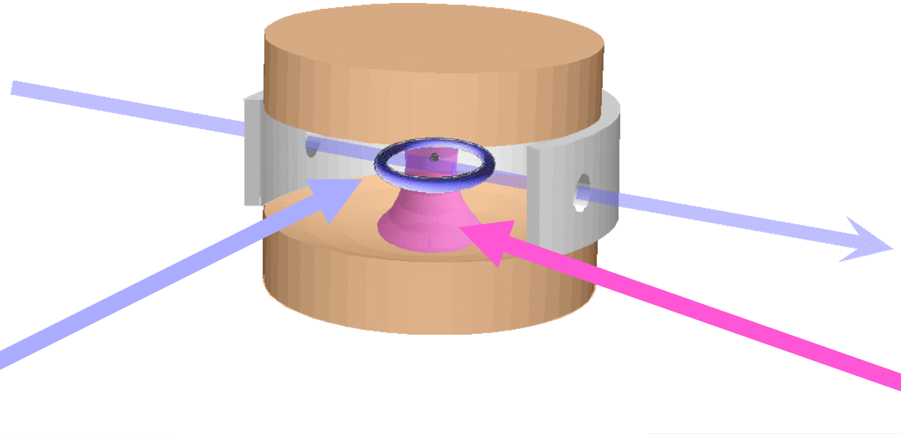
- The experimental setup
- Vacuum Rabi oscillations

3. Rabi oscillation in a small coherent field

- Direct observation of field graininess
- Preparation of a "44 photons" Schrödinger cat state

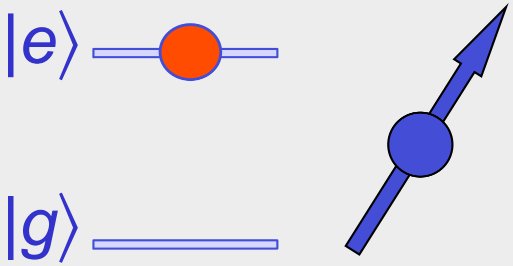
1. One atom, one mode, the Jaynes-Cummings model

Cavity QED: spin and spring



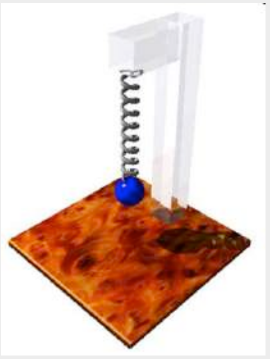
A cavity QED experiment

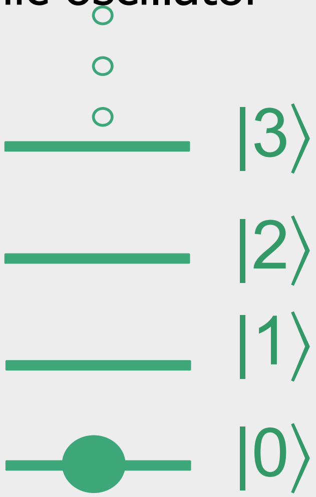
The SPIN:
One atom, two levels



Electric dipole coupling Ω_0

The SPRING:
One high Q cavity mode as an harmonic oscillator





➔ Nearly ideal realization of a simple generic system

Field quantization in a cavity

- Same procedure as in free space:

1- Find the classical eigenmodes of the resonator satisfying the boundary conditions.

→ Classical electric field: $\boxed{\vec{E}_\alpha(\vec{r},t)=E_\omega.\vec{f}_\alpha(\vec{r}).e^{i\omega t}+cc}$

2- Each mode is quantized as an harmonic oscillator.

→ Electric field operator: $\boxed{\hat{\vec{E}}_\alpha(\vec{r},t)=E_\omega.\left(\vec{f}_\alpha(\vec{r}).\hat{a}_\alpha+\vec{f}^{\,*}_\alpha(\vec{r}).\hat{a}^+_\alpha\right)}$ $\boxed{\left[a,a^+\right]=1}$

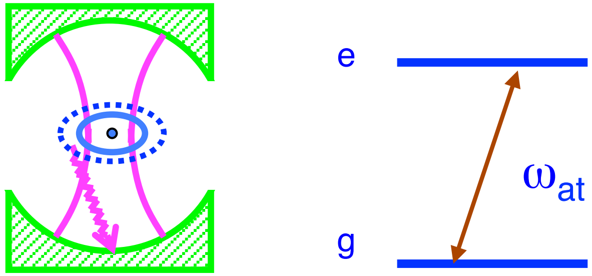
Where:

- $E_\omega=\sqrt{\dfrac{\hbar\omega}{2\varepsilon_0 V_{cav}}}$ "vacuum electric field".
- $V_{cav}=\int\limits_{Cavity}\left|\vec{f}_\alpha(\vec{r})\right|^2 d^3\vec{r}$ volume of the mode. V_{cav} is really a physical volume.
- $\vec{f}_\alpha(\vec{r})$ complex function of Normalization: $Max\left|\vec{f}_\alpha(\vec{r})\right|=1$

  (real functions will be enough for us)

- Here the quantized object is a **collective excitation of the field and all the electric charges** at the surface of the mirror.
- We now consider a single mode and drop the index α.

The Jaynes Cummings model:



+ a single two level atom, frequency ω_{at}
+ a single field mode, frequency ω_c
+ dipole coupling
+ negligible damping

- Atom-field Hamiltonian: $H = H_{at}+H_{cav}+V_{at\text{-}cav}$

$$\boxed{\begin{aligned} H_{at} &= \frac{\hbar\omega_{at}}{2}\left[|e\rangle\langle e| - |g\rangle\langle g|\right] \\ H_{cav} &= \hbar\omega_c\left[a^+a + 1/2\right] \\ V_{at-cav} &= -\vec{d}.\hat{\vec{E}}(\vec{r}) \\ \vec{d} &= \vec{d}_{eg}\left[|e\rangle\langle g| + |g\rangle\langle e|\right] \end{aligned}}$$

Condition of validity:

- ω_c close to a single atomic transition: $\boxed{|\delta| = |\omega_c - \omega_{at}| << \omega_c, \omega_{at}}$

- small cavity: only one mode close to resonance $\boxed{FSR >> \delta}$

The Jaynes Cummings hamiltonian

- Rotating wave approximation (RWA):

$$V_{at-cav} = \hbar\Omega(\vec{r})/2\left[a|e\rangle\langle g| + a|g\rangle\langle e| + a^{+}|g\rangle\langle e| + a^{+}|e\rangle\langle g|\right]$$

Non-resonant terms are neglected

$$\boxed{V_{at-cav} \approx \hbar\Omega(\vec{r})/2\left[a|e\rangle\langle g| + a^{+}|g\rangle\langle e|\right]}$$

Jaynes-Cummings
Hamiltonian

- Vacuum Rabi frequency:

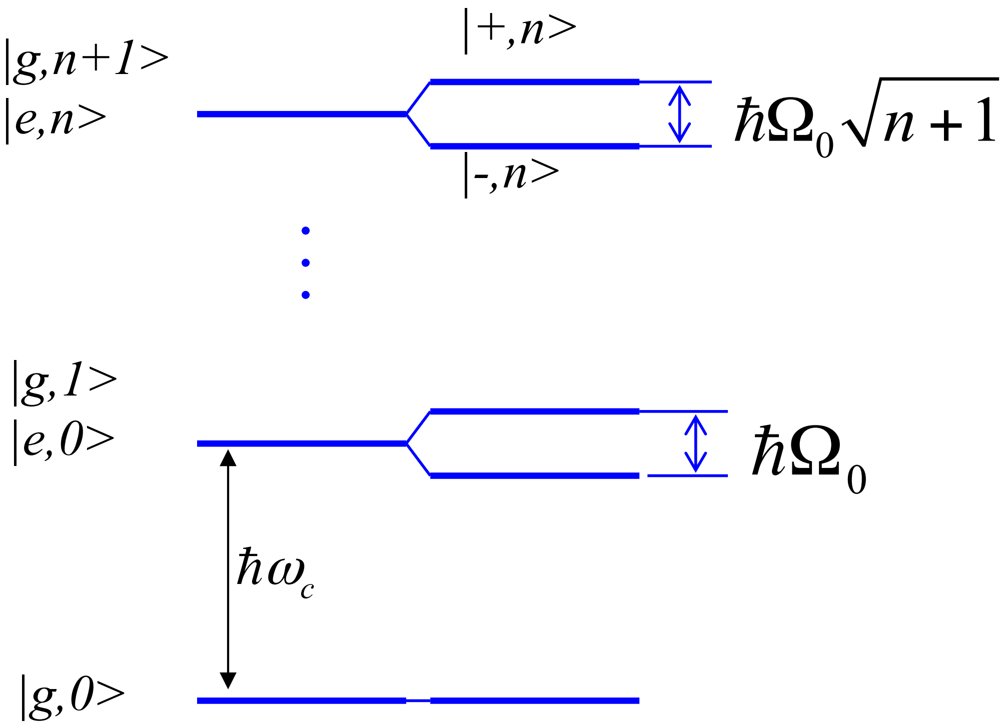
$$\boxed{\Omega(\vec{r}) = -2d_{eg}.\vec{f}(\vec{r}).E_{\omega} = \Omega_0.\left|\vec{f}(\vec{r})\right|}$$

$$\boxed{\Omega_0 = 2d_{eg}.\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{cav}}}}$$

- Validity of RWA: $\boxed{\Omega_0 \ll \omega_{at}, \omega_c}$

Dressed energy levels at resonance $(\omega_{at}=\omega_c)$

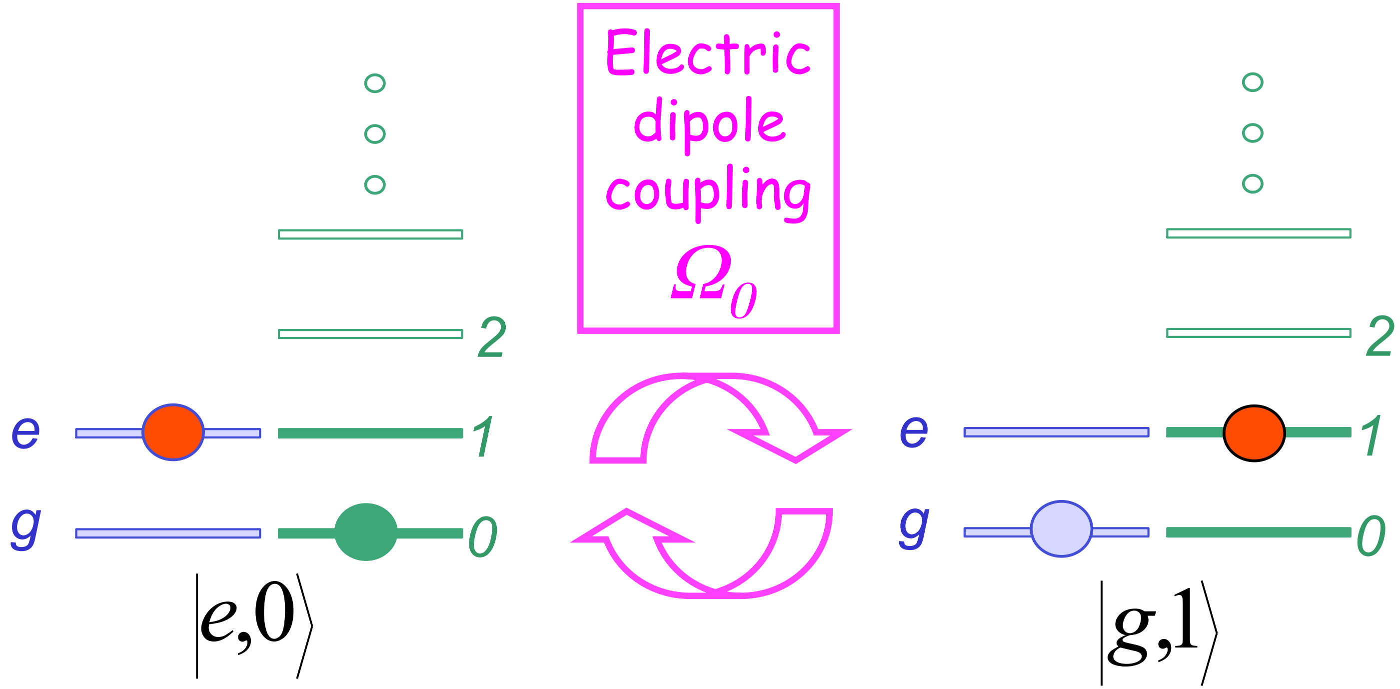
- Eigenvalues: $\boxed{E_{\pm n} = \hbar\omega_c(n+1/2) + \hbar\omega_{at} \pm \hbar\Omega_0/2\sqrt{n+1}}$
- Eigenstates: $\boxed{\left|\pm n\right\rangle = 1/\sqrt{2}\,\left[\left|e,n\right\rangle \pm \left|g,n+1\right\rangle\right]}$



- Levels just couple by pairs (except the ground state)

- level splitting scales as the Field amplitude

Resonant atom-field coupling: dynamic point of view



Electric dipole coupling Ω_0

$|e,0\rangle$

$|g,1\rangle$

$\Omega_0/2\pi = 50\ kHz$
$T_{\text{rabi}} = 20\ \mu s$

$$\boxed{|e,0\rangle \rightarrow \cos\left(\frac{\Omega_0 t}{2}\right)\cdot|e,0\rangle - i\sin\left(\frac{\Omega_0 t}{2}\right)\cdot|g,1\rangle}$$

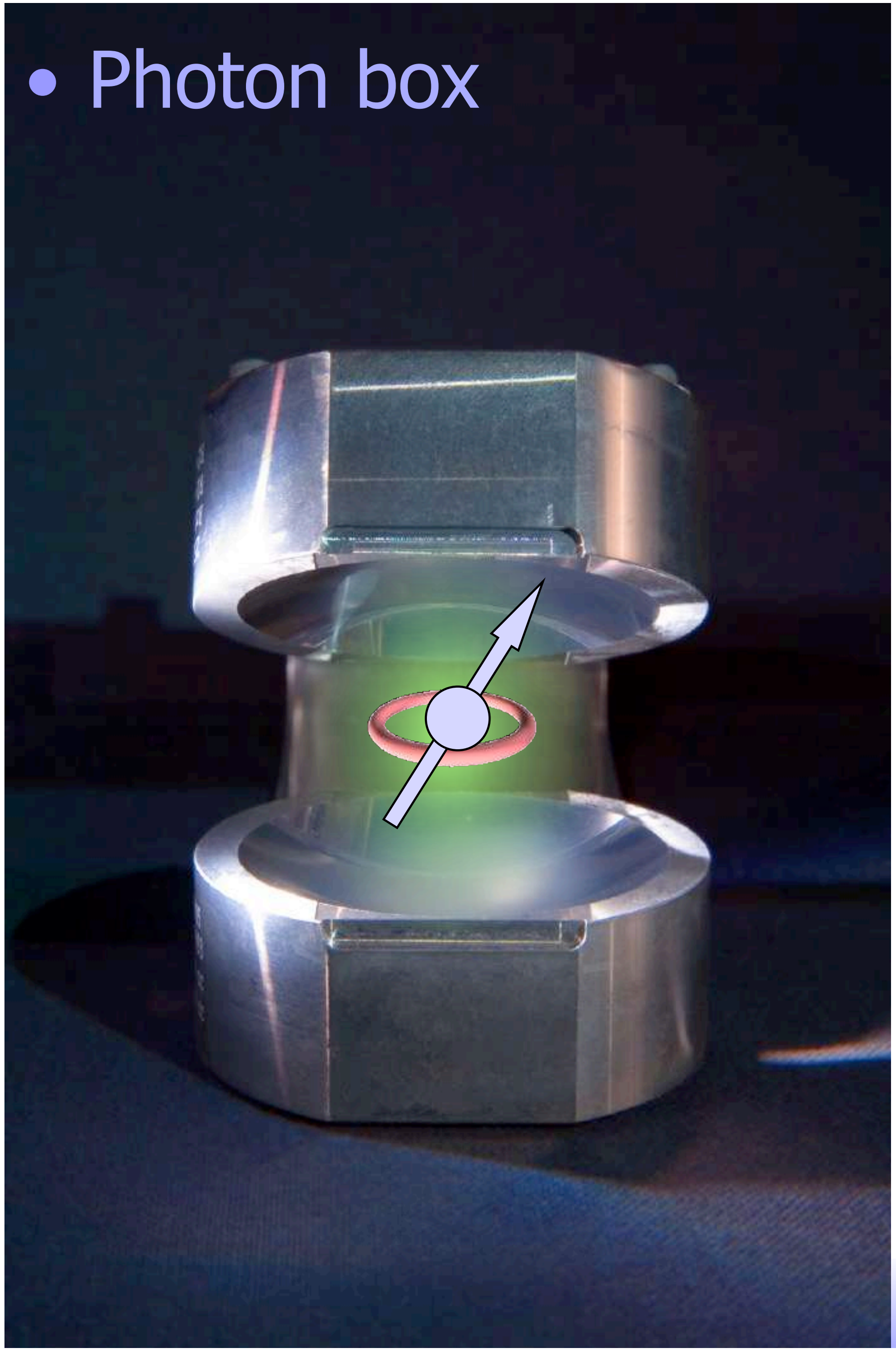
→ Coherent Rabi oscillation

2. Rydberg atoms in a cavity: achieving the strong coupling regime

One photon and one atom in a box:

- Photon box: superconducting microwave cavity
- “circular" Rydberg atoms

Microwave Rydberg atom CQED



Two essential ingredients:

- Photon trap:

the "spring"

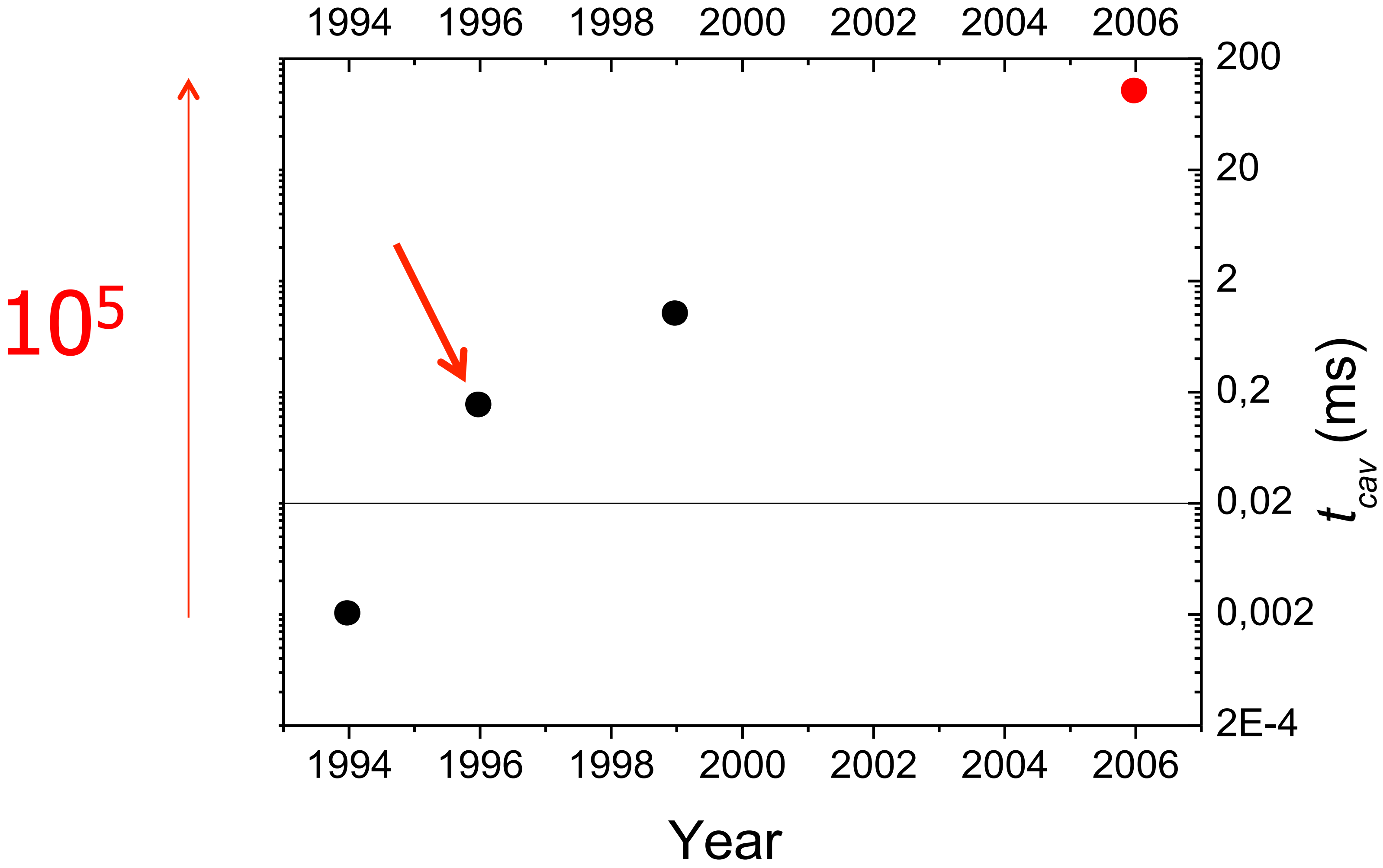
- Photon probe:

the spin

Single Rydberg atoms

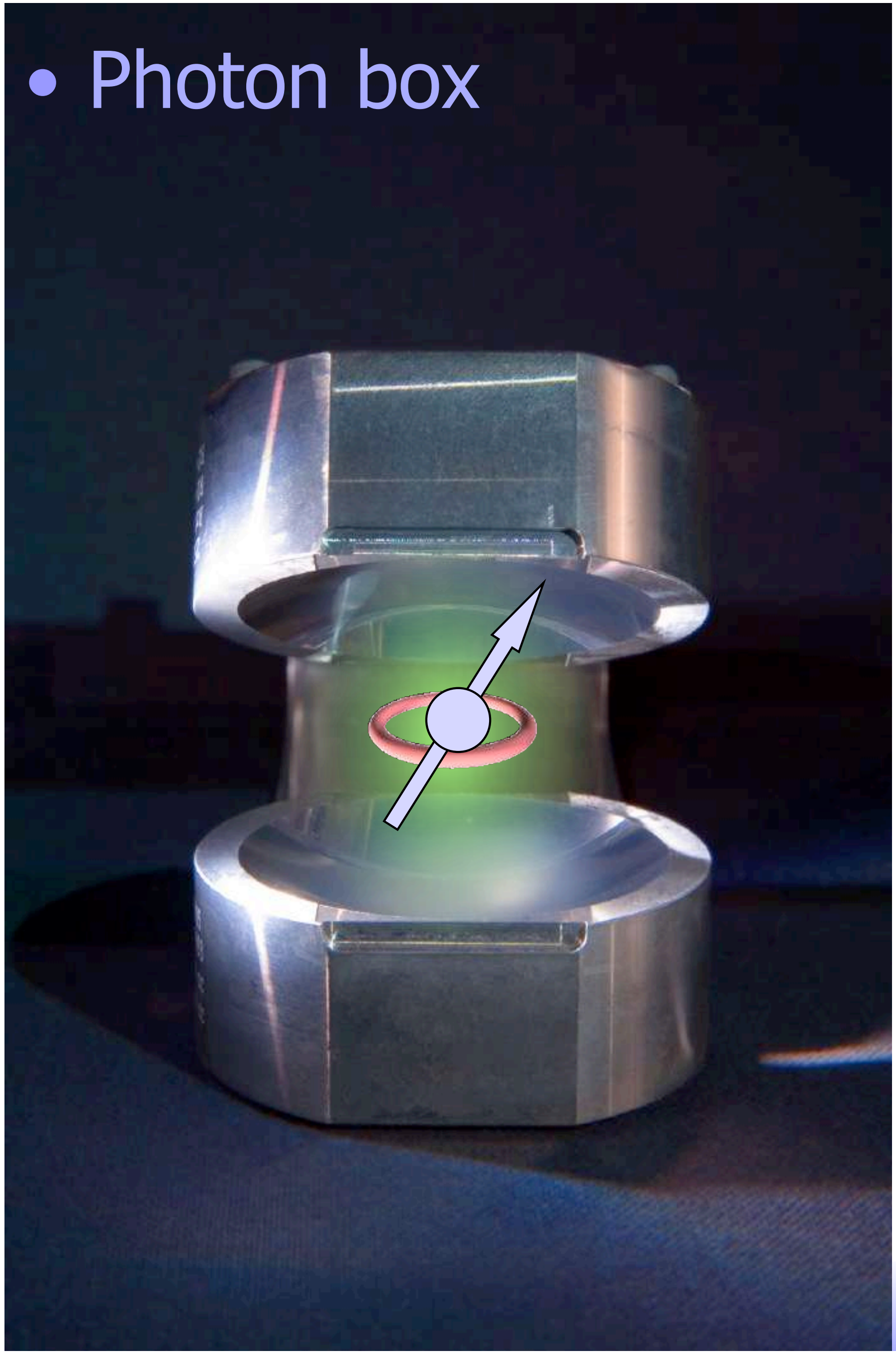
Superconducting cavity technology

- Our version of Moore’s law:



The “Spin”: Circular Rydberg atoms

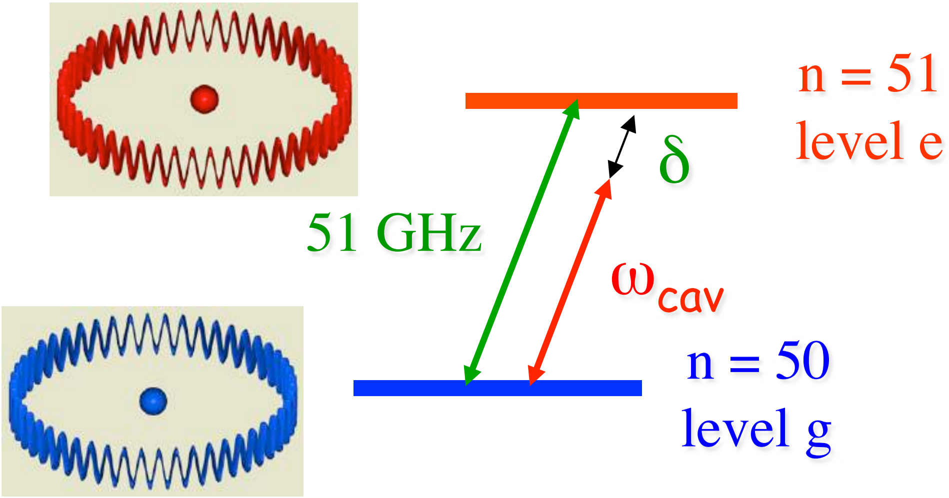
- Photon box



- Photon probes

Circular Rydberg atoms:

$l=|m|=n-1$

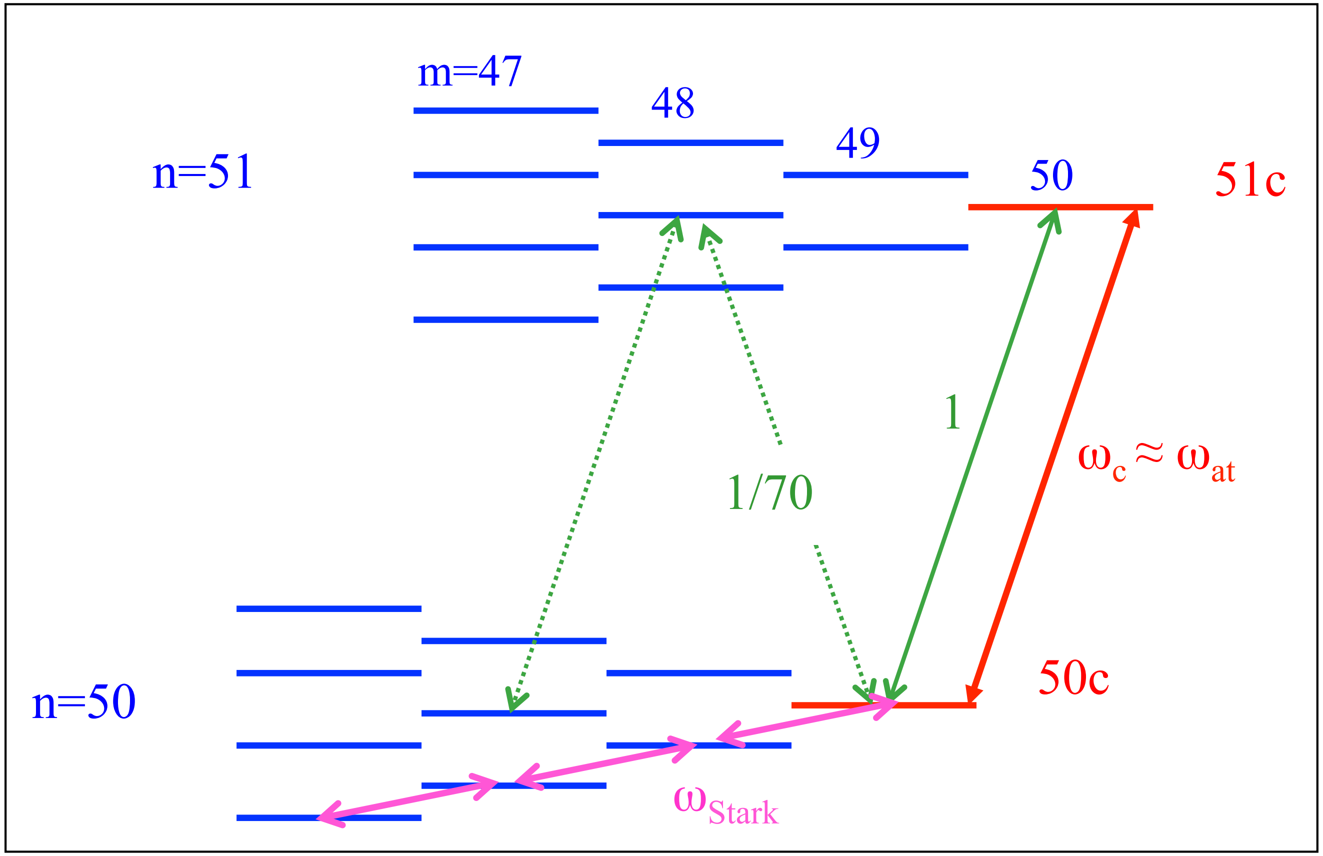


- Large dipole 1500 au
- Long lifetime: 30ms
- detected une by one

"Circular" atoms as two level atoms

- Stark diagram of Rydberg levels:

Good quantum number: m

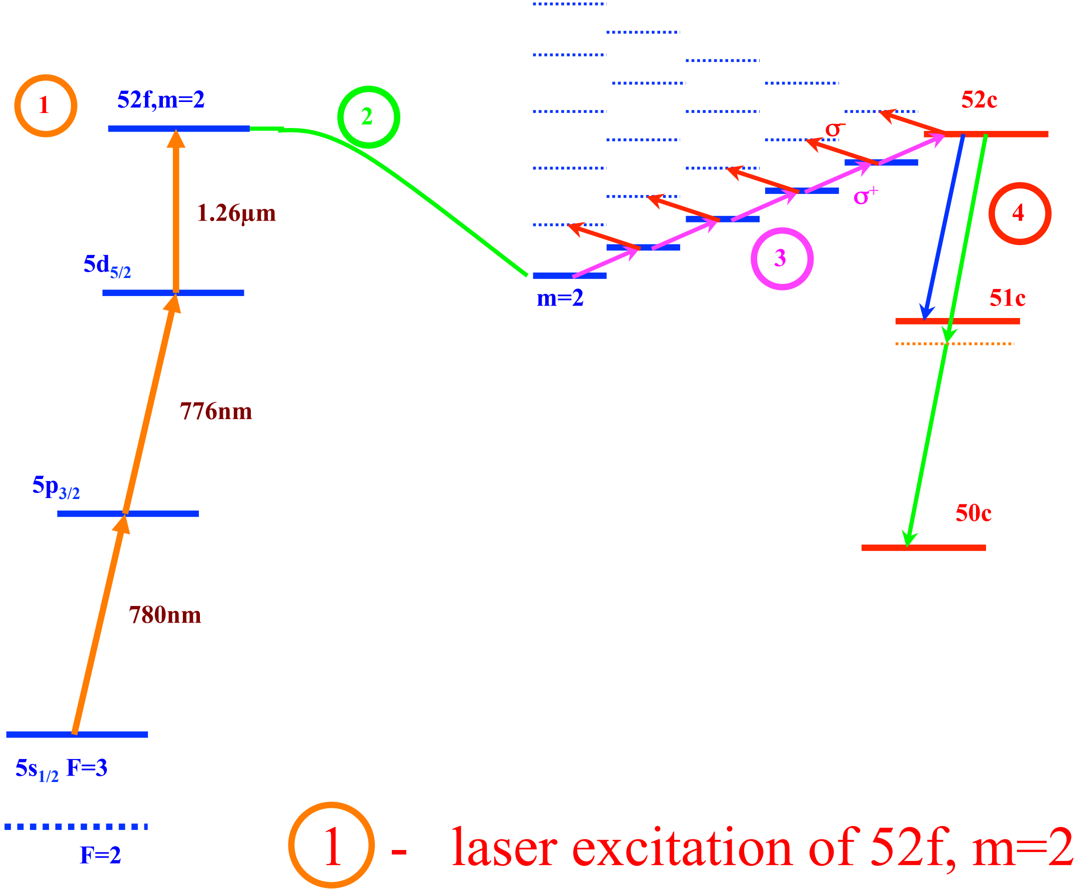


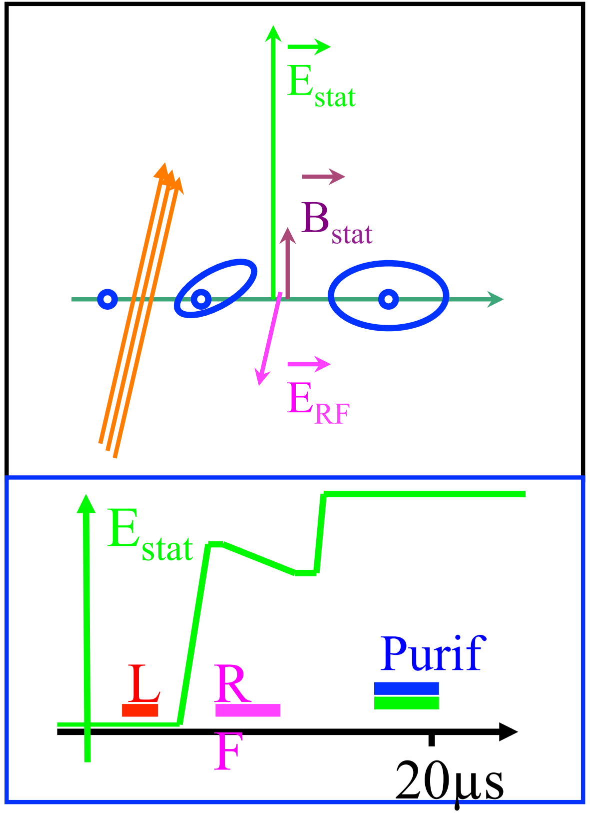
The electric field
removes
the degeneracy
between transitions:

good isolation
of the 51c to 50c
transition

- Linear Stark effect: $\omega_{Stark}/2\pi = 100$ *MHz/(V/cm)*
- Quadratic Stark shift of the 51c-50c transition: 255 *kHz/(V/cm)*^{2}
used for fast tuning of the atom in or out of resonance

Preparation of circular atoms





^{85}Rb

① - laser excitation of 52f, m=2.

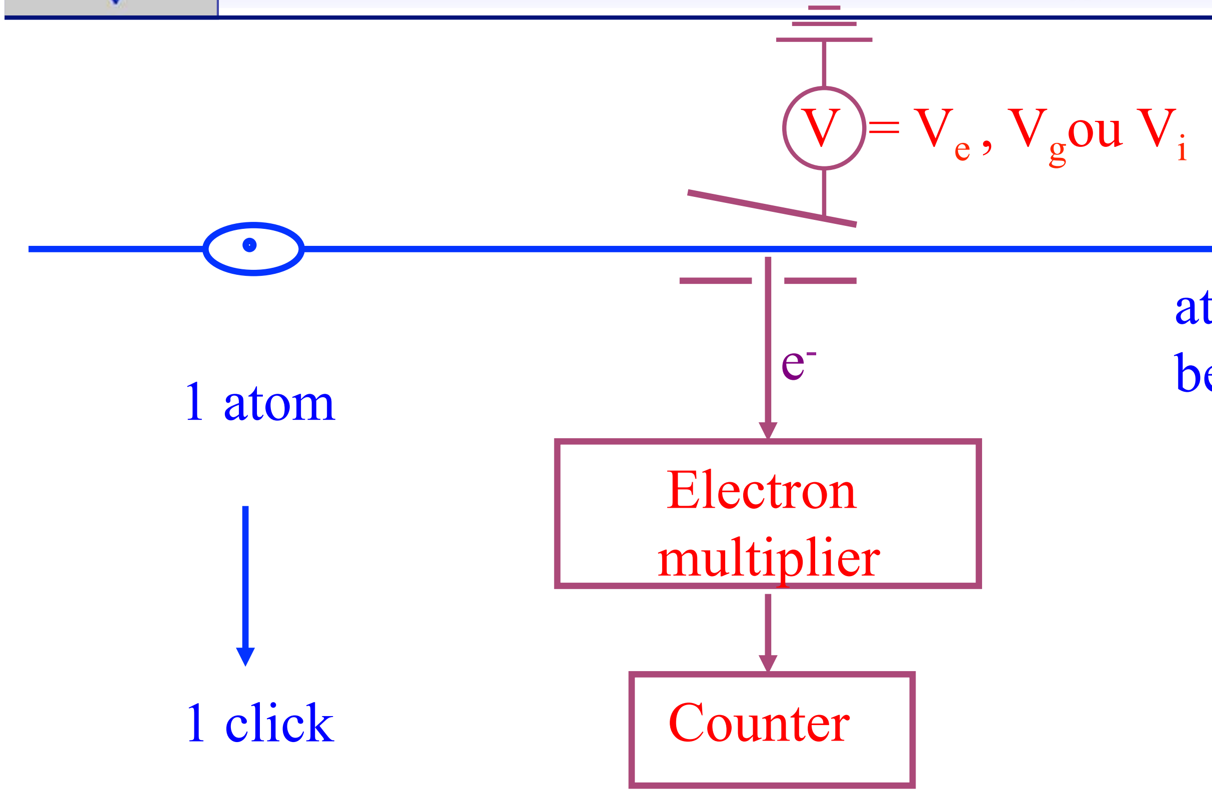
② - "Stark switching": E_{stat}=0➔2.5V/cm.

③ - 49 photons adiabatic transfer to 52c in duced by E_{RF}(ν=250MHz). B_{stat} removes degeneracy betwen σ^+ and σ^-.

④ - "Purification": selective transfer to 51c or 50c.

➔ 300 circular atom/laser pulse. Purity > 99%

Detection of Rydberg atoms (1)



V = V_e, V_g ou V_i

e^-

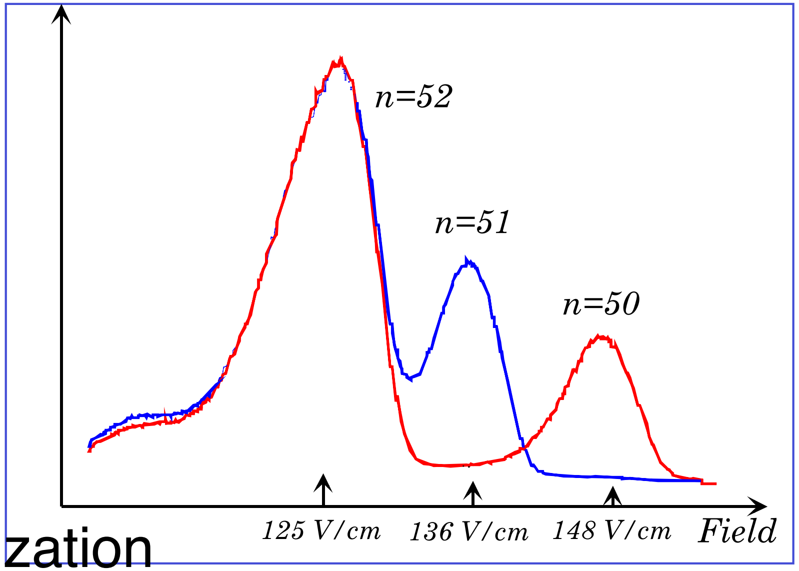
atomic beam

1 atom

1 click

Electron multiplier

Counter

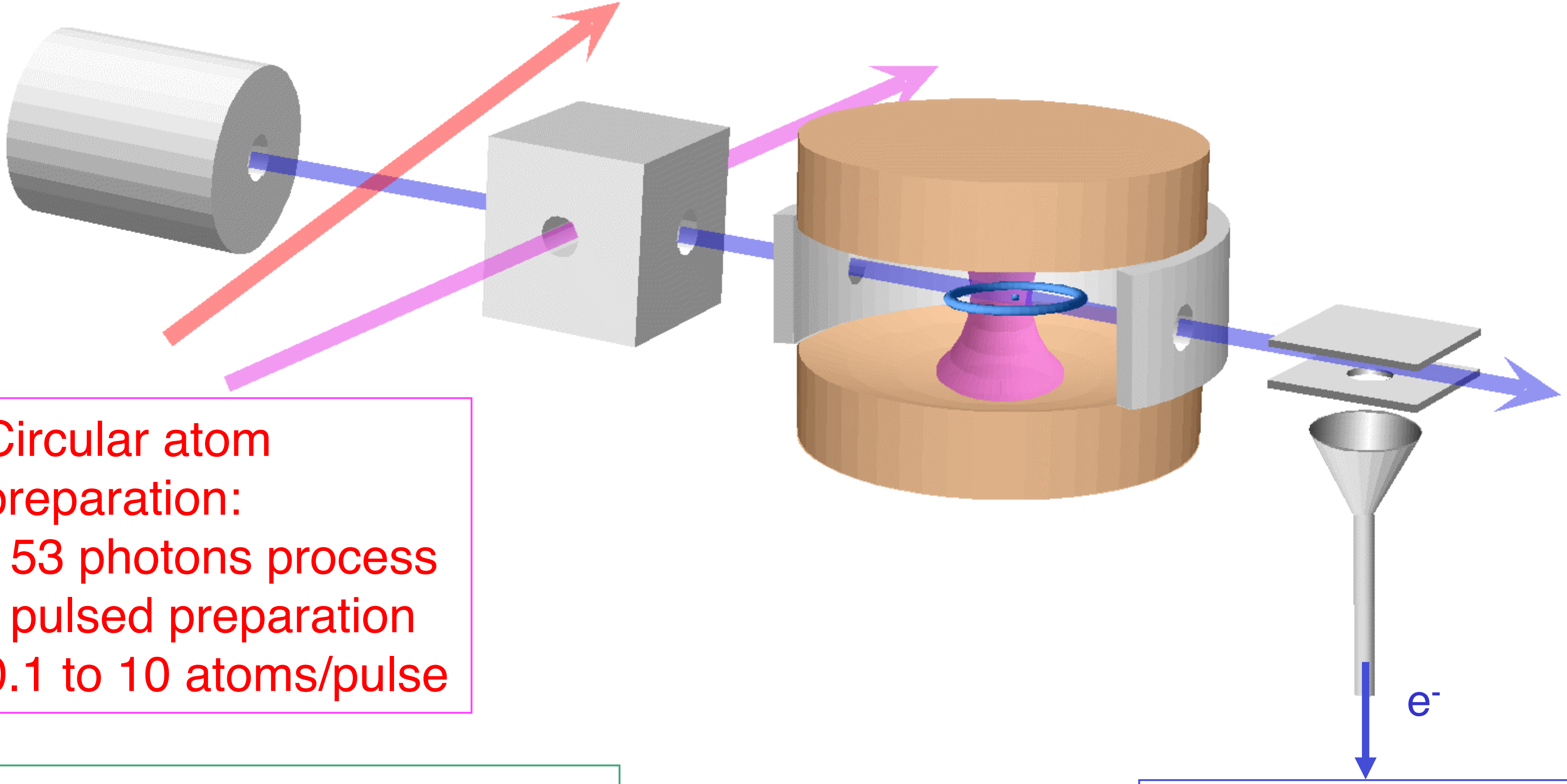


- atoms detected one by one by selective ionization in an electric field

→ measurement of internal energy state of the atom after interaction with C

CW detection in a field gradient: efficiency 80-40%

Experimental set-up



^{85}Rb

Laser velocity selection

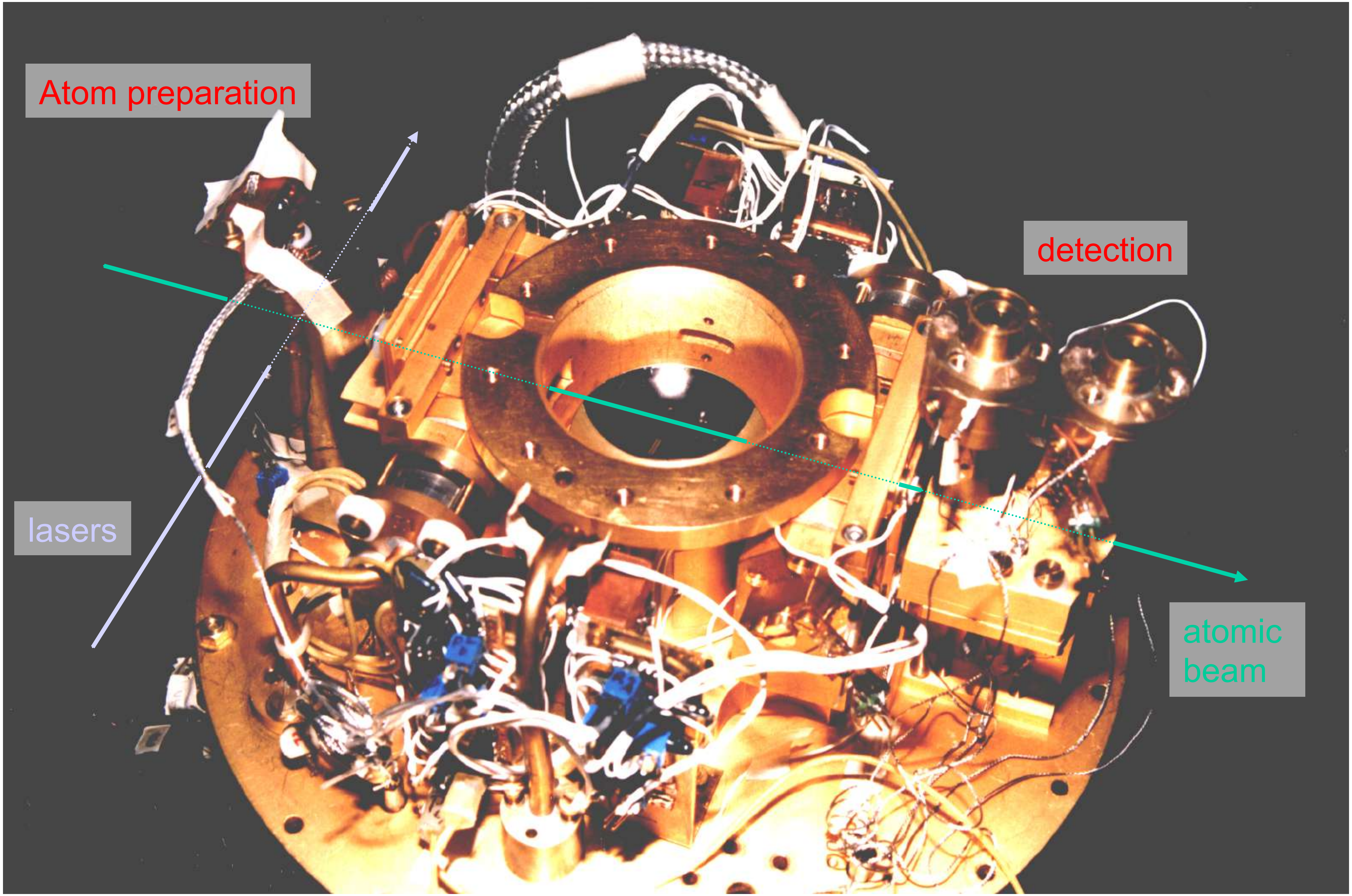
Circular atom preparation:
- 53 photons process
- pulsed preparation

0.1 to 10 atoms/pulse

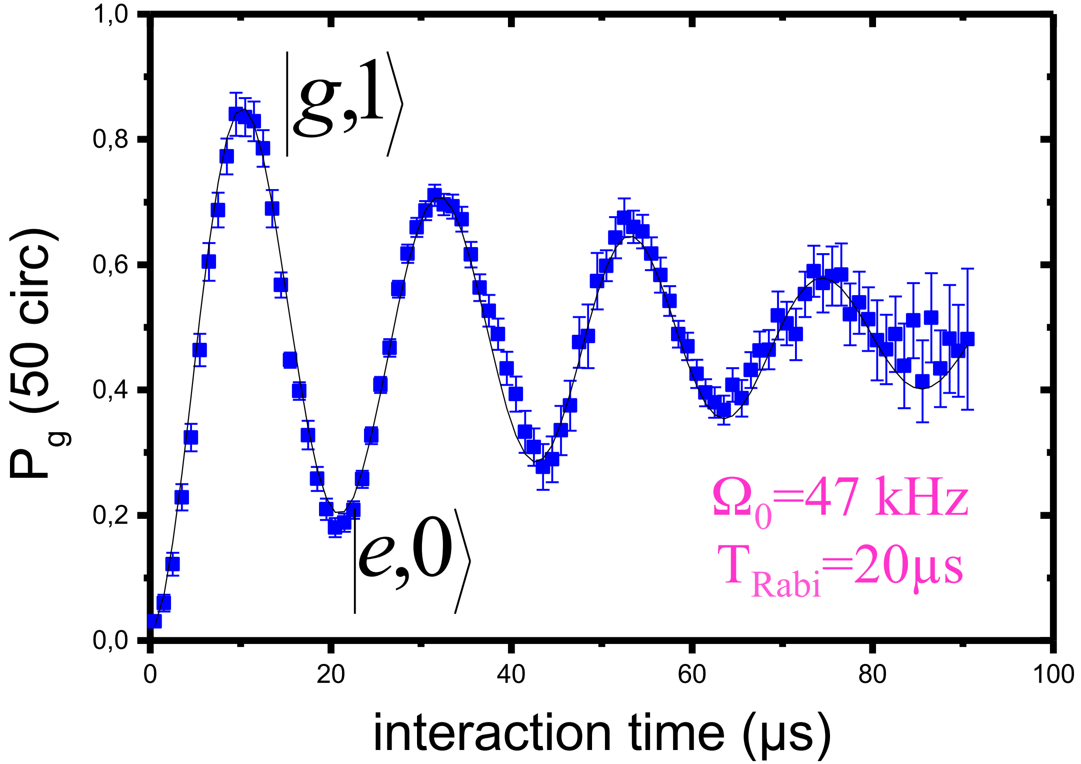
Cryogenic environment
T=0.6 to 1.3 K
→weak blackbody radiation

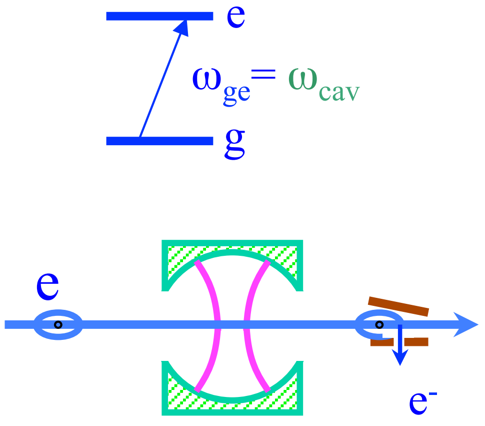
State selective detector
One atom = one click

Experimental setup



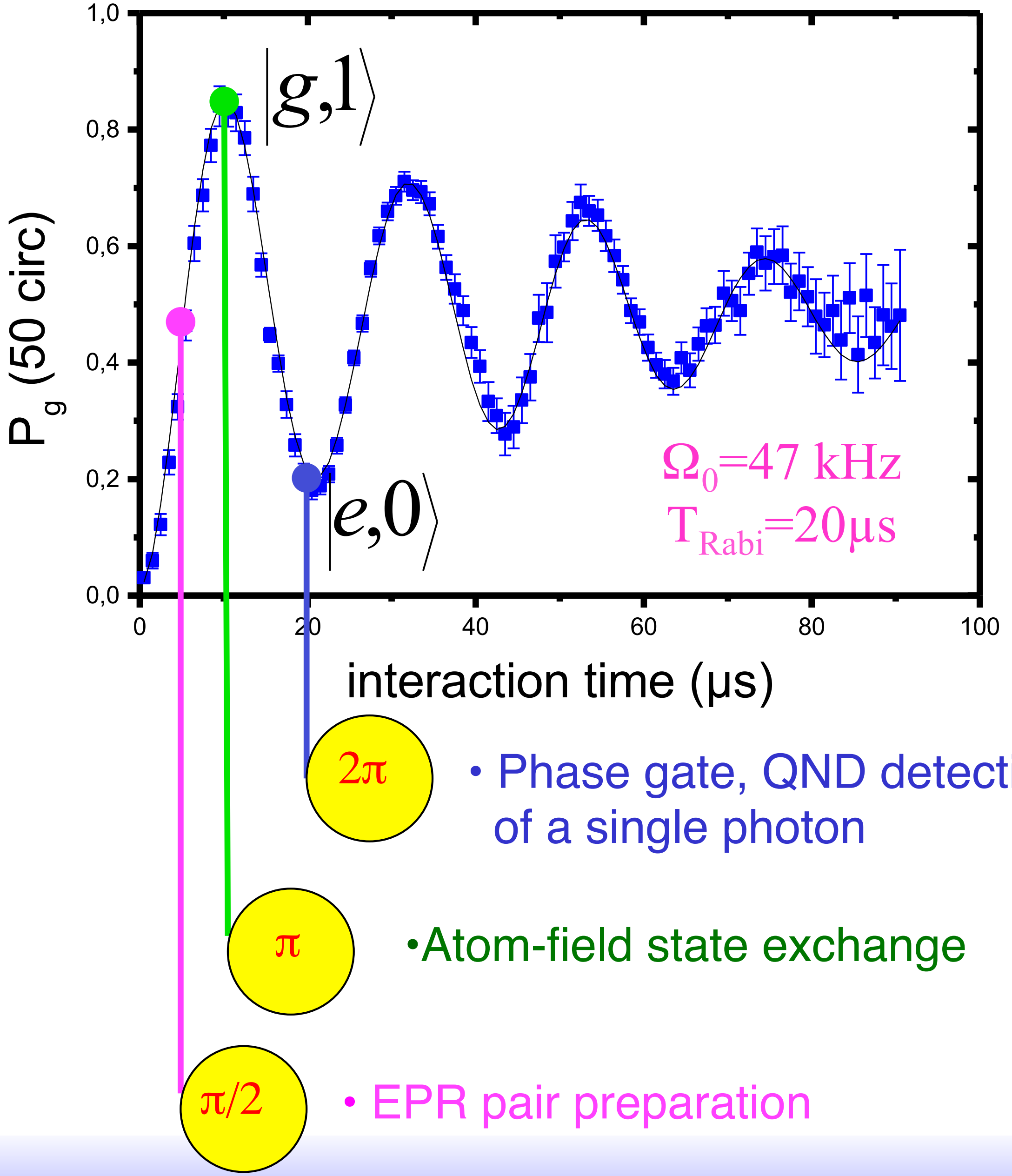
Single photon induced Rabi oscillation





Coherent Rabi oscillation
replaces irreversible damping
by spontaneous emission

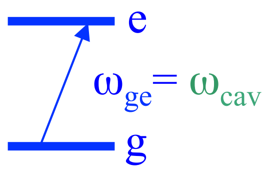
Vacuum Rabi oscillation and quantum gates

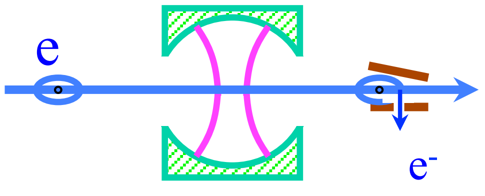


- Phase gate, QND detection of a single photon

•Atom-field state exchange

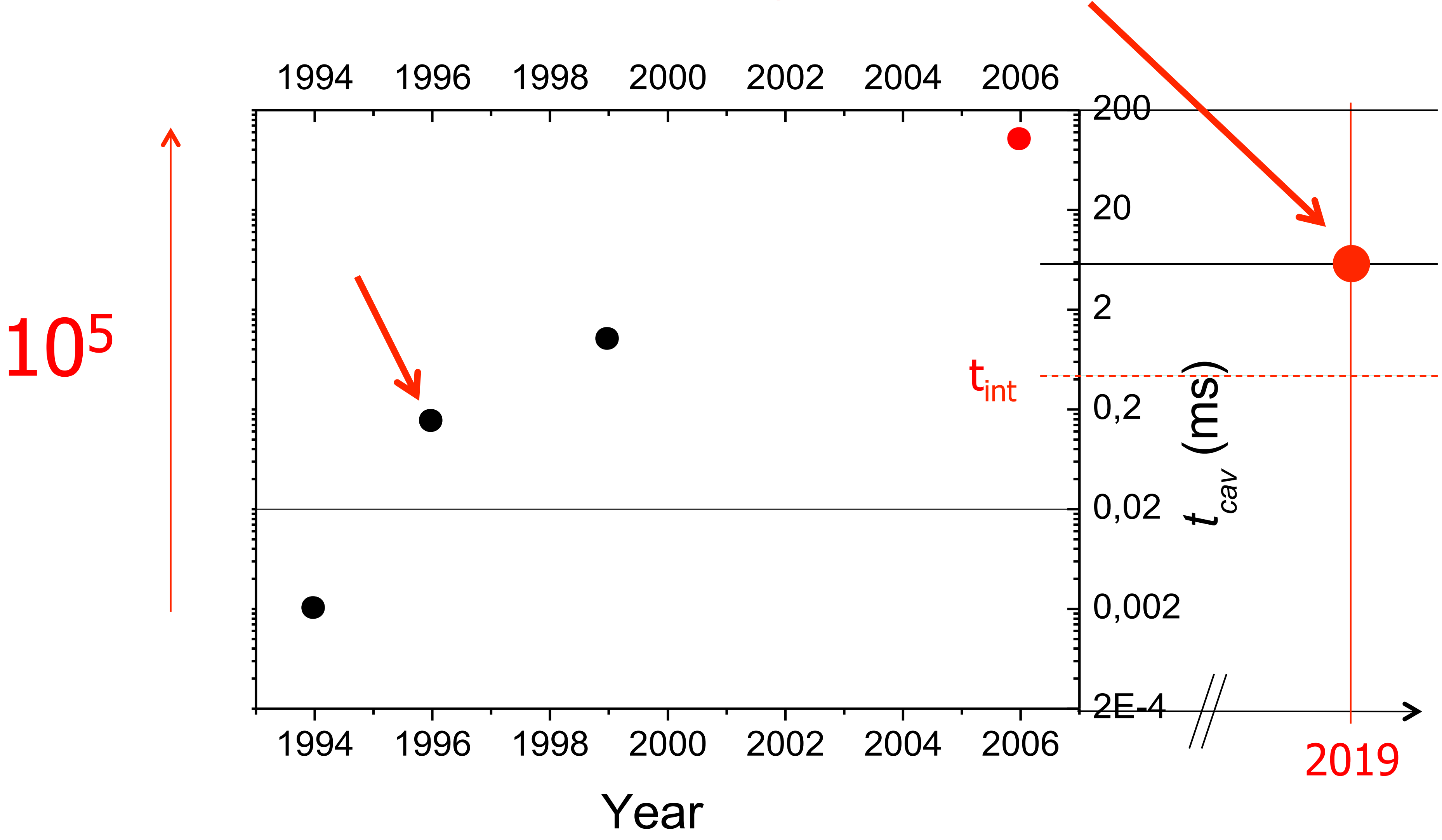
- EPR pair preparation



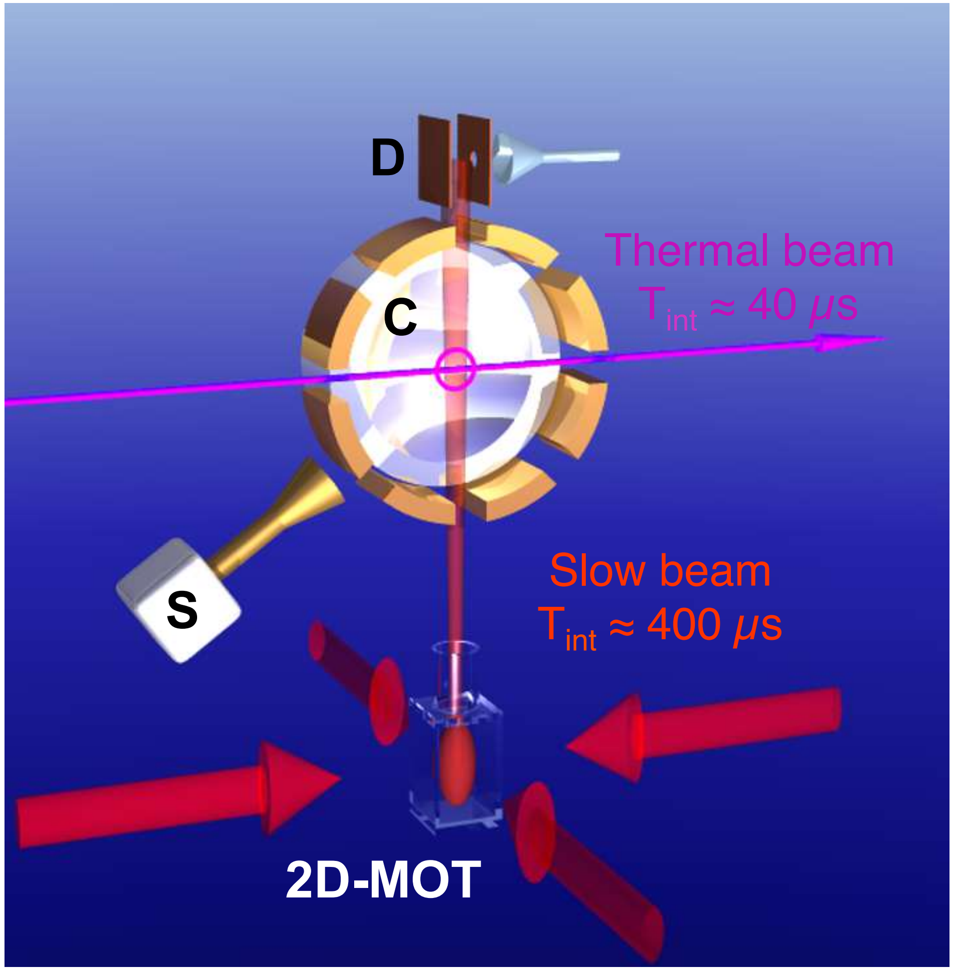


Superconducting cavity technology

- Last version of the expriment using slow atoms



A new, slow-atoms cavity QED setup



- Technical challenges

- preparation of Circular Rydberg atoms inside the cavity

- detection of Rydberg atoms inside the cavity: not yet implemented

- fabrication of a new superconducting cavity setup

$$\boxed{T_{cav} = 8\ \mathrm{ms}}$$

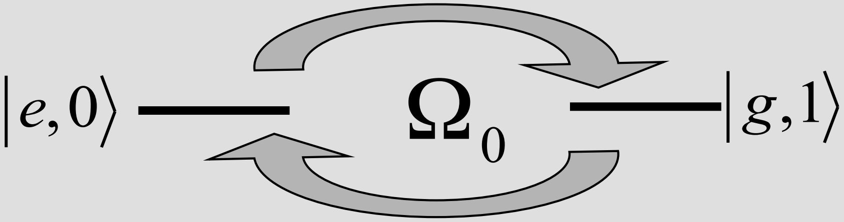
Not the best ever … but good-enough for what follows

⇒ Cavity QED experiment in a new regime with 10 m/s atoms:

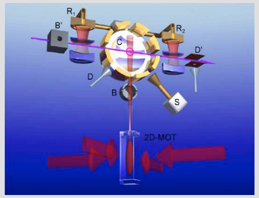
- Resolution of atom-cavity dressed states by microwave spectroscopy using the classical source S
- Observation of resonant interaction over unprecedented timescale

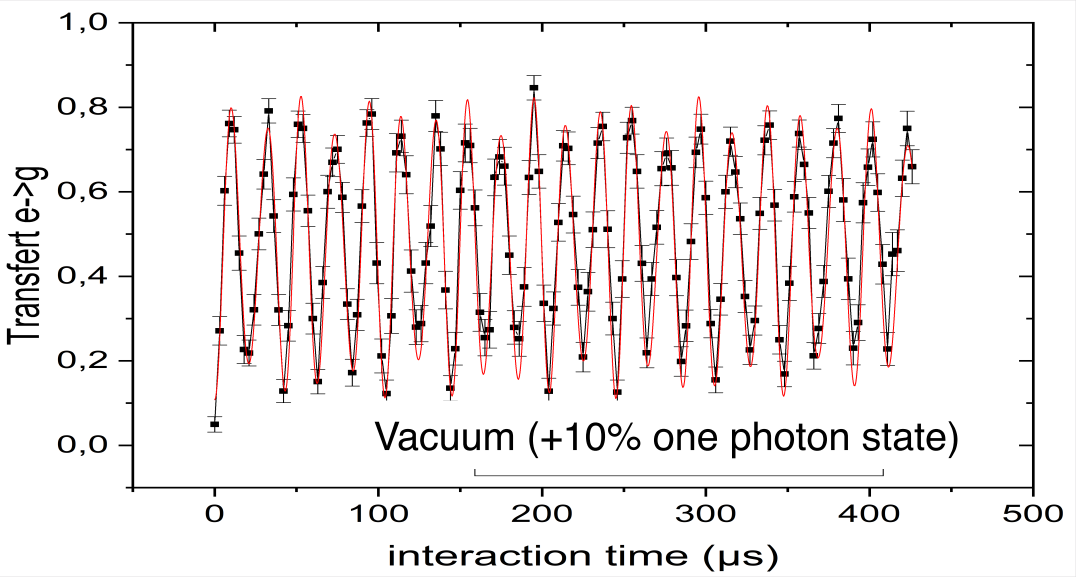
➞ Preparation of large "Schrödinger cat" sates

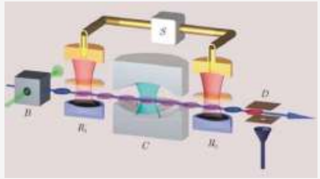
Rabi oscillation in vacuum

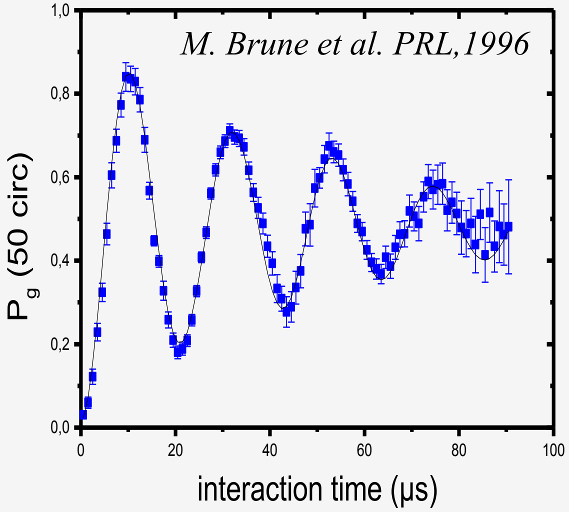


$$|e,0\rangle \longleftrightarrow \Omega_0 \longleftrightarrow |g,1\rangle$$









Our 1996 initial experiment

- No damping visible

(T_{cav}= 8 ms)

- 0.1photon residual blackbody field

Very pure coherent evolution

F. Assémat et al., Arxiv, accepted PRL(2018)

3. Rabi oscillation in a small coherent field

Direct observation

of discrete Rabi frequencies

Coherent field states

- Number state: $|N\rangle$
- Quasi-classical state: defined as eigenvectors of $\hat{a}$

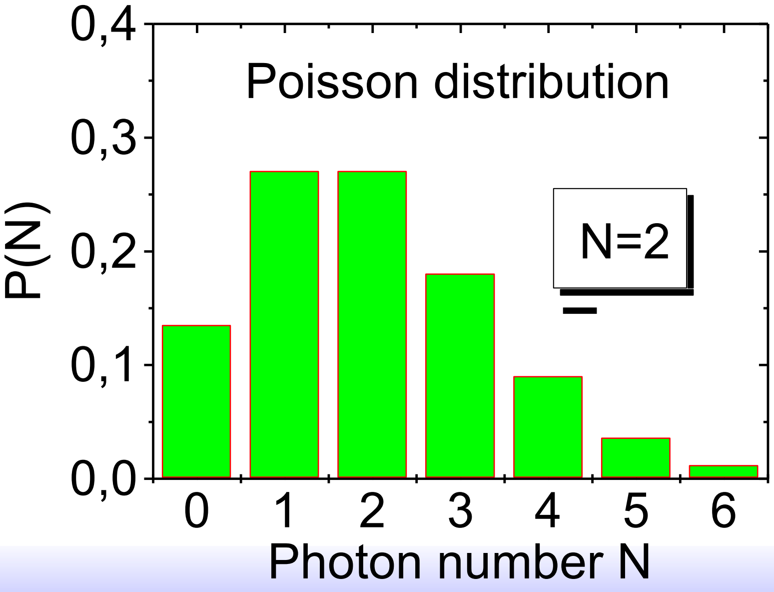
Field radiated by a classical source

$$\boxed{\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \text{ and } \langle\alpha|\hat{a}^+ = \alpha^*\langle\alpha|}$$

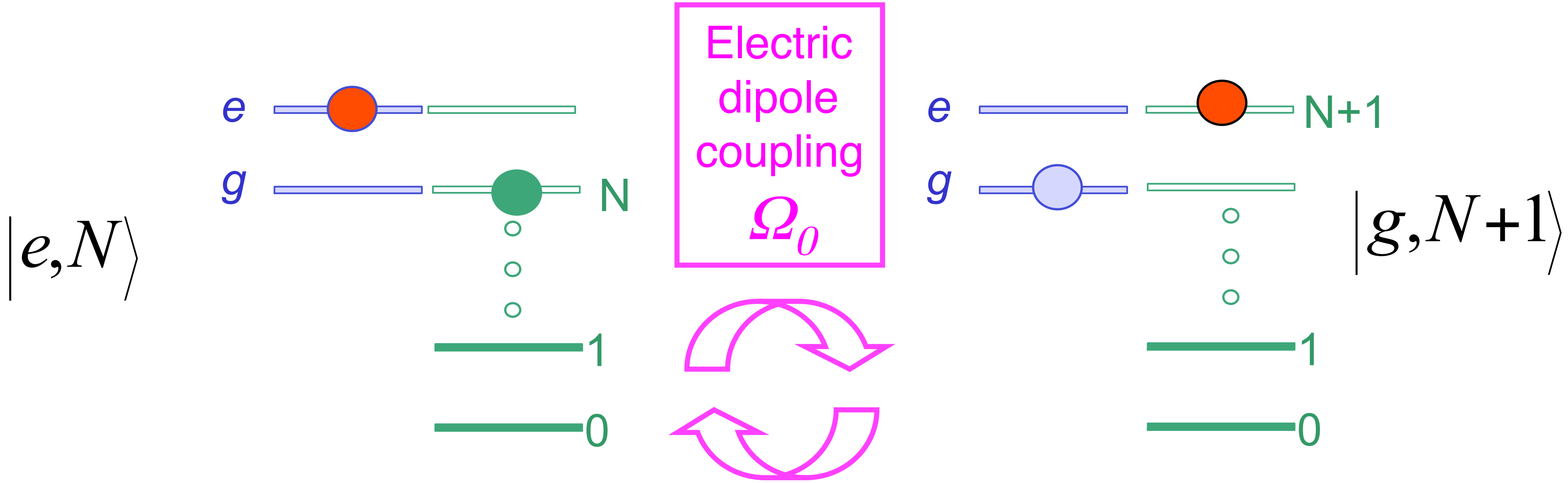
$$\boxed{|\alpha\rangle = e^{-|\alpha|^2/2}\sum_N \frac{\alpha^N}{\sqrt{N!}}|N\rangle \quad ; \quad \alpha = |\alpha|e^{i\Phi}}$$

Photon number distribution:

$$P(N) = e^{-\bar{N}}\frac{\bar{N}^N}{N!} \quad ; \quad \bar{N} = |\alpha|^2$$



Resonant atom-field coupling: Cavity containing N photons



$$|e,N\rangle$$

Electric dipole coupling

$$\Omega_0$$

$$|g,N+1\rangle$$

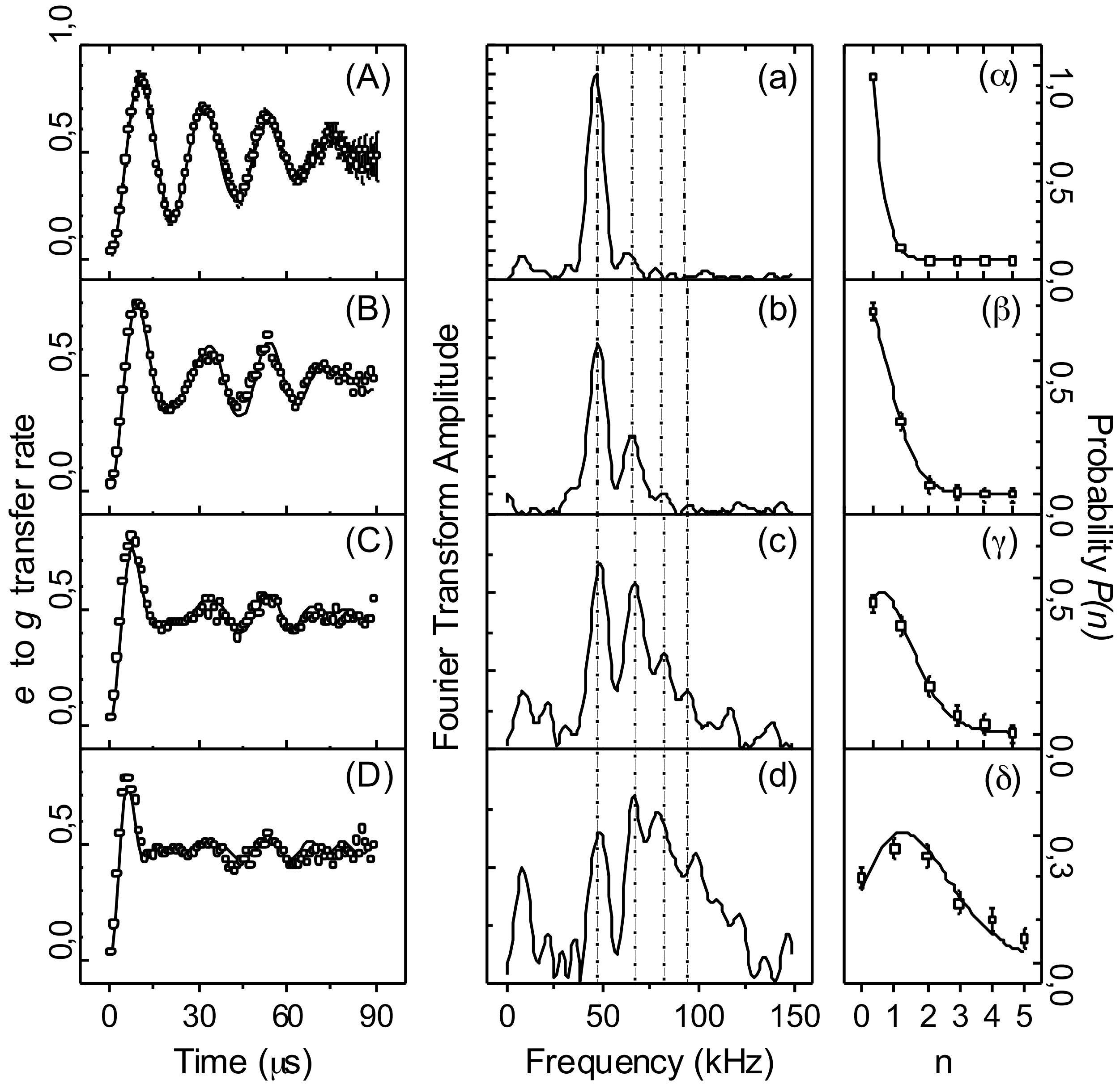
Coupling

$$\Omega_0\sqrt{N+1}$$

$$|e,0\rangle \rightarrow \cos\left(\frac{\Omega_0\sqrt{N+1}\,t}{2}\right)\cdot|e,0\rangle - i\sin\left(\frac{\Omega_0\sqrt{N+1}\,t}{2}\right)\cdot|g,1\rangle$$

$$P_g(t)=\sum_N P(N)\frac{1}{2}\left(1-\cos\left(\Omega_0 t\sqrt{N+1}\right)\right)$$

Rabi oscillation in small coherent fields



<n>=0.06

<n>=0.4

<n>=0.85

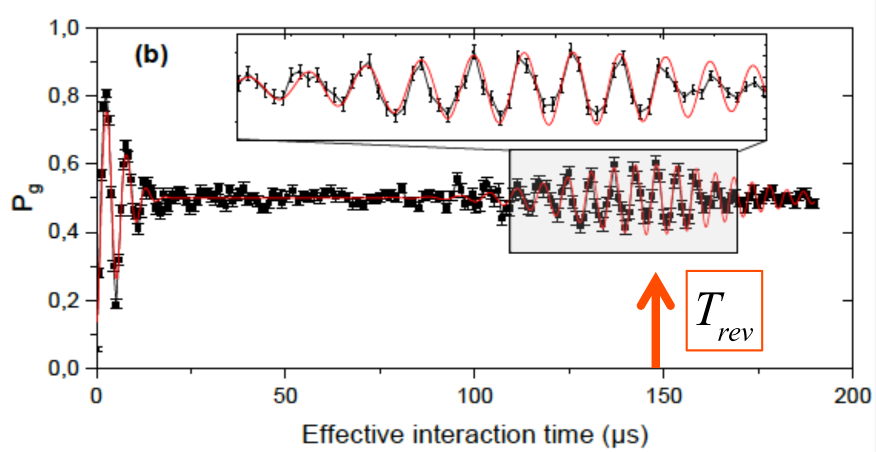
<n>=1.77

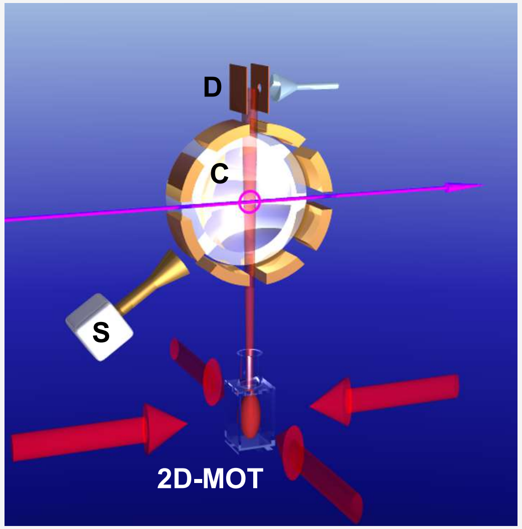
Phys. Rev. Lett. ***76****, 1800 (1996)*

Rabi oscillation in a 13 photon coherent field

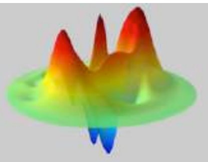
- Rabi oscillation in a coherent state

Eberly et al. PRL **44**, 1323 (1980)



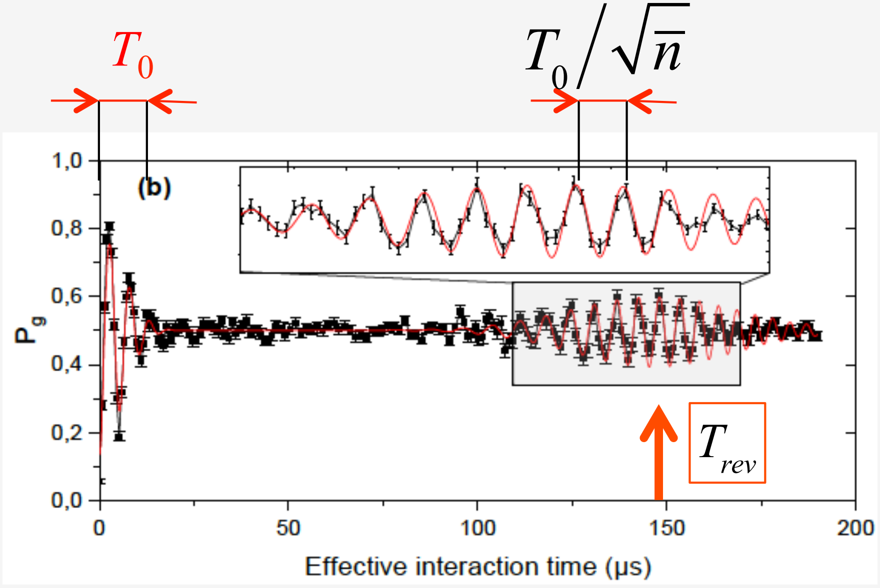


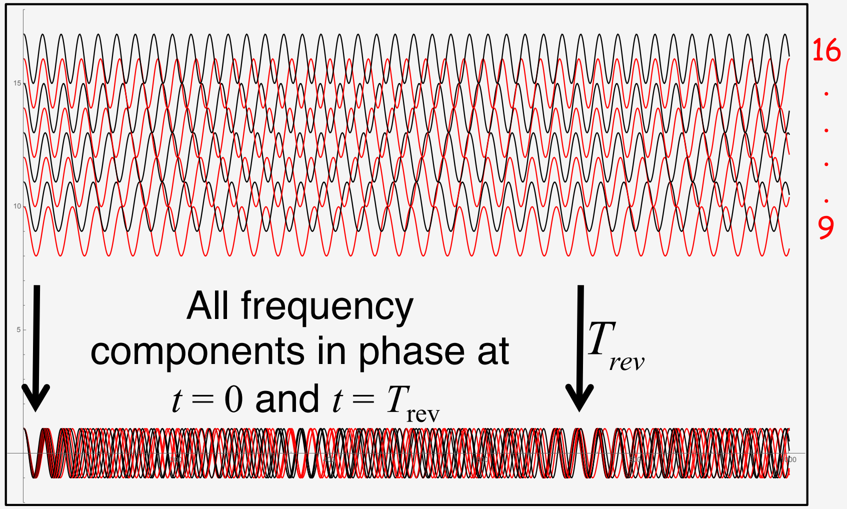
F. Assemat et al.,accepted PRL, arXiv:1905.05247



Rabi oscillation in a 13 photon coherent field

- Rabi oscillation in a coherent state





All frequency components in phase at $t = 0$ and $t = T_{\text{rev}}$

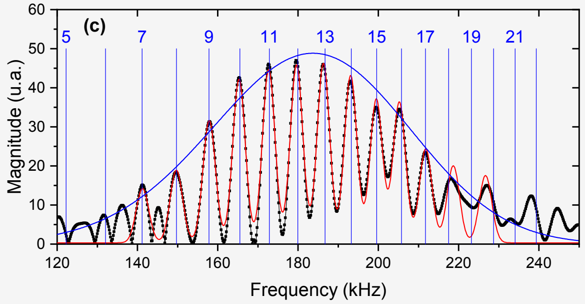
Eberly et al. PRL **44**, 1323 (1980)

Revival time $\boxed{T_{rev} = 2T_0\sqrt{\bar{n}}}$

Usual interpretation :

Collapse – revival is due to beat note between different Rabi frequencies

Fourier transform of the signal

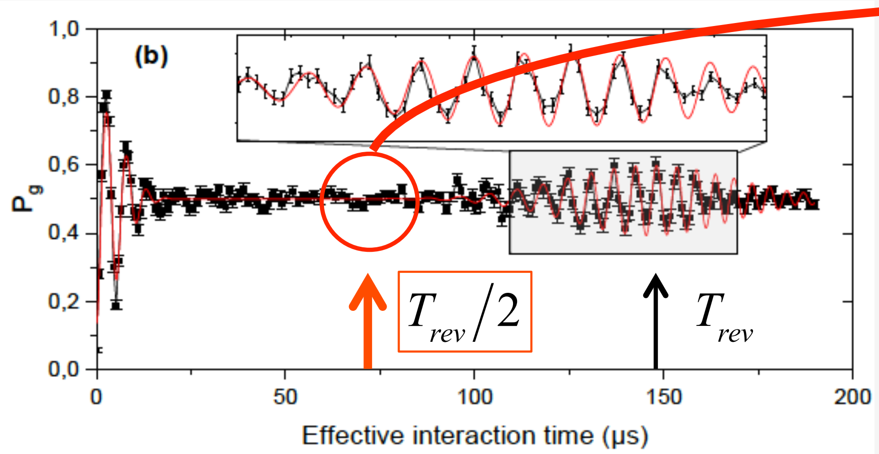


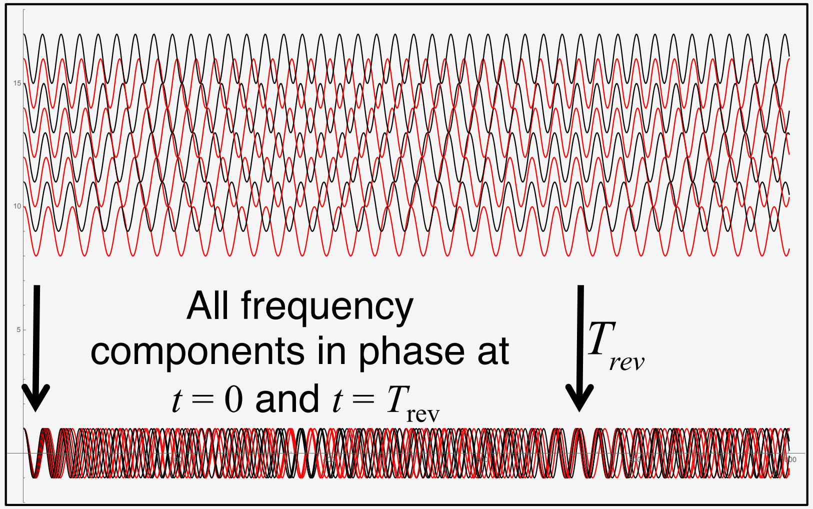
→ a direct manifestation of field quantization

F. Assemat et al.,accepted PRL, arXiv:1905.05247

Field state at "half revival"

- Rabi oscillation in a coherent state





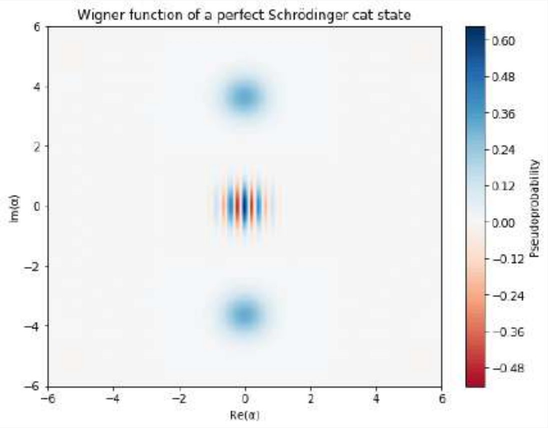
All frequency components in phase at $t = 0$ and $t = T_{\mathrm{rev}}$

Eiselt, et al. Opt. Comm. 72, 351 (1989)
Gea-Banacloche, PRL 65, 3385 (1990)

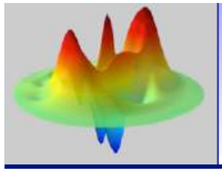
Atom-field state at half revival:

$$|\psi_{at,field}\rangle \approx |\psi_{at}\rangle \otimes |\psi_{cat}\rangle$$

$$\boxed{|\psi_{cat}\rangle = \frac{1}{\sqrt{2}}\left(|i\beta\rangle - |-i\beta\rangle\right)}$$

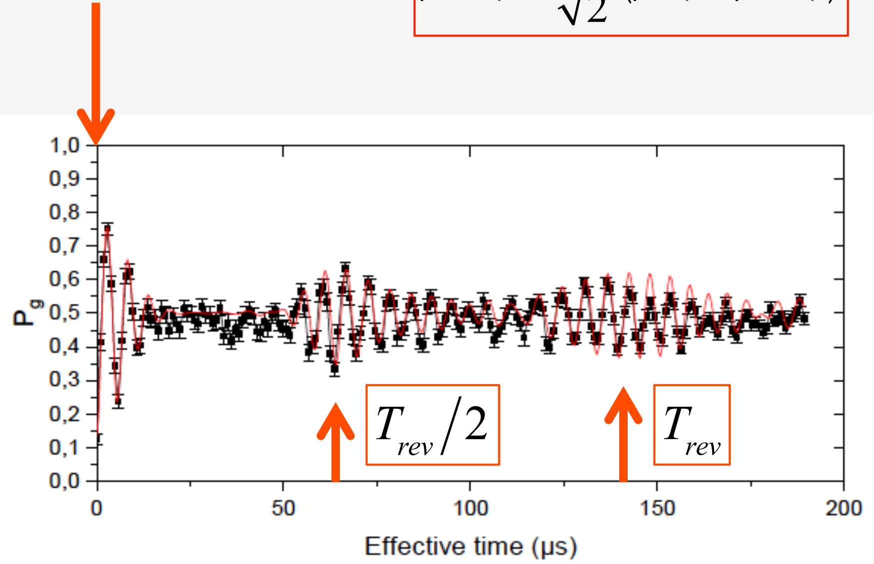


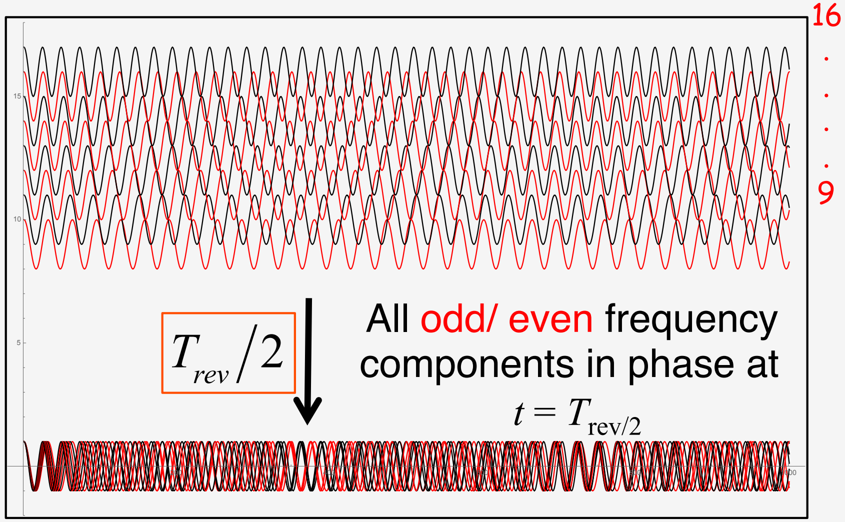
Due to destructive interference: cat state with only odd photon numbers



Rabi oscillation in the cat state

- Initial state: $|\psi_{cat}\rangle = \frac{1}{\sqrt{2}}\left(|i\beta\rangle - |-i\beta\rangle\right)$



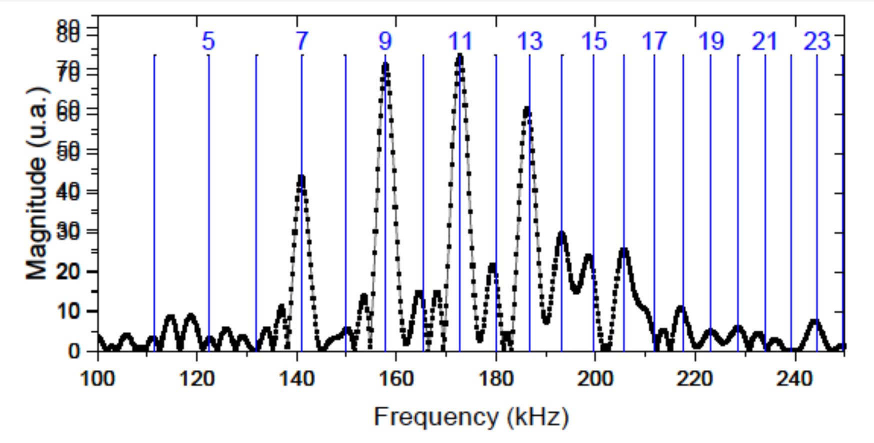


All odd/ even frequency components in phase at $t = T_{rev/2}$

"Odd" cat state: the "half revival" of Rabi oscillation spectrum reveals the cat parity

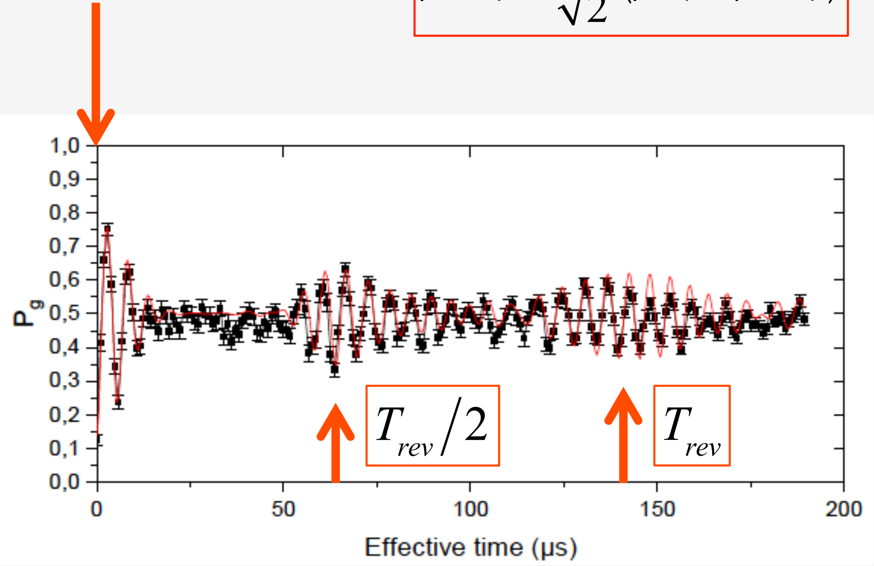
→ signature of quantum interference between the two classical components

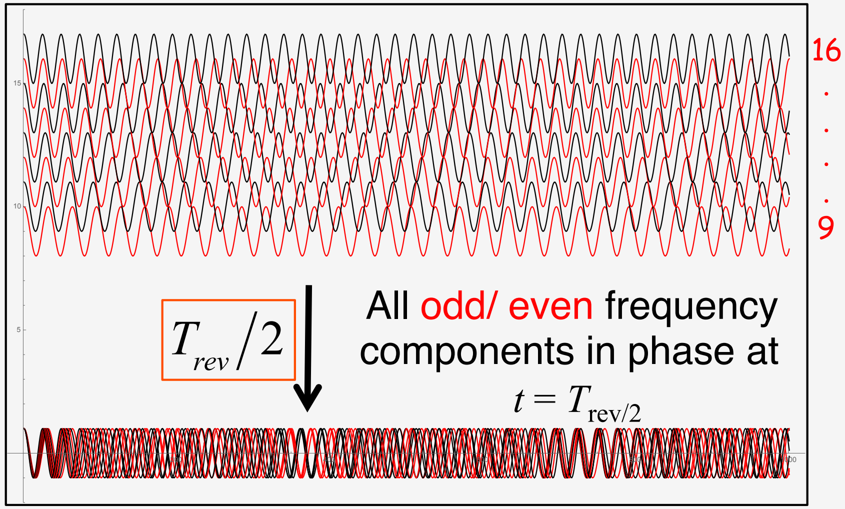
Fourier transform



Revealing the cat state coherence

- Initial state: $|\psi_{cat}\rangle = \frac{1}{\sqrt{2}}\left(|i\beta\rangle - |-i\beta\rangle\right)$

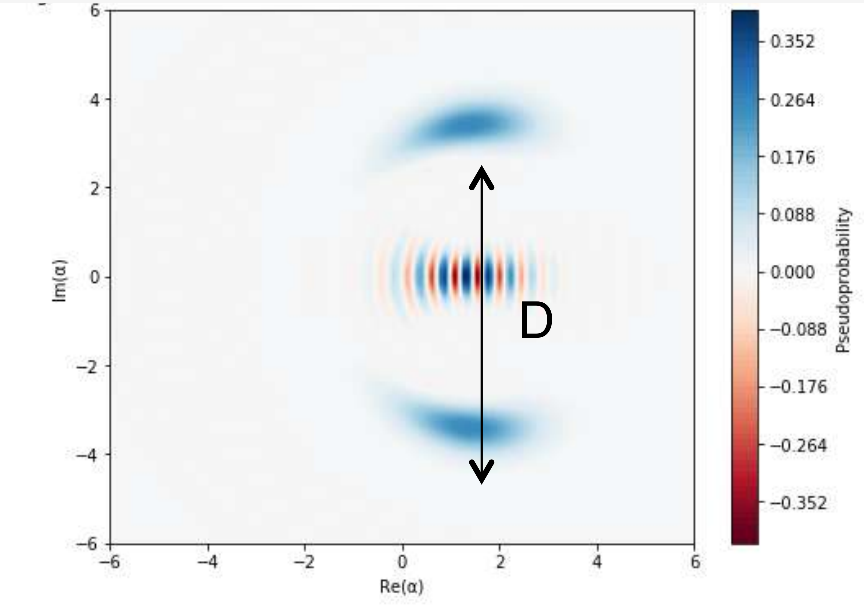




All odd/ even frequency components in phase at $t = T_{\text{rev/2}}$

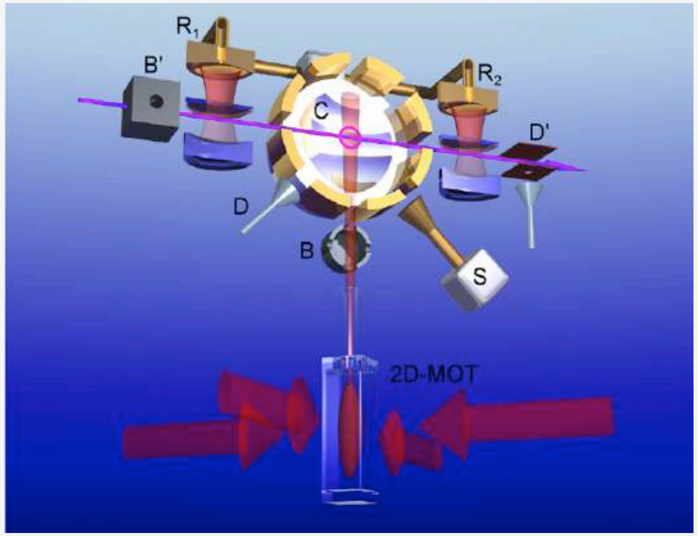
"Odd" cat state: the "half revival" of Rabi oscillation spectrum reveals the cat parity
→ signature of quantum interference between the two classical components

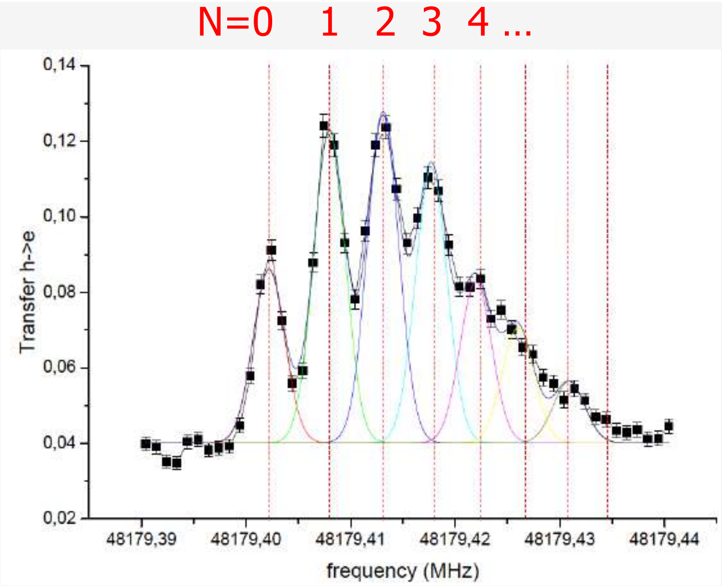
Cat size: D^2 = 46 photons



Dressed levels spectroscopy with slow Rydberg atoms

- A limitation of presented experiments
  - ❑ Atom-cavity interaction time << both systems lifetime
    - → 100 μs << 30ms, 0.13 s
- Achieving long interaction times
  - ❑ A set-up with a nearly stationary Rydberg atom in a cavity
  - ❑ Interaction time: ms range





Dressed states spectroscopy

Conclusion of lecture 1:

Cavity QED with microwave photons and circular Rydberg atoms:

…. a powerfull tool for:

- Achieving strong coupling between single atoms and single photons
- Demonstrating quantum features: collapse and revival of Rabi oscillations, Schrödinger cat state preparation

…. next lecture:

- CQED in the dispersive regime
    - ❑ Non-destructive photon counting

References

- Strong coupling regime in Rydberg atom CQED experiments:
    - F. Bernardot, P. Nussenzveig, M. Brune, J.M. Raimond and S. Haroche. "Vacuum Rabi Splitting Observed on a Microscopic atomic sample in a Microwave cavity". Europhys. lett. **17**, 33-38 (1992).
    - P. Nussenzveig, F. Bernardot, M. Brune, J. Hare, J.M. Raimond, S. Haroche and W. Gawlik. "Preparation of high principal quantum number "circular" states of rubidium". Phys. Rev. **A48**, 3991 (1993).
    - M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J. M. Raimond and S. Haroche: "Quantum Rabi oscillation: a direct test of field quantization in a cavity". Phys. Rev. Lett. **76**, 1800 (1996).
    - J.M. Raimond, M. Brune and S. Haroche : "Manipulating quantum entanglement with atoms and photons in a cavity", Rev. Mod. Phys. vol.73, p.565-82 (2001).
    - P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J.M. Raimond and S. Haroche : "Interference with beam splitters evolving from quantum to classical : a complementarity experiment". Nature 411, 166 (2001).
    - E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J.M. Raimond and S. Haroche: "Generation of Einstein-Podolsky-Rosen pairs of atoms", PRL 79,1 (1997).
    - F. Assemat et al.,accepted PRL, arXiv:1905.05247