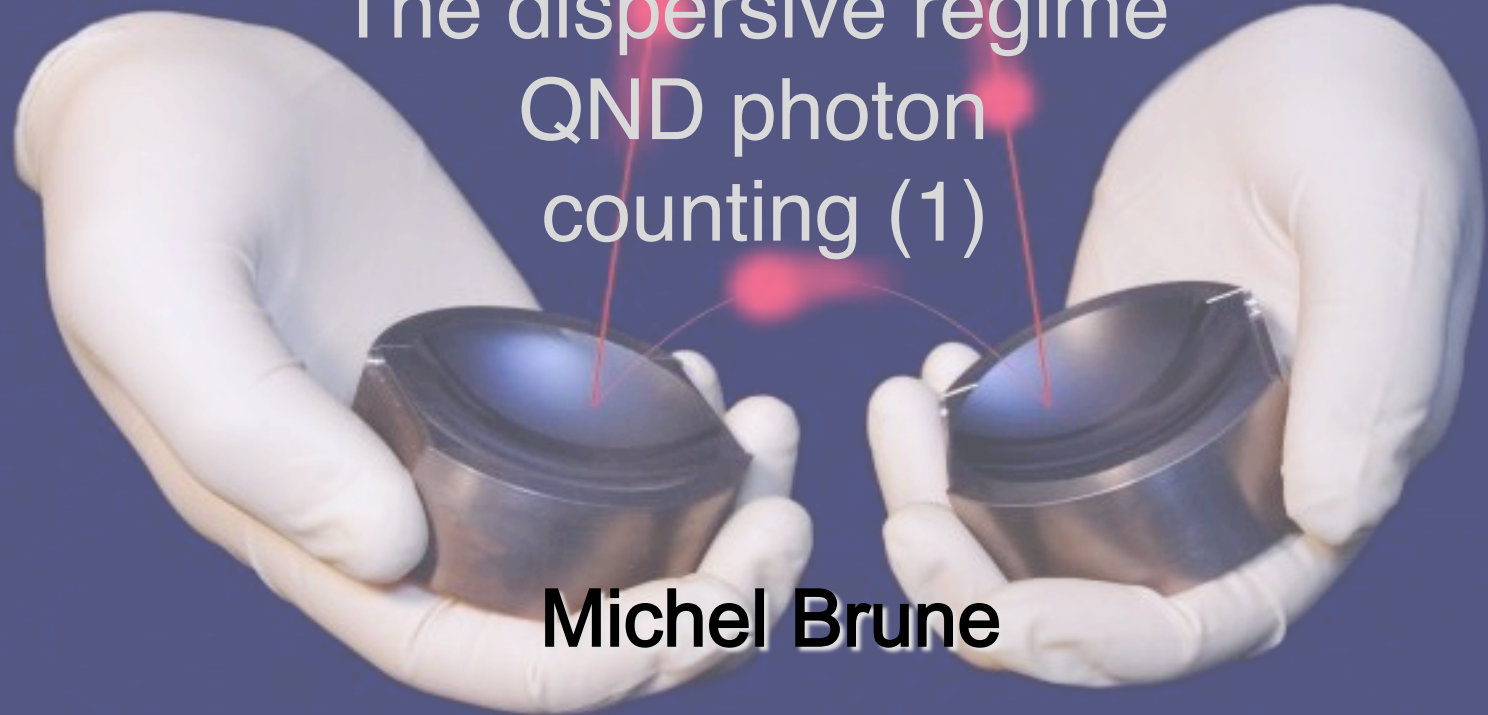


From cavity QED to quantum simulations with Rydberg atoms

Lecture 2

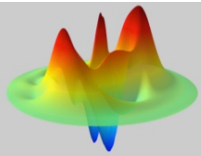
The dispersive regime QND photon counting (1)



Michel Brune

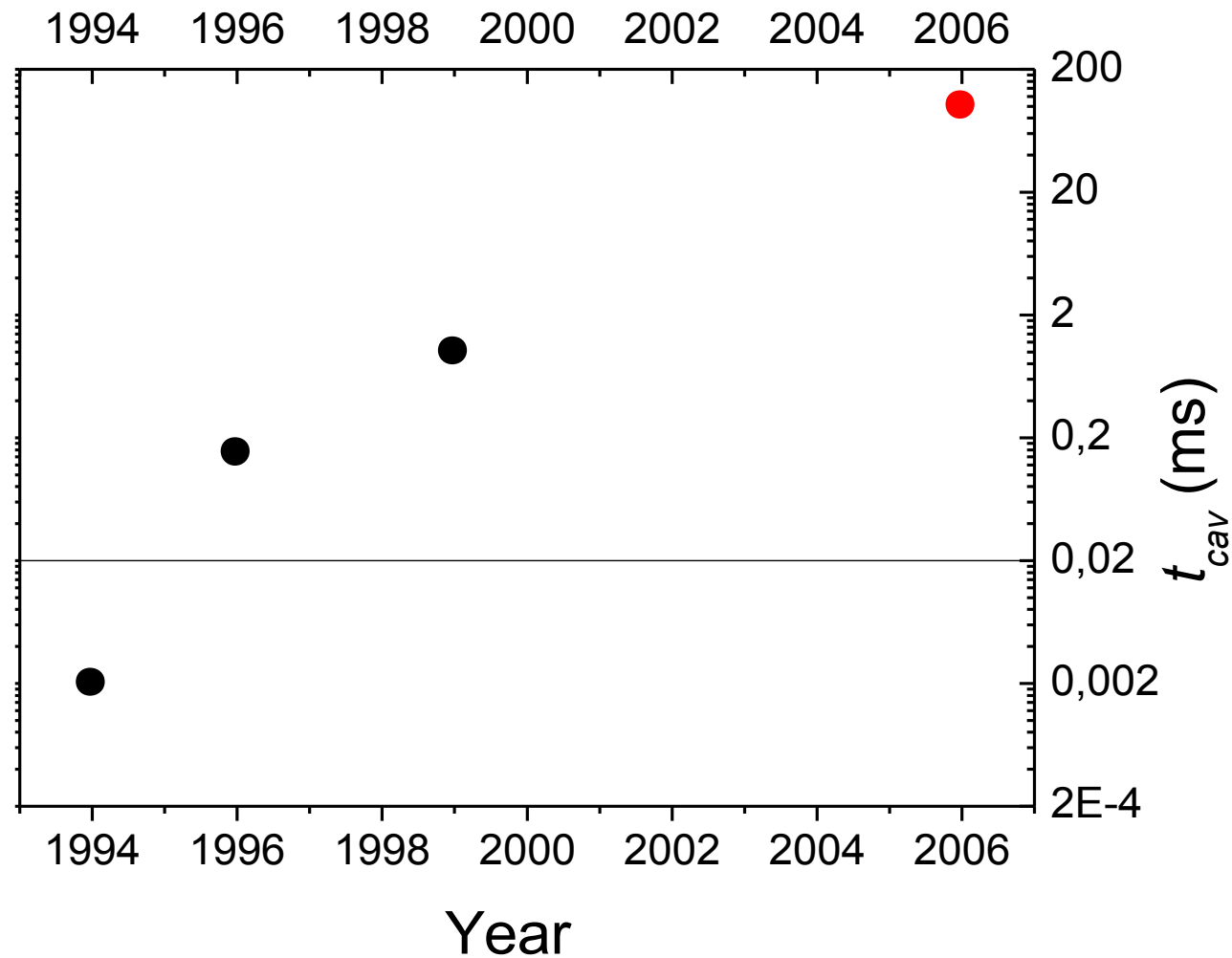


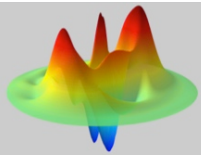
École Normale Supérieure, CNRS,
Université Pierre et Marie Curie,
Collège de France, Paris



The photon box for QND photon counting

- Our version of Moore's law:





QND photon counting: The beginning of the story ...

VOLUME 65, NUMBER 8

PHYSICAL REVIEW LETTERS

20 AUGUST 1990

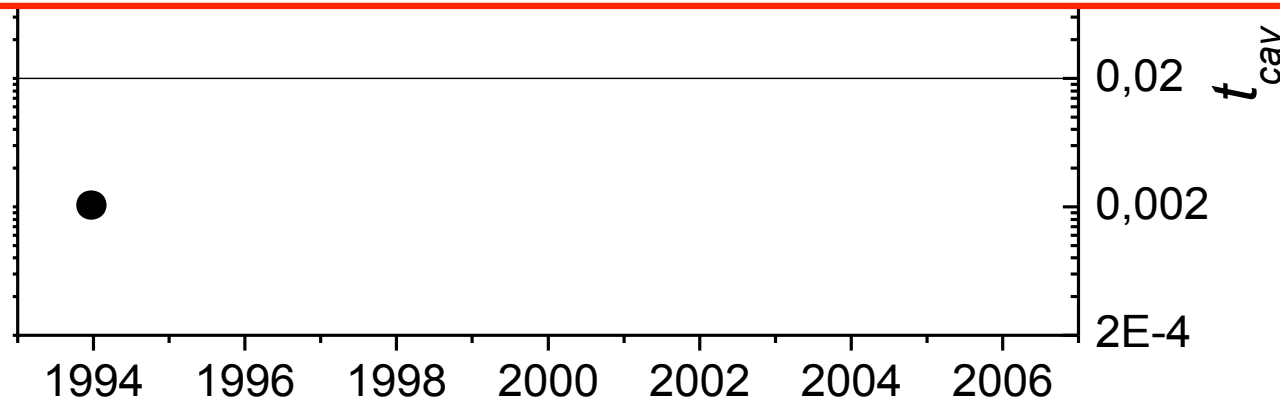
Quantum Nondemolition Measurement of Small Photon Numbers by Rydberg-Atom Phase-Sensitive Detection

M. Brune, S. Haroche, V. Lefevre, J. M. Raimond, and N. Zagury^(a)

*Département de Physique de l'Ecole Normale Supérieure, Laboratoire de Spectroscopie Hertzienne,
24 rue Lhomond, F-75231 Paris CEDEX 05, France*

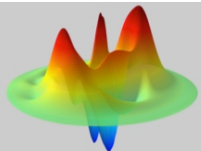
(Received 18 April 1990)

We describe a new quantum nondemolition method to monitor the number N of photons in a microwave cavity. We propose coupling the field to a quasiresonant beam of Rydberg atoms and measuring the resulting phase shift of the atom wave function by the Ramsey separated-oscillatory-fields technique. The detection of a sequence of atoms reduces the field into a Fock state. With realistic Rydberg atom-cavity systems, small-photon-number states down to $N=0$ could be prepared and continuously monitored.

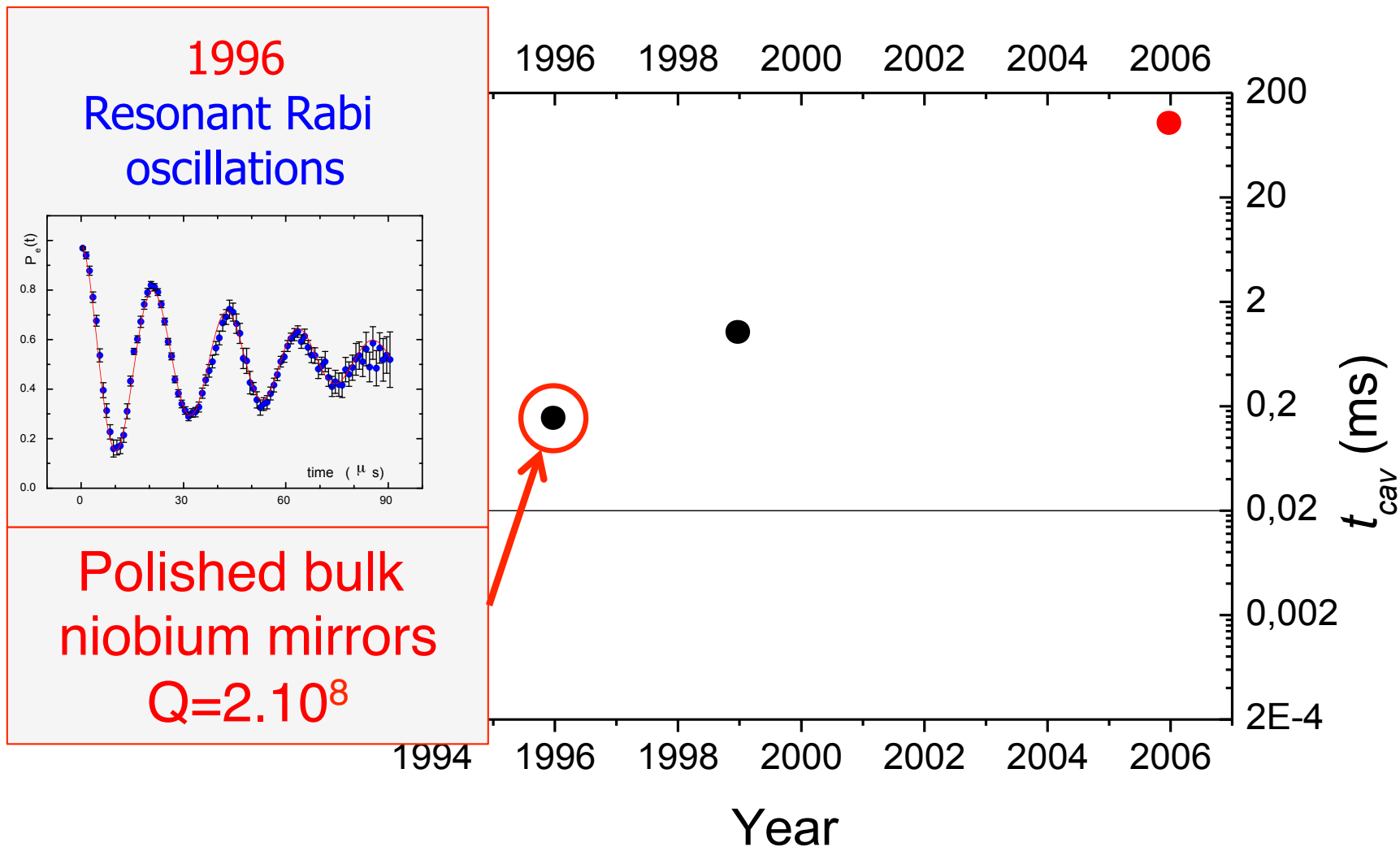


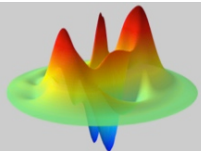
1990

Initial QND measurement
proposal



The vacuum Rabi oscillation

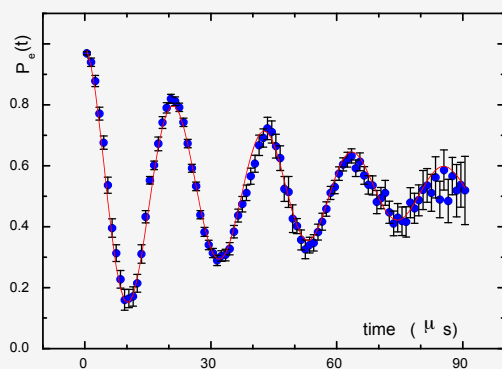




New cavity technology

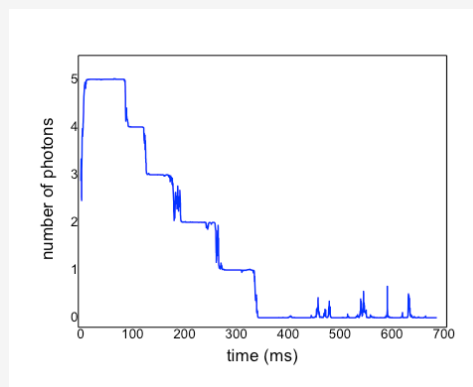
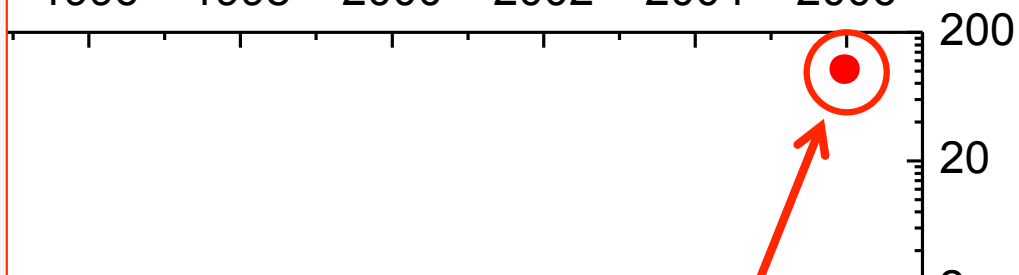
1996

Resonant Rabi oscillations



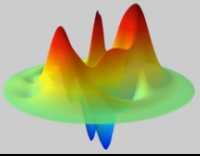
Polished bulk niobium mirrors
 $Q=2 \cdot 10^8$

1996 1998 2000 2002 2004 2006

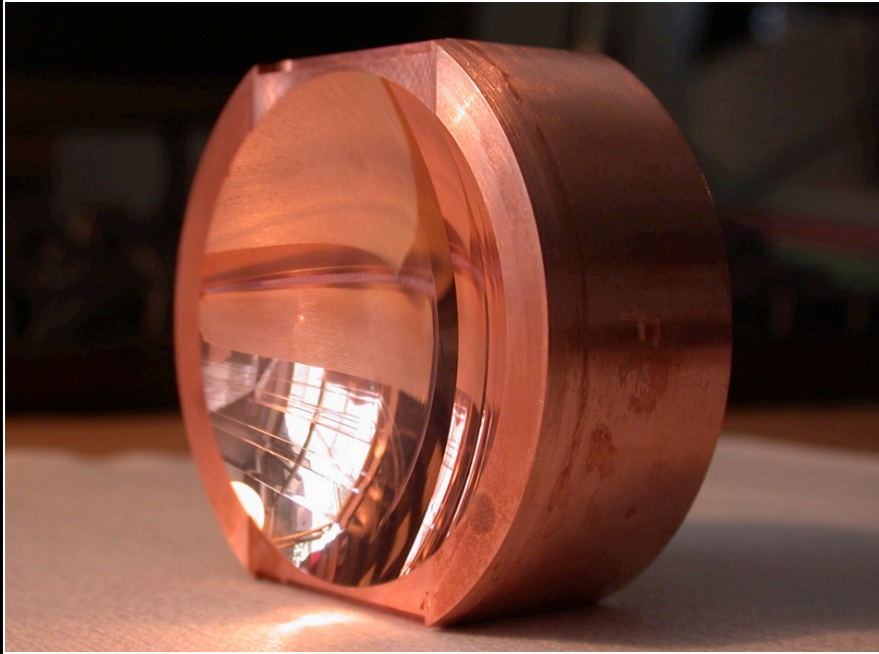


Sputtered niobium mirrors
 $Q=5 \cdot 10^{10}$

1994 1996 1998



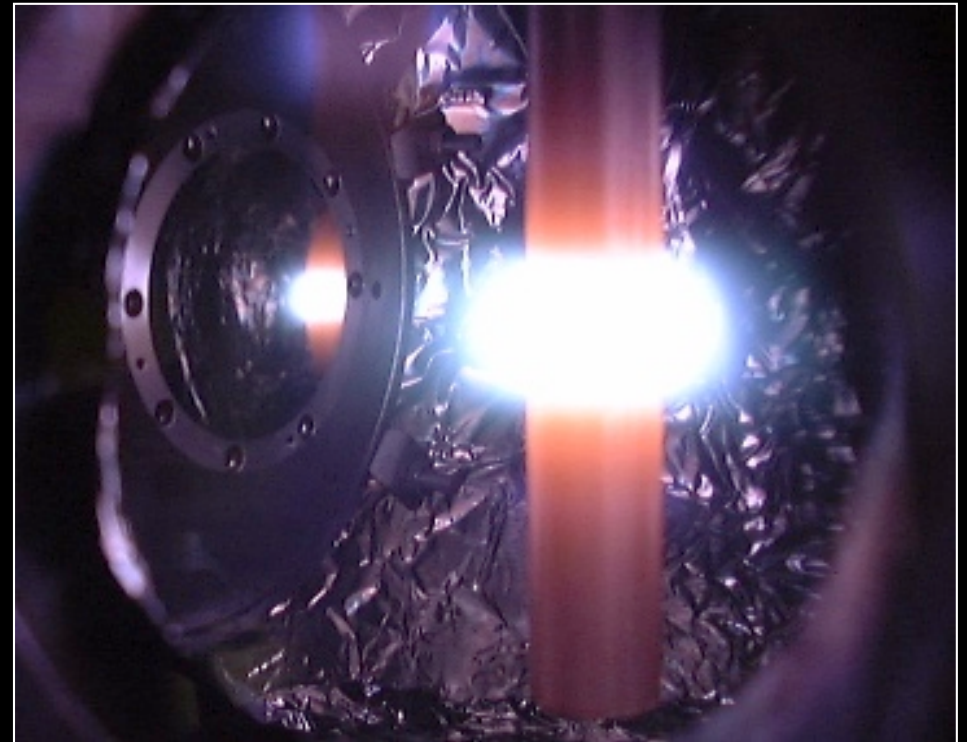
Niobium coated copper mirrors

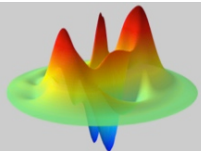


- Copper mirrors
 - Diamond machined
 - $\sim 1 \mu\text{m}$ ptv form accuracy
 - $\sim 10 \text{ nm}$ roughness
- Toroidal è single mode**

- Sputter $12 \mu\text{m}$ of Nb
 - Particles accelerator technique
 - Process done at CEA, Saclay

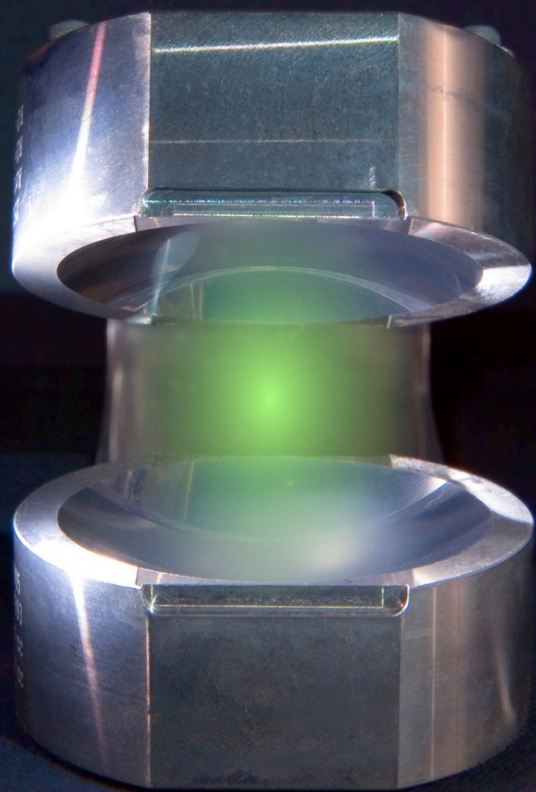
[E. Jacques, B. Visentin, P. Bosland]



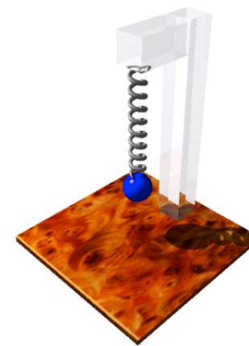


The best photon box

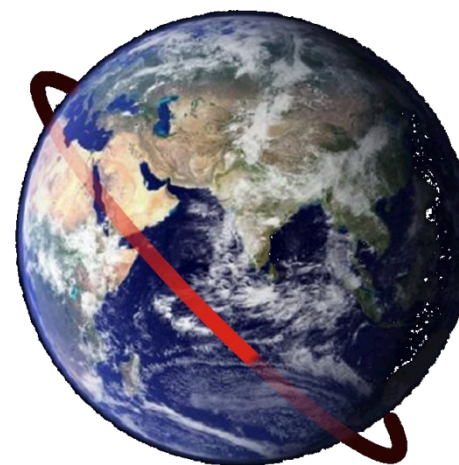
Superconducting cavity
resonance: $\nu_{\text{cav}} = 51 \text{ GHz}$



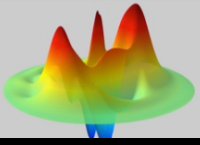
$$T_{\text{cav}} = 130 \text{ ms}$$



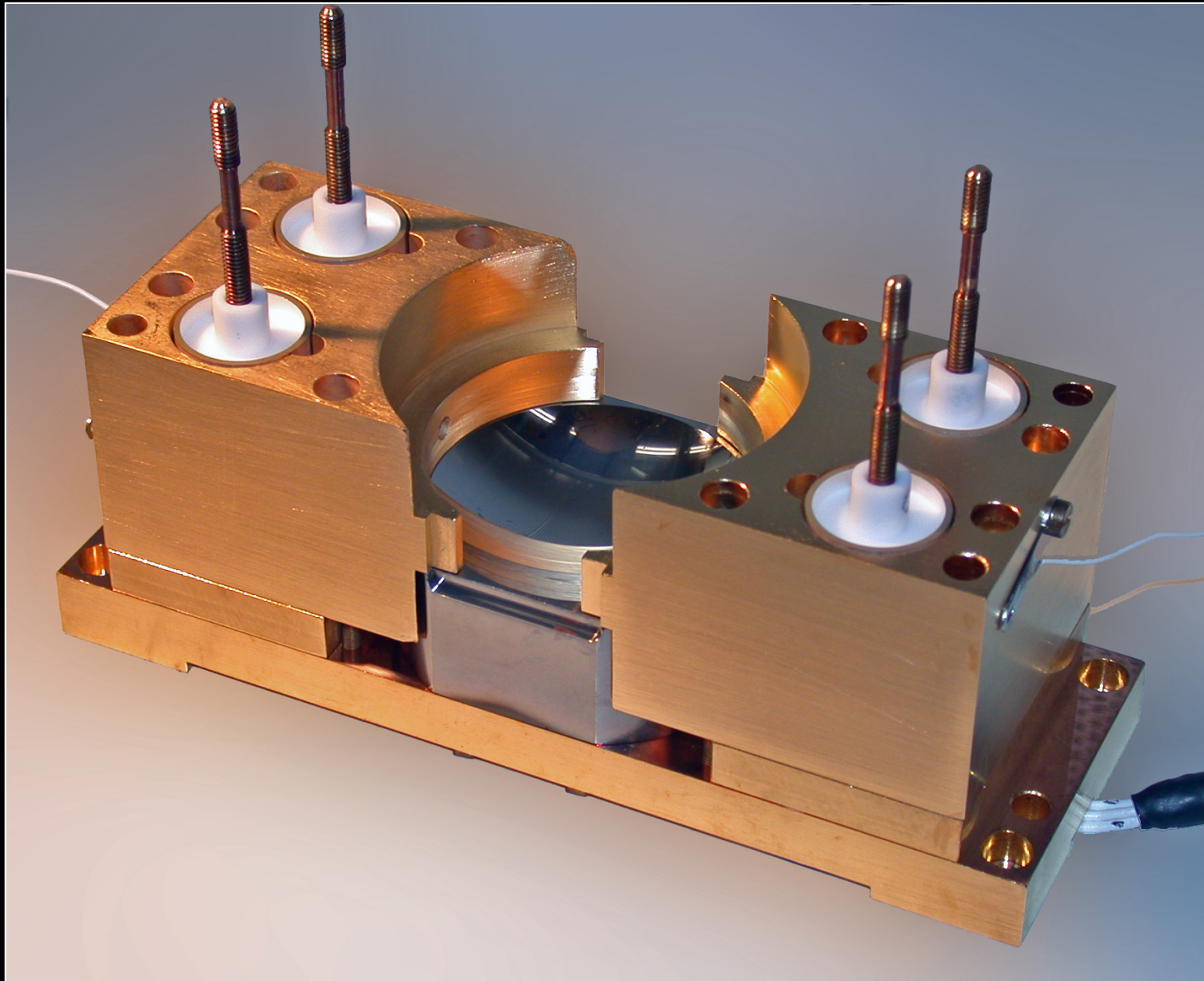
- Q factor = $4.2 \cdot 10^{10}$
- finesse = $4 \cdot 10^9$



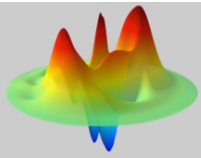
Photons running for 39 000 km
in the box before dying!



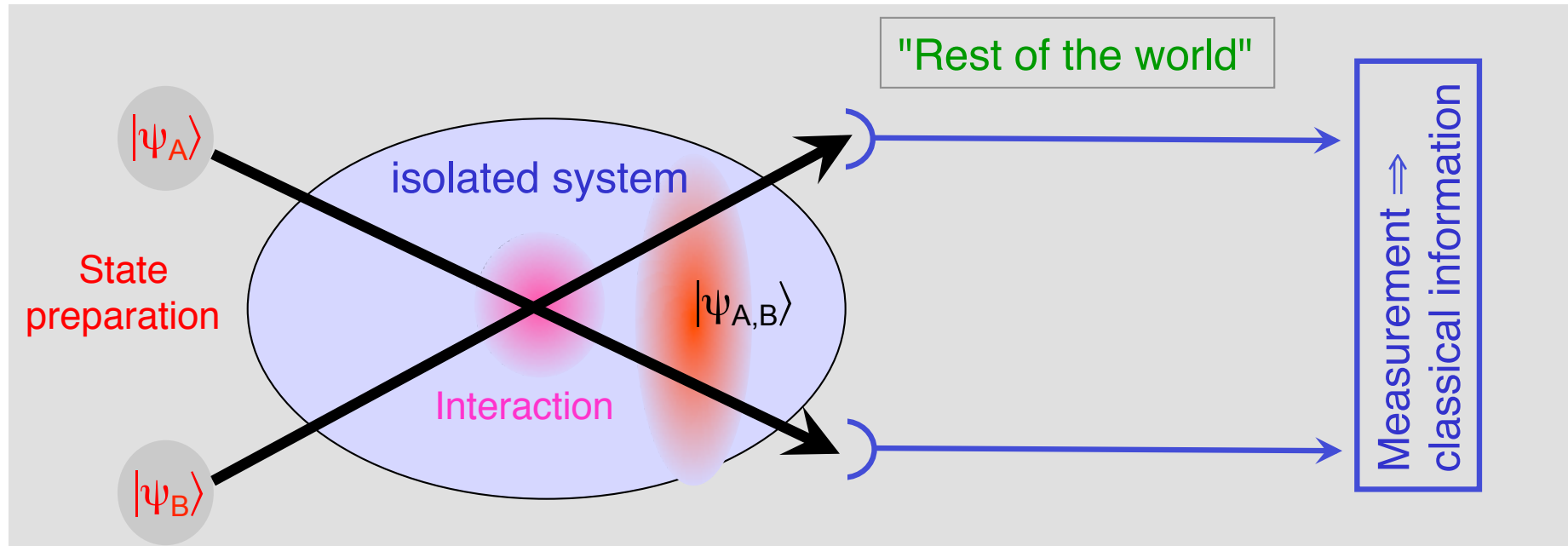
A new cavity setup



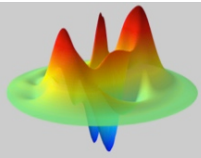
1. Basic reminder on ideal quantum measurement



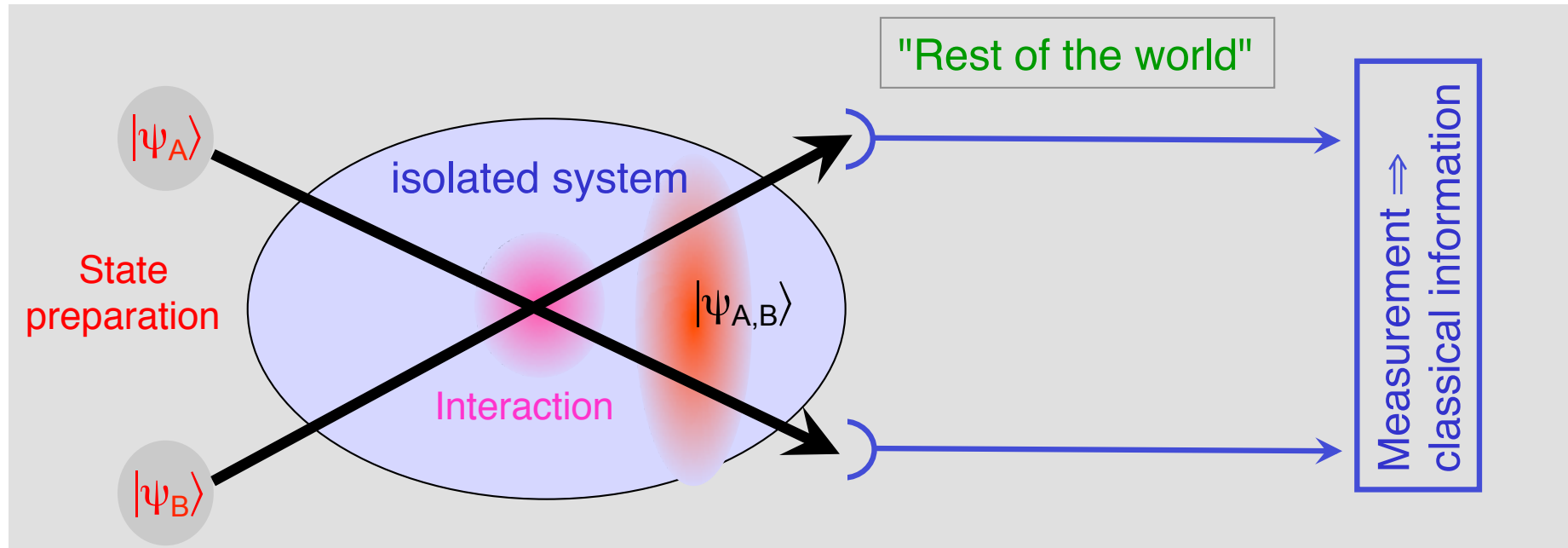
Quantum physics



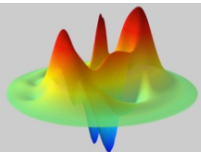
- Description of quantum objects
 - **interaction:** Schrödinger equation.
 - **measurements:** the state determines the statistics of results.
- Quantum theory: the **art of extracting classical information** out of microscopic systems.



Quantum measurement: basic ingredients



- **Entanglement:** "The essence of quantum physics" (Heisenberg)
Created by interaction, describes all correlations between quantum systems.
- **irreversibility introducing dissipation:** macroscopic systems are dissipative. Dissipation plays a fundamental role in the coherence of quantum theory: explains the "decoherence" step during a quantum measurement



Ideal quantum measurement

- The postulates:

- Possible results: eigenvalues a_n of an hermitian operator \hat{A} (observable).

- **Fundamentally random result of individual measurements**

- Probability of results if system in state $|\psi\rangle$:

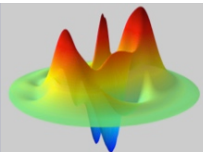
$$p(a_n) = \langle \psi | P_n | \psi \rangle$$

where $P_n =$ projector on the eigenspace associated to a_n .

- State after measurement:

$$|\psi_{after}\rangle = \frac{P_n |\psi\rangle}{\sqrt{p(a_n)}}$$

→ **state collapse**: the system's states changes discontinuously during the measurement process



The postulates, comments

- locks like a recipe:

- does not tell what is a measurement apparatus
- does not tell how to built an apparatus measuring a given observable

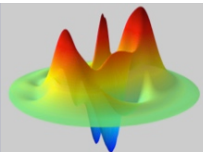
- locks like a strange recipe:

a quantum system seems to be subjected to two kinds of evolution:

- continuous evolution according to Schrödinger equation between measurements
- state collapse during measurements

But a measurement apparatus is made of quantum objects obeying to Schrödinger equation:

Why should evolution during measurement deserve a special treatment?



Outline of lectures 2-4

- Lecture 2:

 - The projection postulate at work

 - an experimental realization: measuring the photon number in a high Q cavity
 - observing the quantum jumps of light in a cavity

- Lecture 3 : applications of QND photon counting

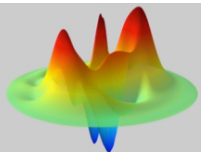
 - Quantum feedback
 - Past-quantum state analysis of a quantum trajectory

- Lecture 4:

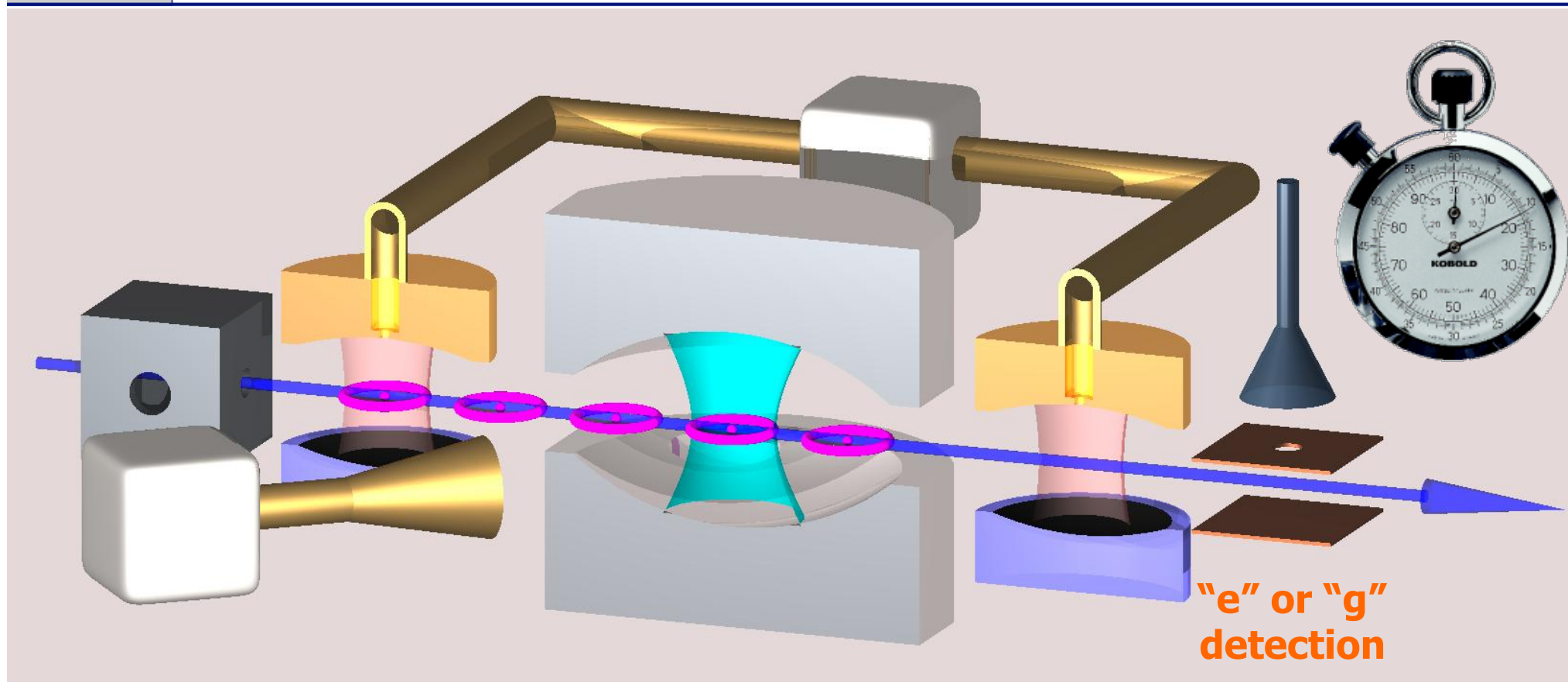
 - The role of dissipation: Schrödinger cat and decoherence

 - The "problem" of quantum measurement
 - The decoherence approach
 - Observing the decoherence of a Schrödinger cat state

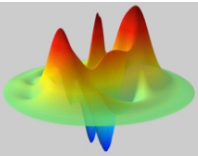
2. Non-destructive single photon counting



Experimental setup: an atomic clock

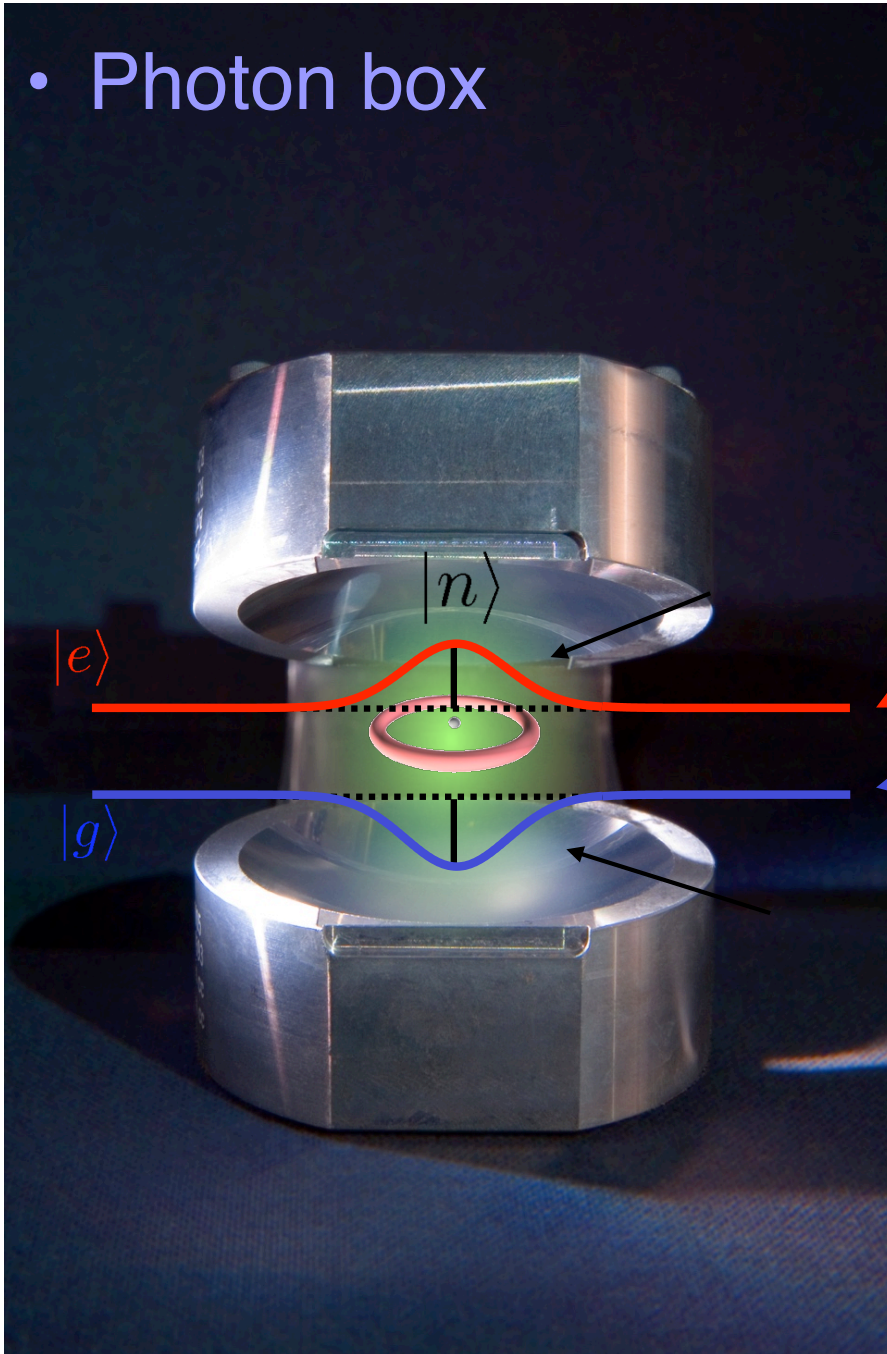


- An atomic clock (Ramsey setup) made of Rydberg for probing light-shifts induced by "trapped" photons
- State selective detection of atoms by field ionization: Atoms detected on "e" or "g" one by one



QND detection of photons: the principle

- Photon box



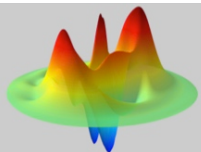
- Photon probes
Circular Rydberg atoms
- Non-resonant interaction
⇒ light shifts

$$\Delta E_e = \hbar \frac{\Omega_0^2}{4\delta} (n + 1)$$

$$\Delta E_g = -\hbar \frac{\Omega_0^2}{4\delta} n$$

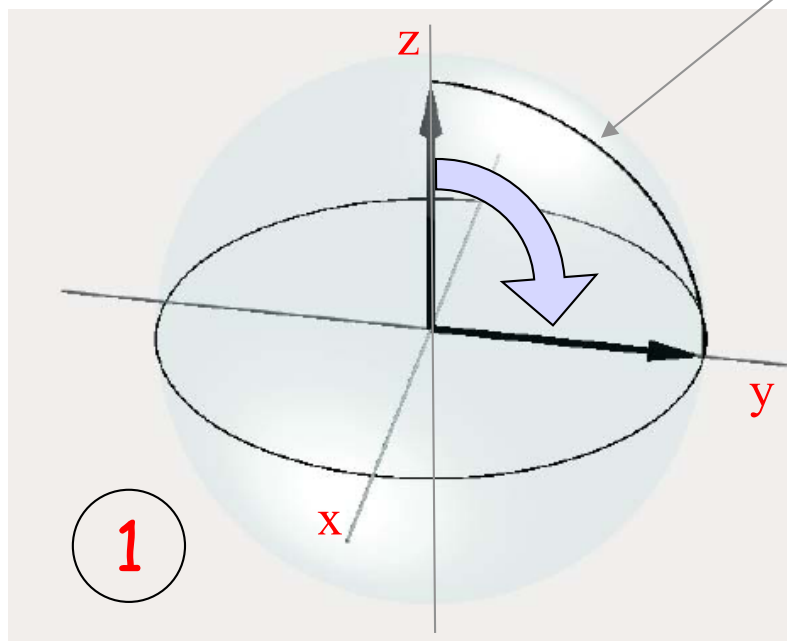
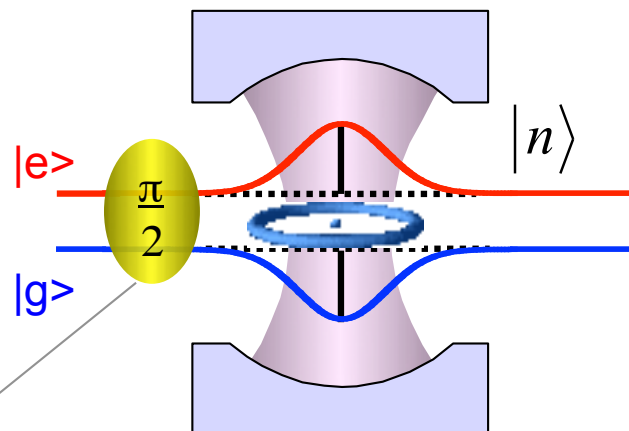
Atoms used as clock
for counting n by
measuring light shifts





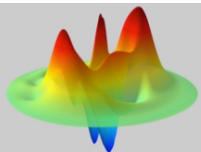
QND detection of 0 or 1 photon

1. Trigger of the clock.



$$|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle + i|g\rangle) = |+_x\rangle$$

In term of a spin $1/2$, this is a $\pi/2$ rotation around the Ox axis



QND detection of 0 or 1 photon

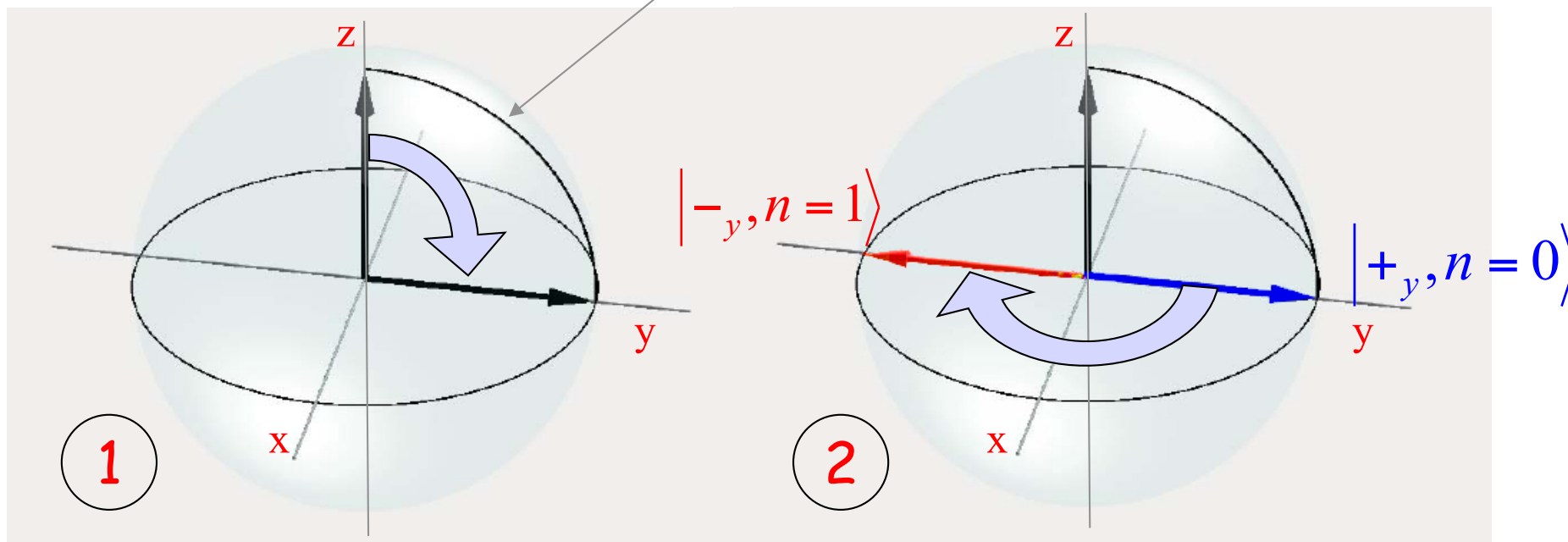
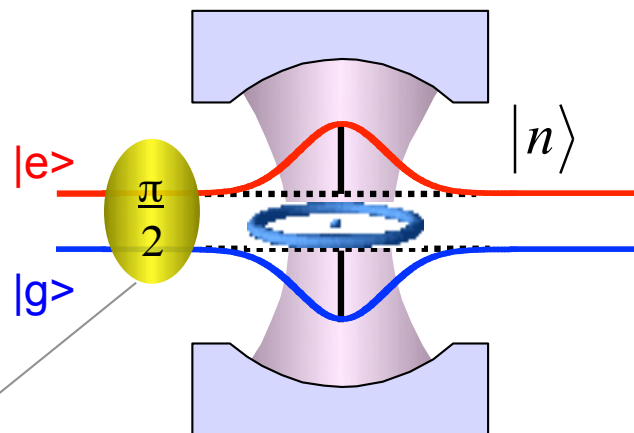
1. Trigger of the clock.



2. precession of the spin through the cavity during T

Phase shift per photon

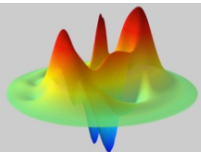
$$\Phi_0 = \pi$$



$$\rightarrow \frac{1}{\sqrt{2}} (|e\rangle + ie^{i\delta_{mw}T} |g\rangle) = |+\phi\rangle$$

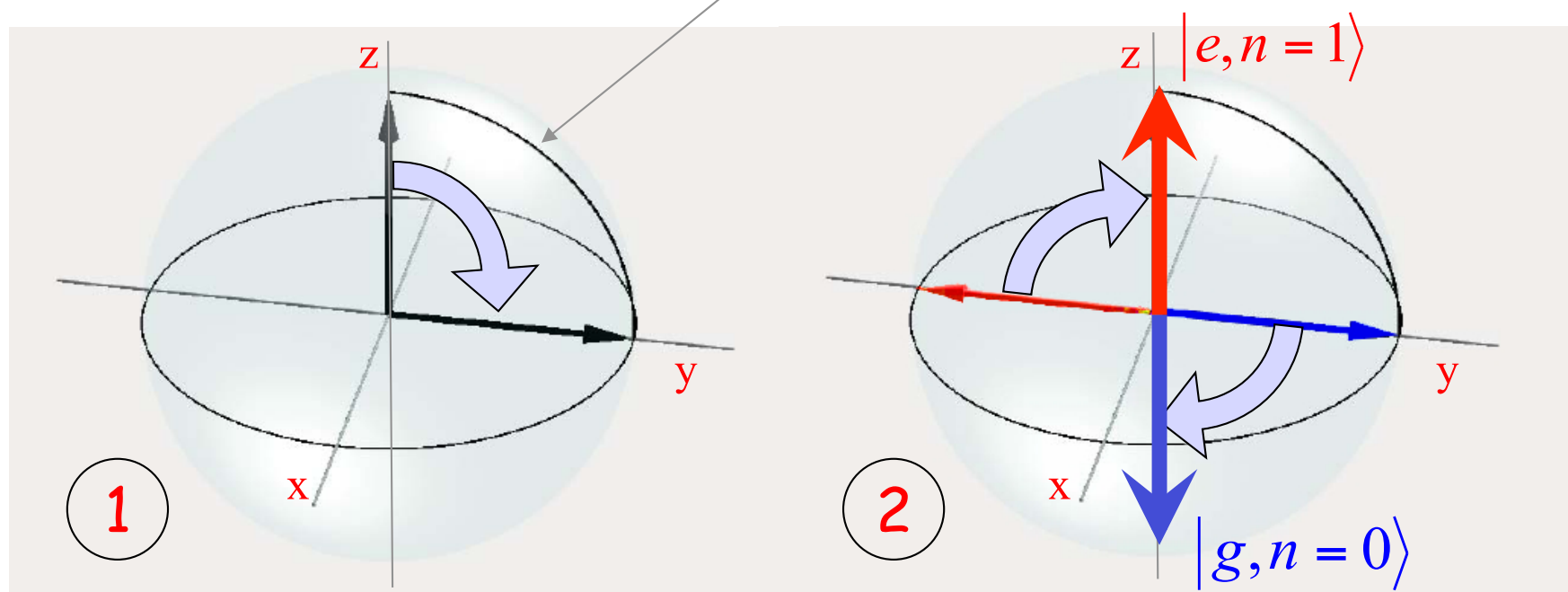
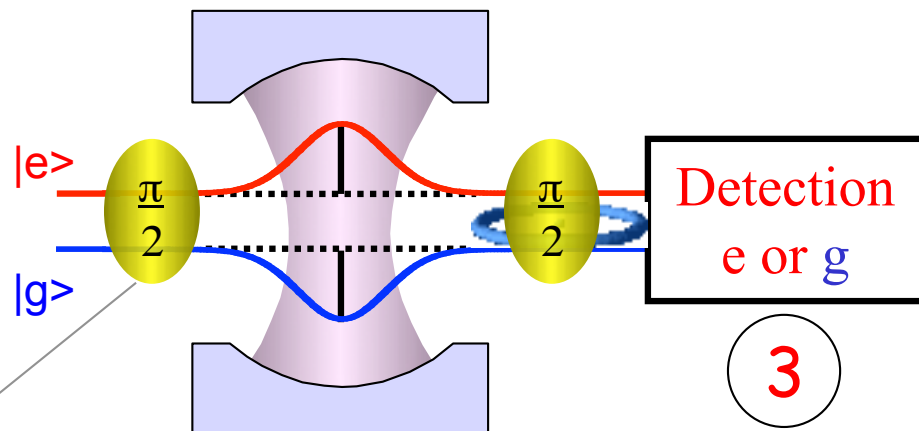
$$\delta_{mw} = \omega_{mw} - \omega_{at}$$

rotation by angle $\phi = \delta_{mw}T$ around the Oz axis

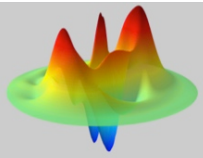


QND detection of 0 or 1 photon

1. Trigger of the clock.
2. precession of the spin through the cavity.
3. Detection of S_y : second $\pi/2$ rotation + detection of e-g



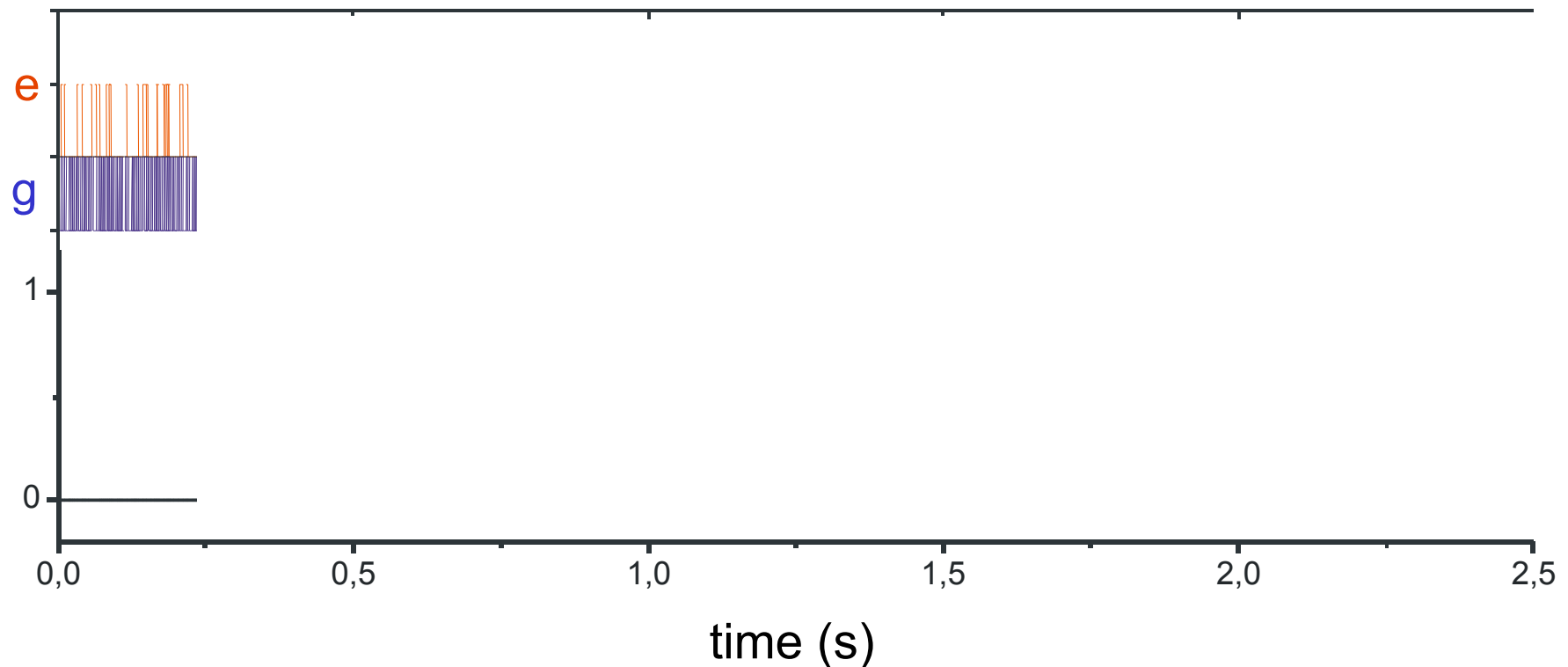
Atom detected in $e \Rightarrow$ field projected on $|1\rangle$
 $g \Rightarrow$ field projected on $|0\rangle$



Detecting blackbody photons

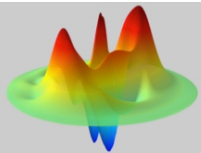
g \rightarrow field projected on $|0\rangle$

e \rightarrow field projected on $|1\rangle$



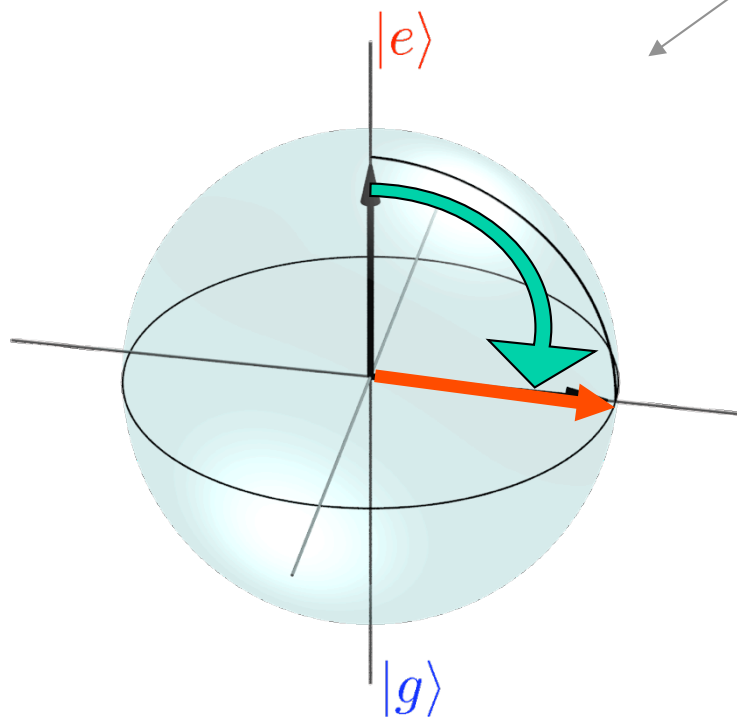
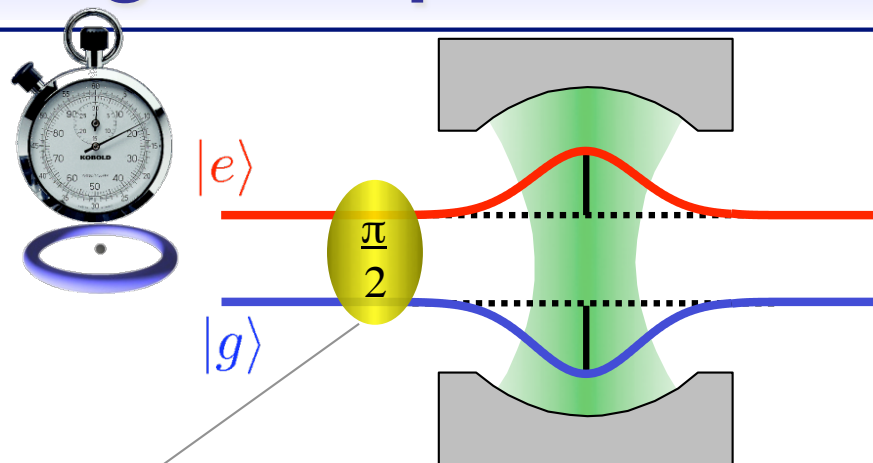
$$T = 0.8 \text{ K} \rightarrow n_{th} = 0.05 \quad (\text{proba. of } n=2 \text{ is negligible})$$

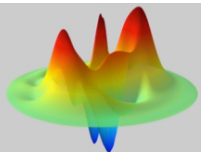
3. Counting more photons



Seeing more photons

1. Trigger of the atom clock:
resonant $\pi/2$ pulse

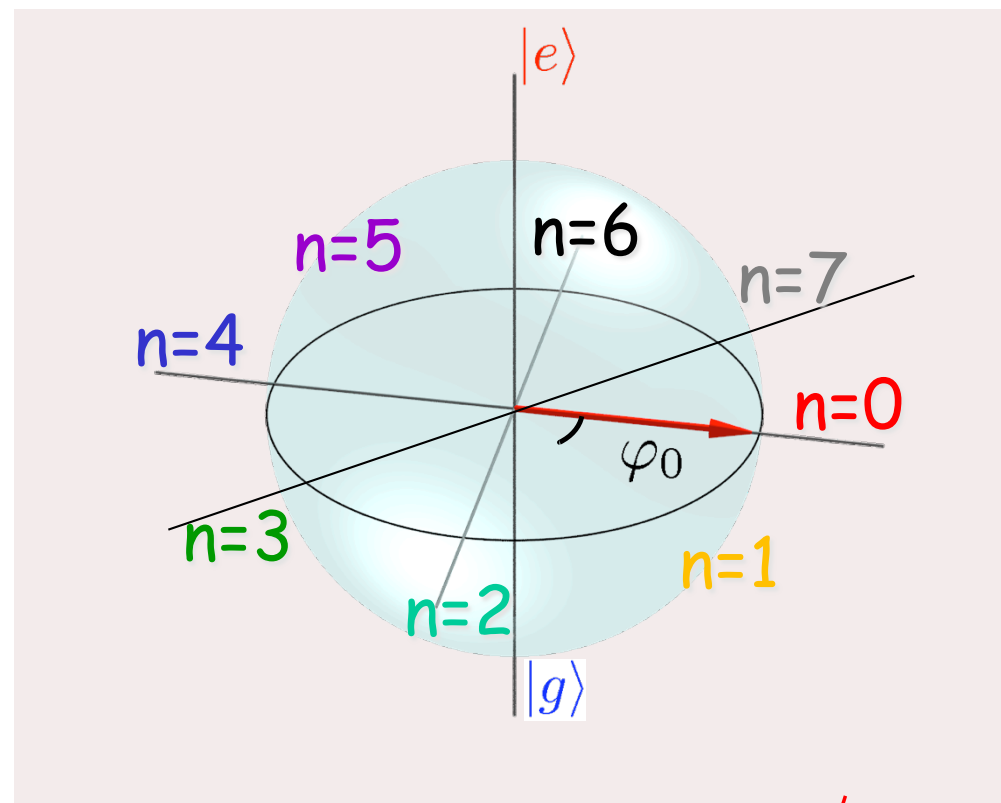
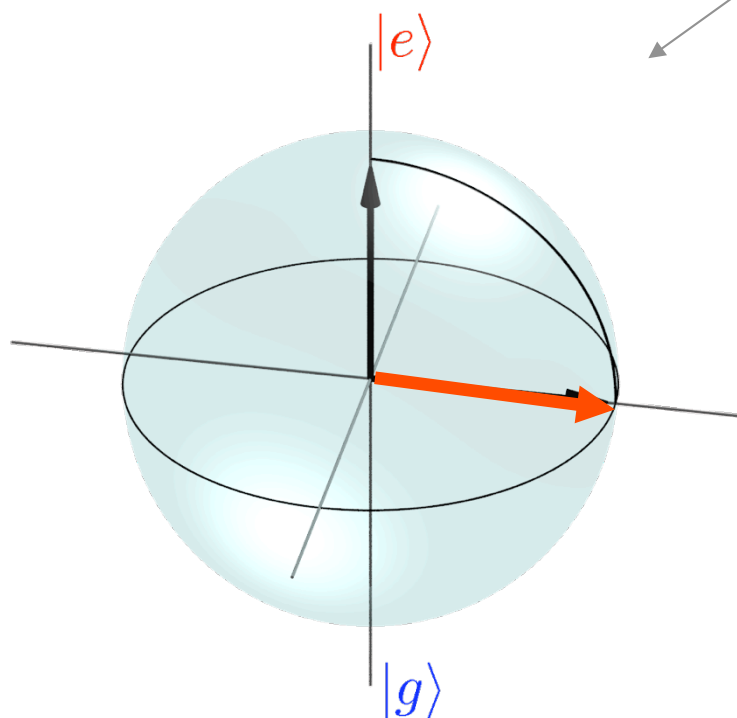
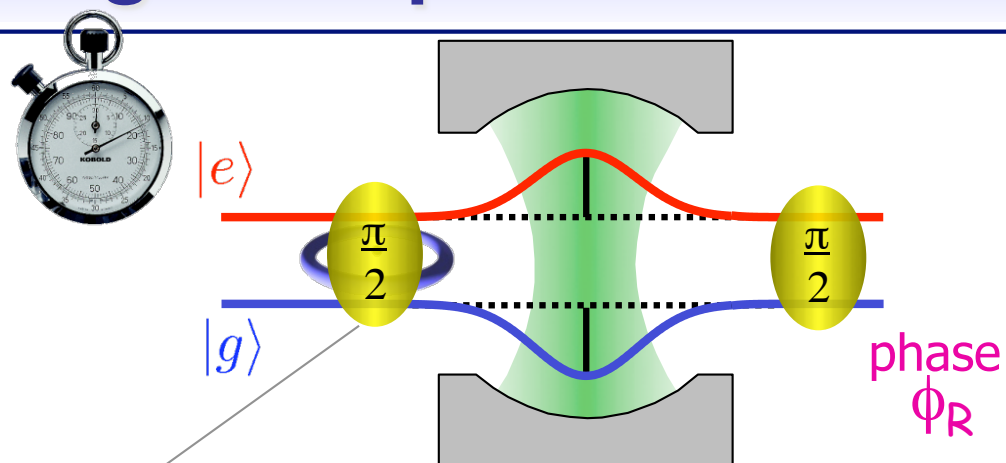




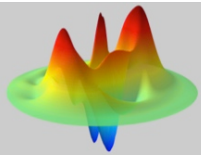
Seeing more photons

1. Trigger of the atom clock:
resonant $\pi/2$ pulse

2. Dephasing of the clock:
interaction with the cavity field



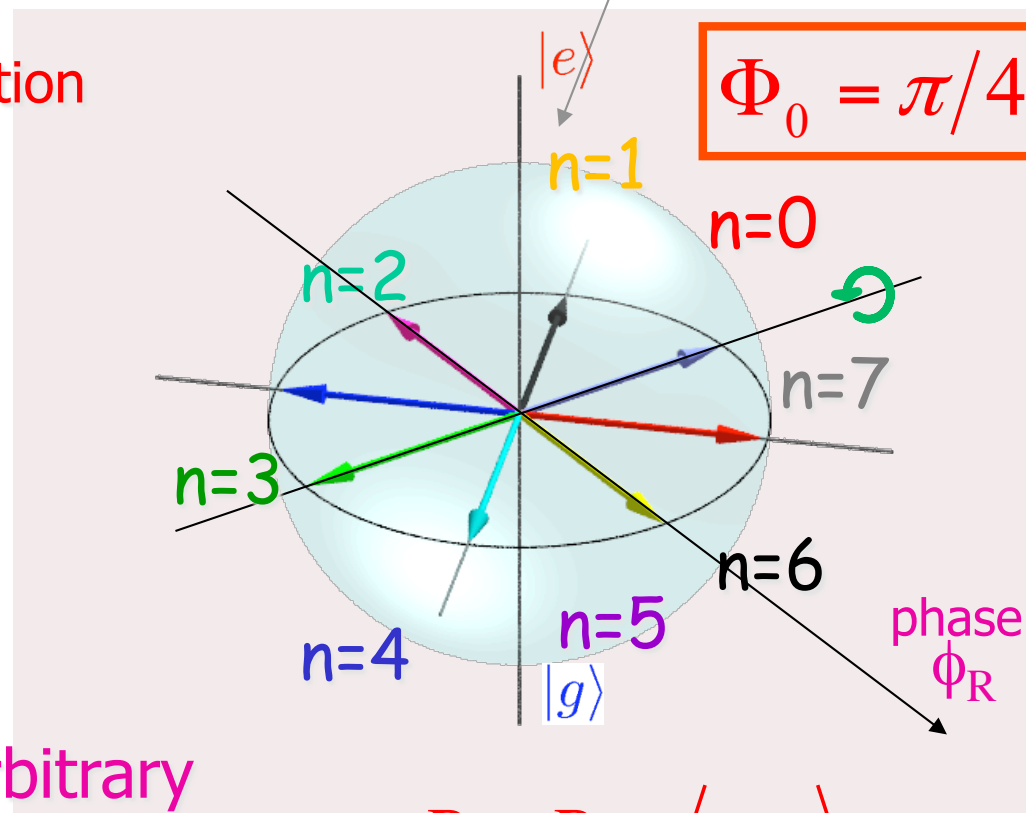
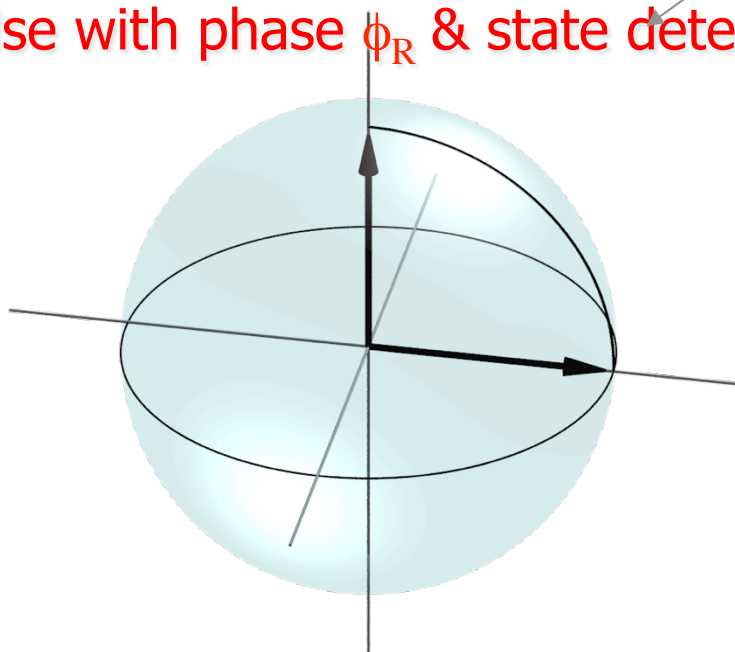
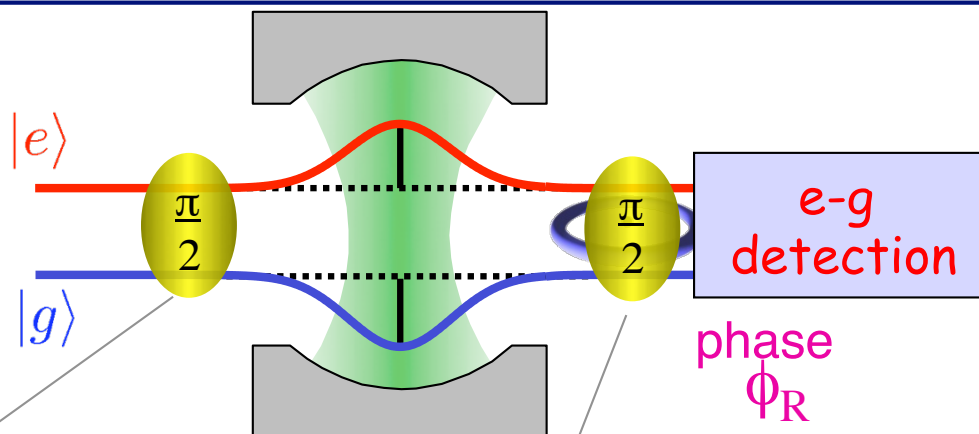
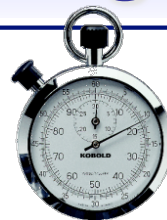
Phase shift per photon $\Phi_0 = \pi/4$



Seeing more photons

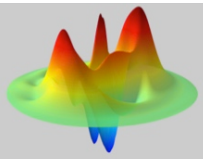
1. Trigger of the atom clock: resonant $\pi/2$ pulse
2. Dephasing of the spin: interaction with the cavity field

3. Measurement of the spin: $\pi/2$ pulse with phase ϕ_R & state detection

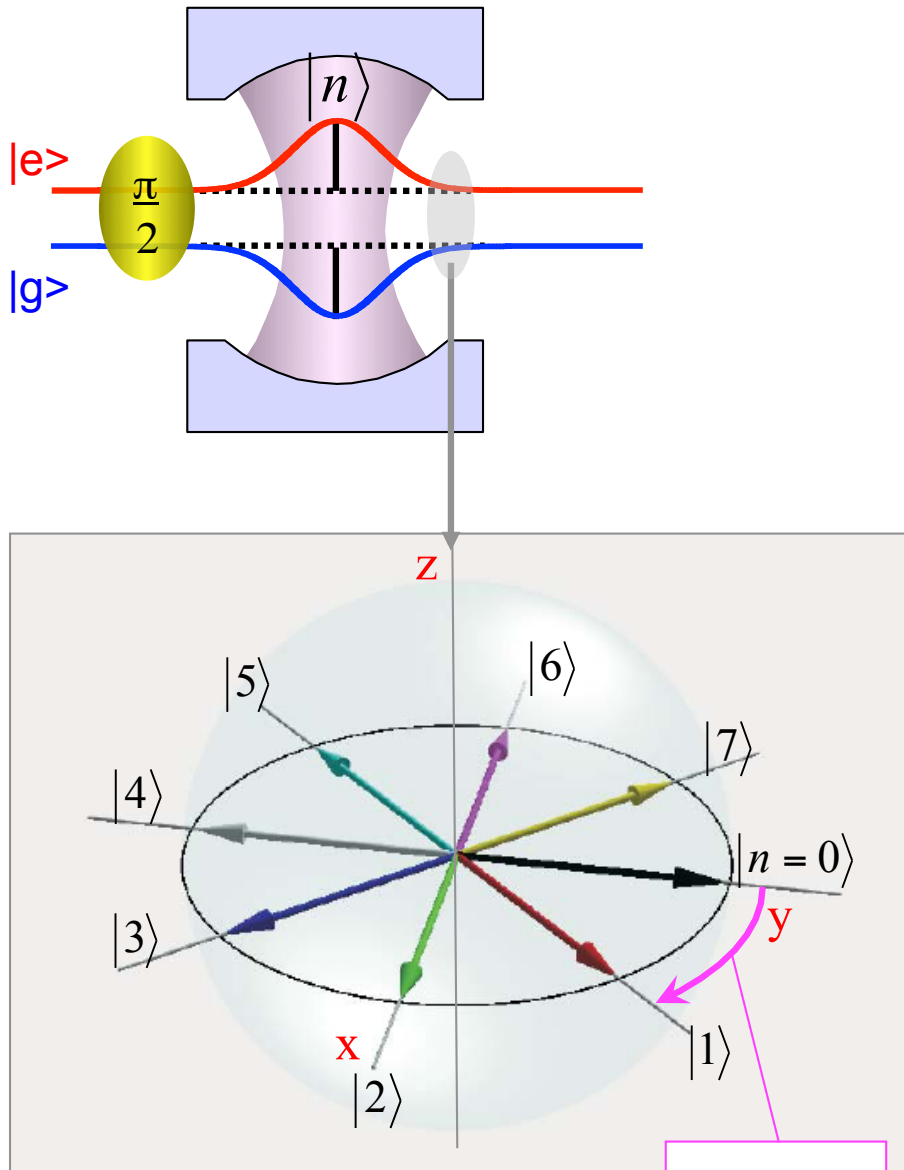


Pseudo-spin measurement in arbitrary direction determined by ϕ_R

$$P_e - P_g = \langle \sigma_{\phi_R} \rangle$$



Detection of $n > 1$



Chose

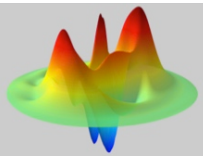
$$\Phi_0 = \frac{\pi}{4}$$

⇒ Photon numbers from 0 to 7 correspond to 8 different final position of the atom "spin"

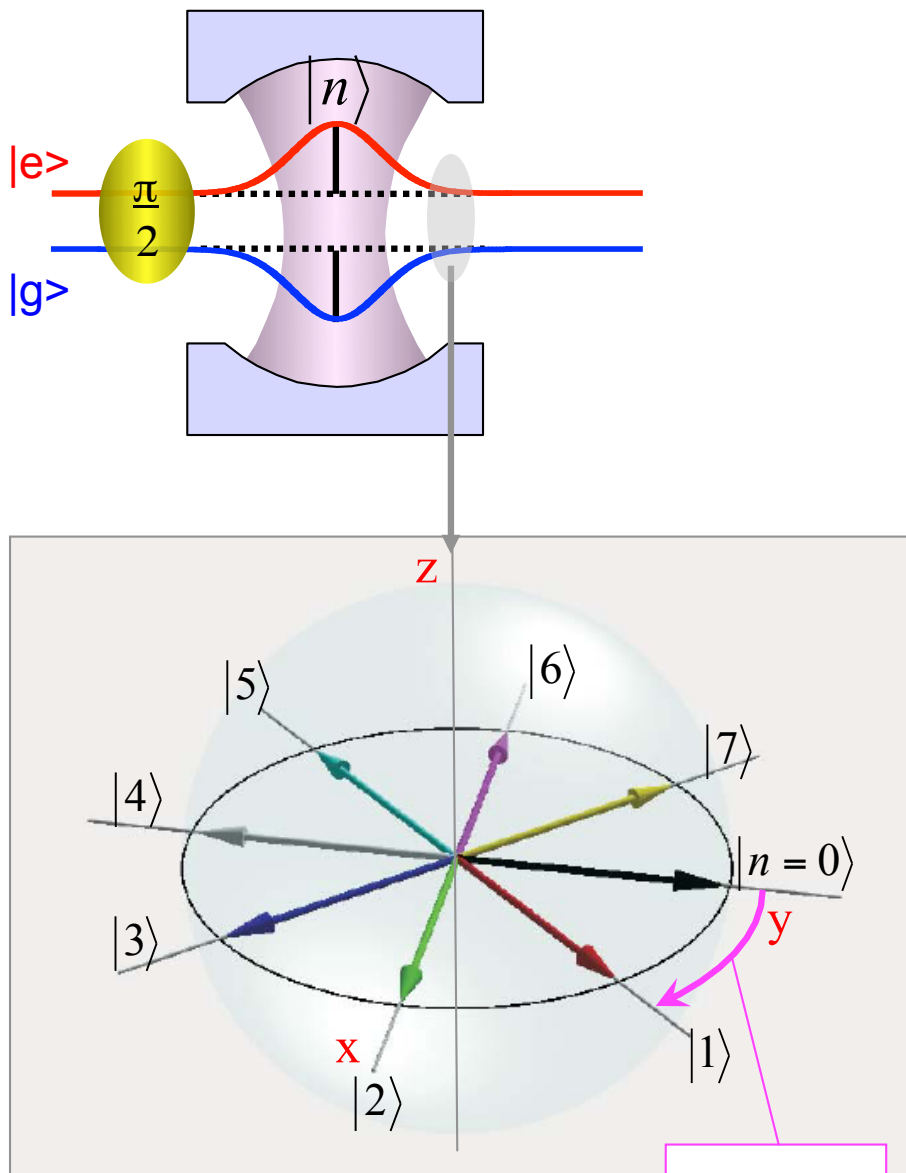
But these states are not orthogonal

⇒ detecting one atom is not enough to determine n .

$$\Phi_0 = \frac{\pi}{4}$$



Detection of $n > 1$



$$\Phi_0 = \frac{\pi}{4}$$

Interaction with one atom prepares:

$$|\Psi\rangle = \sum_n C_n |+_n \Phi_0\rangle \otimes |n\rangle$$

⇒ Repeat measurement

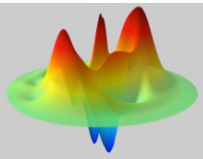
↓ N atoms

$$|\Psi\rangle = \sum_n |+_n \Phi_0\rangle^N \otimes |n\rangle$$

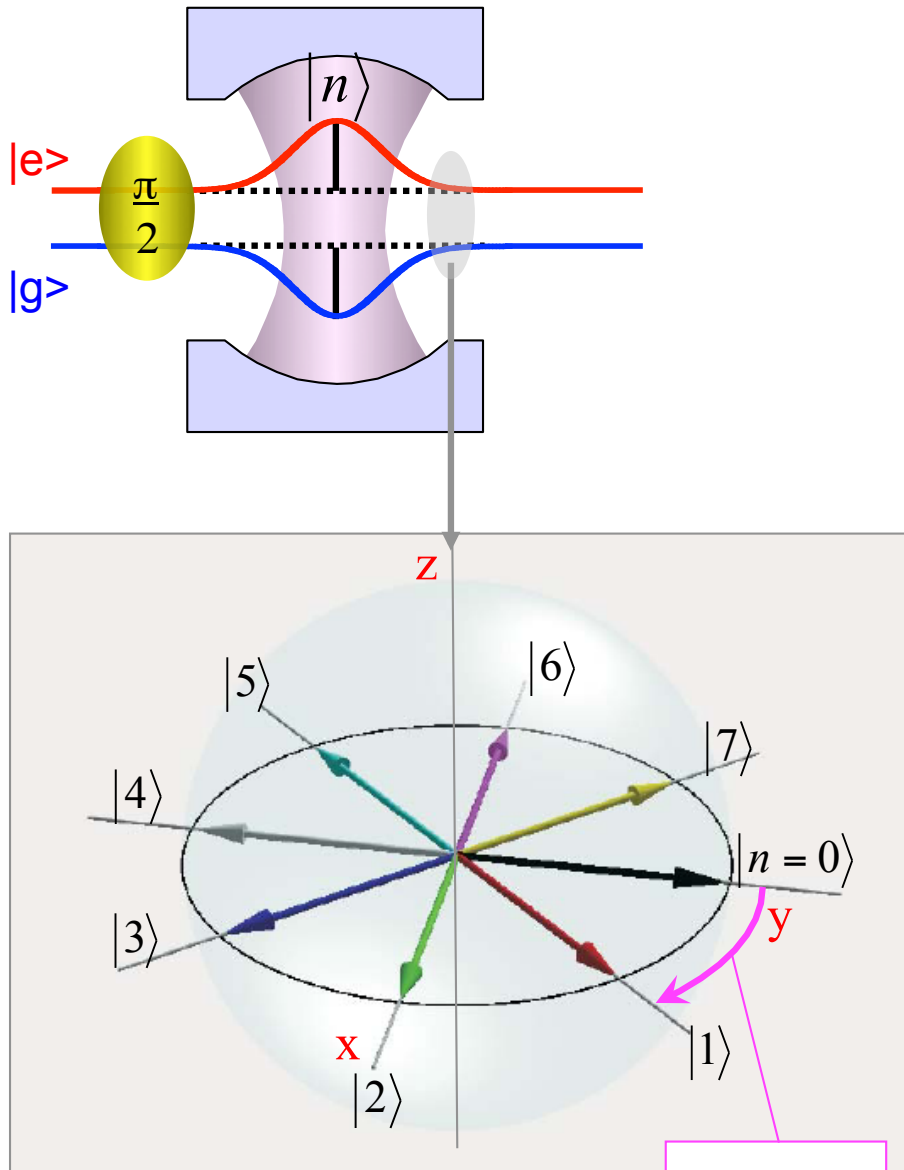
The photon number is now encoded in a mesoscopic sample of atoms.

$$\left| \langle +_{n'} \Phi_0 | +_n \Phi_0 \rangle \right|^N \approx 0$$

Orthogonal states if N large enough



Detection of $n > 1$



$$\Phi_0 = \frac{\pi}{4}$$

Interaction with one atom prepares:

$$|\Psi\rangle = \sum_n C_n |+_n \Phi_0\rangle \otimes |n\rangle$$

⇒ Repeat measurement

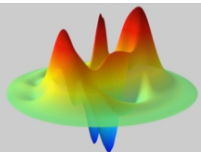
↓ N atoms

$$|\Psi\rangle = \sum_n |+_n \Phi_0\rangle^N \otimes |n\rangle$$

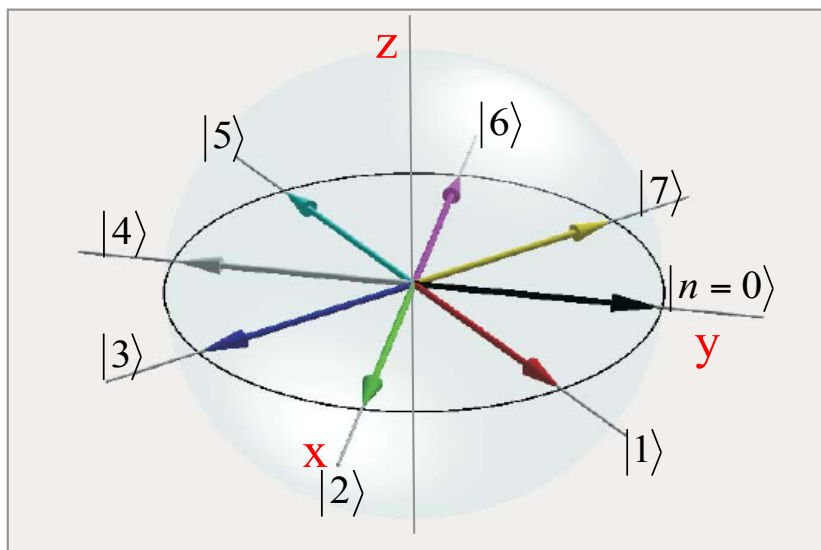
The photon number is now encoded in a mesoscopic sample of atoms.

That is a Schrödinger cat state:

the N atom collective spin points in a direction indicating the photon number



Décoding the photon number



$$|\Psi\rangle = \sum_n C_n \left| +_n \Phi_0 \right\rangle^N \otimes |n\rangle$$

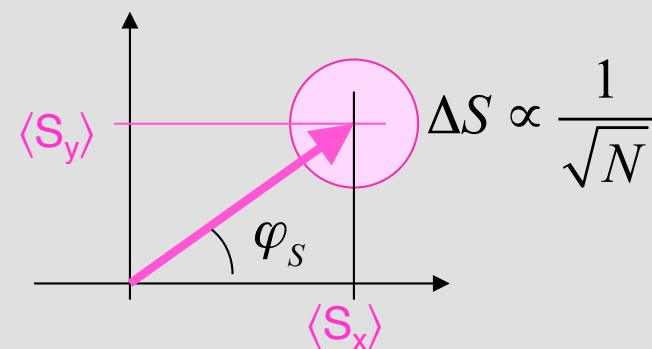
For each n , one detects N identical copies of the atomic state

$$\left| +_n \Phi_0 \right\rangle$$

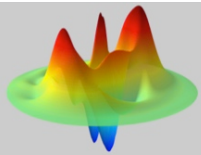
Determination of atom spin by « tomography »:

N atoms $\rightarrow N/4$ atoms: measure $\langle S_{\phi_R} \rangle$
with 4 different settings of ϕ_R

\rightarrow calculate $\langle S_x \rangle$ and $\langle S_y \rangle$

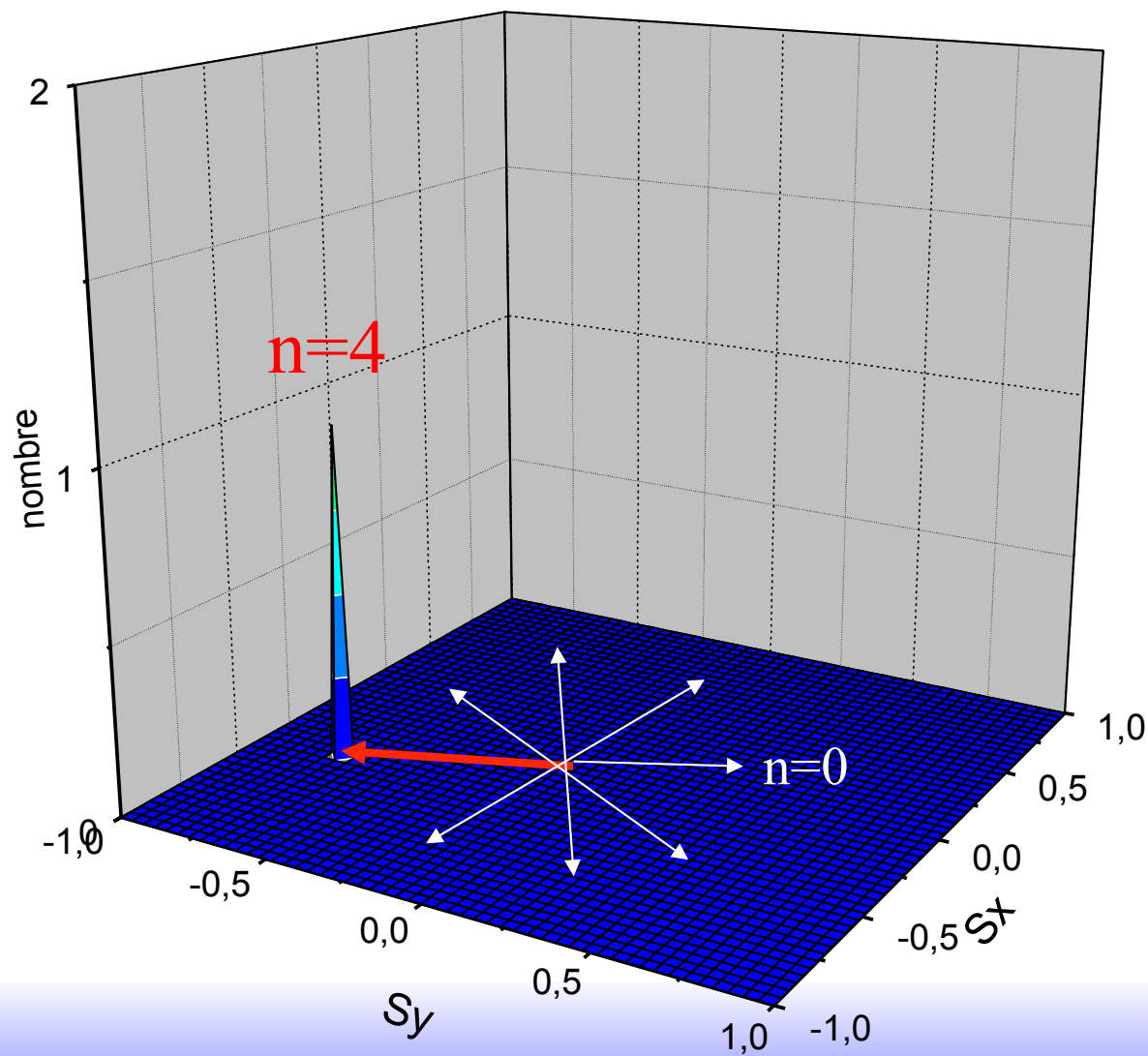


For large enough N , $\Delta\varphi_S \propto \frac{1}{\sqrt{N}} < \Phi_0$ and different photon numbers should be distinguished



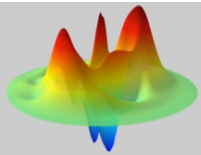
Atom spin state tomography

- Method:
- 1- inject a coherent field $\langle n \rangle = 3.5$ photons.
 - 2- detection of 110 consecutive atoms, $T_{\text{measure}} = 26$ ms



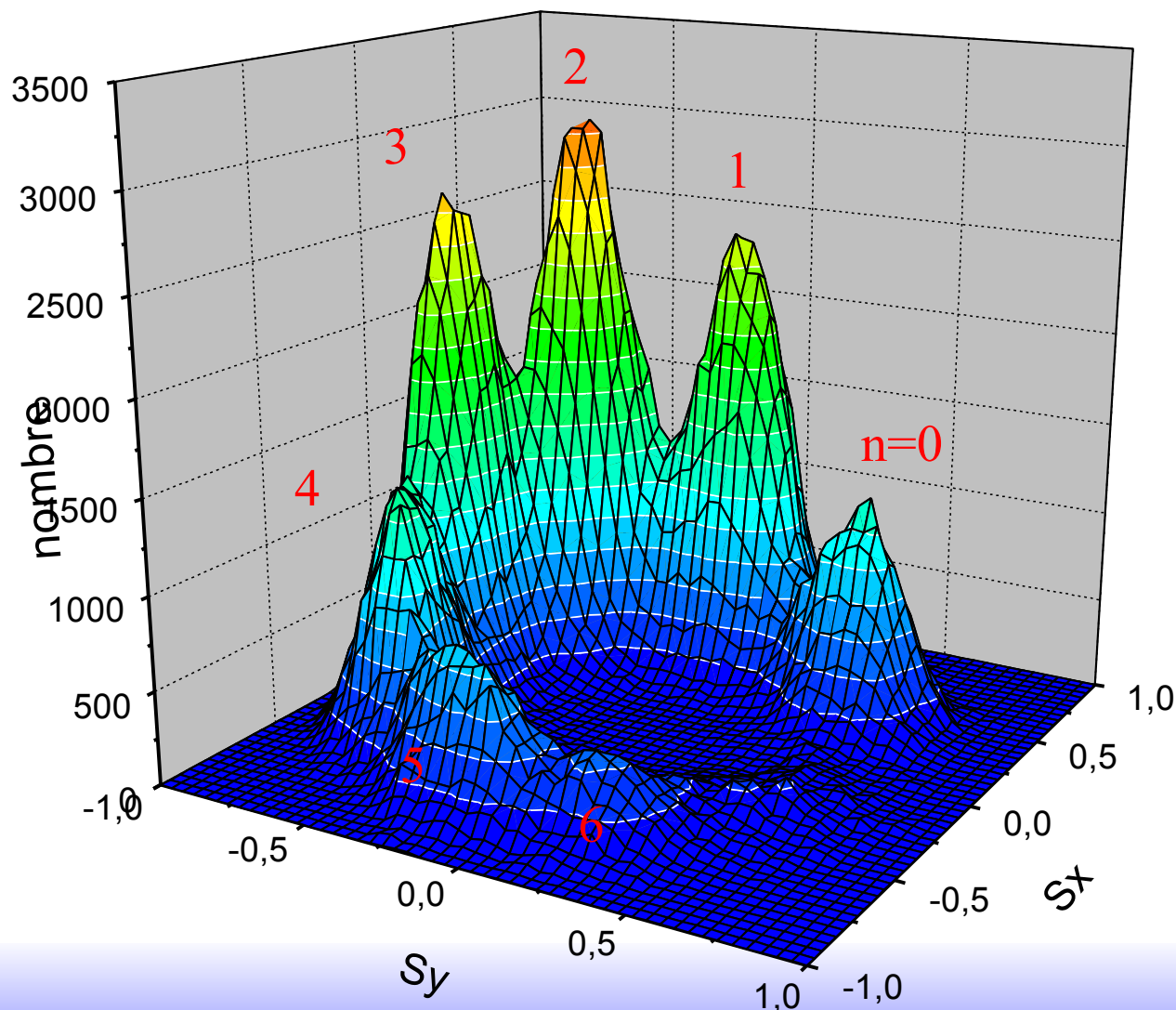
One measurement

$\rightarrow n = 4$



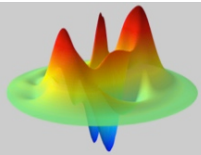
Tomographie de l'état atomique

- Method: 1- inject a coherent field $\langle n \rangle = 3.5$ photons.
2- detection of 110 consecutive atoms, $T_{\text{measure}} = 26$ ms

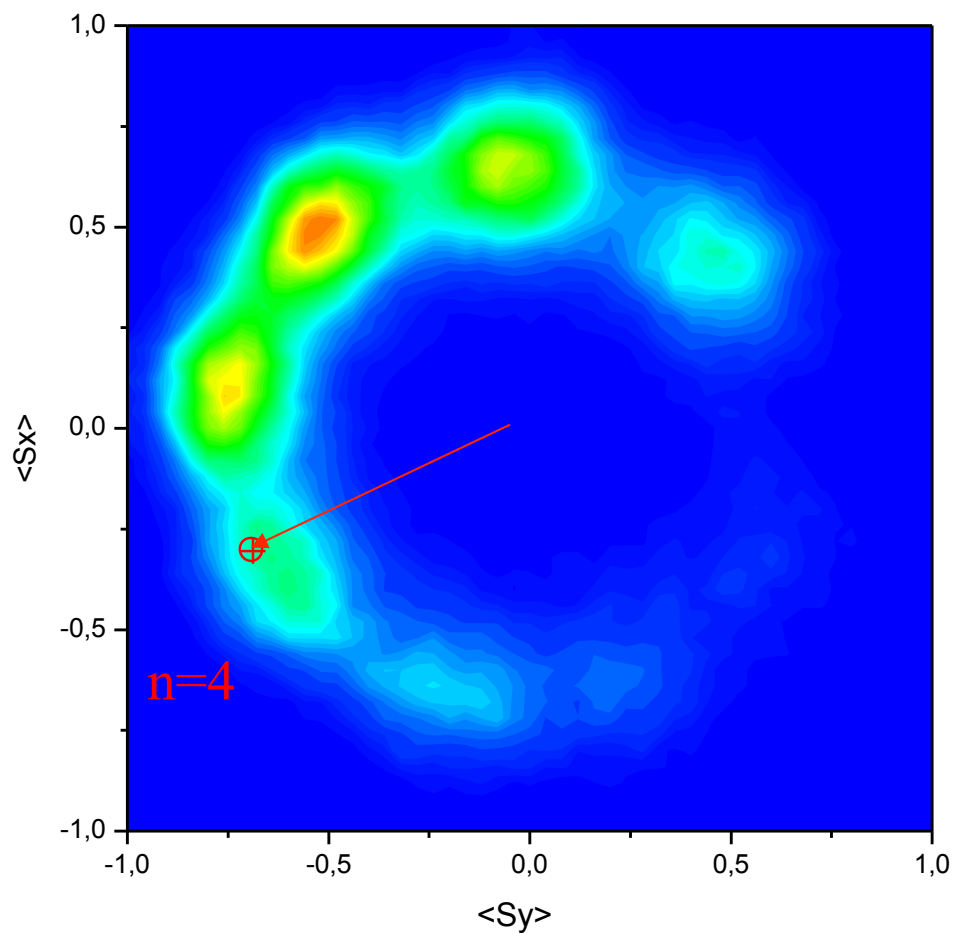


$\langle n \rangle = 2.4$ photons

The collective spin of N atoms points in discrete direction
 $\Rightarrow n$ is obviously quantized
Detecting a collection of 110 atoms is enough to fully determine the photon number

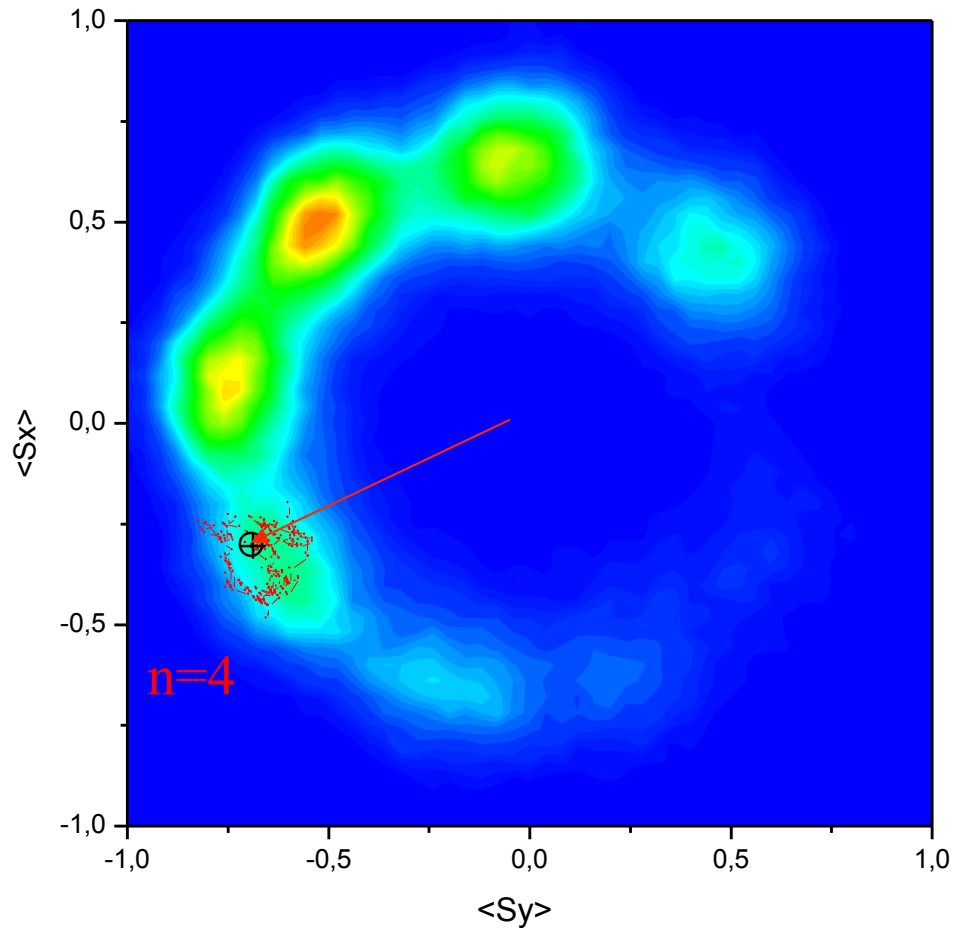
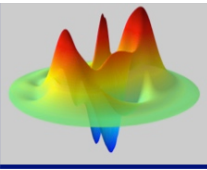


Observing field decay directly on the collective atom spin



- Preparation of an initially coherent 3.5 photons field

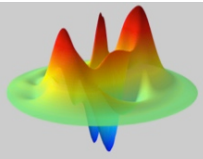
- First measurement: projection on $n=4$



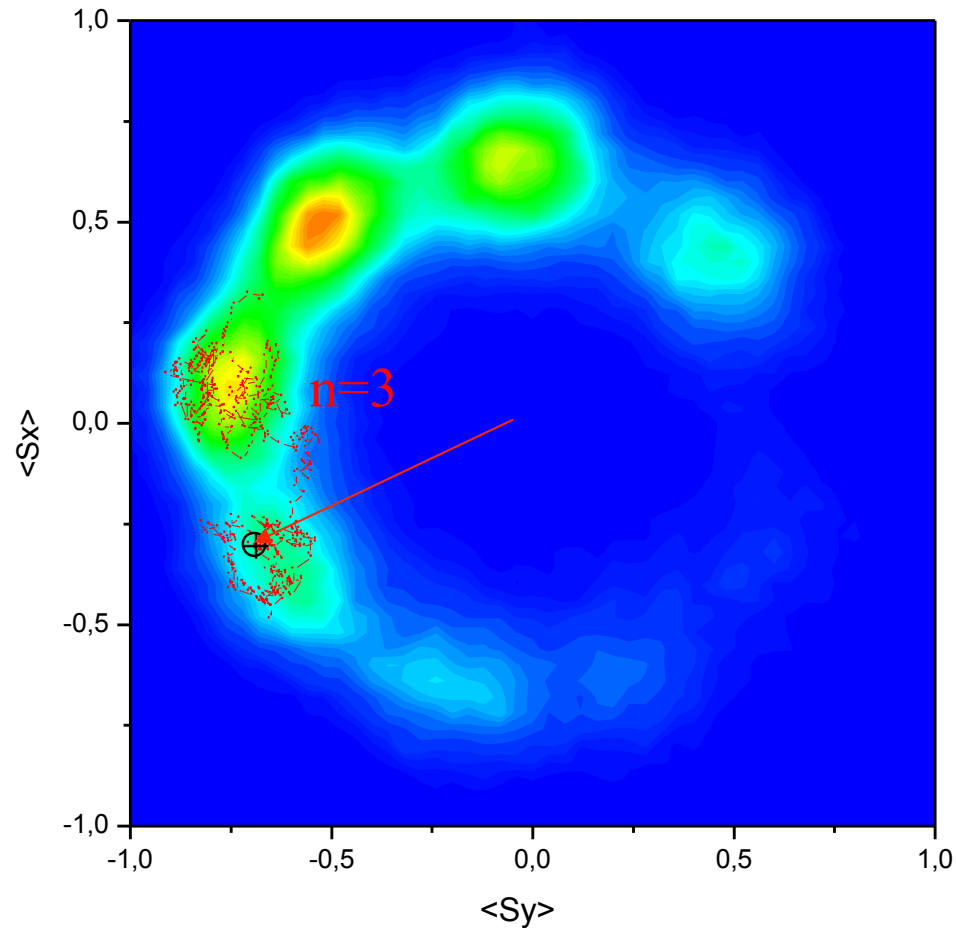
- Preparation of an initially coherent 3.5 photons field

- First measurement:
projection on $n=4$

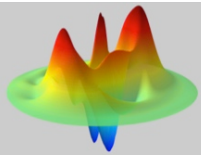
Due to statistical noise the spin fluctuates around $n=4$



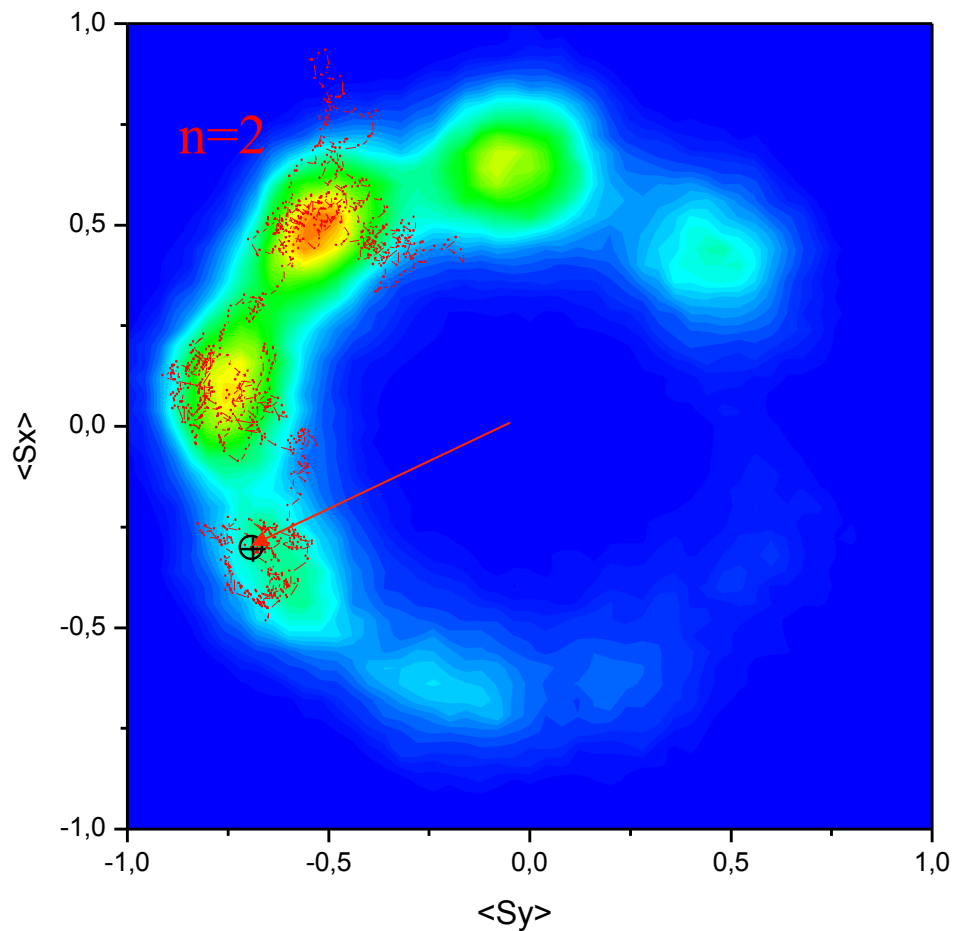
Observing field decay directly on the collective atom spin



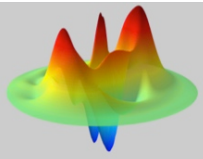
- Preparation of an initially coherent 3.5 photons field
- First measurement: projection on $n=4$
- quantum jump to $n=3$



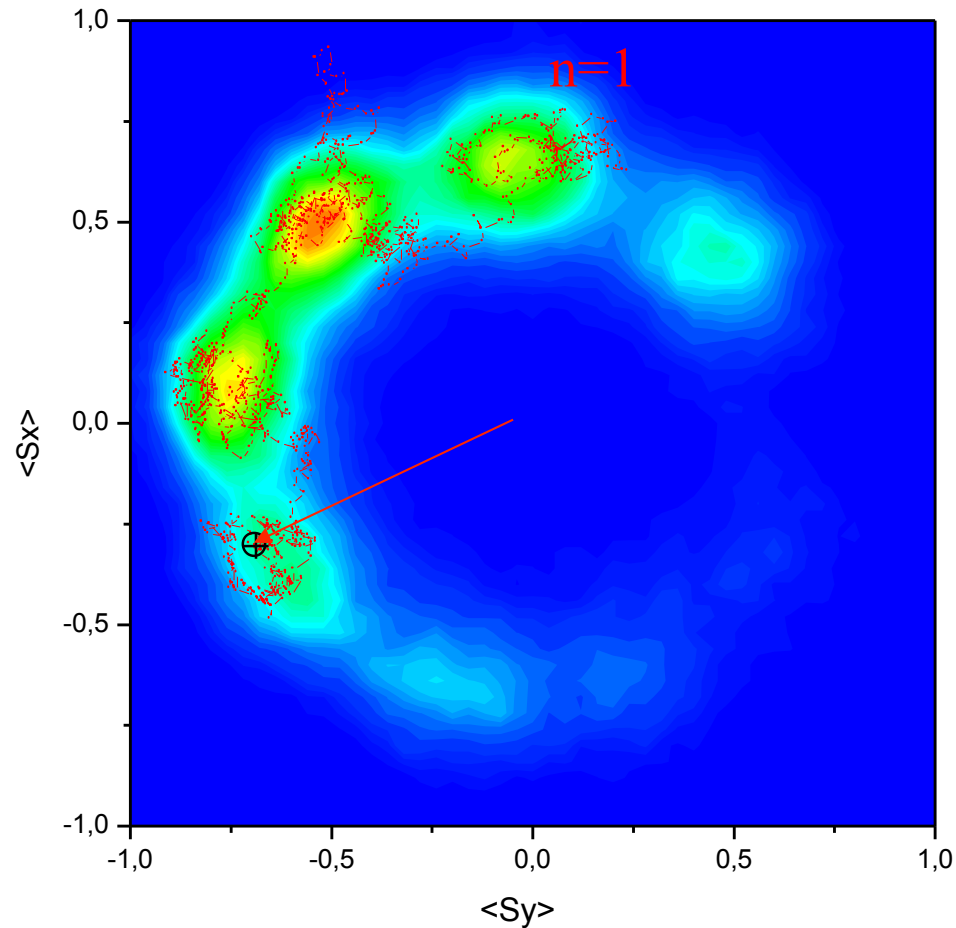
Observing field decay directly on the collective atom spin



- Preparation of an initially coherent 3.5 photons field
- First measurement: projection on $n=4$
- quantum jump to $n=3, 2$



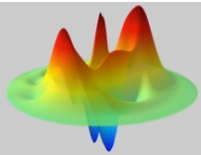
Observing field decay directly on the collective atom spin



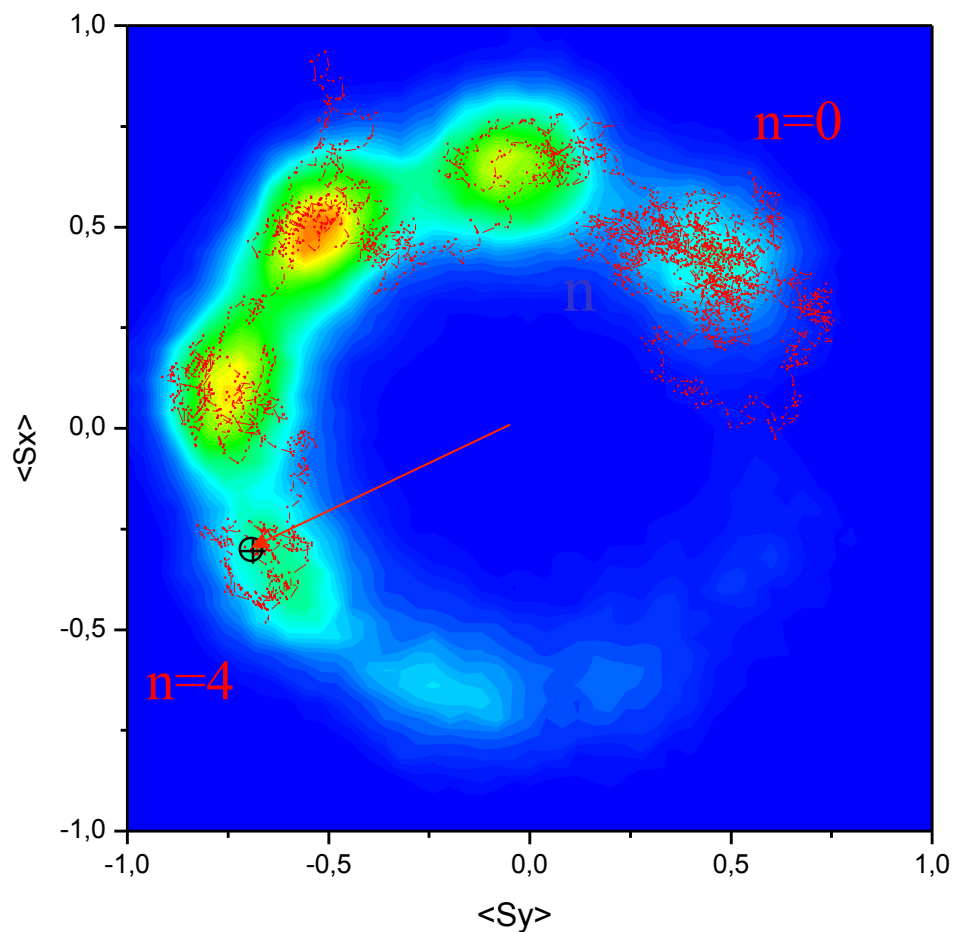
- Preparation of an initially coherent 3.5 photons field

- First measurement:
projection on $n=4$

- quantum jump to $n=3, 2, 1, \dots$



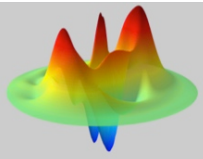
Observing field decay directly on the collective atom spin



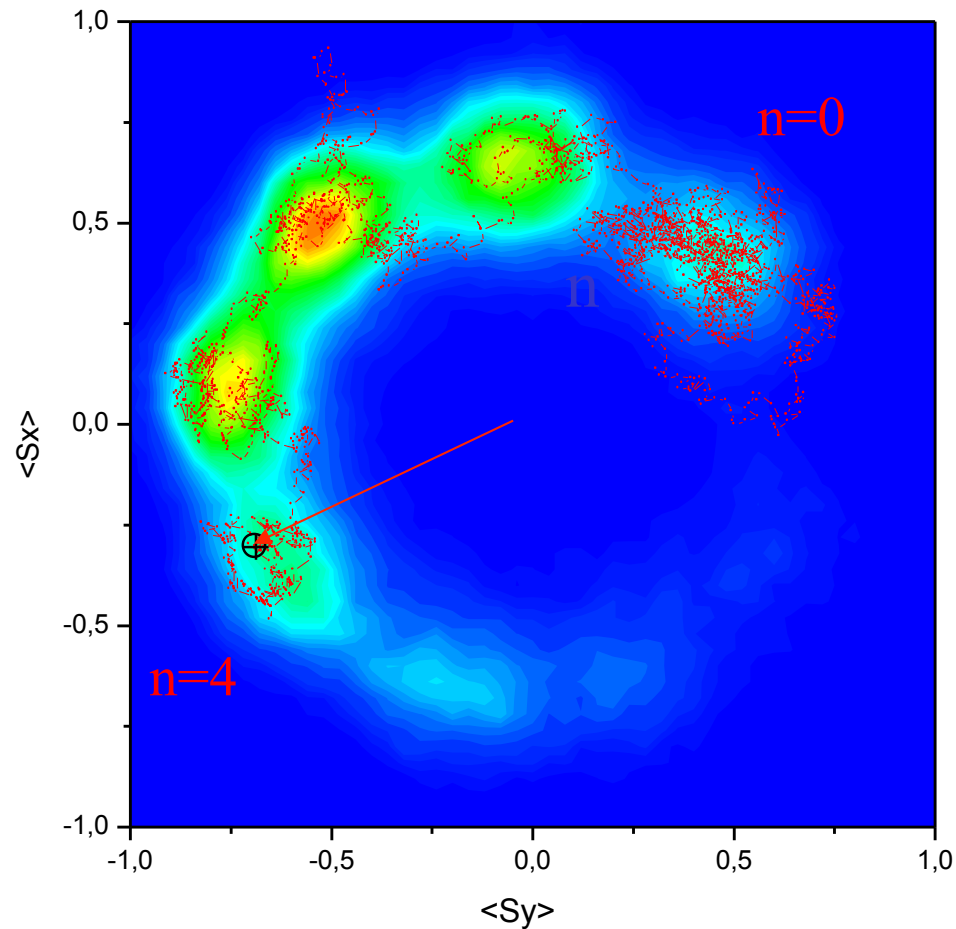
- Preparation of an initially coherent 3.5 photons field

- First measurement:
projection on $n=4$

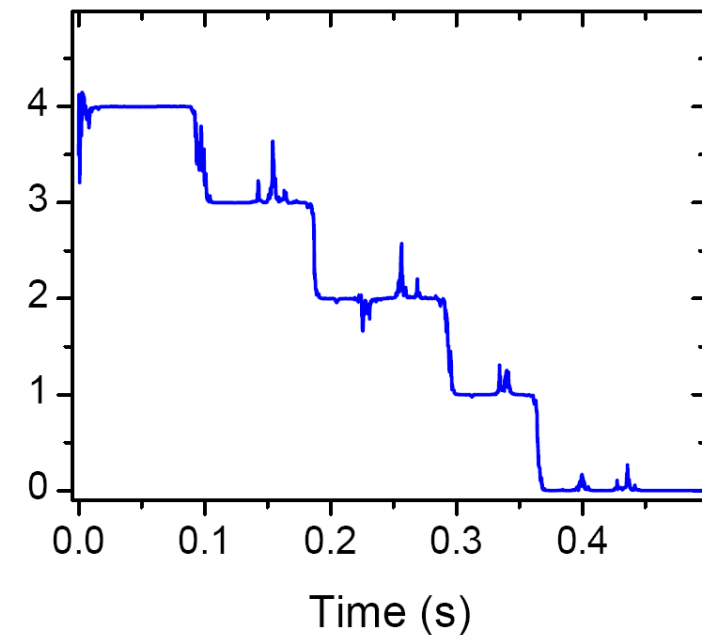
- quantum jump to $n=3, 2, 1, \dots, 0$



Observing field decay directly on the collective atom spin

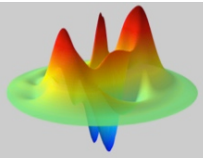


Average photon number evolution



Quantum jumps down to $n=0$.

→ now describe this process in term of progressive acquisition of information by applying the projection postulate atom by atom



Progressive accumulation of information

Apply the projection postulate at each atom detection.

$P_0(n)$ 1- Initial field state



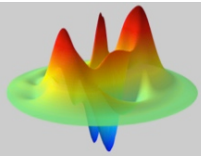
$P_1(n)$



$P_2(n)$

....

$P_N(n)$



Progressive accumulation of information

Apply the projection postulate at each atom detection.

$P_0(n)$

1- Initial field state

ϕ_R randomly chosen among 4 values
aligned on the 4 possible spin directions

2- First measurement of $\hat{S}_{\phi_R} \rightarrow$ résultat: $+j$ or $-j$
 \rightarrow Projection of the atom-field state:

\Downarrow

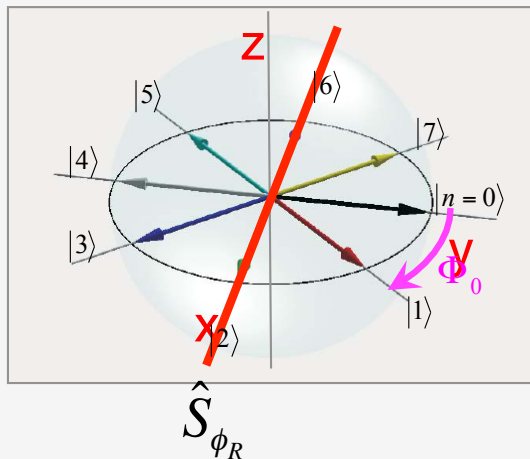
$P_1(n)$

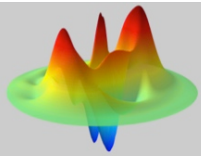
\Downarrow

$P_2(n)$

....

$P_N(n)$





Progressive accumulation of information

Apply the projection postulate at each atom detection.

$P_0(n)$

1- Initial field state

ϕ_R randomly chosen among 4 values aligned on the 4 possible spin directions

2- First measurement of $\hat{S}_{\phi_R} \rightarrow$ résultat: $+_j$ or $-_j$
 \rightarrow Projection of the atom-field state:



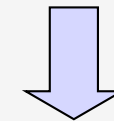
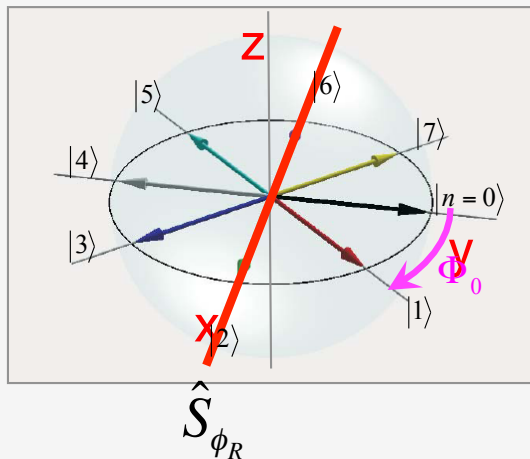
$P_1(n)$



$P_2(n)$

....

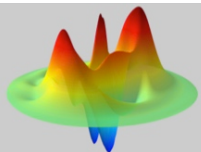
$P_N(n)$



Bayse law

$$P_1(n) = P(n / \pm_{\varphi}) = P_0(n) \cdot P(\pm_{\varphi} / n) \cdot \frac{1}{Z}$$

$Z = P(\pm_{\varphi})$ norm. factor



Progressive accumulation of information

Apply the projection postulate at each atom detection.

$P_0(n)$

1- Initial field state

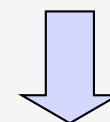
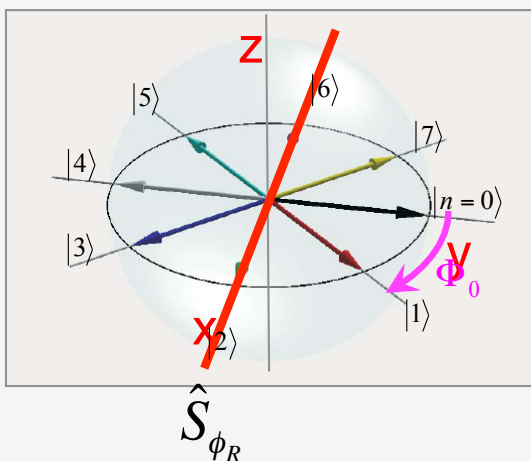
ϕ_R randomly chosen among 4 values aligned on the 4 possible spin directions

2- First measurement of $\hat{S}_{\phi_R} \rightarrow$ résultat: $+_j$ or $-_j$

\rightarrow Projection of the atom-field state:



$P_1(n)$



Bayse law

$$P_1(n) = P(n / \pm_{\varphi}) = P_0(n) \cdot P(\pm_{\varphi} / n) \cdot \frac{1}{Z}$$

$Z = P(\pm_{\varphi})$ norm. factor



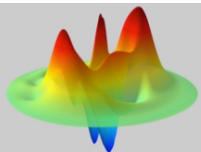
$P_2(n)$

....

3- iterate the process until detection of N atoms

$P_N(n)$

Note: field coherence do not play any role, P((n) is enough here.



Information acquisition by detecting 1 atom

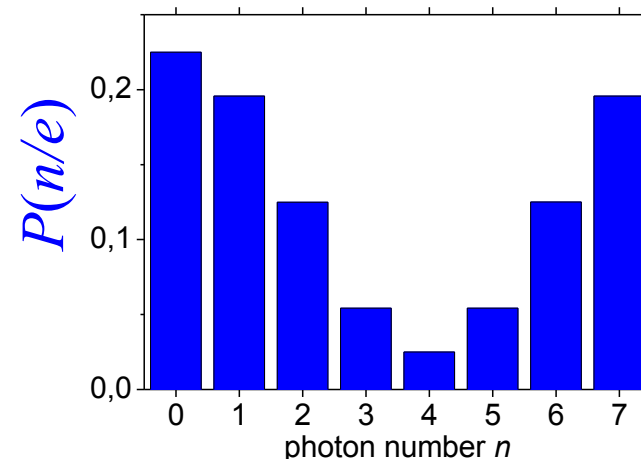
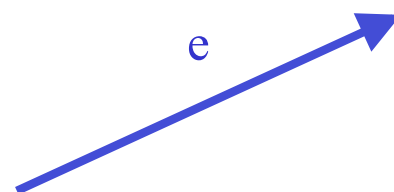
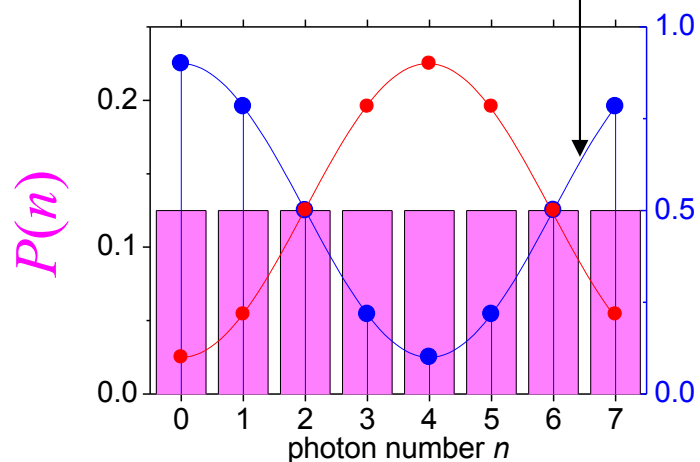
Bayes law:

$$P_{after}(n) = P(n / j_{\phi_R}) = P_{before}(n) \cdot \frac{P(j_{\phi_R} / n)}{P(j_{\phi_R})}$$

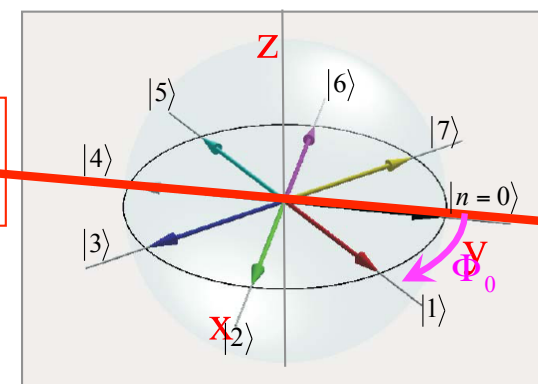
$$j_{\phi_R} = 1 \text{ or } -1 \\ = e \text{ or } g$$

$$\phi_R = 0$$

$$P(+_{\phi_R} / n) = |\langle +_{\phi_R} | +n \rangle|^2$$

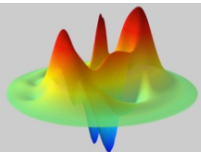


$$\phi_R = 0$$



Probability of n that are incompatible with the measurement result are cancelled.

Repeating the measurement with other values of j decimates other photon numbers



Information acquisition by detecting 1 atom

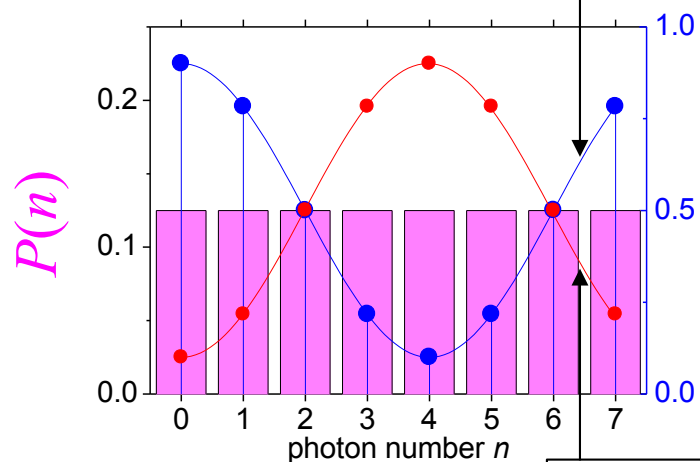
Bayes law:

$$P_{after}(n) = P(n / j_{\phi_R}) = P_{before}(n) \cdot \frac{P(j_{\phi_R} / n)}{P(j_{\phi_R})}$$

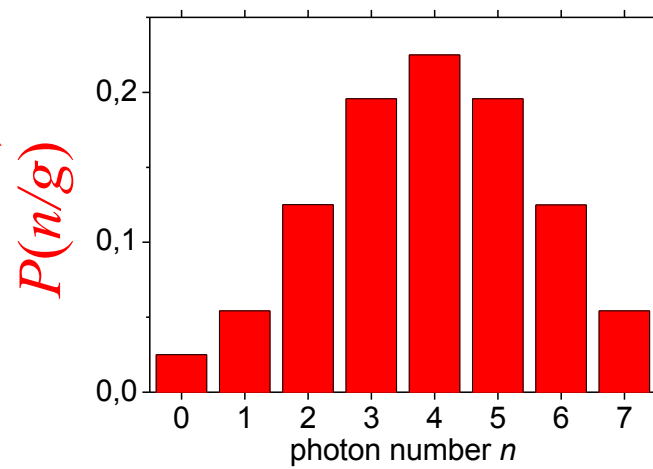
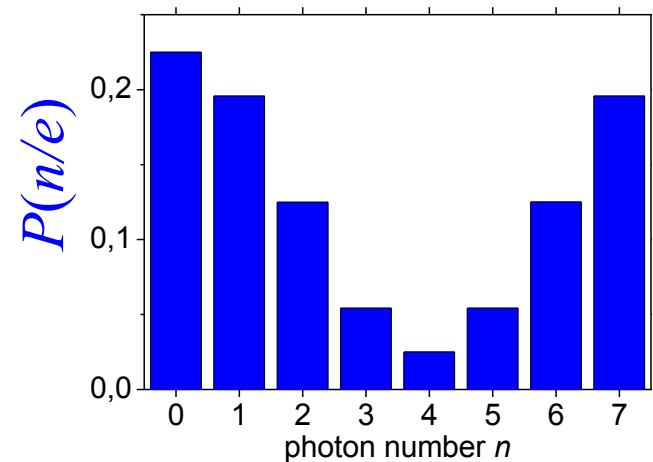
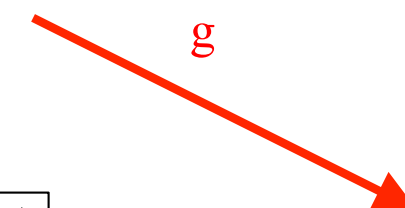
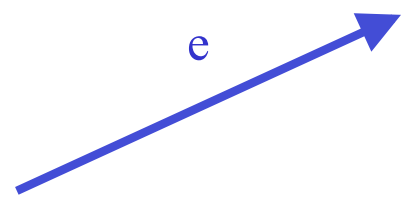
$$j_{\phi_R} = 1 \text{ or } -1 \\ = e \text{ or } g$$

$$\phi_R = 0$$

$$P(+_{\phi_R} / n) = |\langle +_{\phi_R} | +n \rangle|^2$$

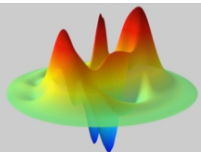


$$P(-_{\phi_R} / n)$$



Probability of n that are incompatible with the measurement result are cancelled.

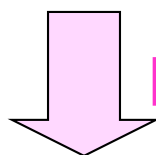
Repeating the measurement with other values of j decimates other photon numbers



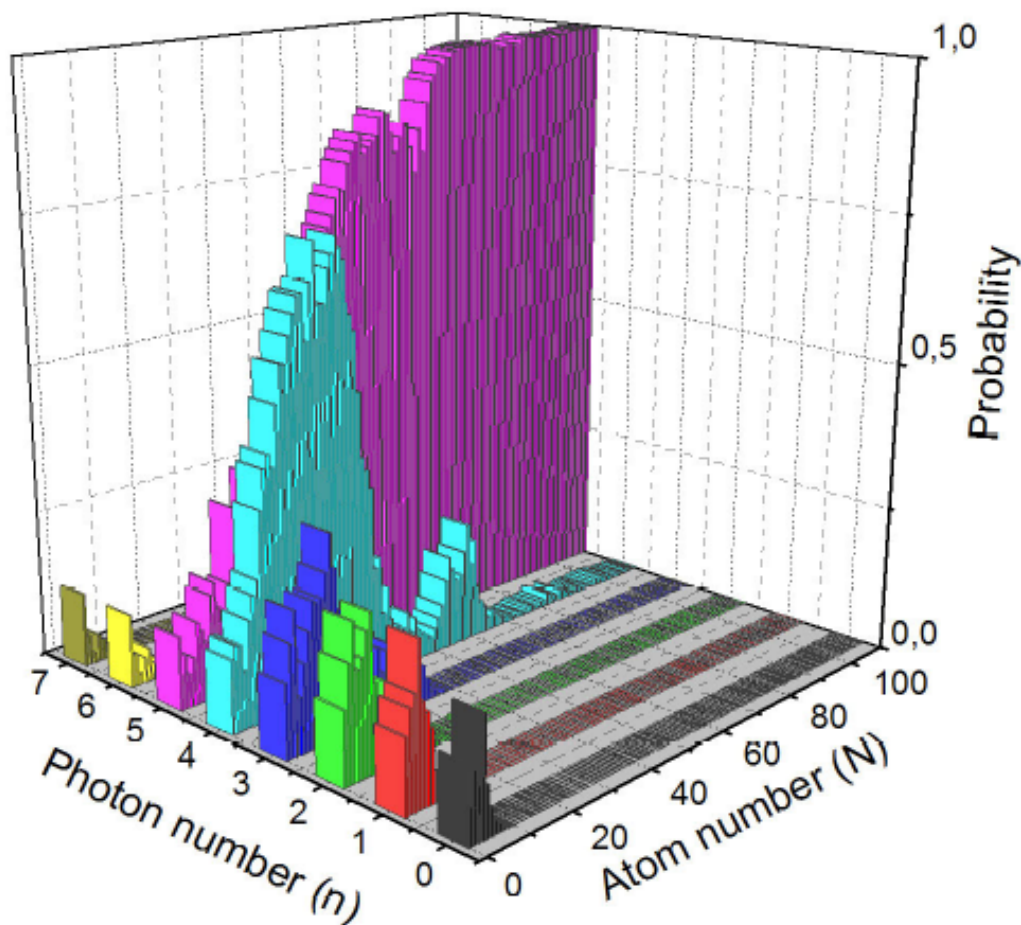
Progressive field collapse

$j(k)$ 11011111111100111011011110101001101010101101011111
 $\phi_R(k)$ ddcbccabcdaadaabadddbadbcdababbaacbccdadccdcbaaacc

k = atom index



Decoding (real data, not simulation)

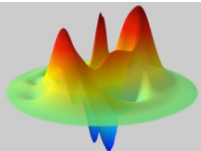


Initial coherent state
 $\langle n \rangle = 3.7 (\pm 0.008)$

Flat initial photon number distribution.

The measurement result is determined by the real field

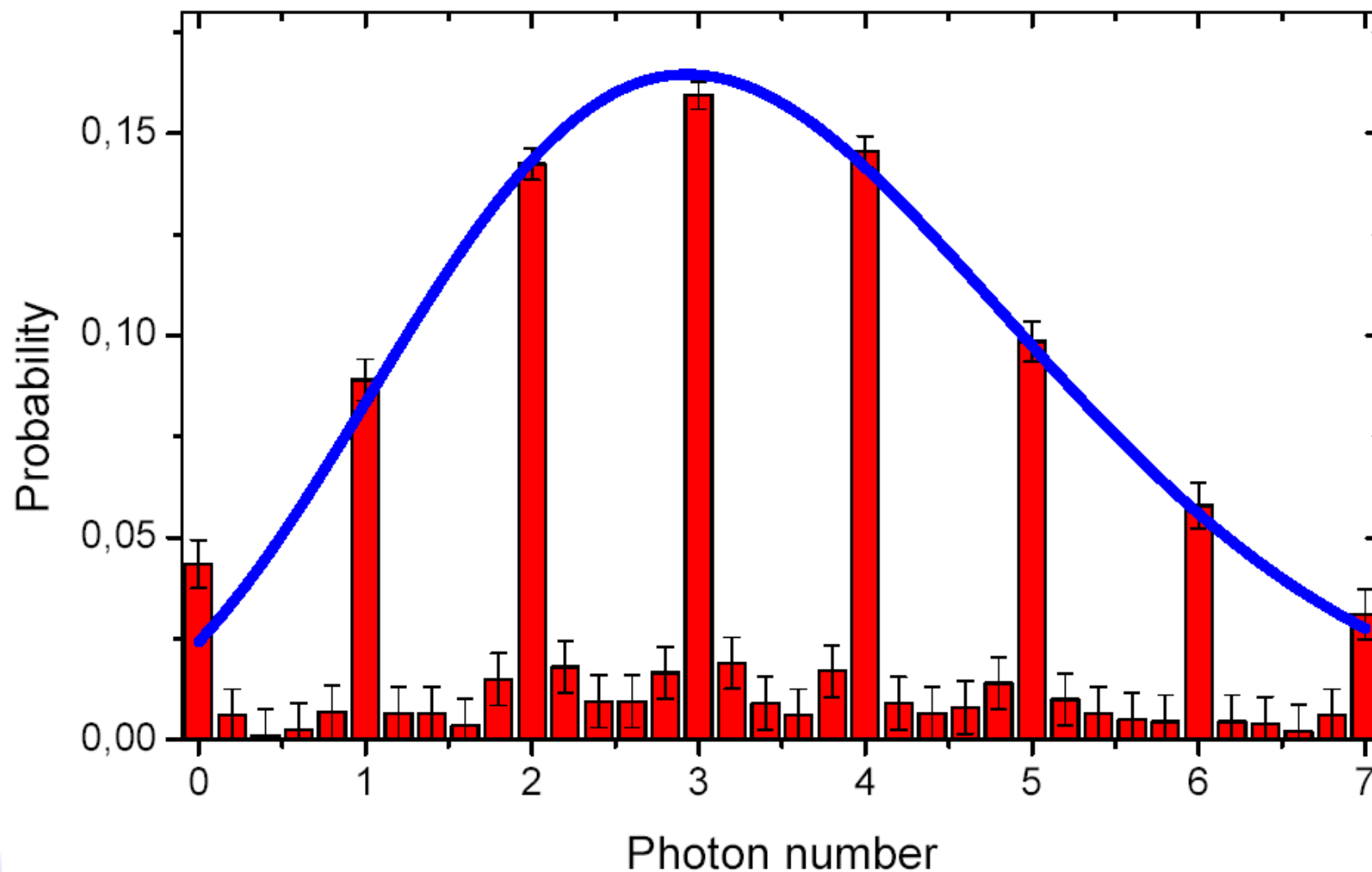
Progressive projection of the field on $n=5$ number state

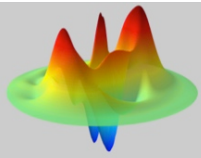


Reconstructing the photon number statistics

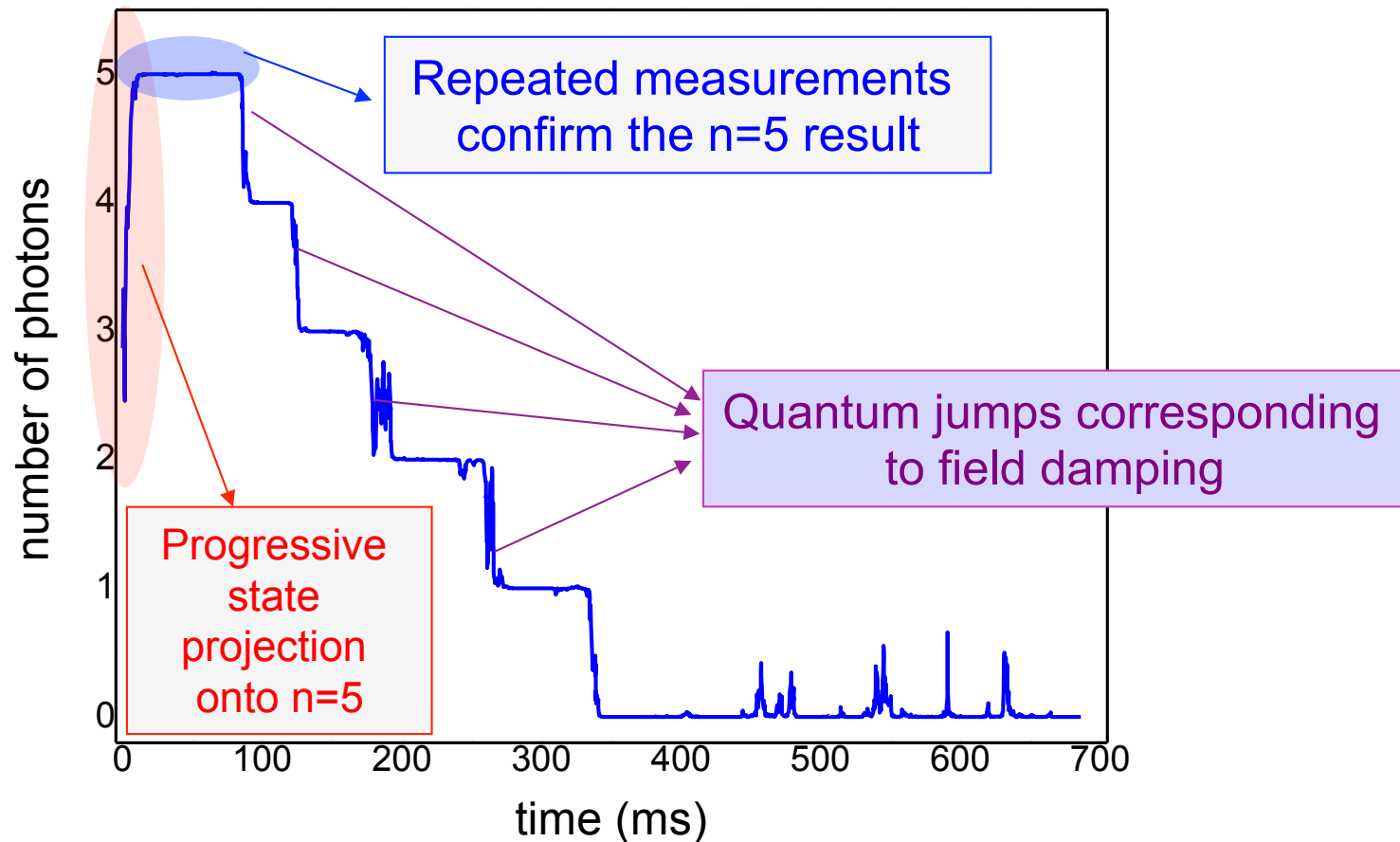
Coherent field at measurement time

$$\langle n \rangle = 3.4 \pm 0.008$$



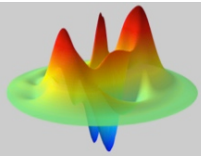


Repeated measurements: evolution of a continuously monitored field



Field evolution due to cavity damping: not to QND measurement

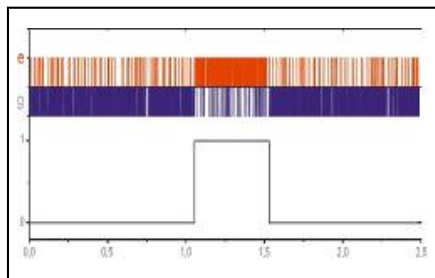
- Exhibits all features of quantum theory of measurement:
 - State collapse / Random result / repeatability



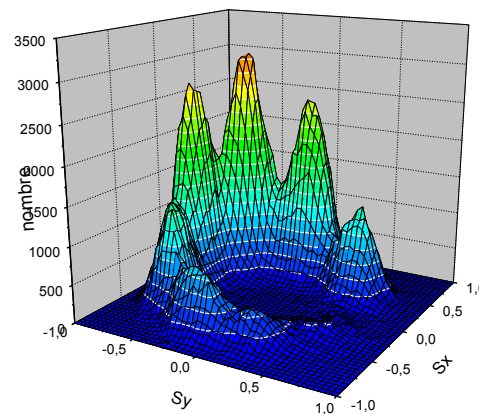
Conclusion of lecture 2:

Cavity QED with microwave photons and circular Rydberg atoms:
a powerful tool for:

- Performing QND measurement of the field state

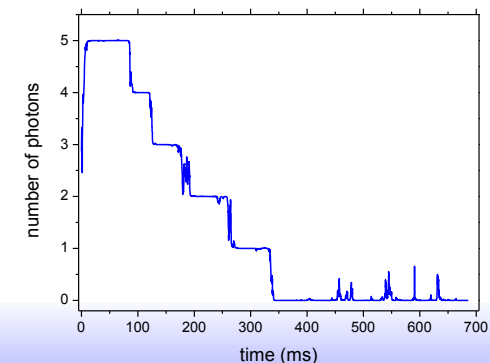


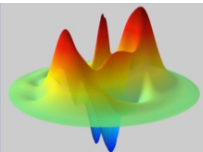
500 atoms "seeing"
the birth and death of
a single photon



A 110 atoms spin arrow
pointing on the photon
number

Quantum jumps
of light

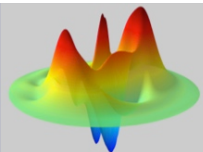




References (1)

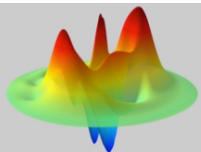
- **Strong coupling regime in CQED experiments:**

- ❑ F. Bernardot, P. Nussenzveig, M. Brune, J.M. Raimond and S. Haroche. "Vacuum Rabi Splitting Observed on a Microscopic atomic sample in a Microwave cavity". *Europhys. Lett.* **17**, 33-38 (1992).
- ❑ P. Nussenzveig, F. Bernardot, M. Brune, J. Hare, J.M. Raimond, S. Haroche and W. Gawlik. "Preparation of high principal quantum number "circular" states of rubidium". *Phys. Rev.* **A48**, 3991 (1993).
- ❑ M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J. M. Raimond and S. Haroche: "Quantum Rabi oscillation: a direct test of field quantization in a cavity". *Phys. Rev. Lett.* **76**, 1800 (1996).
- ❑ J.M. Raimond, M. Brune and S. Haroche : "Manipulating quantum entanglement with atoms and photons in a cavity", *Rev. Mod. Phys.* vol.73, p.565-82 (2001).
- ❑ P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J.M. Raimond and S. Haroche : "Interference with beam splitters evolving from quantum to classical : a complementarity experiment". *Nature* **411**, 166 (2001).
- ❑ E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J.M. Raimond and S. Haroche: "Generation of Einstein-Podolsky-Rosen pairs of atoms", *PRL* **79**,1 (1997).
- ❑ P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J.M. Raimond and S. Haroche : "Interference with beam splitters evolving from quantum to classical : a complementarity experiment". *Nature* **411**, 166 (2001).



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 - ❑ M. Brune et al., Phys. Rev. Lett, **72**, 3339(1994).
 - ❑ Q.A. Turchette et al., Phys. Rev. Lett. **75**, 4710 (1995).
 - ❑ C. Monroe et al., Phys. Rev. Lett. **75**, 4714 (1995).
 - ❑ A. Reuschenbeutel et al., PRL. G. Nogues et al. Nature **400**, 239 (1999).
 - ❑ S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J.M. Raimond and S. Haroche, Phys. Rev. Lett. **87**, 037902 (2001)
 - ❑ F. Yamaguchi, P. Milman, M. Brune, J-M. Raimond, S. Haroche: "Quantum search with two-atom collisions in cavity QED", PRA **66**, 010302 (2002).
- **Q. memory:**
 - ❑ X. Maître et al., Phys. Rev. Lett. **79**, 769 (1997).
- **Atom EPR pairs:**
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 - ❑ Ions: Q.A. Turchette et al., Phys. Rev. Lett. **81**, 3631 (1998).



Reference (3)

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 - S. Gleyzes, S. Kuhr, C. Guerlin, J. Bernu, S. Deléglise, U. Busk Hoff, M. Brune, J.-M. Raimond and S. Haroche, Nature 446, 297-300 (2007): "Quantum jumps of light recording the birth and death of a photon in a cavity".
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 - S. Kuhr, S. Gleyzes, C. Guerlin, J. Bernu, U. B. Hoff, S. Deleglise, S. Osnaghi, M. Brune, J.-M. Raimond, S. Haroche, E. Jacques, P. Bosland, and B. Visentin, Appl. Phys. Lett. 90, 164101 (2007): "Ultrahigh finesse Fabry-Pérot superconducting resonator".

