From cavity QED to quantum simulations with Rydberg atoms

Lecture 2
The dispersive regime
QND photon counting (1)

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Our version of Moore’s law:
QND photon counting:
The beginning of the story ...

Quantum Non demolition Measurement of Small Photon Numbers by Rydberg-Atom Phase-Sensitive Detection

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We describe a new quantum nondemolition method to monitor the number \( N \) of photons in a microwave cavity. We propose coupling the field to a quasiresonant beam of Rydberg atoms and measuring the resulting phase shift of the atom wave function by the Ramsey separated-oscillatory-fields technique. The detection of a sequence of atoms reduces the field into a Fock state. With realistic Rydberg atom-cavity systems, small-photon-number states down to \( N = 0 \) could be prepared and continuously monitored.

Initial QND measurement proposal

1990
The vacuum Rabi oscillation

1996
Resonant Rabi oscillations

Polished bulk niobium mirrors
$Q=2.10^8$
New cavity technology

1996
Resonant Rabi oscillations

Polished bulk niobium mirrors
$Q=2 \times 10^8$

1996
Sputtered niobium mirrors
$Q=5 \times 10^{10}$
Niobium coated copper mirrors

- Copper mirrors
  - Diamond machined
    - \( \sim 1 \, \mu m \) ptyv form accuracy
    - \( \sim 10 \, nm \) roughness
  - Toroidal è single mode

- Sputter 12 \( \mu m \) of Nb
  - Particles accelerator technique
  - Process done at CEA, Saclay

[E. Jacques, B. Visentin, P. Bosland]
Superconducting cavity resonance: $\nu_{\text{cav}} = 51 \text{ GHz}$

- Q factor = $4.2 \cdot 10^{10}$
- finesse = $4 \cdot 10^9$

$T_{\text{cav}} = 130 \text{ ms}$

Photons running for 39,000 km in the box before dying!
A new cavity setup
1. Basic reminder on ideal quantum measurement
Quantum physics

- **Description of quantum objects**
  - interaction: Schrödinger equation.
  - measurements: the state determines the statistics of results.

- **Quantum theory:** the **art of extracting classical information** out of microscopic systems.
Quantum measurement: basic ingredients

- **Entanglement**: "The essence of quantum physics" (Heisenberg)
  Created by interaction, describes all correlations between quantum systems.

- **irreversibility introducing dissipation**: macroscopic systems are dissipative. Dissipation plays a fundamental role in the coherence of quantum theory: explains the "decoherence" step during a quantum measurement.
Ideal quantum measurement

• The postulates:
  - Possible results: eigenvalues $a_n$ of an hermitian operator $\hat{A}$ (observable).
  - Fundamentally random result of individual measurements
  - Probability of results if system in state $|\psi\rangle$:

    \[
    p(a_n) = \langle \psi | P_n | \psi \rangle
    \]
    where $P_n$ = projector on the eigenspace associated to $a_n$.

  - State after measurement:

    \[
    |\psi_{after}\rangle = \frac{P_n |\psi\rangle}{\sqrt{p(a_n)}}
    \]

→ state collapse: the system's states changes discontinuously during the measurement process
• locks like a recipe:
  - does not tell what is a measurement apparatus
  - does not tell how to build an apparatus measuring a given observable

• locks like a strange recipe:
  a quantum system seems to be subjected to two kinds of evolution:
    - continuous evolution according to Schrödinger equation between measurements
    - state collapse during measurements

But a measurement apparatus is made of quantum objects obeying to Schrödinger equation:

Why should evolution during measurement deserve a special treatment?
Outline of lectures 2-4

• Lecture 2:
  The projection postulate at work
  • an experimental realization: measuring the photon number in a high Q cavity
  • observing the quantum jumps of light in a cavity

• Lecture 3: applications of QND photon counting
  • Quantum feedback
  • Past-quatum state analysis of a quantum trajectory

• Lecture 4:
  The role of dissipation: Schrödinger cat and decoherence
  • The "problem" of quantum measurement
  • The decoherence approach
  • Observing the decoherence of a Schrödinger cat state
2. Non-destructive single photon counting
Experimental setup: an atomic clock

- An atomic clock (Ramsey setup) made of Rydberg for probing light-shifts induced by “trapped” photons

- State selective detection of atoms by field ionization: Atoms detected on “e” or “g” one by one
QND detection of photons: the principle

- Photon box

- Photon probes

Circular Rydberg atoms

- Non-resonant interaction

\[ \Rightarrow \text{light shifts} \]

\[ \Delta E_e = \hbar \frac{\Omega_0^2}{4\delta} (n + 1) \]

\[ \Delta E_g = -\hbar \frac{\Omega_0^2}{4\delta} n \]

Atoms used as clock for counting \( n \) by measuring light shifts
1. Trigger of the clock.

In term of a spin $\frac{1}{2}$, this is a $\pi/2$ rotation around the Ox axis.
QND detection of 0 or 1 photon

1. Trigger of the clock.

2. Precession of the spin through the cavity during $T$
   Phase shift per photon
   \[ \Phi_0 = \pi \]

\[ \rightarrow \frac{1}{\sqrt{2}} (|e\rangle + ie^{i\delta_{mw}T}|g\rangle) = |+\phi\rangle \]

\[ \delta_{mw} = \omega_{mw} - \omega_{at} \]

rotation by angle $\phi = \delta_{mw}T$ around the Oz axis
QND detection of 0 or 1 photon

1. Trigger of the clock.
2. precession of the spin through the cavity.
3. Detection of $S_y$: second $\pi/2$ rotation + detection of e-g

Atom detected in $e \Rightarrow$ field projected on $|1\rangle$
$g \Rightarrow$ field projected on $|0\rangle$
Detecting blackbody photons

\[ g \rightarrow \text{field projected on } |0> \]

\[ e \rightarrow \text{field projected on } |1> \]

\[ T = 0.8 \text{ K} \rightarrow n_{th} = 0.05 \quad \text{(proba. of } n=2 \text{ is negligible)} \]

3. Counting more photons
1. Trigger of the atom clock: resonant $\pi/2$ pulse
1. Trigger of the atom clock: resonant $\pi/2$ pulse
2. Dephasing of the clock: interaction with the cavity field

Phase shift per photon $\Phi_0 = \pi/4$
1. Trigger of the atom clock: resonant $\pi/2$ pulse
2. Dephasing of the spin: interaction with the cavity field
3. Measurement of the spin: $\pi/2$ pulse with phase $\phi_R$ & state detection

Pseudo-spin measurement in arbitrary direction determined by $\phi_R$

$P_e - P_g = \langle \sigma_{\phi_R} \rangle$
Detection of $n>1$

\[ n = \frac{\pi}{2} \]

Chose

\[ \Phi_0 = \frac{\pi}{4} \]

⇒ Photon numbers from 0 to 7 correspond to 8 different final position of the atom "spin"

But these states are not orthogonal

⇒ detecting one atom is not enough to determine $n$. 
Detection of $n>1$

Interaction with one atom prepares:

$$|\Psi\rangle = \sum_n C_n \left| +_n \Phi_0 \right\rangle \otimes |n\rangle$$

$\Rightarrow$ Repeat measurement

The photon number is now encoded in a mesoscopic sample of atoms.

Orthogonal states if $N$ large enough

$$\left\langle +_{n'} \Phi_0 | +_n \Phi_0 \right\rangle^N \approx 0$$

Orthogonal states if $N$ large enough
Detection of $n>1$

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$$|\Psi\rangle = \sum_n C_n \left| +_n \Phi_0 \right\rangle \otimes |n\rangle$$

$\Rightarrow$ Repeat measurement

The photon number is now encoded in a mesoscopic sample of atoms.

**That is a Schrödinger cat state:**

the N atom collective spin points in a direction indicating the photon number.
Décoding the photon number

\[ |\Psi\rangle = \sum_n C_n |+_n \Phi_0 \rangle^N \otimes |n\rangle \]

For each \( n \), on detects \( N \) identical copies of the atomic state

\[ |+_n \Phi_0 \rangle \]

Determination of atom spin by « tomography »:

\( N \) atoms \( \rightarrow \) \( N/4 \) atoms: measure \( \langle S_{\Phi_R} \rangle \)

\( \rightarrow \) calculate \( \langle S_x \rangle \) and \( \langle S_y \rangle \)

For large enough \( N \), \( \Delta \varphi_s \propto \frac{1}{\sqrt{N}} \) and different photon numbers should be distinguished.
Atom spin state tomography

Method:
1- inject a coherent field $\langle n \rangle = 3.5$ photons.
2- detection of 110 consecutive atoms, $T_{\text{measure}} = 26$ ms

One measurement
$\rightarrow n = 4$
Tomographie de l’état atomique

Method:  
1- inject a coherent field $\langle n \rangle = 3.5$ photons.  
2- detection of 110 consecutive atoms, $T_{\text{measure}} = 26$ ms

$\langle n \rangle = 2.4$ photons

The collective spin of $N$ atoms points in discrete direction  
$\Rightarrow n$ is obviously quantized

Detecting a collection of 110 atoms is enough to fully determine the photon number
Observing field decay directly on the collective atom spin

- Preparation of an initially coherent 3.5 photons field

- First measurement: projection on $n=4$
- Preparation of an initially coherent 3.5 photons field

- First measurement: projection on n=4

Due to statistical noise the spin fluctuates around n=4
- Preparation of an initially coherent 3.5 photons field

- First measurement: projection on n=4
- quantum jump to n=3
Observing field decay directly on the collective atom spin

- Preparation of an initially coherent 3.5 photons field

- First measurement: projection on $n=4$
  - quantum jump to $n=3, 2$
Observing field decay directly on the collective atom spin

- Preparation of an initially coherent 3.5 photons field

- First measurement: projection on n=4
  - quantum jump to n=3, 2, 1….
Observing field decay directly on the collective atom spin

- Preparation of an initially coherent 3.5 photons field

- First measurement: projection on $n=4$
- quantum jump to $n=3$, $2$, $1$, ..., $0$
Observing field decay directly on the collective atom spin

Quantum jumps down to $n=0$.

→ now describe this process in term of progressive acquisition of information by applying the projection postulate atom by atom
Progressive accumulation of information

Apply the projection postulate at each atom detection.

\[ P_0(n) \quad 1\text{- Initial field state} \]

\[ P_1(n) \]

\[ P_2(n) \]

\[ P_N(n) \]
Progressive accumulation of information

Apply the projection postulate at each atom detection.

\[ P_0(n) \rightarrow \text{1- Initial field state} \]

\[ \phi_R \text{ randomly chosen among 4 values} \]

\[ \text{aligned on the 4 possible spin directions} \]

\[ \Downarrow \]

\[ P_1(n) \rightarrow \text{2- First measurement of } \hat{S}_{\phi_R} \rightarrow \text{résult: } +j \text{ or } -j \]

\[ \rightarrow \text{Projection of the atom-field state:} \]

\[ P_2(n) \]

\[ \Downarrow \]

\[ P_N(n) \]

\[ \vdots \]
**Progressive accumulation of information**

Apply the projection postulate at each atom detection.

\[ P_0(n) \]

1- Initial field state

\[ \phi_R \]

randomly chosen among 4 values aligned on the 4 possible spin directions

2- First measurement of \( \hat{S}_{\phi_R} \): résultat: \(+_j\) or \(-_j\)

\[ P_1(n) \]

\[ P_2(n) \]

\[ \ldots \]

\[ P_N(n) \]

Bayse law

\[ P_1(n) = P(n/\pm_\varphi) = P_0(n).P(\pm_\varphi/n).\frac{1}{Z} \]

\[ Z = P(\pm_\varphi) \text{ norm. factor} \]
Progressive accumulation of information

Apply the projection postulate at each atom detection.

\[ P_0(n) \] 1- Initial field state

\[ \phi_R \] randomly chosen among 4 values aligned on the 4 possible spin directions

\[ \downarrow \]

\[ \phi_R \] Projection of the atom-field state:

\[ \downarrow \]

\[ P_1(n) \] 2- First measurement of \( \hat{S}_{\phi_R} \) → résult: +_j or -_j

\[ \downarrow \]

\[ P_2(n) \] → Projection of the atom-field state:

\[ \downarrow \]

\[ \cdots \]

\[ P_N(n) \] 3- Iterate the process until detection of N atoms

Note: field coherence do not play any role, \( P((n) \) is enough here.
Information acquisition by detecting 1 atom

Bayes law:

\[ P_{\text{after}}(n) = P(n / j_{\phi_R}) = P_{\text{before}}(n) \cdot \frac{P(j_{\phi_R} / n)}{P(j_{\phi_R})} \]

\( j_{\phi_R} = 1 \) or \(-1\) = e or g

\( \phi_R = 0 \)

\[ P\left(\phi_R / n\right) = \left|\langle \phi_R | +n \rangle\right|^2 \]

Probability of \( n \) that are incompatible with the measurement result are cancelled.

Repeating the measurement with other values of \( j \) decimates other photon numbers.
Information acquisition by detecting 1 atom

Bayes law:

\[ P_{after}(n) = P(n / j_{\phi_R}) = P_{before}(n) \cdot \frac{P(j_{\phi_R} / n)}{P(j_{\phi_R})} \]

\( j_{\phi_R} = 1 \) or \(-1 \)

\( = e \) or \( g \)

\( \phi_R = 0 \)

\[ P(\pm_{\phi_R} / n) = \left| \left\langle \pm_{\phi_R} | +n \right\rangle \right|^2 \]

Probability of \( n \) that are incompatible with the measurement result are cancelled.

Repeating the measurement with other values of \( j \) decimates other photon numbers.
Progressive field collapse

\[ j(k) = 1101111111100111011011101001101010110101101111 \]
\[ \phi_R(k) = ddcbbcaabcdadaabaaddbaddbcdbabbaacbcddadcadcdaacc \]

\( k \) = atom index

Decoding (real data, not simulation)

Initial coherent state
\[ <n> = 3.7 \ (\pm 0.008) \]

Flat initial photon number distribution.
The measurement result is determined by the real field

Progressive projection of the field on \( n=5 \) number state

Reconstructing the photon number statistics

Coherent field at measurement time

\[ \langle n \rangle = 3.4 \pm 0.008 \]
Repeated measurements:
evolution of a continuously monitored field

- Exhibits all features of quantum theory of measurement:
  - State collapse / Random result / repeatability

Field evolution due to cavity damping: not to QND measurement
Conclusion of lecture 2:

Cavity QED with microwave photons and circular Rydberg atoms:

a powerful tool for:

- Performing QND measurement of the field state

500 atoms "seeing" the birth and death of a single photon

A 110 atoms spin arrow pointing on the photon number

Quantum jumps of light
• Strong coupling regime in CQED experiments:

References (2)

- **Gates: QPG or C-Not, algorithm:**

- **Q. memory:**

- **Atom EPR pairs:**
Reference (3)

- **QND detection of photons:**

- **High Q superconducting cavity:**