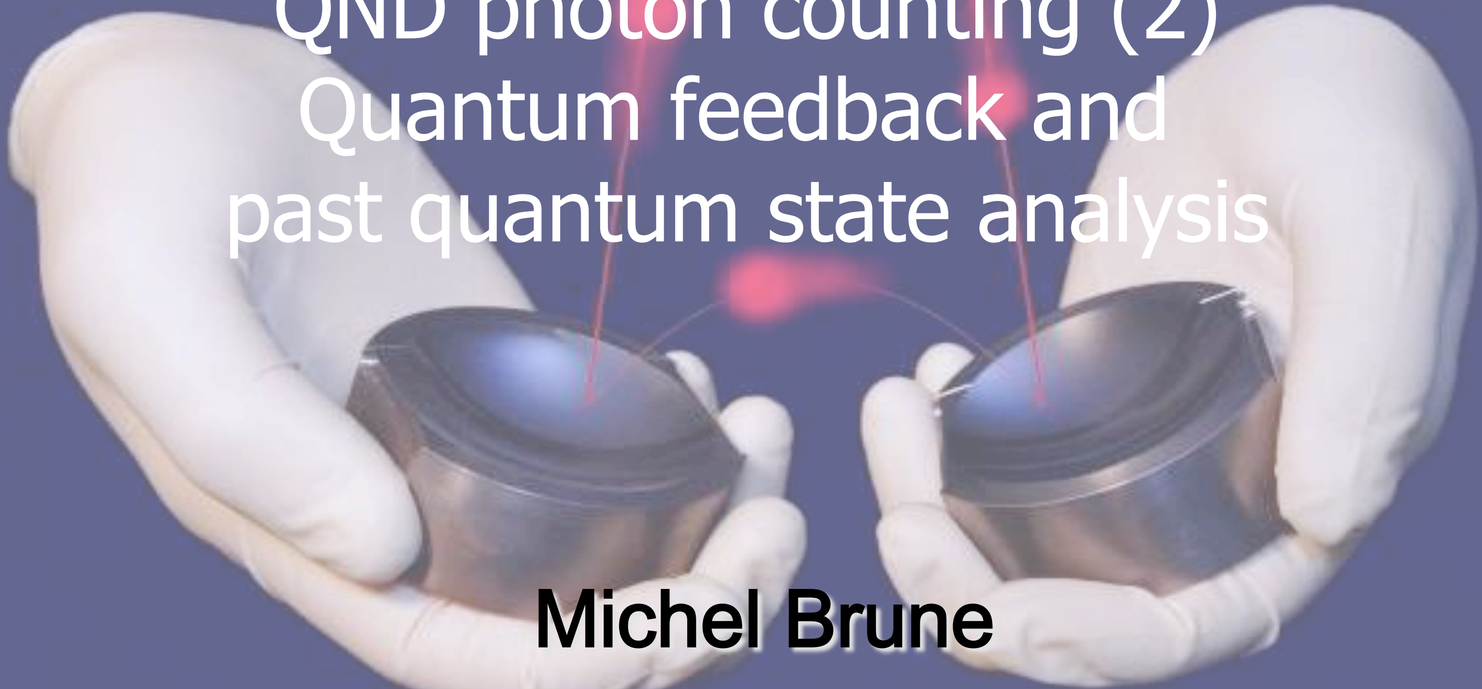
From cavity QED to quantum simulations with Rydberg atoms

Lecture 3
QND photon counting (2)
Quantum feedback and past quantum state analysis



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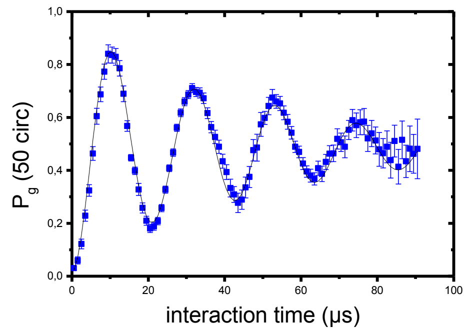
Previous lectures

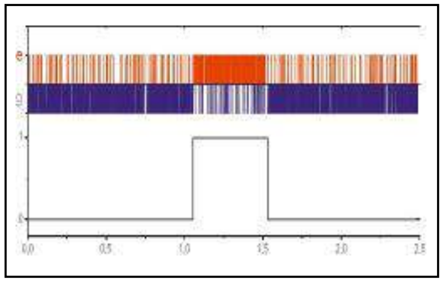
Cavity QED with microwave photons and circular Rydberg atoms:

a powerful tool for:

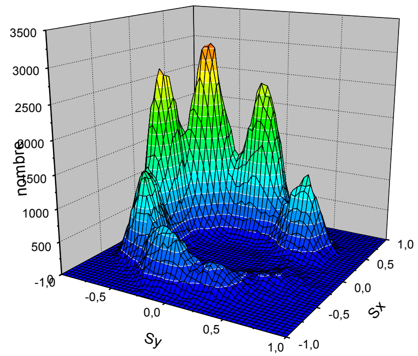
- Achieving strong coupling between single atoms and single photons



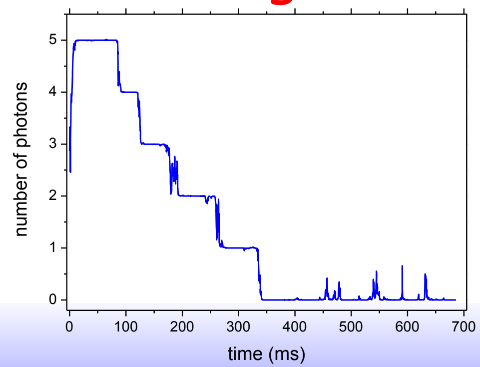
- Performing QND measurement of the field state

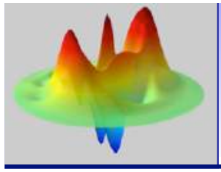


500 atoms "seeing" the same photon

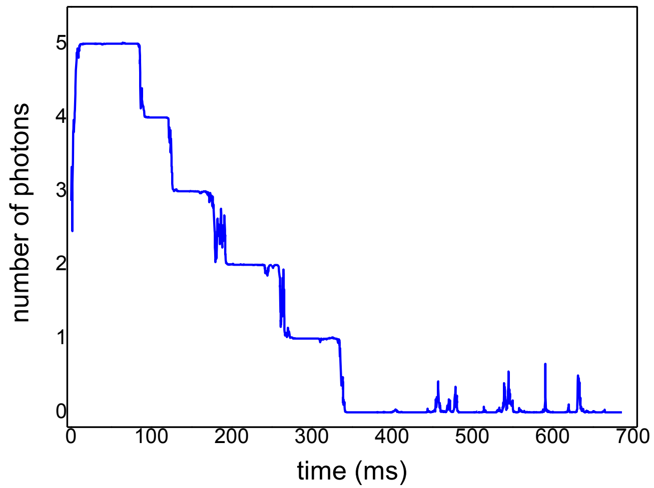


Quantum jumps of light





Repeated measurements: evolution of a continuously monitored field



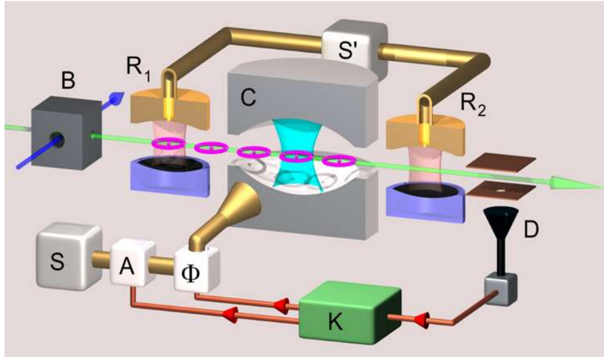
Field evolution due to cavity damping

Topic of lecture 2:

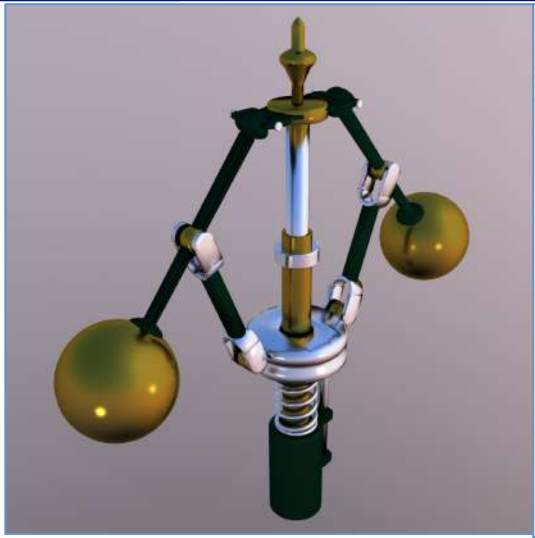
1- Using QND information for performing quantum feedback

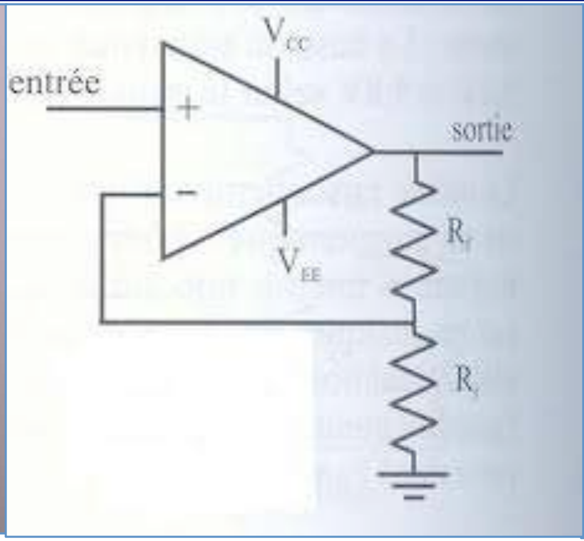
2- Improving the fidelity of quantum trajectories
by "past-quantum state" analysis

I. Stabilization of number states by quantum feedback



Classical versus quantum feedback





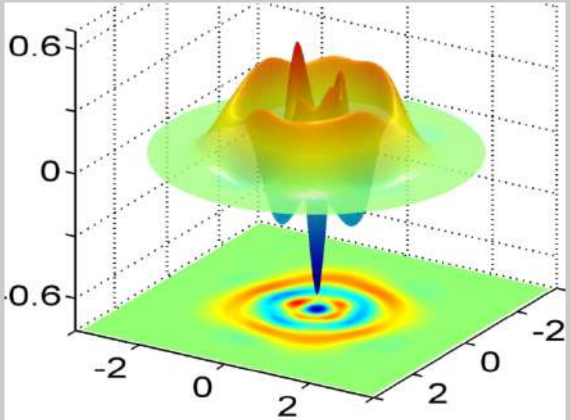
- Classical feedback is present in nearly all control systems
  - A **SENSOR** measures the system's state
  - A **CONTROLLER** compares the measured quantity with a target value
  - An **ACTUTATOR** reacts on the system to bring it closer to the target

- Quantum feedback has same aims for a quantum system
  - Must face a **fundamental difficulty**:

    ➞ Measurement back-action changes the system state

Stabilizing photon number states in a cavity

Number states



S. Deléglise *et al*.
Nature **455**, 510 (2008)

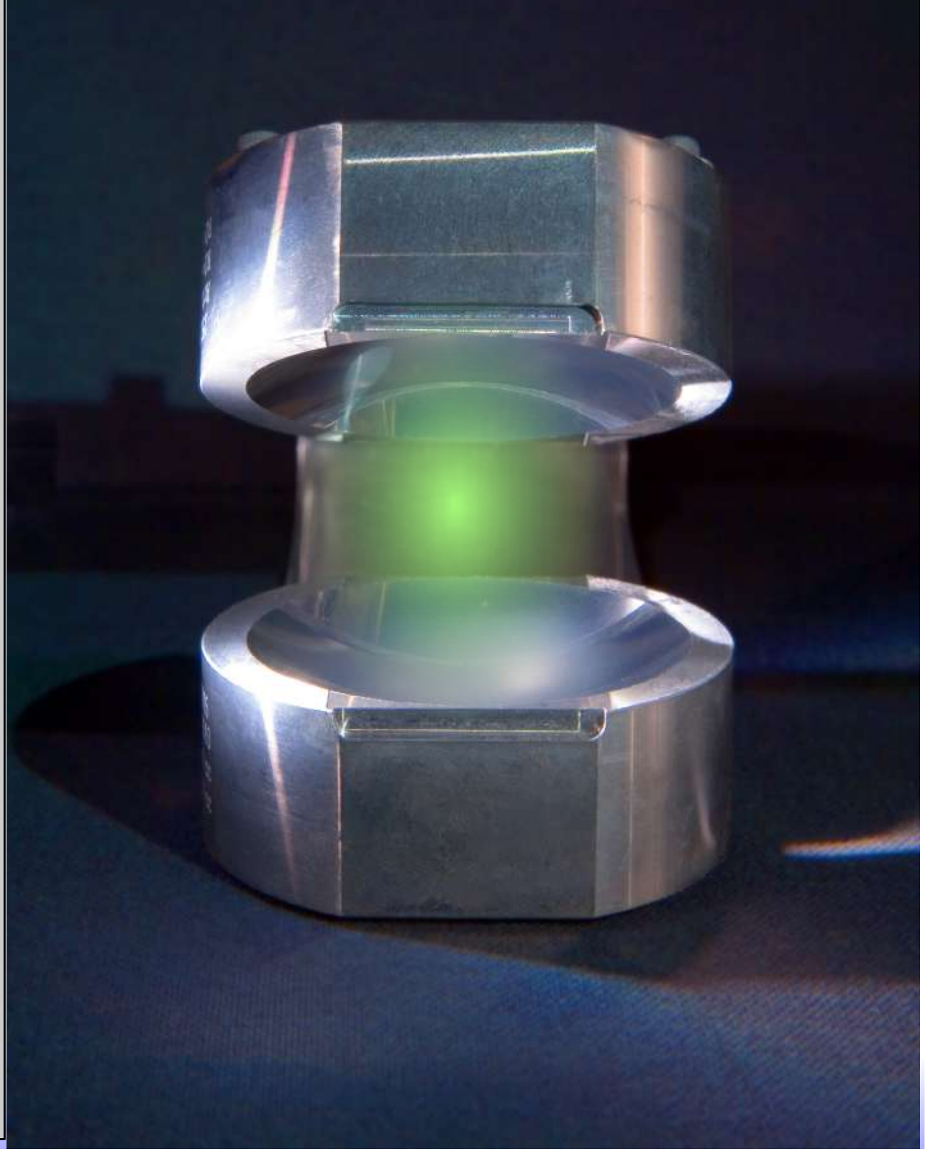
Highly non-classical:

- Negative Wigner Function
- Fast decoherence

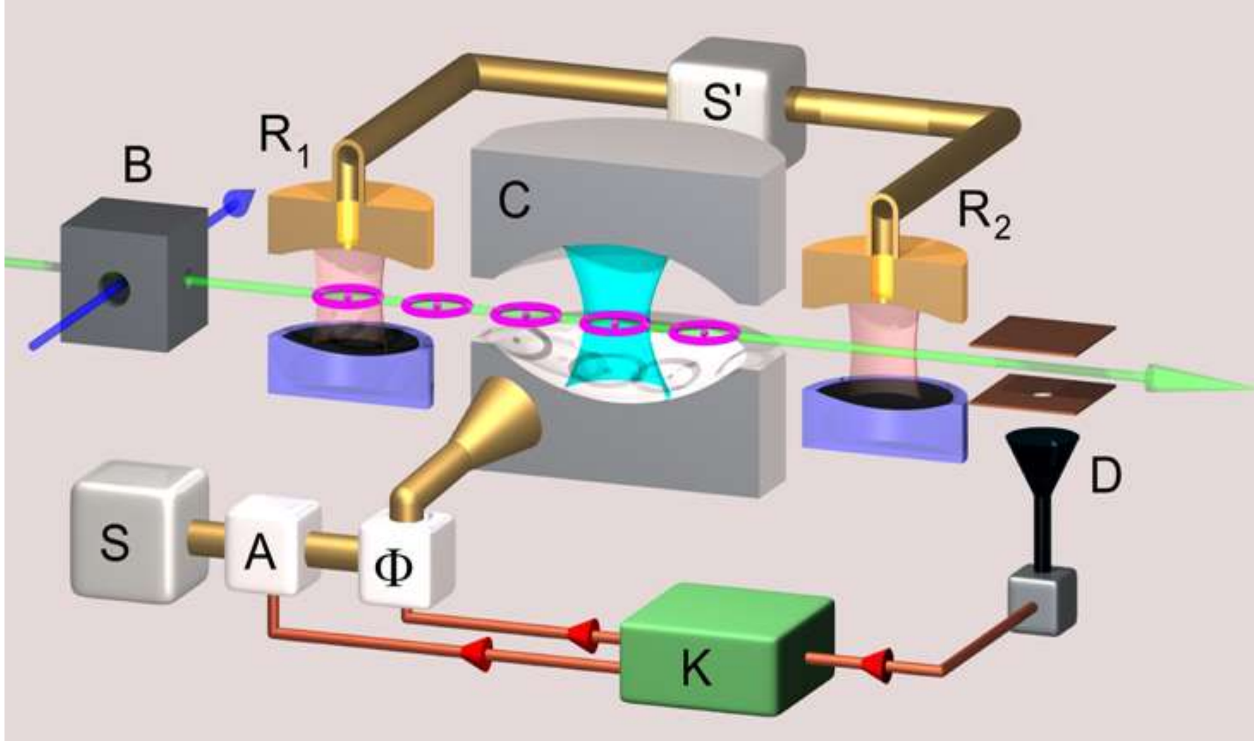
T_{dec}=Tc/n

M. Brune *et al*. PRL **101**, 240402 (2008)

high Q cavity
Field storage time
T_c

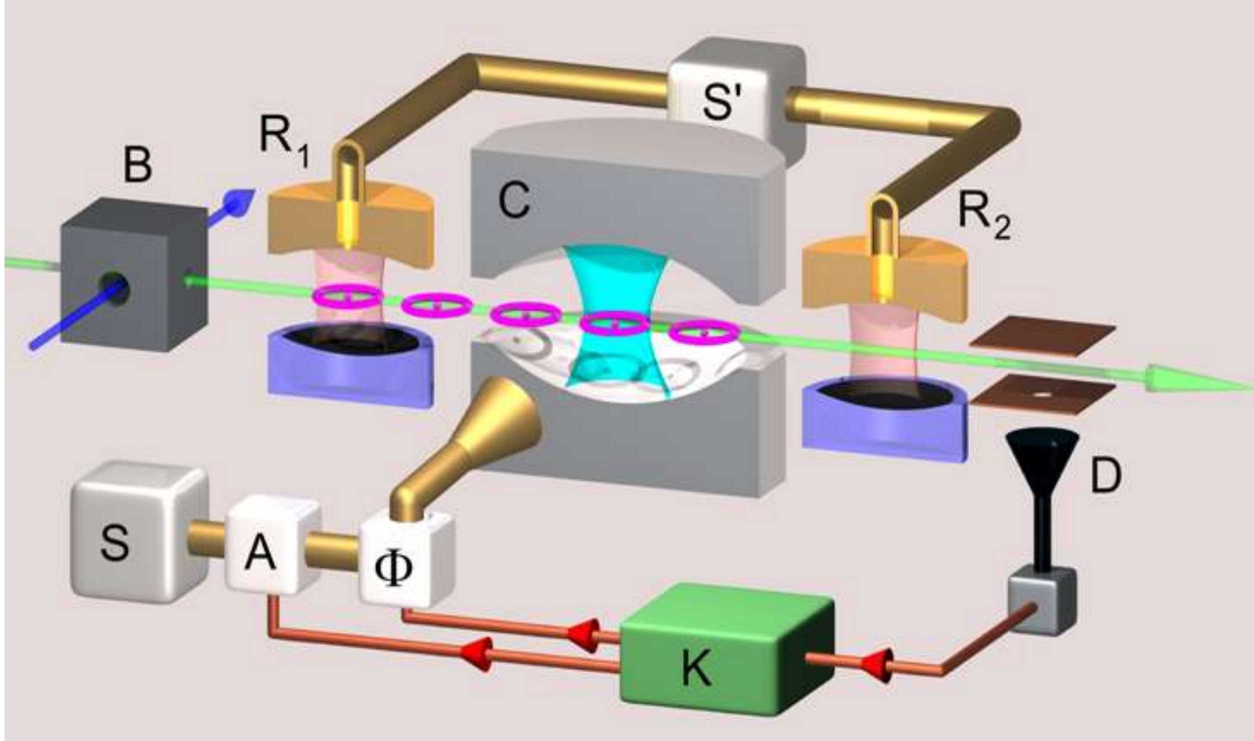


Feedback loop



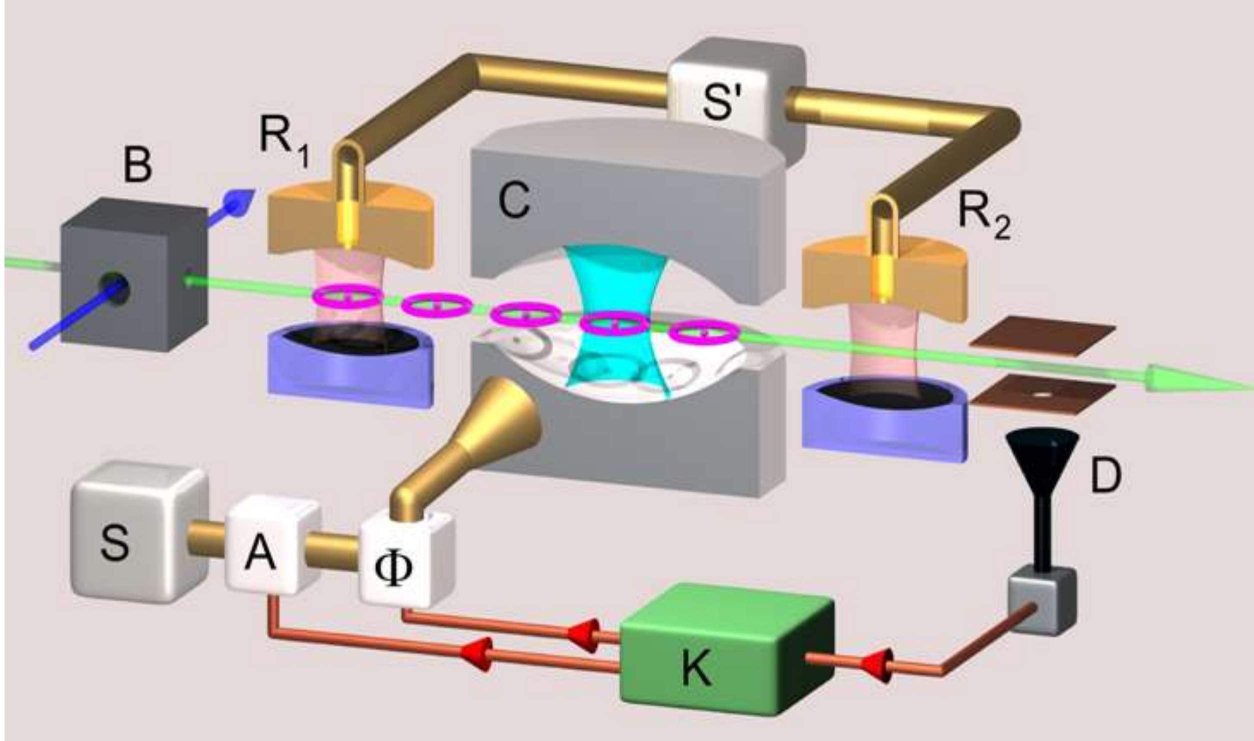
- Sensor: atom performing QND photon counting
- Controller: a classical computer (Adwin)
  - → Estimates cavity state taking into account all information
  - → Determines optimal actuator action
- Actuator
  - \- Classical field pulse
  - \- Quantum: single photon emission/absorption

Feedback loop: classical actuator



- Actuator
  - Classical source performing displacement of cavity field
  - Real amplitude α
  - K choses A and Φ after each atom detection
    - → Use second order approximation of displacement operator for fast calculations (computing time <82 μs)
    - → Action limited in the $|\alpha|<0.1$ interval

complete feedback algorithm



- Controller tasks: at each atom sample detection:
  - ➞ **Projects** the field on the measurement result
  - ➞ Take onto account **field damping**
  - ➞ Take into account various imperfections: detection efficiency, two atom samples, detection errors…
  - ➞ **Choses the best actions** for next actuator atoms

Estimate distance with respect to target state?

- A simple measure of distance to target $\rho_t = |n_t\rangle\langle n_t|$

$$d = 1 - Tr(\rho\rho_t)$$

Fidelity of state

- ❑ Equal to zero for the target state
- ❑ Equal to 1 (maximal) for all other Fock states

$\Rightarrow$ Does not discriminate properly the 'distance' to the target

- A more sophisticated distance

$$d(n_t, \rho) = 1 - \sum_{n=0}^{n_{\max}} \Lambda_{nn}^{(n_t)} \langle n|\rho|n\rangle = 1 - \mathrm{Tr}\left(\Lambda^{(n_t)}\rho\right) \qquad \Lambda_{n_t n_t}^{(n_t)} = 1$$

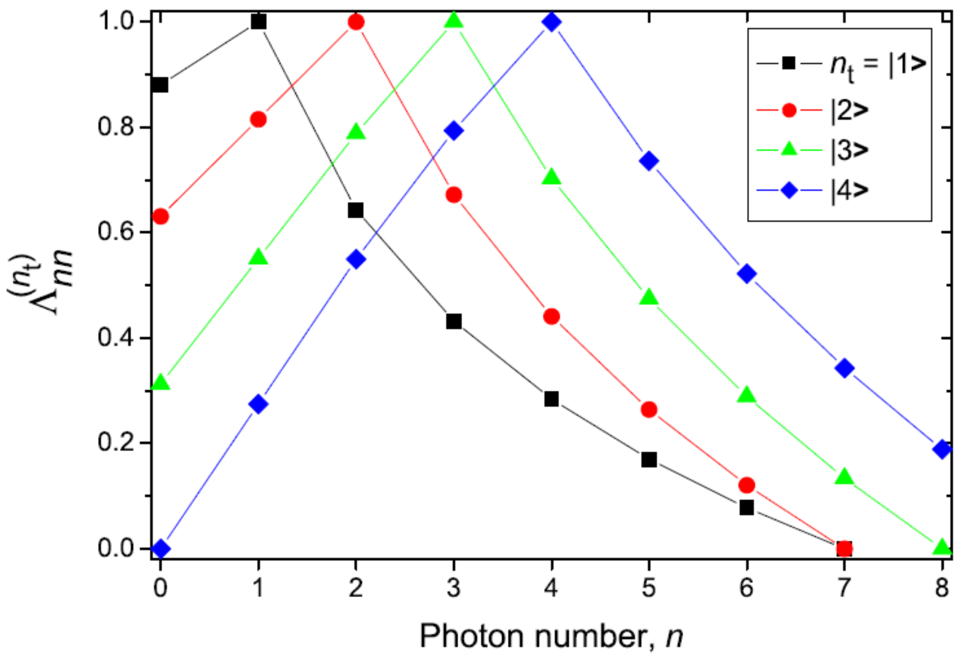
- ❑ provides sensitivity to photon numbers different from n_t
- ❑ Optimization of Λ ?

I. Dotsenko et al. Rev. A 80, 013805 (2009)

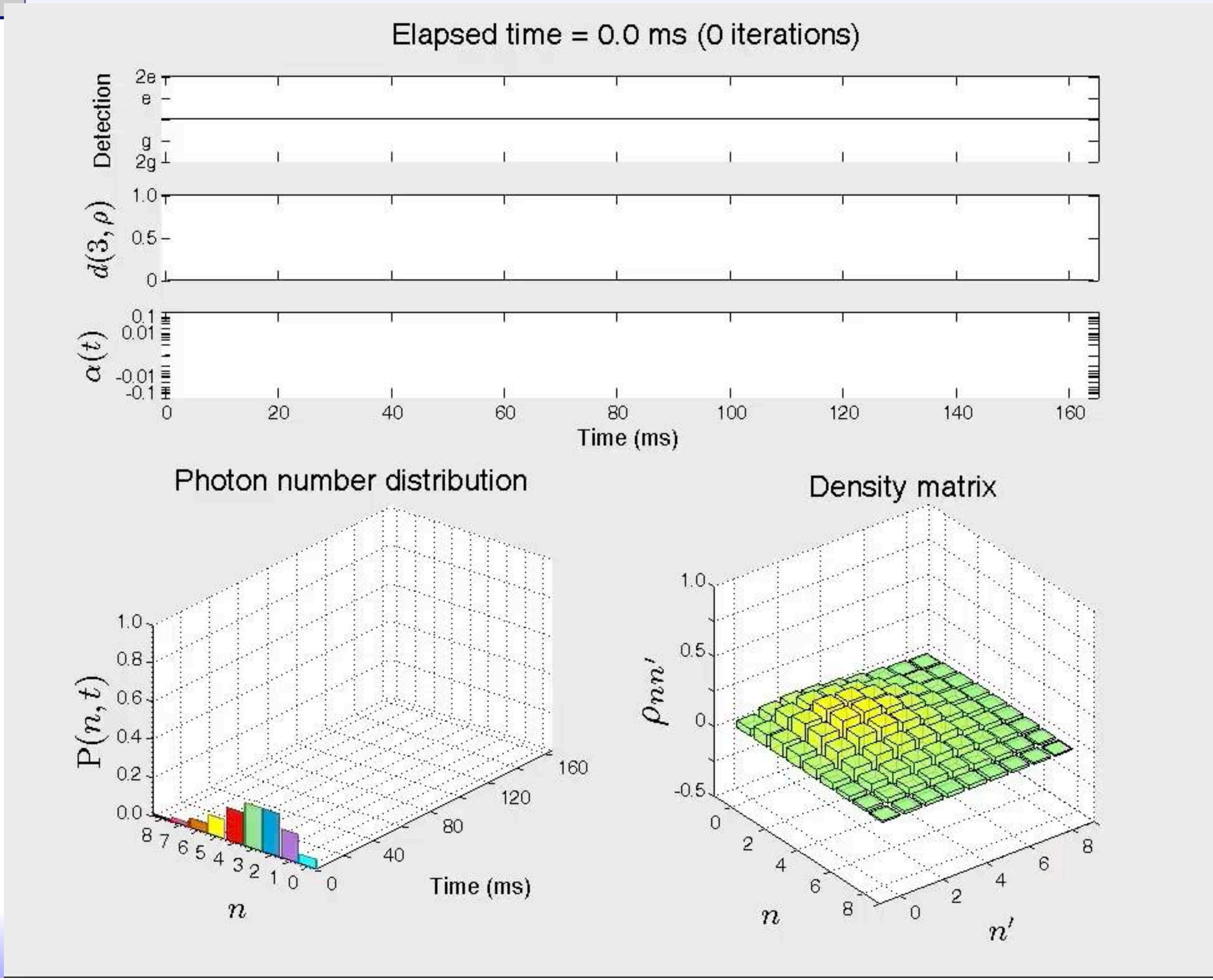
Estimate: distance wrt target state

$$d(n_\mathrm{t},\rho) = 1 - \sum_{n=0}^{n_\mathrm{max}} \Lambda_{nn}^{(n_\mathrm{t})} \langle n|\rho|n\rangle = 1 - \mathrm{Tr}\left(\Lambda^{(n_\mathrm{t})}\rho\right)$$

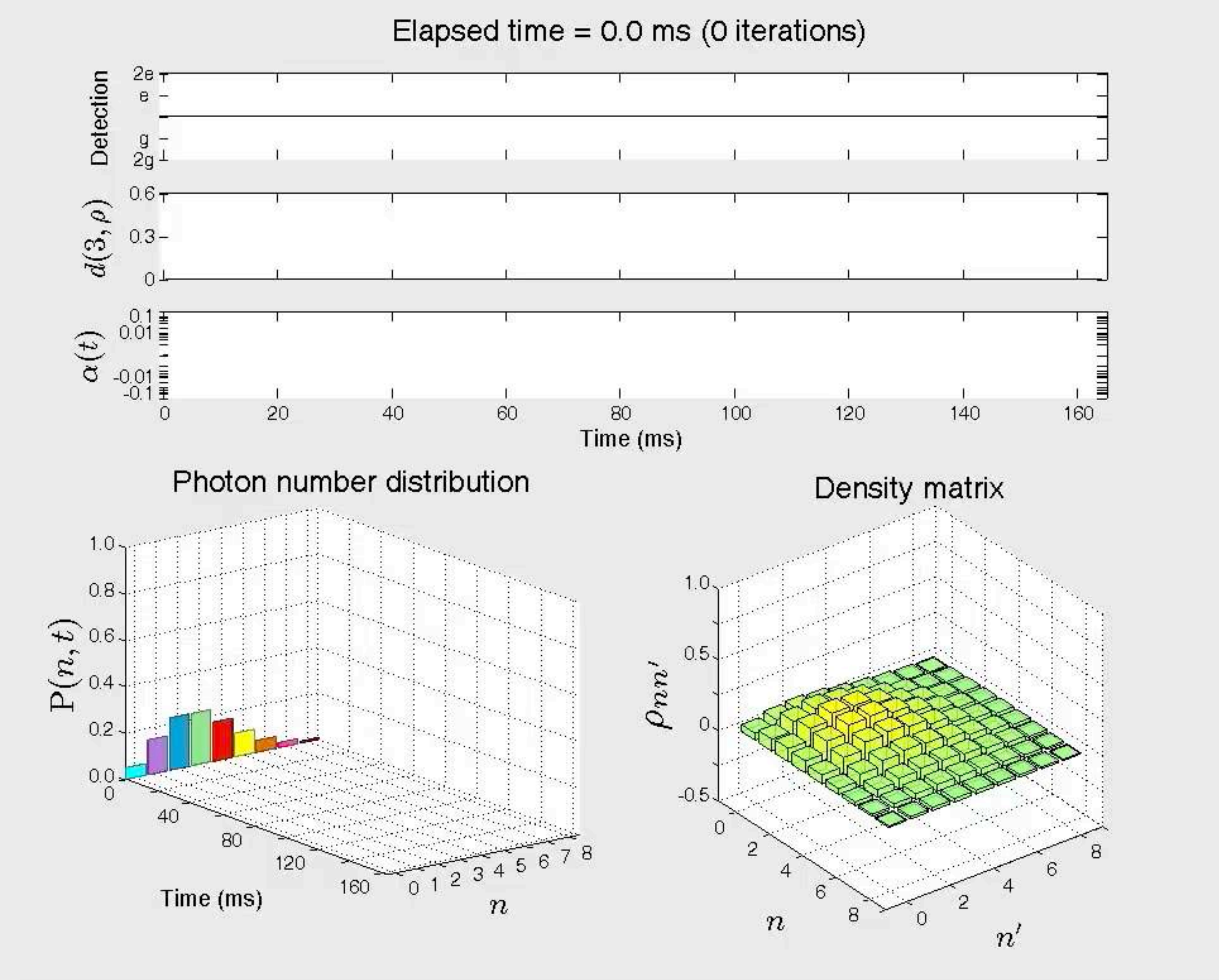
- Λ coefficients optimization
  - Distance has an absolute minimum (0) close to the target state
  - Distance has a local maximum on all other Fock states
  - Final optimization by numerical simulations of the convergence process



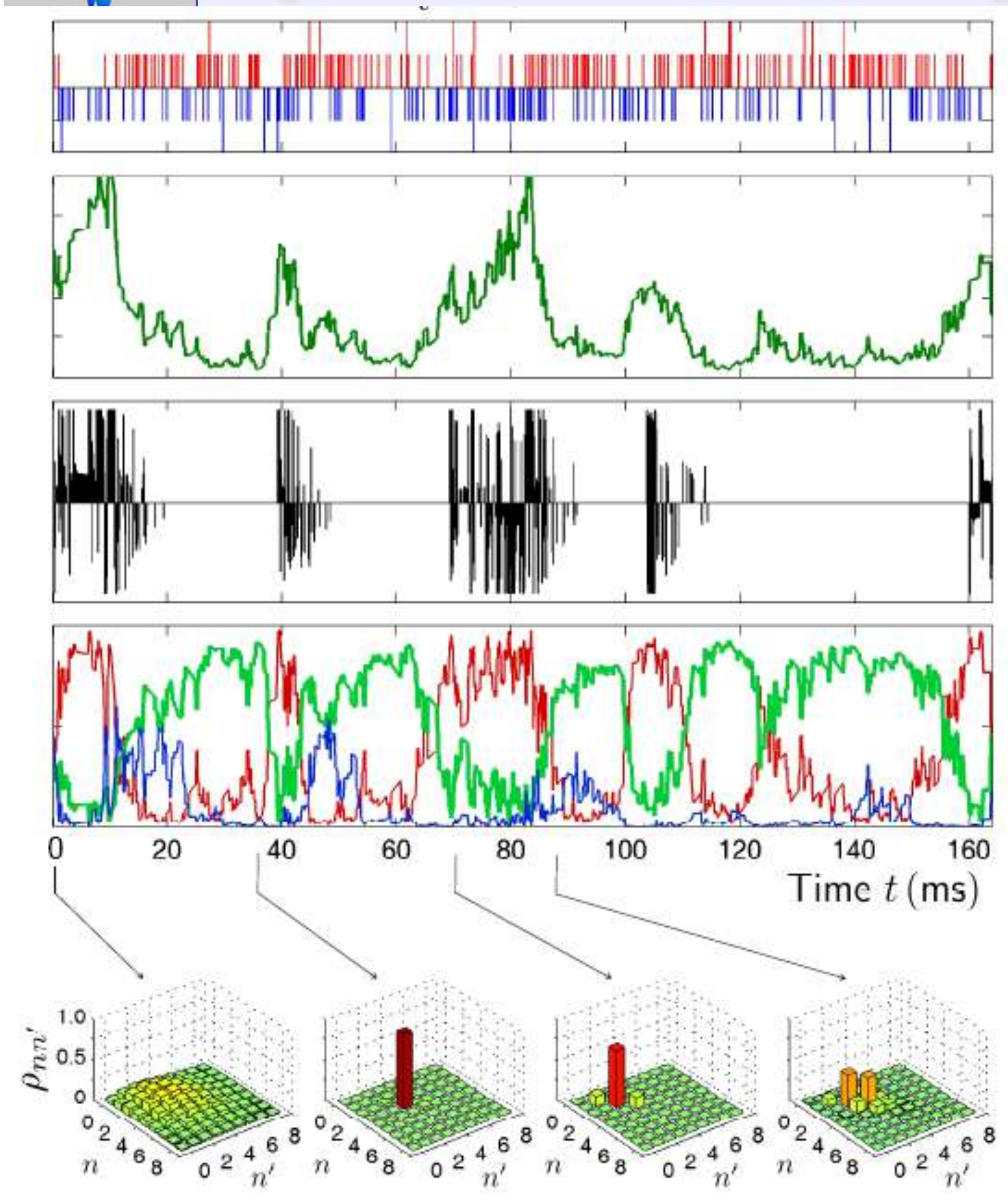
Open loop operation: QND and quantum jumps



Quantum feedback trajectory: 3 photon target



Quantum feedback trajectory: 3 photon target



- Detection outcomes

- Distance to the target

- Control injection: α real

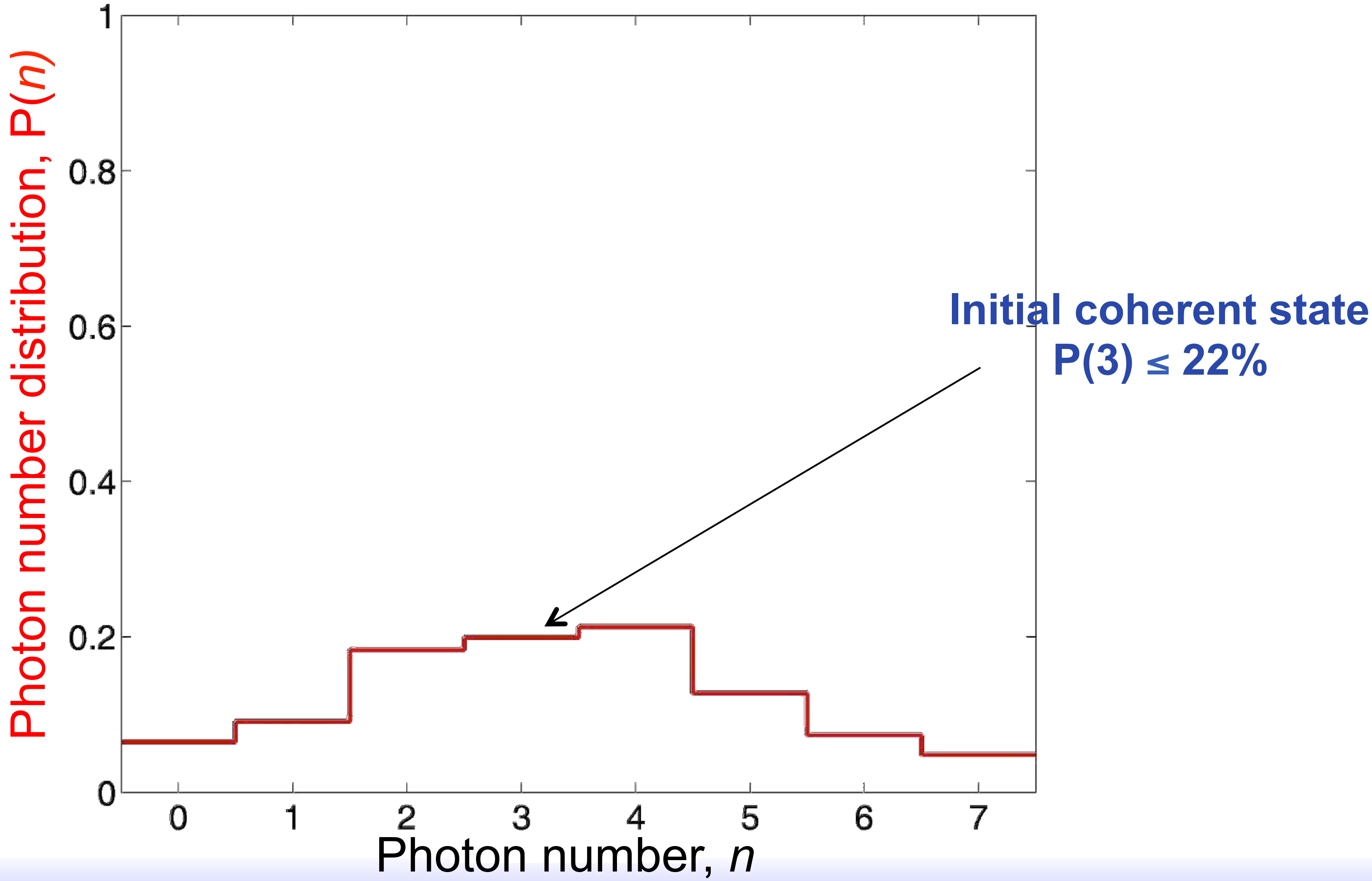
- Photon-number distribution:

  $P(n<n_t)$, $P(n=n_t)$
  and $P(n>n_t)$

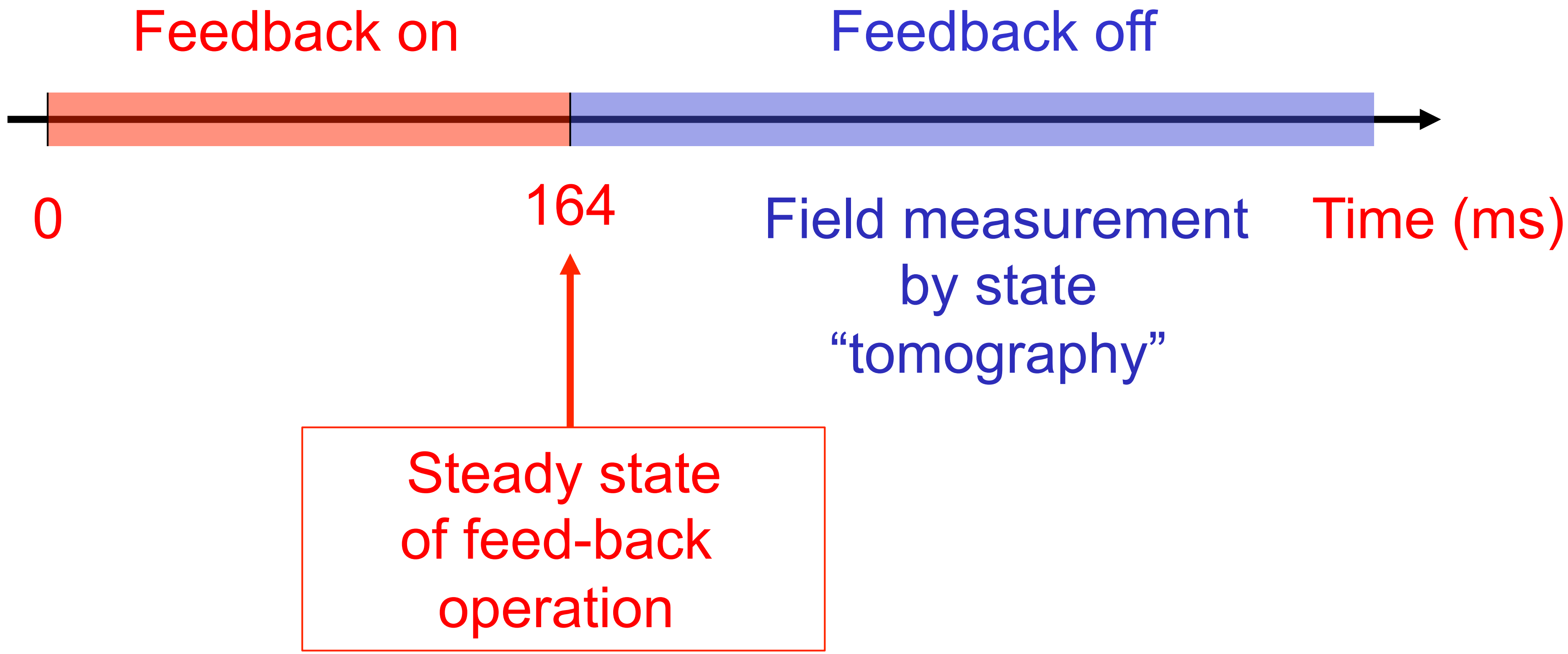
- Density matrices

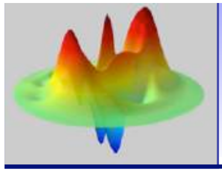
Fidelity of the state stabilization

- Average over 4000 trajectories
- Feedback sequence immediately followed by a "standard" QND measurement



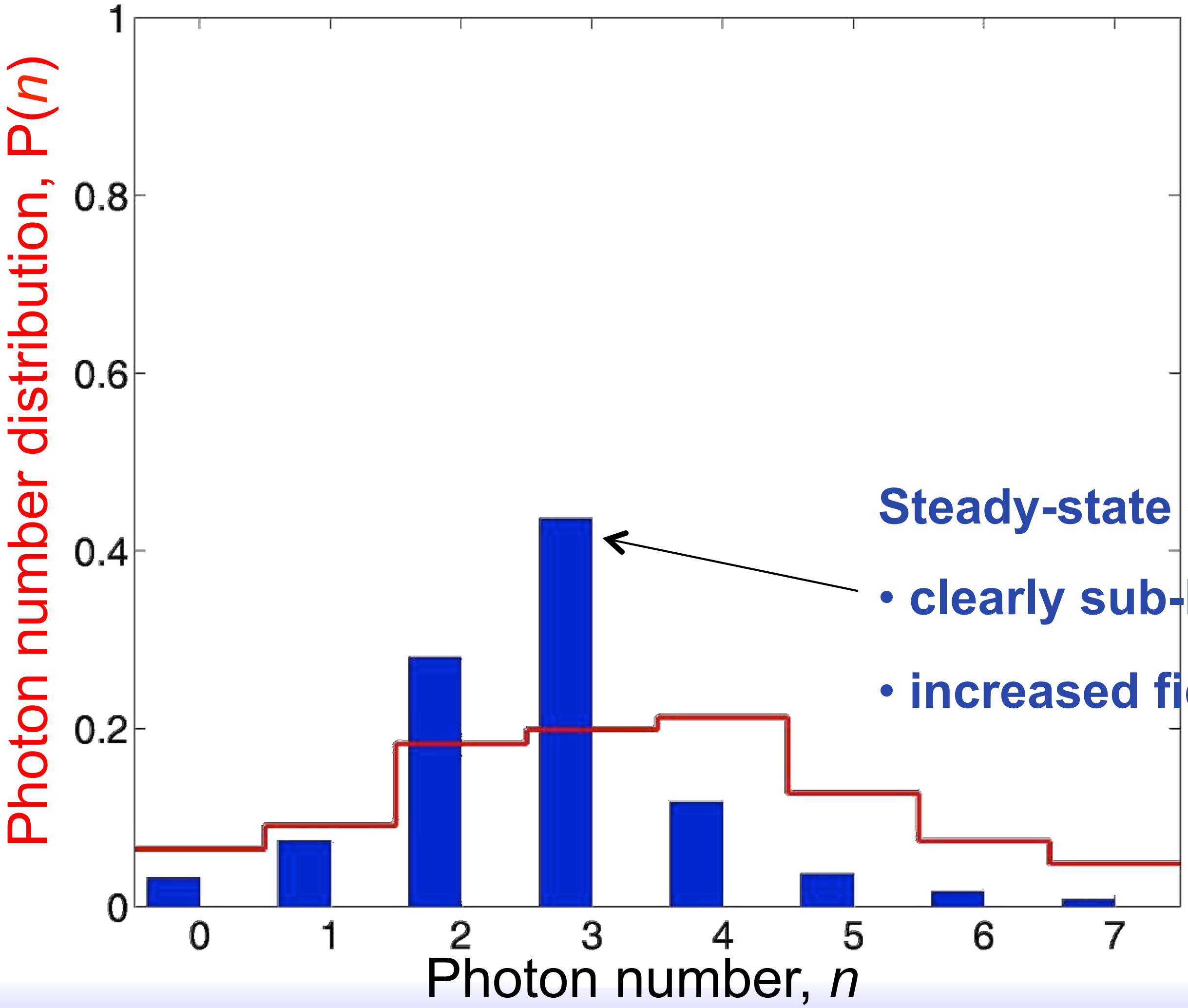
Checking the fidelity of the prepared state





Fidelity of the state stabilization

- Average over 4000 trajectories stopped at t = 164 ms > T_{cav}
- Feedback sequence immediately followed by a "standard" QND measurement

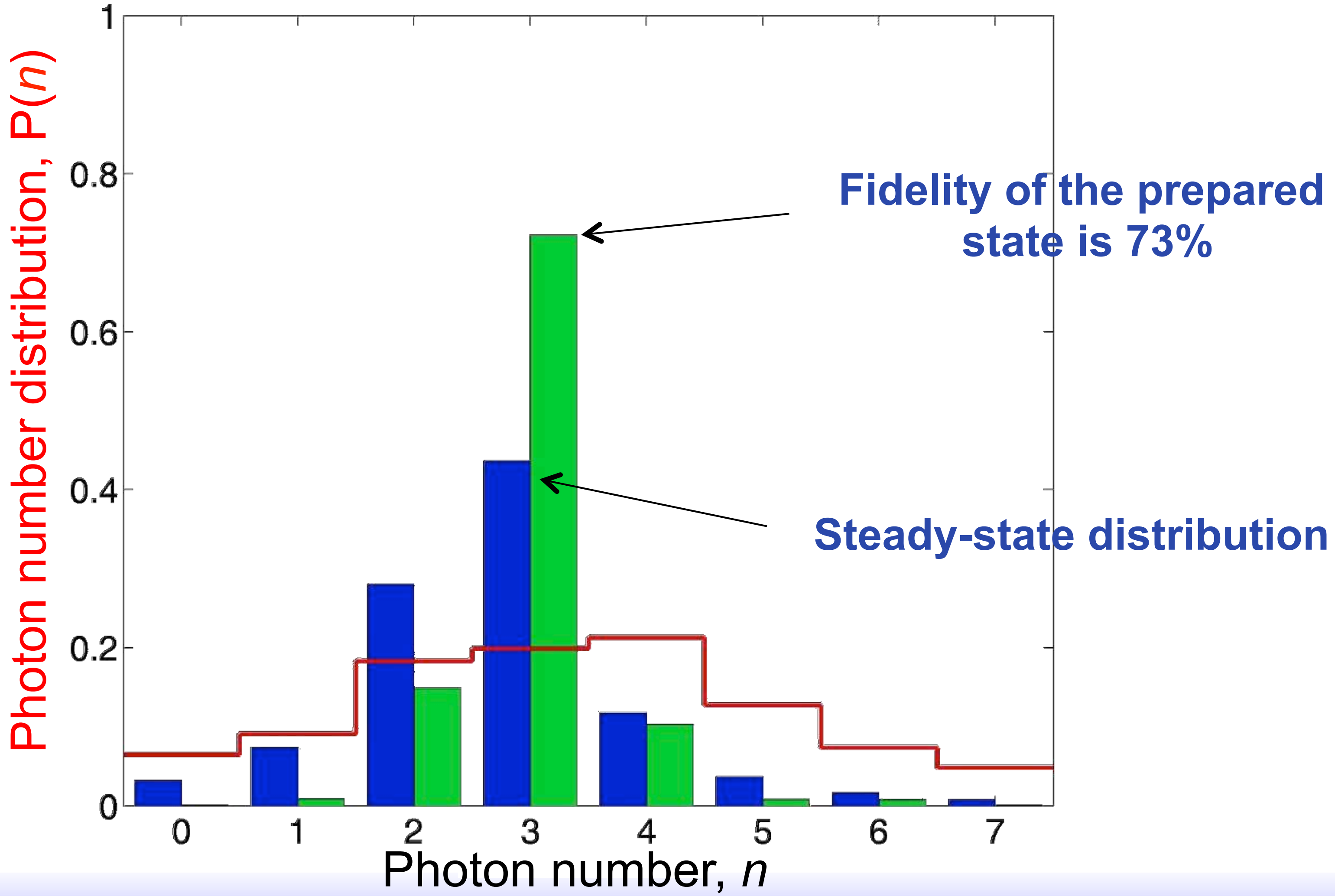


Steady-state distribution:

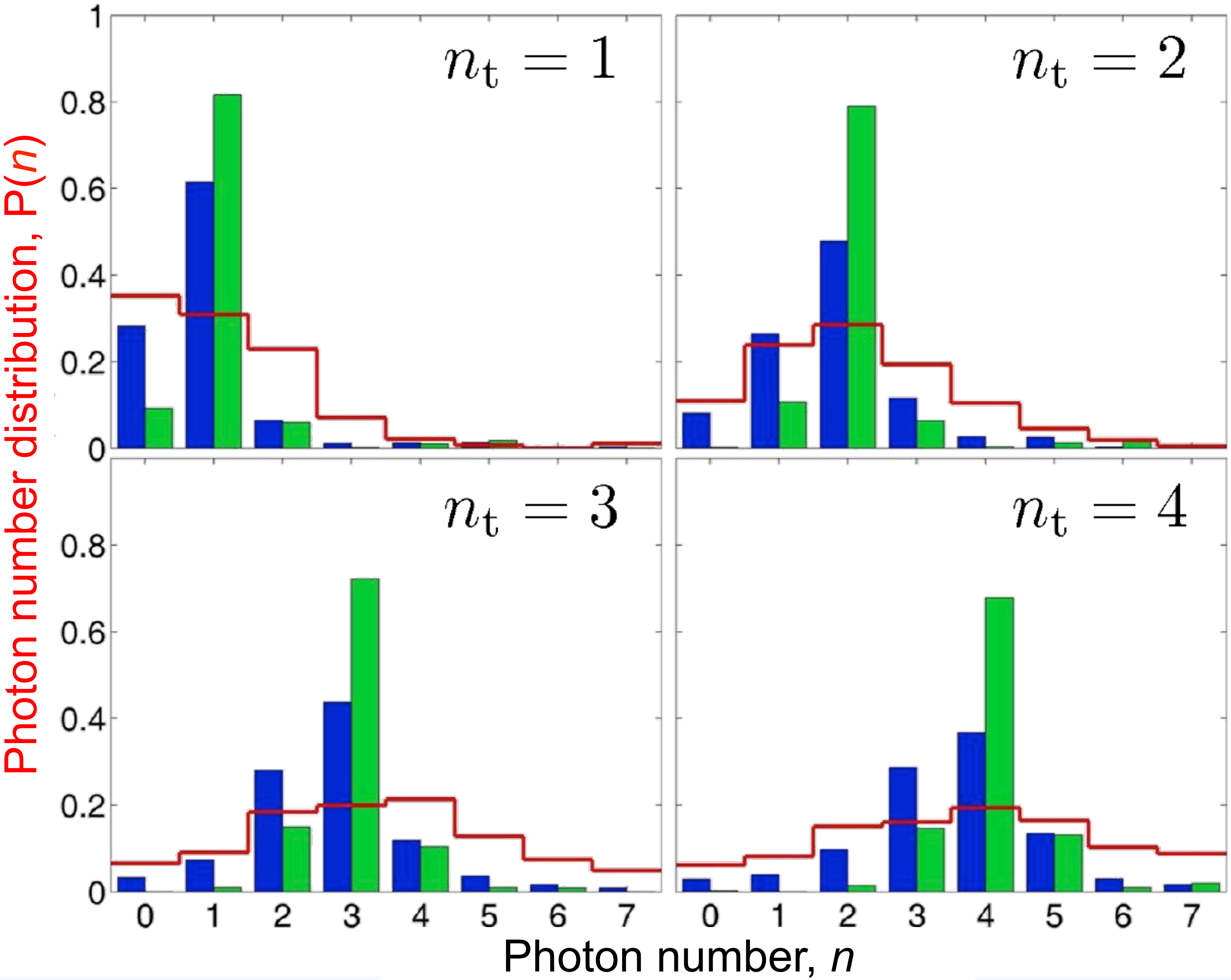
- **clearly sub-Poissonian**
- **increased fidelity: P(3) = 43%**

Fidelity of the state stabilization

- Average over 4000 trajectories stopped **when $P(n_t)$ reaches 80%**
- Feedback sequence immediately followed by a QND measurement based on the maximum likelihood reconstruction of P(n)



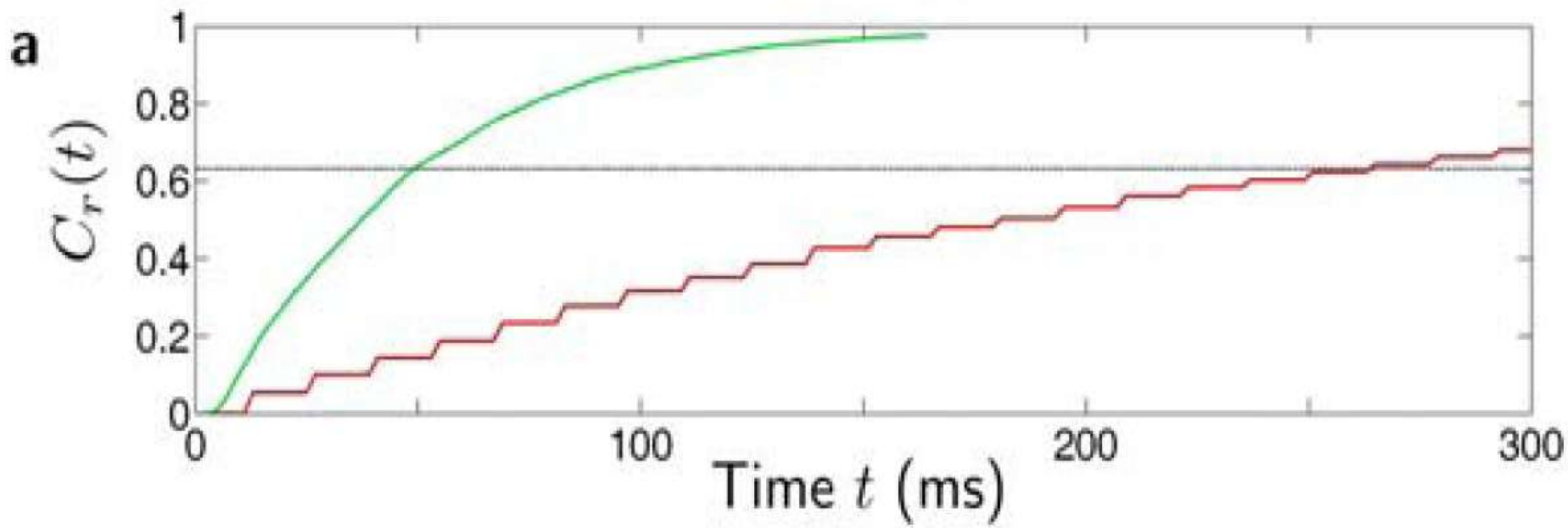
Fidelity of the state stabilization



C. Sayrin et al Nature **477**, 73 (2011), arxiv1107.4027

Rate of convergence

- Fraction of experiments reaching 80% fidelity versus time
- Compare feedback with a fail and retry method
  - ❑ Measure QND for 10 ms
  - ❑ If n=3 success
  - ❑ If not reset field and retry



➞ Feedback operation is much more efficient

Classical and quantum actuators

- Many injections to compensate for a quantum jump
  - ❑ Mismatch between the classical source and the nature of the single-photon quantum jumps
  - ❑ Slow recovery from jumps (15 ms)
    - ➞ Method limited to 4 photons
      - –Lifetime of |4>=15 ms

- Quantum feedback with a quantum actuator
  - ❑ Single atom, interacts resonantly with the cavity mode
    - ➞ Prepared in e: ideally emits a single photon
    - ➞ Prepared in g: ideally absorbs a single photon
  - ❑ Ideally compensates for jumps in a single operation
    - ➞ Fast recovery
    - ➞ Stabilization of higher-lying Fock states

Single atom actuator action

Rabi oscillation in n photons

$$|e,n\rangle \longleftrightarrow |g,n+1\rangle$$

Single atom actuator action

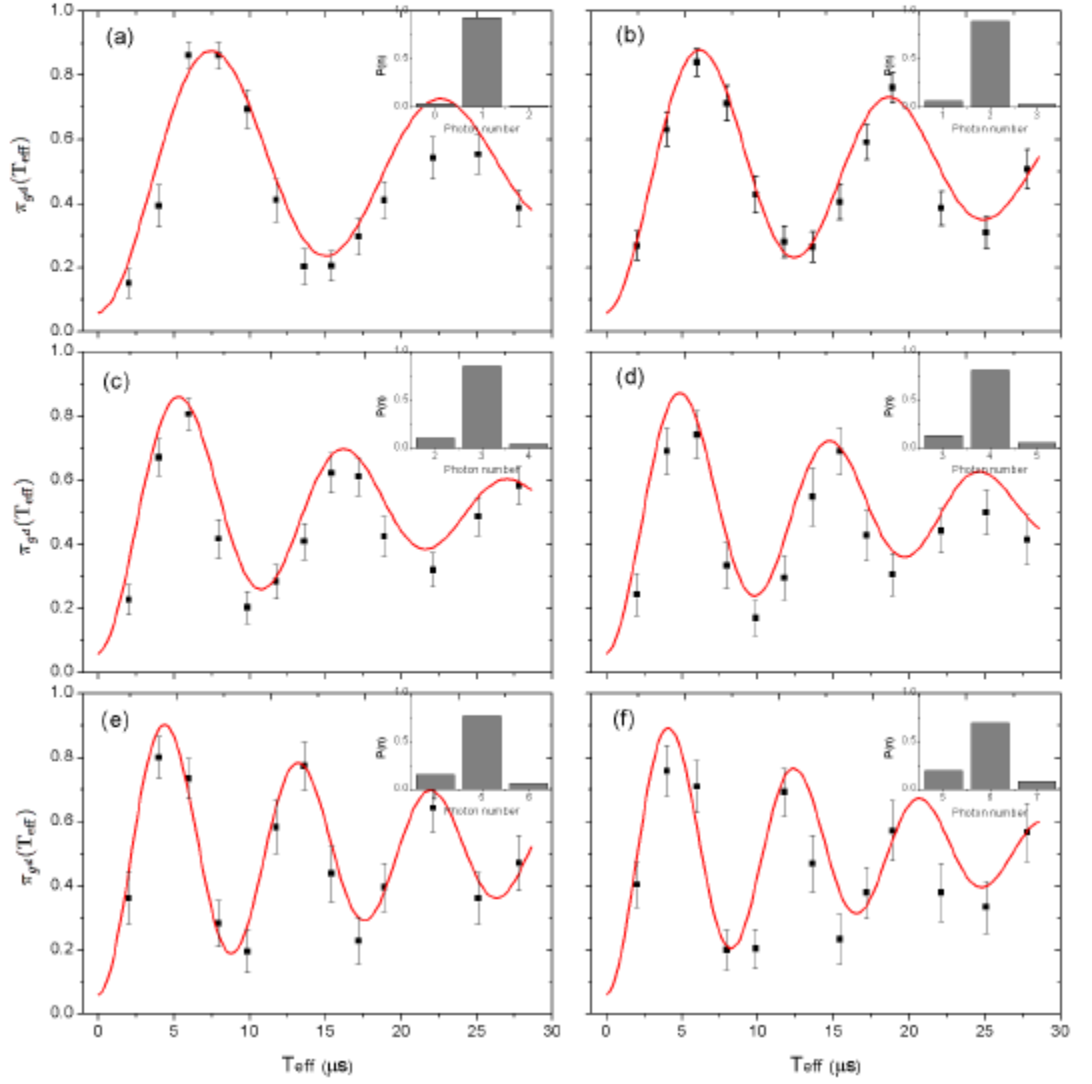
Rabi oscillation in n photons

$$|e,n\rangle \longleftrightarrow |g,n+1\rangle$$

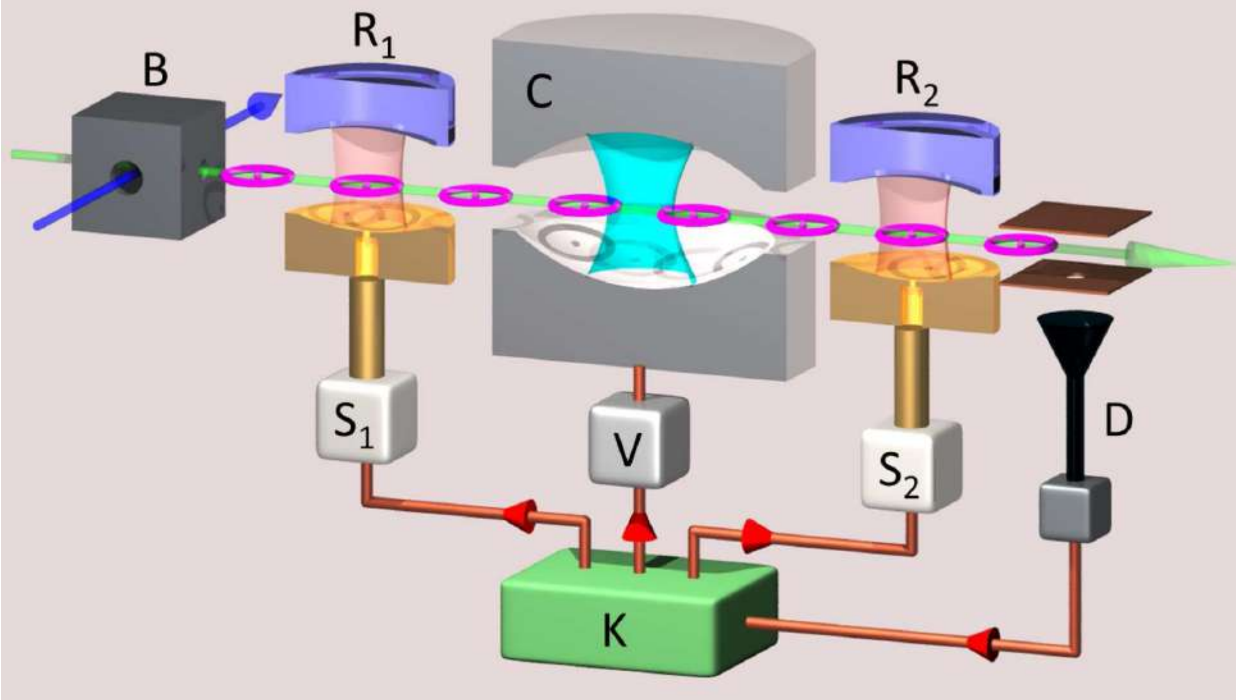
$$P_{res}(\mathrm{em}/n)$$

$$P_{res}(\mathrm{abs}/n)$$

Are obtained
by fitting these data
K models the interaction
using these calibrations

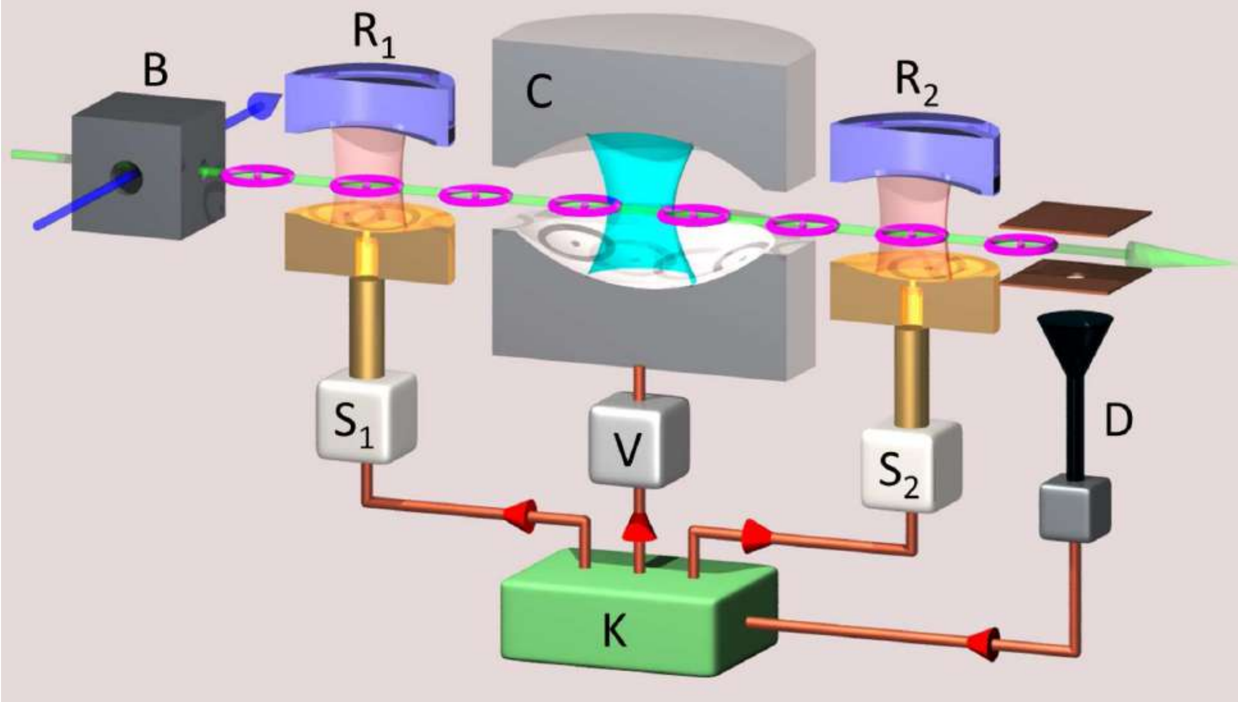


Feedback with atom actuator



- Controller action: (1) 4 possible choices
  - ❑ Absorber: no pulse in R_1, atom set on resonance
  - ❑ Emitter: π pulse in R1, atom set on resonance
  - ❑ QND sensor: $\pi/2$ pulses in R1 and R2, atom detuned
  - ❑ At last time: K can decide while the atom is flying not to set the atom on resonance if this became a better choice.

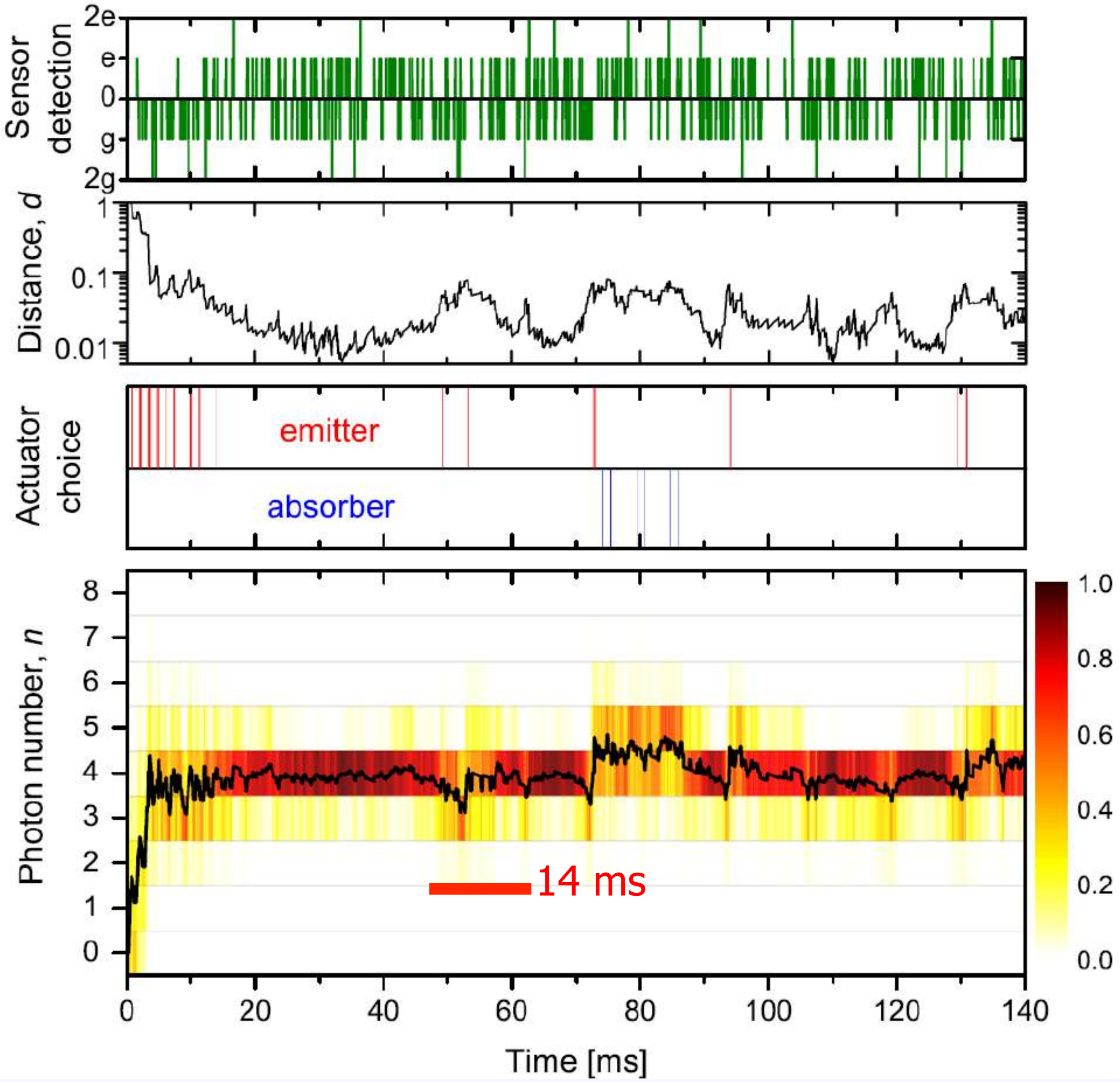
Feedback with atom actuator



- Controller action (2)
  - For atomic emitter/absorber there are no coherences in the field. Is is enough to estimate the photon number distribution P(n).
  - Used distance

$$d = \sum_n (n - n_t)^2 p(n) = \Delta n^2 + (\overline{n} - n_t)^2$$

Single closed loop trajectory

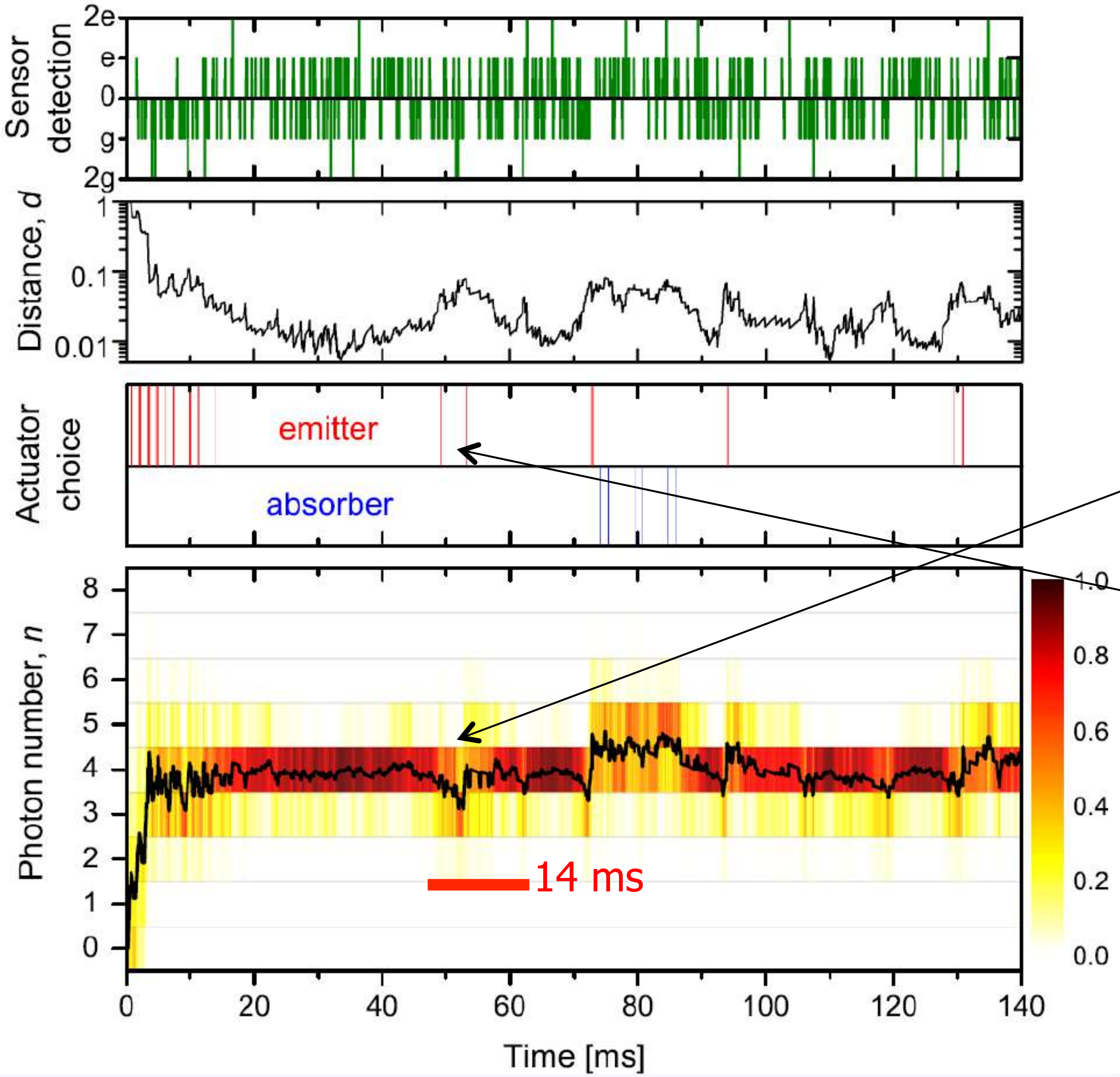


QND probe atoms

$$d = \Delta n^2 + \left(\overline{n} - n_t\right)^2$$

➙ much faster than the 14 ms convergence time of coherent injection method

Single closed loop trajectory



QND probe atoms

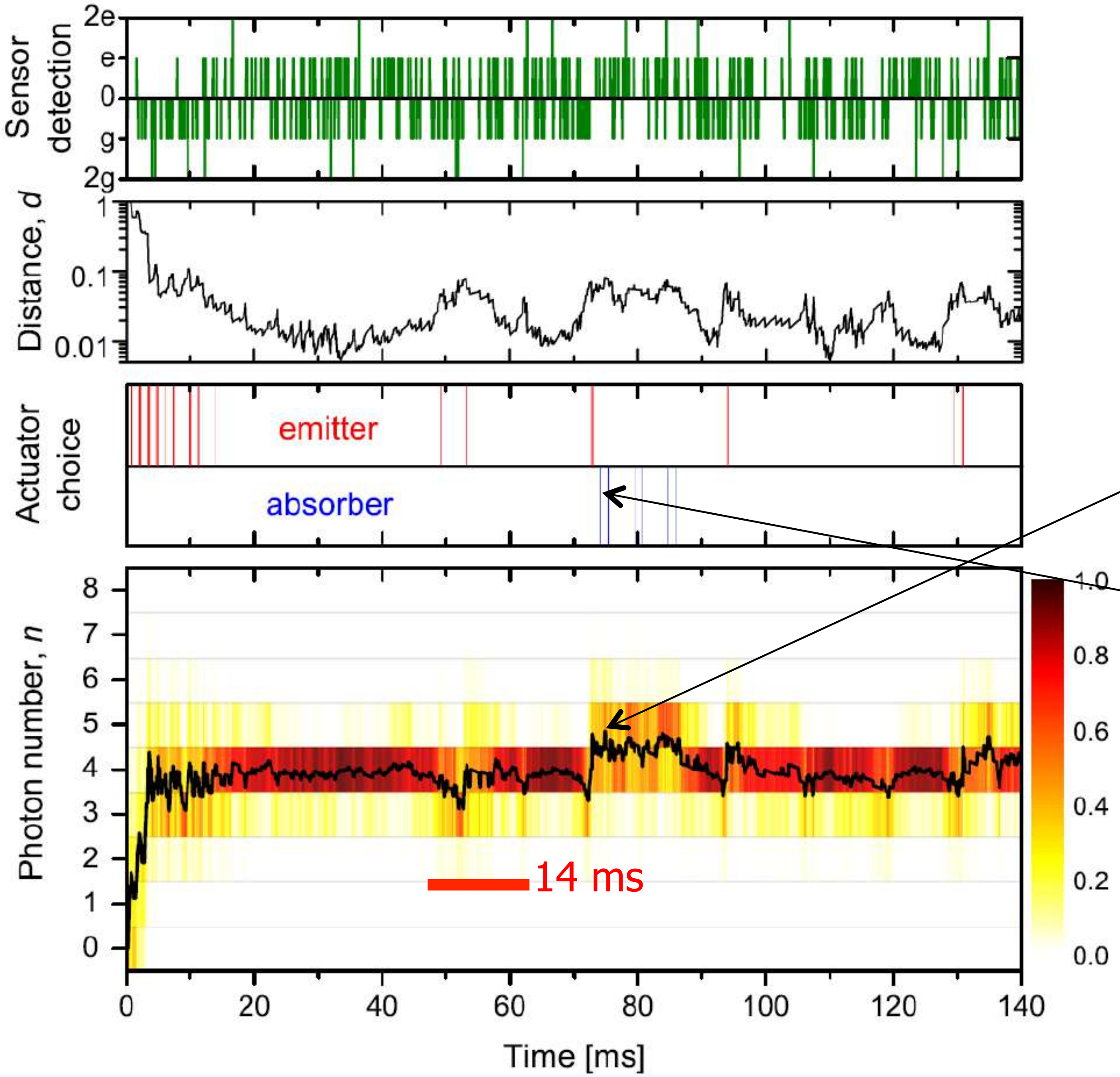
$$d = \Delta n^2 + (\bar{n} - n_t)^2$$

One lost photon

Correction by emitter atoms

➙ much faster than the 14 ms convergence time of coherent injection method

Single closed loop trajectory



QND probe atoms

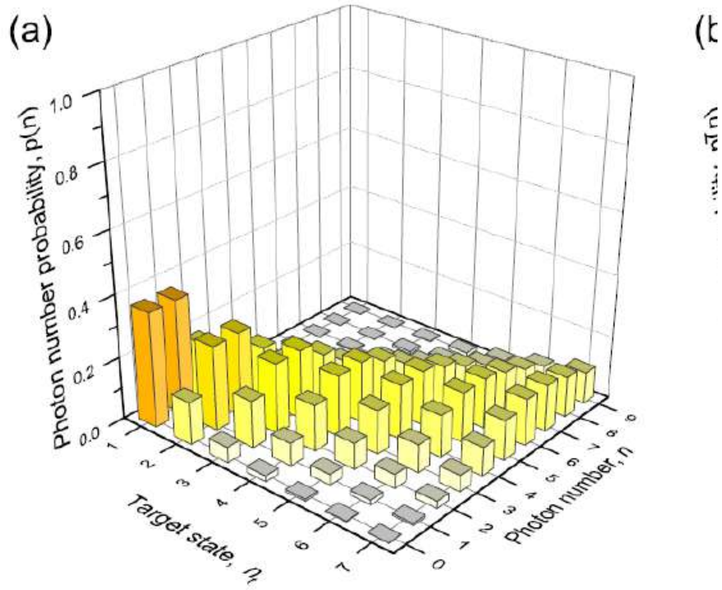
$$d = \Delta n^2 + \left(\bar{n} - n_t\right)^2$$

One more photon

Correction by absorber

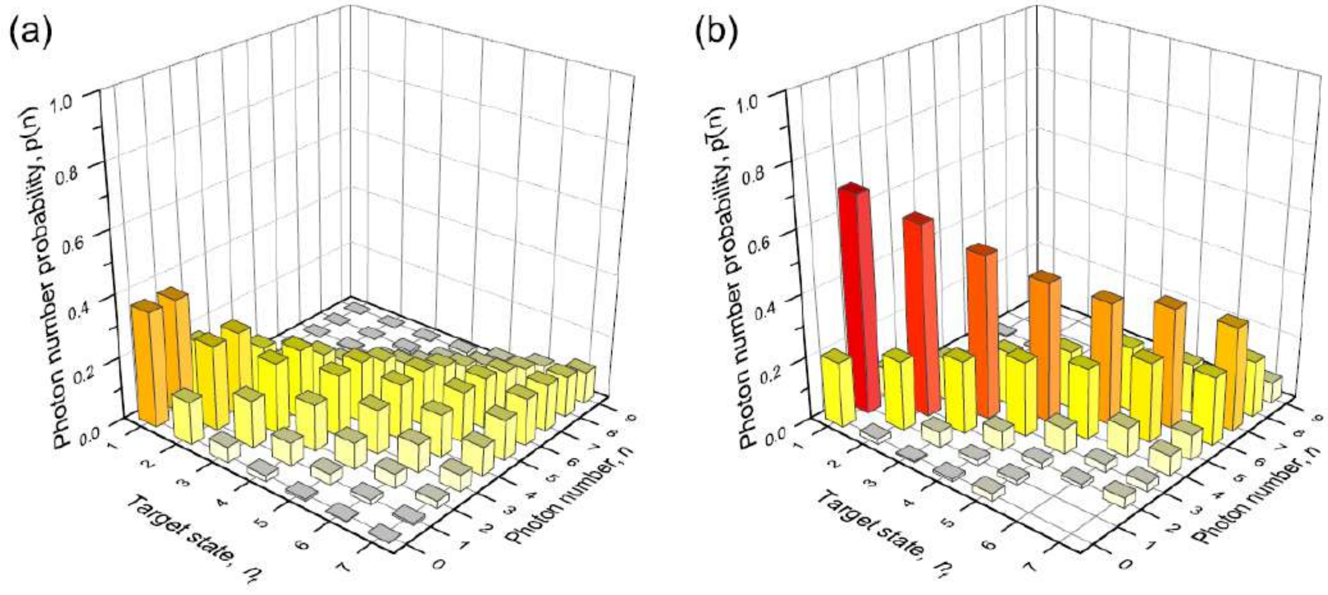
➙ much faster than the 14 ms convergence time of coherent injection method

Feedback for high photon numbers



Reference
coherent state with
n_t photons on the average

Feedback for high photon numbers



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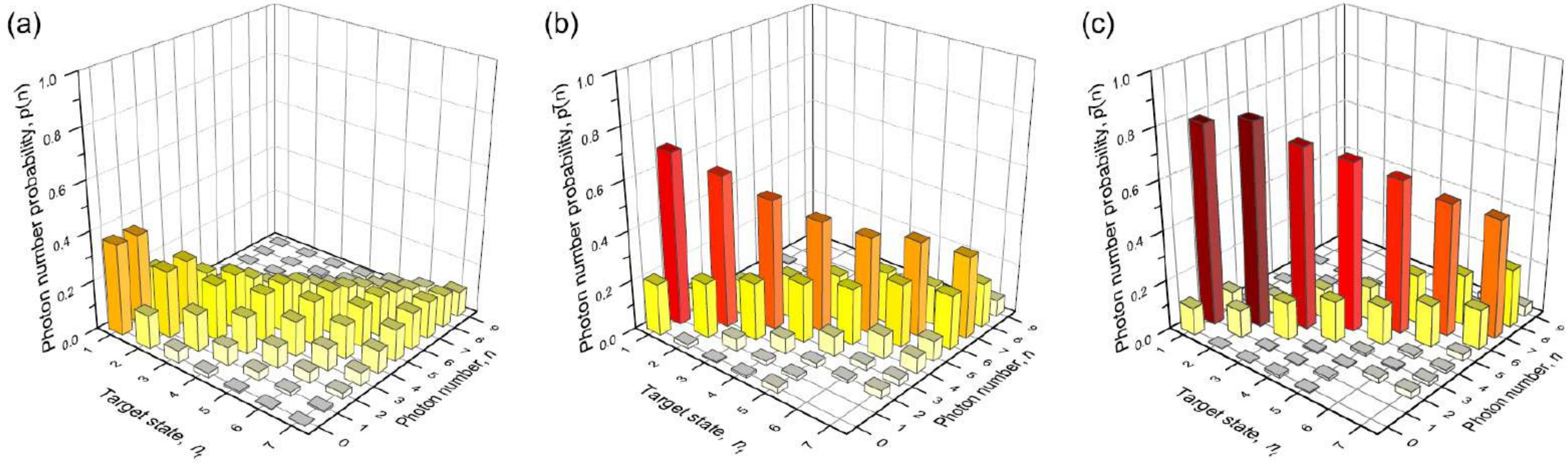
coherent state with
n_t photons on the average

Steady state
- stops loop at 140 ms
- independent QND estimation of average photon number distribution *P(n)*

Feedback for high photon numbers



Reference
coherent state with
n_t photons on the average

Steady state
- stops loop at 140 ms
- independent QND estimation of average photon number distribution *P(n)*

Optimal stop
- Stops loop when $p(n_t)>0.8$
- Independent QND estimation of *P(n)*

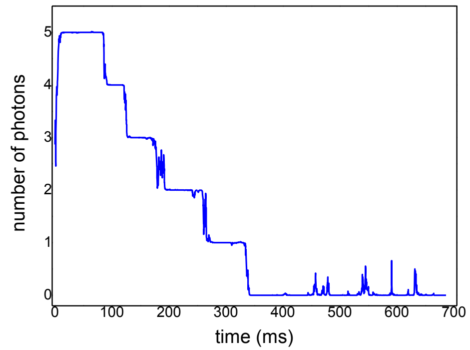
Stabilization of photon numbers up to 7

X. Xhou et al., Phys. Rev. Lett. 108, 243602 (2012)

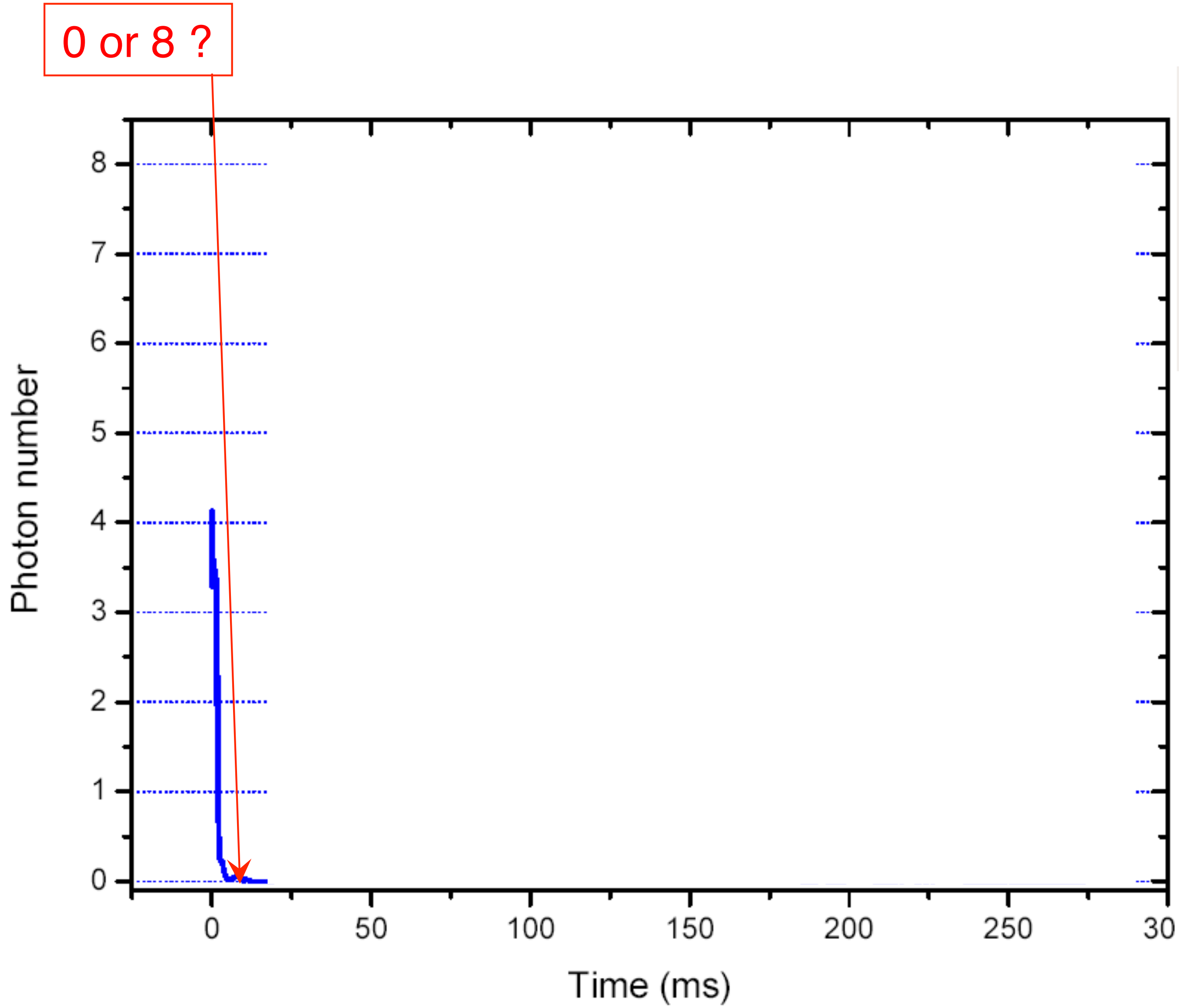
Next sections where not presented during lecture

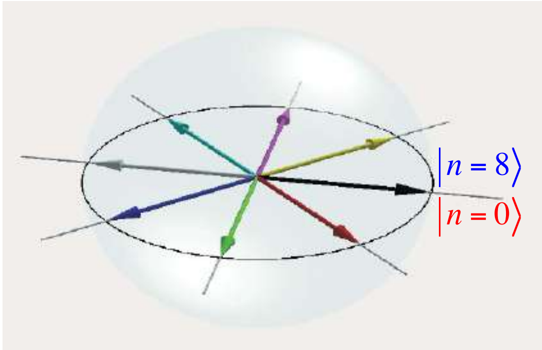
II. The past quantum state trajectory reconstruction method

1. Principle of the method

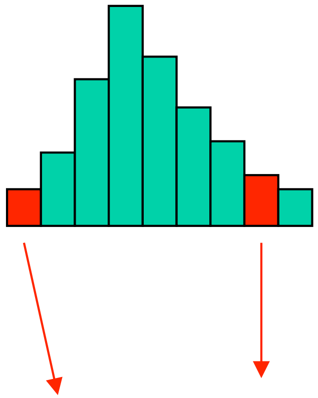


A particular field trajectory





Interferometer counts n modulo 8: does not distinguish 0 and 8

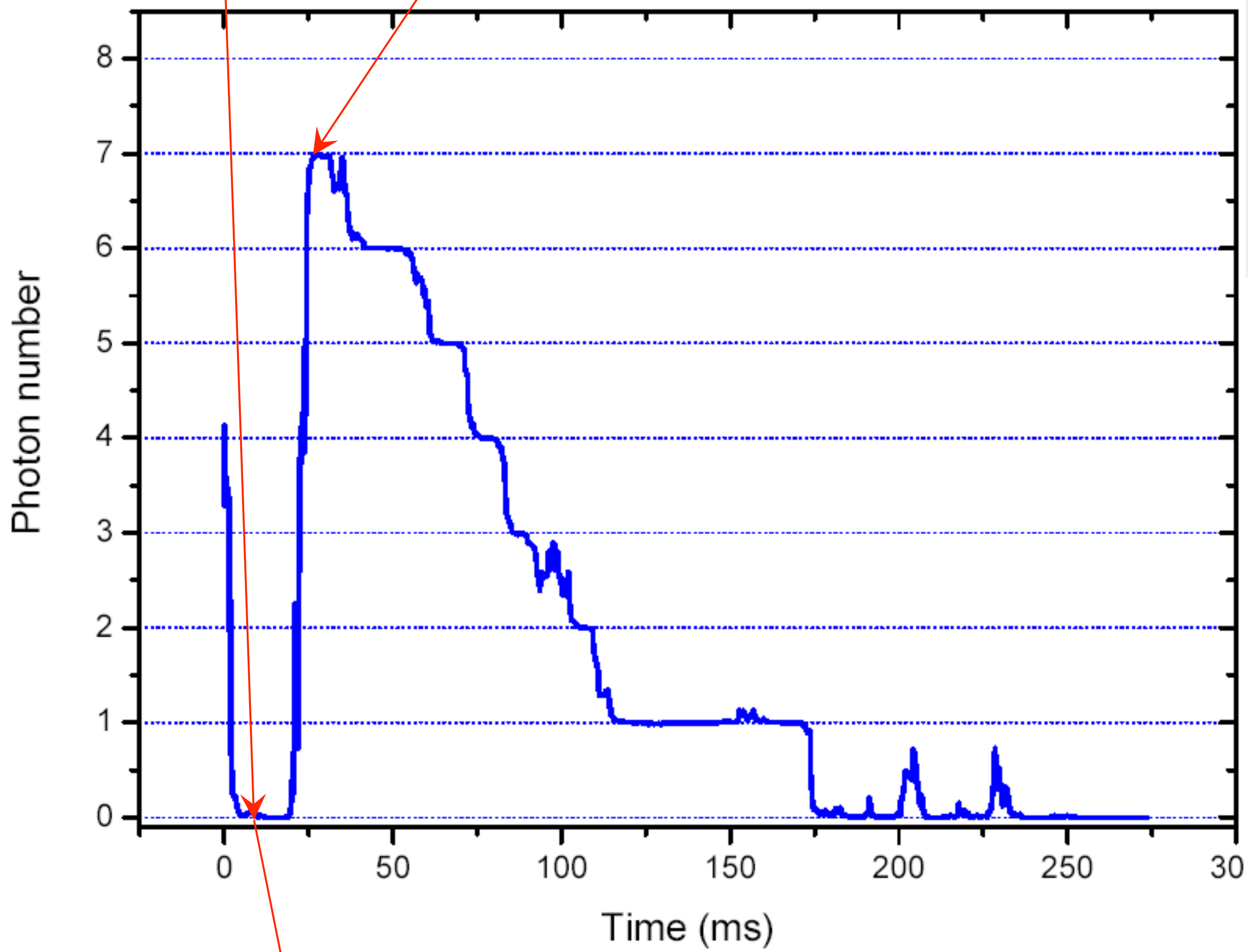


$$c_0 |0\rangle + c_8 |8\rangle$$

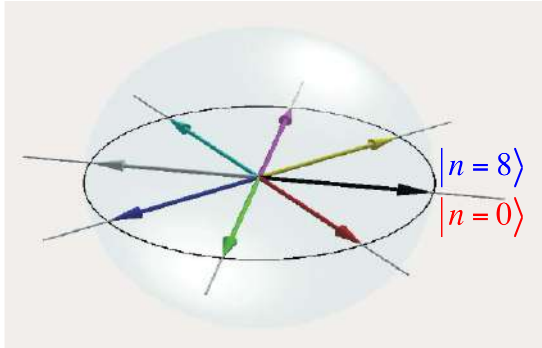
A particular field trajectory

0 or 8 ?

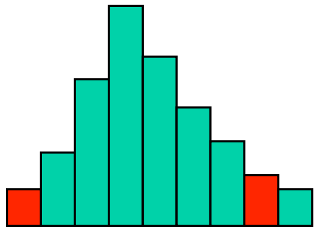
Finaly jumps to 7: if was not 0 but 8!



Looking at the future from that time tells you that 0 was indeed 8 photons!



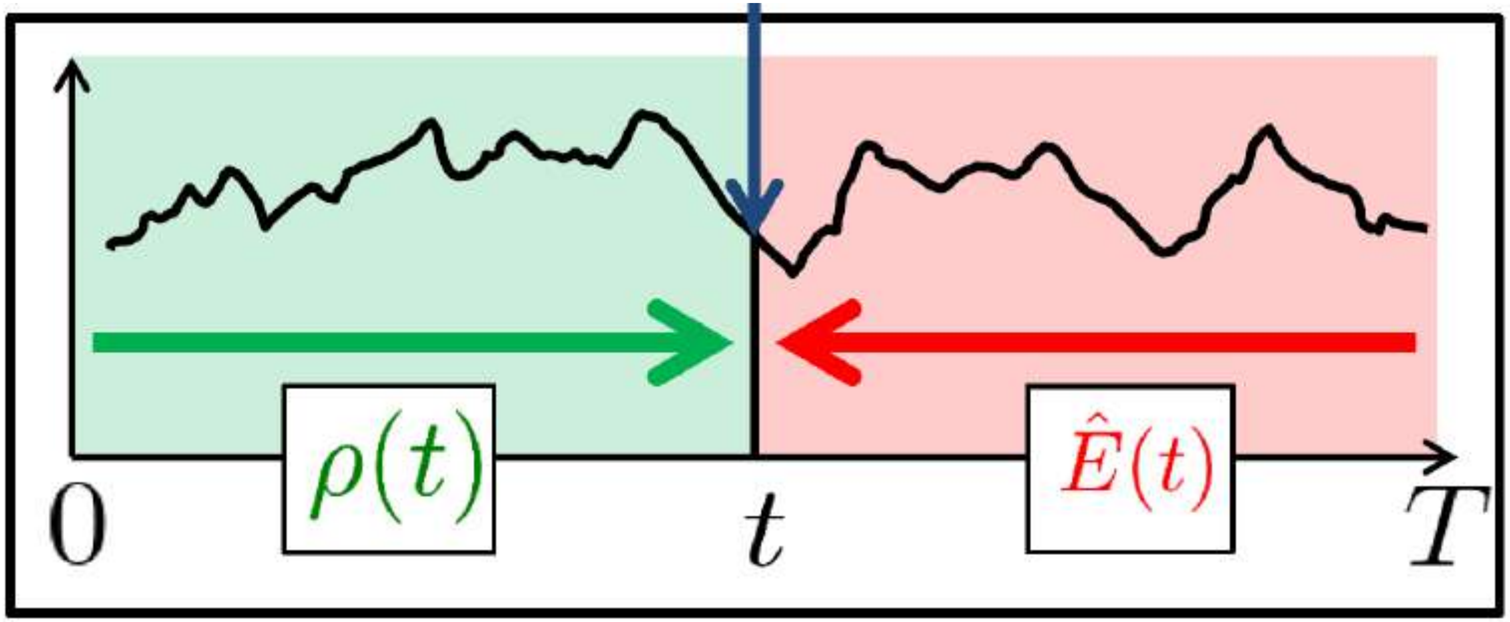
Interferometer counts n modulo 8: does not distinguish 0 and 8



$$c_0\,|0\rangle + c_8\,|8\rangle$$

The Past Quantum State approach

- A posteriori estimation of the photon number at *t* based on all available information, gathered from 0 to *t* AND from *t* to *T*
  - From the journalist's to the historian's perspective
- A quantum formalism: *(S. Gammelmak et al. PRL 111, 160401)*
  - The Past quantum state



- Best estimate for the results of a quantum measurement at *t* based on the **density matrix** $\rho(t)$ computed forward in time

AND on an **"effect matrix"** $E(t)$ computed backwards in time.

Forward-backward estimation

- Forward estimation: usual calculation of the density matrix $\rho(t)$ taking into account projection at measurement and relaxation → $P^f(n,t)$

- Backward estimation: calculation effect matrix $E(t)$ ← $P^b(n,t)$
  - Flat distribution at final time T
  - Same measurement operators as forward
  - 'inverse' relaxation (annihilation and creation operators exchanged)
    - → Exponential growth of the photon number

Forward-backward estimation

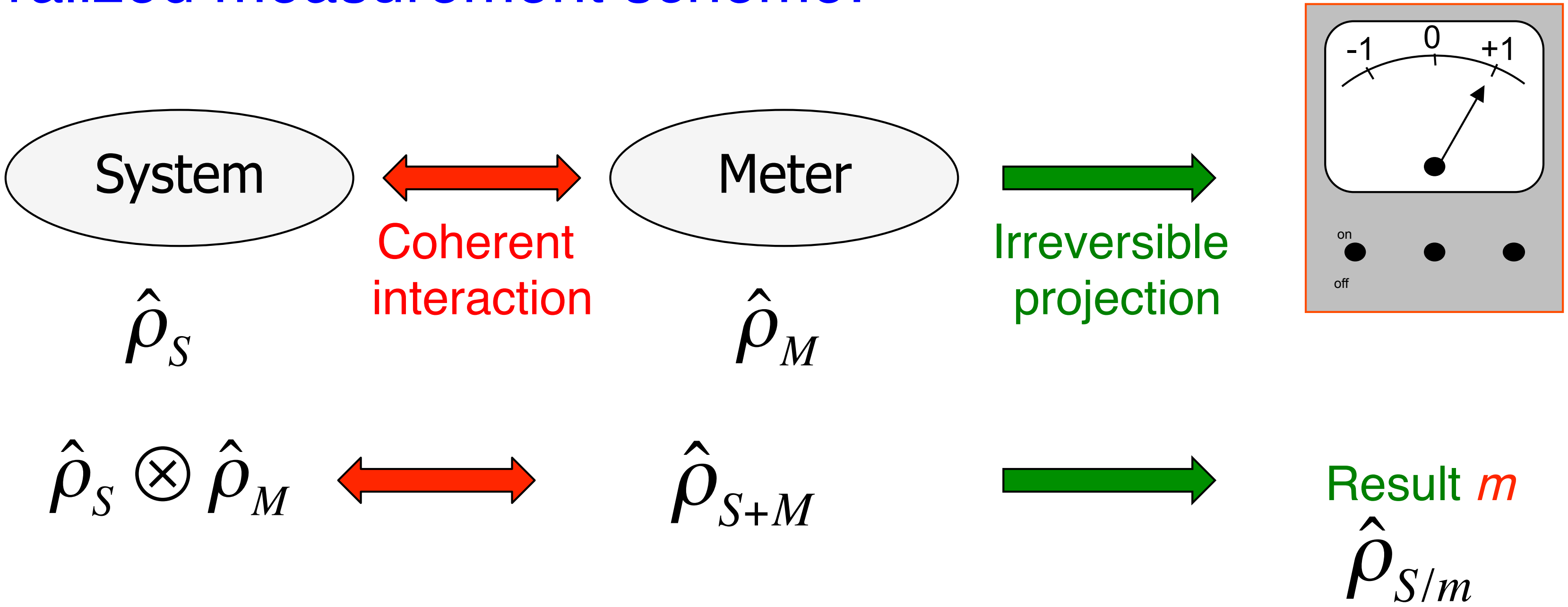
- Forward estimation: usual calculation of the density matrix $\rho(t)$ taking into account projection at measurement and relaxation → $P^{f}(n,t)$
- Backward estimation: calculation effect matrix $E(t)$ ← $P^{b}(n,t)$
  - Flat distribution at final time T
  - Same measurement operators as forward
  - 'inverse' relaxation (annihilation and creation operators exchanged)
    - → Exponential growth of the photon number
- Foreward/backward for diagonal measurement/relaxation operators

$$P^{fb}(n,t) = \frac{P^f(n,t)P^b(n,t)}{\sum_m P^f(m,t)P^b(m,t)}$$

  - PQS reduces to the "forward/backward smoothing algorithm", which can be safely used in this quantum context
  - P(n) is the product of two photon number distributions computed forward and backward in time.

Extracting information from a measurement

- Generalized measurement scheme:



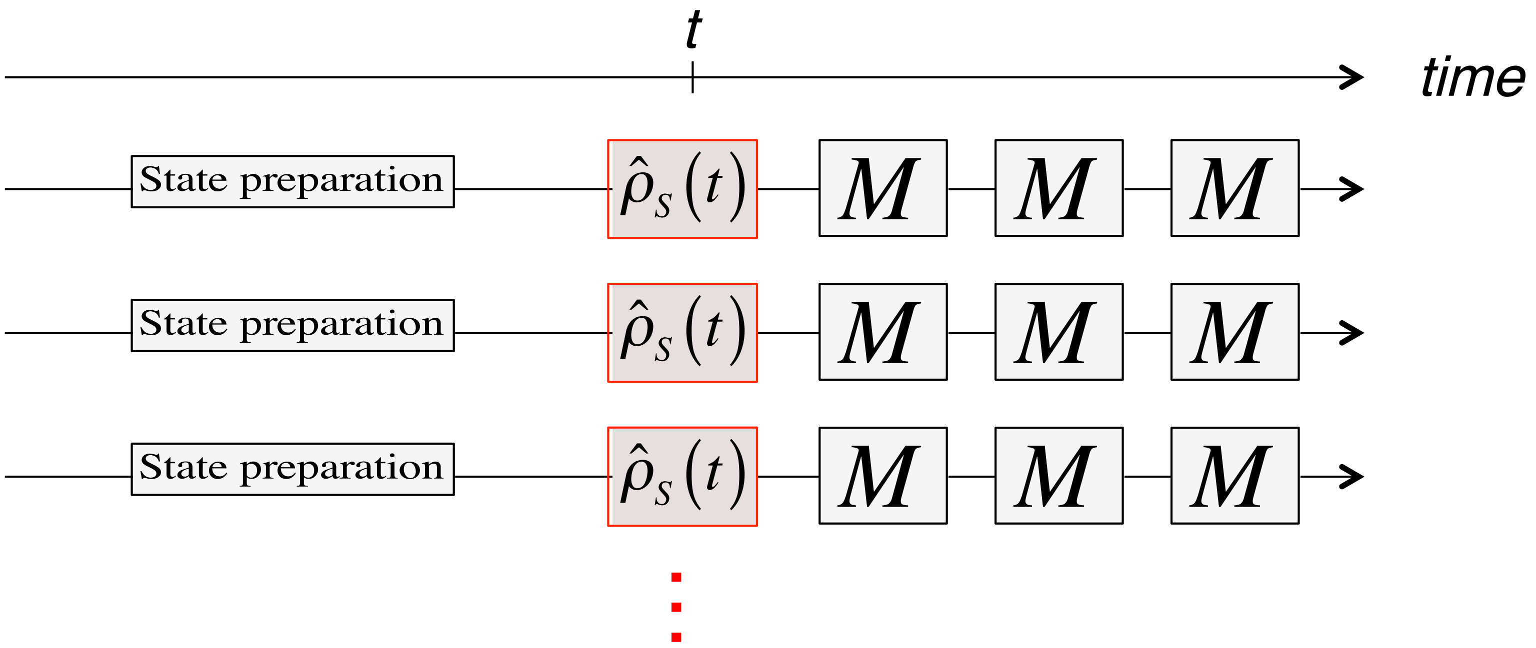
$$\hat{\rho}_S \longrightarrow \hat{\rho}_{S/m}$$

The measurement result provides (partial) information on S

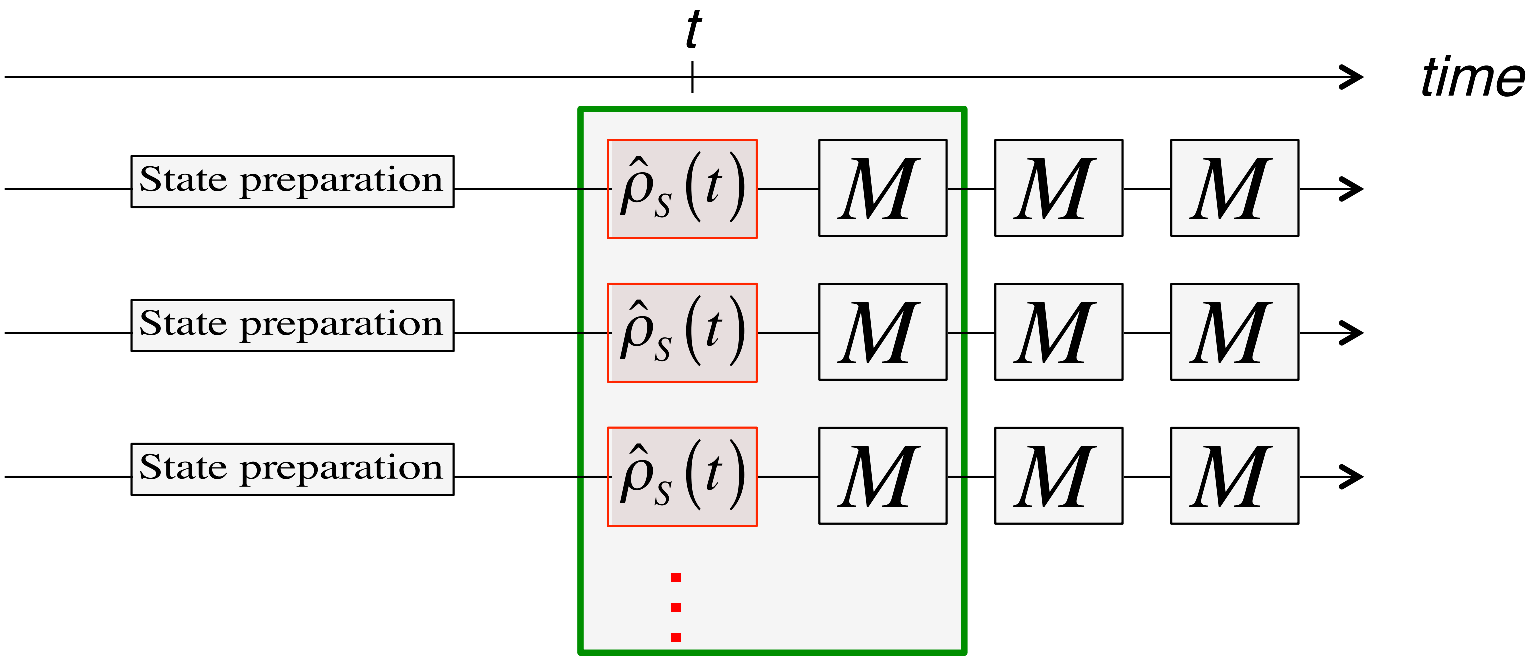
General state reconstruction problem:

- optimize the amount of information extracted on S
- get the best estimate of the state after a measurement

Quantum state reconstruction and time evolution



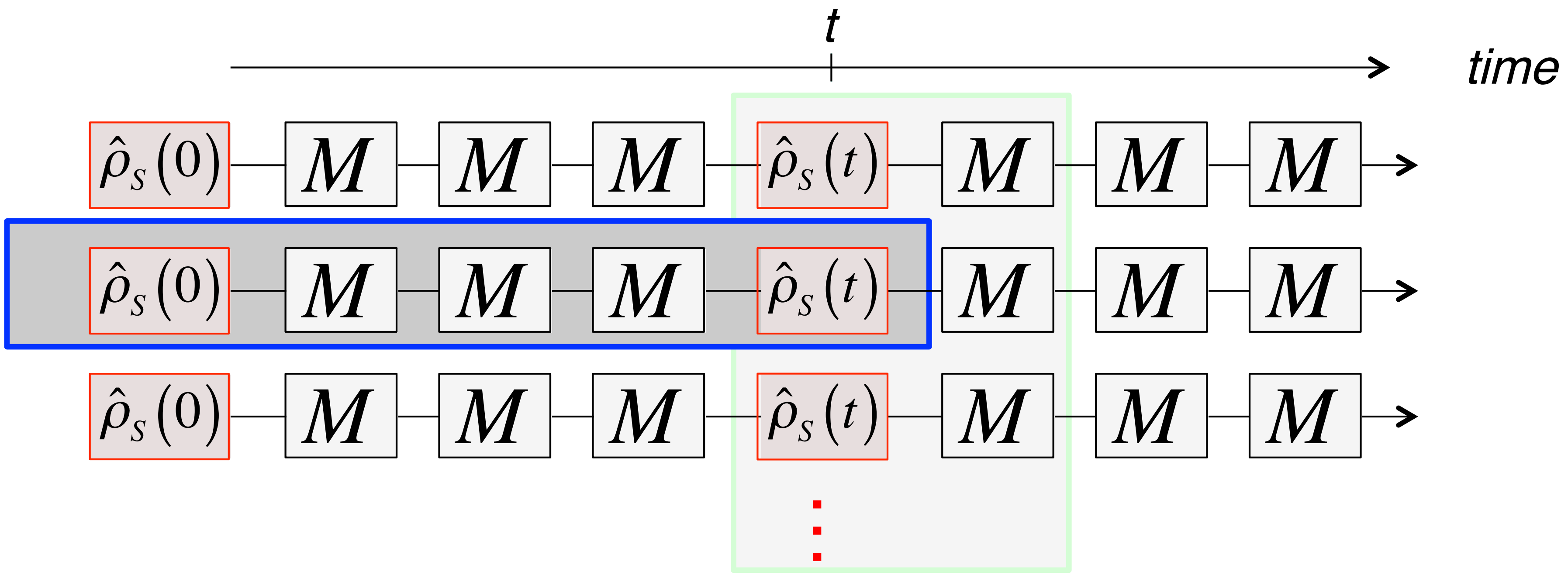
Quantum state reconstruction and time evolution



- Reconstruct $\hat{\rho}_S(t)$ given a large number of identical preparation
  → quantum state tomography

→ topic of lecture 4

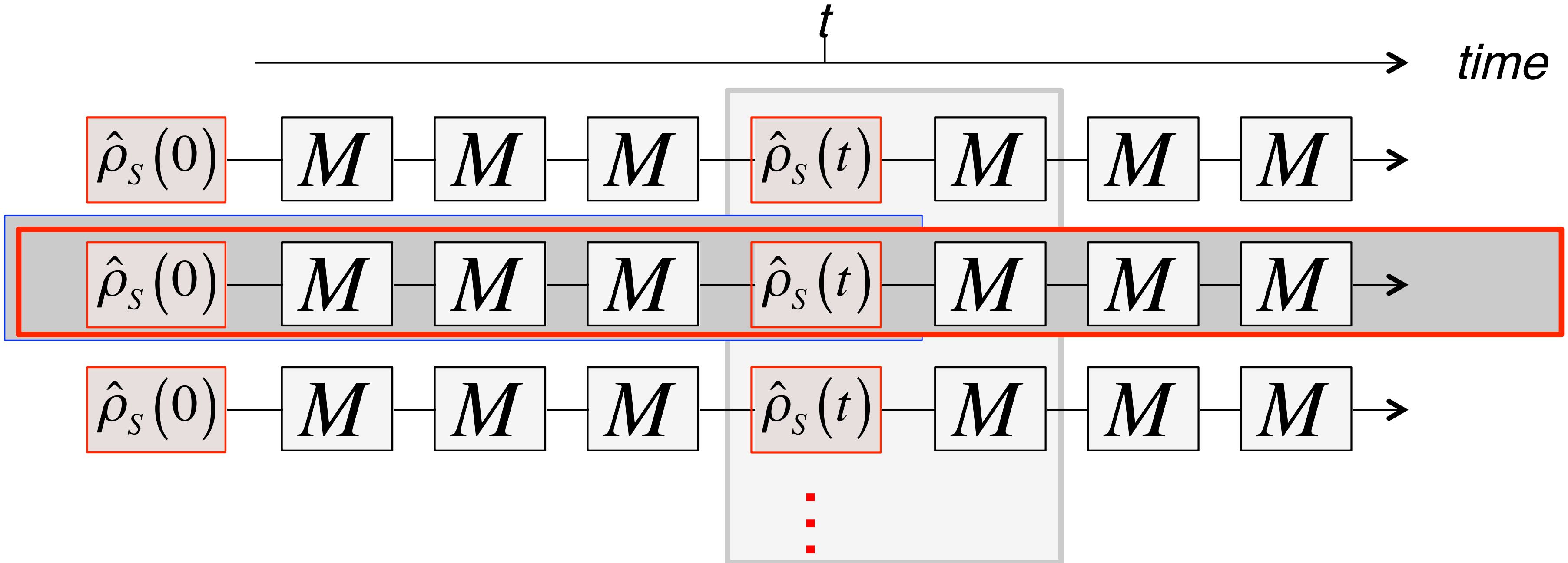
Quantum state reconstruction and time evolution



- Reconstruct $\hat{\rho}_S(t)$ given a large number of identical preparation
  ➞ quantum state tomography

- Estimate $\hat{\rho}_S(t)$ in a given realization knowing measurement results before t_0 ➞ quantum trajectory reconstruction
  "standard approach"

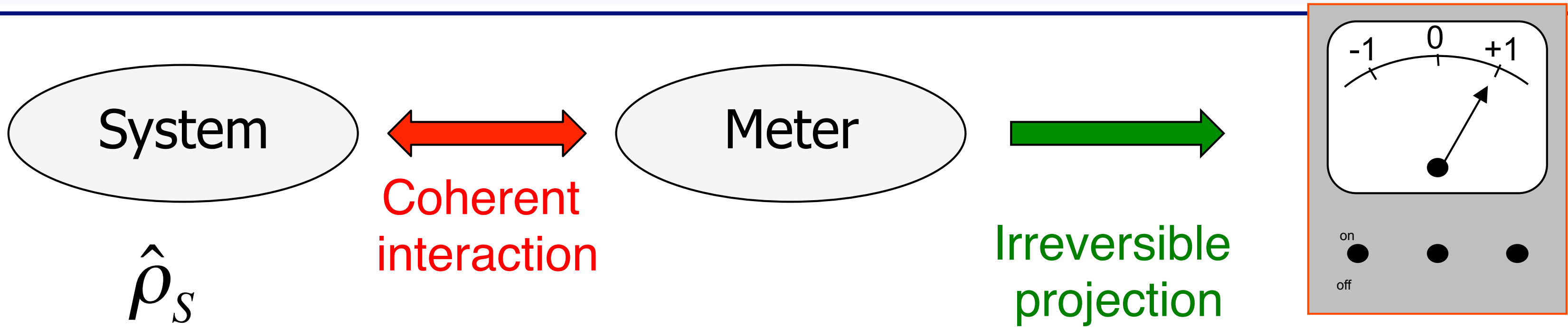
Optimal quantum state reconstruction and time evolution



- Reconstruct $\hat{\rho}_S(t)$ given a large number of identical preparation
  → quantum state tomography

- Estimate $\hat{\rho}_S(t)$ in a given realization knowing measurement results before t_0 → quantum trajectory reconstruction
  "standard approach"
- Estimate $\hat{\rho}_S(t)$ in a given realization knowing measurement results **before and after** t_0 → Past quantum state *(Mölmer PRL 2013)*

Generalized quantum measurement

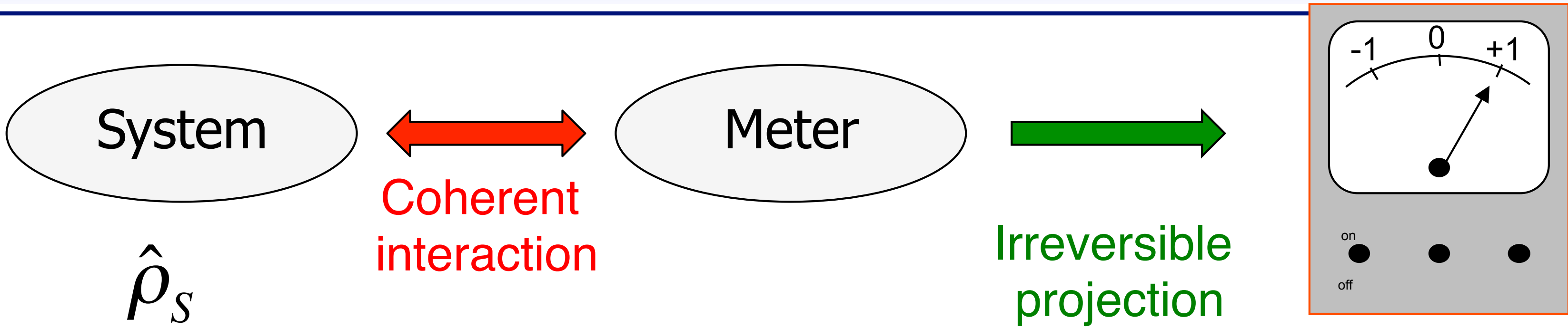


$$\hat{\rho}_S$$

$$\hat{\rho}_S \longrightarrow \hat{\rho}_{S/m} = \frac{\hat{M}_m \hat{\rho}_S \hat{M}_m^+}{Norm}$$

- Operators $\left\{\hat{M}_m\right\}$: set of operators of S such that $\sum_m \hat{M}_m^+ \hat{M}_m = \hat{1}$.
- Proba of result m: $P(m) = tr\ \hat{M}_m \hat{\rho}_S \hat{M}_m^+$

Generalized quantum measurement



$\hat{\rho}_S$

$$\hat{\rho}_S \longrightarrow \hat{\rho}_{S/m} = \frac{\hat{M}_m \hat{\rho}_S \hat{M}_m^+}{Norm}$$

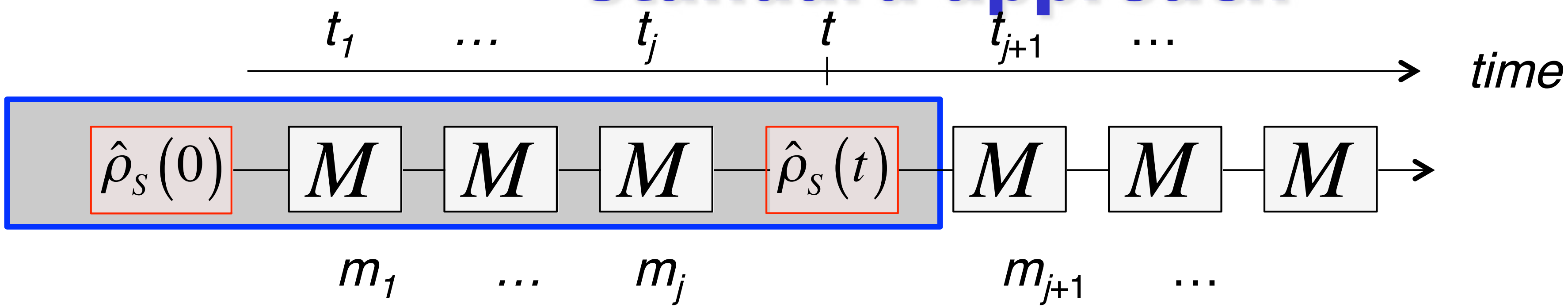
(See T. Gorin lecture)

- Operators $\left\{\hat{M}_m\right\}$: set of operators of S such that $\sum_m \hat{M}_m^+ \hat{M}_m = \hat{1}$.
- Proba of result m: $P(m) = tr\, \hat{M}_m \hat{\rho}_S \hat{M}_m^+$

➙ **describes any evolution**:

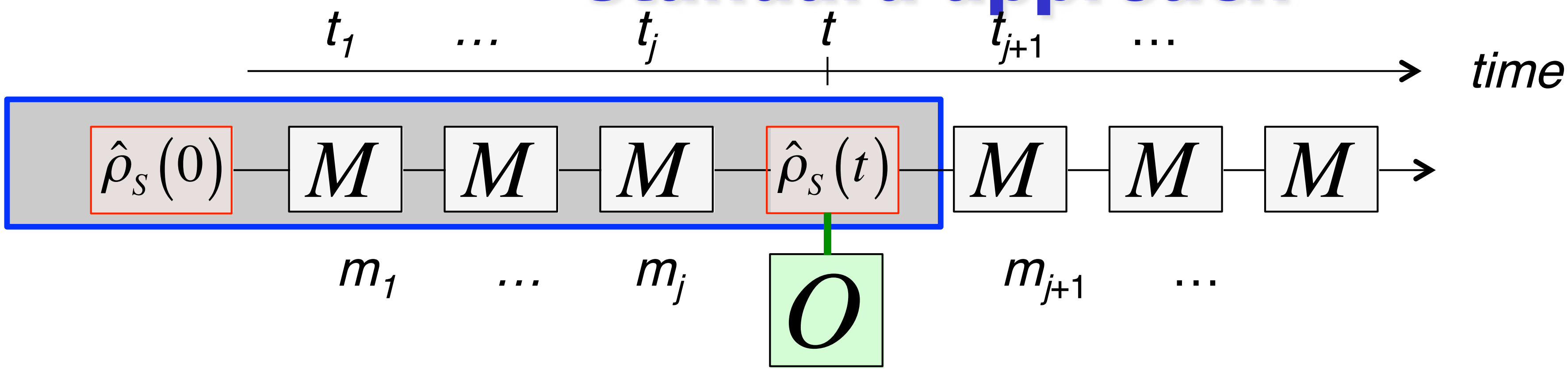
- any measurement
- unitary: only one operator $\hat{M}_0 = \hat{U}(t_0, t)$
- relaxation can be seen as unread measurement in some environment ➙ also described by the action of $\hat{M}_m$ on $\hat{\rho}_S$

Quantum trajectory reconstruction: "standard approach"



$$\hat{\rho}_S(t) = \hat{\rho}_{S/\{m_j\}} = \frac{\hat{M}_{m_j} \dots \hat{M}_{m_0} \hat{\rho}_S(0) \hat{M}^{+}_{m_0} \dots \hat{M}^{+}_{m_j}}{Norm}$$

Quantum trajectory reconstruction: "standard approach"



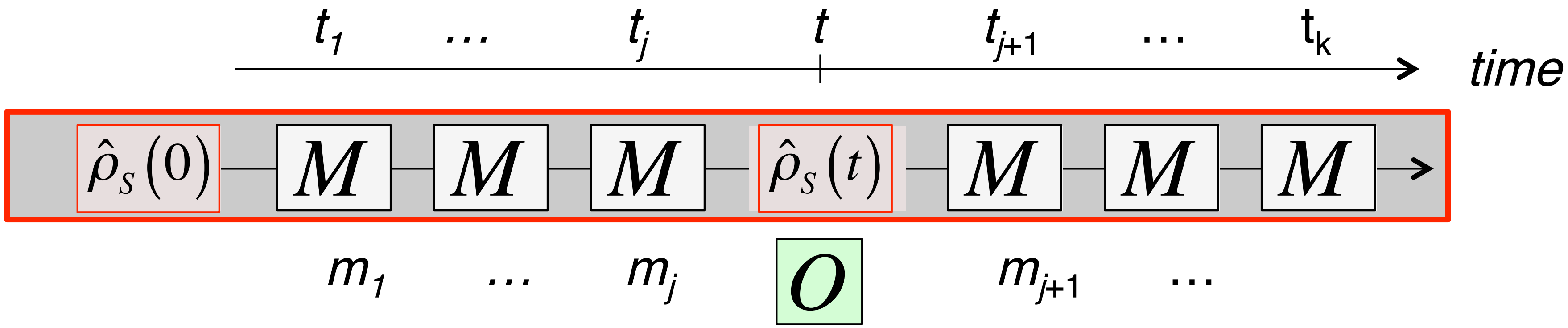
$$\hat{\rho}_S(t) = \hat{\rho}_{S/\{m_j\}} = \frac{\hat{M}_{m_j} \dots \hat{M}_{m_0} \hat{\rho}_S(0) \hat{M}^+_{m_0} \dots \hat{M}^+_{m_j}}{Norm}$$

With $\hat{\rho}_{S/\{m_j\}}$ one can describe the results of any measurement $\{\hat{O}_i\}$ performed at time *t*.

➞ one gets the probability of the measurement result o_i conditional to previous measurements

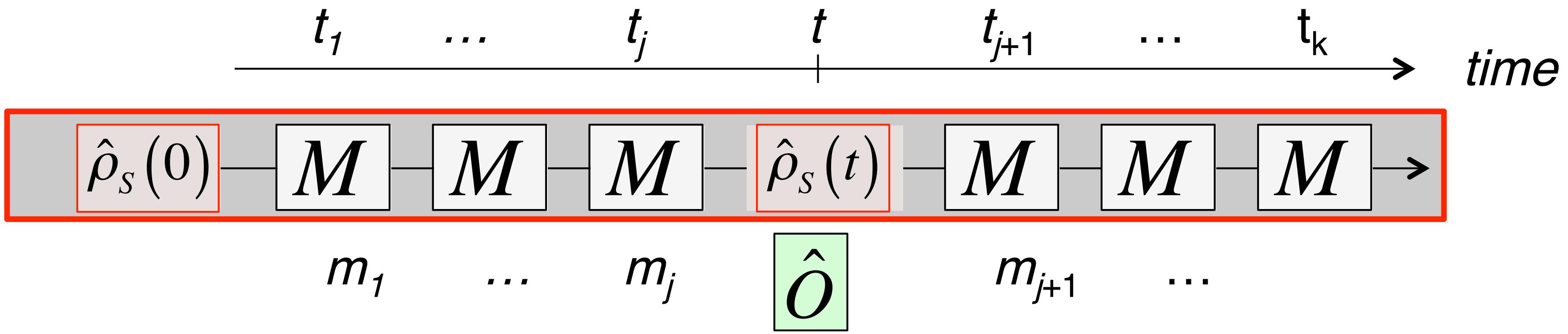
$$P\left(o_i, t / \{m_{1\dots j}\}\right) = \frac{tr\, \hat{O}_i\, \hat{\rho}_S(t_j)}{Norm}$$

The "past quantum state approach"



We are now interested in another conditional probability: description of the measurement of $\{\hat{O}_i\}$ knowing the past and future measurement results.

The "past quantum state approach"



We are now interested in another conditional probability: probability of the measurement results $\{o_i\}$ knowing the past and future measurement results.

$$P\left(o_i, t / \{m_{1\ldots k}\}\right) = \frac{tr\, \hat{O}_i\, \hat{\rho}_S(t)\, \hat{O}_i^+\, \hat{E}_S(t)}{Norm}$$

Mölmer PRL 2013

$$\hat{\rho}_S(t) = \hat{\rho}_{S/\{m_k\}} = \frac{\hat{M}_{m_j} \ldots \hat{M}_{m_0} \hat{\rho}_S(0) \hat{M}_{m_0}^+ \ldots \hat{M}_{m_j}^+}{Norm}$$

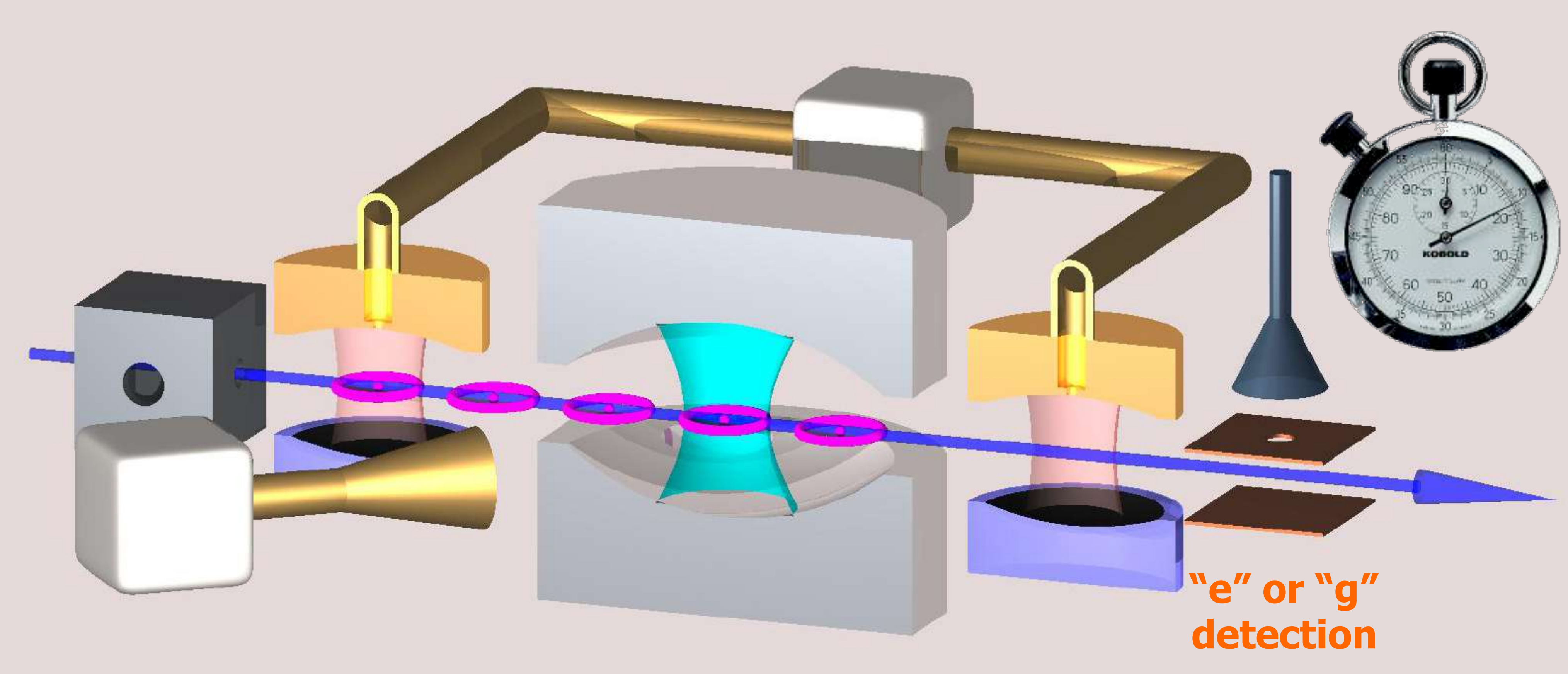
$$\hat{E}_S(t) = \frac{\hat{M}_{m_{j+1}}^+ \ldots \hat{M}_{m_k}^+\, \hat{1}\, \hat{M}_{m_k} \ldots \hat{M}_{m_{j+1}}}{Norm}$$

The "effect" matrix $\hat{E}_S(t)$ is similar to $\hat{\rho}_S(t)$, it involves the same measurement operators but in opposite order.

II. The past quantum state trajectory reconstruction method

2. Experimental implementation

Experimental setup: an atomic clock

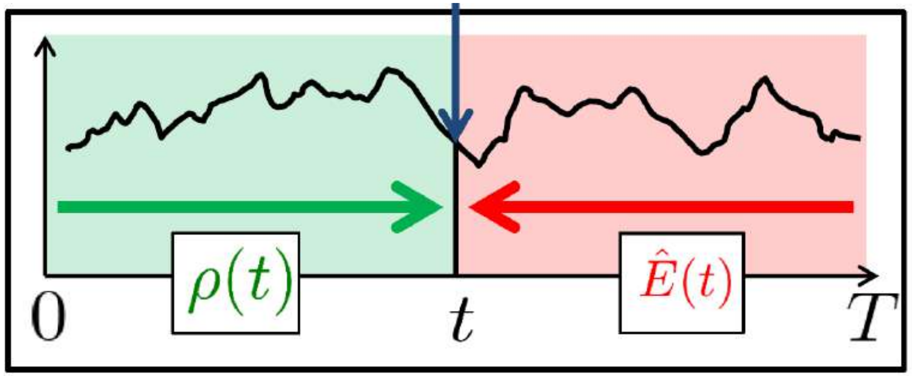


- An atomic clock (Ramsey setup) made of Rydberg for probing light-shifts induced by "trapped" photons

- State selective detection of atoms by field ionization: Atoms detected on "e" or "g" one by one

Following a quantum trajectory

S. Gammelmak et al. PRL 111, 160401(2013)

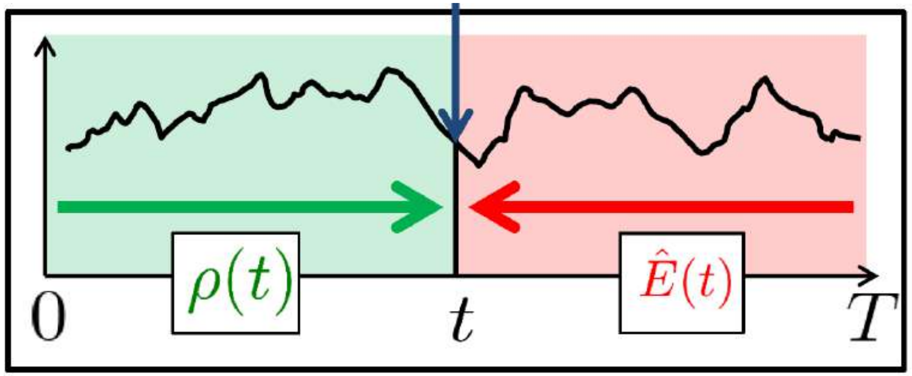


$$P\left(o_i, t / \{m_k\}\right) = \frac{tr\, \hat{O}_i\, \hat{\rho}_S(t)\, \hat{O}_i^+\, \hat{E}_S(t)}{Norm}$$

Apply to photon number operator $\hat{O} = \hat{N}$: $\hat{O}_n = |n\rangle\langle n|$

Following a quantum trajectory

S. Gammelmak et al. PRL 111, 160401(2013)



$$P\left(o_i, t/\{m_k\}\right) = \frac{tr\,\hat{O}_i\,\hat{\rho}_S(t)\,\hat{O}_i^{+}\,\hat{E}_S(t)}{Norm}$$

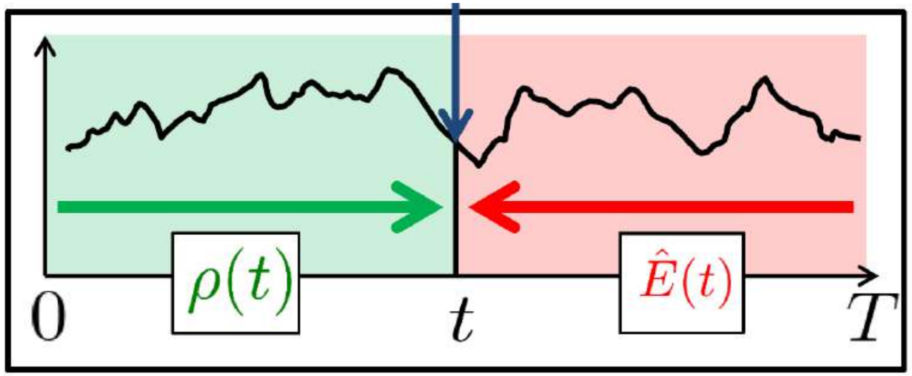
Apply to photon number operator $\hat{O} = \hat{N}$: $\hat{O}_n = |n\rangle\langle n|$

$$P\left(n, t/\{m_k\}\right) = \frac{tr\,|n\rangle\langle n|\,\hat{\rho}_S(t)\,|n\rangle\langle n|\,\hat{E}_S(t)}{Norm}$$

$$\Longrightarrow \quad P\left(n, t/\{m_k\}\right) = \frac{\hat{\rho}^S_{n,n}(t)\,\hat{E}^S_{n,n}(t)}{Norm}$$

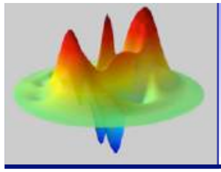
→ implies only diagonal matrix elements

Following a quantum trajectory

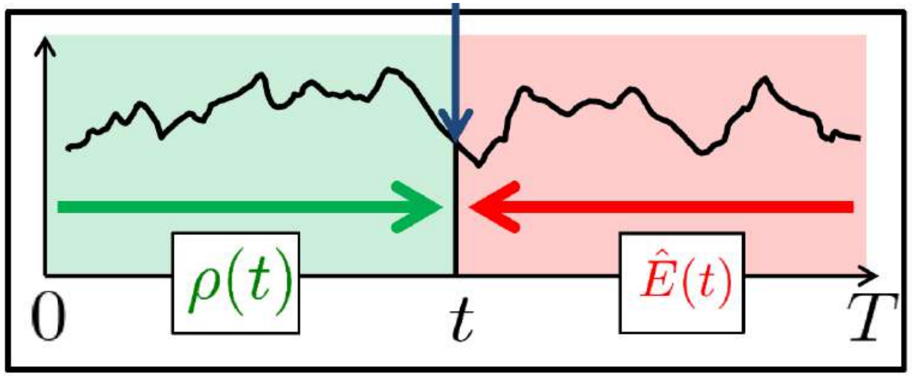


$$P\left(n,t/\{m_k\}\right)=\frac{tr\ \hat{\rho}^{S}_{n,n}(t)\,\hat{E}^{S}_{n,n}(t)}{Norm}$$

→Photon number distributions:



Following a quantum trajectory



$$P\left(n,t/\{m_k\}\right) = \frac{tr\ \hat{\rho}^S_{n,n}(t)\ \hat{E}^S_{n,n}(t)}{Norm}$$

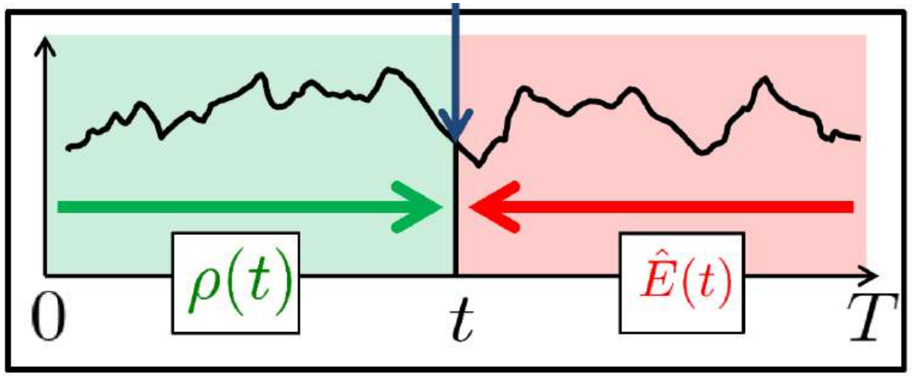
→ Photon number distributions:

- Forward estimation: → $P^f(n,t) = \hat{\rho}^S_{n,n}(t)$

standard calculation of the density matrix *ρ(t)* taking into account

- projection at measurement
- relaxation between measurements

Following a quantum trajectory



$$P\left(n,t/\{m_k\}\right)=\frac{tr\ \hat{\rho}^S_{n,n}(t)\ \hat{E}^S_{n,n}(t)}{Norm}$$

→Photon number distributions:

- Forward estimation: → $P^f(n,t)=\hat{\rho}^S_{n,n}(t)$

standard calculation of the density matrix $\rho(t)$ taking into account

- projection at measurement
- relaxation between measurements

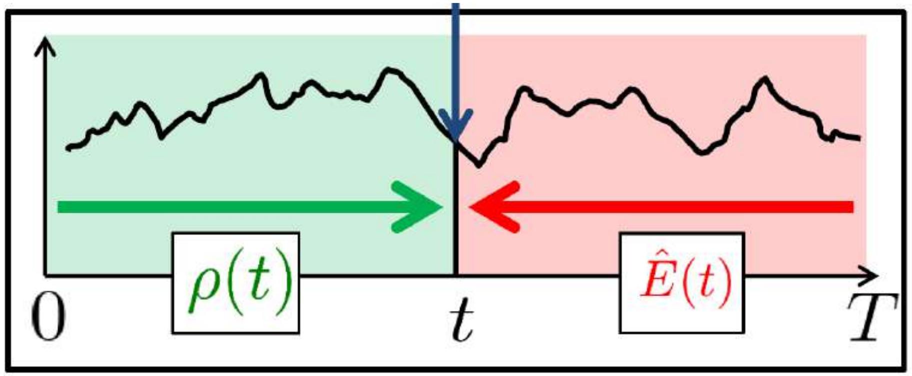
- Backward estimation: ← $P^b(n,t)=\hat{E}^S_{n,n}(t)$

calculation effect matrix $E(t)$:

- Flat distribution at final time T: describes an unknown final state
- Same measurement operators as forward
- 'inverse' relaxation (annihilation and creation operators exchanged)

→ Exponential growth of the photon number in "backward time"

Following a quantum trajectory



$$P\left(n,t/\{m_k\}\right) = \frac{tr\ \hat{\rho}^S_{n,n}(t)\ \hat{E}^S_{n,n}(t)}{Norm}$$

→ Photon number distributions:

- Forward estimation: → $P^f(n,t) = \hat{\rho}^S_{n,n}(t)$
- Backward estimation: ← $P^b(n,t) = \hat{E}^S_{n,n}(t)$
- Past quantum state / forward-backward estimation

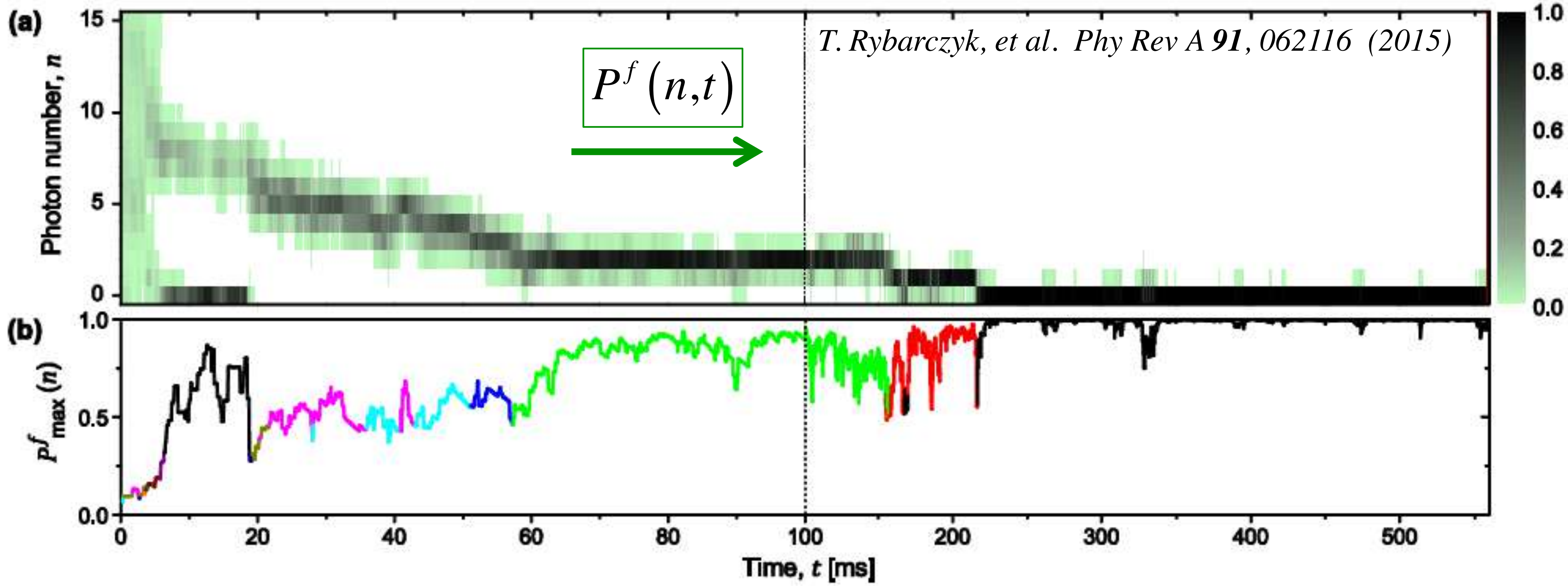
$$P^{fb}(n,t) = \frac{P^f(n,t).P^b(n,t)}{Norm}$$

→ P(n) is the product of two photon number distributions computed forward and backward in time.

In our case PQS reduces to the "forward/backward smoothing algorithm", which can be safely used in this quantum context

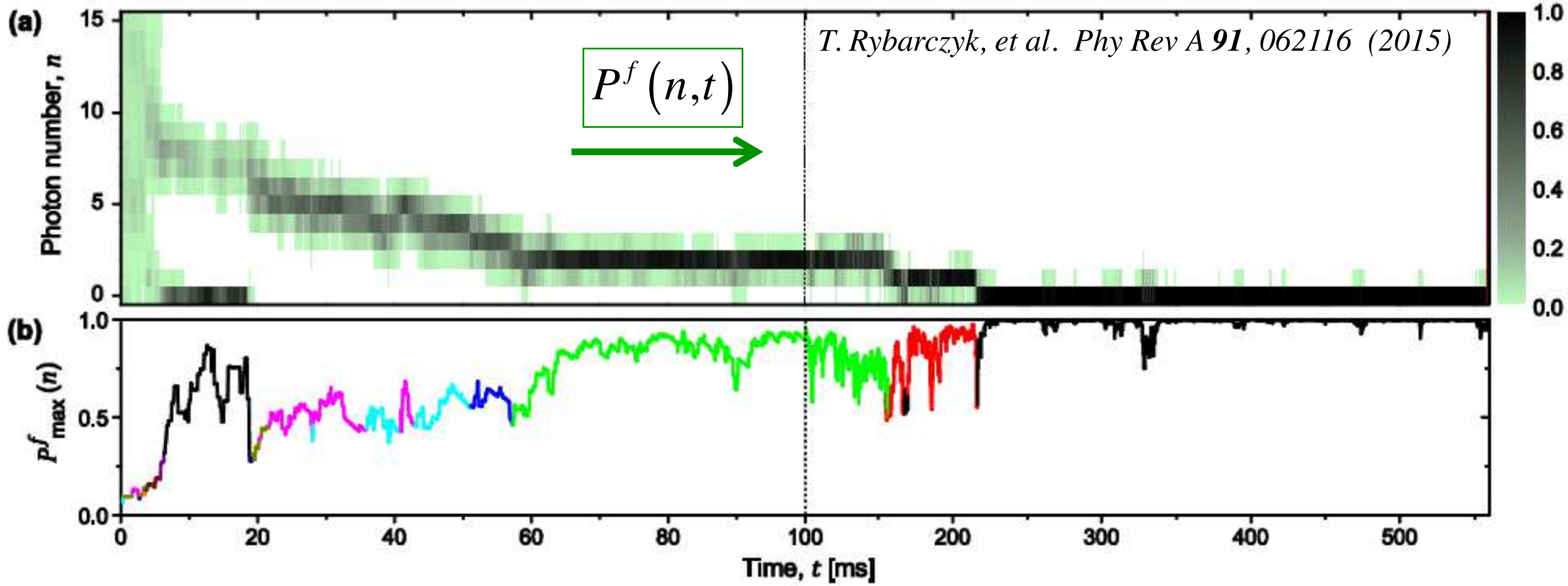
Quantum trajectory for a larger initial field

- Forward estimation of the field at time *t*



Quantum trajectory for a larger initial field

- Forward estimation of the field at time *t*



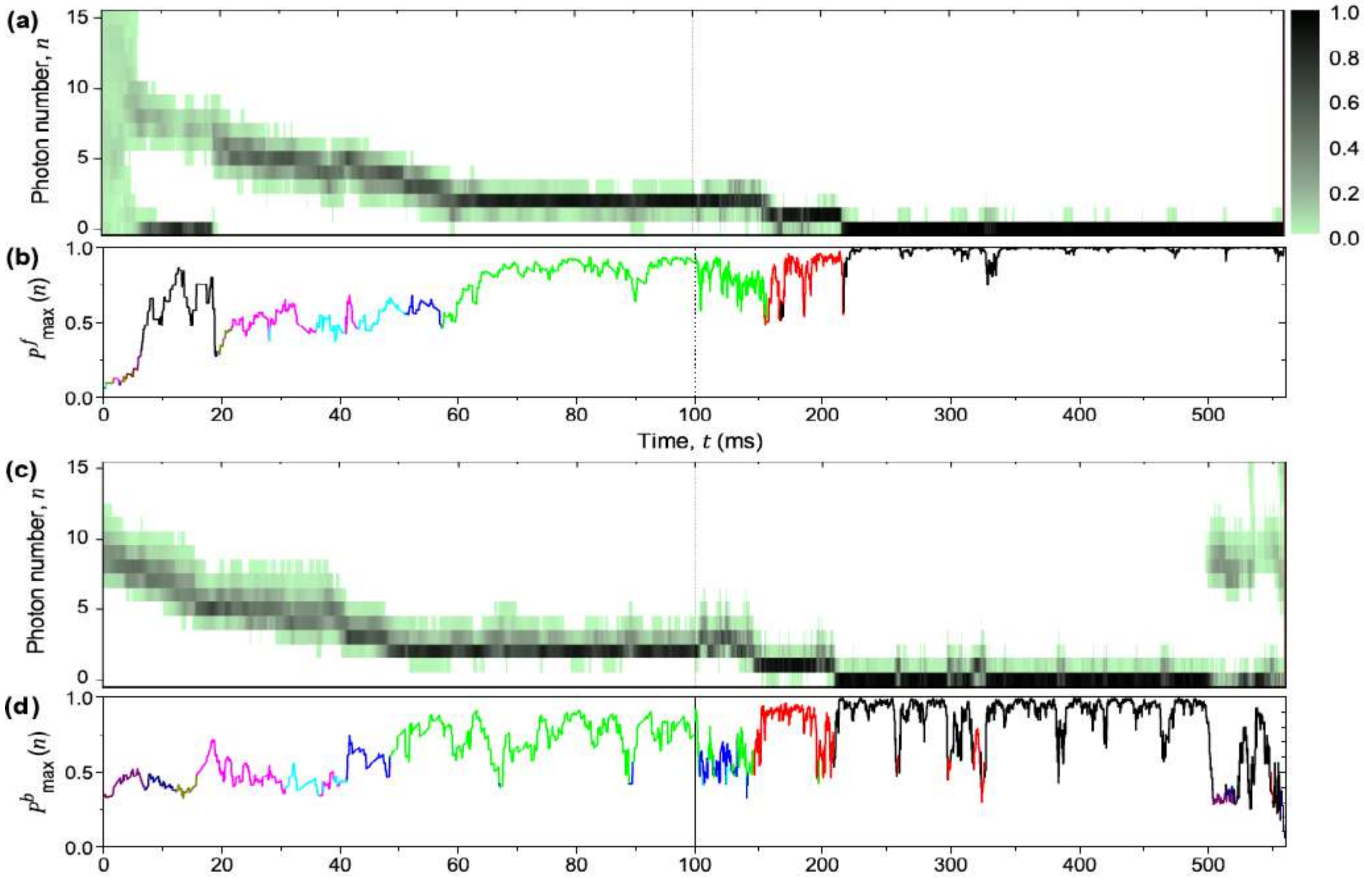
Obvious limitations

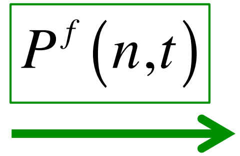
→ Noise due to statistical fluctuations of atomic detections

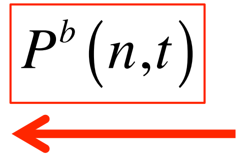
→ Initial ambiguity in the photon number due to the periodicity of the measurement operators

– Absurd photon number jumps (from 0 to 7)

Forward and backward estimations







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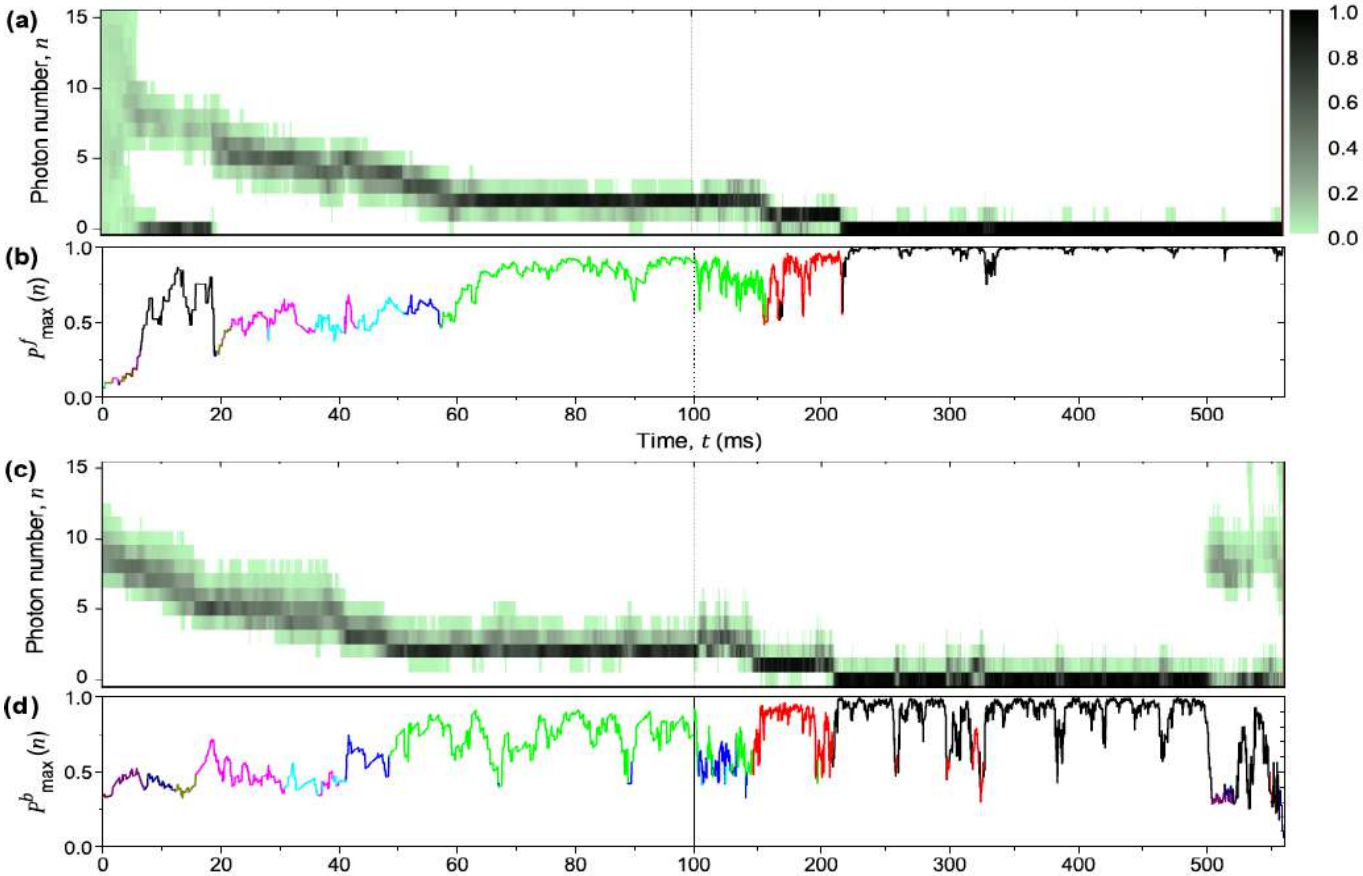
Forward and backward estimations



$P^f(n,t)$ →

$P^b(n,t)$ ←

➙ Noise due to statistical fluctuations of atomic detections

➙ Final ambiguity in the photon number due flat distribution at T and to the periodicity of the measurement operators

➙ "Reverse" relaxation makes a good job!

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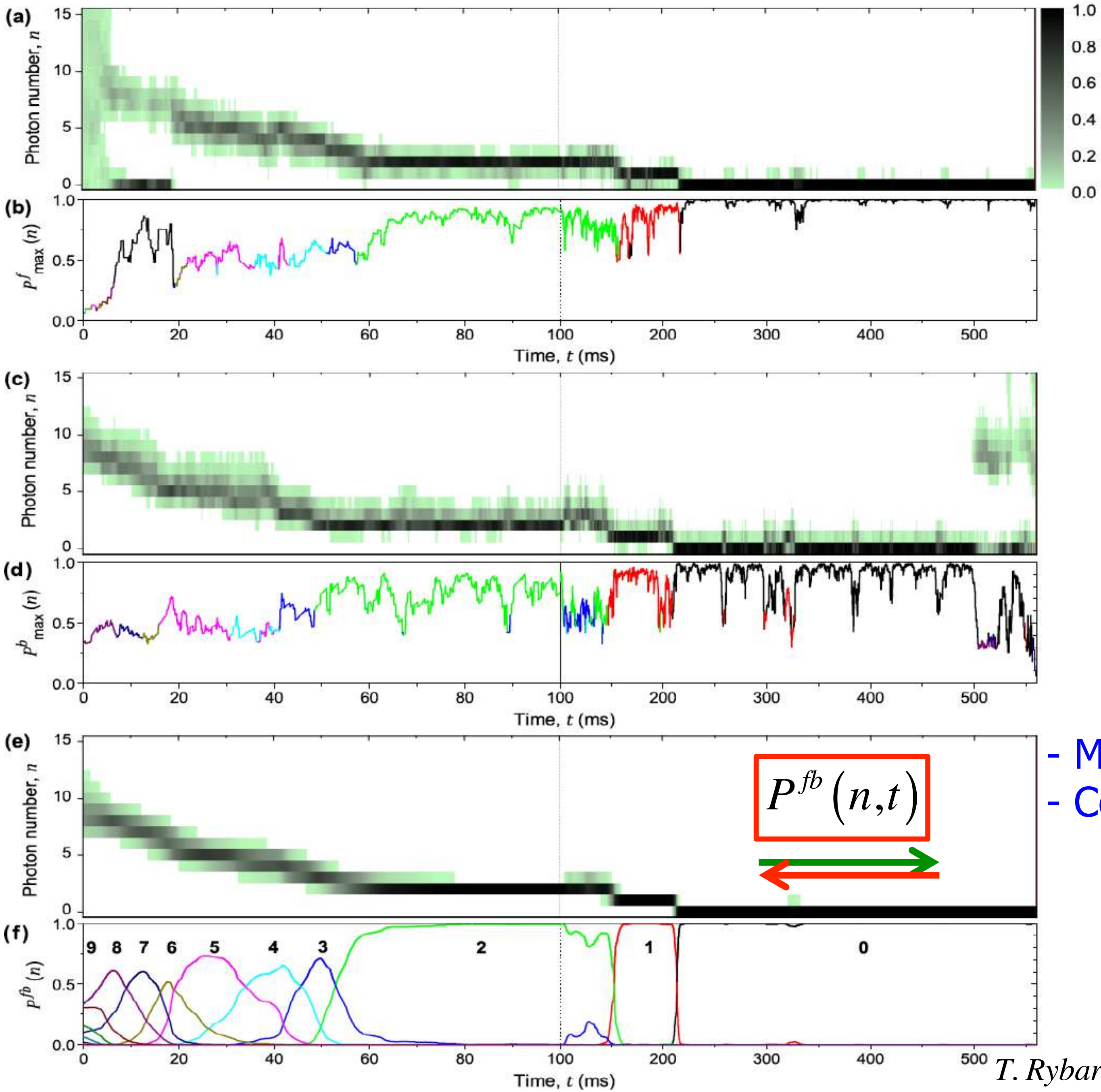
Forward and backward estimations



$P^f(n,t)$

$P^b(n,t)$

$P^{fb}(n,t)$

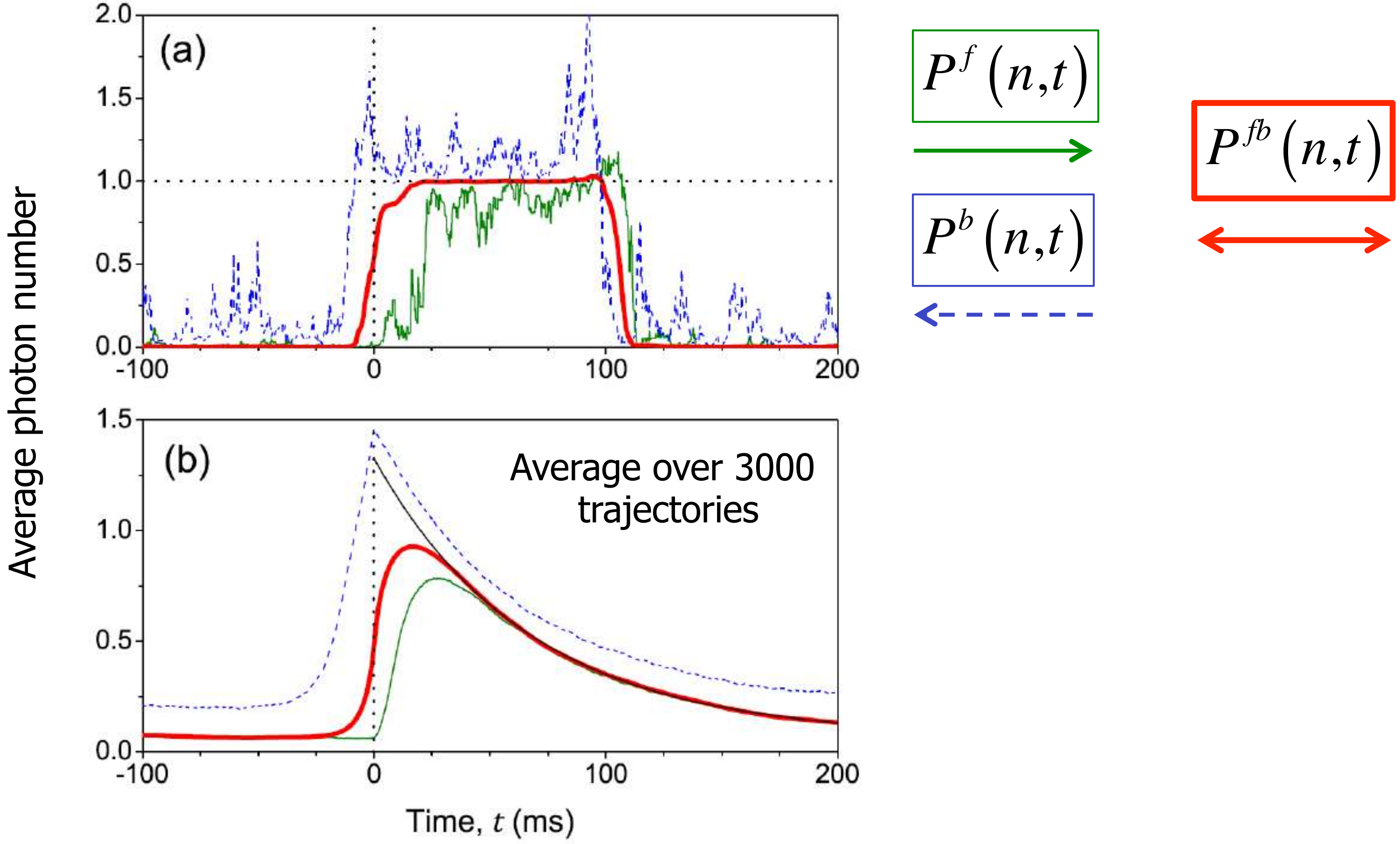
- Measurement ambiguities lifted
- Considerable noise reduction:

  All estimations take into account **ALL** available information

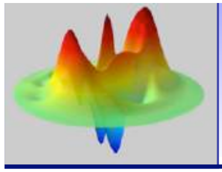
*T. Rybarczyk, et al. Phy Rev A **91**, 062116 (2015)*

PQS estimation of a single-photon quantum jump

- A single photon is emitted by a resonant atom at t=0
- The estimator only knows QND measurement results

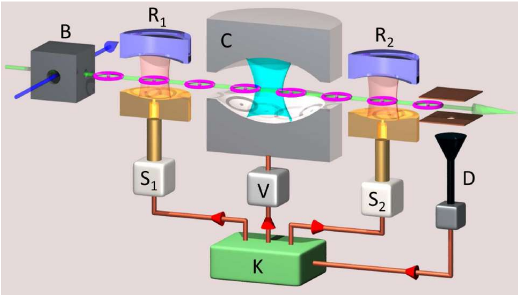


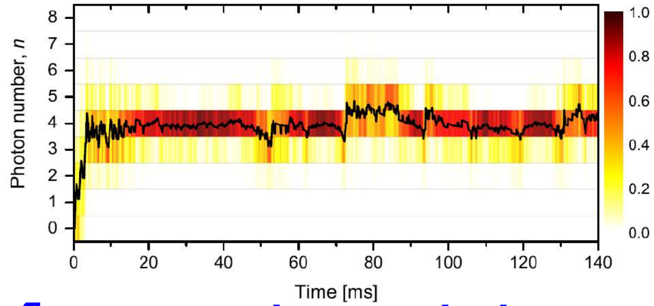
*T. Rybarczyk, et al. Phy Rev A **91**, 062116 (2015)*



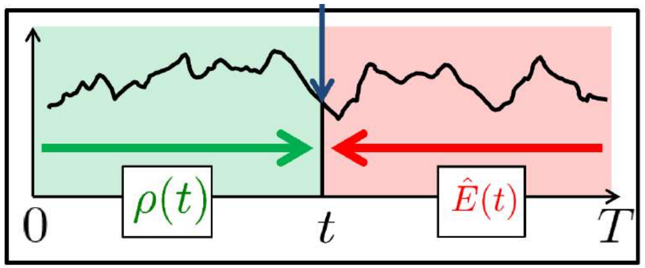
Conclusion of lecture 2

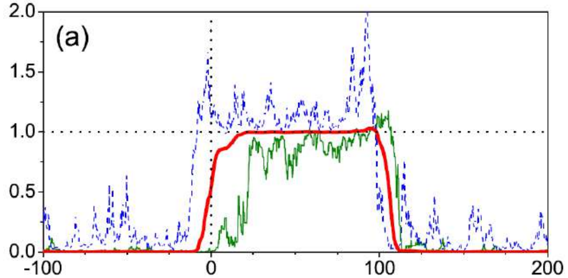
- Quantum feedback stabilization of number states





- PQS analysis is a fruitful tool for quantum state estimation:
  - ➞ reconstruction of quantum trajectories with much better fidelity





Also performed for spin 1/2-like systems

- ➞ Gammelmark et al., PRA **89**, 043839
- ➞ Armen et al., PRL **103**, 173601
- ➞ Kerkhoff et al. Opt. Expr. **19**, 6478
- ➞ Tan et al., PRL **114**, 040903

Reference (4)

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- C. Sayrin, I. Dotsenko, S. Gleyzes, M. Brune, J.M. Raimond ans S. Haroche, New J. Phys. **14** 115007 (2012) doi:10.1088/1367-2630/14/11/115007: “Optimal time-resolved photon number distribution reconstruction of a cavity field by maximum likelihood”
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- Past quantum state:

- S.Gammelmark, B. Julsgaard, K. Molmer, PRL 111 160401(2013); "Past Quantum States of a Monitored System".
- T. Rybarczyk, B. Peaudecerf, M. Penasa, S. Gerlich, B. Julsgaard, K. Molmer, S. Gleyzes, M. Brune, J.M. Raimond, S. Haroche, and I. Dotsenko, Phy Rev A 91, 062116 (2015):" Forward-backward analysis of the photon-number evolution in a cavity".