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From cavity QED to quantum simulations with Rydberg atoms

Lecture 4 Quantum measurement, Schrödinger cat and decoherence

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#### Cavity QED with microwave photons and circular Rydberg atoms:

- L1:Achieving strong coupling between single atoms and single photons
- L2: Performing QND measurement of the field state
- L3: application to quantum feedback and past quantum state analysis of a quantum trajectory
- L4: The same experiment seen from the point of view of the field:
- → Schrödinger cat preparation and monitoring of its decoherence







## Lecture 4: Quantum measurement, Schrödinger cat and decoherence

#### **Quantum measurement: basic ingredients**



- We have shown how to built an ideal QND meter of the photon number
- □ This detector is based on a destructive detector of the atom energy.
- Let us now built a more complete, fully quantum, model of detector including the dissipative part

# **1. The "Schrödinger cat"** and the quantum measurement

# The border separation quantum and classical behavior





#### **Quantum description of a meter:** the "Schrödinger cat" problem



One encloses in a box a cat whose fate is linked to the evolution of a quantum system: one radioactive atom.



# The "Schrödinger cat"

 One closes the box and wait until the atom is desintegrated with a probability 1/2



• When opening the box is the cat dead, alive or in a superposition of both?





- Before opening the box, the system is isolated and unitary evolution prepares a maximally atom-meter entangled state
- Does this state "really" exists?
  - → a more relevant question: can one perform experiments demonstrating cat superposition state? Up to which limit?
- That is a fundamental question for the quantum theory of measurement: how does the unphysical entanglement of SC state vanishes at the macroscopic scale. That is the problem of the transition between quantum and classical world

$$\frac{1}{\sqrt{2}}(|a\rangle + |b\rangle) \implies \frac{1}{\sqrt{2}}(|a, |a, |b\rangle) \implies \frac{1}{\sqrt{2}}(|a, |b\rangle) \implies \frac{1}{\sqrt{2}}(|b\rangle) \implies \frac{1}{\sqrt{2}}(|a, |b\rangle) \implies \frac{1}{\sqrt{2}}(|b\rangle) \implies \frac{1}{\sqrt{2$$



$$\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \implies \frac{1}{\sqrt{2}}(|e, |e, |e, |g, |h|) \Rightarrow \frac{1}{\sqrt{2}}(|e, |e, |h|) \Rightarrow \frac{1}{\sqrt{2}}(|e, |e, |h|) \Rightarrow \frac{1}{\sqrt{2}}(|e, |h|)$$

- Real measurement provide one definite result and not superposition of results: SC states are unphysical ?
- Schrödinger: unitary evolution should "obviously" not apply any more at "some scale".
- It seems that the atom-meter space contains to many states for describing reality
- Including dissipation due to the coupling of the meter to the environment will provide a physical mechanism "selecting" the physically acceptable states.
- Let's look at this in a real experiment using a meter whose size can be varied continuously from microscopic to macroscopic world.

### 2. A mesoscopic field as atomic state measurement aparatus



## A mesoscopic "meter": coherent field states

- Number state:  $|N\rangle$
- Quasi-classical state:

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{N} \frac{\alpha^N}{\sqrt{N!}} |N\rangle$$
;  $\alpha = |\alpha| e^{i\Phi}$ 

Photon number distribution



Phase space representation











→ The field phase "points" on the atomic state





The field phase "points" on the atomic state



#### **Atom-meter entanglement**



$$\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \implies \frac{1}{\sqrt{2}}(|e, |f|) + |g, |f|)$$

#### This is a "Schrödinger cat state"

Let us now consider the effect of coupling of the cavity to the "environment"





- For long atom-cavity interaction time field damping couples the system to the outside world
- → a complete description of the system must take into account the state of the field energy "leaking" in the environment.
- General method for describing the role of the environment:

$$\frac{d\rho^{field}}{dt} = -\frac{1}{2T_{cav}} \left[a^{+}a, \rho^{field}\right]_{+} + \frac{1}{T_{cav}}a\rho^{field}a^{+}$$

master equation of the field density matrix

• Physical result: decoherence

$$au_{dec} pprox rac{ au_{cav}}{\overline{N}}$$





• Decay of a coherent field:

$$\begin{aligned} |\alpha(0)\rangle \otimes |vacuum\rangle_{env} \rightarrow |\alpha(t)\rangle \otimes |\beta(t)\rangle_{env} \\ \alpha(t) = \alpha(0).e^{-t/2\tau_{cav}} \end{aligned}$$

 the cavity field remains coherent
 the leaking field has the same phase as α

□ no entanglement during decay:

That is a property defining coherent states: coherent state are the only one which do not get entangled while decaying









• Decay of a "cat" state:  

$$|\Psi_{cat}\rangle \otimes |vacuum\rangle_{env}$$
  
 $+ 1/\sqrt{2} (|\alpha_{+}(t)\rangle \otimes |\beta_{+}(t)\rangle_{env} + |\alpha_{-}(t)\rangle \otimes |\beta_{-}(t)\rangle_{env})$ 

- cavity-environment entanglement:
   the leaking field "broadcasts" phase information
- □ trace over the environment
- ⇒ decoherence (=diagonal field reduced density matrix) as soon as:

$$\left< \beta_{-}(t) \middle| \beta_{+}(t) \right>_{env} \approx 0$$





• Decay of a "cat" state:  

$$|\Psi_{cat}\rangle \otimes |vacuum\rangle_{env}$$

$$\overline{2}\left(|\alpha_{+}(t)\rangle \otimes |\beta_{+}(t)\rangle_{env} + |\alpha_{-}(t)\rangle \otimes |\beta_{-}(t)\rangle_{env}\right)$$

$$\left|\langle\beta_{+}(t)|\beta_{-}(t)\rangle = e^{-|\beta|^{2}(1-e^{2i\Phi_{0}})}$$

$$|\alpha(t)|^{2} + |\beta(t)|^{2} = |\alpha_{0}|^{2}$$

$$\Rightarrow |\beta(t)|^{2} = |\alpha_{0}|^{2}(1-e^{-t/T_{cav}}) \approx |\alpha_{0}|^{2} \cdot t/T_{cav}$$
The two states of the environment become orthogonal as soon as
$$\left|\beta(t)\right|^{2} \approx 1 \Rightarrow t > \frac{T_{cav}}{\overline{N}} \approx T_{dec}$$



### The decoherence time





# Quantum measurement: the role of the environment 1

 $\Rightarrow$  Physical origin of decoherence:

leak of information into the environment.

⇒ The experimentalist does not kill the cat when opening the box: the environment "knows" whether the cat is dead or alive well before one opens the box.

⇒ The environment performs continuously unread repeated measurement of the cat state

The "collapse" of the quantum state can be considered as a shortcut to describe this complex physical process

#### Does it solves "the measurement problem"?

No: if the problem consists in telling how or why nature chooses randomly one classical state.

Yes: once one a priori accepts the statistical nature of quantum theory, dechoherence is the mechanism providing classical probabilities



# Quantum measurement: the role of the environment 2

#### $\Rightarrow$ Definition of "pointer basis" of a meter: (Zurek)

- □ the pointer state of the meter is a classical state
- once decoherence occurs, the physical state of a meter is described by a diagonal density matrix in the pointer basis:



⇒ at this level, quantum description only involves classical probabilities and no macroscopic superposition states.

The decoherence approach shows that quantum theory is consistent with classical logic at macroscopic scale: it only provides classical statistics at the macroscopic scale.

## 3. Observing decoherence experimentally

# **Probing the coherence of the cat state**



"cat" state coherence Interference term in two atom correlation



# **Decoherence signal**



Brune et al. Phys. Rev. Lett. 77, 4887 (1996)

## 4. Full tomography of the field state



This correlation signal is a very partial information on the field state

One can reconstruct the full density matrix of the cavity field







# **Principle of stet reconstruction**



→ Each measurement sets a constrain to the density operator

$$\operatorname{Tr}(\hat{\rho}\,\hat{G}_i) = g_i$$

#### Problems to face:

- Having a complete set of observable  $\{\hat{G}_i\}$
- Statistical noise on  $\{g_i\}$  may lead to unphysical/very noisy density operators



# **Measuring the field density operator?**

#### General field state description: density operator

$ ho_{field}$ =	$ ho_{00}$	$ ho_{_{01}}$	$ ho_{02}$	•]
	$ ho_{10}$	$ ho_{\scriptscriptstyle 11}$	$ ho_{12}$	•
	$ ho_{ m 20}$	$ ho_{_{21}}$	$ ho_{\scriptscriptstyle 22}$	•
	•	•	•	•

Displacement operator

 $ho_{\it field}$ 

 $\rho_{\text{field}}^{(\alpha)} = \hat{D}(\alpha) \rho_{\text{field}} \hat{D}^{+}(\alpha)$ 

 $\hat{D}(\alpha) = e^{\alpha a^{+} - \alpha^{*}a}$ 

• Previous lectures: QND counting of photons  $\Rightarrow$  measurement of diagonal elements  $\rho_{nn}$   $\Rightarrow$  How to measure the offdiagonal elements of  $\rho_{field}$ ?

⇒ by counting photons after applying "displacement"

The displacement operator is the unitary transform corresponding to the coupling to a classical source. It mixes diagonal and off-diagonal matrix elements of  $\rho_{field}$ . Measuring the photon number after displacement for a large number of different a gives information about all matrix elements of  $\rho_{field}$ .



#### • Various possibilities:

 $\Box \text{ Direct fit of } \rho_{field} \text{ on the measured data } g_i = tr \left[ \rho_{field} \cdot \cos(\phi(\hat{n}) + \varphi) \right]$ 

- □ Maximum likelihood: find  $\rho_{field}$  which maximizes the probability of finding the actually measured results  $g_i$ .
- Maximum entropy principle: find ρ<sub>field</sub> which fits the measurements and additionally maximizes entropy
   S=ρ<sub>field</sub>log(ρ<sub>field</sub>).
   V. Bužek and G. Drobný, *Quantum tomography via the MaxEnt principle*

*via the MaxEnt principle*, Journal of Modern Optics **47**, 2823 (2000)

Estimates the state only on the basis of measured information: in case of incomplete set of measurements, gives a "worse estimate of  $\rho_{field}$ .

In practice the two last methods give the same result provided one measures enough data completely determining the state.



- 1- prepare the state to be measured  $|\psi_{field}\rangle$ 2- measure  $\hat{G}(\alpha)$  for a large number of different values of  $\alpha$  (400 to 600 points).
- 3- reconstruct  $ho_{\it field}$  by maximum entropy method
- 4- calculate Wigner function from  $\rho_{\it field}$ .



- Measurement for 161 values of  $\alpha$  (<1 hour measurement)
- 7000 detected atoms in 600 repetition of the experimental sequence for each  $\alpha$ .





# **Reconstruction of number states**

- Prepare a coherent state  $\beta^2 = 1.3$  or 5.5 photons.
- Select pure number state by QND measurement of *n*.
   Phase shift per photon φ<sub>0</sub>≈π/2 : measurement of n modulo 4.
- Measurements of  $G(\alpha)$  for 2 different values of  $\phi$  and ~400 values of  $\alpha$ .



#### 5. Schrödinger cat states reconstruction

state preparationa movie of decoherence



# **Preparation of the cavity cat state**

Phase shift per photon  $\Phi_{\circ}$ 



$$\frac{1}{\sqrt{2}} \left( \left| e \right\rangle + \left| g \right\rangle \right) \otimes \left| \alpha \right\rangle \quad \Rightarrow \quad \frac{1}{\sqrt{2}} \left( \left| e \right\rangle \otimes \left| \alpha \cdot e^{i\Phi_0/2} \right\rangle + \left| g \right\rangle \otimes \left| \alpha \cdot e^{-i\Phi_0/2} \right\rangle \right)$$



Phase shift per photon  $\Phi_{\rm c}$ 



$$\frac{1}{\sqrt{2}} \left( \left| e \right\rangle + \left| g \right\rangle \right) \otimes \left| \alpha \right\rangle \quad \Rightarrow \quad \frac{1}{\sqrt{2}} \left( \left| e \right\rangle \otimes \left| \alpha \cdot e^{i\Phi_0/2} \right\rangle + \left| g \right\rangle \otimes \left| \alpha \cdot e^{-i\Phi_0/2} \right\rangle \right)$$

• Field state after detection:



Depending on the detected atomic state the cat has a well defined photon number parity.

For  $\pi$  per photon phase shift, one atom measures just the field parity. Projection on a cat state is the "back-action" of parity measurement.



#### **Measured raw data**

> We perform measurements at about 600 points in the phase space (about 10 scans)  $\succ \approx 100$  state preparations for each point  $\succ \approx$  12 probe atoms for each state preparation



1,0

#### measured raw data





#### Even (odd) cat has even (odd) photon number statistics



Fidelity of the preparation and reconstruction - 66% (71% for the odd state)



## **Reconstructed Wigner function**



Deleglise et al. Nature **455**, 510 (2008)



# **Reconstructed Wigner function**

**Classical components** 

 ≈2.1 photons in each classical component
 (amplitude of the initial coherent field)

cat size  $D^2 \approx 7.5$  photons

coherent components are completely separated (D > 1)



Deleglise et al. Nature **455**, 510 (2008)



# **Reconstructed Wigner function**

quantum superposition of two classical fields *(interference fringes)* 

quantum signature of the prepared state (negative values of Wigner function)



Deleglise et al. Nature **455**, 510 (2008)



# A larger cat for observing decoherence

- Initial coherent field  $\beta^2 = 3.5$  photons
- Measurement for 400 values of  $\alpha$ .



State fidelity with respect to the expected state including phase shift non-lineariry (insets)

*F*= 0.72

Deleglise et al. Nature **455**, 510 (2008)



#### **Movie of decoherence**



# **Decoherence of a** D<sup>2</sup>=11.8 **photon cat state**





Theory:  $T_{dec} = 2T_{cav}/D^2 = 22 \text{ ms}$ 

+ small blackbody contribution @ 0.8 K

 $T_{dec} = 19.5 \text{ ms}$ 

M.S. Kim and V. Bužek, Schrödinger-cat state at finite temperature, Phys. Rev. A 46, 4239 (1992)



#### Summary

- Generation of cat states in a cavity
- State measurement (QND) and reconstruction (MaxEnt)
- Wigner function of the cat states
- Time evolution and decoherence of the cats







# Perspectives



### **CQED** with two cavities



 $\frac{1}{\sqrt{2}}\left(|\alpha\rangle|0\rangle+|0\rangle|-\alpha\rangle\right)$  $\frac{1}{\sqrt{2}}\left(|8\rangle + |6\rangle\right)$ 

→ alive-here-and-dead-there state



#### Exploring non-local states with two cavities



#### • Full state reconstruction by "quantum trajectory tomography Method proposed by P. Rouchon

P. Six et al., Phys. Rev. A 93, 012109 (2016)

Quantum state tomography with non-instantaneous measurements, imperfections, and decoherence.



# A two-cavity experiment: exploring quantum thermodynamics

 Fast atoms crossing two microwave high-Q cavities



• Projects

#### Quantum thermodynamics

(ANR with A. Auffeves and P. Sénellart)



Heat going from cold to hot using information! Exp. In progress



• People: Igor Dostenko, collab. A. Auffeves



1,0

0,8

0.6

0,4

0,2

0,0 -0

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# Slow atoms cavity QED set-up



J.M. Raimond et al PRL **105**, 213601 (2010)

□ Reservoir engineering A. Sarlette, A. et al. PRL **107**, 010402 (2011)



spectroscopy



# Engineering quantum state of the Rydberg electron motion

 $|n,l,m\rangle$ : n<sup>2</sup> levels with same n



One Rydberg atom → use multi-level structure of for quantum metrology

 $\rightarrow$  more about this in the coloquium





Trapped Rydberg atoms with dipole interactions →quantum simulations with circular atoms

→ topic of lecture 5

# **The LKB-ENS cavity QED team**

#### Staring, in order of apparition

Serge Haroche Michel Gross Claude Fabre Philippe Goy Pierre Pillet Jean-Michel Raimond **Guy Vitrant** Yves Kaluzny Jun Liang Michel Brune Valérie Lefèvre-Seguin Jean Hare **Jacques** Lepape Aephraim Steinberg Andre Nussenzveig Frédéric Bernardot Paul Nussenzveig Laurent Collot Matthias Weidemuller François Treussart Abdelamid Maali **David Weiss** Vahid Sandoghdar Jonathan Knight Nicolas Dubreuil Peter Domokos Ferdinand Schmidt-Kaler Jochen Drever 

- Peter Domokos
- Ferdinand Schmidt-
- Kaler

- □ Ed Hagley
- Xavier Maître
- Christoph Wunderlich
- Gilles Nogues
- Vladimir Ilchenko
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- Stefano Osnaghi
- Arno Rauschenbeutel
- Wolf von Klitzing
- Erwan Jahier
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