From cavity QED to quantum simulations with Rydberg atoms

Lecture 4
Quantum measurement, Schrödinger cat and decoherence

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Previous lectures

Cavity QED with microwave photons and circular Rydberg atoms:

• L1: Achieving strong coupling between single atoms and single photons
• L2: Performing QND measurement of the field state
• L3: Application to quantum feedback and past quantum state analysis of a quantum trajectory
• L4: The same experiment seen from the point of view of the field:
  ➔ Schrödinger cat preparation and monitoring of its decoherence
Lecture 4:
Quantum measurement, Schrödinger cat and decoherence
Quantum measurement: basic ingredients

We have shown how to built an ideal QND meter of the photon number. This detector is based on a destructive detector of the atom energy. Let us now built a more complete, fully quantum, model of detector including the dissipative part.

- Now focus on this part
- We have shown how to built an ideal QND meter of the photon number
- This detector is based on a destructive detector of the atom energy.
- Let us now built a more complete, fully quantum, model of detector including the dissipative part.
1. The “Schrödinger cat” and the quantum measurement

The border separation quantum and classical behavior
One encloses in a box a cat whose fate is linked to the evolution of a quantum system: one radioactive atom.
The "Schrödinger cat"

• One closes the box and wait until the atom is desintegrated with a probability 1/2

• When opening the box is the cat dead, alive or in a superposition of both?
Schrödinger cat and quantum measurement

Before opening the box, the system is isolated and unitary evolution prepares a maximally atom-meter entangled state.

Does this state "really" exists?

→ a more relevant question: can one perform experiments demonstrating cat superposition state? Up to which limit?

That is a fundamental question for the quantum theory of measurement: how does the unphysical entanglement of SC state vanishes at the macroscopic scale. That is the problem of the transition between quantum and classical world.

\[
\frac{1}{\sqrt{2}} (|a\rangle + |b\rangle) \Rightarrow \frac{1}{\sqrt{2}} (|a, \text{off}\rangle + |b, \text{on}\rangle)
\]
Schrödinger cat and quantum measurement

\[ \frac{1}{\sqrt{2}} (|e\rangle + |g\rangle) \Rightarrow \frac{1}{\sqrt{2}} (|e, \text{on}\rangle + |g, \text{off}\rangle) \]

- Real measurement provide one definite result and not superposition of results: SC states are unphysical?
- Schrödinger: unitary evolution should "obviously" not apply any more at "some scale".
- It seems that the atom-meter space contains too many states for describing reality.
- Including dissipation due to the coupling of the meter to the environment will provide a physical mechanism "selecting" the physically acceptable states.

Let's look at this in a real experiment using a meter whose size can be varied continuously from microscopic to macroscopic world.
2. A mesoscopic field as atomic state measurement apparatus
A mesoscopic "meter": coherent field states

- **Number state:** \( |N\rangle \)
- **Quasi-classical state:**
  \[
  |\alpha\rangle = e^{-|\alpha|^2/2} \sum \frac{\alpha^N}{\sqrt{N!}} |N\rangle \quad ; \quad \alpha = |\alpha| e^{i\Phi}
  \]

- **Photon number distribution**
  \[
  P(N) = e^{-\overline{N}} \frac{\overline{N}^N}{N!} \quad ; \quad \overline{N} = |\alpha|^2
  \]

- **Phase space representation**
  \[
  \Delta N \cdot \Delta \Phi > 1
  \]
QND detection of atoms using non-resonant interaction with a coherent field

\[ |e\rangle \]

\[ \text{Im}(\alpha) \]

\[ \text{Re}(\alpha) \]
QND detection of atoms using non-resonant interaction with a coherent field

\[ |e\rangle \otimes |\alpha\rangle \rightarrow |e\rangle \otimes |\alpha e^{i\Phi_0}\rangle \]
QND detection of atoms using non-resonant interaction with a coherent field

A single atom controls the phase of the field

The field phase "points" on the atomic state
QND detection of atoms using non-resonant interaction with a coherent field

\[ \text{Re}(\alpha) \]

\[ \text{Im}(\alpha) \]

\[ |e\rangle \]

\[ |g\rangle \]

\[ \pi/2 \]

pulse \( R_1 \)

\[ |e\rangle \otimes |\alpha\rangle \rightarrow |e\rangle \otimes |\alpha e^{i\Phi_0}\rangle \]

\[ |g\rangle \otimes |\alpha\rangle \rightarrow |g\rangle \otimes |\alpha e^{-i\Phi_0}\rangle \]

\[ \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \otimes |\alpha\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle \otimes |\alpha e^{i\Phi_0}\rangle + |g\rangle \otimes |\alpha e^{-i\Phi_0}\rangle) \]

→ The field phase "points" on the atomic state

a single atom controls the phase of the field
Atom-meter entanglement

\[
\frac{1}{\sqrt{2}} ( |e\rangle + |g\rangle ) \otimes |\alpha\rangle \rightarrow \frac{1}{\sqrt{2}} ( |e\rangle \otimes |\alpha e^{i\Phi_0}\rangle + |g\rangle \otimes |\alpha e^{-i\Phi_0}\rangle )
\]

\[
\frac{1}{\sqrt{2}} ( |e\rangle + |g\rangle ) \Rightarrow \frac{1}{\sqrt{2}} ( |e, \text{on}\rangle + |g, \text{off}\rangle )
\]

This is a "Schrödinger cat state"

Let us now consider the effect of coupling of the cavity to the "environment"
The role of the "environment":

- For long atom-cavity interaction time field damping couples the system to the outside world → a complete description of the system must take into account the state of the field energy "leaking" in the environment.

- General method for describing the role of the environment:

\[
\frac{d \rho_{\text{field}}}{dt} = -\frac{1}{2 T_{\text{cav}}} \left[ a^+ a, \rho_{\text{field}} \right]_+ + \frac{1}{T_{\text{cav}}} a \rho_{\text{field}} a^+
\]

master equation of the field density matrix

- Physical result: decoherence

\[
\tau_{\text{dec}} \approx \frac{\tau_{\text{cav}}}{N}
\]
The origin of decoherence:
entanglement with the environment

- Decay of a coherent field:

\[
\begin{align*}
|\alpha(0)\rangle \otimes |\text{vacuum}\rangle_{env} & \rightarrow |\alpha(t)\rangle \otimes |\beta(t)\rangle_{env} \\
\alpha(t) &= \alpha(0)e^{-t/2\tau_{cav}}
\end{align*}
\]

- the cavity field remains coherent
- the leaking field has the same phase as \(\alpha\)
- no entanglement during decay:

That is a property defining coherent states: coherent state are the only one which do not get entangled while decaying.
The origin of decoherence: entanglement with the environment

- Decay of a "cat" state:

\[
\Psi_{cat} \otimes \text{vacuum}_{\text{env}}
\]

\[
\Rightarrow \frac{1}{\sqrt{2}} \left( |\alpha_+ (t)\rangle \otimes |\beta_+ (t)\rangle_{\text{env}} + |\alpha_- (t)\rangle \otimes |\beta_- (t)\rangle_{\text{env}} \right)
\]
The origin of decoherence: entanglement with the environment

- Decay of a "cat" state:

\[
\left| \Psi_{\text{cat}} \right\rangle \otimes \left| \text{vacuum} \right\rangle_{\text{env}} \\
\Rightarrow 1/\sqrt{2} \left( \left| \alpha_+ (t) \right\rangle \otimes \left| \beta_+ (t) \right\rangle_{\text{env}} + \left| \alpha_- (t) \right\rangle \otimes \left| \beta_- (t) \right\rangle_{\text{env}} \right)
\]

- cavity-environment entanglement: the leaking field "broadcasts" phase information

- trace over the environment

\[
\Rightarrow \text{decoherence (}=\text{diagonal field reduced density matrix)} \text{ as soon as:} \\
\left< \beta_- (t) | \beta_+ (t) \right|_{\text{env}} \approx 0
\]
The origin of decoherence: entanglement with the environment

- Decay of a "cat" state:

\[ |\Psi_{cat}\rangle \otimes |\text{vacuum}\rangle_{\text{env}} \]

\[ \Rightarrow \frac{1}{\sqrt{2}} \left( |\alpha_+ (t)\rangle \otimes |\beta_+ (t)\rangle_{\text{env}} + |\alpha_- (t)\rangle \otimes |\beta_- (t)\rangle_{\text{env}} \right) \]

\[ \langle \beta_+(t)|\beta_-(t)\rangle = e^{-|\beta|^2 (1-e^{2i\Phi_0})} \]

\[ |\alpha(t)|^2 + |\beta(t)|^2 = |\alpha_0|^2 \]

\[ \Rightarrow |\beta(t)|^2 = |\alpha_0|^2 \left( 1 - e^{-t/T_{cav}} \right) \approx |\alpha_0|^2 . t / T_{cav} \]

The two states of the environment become orthogonal as soon as

\[ |\beta(t)|^2 \approx 1 \Rightarrow t > \frac{T_{cav}}{N} \approx T_{dec} \]
The decoherence time

Environment

$T_{decoh} = \frac{2T_{cav}}{D^2} = \frac{T_{cav}}{\bar{N}.2\sin^2(\Phi)}$

$D$: "Distance" between the two fields components.

Infinitely short decoherence time for macroscopic fields

→ The Schrödinger cat does not exist for "long" time

Detailed calculation in PHYSICA SCRIPTA T78, 29 (1998)
Quantum measurement: the role of the environment 1

⇒ Physical origin of decoherence:
   leak of information into the environment.

⇒ The experimentalist does not kill the cat when opening the box: the environment “knows” whether the cat is dead or alive well before one opens the box.

⇒ The environment performs continuously unread repeated measurement of the cat state

The “collapse” of the quantum state can be considered as a shortcut to describe this complex physical process

Does it solves “the measurement problem”?  

No: if the problem consists in telling how or why nature chooses randomly one classical state.

Yes: once one a priori accepts the statistical nature of quantum theory, decoherence is the mechanism providing classical probabilities
⇒ Definition of "pointer basis" of a meter: (Zurek)

- the pointer state of the meter is a classical state
- once decoherence occurs, the physical state of a meter is described by a diagonal density matrix in the pointer basis:

\[
\begin{pmatrix}
|e, \makebox{\tiny\text{meter}}\rangle & |g, \makebox{\tiny\text{meter}}\rangle
\end{pmatrix}
\]

\[
\rho_{\text{dec}} = \begin{pmatrix}
P_e & 0 \\
0 & P_g
\end{pmatrix}
\]

⇒ at this level, quantum description only involves classical probabilities and no macroscopic superposition states.

The decoherence approach shows that quantum theory is consistent with classical logic at macroscopic scale: it only provides classical statistics at the macroscopic scale.
3. Observing decoherence experimentally
Probing the coherence of the cat state

- Non resonant phase shift in C

"cat" state coherence
Interference term in two atom correlation
Decoherence signal

\[ \eta(\tau) \]

Time delay between atoms
\[ \tau = t/\tau_{\text{cav}} \]

\( n = 3.3 \) photons
\( \delta/2\pi = 70 \) and 170 kHz

4. Full tomography of the field state

This correlation signal is a very partial information on the field state

One can reconstruct the full density matrix of the cavity field
Principle of stet reconstruction

System

\[ \hat{\rho} \]

unknown state (density operator)

Measurement

\[ \{ \hat{G}_i \} \]

set of observables

Results

\[ \{ g_i \} \]

set of mean values

→ Each measurement sets a constrain to the density operator

\[ \text{Tr}(\hat{\rho} \hat{G}_i) = g_i \]

Problems to face:

• Having a complete set of observable \( \{ \hat{G}_i \} \)
• Statistical noise on \( \{ g_i \} \) may lead to unphysical/very noisy density operators
Measuring the field density operator?

General field state description: density operator

\[
\rho_{\text{field}} = \begin{bmatrix}
\rho_{00} & \rho_{01} & \rho_{02} \\
\rho_{10} & \rho_{11} & \rho_{12} \\
\rho_{20} & \rho_{21} & \rho_{22} \\
. & . & .
\end{bmatrix}
\]

- Previous lectures:
  - QND counting of photons
  - ⇒ measurement of diagonal elements $\rho_{nn}$
  - ⇒ How to measure the off-diagonal elements of $\rho_{\text{field}}$?

\[
\rho^{(\alpha)}_{\text{field}} = \hat{D}(\alpha) \rho_{\text{field}} \hat{D}^+(\alpha)
\]

$\hat{D}(\alpha) = e^{\alpha a^* - \alpha^* a}$  Displacement operator

⇒ by counting photons after applying "displacement"

The displacement operator is the unitary transform corresponding to the coupling to a classical source. It mixes diagonal and off-diagonal matrix elements of $\rho_{\text{field}}$.

Measuring the photon number after displacement for a large number of different $\alpha$ gives information about all matrix elements of $\rho_{\text{field}}$.
Choice of reconstruction method

• Various possibilities:
  - Direct fit of $\rho_{\text{field}}$ on the measured data
    $$ g_i = \text{tr} \left[ \rho_{\text{field}} \cdot \cos(\phi(\hat{n}) + \varphi) \right] $$
  - Maximum likelihood: find $\rho_{\text{field}}$ which maximizes the probability of finding the actually measured results $g_i$.
  - Maximum entropy principle: find $\rho_{\text{field}}$ which fits the measurements and additionally maximizes entropy
    $$ S = \rho_{\text{field}} \log(\rho_{\text{field}}). $$

Estimates the state only on the basis of measured information: in case of incomplete set of measurements, gives a "worse estimate of $\rho_{\text{field}}$.

In practice the two last methods give the same result provided one measures enough data completely determining the state.

State reconstruction: experimental method

1- prepare the state to be measured $|\psi_{\text{field}}\rangle$
2- measure $\hat{G}(\alpha)$ for a large number of different values of $\alpha$ (400 to 600 points).
3- reconstruct $\rho_{\text{field}}$ by maximum entropy method
4- calculate Wigner function from $\rho_{\text{field}}$. 
reconstruction of a coherent field

• Measurement for 161 values of $\alpha$ (<1 hour measurement)
• 7000 detected atoms in 600 repetition of the experimental sequence for each $\alpha$.

$F = \text{tr} \left( |\beta\rangle \langle \beta| \rho_{\text{field}} \right)$

$F = 98\%$ for $\beta^2 = 2.5$ photons
Reconstruction of number states

- Prepare a coherent state $\beta^2 = 1.3$ or $5.5$ photons.
- Select pure number state by QND measurement of $n$.
  Phase shift per photon $\phi_0 \approx \pi/2$ : measurement of $n$ modulo 4.
- Measurements of $G(\alpha)$ for 2 different values of $\varphi$ and $\sim 400$ values of $\alpha$.

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<td>4</td>
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5. Schrödinger cat states reconstruction

- state preparation
- a movie of decoherence
Preparation of the cavity cat state

\[ \frac{1}{\sqrt{2}} (|e\rangle + |g\rangle) \otimes |\alpha\rangle \Rightarrow \frac{1}{\sqrt{2}} (|e\rangle \otimes |\alpha.e^{-i\Phi_0/2}\rangle + |g\rangle \otimes |\alpha.e^{i\Phi_0/2}\rangle) \]

Phase shift per photon \( \Phi_0 \)
Preparation of the cavity cat state

Phase shift per photon \( \Phi_0 \)

\[
\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \otimes |\alpha\rangle \Rightarrow \frac{1}{\sqrt{2}}(|e\rangle \otimes |\alpha e^{i\Phi_0/2}\rangle + |g\rangle \otimes |\alpha e^{-i\Phi_0/2}\rangle)
\]

- **Field state after detection:**

\[
\Rightarrow \frac{1}{\sqrt{2}}\left(|\alpha e^{i\Phi_0/2}\rangle + |\alpha e^{-i\Phi_0/2}\rangle\right) \text{ if } "e" \text{ detected}
\]

\[
+ \frac{1}{\sqrt{2}}\left(|\alpha e^{i\Phi_0/2}\rangle - |\alpha e^{-i\Phi_0/2}\rangle\right) \text{ if } "g" \text{ detected}
\]

\( \Phi_0 = \pi \)

- \( e \rightarrow \) even cat state
- \( g \rightarrow \) odd cat state

Depending on the detected atomic state the cat has a well defined photon number parity.

For \( \pi \) per photon phase shift, one atom measures just the field parity. Projection on a cat state is the "back-action" of parity measurement.
We perform measurements at about 600 points in the phase space (about 10 scans)
≈ 100 state preparations for each point
≈ 12 probe atoms for each state preparation

**measured raw data**
Reconstructed density matrix (real part)

Even (odd) cat has even (odd) photon number statistics

(expectation (even cat))

(reconstruction (even cat))

($\bar{n}_{\text{inj}} \approx 2.1$ photons)

$(<n> = 2.2$ photons$)$

Fidelity of the preparation and reconstruction - 66%
(71% for the odd state)
Reconstructed Wigner function

\[ W(\alpha) = \text{Tr} \left[ \hat{\rho}_{\text{me}} \hat{D}(\alpha) e^{i\pi \hat{a}^\dagger \hat{a}} \hat{D}(\alpha) \right] \]

\[ \alpha = x + i p \]

No a priori knowledge on a prepared state except for the size of the Hilbert space of
\[ N_{\text{Hilbert}} = 9 \]

Reconstructed Wigner function

≈2.1 photons in each classical component (amplitude of the initial coherent field)

cat size $D^2 \approx 7.5$ photons

coherent components are completely separated ($D > 1$)

Reconstructed Wigner function

Quantum coherence

quantum superposition of two classical fields
(interference fringes)

quantum signature of the prepared state
(negative values of Wigner function)

A larger cat for observing decoherence

- Initial coherent field $\beta^2 = 3.5$ photons
- Measurement for 400 values of $\alpha$.

Even cat  
Odd cat  
Sum of two WF: Statistical mixture

State fidelity with respect to the expected state including phase shift non-linearity (insets) 

$F = 0.72$

Movie of decoherence

Decoherence of a $D^2 = 11.8$ photon cat state

$T_{\text{dec}} = 2T_{\text{cav}}/D^2 = 22$ ms

Theory:

$T_{\text{dec}} = 17 \pm 3$ ms

$T_{\text{meas}} \approx 4$ ms $< T_{\text{dec}, \bar{n}_{\text{th}}}$

+ small blackbody contribution @ 0.8 K

$T_{\text{dec}} = 19.5$ ms

- Generation of cat states in a cavity
- State measurement (QND) and reconstruction (MaxEnt)
- Wigner function of the cat states
- Time evolution and decoherence of the cats
Perspectives
CQED with two cavities

\[ \frac{1}{\sqrt{2}} \left( |\alpha\rangle |0\rangle + |0\rangle |-\alpha\rangle \right) \]

\[ \frac{1}{\sqrt{2}} \left( | \begin{array}{c} \text{\#} \\ \text{\#} \end{array} \rangle + | \begin{array}{c} \text{\#} \\ \text{\#} \end{array} \rangle \right) \]

\[ \rightarrow \text{alive-here-and-dead-there state} \]
• Full state reconstruction by "quantum trajectory tomography
Method proposed by P. Rouchon

Quantum state tomography with non-instantaneous measurements, imperfections, and decoherence.

\[ \frac{1}{\sqrt{2}} \left( |1,0\rangle + |0,1\rangle \right) \]

V. Météillon et al., accepted PRL
arXiv:1904.04681
A two-cavity experiment: exploring quantum thermodynamics

- Fast atoms crossing two microwave high-Q cavities

- Projects

Quantum thermodynamics
(ANR with A. Auffeves and P. Sénellart)

Heat going from cold to hot using information!
Exp. In progress

- People: Igor Dostenko, collab. A. Auffeves
Slow atoms cavity QED set-up

• **parspectives:**
  - Large cats, metrology of decoherence
  - Quantum Zeno dynamics
  - Reservoir engineering

A. Sarlette, A. et al. PRL 107, 010402 (2011)

Collapse and revival of Rabi oscillation

Preparation of a 44 photon cat state

Dressed states spectroscopy

V = 10 m/s
Another direction: Rydberg atoms without cavities

Engineering quantum state of the Rydberg electron motion

\[ | n, l, m \rangle : n^2 \text{ levels with same } n \]

One Rydberg atom

\[ \rightarrow \text{ use multi-level structure of} \]

\[ \text{for quantum metrology} \]

\[ \rightarrow \text{more about this in the colloquium} \]

Trapped Rydberg atoms with dipole interactions

\[ \rightarrow \text{quantum simulations with circular atoms} \]

\[ \rightarrow \text{topic of lecture 5} \]
The LKB-ENS cavity QED team

- Staring, in order of apparition
  - Serge Haroche
  - Michel Gross
  - Claude Fabre
  - Philippe Goy
  - Pierre Pillet
  - Jean-Michel Raimond
  - Guy Vitrant
  - Yves Kaluzny
  - Jun Liang
  - Michel Brune
  - Valérie Lefèvre-Seguin
  - Jean Hare
  - Jacques Lepape
  - Aephraim Steinberg
  - André Nussenzveig
  - Frédéric Bernardot
  - Paul Nussenzveig
  - Laurent Collot
  - Matthias Weidemuller
  - François Treussart
  - Abdelamid Maali
  - David Weiss
  - Vahid Sandoghdar
  - Jonathan Knight
  - Nicolas Dubreuil
  - Peter Domokos
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  - Jochen Dreyer
  - Peter Domokos
  - Ferdinand Schmidt-Kaler
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  - Xavier Maître
  - Christoph Wunderlich
  - Gilles Nogues
  - Vladimir Ilchenko
  - Jean-François Roch
  - Stefano Osnaghi
  - Arno Rauschenbeutel
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  - Valentin Métillon
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