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From cavity QED to quantum simulations with Rydberg atoms

Lecture 4 Quantum measurement, Schrödinger cat and decoherence

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Cavity QED with microwave photons and circular Rydberg atoms:

- L1:Achieving strong coupling between single atoms and single photons
- L2: Performing QND measurement of the field state
- L3: application to quantum feedback and past quantum state analysis of a quantum trajectory
- L4: The same experiment seen from the point of view of the field:
- → Schrödinger cat preparation and monitoring of its decoherence







Lecture 4: Quantum measurement, Schrödinger cat and decoherence

Quantum measurement: basic ingredients



- We have shown how to built an ideal QND meter of the photon number
- □ This detector is based on a destructive detector of the atom energy.
- Let us now built a more complete, fully quantum, model of detector including the dissipative part

1. The "Schrödinger cat" and the quantum measurement

The border separation quantum and classical behavior





Quantum description of a meter: the "Schrödinger cat" problem



One encloses in a box a cat whose fate is linked to the evolution of a quantum system: one radioactive atom.



The "Schrödinger cat"

 One closes the box and wait until the atom is desintegrated with a probability 1/2



• When opening the box is the cat dead, alive or in a superposition of both?





- Before opening the box, the system is isolated and unitary evolution prepares a maximally atom-meter entangled state
- Does this state "really" exists?
 - → a more relevant question: can one perform experiments demonstrating cat superposition state? Up to which limit?
- That is a fundamental question for the quantum theory of measurement: how does the unphysical entanglement of SC state vanishes at the macroscopic scale. That is the problem of the transition between quantum and classical world

$$\frac{1}{\sqrt{2}}(|a\rangle + |b\rangle) \Rightarrow \frac{1}{\sqrt{2}}(|a, |a, |b\rangle) \Rightarrow \frac{1}{\sqrt{2}}(|a, |b\rangle) \Rightarrow \frac{1}{\sqrt{2}}(|b\rangle) \Rightarrow \frac{1}{\sqrt{2}}(|a, |b\rangle) \Rightarrow \frac{1}{\sqrt{2}}(|b\rangle) \Rightarrow \frac{1}{\sqrt{2$$



$$\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \implies \frac{1}{\sqrt{2}}(|e, |e, |e, |g, |h|) \Rightarrow \frac{1}{\sqrt{2}}(|e, |e, |h|) \Rightarrow \frac{1}{\sqrt{2}}(|e, |e, |h|) \Rightarrow \frac{1}{\sqrt{2}}(|e, |h|)$$

- Real measurement provide one definite result and not superposition of results: SC states are unphysical ?
- Schrödinger: unitary evolution should "obviously" not apply any more at "some scale".
- It seems that the atom-meter space contains to many states for describing reality
- Including dissipation due to the coupling of the meter to the environment will provide a physical mechanism "selecting" the physically acceptable states.
- Let's look at this in a real experiment using a meter whose size can be varied continuously from microscopic to macroscopic world.

2. A mesoscopic field as atomic state measurement aparatus



A mesoscopic "meter": coherent field states

- Number state: $|N\rangle$
- Quasi-classical state:

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{N} \frac{\alpha^N}{\sqrt{N!}} |N\rangle$$
; $\alpha = |\alpha| e^{i\Phi}$

Photon number distribution



Phase space representation











→ The field phase "points" on the atomic state





The field phase "points" on the atomic state



Atom-meter entanglement



$$\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \implies \frac{1}{\sqrt{2}}(|e, |f|) + |g, |f|)$$

This is a "Schrödinger cat state"

Let us now consider the effect of coupling of the cavity to the "environment"





- For long atom-cavity interaction time field damping couples the system to the outside world
- → a complete description of the system must take into account the state of the field energy "leaking" in the environment.
- General method for describing the role of the environment:

$$\frac{d\rho^{field}}{dt} = -\frac{1}{2T_{cav}} \left[a^{+}a, \rho^{field}\right]_{+} + \frac{1}{T_{cav}}a\rho^{field}a^{+}$$

master equation of the field density matrix

• Physical result: decoherence

$$au_{dec} pprox rac{ au_{cav}}{\overline{N}}$$





• Decay of a coherent field:

$$\begin{aligned} |\alpha(0)\rangle \otimes |vacuum\rangle_{env} \rightarrow |\alpha(t)\rangle \otimes |\beta(t)\rangle_{env} \\ \alpha(t) = \alpha(0).e^{-t/2\tau_{cav}} \end{aligned}$$

 the cavity field remains coherent
 the leaking field has the same phase as α

□ no entanglement during decay:

That is a property defining coherent states: coherent state are the only one which do not get entangled while decaying









• Decay of a "cat" state:

$$|\Psi_{cat}\rangle \otimes |vacuum\rangle_{env}$$

 $+ 1/\sqrt{2} (|\alpha_{+}(t)\rangle \otimes |\beta_{+}(t)\rangle_{env} + |\alpha_{-}(t)\rangle \otimes |\beta_{-}(t)\rangle_{env})$

- cavity-environment entanglement:
 the leaking field "broadcasts" phase information
- □ trace over the environment
- ⇒ decoherence (=diagonal field reduced density matrix) as soon as:

$$\left< \beta_{-}(t) \right| \beta_{+}(t) \right>_{env} \approx 0$$





• Decay of a "cat" state:

$$|\Psi_{cat}\rangle \otimes |vacuum\rangle_{env}$$

$$\overline{2} \left(|\alpha_{+}(t)\rangle \otimes |\beta_{+}(t)\rangle_{env} + |\alpha_{-}(t)\rangle \otimes |\beta_{-}(t)\rangle_{env} \right)$$

$$\left| \left\langle \beta_{+}(t) |\beta_{-}(t)\rangle \right\rangle = e^{-|\beta|^{2} \left(1 - e^{2i\Phi_{0}}\right)}$$

$$|\alpha(t)|^{2} + |\beta(t)|^{2} = |\alpha_{0}|^{2}$$

$$\Rightarrow |\beta(t)|^{2} = |\alpha_{0}|^{2} \left(1 - e^{-t/T_{cav}}\right) \approx |\alpha_{0}|^{2} \cdot t/T_{cav}$$
The two states of the environment become orthogonal as soon as

10.00

$$\left|\beta(t)\right|^{2} \approx 1 \Rightarrow t > \frac{T_{cav}}{\overline{N}} \approx T_{dec}$$



The decoherence time





Quantum measurement: the role of the environment 1

 \Rightarrow Physical origin of decoherence:

leak of information into the environment.

⇒ The experimentalist does not kill the cat when opening the box: the environment "knows" whether the cat is dead or alive well before one opens the box.

⇒ The environment performs continuously unread repeated measurement of the cat state

The "collapse" of the quantum state can be considered as a shortcut to describe this complex physical process

Does it solves "the measurement problem"?

No: if the problem consists in telling how or why nature chooses randomly one classical state.

Yes: once one a priori accepts the statistical nature of quantum theory, dechoherence is the mechanism providing classical probabilities



Quantum measurement: the role of the environment 2

\Rightarrow Definition of "pointer basis" of a meter: (Zurek)

- □ the pointer state of the meter is a classical state
- once decoherence occurs, the physical state of a meter is described by a diagonal density matrix in the pointer basis:



⇒ at this level, quantum description only involves classical probabilities and no macroscopic superposition states.

The decoherence approach shows that quantum theory is consistent with classical logic at macroscopic scale: it only provides classical statistics at the macroscopic scale.

3. Observing decoherence experimentally

Probing the coherence of the cat state



"cat" state coherence Interference term in two atom correlation



Decoherence signal



Brune et al. Phys. Rev. Lett. 77, 4887 (1996)

4. Full tomography of the field state



This correlation signal is a very partial information on the field state

One can reconstruct the full density matrix of the cavity field







Principle of stet reconstruction



→ Each measurement sets a constrain to the density operator

$$\operatorname{Tr}(\hat{\rho}\,\hat{G}_i) = g_i$$

Problems to face:

- Having a complete set of observable $\{\hat{G}_i\}$
- Statistical noise on $\{g_i\}$ may lead to unphysical/very noisy density operators



Measuring the field density operator?

General field state description: density operator

$ ho_{field} =$	ρ_{00}	$ ho_{\scriptscriptstyle 01}$	$ ho_{\scriptscriptstyle 02}$.]
	$ ho_{10}$	$ ho_{\!11}$	$ ho_{12}$	•
	$ ho_{ m 20}$	$ ho_{11}$ $ ho_{21}$	$ ho_{\scriptscriptstyle 22}$	•
		•	•	•

Displacement operator

 $ho_{\it field}$

 $\rho_{\text{field}}^{(\alpha)} = \hat{D}(\alpha) \rho_{\text{field}} \hat{D}^{+}(\alpha)$

 $\hat{D}(\alpha) = e^{\alpha a^{+} - \alpha^{*}a}$

• Previous lectures: QND counting of photons \Rightarrow measurement of diagonal elements ρ_{nn} \Rightarrow How to measure the offdiagonal elements of ρ_{field} ?

⇒ by counting photons after applying "displacement"

The displacement operator is the unitary transform corresponding to the coupling to a classical source. It mixes diagonal and off-diagonal matrix elements of ρ_{field} . Measuring the photon number after displacement for a large number of different a gives information about all matrix elements of ρ_{field} .



• Various possibilities:

 $\Box \text{ Direct fit of } \rho_{field} \text{ on the measured data } g_i = tr \left[\rho_{field} \cdot \cos(\phi(\hat{n}) + \varphi) \right]$

- □ Maximum likelihood: find ρ_{field} which maximizes the probability of finding the actually measured results g_i .
- Maximum entropy principle: find ρ_{field} which fits the measurements and additionally maximizes entropy
 S=ρ_{field}log(ρ_{field}).
 V. Bužek and G. Drobný, *Quantum tomography via the MaxEnt principle*

via the MaxEnt principle, Journal of Modern Optics **47**, 2823 (2000)

Estimates the state only on the basis of measured information: in case of incomplete set of measurements, gives a "worse estimate of ρ_{field} .

In practice the two last methods give the same result provided one measures enough data completely determining the state.



- 1- prepare the state to be measured $|\psi_{field}\rangle$ 2- measure $\hat{G}(\alpha)$ for a large number of different values of α (400 to 600 points).
- 3- reconstruct $ho_{\it field}$ by maximum entropy method
- 4- calculate Wigner function from $\rho_{\it field}$.



- Measurement for 161 values of α (<1 hour measurement)
- 7000 detected atoms in 600 repetition of the experimental sequence for each α .





Reconstruction of number states

- Prepare a coherent state $\beta^2 = 1.3$ or 5.5 photons.
- Select pure number state by QND measurement of *n*.
 Phase shift per photon φ₀≈π/2 : measurement of n modulo 4.
- Measurements of $G(\alpha)$ for 2 different values of ϕ and ~400 values of α .



5. Schrödinger cat states reconstruction

state preparationa movie of decoherence



Preparation of the cavity cat state

Phase shift per photon Φ_{\circ}



$$\frac{1}{\sqrt{2}} \left(\left| e \right\rangle + \left| g \right\rangle \right) \otimes \left| \alpha \right\rangle \implies \frac{1}{\sqrt{2}} \left(\left| e \right\rangle \otimes \left| \alpha \cdot e^{i\Phi_0/2} \right\rangle + \left| g \right\rangle \otimes \left| \alpha \cdot e^{-i\Phi_0/2} \right\rangle \right)$$


Phase shift per photon $\Phi_{\rm c}$



$$\frac{1}{\sqrt{2}} \left(\left| e \right\rangle + \left| g \right\rangle \right) \otimes \left| \alpha \right\rangle \quad \Rightarrow \quad \frac{1}{\sqrt{2}} \left(\left| e \right\rangle \otimes \left| \alpha \cdot e^{i\Phi_0/2} \right\rangle + \left| g \right\rangle \otimes \left| \alpha \cdot e^{-i\Phi_0/2} \right\rangle \right)$$

• Field state after detection:



Depending on the detected atomic state the cat has a well defined photon number parity.

For π per photon phase shift, one atom measures just the field parity. Projection on a cat state is the "back-action" of parity measurement.



Measured raw data

> We perform measurements at about 600 points in the phase space (about 10 scans) $\succ \approx 100$ state preparations for each point $\succ \approx$ 12 probe atoms for each state preparation



1,0

measured raw data





Even (odd) cat has even (odd) photon number statistics



Fidelity of the preparation and reconstruction - 66% (71% for the odd state)



Reconstructed Wigner function



Deleglise et al. Nature **455**, 510 (2008)



Reconstructed Wigner function

Classical components

 ≈2.1 photons in each classical component
(amplitude of the initial coherent field)

cat size $D^2 \approx 7.5$ photons

coherent components are completely separated (D > 1)



Deleglise et al. Nature **455**, 510 (2008)



Reconstructed Wigner function

quantum superposition of two classical fields *(interference fringes)*

quantum signature of the prepared state (negative values of Wigner function)



Deleglise et al. Nature **455**, 510 (2008)



A larger cat for observing decoherence

- Initial coherent field $\beta^2 = 3.5$ photons
- Measurement for 400 values of α .



State fidelity with respect to the expected state including phase shift non-lineariry (insets)

F= 0.72

Deleglise et al. Nature **455**, 510 (2008)



Movie of decoherence



Decoherence of a D²=11.8 **photon cat state**





Theory: $T_{dec} = 2T_{cav}/D^2 = 22 \text{ ms}$

+ small blackbody contribution @ 0.8 K

 $T_{dec} = 19.5 \text{ ms}$

M.S. Kim and V. Bužek, Schrödinger-cat state at finite temperature, Phys. Rev. A 46, 4239 (1992)



Summary

- Generation of cat states in a cavity
- State measurement (QND) and reconstruction (MaxEnt)
- Wigner function of the cat states
- Time evolution and decoherence of the cats







Perspectives



CQED with two cavities



 $\frac{1}{\sqrt{2}}\left(|\alpha\rangle|0\rangle+|0\rangle|-\alpha\rangle\right)$ $\frac{1}{\sqrt{2}}\left(|8\rangle + |6\rangle\right)$

→ alive-here-and-dead-there state



Exploring non-local states with two cavities



• Full state reconstruction by "quantum trajectory tomography Method proposed by P. Rouchon

P. Six et al., Phys. Rev. A 93, 012109 (2016)

Quantum state tomography with non-instantaneous measurements, imperfections, and decoherence.



A two-cavity experiment: exploring quantum thermodynamics

 Fast atoms crossing two microwave high-Q cavities



• Projects

Quantum thermodynamics

(ANR with A. Auffeves and P. Sénellart)



Heat going from cold to hot using information! Exp. In progress



• People: Igor Dostenko, collab. A. Auffeves



1,0

0,8

0.6

0,4

0,2

0,0 -0

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Slow atoms cavity QED set-up



J.M. Raimond et al PRL **105**, 213601 (2010)

Reservoir engineering A. Sarlette, A. et al. PRL **107**, 010402 (2011)



Dressed states spectroscopy



Engineering quantum state of the Rydberg electron motion

 $|n,l,m\rangle$: n² levels with same n



One Rydberg atom → use multi-level structure of for quantum metrology

 \rightarrow more about this in the coloquium





Trapped Rydberg atoms with dipole interactions →quantum simulations with circular atoms

→ topic of lecture 5

The LKB-ENS cavity QED team

Staring, in order of apparition

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