Dipolar quantum gases

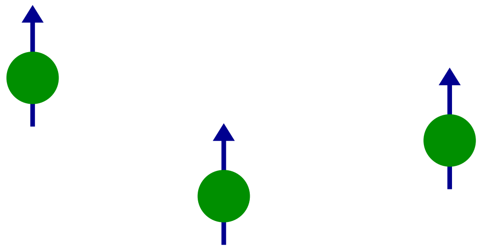
Gora Shlyapnikov
LPTMS, Orsay, France
Russian Quantum Center, Moscow, Russia
University of Amsterdam, The Netherlands

- Introduction. Dipolar particles
- First experiments
- BEC in dipolar quantum gases
- Quantum dipolar droplets
- Supersolid states of bosons

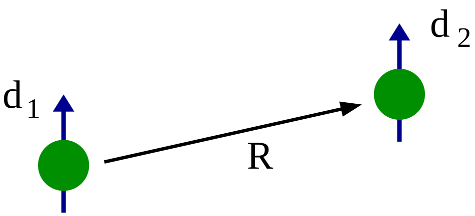
Sao Paulo, Brazil, September 23, 2019

Dipolar particleas

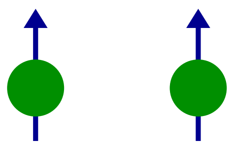
Polar molecules or atoms with a large magnetic moment



Dipole-dipole interaction $V_d = \dfrac{\vec{d}_1 \vec{d}_2 R^2 - 3(\vec{d}_1 \vec{R})(\vec{d}_2 \vec{R})}{R^5} \sim \dfrac{1}{R^3}$



long-range, anisotropic



repulsion

attraction

Different physics compared to ordinary atomic ultracold gases

Alkali-atom molecules d from $0.6\,D$ for KRb to $5.5\,D$ for LiCs

Atoms with large μ

First experiments

Remarkable experiments with Cr atoms ($\mu = 6\mu_B \Rightarrow d \approx 0.05$ D)

T. Pfau group (Stuttgart)

Effects of the dipole-dipole interaction in the dynamics

Stability diagram of trapped dipolar BEC

Spinor physics in Cr experiments at Villetaneuse, B. Laburthe-Tolra

Dysprosium ($\mu = 10\mu_B$, (B. Lev))

Erbium ($\mu = 7\mu_B$, (F. Ferlaino))

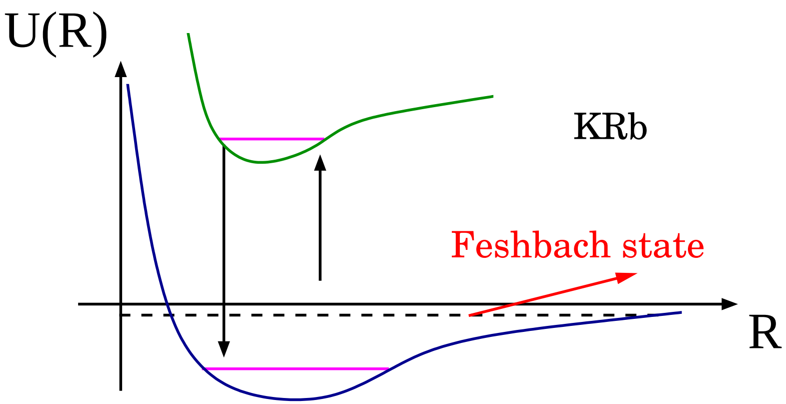
Polar molecules. Creation of ultracold clouds

Photoassociation

Transfer of weakly bound KRb fermionic molecules

to the ground rovibrational state

JILA, D. Jin, J. Ye groups



$$n \sim 10^{12} - 10^{13}\ \text{cm}^{-3}$$
$$T \approx 200nK \ \sim E_F$$

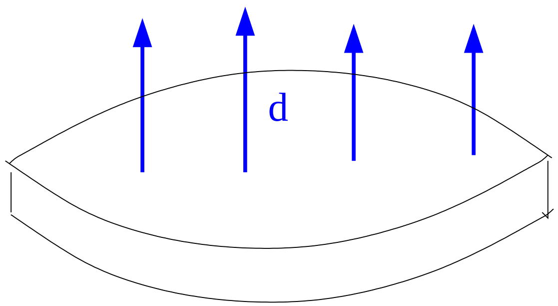
Ultracold chemistry

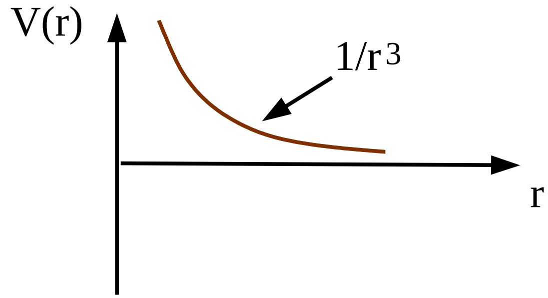
Ultracold chemical reactions $\mathrm{KRb} + \mathrm{KRb} \Rightarrow \mathrm{K_2} + \mathrm{Rb_2}$

New trends in ultracold chemistry

Suppress instability $\rightarrow$ induce intermolecular repulsion

For example, 2D geometry with dipoles perpendicular to the plane





Reduction of the decay rate by 2 orders of magnitude at JILA

Select non-reactive molecules, like NaK, KCs, RbCs

Present experiments

First generation of magnetic atom experiments

Cr ($6\mu_B$) Stuttgart; Villetaneuse, etc.

Dy ($10\mu_B$) Stanford; Stuttgart, etc.

Er ($7\mu_b$) Innsbruck

First generation of ground-state polar molecule experiments

KRb JILA

CsRb Innsbruck

NaK and NaLi at MIT; NaK in Munich

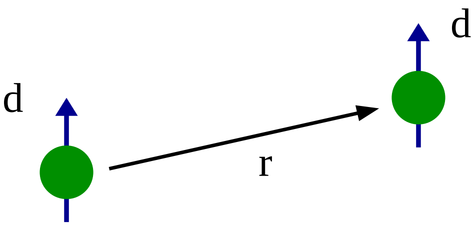
NaRb in Hong Kong

LiCs and CaF at Harvard

The main initial goal $\Rightarrow$ Reveal the role of dipolar interactions

Stability diagram and the shape of a trapped cloud

Radius of the dipole-dipole interaction



$$\left(-\frac{\hbar^2}{m}\Delta + V_d(\vec{r})\right)\psi(\vec{r}) = \frac{\hbar^2 k^2}{m}\psi(\vec{r})$$

$$\frac{\hbar^2}{m r_*^2} = \frac{d^2}{r_*^3} \Rightarrow r_* \approx \frac{m d^2}{\hbar^2}$$

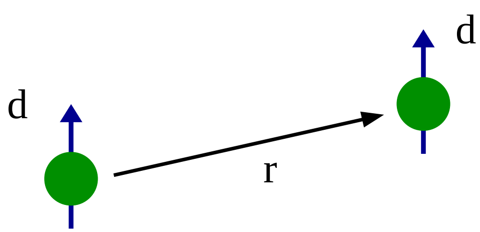
$r \gg r_*$ $\rightarrow$ free relative motion

$r_* \sim 10^6 \div 10^3 a_0$ polar molecules

$r_* \approx 50 a_0 \rightarrow$ chromium atoms

$$k r_* \ll 1 \quad \rightarrow \underbrace{\text{Ultracold limit}}_{T \ll 1mK \text{ for Cr}}$$

Scattering amplitude



$$V(\vec{r}) = \mathcal{U}(\vec{r}) + V_d(\vec{r})$$

$$f = \int \psi^*_{k_i}(\vec{r}) V(\vec{r}) e^{i\vec{k}_f \vec{r}} d^3 r$$

Ultracold limit $kr_* \ll 1$

$$V_d = 0 \Rightarrow f = g = \frac{4\pi\hbar^2}{m} a$$

What V_d does?

$$k = 0 \rightarrow g = \int \psi^*_0(\vec{r})(\mathcal{U}(\vec{r}) + V_d(\vec{r}))d^3 r = \text{const}; \qquad r \lesssim r_*$$

g may depend on d

Scattering amplitude

$$k \neq 0$$

$$f = \int \psi^*_{k_i}(\vec{r}) V(\vec{r}) e^{i\vec{k}_f \vec{r}} d^3 r$$

$$r \lesssim r_* \rightarrow \text{put } k = 0 \rightarrow g$$

$$r \gg r_* \rightarrow \psi_{k_i} = e^{i\vec{k}_i \vec{r}}$$

$$f = \int V_d(\vec{r}) e^{i\vec{q}\vec{r}} d^3 r \longrightarrow \frac{4\pi d^2}{3}(3\cos^2\theta_{qd} - 1);\ \vec{q} = \vec{k}_f - \vec{k}_i$$

$$f = g + \frac{4\pi d^2}{3}(3\cos^2\theta_{qd} - 1)$$

Dipolar BEC in free space

Uniform gas

$$H = \int d^3 \left[\psi^\dagger(\vec{r}) \left(-\frac{\hbar^2}{2m}\Delta \right) \psi(\vec{r}) + \frac{1}{2} g \psi^\dagger(\vec{r})\psi^\dagger(\vec{r})\psi(\vec{r})\psi(\vec{r}) \right.$$
$$\left. + \frac{1}{2} \int d^3 r' \psi^\dagger(\vec{r})\psi^\dagger(\vec{r}')V_d(\vec{r}-\vec{r}')\psi(\vec{r}')\psi(\vec{r}) \right]$$

Bogoliubov approach $\psi = \psi_0 + \delta\Psi \rightarrow$ biliniear Hamiltonian

$$H_B = \frac{N^2}{2V} g + \sum_k \left[\frac{\hbar^2 k^2}{2m} a_k^\dagger a_k + n \left(g + \frac{4\pi d^2}{3} \left(3\cos^2\theta_k - 1\right) \right) a_k^\dagger a_k \right.$$
$$\left. + \frac{n}{2} \left(g + \frac{4\pi d^2}{3} \left(3\cos^2\theta_k - 1\right) \right) \left(a_k^\dagger a_{-k}^\dagger + a_k a_{-k} \right) \right]$$

Dipolar BEC in free space

Excitation spectrum

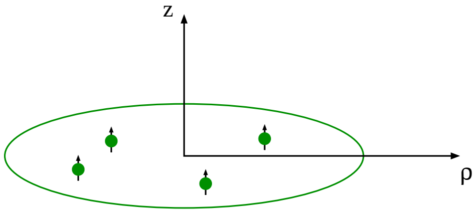
$$\epsilon_k = \sqrt{E_k^2 + 2E_k n \left(g + \frac{4\pi d^2}{3} \left(3\cos^2\theta_k - 1\right) \right)}$$

$$g > \frac{4\pi d^2}{3} \rightarrow \text{dynamically stable BEC}$$

$$g < \frac{4\pi d^2}{3} \rightarrow \text{complex frequencies at small } k$$

$$\cos^2\theta_k < \frac{1}{3} \rightarrow \text{collapse}$$

Trapped dipolar BEC



Cylindrical trap

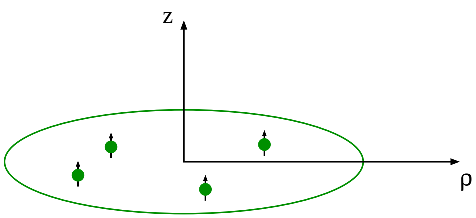
$$V_h = \frac{m}{2}\left(\omega_\rho^2 \rho^2 + \omega_z^2 z^2\right)$$

Gross-Pitaevskii equation

$$\left[-\frac{\hbar^2}{2m}\Delta + V_h(\vec{r}) + g\psi_0^2 + \int \psi_0(\vec{r}')^2 V_d(\vec{r}-\vec{r}')d^3r'\right]\psi_0(\vec{r}) = \mu\psi_0(\vec{r})$$

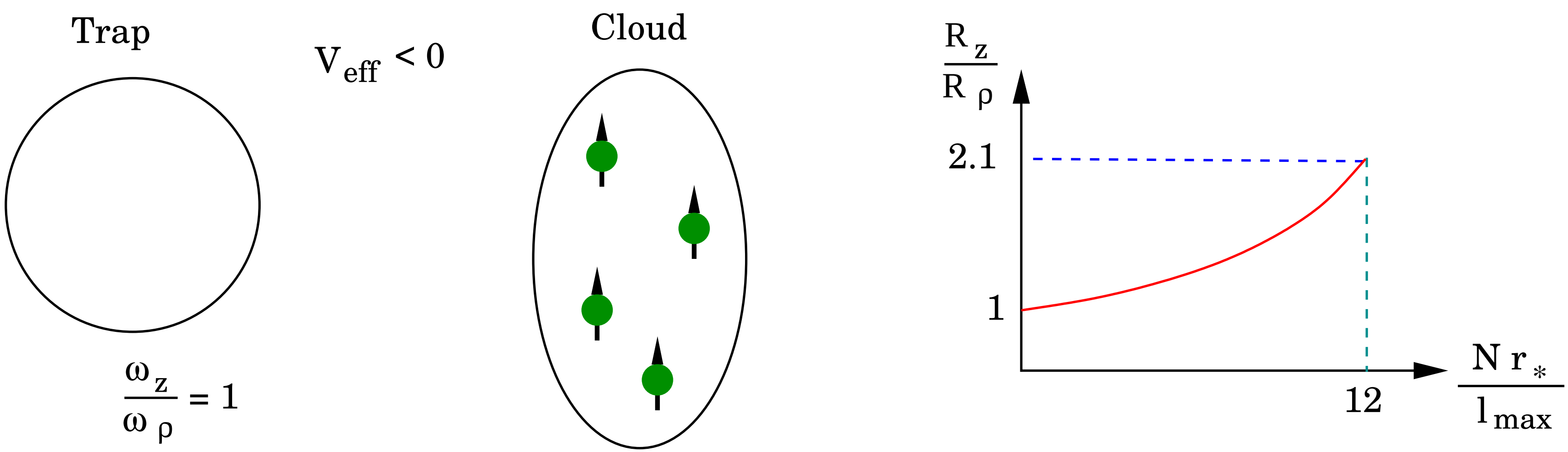
Important quantity

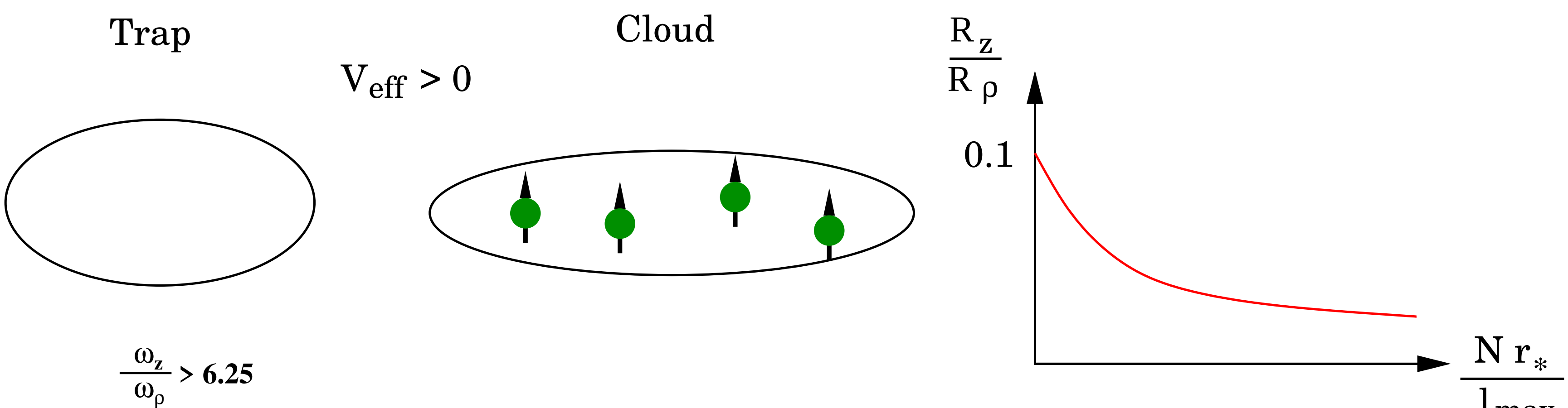
$$V_{eff} = g\int \psi_0^4(\vec{r})d^3r + \int \psi_0^2(\vec{r}')V_d(\vec{r}-\vec{r}')\psi_0^2(\vec{r})d^3r d^3r'$$



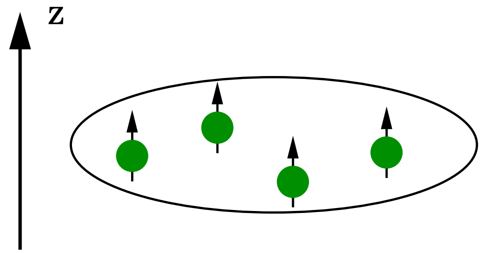
$V_{eff} > 0$ or $V_{eff} < 0$ and $|V| < \hbar\omega$
$g = 0 \rightarrow N < N_c \rightarrow$ suppressed
low k instability
(Santos et.al, 2000)

Trapped dipolar BEC





Stability problem



Dipolar BEC

$$\langle V_d \rangle = \int n_0(\vec{r}')V_d(\vec{r}' - \vec{r})d^3r = -d^2 \frac{\partial^2}{\partial z^2} \int \frac{n_0(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r$$

$$V_d = -d^2 \frac{\partial^2}{\partial z^2} \frac{1}{|\vec{r}-\vec{r}'|} - \frac{4\pi d^2}{3}\delta(\vec{r} - \vec{r}')$$

Large $N \Rightarrow$ Thomas-Fermi BEC

$$n_0 = n_{0\,max}\left(1 - \frac{z^2}{R_z^2} - \frac{\rho^2}{R_\rho^2}\right)$$ Eberlein et. al (2005)

$g > \frac{4\pi d^2}{3} \rightarrow$ stable at any N

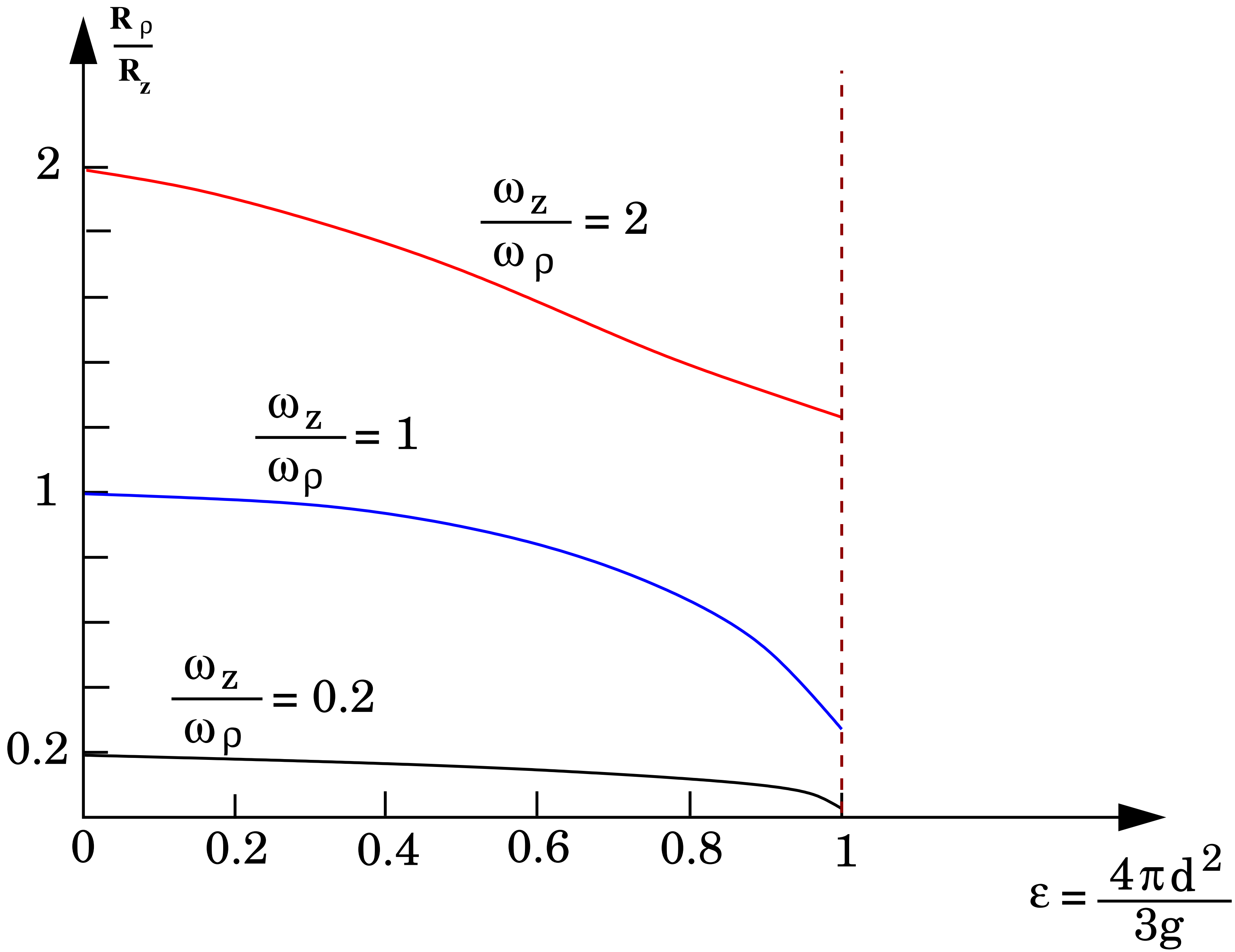
Papers to look in

Exact solution of the Thomas-Fermi equation
for a trapped Bose-Einstein condensate with dipole-dipole interactions

Claudia Eberlein, Stefano Giovanazzi, and Duncan H. J. OÕDell

Phys. Rev. A 71, 033618 (2005)

Example



$\frac{R_\rho}{R_z}$
2
1
0.2
$\frac{\omega_z}{\omega_\rho} = 2$
$\frac{\omega_z}{\omega_\rho} = 1$
$\frac{\omega_z}{\omega_\rho} = 0.2$
0
0.2
0.4
0.6
0.8
1
$\varepsilon = \frac{4\pi d^2}{3g}$

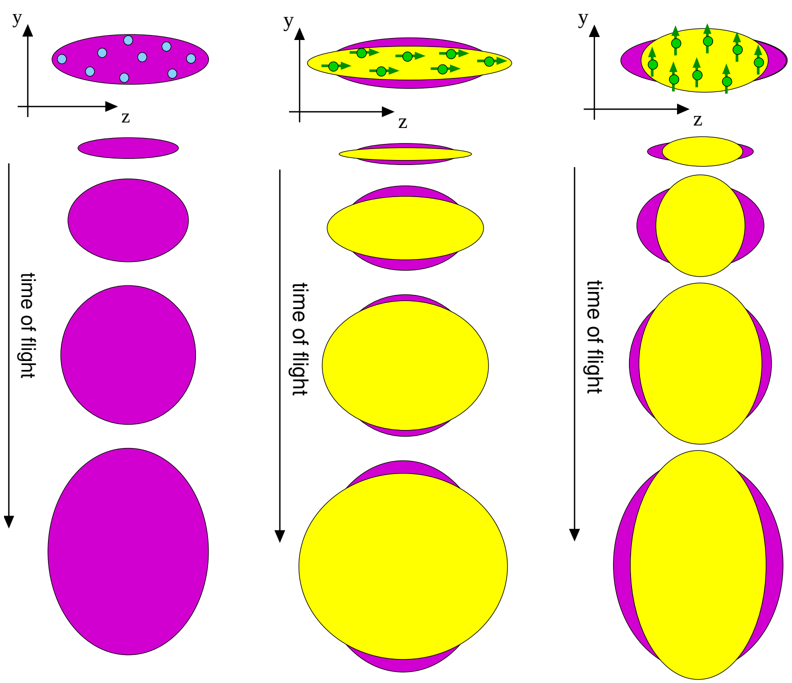
Experiment with Cr

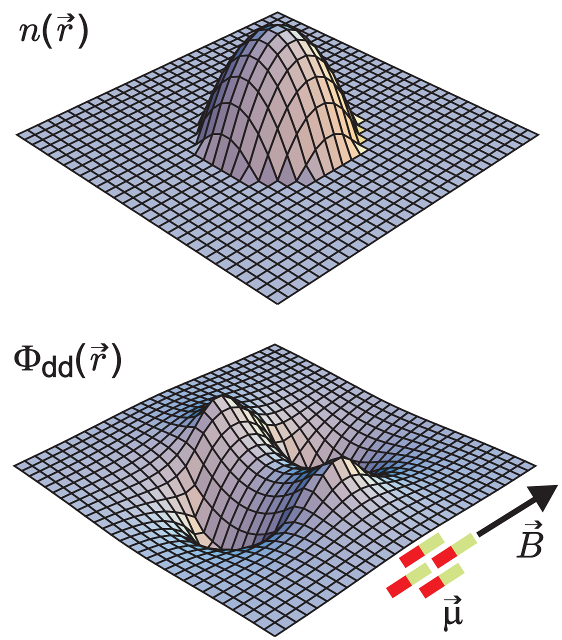
$$g > \frac{4\pi d^2}{3}$$

$(\mu = 6\mu_B!)$

(T. Pfau, Stuttgart) BEC $(n \sim 10^{14} cm^{-3})$

effect of the dipole-dipole interaction (small)



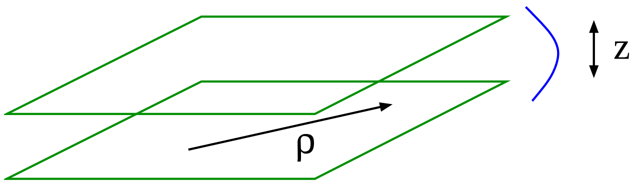


Pancake dipolar BEC

$$g < g_d = \frac{4\pi d^2}{3} \quad ?$$

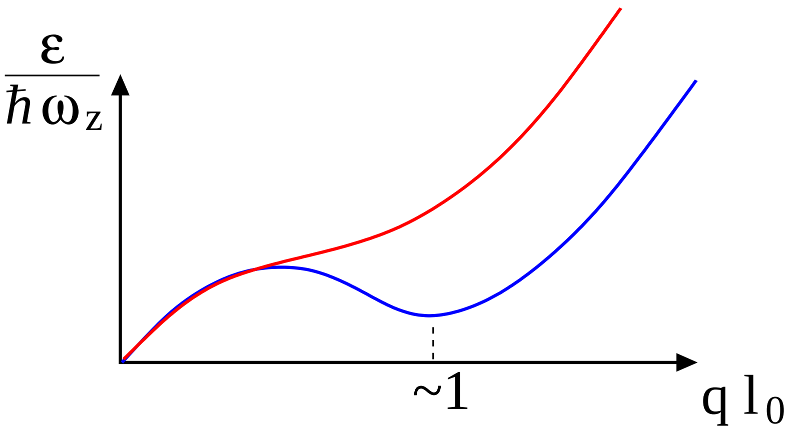
Thomas-Fermi in the z direction

Extreme pancake ($\omega_\rho = 0$)



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$V_d + g$ (short-range) $\quad g > 0$

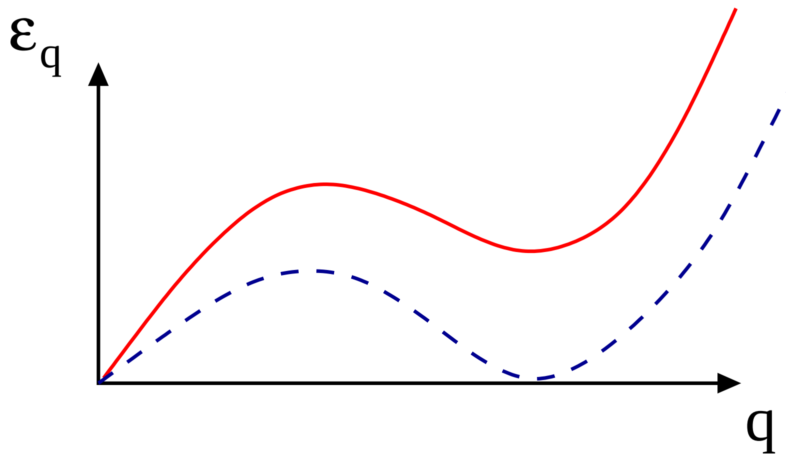


$g/g_d = 1.06 \qquad \mu/\hbar\omega_z = 46$

$g/g_d = 0.94 \qquad \mu/\hbar\omega_z = 53$

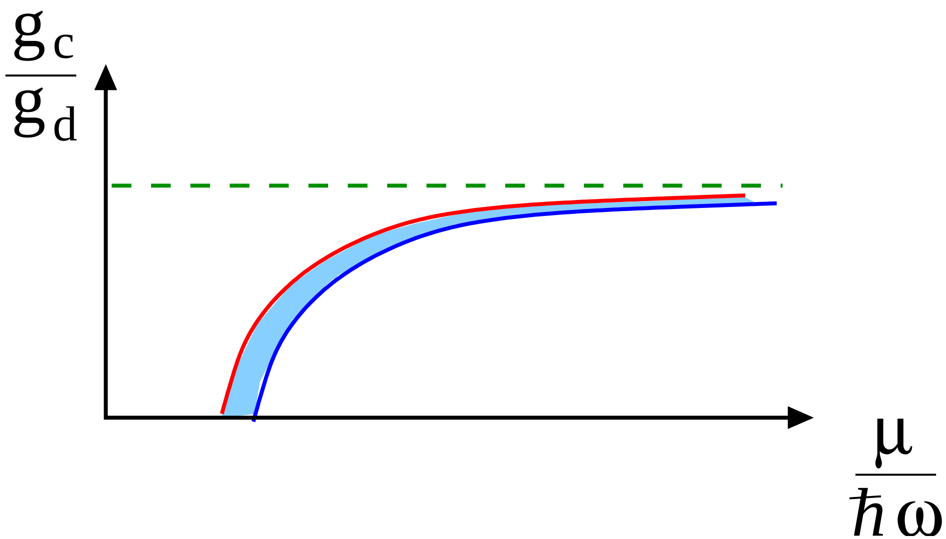
Roton structure ⇒ decrease of the interaction amplitude

Roton-maxon structure



Roton minimum can be but at zero

Instability!



rotonization

instability

(L. Santos et al., 2003)

Papers to look in

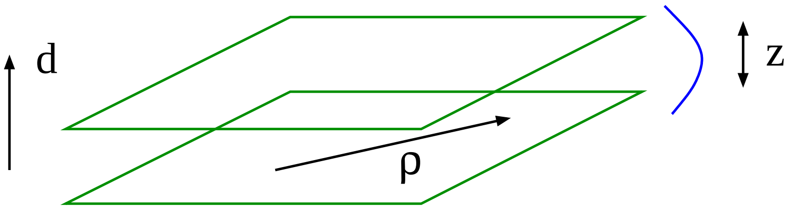
Roton-Maxon Spectrum and Stability

of Trapped Dipolar Bose-Einstein Condensates

L. Santos, G. V. Shlyapnikov, and M. Lewenstein

Phys. Rev. Lett. 90, 250403 (2003)

Quasi2D dipolar BEC at $T = 0$



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$$\varphi_0(z) = \frac{1}{\pi^{1/4} l_0^{1/2}} \exp\left\{\frac{-z^2}{2l_0^2}\right\}$$

short range interaction (g) + dipole-dipole

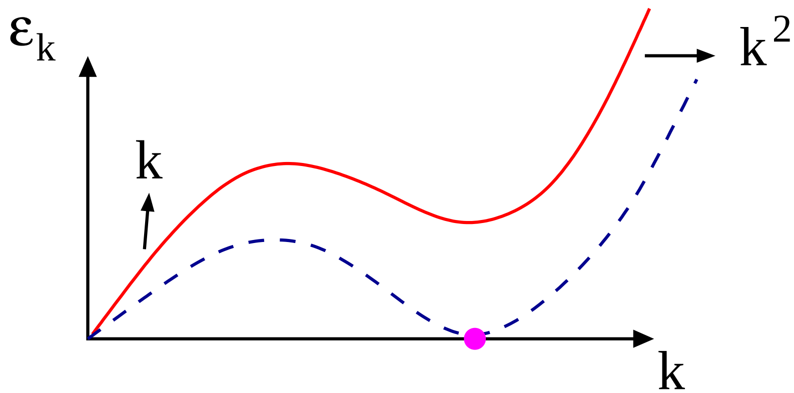
Consider $0 < g \ll \frac{4\pi d^2}{3}$. Then, for $qr_* \ll 1$

$$V_{\vec{q}\vec{p}} = g(1 - C|\vec{q} - \vec{p}|)$$

$$C = \frac{2\pi d^2}{g}$$

Spectrum

$$\hat{H} = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m} a^\dagger_{\vec{k}} a_{\vec{k}} + \frac{g}{2} \sum_{\vec{k},\vec{q},\vec{p}} (1 - C|\vec{q} - \vec{p}|) a^\dagger_{\vec{k}+\vec{q}} a^\dagger_{\vec{k}-\vec{q}} a_{\vec{k}+\vec{p}} a_{\vec{k}-\vec{p}}$$



$$\epsilon_k^2 = E_k^2 + 2\mu E_k (1 - Ck)$$

$$k_r = \frac{3}{2} \left(1 + \sqrt{\frac{C^2}{\xi^4} - \frac{8}{9\xi^2}} \right) \quad \xi = \frac{\hbar}{mng}.$$

Rotonization

$$\xi \geq C \geq \frac{\sqrt{8}}{3}\xi$$

The roton minimum touches zero for

$$C = \xi \Rightarrow \quad k_r = \frac{2C}{\xi}$$

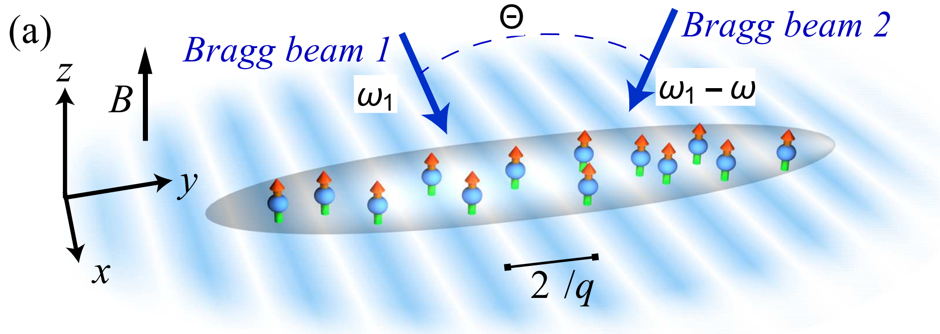
For $C > \xi$ we have collapse. No stable supersolid state

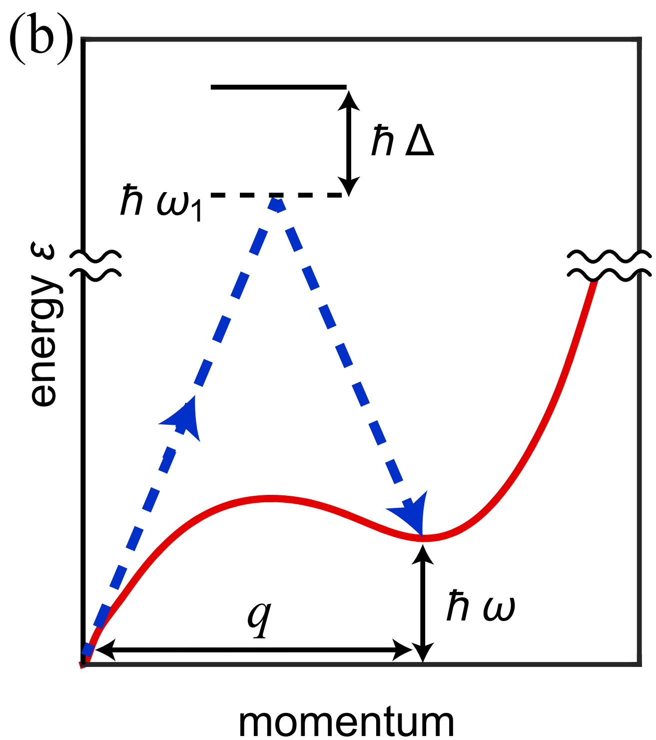
Pedri/Shlyapnikov; Cooper/Komineas (2007)

Roton-maxon structure. Experiment

Innsbruck experiment of F. Ferlaino group (2017)

with Er atoms in a very elongated geometry





Dipolar droplets

Free space (Baillie et al, 2016)

$$H = -\int d^3r\psi_0\frac{\hbar^2}{2m}\Delta\psi_0 + \frac{g}{2}\int d^3r|\psi_0|^4$$
$$+\frac{1}{2}\int d^3rd^3r'V_d(\mathbf{r}-\mathbf{r}')|\psi_0(\mathbf{r})\psi_0(\mathbf{r}')|^2 + \frac{2}{5}\gamma_{QF}\int d^3r|\psi_0|^5$$

$\gamma_{QF} \rightarrow$ quantum fluctuations

Originates from the terms

$$g\langle\int\psi'^{\dagger}(\mathbf{r})\psi'(\mathbf{r})\psi_0^2(\mathbf{r})d^3r\rangle + \frac{g}{2}\langle[\int\psi'^2(\mathbf{r})\psi_0^2d^3r + \int\psi'^{\dagger 2}(\mathbf{r})\psi_0^2(\mathbf{r})d^3r]\rangle$$

in the energy functional

$$\gamma_{QF} = \frac{32}{3}g\sqrt{\frac{a^3}{\pi}}\left(1+\frac{3}{2}\epsilon_{dd}^2\right)$$

$a \rightarrow$ 3D scattering length for short-range interaction

$$g = \frac{4\pi\hbar^2}{m}a; \quad \epsilon_{dd} = \frac{a_{dd}}{a}; \quad a_{dd} = \frac{md^2}{12\hbar^2}$$

Papers to look in

Quantum mechanical stabilization of a collapsing Bose-Bose mixture

D.S. Petrov

Physical Review Letters 115, 155302 (2015)

Dipolar droplets

Trivial solution $\psi_0 = \sqrt{n}$ $(E = 0)$

Gaussian Ansatz

$$\psi_0(\mathbf{r}) = \sqrt{\frac{8N}{\pi^{3/2}\sigma_\rho^2\sigma_z}} \exp\left\{-2\left(\frac{\rho^2}{\sigma_\rho^2} + \frac{z^2}{\sigma_z^2}\right)\right\}$$

$$E(\sigma_\rho, \sigma_z) = \frac{\hbar^2 N}{m}\left(\frac{2}{\sigma_\rho^2} + \frac{1}{\sigma_z^2}\right) + \frac{8N^2 a_{dd}}{\sqrt{2\pi}\sigma_\rho^2\sigma_z}\left[\epsilon_{dd}^{-1} - f\left(\frac{\sigma_\rho}{\sigma_z}\right)\right]$$

$$c\frac{\hbar^2}{m}\,\frac{1 + 3\epsilon_{dd}^2 N^{5/2}/2}{\sigma_\rho^3\sigma_z^{3/2}} a^{5/2}; \quad c \approx 13.8$$

$$f(x) = \frac{1 + 2x^2}{1 - x^2} - \frac{3x^2}{(1 - x^2)^{3/2}} \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}$$

Droplets elongated in the direction of the dipoles

$\sigma_z \gg \sigma_\rho$ and $f \simeq 1$

Dipolar droplets

Minimize E

$$\sigma_\rho^2 \sigma_z \propto N; \qquad E \propto -N$$

$n \simeq N/\sigma_\rho^2 \sigma_z \rightarrow$ does not depend on N. Liquid droplets

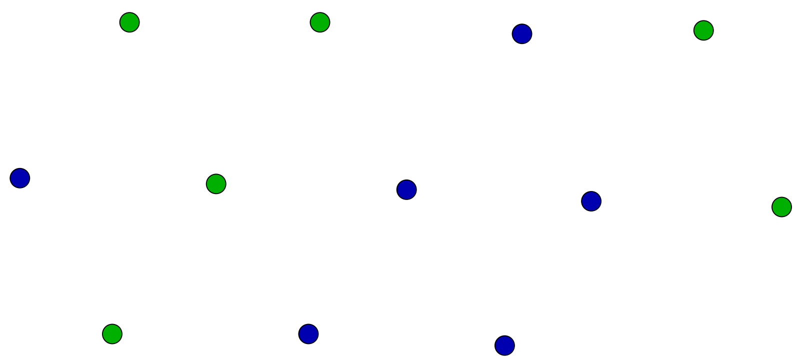
Increasing $N \Rightarrow \quad E$ the same for 2 or more droplets far from each other

Structure of f, kinetic energy, trapping potential $\rightarrow$

N above a critical value leads to 2 or more droplets

Remarkable experiments in the group of T. Pfau (last 2 years, Stuttgart)

Dipolar droplets



Numerical calculations: Wachtler/Santos; Saito (2016)

Distance between droplets $\rightarrow$ several μm. Not a supersolid

Stripe supersolid

Dy atoms $\rightarrow$ change the ratio of the dipole-dipole to short-range interaction

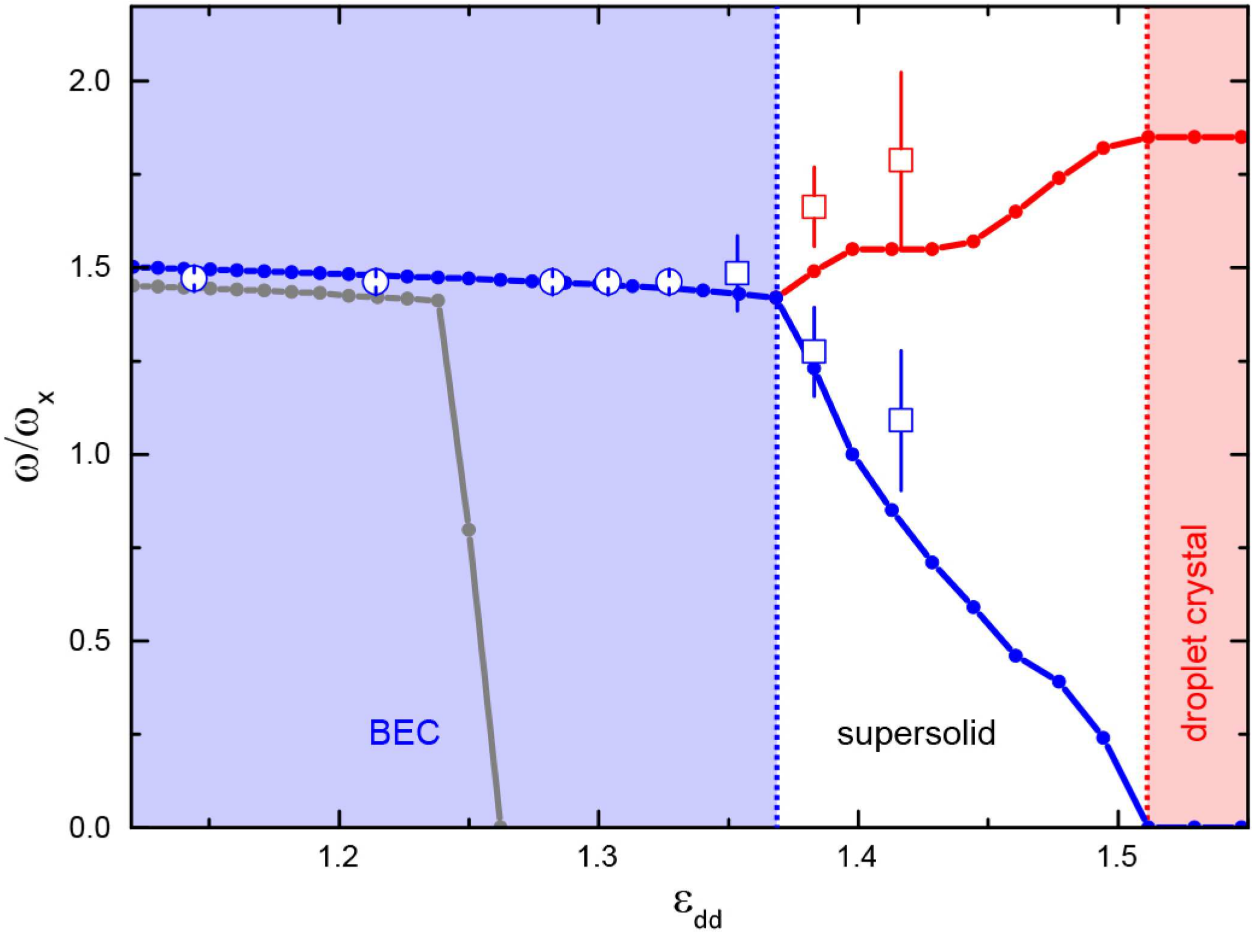
G.Modugno group (Florence, Italy)

T. Pfau group (Stuttgart, Germany)

F. Ferlaino group (Innsbruck, Austria)

Phase coherent arrays of droplets

Spectrum of low-energy excitations



The system is likely a supersolid

Papers to look in

Supersolid symmetry breaking

from compressional oscillations in a dipolar quantum gas

L. Tanzi, S. M. Roccuzzo, E. Lucioni, F. Fama, A. Fioretti, C. Gabbanini
G. Modugno, A. Recati, S. Stringari

arXiv:1906.02791

The low-energy Goldstone mode in a trapped dipolar supersolid

Mingyang Guo, Fabian Bottcher, Jens Hertkorn, Jan-Niklas Schmidt
Matthias Wenzel, Hans Peter Buchler, Tim Langen, Tilman Pfau

arXiv:1906.04633

Excitation spectrum of a trapped dipolar supersolid
and its experimental evidence

G. Natale, R. M. W. van Bijnen, A. Patscheider, D. Petter
M. J. Mark, L. Chomaz, F. Ferlaino

arXiv:1907.01986