Dipolar Fermi gases

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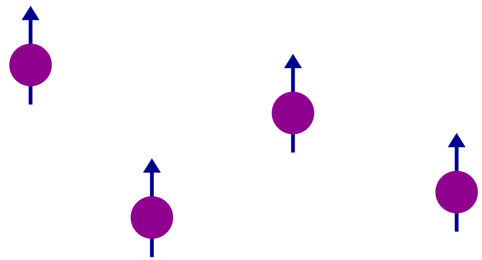
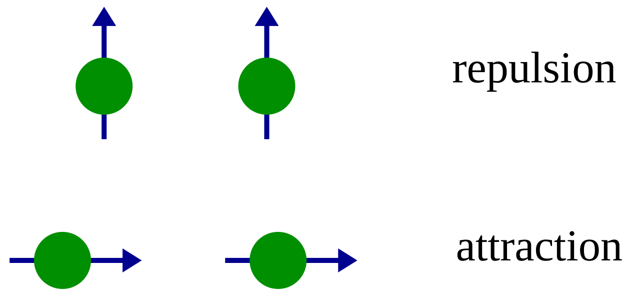
- Introduction
- Topological $p_x + ip_y$ phase in 2D
- BCS-BEC crossover in bilayered systems
- Novel Fermi liquid
- Conclusions and outlook

Sao Paulo, September 25, 2019

Dipolar Fermi gas



What does the dipole-dipole interaction do in a Fermi gas?



p-wave superfluid pairing in a single-component gas

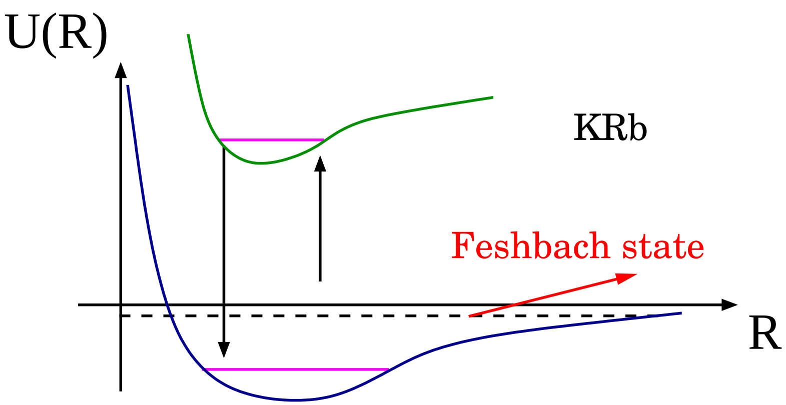
Novel Fermi liquid in two and three dimensions?

Alkali-atom molecules d from $0.6\,D$ for KRb to $5.5\,D$ for LiCs

Polar molecules. Creation of ultracold clouds

Transfer of weakly bound KRb molecules to the ground rovibrational state

JILA, D. Jin, J. Ye groups



$\Rightarrow$ quantum degeneracy

Ultracold chemical reactions $\mathrm{KRb} + \mathrm{KRb} \Rightarrow \mathrm{K}_2 + \mathrm{Rb}_2$

Suppress instability $\rightarrow$ induce intermolecular repulsion

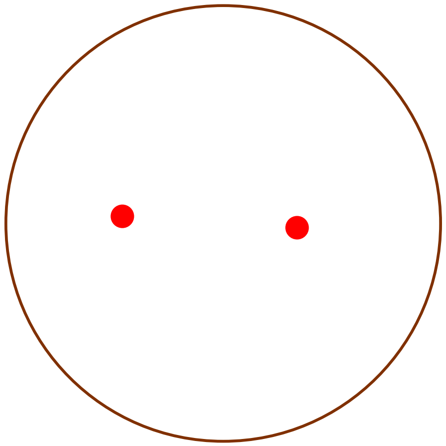
2D geometry with dipoles perpendicular to the plane. JILA experiment

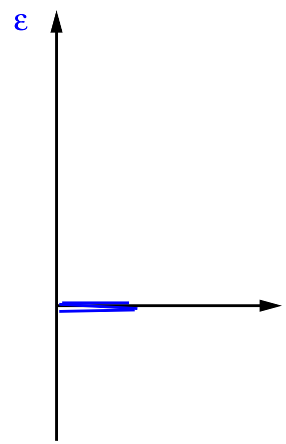
What are prospects for novel physics ?

Why single-component fermions are interesting?

Topological aspects of $p_x + ip_y$ state in 2D

Vortices. Zero-energy mode related to two vortices. (Read/Green, 2000)





The number of zero-energy states exp. grows with the number of vortices $2^{(N_v/2-1)}$

Non-abelian statistics $\Rightarrow$ Exchanging vortices creates a different state!

Non-local character of the state. Local perturbation does not cause decoherence

Topologically protected state for quantum information processing

p-wave resonance for fermionic atoms

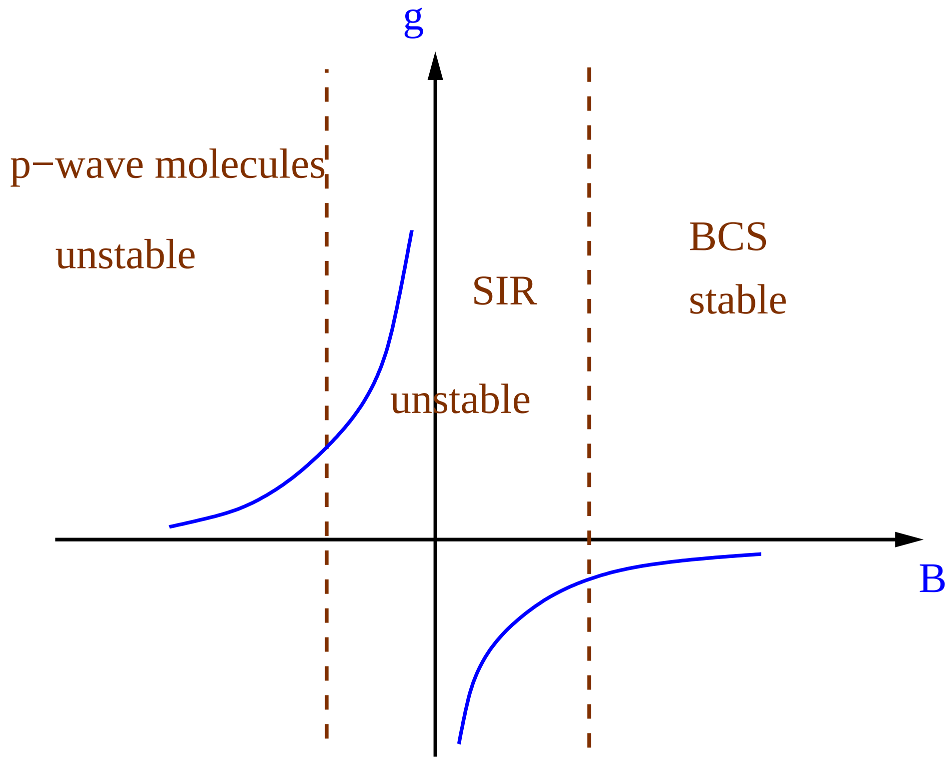
p-wave resonance $\Rightarrow$ JILA, ENS, Melbourne, Tokyo, elsewhere

BCS $\Rightarrow$ $T_c \sim \exp\left(-\frac{1}{(k_F b)^2}\right)$ practically zero

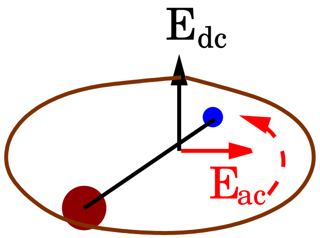
Molecular and strongly interacting regimes $\Rightarrow$ rather high T_c

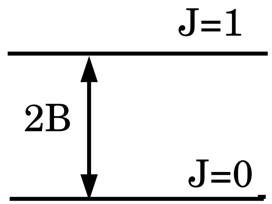
but collisional instability

Gurarie/Radzihovsky; Gurarie/Cooper; Castin/Jona-Lazinio



RF-dressed polar molecules in 2D. Innsbruck idea





σ^+ polarized RF E_{ac}

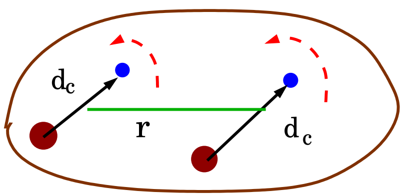
+ constant $E_{dc} \longrightarrow 0$

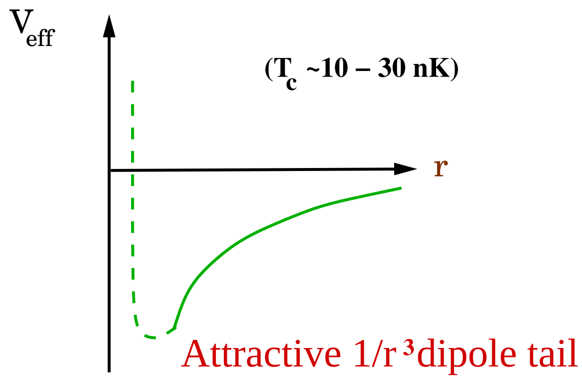
Dressed states $\quad |+\rangle = \alpha|0,0\rangle + \beta|1,1\rangle; \quad ,|-\rangle = \beta|0,0\rangle - \alpha|1,1\rangle$

$$\alpha = -\frac{A}{\sqrt{A^2+\Omega^2}}; \quad \beta = \frac{\Omega}{\sqrt{A^2+\Omega^2}}; \quad A = \frac{1}{2}(\delta + \sqrt{\delta^2 + 4\Omega^2})$$

Two RFD molecules in 2D. The dipole moment is rotating with RF frequency

N.R. Cooper, J. Levinsen, and G.V. Shlyapnikov
PRL and PRA, 2009−2011





Microwave−dressed polar molecules

Topological $p_x + ip_y$ superfluid phase promising for quantum computing

$$\text{Large } r \rightarrow \quad V_{eff} = \langle (1 - 3\cos^2\phi) \rangle \frac{d_c^2}{r^3} = -\frac{d_c^2}{2r^3}; \quad r_* = m d_c^2 / 2\hbar^2$$

Fermionic RFD molecules. Superfluid transition

Fermionic RFD molecules in a single quantum state in 2D

Attractive interaction for the p-wave scattering ($l = \pm 1$)

$$\hat{H} = \int d^2r\, \hat{\Psi}^\dagger(\mathbf{r})\{-(\hbar^2/2m)\Delta + \int d^2r' \hat{\Psi}^\dagger(\mathbf{r}')V_{eff}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}') - \mu\}\hat{\Psi}(\mathbf{r})$$

$$\Delta(\mathbf{r}-\mathbf{r}') = \langle V_{eff}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}')\rangle$$

Gap equation $\Delta(\mathbf{k}) = -\int \frac{d^2k}{(2\pi)^2} V_{eff}(\mathbf{k}-\mathbf{k}')\Delta(\mathbf{k}')\frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')}$

$$\epsilon(k) = \sqrt{(\hbar^2k^2/2m - \mu)^2 + |\Delta(k)|^2}; \quad \mu \approx E_F$$

$$T_c \approx E_F \exp(-3\pi/4k_F r_*)$$

$\Delta(\mathbf{k}) = \Delta \exp(i\phi_k)$ $\quad p_x + ip_y$ state ($l = \pm 1$)

Superfluid transition. Role of anomalous scattering

For short-range potentials should be $V_{eff} \propto k^2$ and $T_c \propto \exp(-1/(k_F b)^2)$
This is the case for the atoms

Anomalous scattering in $1/r^3$ potential $\rightarrow$ Contribution from $r \sim 1/k$

$$V_{eff}(k) = -\frac{8\hbar^2}{3m}(kr_*); \qquad |k| = |k'|$$

$$T_c \propto \exp\left(-\frac{1}{\nu(k_F)|V_{eff}(k_F)|}\right); \qquad \nu = \frac{m}{2\pi\hbar^2}$$

$$T_C \propto \exp\left(-\frac{3\pi}{4k_F r_*}\right)$$

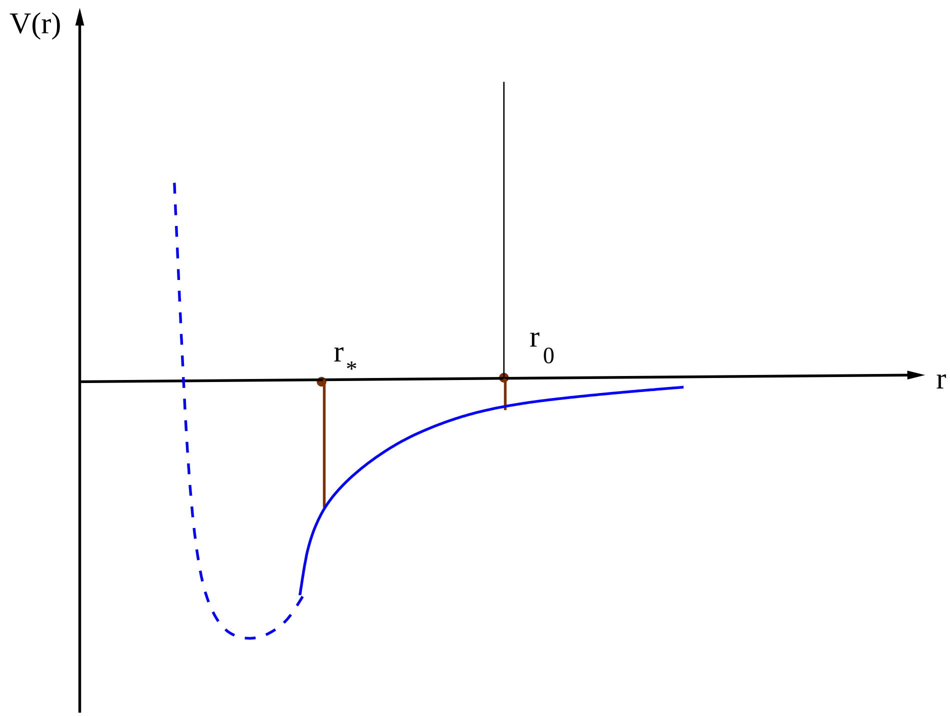
Transition temperature

Do better than simple BCS. Reveal the role of short-range physics

Renormalized gap equation

$$\Delta(\mathbf{k}') = -\int f(\mathbf{k}',\mathbf{k})\Delta(\mathbf{k})\left\{\frac{\tanh[\epsilon(k)/2T]}{2\epsilon(k)} - \frac{1}{(E_k - E_{k'} - i0)}\right\}\frac{d^2k}{(2\pi)^2}$$

$\Delta(\mathbf{k}) = \Delta(k)\exp(i\phi_k)$; $f(\mathbf{k}',\mathbf{k}) = f(k',k)\exp[i(\phi_k - \phi_{k'})]$ scattering amplitude



$$f = -(8\pi/3)d^2k + (\pi/2)d^2r_*k^2\ln ukr_*$$

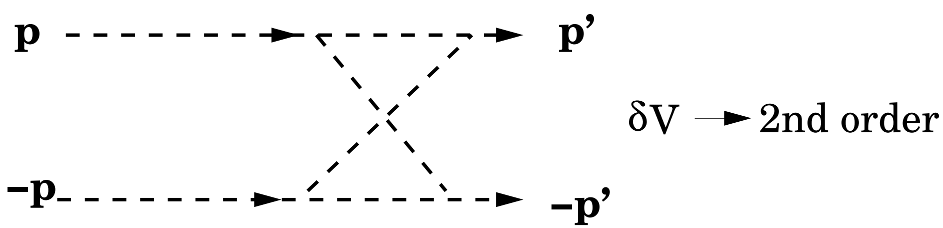
Related results for the off-shell scattering amplitude

Manipulate T_c?

Put $f(k', k)$ and include k^2-term $f = \frac{1}{2}\pi d^2 r_* k^2 \ln[k r_* u]$

$$T_c = \frac{2e^C}{\pi} E_F \exp\left\{-\frac{3\pi}{4k_F r_*} - \frac{9\pi^2}{64}\ln[k_F r_* u]\right\}$$

Take into account second-order Gor'kov-Melik-Barkhudarov processes



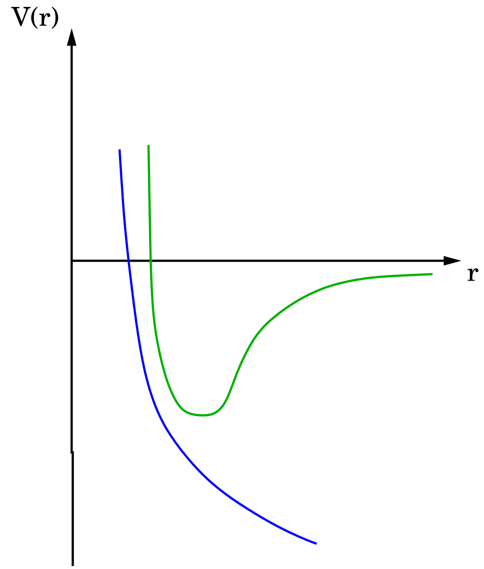
$$T_c = \kappa E_F^{0.3} E_*^{0.7} \exp\left\{-\frac{3\pi}{4k_F r_*}\right\}; \quad E_* = \frac{\hbar^2}{2m r_*^2} \gg E_F$$

κ depends on short-range physics and can be varied within 2 orders of magnitude

Collisional stability and T_c

p-wave atomic superfluids: $\mathrm{BCS} \Rightarrow T_c \to 0$ Resonance $\Rightarrow$ collisional instability

Polar molecules $\Rightarrow$ sufficiently large T_c and collisional stability



$\alpha_{in} = A\frac{\hbar}{m}(kr_*)^2;$ $A \Rightarrow 10^{-3} - 10^{-4}$ $\alpha_{in} \to (10^{-8} - 10^{-9})$ cm^2/s

LiK molecules $\to$ $d \simeq 3.5$ D $r_* \approx 4000 a_0$

$n = 2 \times 10^8$ cm^{-2} $\Rightarrow$ $E_F = 2\pi\hbar^2 n/m = 120$ nK $T_c \approx 10$ nK; $\tau \sim 2$s

Papers to look in

Stable Topological Superfluid Phase
of Ultracold Polar Fermionic Molecules

N. R. Cooper and G. V. Shlyapnikov

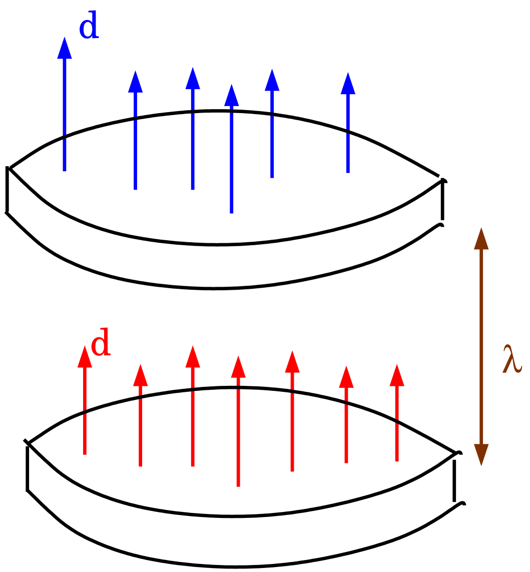
Phys. Rev. Lett. 103, 155302 (2009)

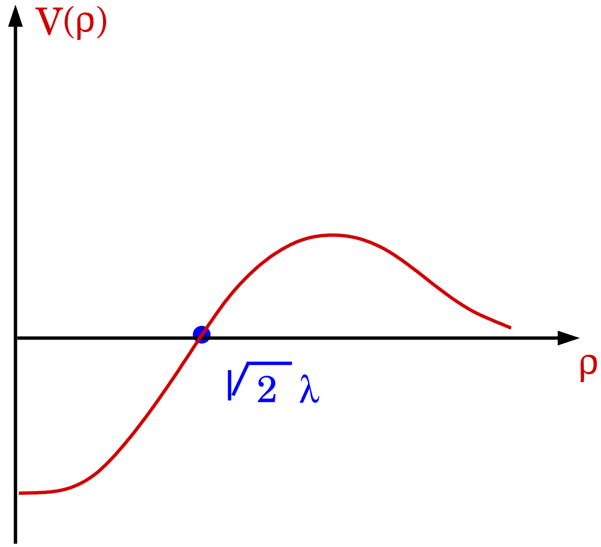
Topological px+ipy superfluid phase of fermionic polar molecules

J. Levinsen, N. R. Cooper, and G. V. Shlyapnikov

Phys. Rev. A 84, 013603 (2011)

Bilayered dipolar fermionic systems. BCS-BEC crossover





$$V(\rho) = d^2 \left\{ \frac{1}{(\rho^2+b^2)^{3/2}} - \frac{3b^2}{(\rho^2+b^2)^{5/2}} \right\}$$ Always a bound state of ↑ and ↑ dipoles

Dipole-dipole length $r_* = md^2/\hbar^2$ Dipole-dipole strength r_*/b.

$$r_* \le b \Rightarrow \quad \epsilon_b \simeq \frac{\hbar^2}{4mb^2} \exp\left[-\frac{8b^2}{r_*^2}\left(1-\frac{r_*}{b}\right) - (5+2\gamma) \right]$$

$\epsilon_b \ll E_F \Rightarrow f < 0 \rightarrow$ s-wave BCS pairing

$\epsilon_b \gg E_F \Rightarrow$ Molecules of ↑ and ↑ dipoles. Molecular BEC

New BCS-BEC crossover (Pikovski, Klawunn, Santos, GS)

Transition temperature

Kosterlitz-Thouless transition $\epsilon_b \ll E_F \to T_{KT}$ is close to T_{BCS}

$k_F r_* \ll r_*^2/b^2 \to$ Short-range contribution $f = -4\pi\hbar^2/m \ln(E_F/\epsilon_b)$

$$T_{KT} \simeq \frac{e^\gamma}{\pi}\sqrt{2E_F\epsilon_b}$$

$k_F r_* \gg r_*^2/b^2 (b \gg r_*) \to$ Anomalous scattering wins

$$f(k) = \frac{\hbar^2}{m}\left\{-8kr_* - \frac{\pi r_*^2}{2b^2} + 4\pi k^2 r_* b + 3\pi(kr_*)^2 \ln \zeta kb\right\}; \quad kb \ll 1 \ \ \zeta \approx 6$$

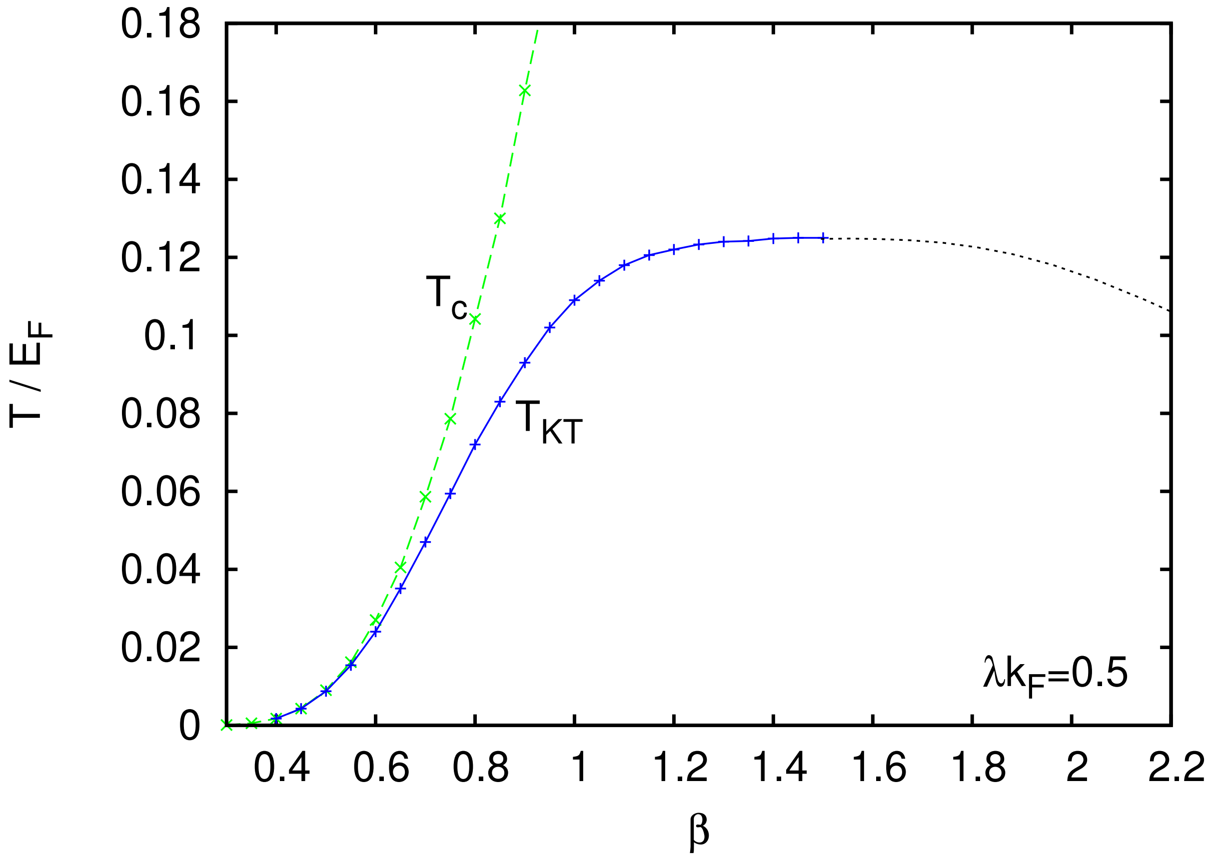
$$T_{KT} \simeq 0.1\left(\frac{E_0}{E_F}\right)^{0.46} \exp\left\{-\frac{\pi}{4k_F r_*} G(k_F b, r_*/b)\right\}$$

$$E_0 = \hbar^2/mb^2; \ \ G(x,y) = (1 - \pi x/2 + \pi y/16x)^{-1}$$

$E_F \ll \epsilon_b \to$ Formation of bound pairs by fermions of different layers

T_{KT} of a weakly interacting Bose gas

Transition temperature



LiCs and KRb molecules $b \simeq 250$ nm, $n \simeq 5\,10^8$ cm^{-2}, $k_F b \simeq 2$, $E_F \simeq 110$ nk

$\Rightarrow T_{KT}$ of a few nanokelvin

Papers to look in

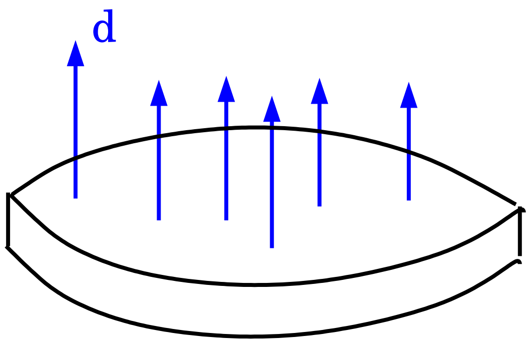
Interlayer Superfluidity in Bilayer Systems of Fermionic Polar Molecules

A. Pikovski, M. Klawunn, G. V. Shlyapnikov, and L. Santos

Phys. Rev. Lett. 105, 215302 (2010)

Fermi liquid behavior

Can one do something interesting with a ”simple” system of $\uparrow$ polar molecules in 2D?



Suppressed ultracold chemistry

Novel Fermi liquid?

’Old” question of beyond mean field effects for weak interactions

Short-range weak repulsion. Two-component Fermi gas

Lee/Huang/Yang and Abrikosov/Khalatnikov

Milestone in this direction $\Rightarrow$ recent ENS experiment (Nature, 2010)

$$\frac{E}{N} = \frac{3\hbar^2 k_F^2}{10m}\left[1 + \frac{10}{9\pi}k_F a + \frac{4(11 - 2\ln 2)}{21\pi^2}(k_F a)^2\right]$$

Measurement of $P = -\partial E/\partial V$. Recover the non-mean field correction

(for $k_F a = -1$)

Novel Fermi liquid

Single-component polar $\uparrow$ molecules

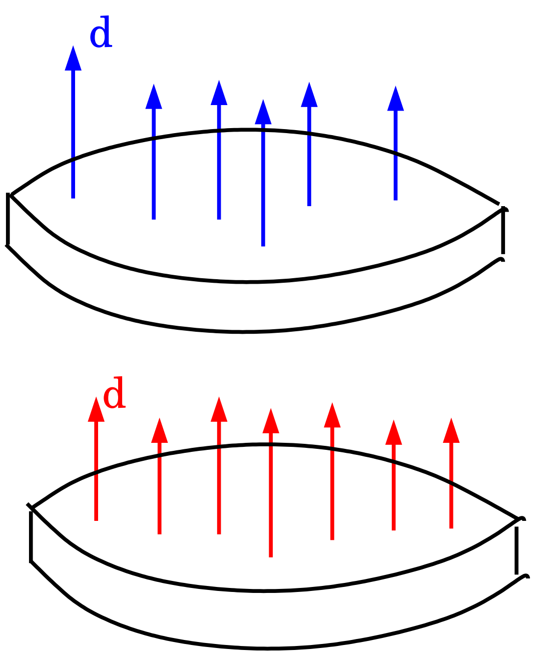
$$\frac{E}{N} = \frac{\hbar^2 k_F^2}{4m}\left[1 + \frac{128}{45\pi} k_F r_* + \frac{1}{4}(k_F r_*)^2 \ln u k_F r_*\right]$$

$u \simeq 1.13$ and $r_* = md^2/\hbar^2$

$k_F r_* \sim 0.5 \Rightarrow$ measure all terms

Can one make a two-component Fermi gas of $\uparrow$ polar molecules?

Single layer $\Rightarrow$ ultracold chemistry may come into play



Bilayer system

Can be made imbalanced

Papers to look in

Fermi liquid of two-dimensional polar molecules

Zhen-Kai Lu and G. V. Shlyapnikov

Phys. Rev. A 85, 023614 (2012)

Conclusions

Creation of ultracold polar molecules opens
wide avenues to make new quantum states

- $p_x + ip_y$ topological state for identical fermions
- BCS-BEC crossover in bilayered fermionic dipolar systems
- Novel Fermi liquid in 2D