Dark Matter and Vector fields in non-trivial representations of SU(2)\_L

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I. Why consider vector fields?

II. A vector resonance in the Dark Sector

III. Spin-1 Dark Matter
   a) Many possibilities
   b) Vectors in the fundamental of SU(2)_L
   c) Vectors in the adjoint of SU(2)_L

IV. Conclusions
Why consider vector fields?
Why consider vector fields?

- Vector fields are wild animals

- Usually they are tamed introducing gauge symmetries (like in the SM)

- However vectors play different roles in Nature

  - In the hadronic sector we have some examples:
    - Rho-meson can be seen as an emergent gauge boson (HLS)
    - The a1 axial-vector behaves like a matter field (transforming homogeneously under HLS)

- Of course this example comes from an effective non-fundamental sector, but nothing prevents its realization in other (dark?) sectors of Nature
A Vector Resonance in the Dark Sector

Based on:
• Chin.Phys. C43 (2019), 063102 C. Callender, AZ
IDM + Vector Resonance

Model Based on $SU(2)_1 \times SU(2)_2 \times U(1)_Y$

Symmetry Breaking Pattern

Hypothesis: $g_1 \ll g_2$

$SU(2)_L$

Inert doublet
### Spectrum

<table>
<thead>
<tr>
<th>$\rho_{\mu}^{0,\pm}$</th>
<th>$M_\rho = 2 - 4.5$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h^{\pm}$</td>
<td></td>
</tr>
<tr>
<td>$h_1$</td>
<td>DM candidate</td>
</tr>
<tr>
<td>$h_2$</td>
<td></td>
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<tr>
<td>$H$</td>
<td>Standard-like Higgs</td>
</tr>
</tbody>
</table>
# Parameters

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \frac{u}{v} )</th>
<th>( \alpha = 3, 4, 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_2 )</td>
<td></td>
<td>( \phi_2 ) Self Interaction</td>
</tr>
<tr>
<td>( \lambda_{345} )</td>
<td>( \lambda_3 + \lambda_4 + \lambda_5 )</td>
<td>Higgs-New Scalars interaction</td>
</tr>
<tr>
<td>( M_\rho )</td>
<td></td>
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<tr>
<td>( M_{h^\pm} )</td>
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<tr>
<td>( M_{h1} )</td>
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<tr>
<td>( M_{h2} )</td>
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</tbody>
</table>
Relic density

\[ M_{h_2} = M_{h^\pm} = M_{h_1} + 1 \text{ GeV}, \ M_{\rho} = 3000 \text{ GeV}, \ a = 2 \]

Relevant parameters:
- \( \lambda_{345} = 1 \)
- \( \lambda_{345} = 0.1 \)
- \( \lambda_{345} = 0.01 \)

Graph shows the relic density \( \Omega h^2 \) as a function of \( M_{h_1} \) (GeV).
Relic density

\[ M_{h_2} = M_{h^\pm} = M_{h_1} + 100 \text{ GeV}, \ M_p=3000 \text{ GeV}, \ a=2 \]

Graph showing relic density \( \Omega h^2 \) vs. \( M_{h_1} \) (GeV) with different values of \( \lambda_{345} \):
- \( \lambda_{345} = 1 \)
- \( \lambda_{345} = 0.1 \)
- \( \lambda_{345} = 0.01 \)

The graph indicates the relic density for different values of \( \lambda_{345} \) with a horizontal line at \( \Omega h^2 = 10^{-1} \) as a reference.
Relic density
Relic density
Neutral vector production at LHC

![Graph showing the production cross-section of neutral vector bosons](image)

- $\sigma(pp \rightarrow B^0)$ (fb)
- $M_B$ (GeV)

- Blue dashed line: $a=3$
- Red line: $a=4$
- Green dotted line: $a=5$
Mono-Z at the LHC
Mono-Z at the LHC
Resonance Searches at CLIC (Heavy Scalars)

\[ M_{h_1}, M_{h_2}, M_{h\pm} > M_\rho / 2 \]

\[ e^+ e^- \rightarrow \mu^+ \mu \]

\[ \sqrt{s} = 3 \text{ TeV} \]
Resonance Searches at CLIC (Heavy Scalars)

\[ M_{h_1}, M_{h_2}, M_{h\pm} > M_\rho / 2 \]

\[ e^+ e^- \rightarrow \mu^+ \mu \]
Mono-Z at CLIC
Vector Dark Matter
Vector Dark Matter
Vector Dark Matter Candidates

- Kaluza-Klein photon
- T-odd heavy photon in Little Higgs with T-parity
- (Abelian) $Z'$ with $Z_2$ symmetry
- Vector part of propagating torsion [A. Belyaev and I. Shapiro]
- Vectors fields in SU(2)$_L$ representations
Minimal Isotriplet Vector Dark Matter

Based on:
• Phys.Rev. D99 (2019), 115003  A.Belyaev, G. Cacciapaglia, J. McKey, D. Marin and AZ
The Lagrangian is consistent with perturbative unitarity at tree level due to two crucial facts:

- It has an accidental $\mathbb{Z}_2$ symmetry
- There is only one coupling constant: the gauge coupling constant

\[ \mathcal{L} = -\frac{1}{2} Tr \left\{ G_{\mu\nu} G^{\mu\nu} \right\} - Tr \left\{ D_\mu V_\nu D^\mu V^\nu \right\} + Tr \left\{ D_\mu V_\nu D^\nu V^\mu \right\} - \frac{g^2}{2} Tr \left\{ \left[ V_\mu, V_\nu \right] \left[ V^\mu, V^\nu \right] \right\} - ig Tr \left\{ G_{\mu\nu} \left[ V^\mu, V^\nu \right] \right\} + M^2 Tr \left\{ V_\nu V^\nu \right\} \]

$V$ is a massive vector field in the adjoint representation of a local SU(N)

This Lagrangian is consistent with perturbative unitarity at tree level due to two crucial facts:

- It has an accidental $\mathbb{Z}_2$ symmetry
- There is only one coupling constant: the gauge coupling constant

\[ G_\mu \rightarrow UG_\mu U^{-1} - \frac{1}{g} (\partial_\mu U) U^{-1} \]

\[ V_\mu \rightarrow UV_\mu U^{-1} \]
A simple rotation

\[ G = \frac{1}{\sqrt{2}} (A1 + A2) \]
\[ V = \frac{1}{\sqrt{2}} (A1 - A2) \]

\[ \mathcal{L} = -\frac{1}{2} Tr \left[ F_{1 \mu \nu} F_{1 \mu \nu}^{\mu \nu} \right] - \frac{1}{2} Tr \left[ F_{2 \mu \nu} F_{2 \mu \nu}^{\mu \nu} \right] + \frac{M^2}{2} Tr \left[ (A_{1 \mu} - A_{2 \mu})^2 \right] \]

\[ F_{1 \mu \nu} = \partial_{\mu} A_{1 \nu} - \partial_{\nu} A_{1 \mu} - i \sqrt{2} g [A_{1 \mu}, A_{1 \nu}] \]

Same coupling constant
(pseudo-) T-parity

\[ F_{2 \mu \nu} = \partial_{\mu} A_{2 \nu} - \partial_{\nu} A_{2 \mu} - i \sqrt{2} g [A_{2 \mu}, A_{2 \nu}] \]

\[ A_{i \mu} \rightarrow U A_{i \mu} U^{-1} - \frac{1}{\sqrt{2} g} (\partial_{\mu} U) U^{-1} \quad (i = 1, 2) \]
This is the Yang-Mills analog of Bigravity  (Thanks to Max Bañados)

\[ S = - \frac{M_g^2}{2} \int d^4x \sqrt{-g} R(g) - \frac{M_f^2}{2} \int d^4x \sqrt{-f} R(f) + m^2 M_g^2 \int d^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(X) - \]

\[ X = \sqrt{g^{-1} f} \]
\[ \mathcal{L} = \mathcal{L}_{SM} - Tr \left\{ D_\mu V_\nu D^\mu V^\nu \right\} + Tr \left\{ D_\mu V_\nu D^\nu V^\mu \right\} - \frac{g^2}{2} Tr \left\{ [V_\mu, V_\nu] [V^\mu, V^\nu] \right\} - ig Tr \left\{ W_{\mu\nu} [V^\mu, V^\nu] \right\} + \tilde{M}^2 Tr \left\{ V_\nu V^\nu \right\} + \alpha (\Phi^\dagger \Phi) Tr \left\{ V_\nu V^\nu \right\} \]

\( D_\mu \) is the covariant derivative of SU(2) in the adjoint representation

Higgs doublet

Only two free parameters: \( a \) and \( M \)

\[ a^{1-loop} = -3 \frac{\alpha^2}{\sin \theta_W^4} \ln \frac{\Lambda}{M_V} \approx -0.0037 \ln \frac{\Lambda}{M_V} \]
Unitarity

\[ \sigma_l(k) \leq \frac{4\pi (2l + 1)}{k^2} \]

\[ \Lambda \approx \frac{8\sqrt{\pi} M_V^2}{\sqrt{4a^2 v^2 + 3g^2 M_W^2}} \]
Unitarity violation scale

Unitarity violation scale, $\Lambda(\text{TeV})$

$M_V$ (GeV)

$a$

$10^{-2}$ $10^{-1}$ $10^{0}$ $10^{1}$ $10^{2}$ $10^{3}$ $10^{4}$ $10^{5}$
Radiative corrections produce mass splitting

\[ \Delta M \approx 210 \text{MeV} \]

\[ V^\pm \rightarrow V^0 e^\pm \nu \quad V^\pm \rightarrow V^0 \pi^\pm \]
Lifetime of the charged components
Interaction between $V^0$ and quarks (low energy)
Experimental Constrains
Relic density

\[ \Omega_{DM} h^2 \]

\[ M_V \text{ (GeV)} \]

- $a = 0$
- $a = 0.5$
- $a = 1$
- $a = 5$
- $a = 4\pi$

Unitarity constraints

$\Omega_{DM} h^2 = \Omega_{\text{PLANCK}} h^2$

$\Omega_{DM} h^2 > \Omega_{\text{PLANCK}}^{\max} h^2$
Direct detection constraints

\[ \sigma_{\text{SI}}^p \] (pb)

\[ M_V \text{ (GeV)} \]

- \( a = 0.1 \)
- \( a = 0.5 \)
- \( a = 1 \)
- \( a = 5 \)
- \( a = 4\pi \)
- XENON1T(2018)
- Unitarity constraints
- \( \Omega_{\text{DM}}h^2 = \Omega_{\text{PLANCK}}h^2 \)
$a = +/- 1$ (dashed) and $a = +/- 5$

The colour code uses red for positive values and blue for negative. The coloured bands are the experimentally allowed regions at 95 % CL from ATLAS (pink) and CMS (yellow), while the orange band shows the overlap.
$R_{SI} < 1, \Omega_{DM} h^2 \leq \Omega h^2_{\text{PLANCK}} h^2, \mu_{\gamma\gamma} \leq \mu_{\gamma\gamma} \leq \mu_{\gamma\gamma}^{\max}$
$R_{SI} < 1$, $\Omega_{\text{PLANCK}}^{\min} h^2 \leq \Omega_{\text{DM}} h^2 \leq \Omega_{\text{PLANCK}}^{\max} h^2$, $\mu_{\gamma\gamma}^{\min} \leq \mu_{\gamma\gamma} \leq \mu_{\gamma\gamma}^{\max}$

![Graph showing a relationship between $a$ and $M_V$ (GeV)]
Indirect Detection Constraints

\[ \langle \sigma v \rangle \text{ (cm}^3\text{s}^{-1}) \]

\[ M_V \text{ (GeV)} \]

- \[ a = 0.1 \]
- \[ a = 0.5 \]
- \[ a = 1 \]
- \[ a = 5 \]
- \[ a = 4\pi \]

\[ \Omega_{DM}h^2 = \Omega_{PLANCK}h^2 \]

- HESS (Einasto)
- HESS (Einasto2)

Unitarity constraints
What about the LHC?

\[ \sigma_{\text{eff}} = \sigma(pp \rightarrow V^\pm V^0) + 2\sigma(pp \rightarrow V^+ V^-) \]
Dark Matter from a Vector Field in the Fundamental Representation of SU(2)_L

Based on:
It is not possible to couple $V$ to standard fermions without introducing exotic vector-like fermions.

Very nice but already invented by M.V. Chizhov and G. Dvali, PLB 703 (2011) 593
LEP

\begin{figure}
\centering
\includegraphics[width=\textwidth]{lep.png}
\caption{LEP II excluded region}
\end{figure}

- $M_{V1}$ (GeV)
- $M_{\nu}$ (GeV)
$M_{V_2} = M_{V^\pm} = M_{V_1} + 1$ GeV

**XENON1T**

Excluded by LEP

$\hat{\sigma}_S (pb)$

$M_{V_1}$ (GeV)
$M_{V_2} = M_{V^\pm} = M_{V_1} + 100 \text{ GeV}$

$\hat{\sigma}_{SI} (\text{pb})$

$M_{V_1} (\text{GeV})$
Conclusions

- Vectors fields can play crucial roles in the dark sector
- We built extensions of the SM incorporating the new vector matter field in SU(2) multiplets and the models contain natural dark matter candidates.
- In both models a new $\mathbb{Z}_2$ symmetry appears motivated by theory and not imposed by hand (specially in the isotriplet case).
- The models are consistent with collider and cosmological data
- A Vector in the Fundamental Representation offers a nice DM candidate but the model is strongly challenged by data and by unitarity constrains
- The model with isotriplet vector fields seems to be more robust.
- The models can be completely rule out (or discovered) at a 100 TeV collider
People’s demand....

We want a 100 TeV collider NOW
Backup
Intriguing Questions

- How do we construct a consistent theory containing only a gauge boson and a massive vector field?

- Is it possible to avoid the introduction of scalar fields?
Poor Man (phenomenologist) Approach

\[ \mathcal{L} = -\frac{1}{2} \text{Tr} \{ G_{\mu\nu} G^{\mu\nu} \} - \text{Tr} \{ D_\mu V_\nu D^\mu V^\nu \} \\
+ (1 + a) \text{Tr} \{ D_\mu V_\nu D^\nu V^\mu \} \\
+ a_1 \text{Tr} \{ (D_\mu V_\nu - D_\nu V_\mu) [V^\mu, V^\nu] \} \\
+ \frac{a_2}{2} \text{Tr} \{ [V_\mu, V_\nu] [V^\mu, V^\nu] \} \\
+ ia_3 \text{Tr} \{ G_{\mu\nu} [V^\mu, V^\nu] \} + M^2 \text{Tr} \{ V_\nu V^\nu \} \]

\[ G_\mu \rightarrow U G_\mu U^{-1} - \frac{1}{g} (\partial_\mu U) U^{-1} \]

\[ V_\mu \rightarrow U V_\mu U^{-1} \]

Compute \[ V_L V_L \rightarrow V_L V_L \quad GG \rightarrow V_L V_L \]

\[
\mathcal{M} = \frac{(a_2 + g^2 + a_1^2) (t^2 - 2tu - 2u^2)(t + u)^2 i tu}{4s (M^2 - s)(M^2 - t)(M^2 - u) M^4} f_{abc} f_{cd} e \\
+ \frac{(a_2 + g^2 + a_1^2) (t^2 + 4tu + u^2)(t + u)^2 i tu}{4s (M^2 - s)(M^2 - t)(M^2 - u) M^4} f_{ace} f_{bde} \\
- \frac{(a_2 + g^2) (t^4 + 11t^3u - 23t^2u^2 - 28tu^3 - 2u^4)(t + u) i}{4s (M^2 - s)(M^2 - t)(M^2 - u) M^2} f_{abe} f_{cde} \\
- \frac{6i(t + u)^2 ag^2 t^2 u}{4s (M^2 - s)(M^2 - t)(M^2 - u) M^2} f_{abe} f_{cde} \\
- \frac{(t^4 + 14t^3u - 20t^2u^2 - 28tu^3 - 2u^4) a_1^2 (t + u) i}{4s (M^2 - s)(M^2 - t)(M^2 - u) M^2} f_{abe} f_{cde} \\
- \frac{(t^4 + 14t^3u + 40t^2u^2 + 14tu^3 + u^4) a_1^2 (t + u) i}{4s (M^2 - s)(M^2 - t)(M^2 - u) M^2} f_{ace} f_{bde} \\
+ \frac{6i(t + u)^3 ag^2 tu}{4s (M^2 - s)(M^2 - t)(M^2 - u) M^2} f_{ace} f_{bde} \\
- \frac{(a_2 + g^2) (t^4 + 17t^3u + 46t^2u^2 + 17tu^3 + u^4)(t + u) i}{4s (M^2 - s)(M^2 - t)(M^2 - u) M^2} f_{ace} f_{bde} \\
+ \mathcal{O} \left( \frac{M^2}{s} \right)
If we choose:

\[
\begin{align*}
a &= 0 \\
a_1 &= 0 \\
a_2 &= -g^2 \\
a_3 &= -g
\end{align*}
\]

The unitary-violating terms disappear

\[
\mathcal{M}(V_L V_L \rightarrow V_L V_L) = 0
\]

\[
\mathcal{M}(G G \rightarrow V_L V_L) = \text{constant} + \mathcal{O}\left(\frac{M^2}{s}\right)
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\]
This Lagrangian is consistent with perturbative unitarity at tree level due to two crucial facts:

- It has an accidental $Z_2$ symmetry
- There is only one coupling constant: the gauge coupling constant

$V$ is a massive vector field in the adjoint representation of a local SU(N)
A simple rotation

\[ G = \frac{1}{\sqrt{2}} (A_1 + A_2) \]

\[ V = \frac{1}{\sqrt{2}} (A_1 - A_2) \]

\[ \mathcal{L} = -\frac{1}{2} Tr [F_{1\mu\nu} F_{1\mu\nu}^\ast] - \frac{1}{2} Tr [F_{2\mu\nu} F_{2\mu\nu}^\ast] + \frac{M^2}{2} Tr [(A_{1\mu} - A_{2\mu})^2] \]

\[ F_{1\mu\nu} = \partial_\mu A_{1\nu} - \partial_\nu A_{1\mu} - i \sqrt{2} g [A_{1\mu}, A_{1\nu}] \]

Same coupling constant (pseudo-) T-parity

\[ F_{2\mu\nu} = \partial_\mu A_{2\nu} - \partial_\nu A_{2\mu} - i \sqrt{2} g [A_{2\mu}, A_{2\nu}] \]

\[ A_{i\mu} \rightarrow U A_{i\mu} U^{-1} - \frac{1}{\sqrt{2} g} (\partial_\mu U) U^{-1} \quad (i = 1, 2) \]
This is the Yang-Mills analog of Bigravity (Thanks to Max Bañados)

\[
S = -\frac{M_g^2}{2} \int d^4x \sqrt{-g} R(g) - \frac{M_f^2}{2} \int d^4x \sqrt{-f} R(f) + m^2 M_g^2 \int d^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(X) -
\]

\[
X = \sqrt{g^{-1} f}
\]

A simple rotation

\[
G = \frac{1}{\sqrt{2}} (A1 + A2)
\]

\[
V = \frac{1}{\sqrt{2}} (A1 - A2)
\]

Bigauge theory
Let’s start from the gauge-like representation

\[
\mathcal{L} = -\frac{1}{2} \text{Tr} [F_{1\mu\nu} F_{1}^{\mu\nu}] - \frac{1}{2} \text{Tr} [F_{2\mu\nu} F_{2}^{\mu\nu}] + \frac{M^2}{2} \text{Tr} \left[ (A_{1\mu} - A_{2\mu})^2 \right]
\]

We have to reproduce this particular structure

The idea is to describe both vector fields as gauge fields and to break down the symmetry to SM
$SU(2)_1 \quad \text{Same coupling constant!} \quad SU(2)_2$

$u = \langle \Phi \rangle \neq 0$

$SU(2)_L$

$\Phi \rightarrow U_1 \Phi U_2^\dagger$

$D\Phi = \partial\Phi - i \frac{g}{\sqrt{2}} A_1 \Phi + i \frac{g}{\sqrt{2}} \Phi A_2$

$\Phi = \frac{u + \tilde{\sigma}(x)}{\sqrt{2}} \Sigma$

Same coupling constant! (T-parity)
But...

- SU(2) x SU(2) has the same algebra than SO(4)
- There is only one coupling constant
- Maybe the underlying theory is based on $SO(4)_L \times U(1)_Y$
A simple idea for coupling the matter fields
(toy model)

- Make the standard left-handed fermions doublets of both SU(2)'s
- Make the SM Higgs field doublet of both SU(2)'s

\[ D\psi_L = \partial\psi_L - i \frac{g}{\sqrt{2}} A_1 \psi_L - i \frac{g}{\sqrt{2}} A_2 \psi_L - ig'Y B\psi_L \]

\[ D\varphi = \partial\varphi - i \frac{g}{\sqrt{2}} A_1 \varphi - i \frac{g}{\sqrt{2}} A_2 \varphi - ig'Y B\varphi \]

In this way, the standard fields only couple to the even combination

\[ W = \frac{1}{\sqrt{2}} (A_1 + A_2) \]
Scalar Potential

\[
V(\Phi, \varphi) = \frac{\mu^2}{2} \text{Tr} [\Phi^\dagger \Phi] + \frac{\lambda_2}{4} \text{Tr} [\Phi^\dagger \Phi]^2 \\
+ \mu_1^2 \varphi^\dagger \varphi + \lambda_1 (\varphi^\dagger \varphi)^2 \\
+ \frac{\lambda_3}{2} (\varphi^\dagger \varphi) \text{Tr} [\Phi^\dagger \Phi]
\]

\[
m_\sigma \approx \sqrt{2 \lambda_2} u \left(1 + \frac{\lambda_3^2 v^2}{32 \lambda_2^2 u^2}\right)
\]

\[
m_h \approx \sqrt{2 \lambda_1 - \frac{\lambda_3^2}{8 \lambda_2}} v
\]
Some prediction and results

- As $M_V$ is about 3-4 TeV, the scale of the new symmetry breaking has to be about 10 TeV
- $\lambda_1$ has to be larger than in the SM
- The $a$ parameter (controlling the hhVV interaction) is small because it is generated only through mixings
- The model preserves unitarity