The Top Quark Mass at Lepton Colliders

and the Inclusive Top Quark Pair Production Cross Section from Threshold to Continuum

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Workshop on Determination of Fundamental QCD Parameters
02/10/2019

Outline

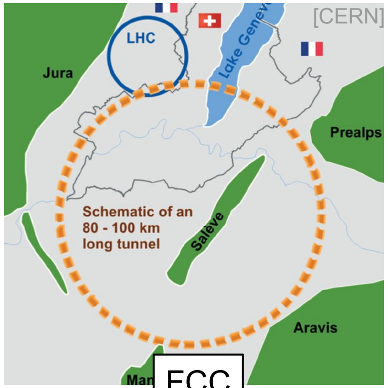
- Top Quark Mass Determination at Lepton Colliders
  - Overview
  - Threshold Scan
  - Radiative Events [Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, AW, Marcel Vos]
- Top Quark Pair Production Cross Section from Threshold to Continuum
  [Dehnadi, Hoang, Mateu, Stahlhofen, AW]
  - Threshold Region
  - Continuum Region
  - Mass Schemes at and above Threshold
  - Matching at $\mathrm{NNLL}_{\mathrm{threshold}} + \mathrm{NNNLO}_{\mathrm{continuum}}$
- Conclusions

Top Quark Mass Determination at Lepton Colliders

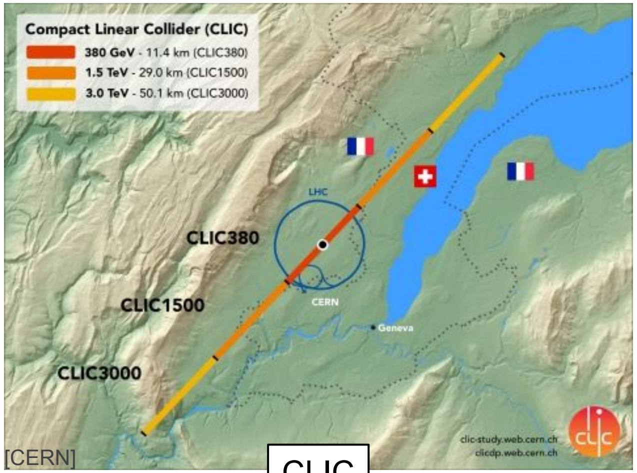
Overview

lepton colliders

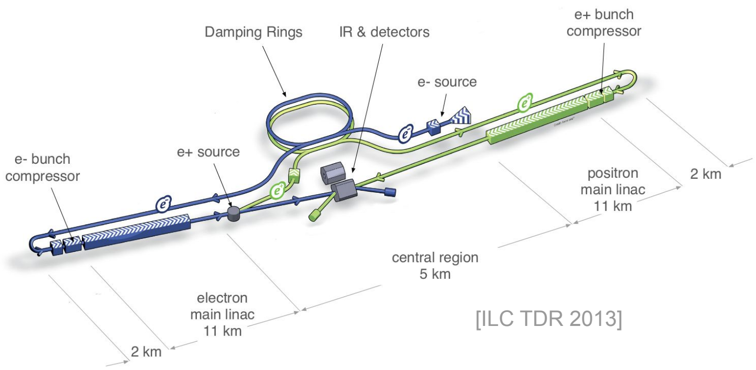
LEP (CERN)	1989 – 2000	209 GeV	
ILC (Japan)	proposed	250 GeV – 1000 GeV	[Baer et al. 2013]
CLIC (CERN)	proposed	350 GeV – 3000 GeV	[CLIC collaboration 2016]
CEPC (China)	proposed	91 GeV – 240 GeV	[CEPC study group 2018]
FCC-ee (CERN)	proposed	90 GeV – 365 GeV	[FCC collaboration 2019]



FCC



CLIC

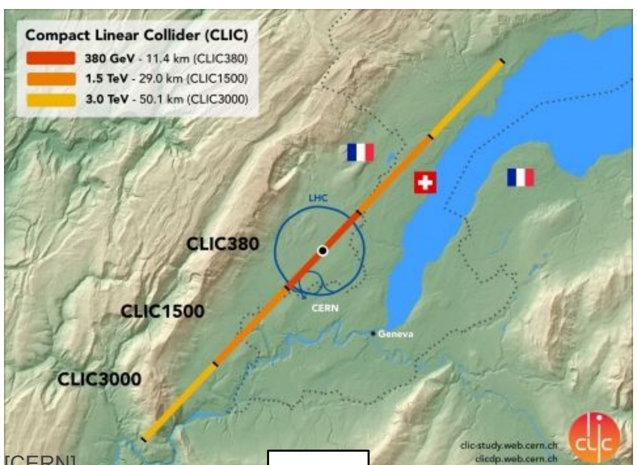


ILC

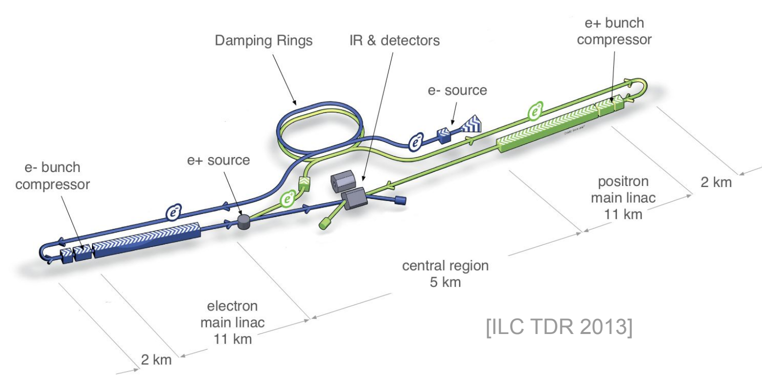
Overview

top quark mass measurements (main methods)

- threshold scan — 350 GeV — $\sigma(e^+e^- \to t\bar{t})$ — < 75 MeV precision [Simon 2019]
- radiative events — 380 GeV, 500 GeV — $\sigma(e^+e^- \to t\bar{t}\gamma)$ — ~ 110 - 150 MeV precision [Boronat et al. 2019 - in preparation]
- direct reconstruction — 380 GeV, 500 GeV — invariant mass — ~ 50 - 100 MeV precision [Abramowicz et al. 2019], [Seidel et al. 2013]



CLIC



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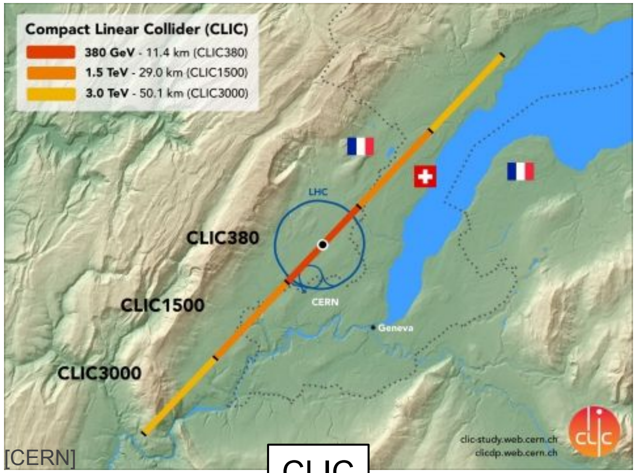
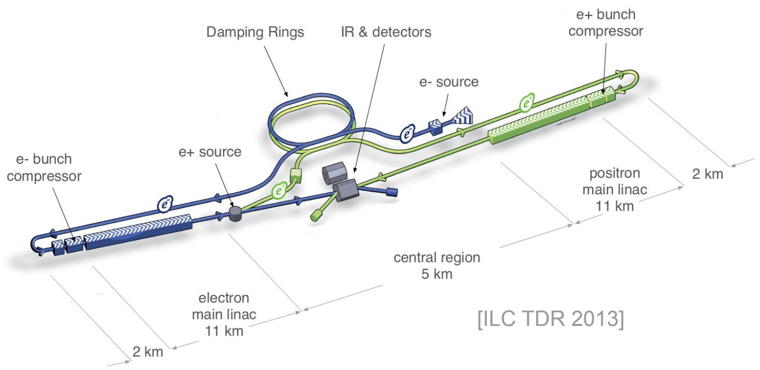
well-defined mass scheme
(low-scale short-distance mass schemes)

Monte Carlo top quark mass
→ talk from Daniel Samitz



[CERN]

CLIC

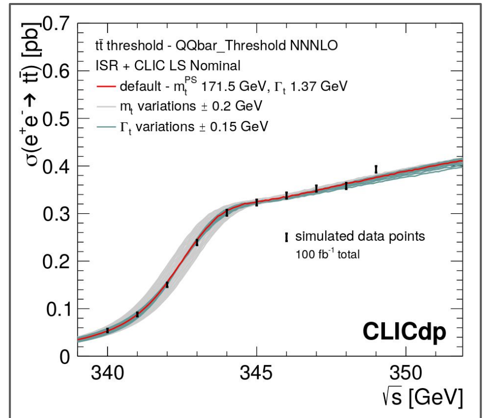


[ILC TDR 2013]

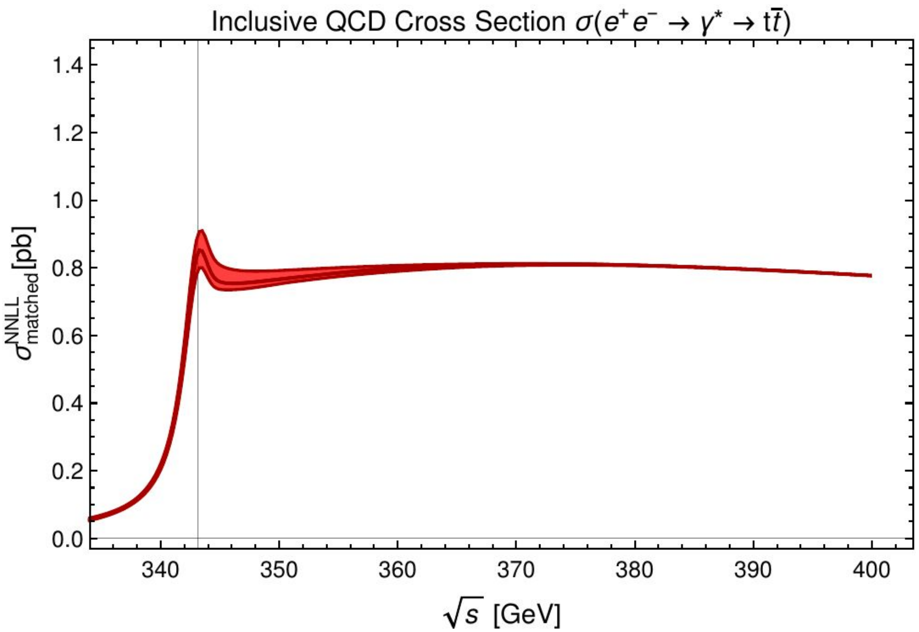
ILC

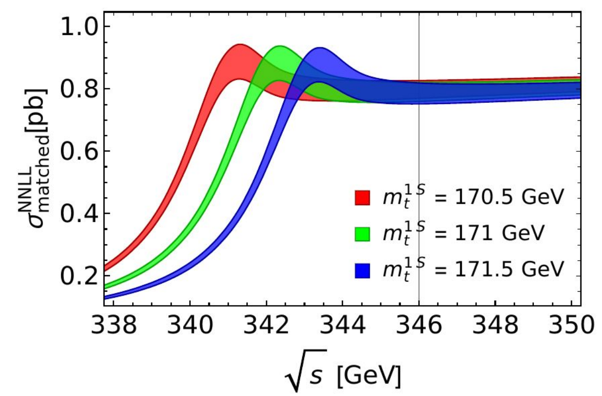
Threshold Scan (340 GeV - 350 GeV)

- top mass: precision < 75 MeV [Simon 2019]
  (now: $m_t^{\mathrm{MC}} = 172.9 \pm 0.4\ \mathrm{GeV}$ [PDG])
- top width: precision < 100 MeV [Simon 2019]
  (now: $\Gamma_t = 1.42^{+0.19}_{-0.15}\ \mathrm{GeV}$ [PDG])
- threshold also sensitive to top Yukawa coupling, strong coupling constant



threshold scan points





mass sensitivity

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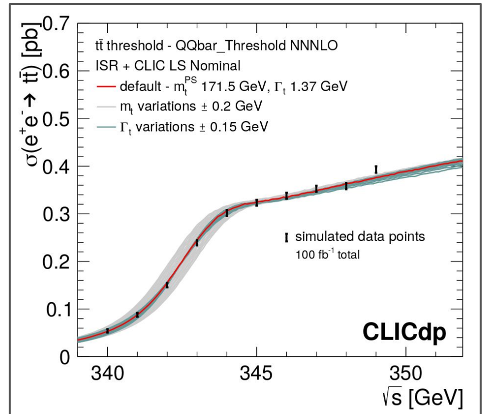
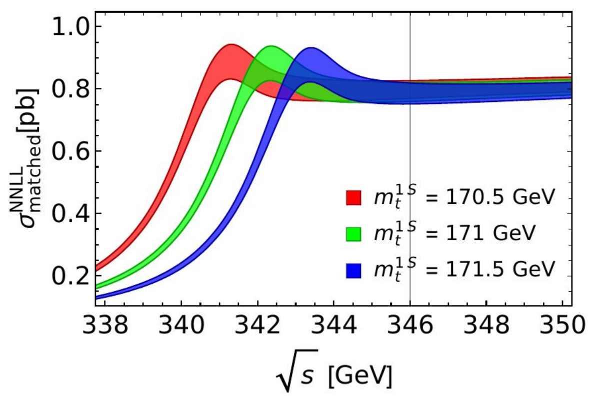


threshold scan points

uncertainties for top quark mass determination

QCD scale variation	~ 40 MeV
parametric α_s	~ 30 MeV (for $\Delta\alpha_s = 0.001$)
statistical	~ 20 MeV
systematic (experimental)	~ 25 - 50 MeV

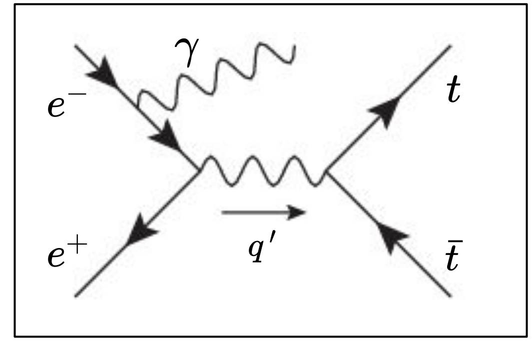
[Abramowicz et al. 2019]



mass sensitivity

Radiative Events (380 GeV, 500 GeV)

[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]



invariant mass of top quark pair: $$(q')^2 = s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$

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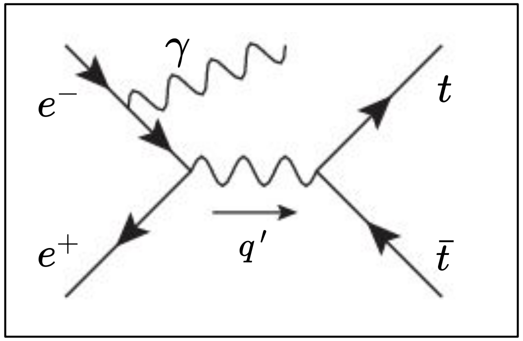

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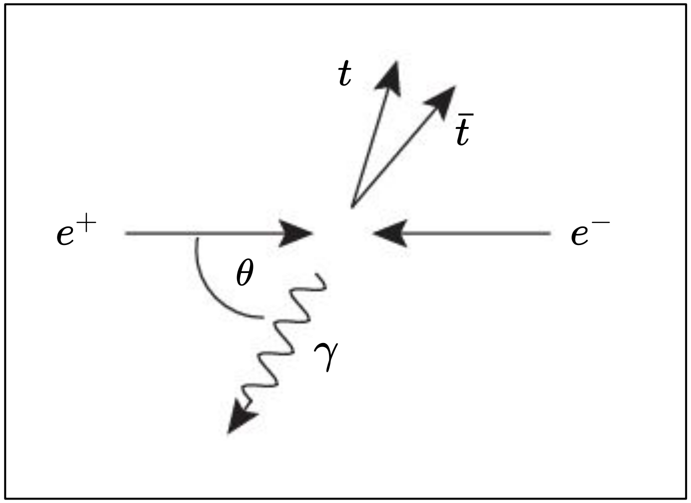
cross section factorizes (in ISR approximation):

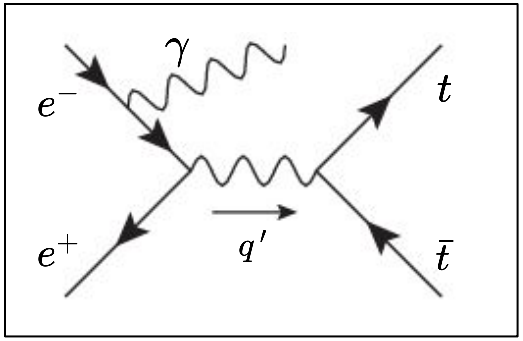
$$\frac{\mathrm{d}\sigma_{t\bar{t}\gamma}}{\mathrm{d}\cos\theta\,\mathrm{d}\sqrt{s'}} = \frac{\alpha_{\rm em}}{\pi\sqrt{s}}\, g(x,\theta)\,\sigma_{t\bar{t}}(s') + \mathcal{O}(\alpha_{\rm em}^2)\,, \quad g(x,\theta) = \frac{2\sqrt{(1-2x)}}{x\sin^2\theta}\left[1-2x+(1+\cos^2\theta)x^2\right], \quad x = \frac{E_\gamma}{\sqrt{s}}$$



- large photon energy $E_\gamma > 5$ GeV
- θ integrated from 8° to 172°
- highest mass sensitivity for collinear top quarks
  - $s' \sim 4\,m_t^2$
  - radiative return to threshold

Radiative Events (380 GeV, 500 GeV)

[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]



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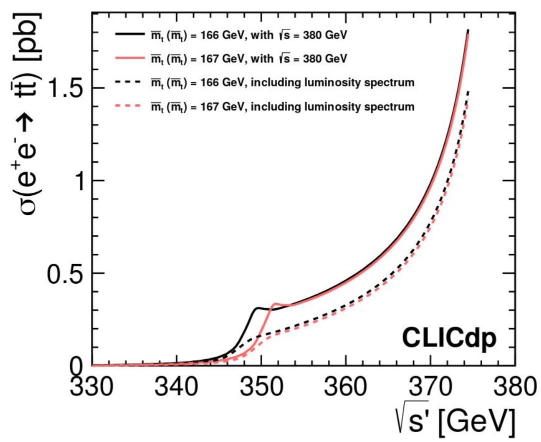

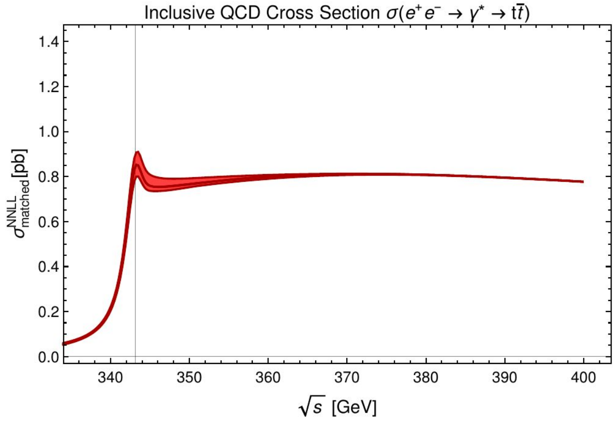

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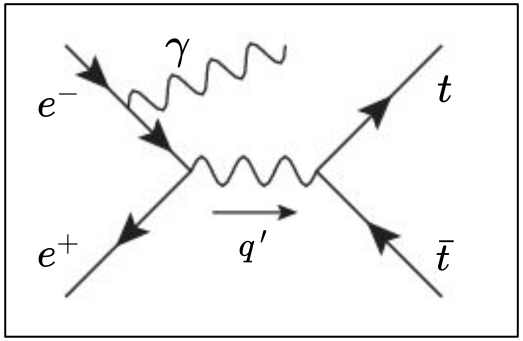

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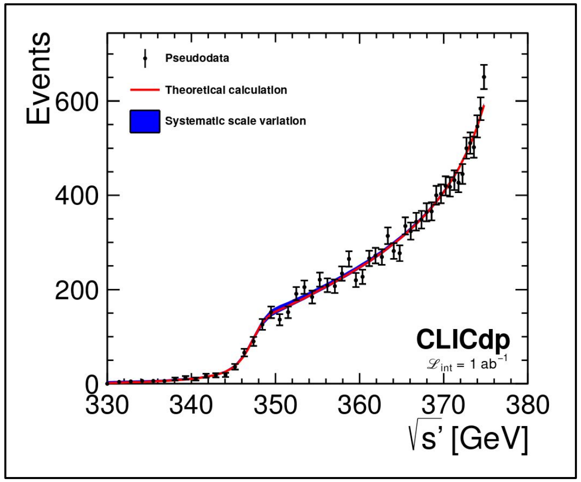
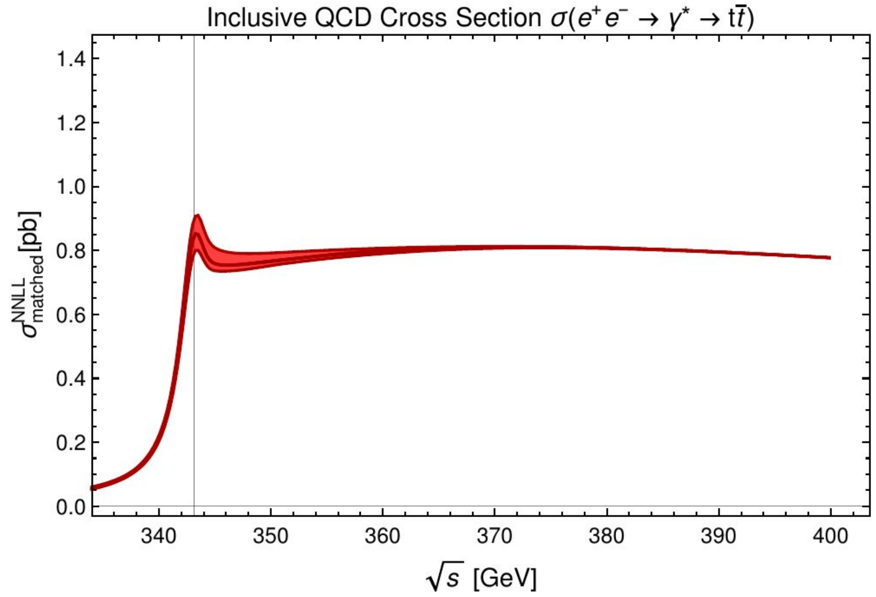


cms energy	CLIC, $\sqrt{s} = 380$ GeV		ILC, $\sqrt{s} = 500$ GeV	
luminosity [fb^{-1}]	500	1000	500	4000
statistical	140 MeV	90 MeV	350 MeV	110 MeV
theory	46 MeV		55 MeV	
lum. spectrum	20 MeV		20 MeV	
photon response	16 MeV		85 MeV	
total	150 MeV	110 MeV	360 MeV	150 MeV

uncertainties for top quark mass

Inclusive Top Quark Pair Production Cross Section from Threshold to Continuum



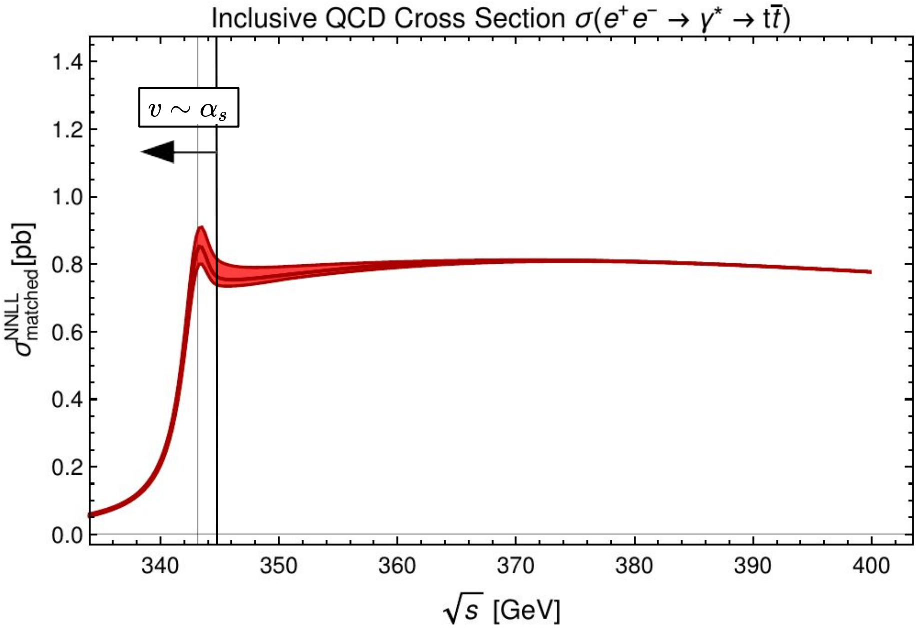
Inclusive Cross Section - Theory Overview

<u>non-relativistic QCD</u>
(NRQCD)

NNLL$_{threshold}$
[Hoang, Stahlhofen 2013]
- QCD + LO electroweak (double-resonant)
- vNRQCD
- 1S mass

NNNLO$_{threshold}$
[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]
[Beneke, Kiyo, Maier, Piclumn]
- QCD + EW + Higgs
- pNRQCD
- PS mass

threshold



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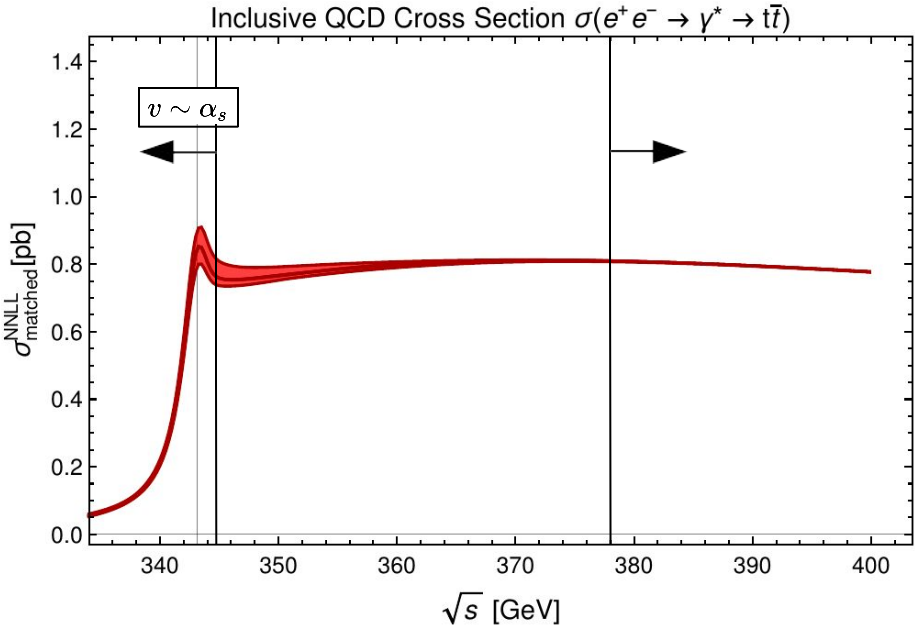

full QCD

$\text{NNNLO}_{\text{continuum}}$

[Hoang, Mateu, Zebarjad 2009]

[Kiyo, Maier, Maierhofer, Marquard 2009]

- QCD corrections

Inclusive Cross Section - Theory Overview

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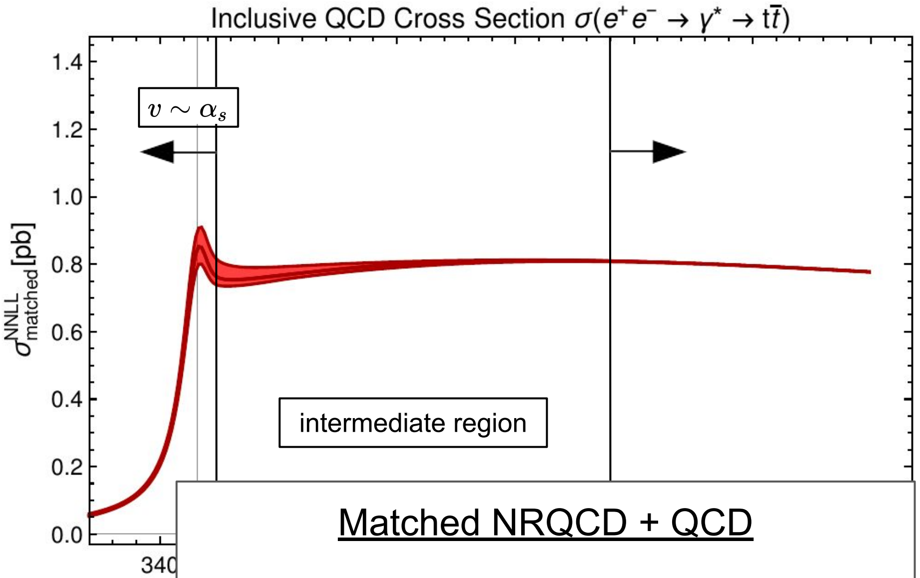


Matched NRQCD + QCD

LL$_{\text{threshold}}$ + NLO$_{\text{continuum}}$ (differential cross section)

[Bach et al. 2017]

NNLL$_{\text{threshold}}$ + NNNLO$_{\text{continuum}}$ this talk

- photon-induced cross section
- QCD + LO EW at threshold
- 1S + MSR mass scheme

full QCD

NNNLO$_{\text{continuum}}$

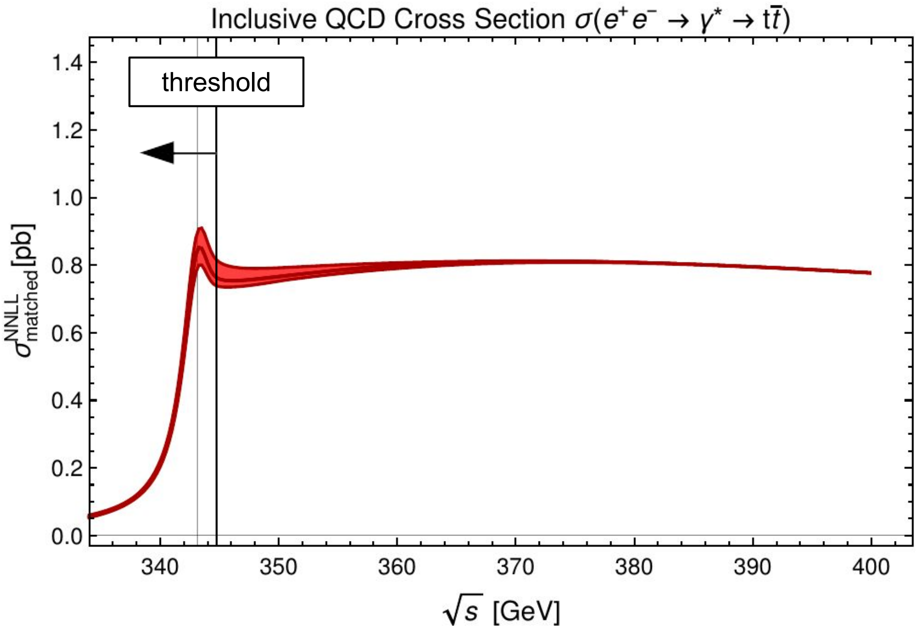
[Hoang, Mateu, Zebarjad 2009]

[Kiyo, Maier, Maierhofer, Marquard 2009]

- QCD corrections

Inclusive Cross Section -

Threshold



Threshold - Coulomb Resummation

At threshold: $v \sim \alpha_s, \, \alpha_s \log(v) \sim 1$

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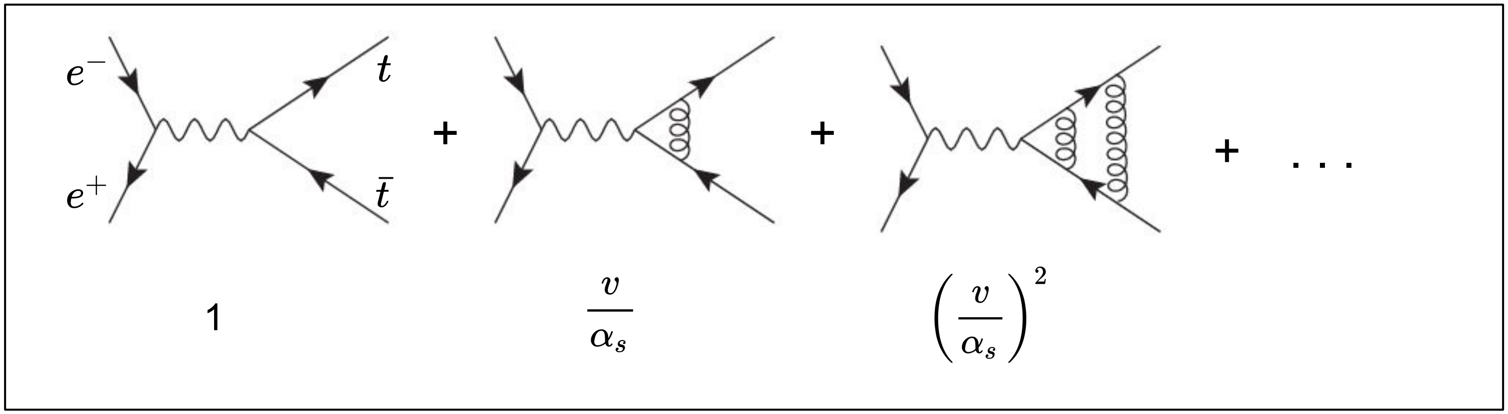
→ ladder diagrams are enhanced



→ resummation of ladder diagrams with Schrödinger equation

→ numerical solution with Toppik [Hoang, Teubner 1999]

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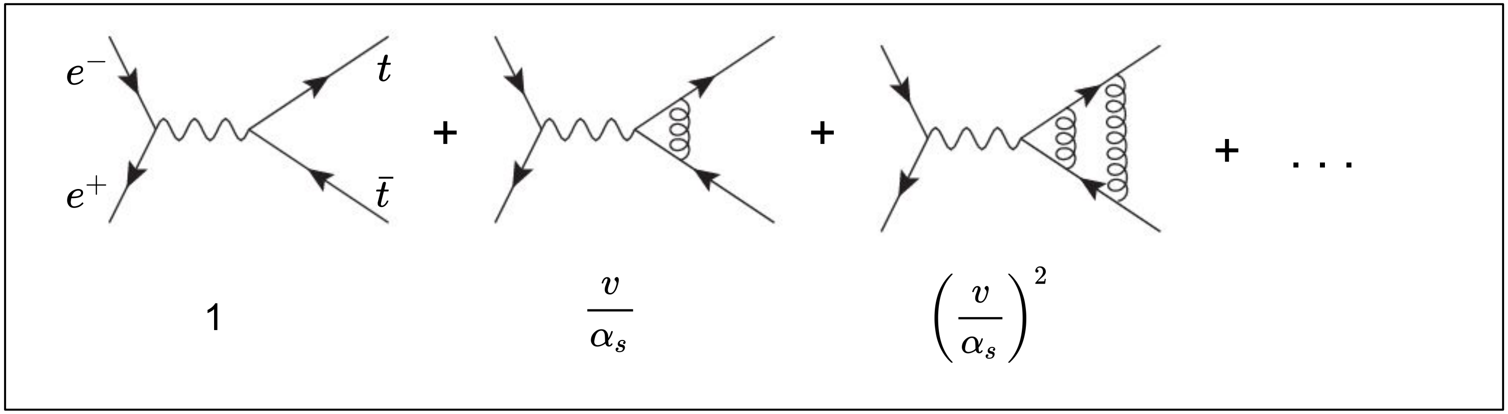
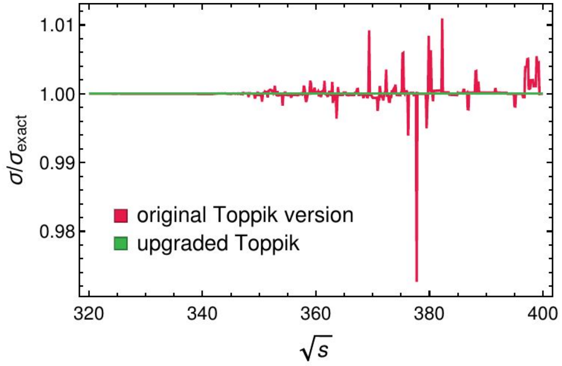


→ resummation of ladder diagrams with Schrödinger equation

→ numerical solution with Toppik [Hoang, Teubner 1999]

→ upgraded version of Toppik

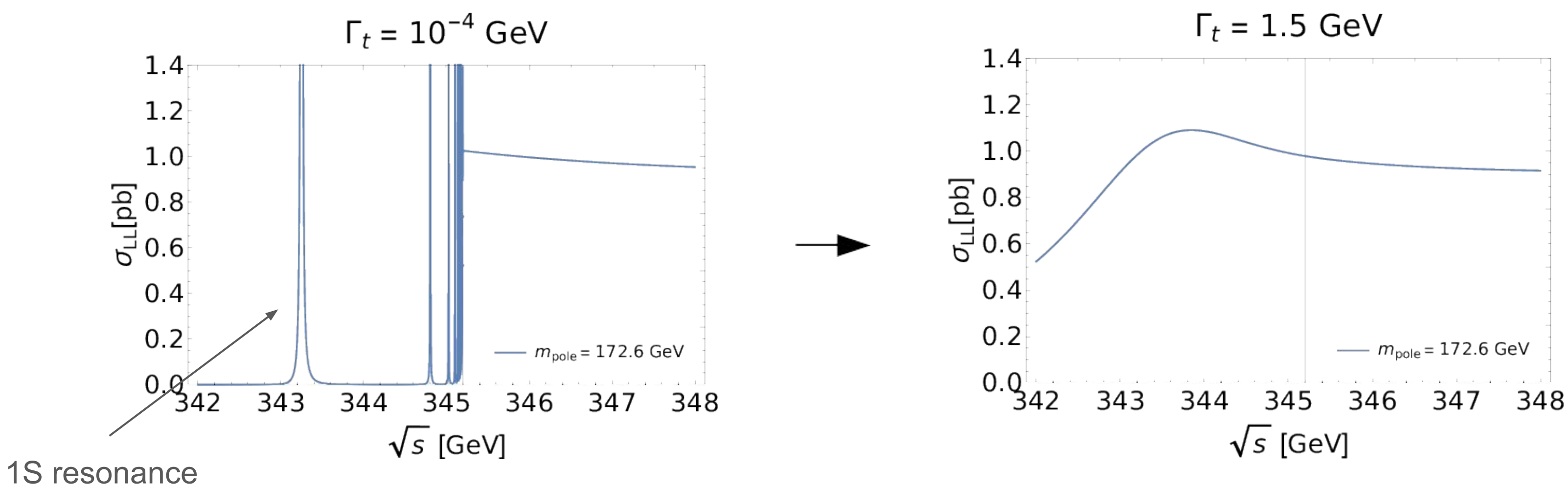
- precision now 10^{-4}
- 10 - 50 times faster than original version



Threshold - Coulomb Resummation

At threshold: $v \sim \alpha_s, \alpha_s \log(v) \sim 1$

- resummation of ladder diagrams gives toponium resonances
- large top quark width smears out the top quark resonances



→ inclusion of width by the replacement $\sqrt{s} + i\epsilon \to \sqrt{s} + i\,\Gamma_t$ (gives LO electroweak contributions at threshold) [Fadin, Khoze 1987]

Threshold - Large Logarithms

At threshold: $v \sim \alpha_s, \boxed{\alpha_s \log(v) \sim 1}$

➞ resummation with vNRQCD (velocity non-relativistic QCD) [Hoang, Stahlhofen 2013]

Contributions to the cross section at threshold :

$$
\begin{aligned}
\sigma_{\text{NRQCD}}^{\text{NNLL}} = v \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \log v)^m & \qquad \text{LL} \\
+ v^2 \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \log v)^m & \qquad \text{NLL} \\
+ v^3 \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \log v)^m & \qquad \text{NNLL}
\end{aligned}
$$

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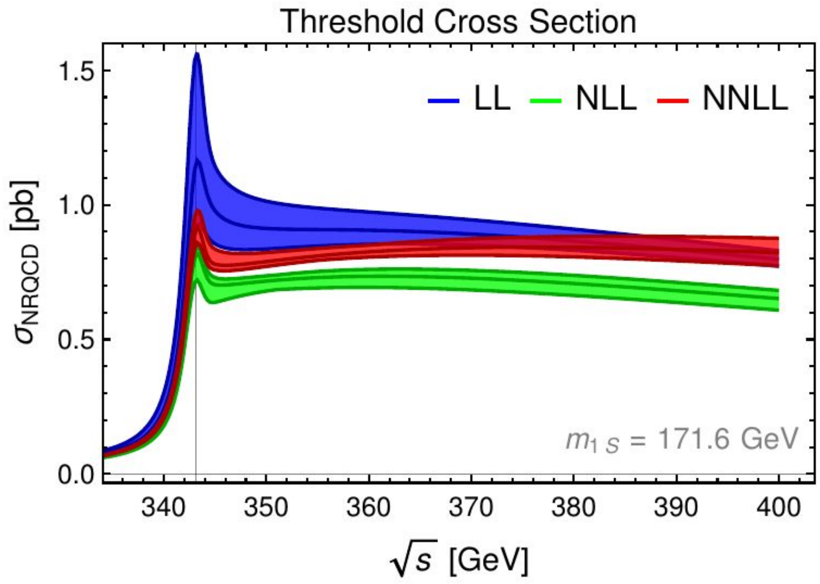
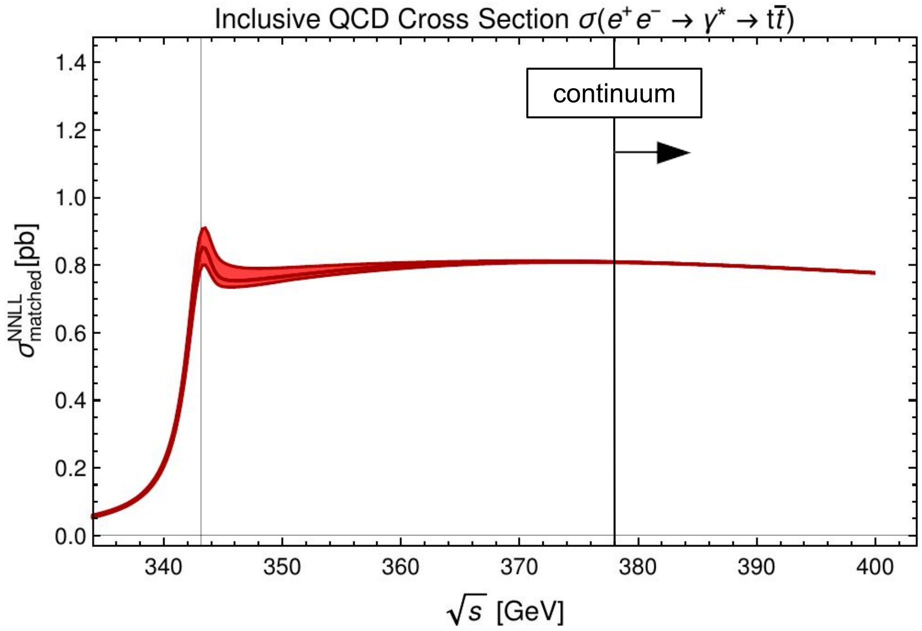


(error bands from variation of renormalization scales)

Inclusive Cross Section -

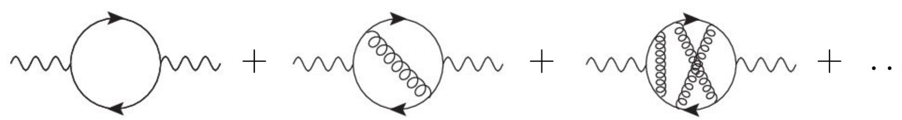
Continuum



Continuum Cross Section

The inclusive cross section is related to the vacuum polarization by the optical theorem:

$$\sigma_{t\bar{t}} = \frac{(4\pi\alpha)^2}{s} Q_t^2 \; \mathrm{Im}\Big[\;\; \ldots \Big]$$



$$= \frac{(4\pi\alpha)^2}{s} Q_t^2 \, \mathrm{Im}\left[\Pi(\sqrt{s} + i\Gamma_t)\right]$$

In the continuum:

$$\sigma_{\mathrm{QCD}}^{\mathrm{N^3LO}} = \frac{(4\pi\alpha)^2}{s} Q_t^2 \cdot \mathrm{Im}\left[\Pi^{(0)} + \alpha_s\,\Pi^{(1)} + \alpha_s^2\,\Pi^{(2)} + \alpha_s^3\,\Pi^{(3)}\right]$$

$\Pi^{(0)}$, $\Pi^{(1)}$ ... known analytically

$\Pi^{(2)}$, $\Pi^{(3)}$ … reconstructed with Padé approximations [Hoang, Mateu, Zebarjad 2009] [Kiyo, Maier, Maierhofer, Marquard 2009]

(validity of Padé approximations for $\Pi^{(2)}$ shown by comparison to exact numerical result in [Maier, Marquard 2017])

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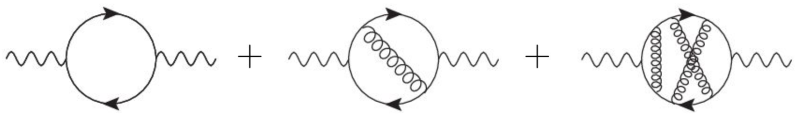
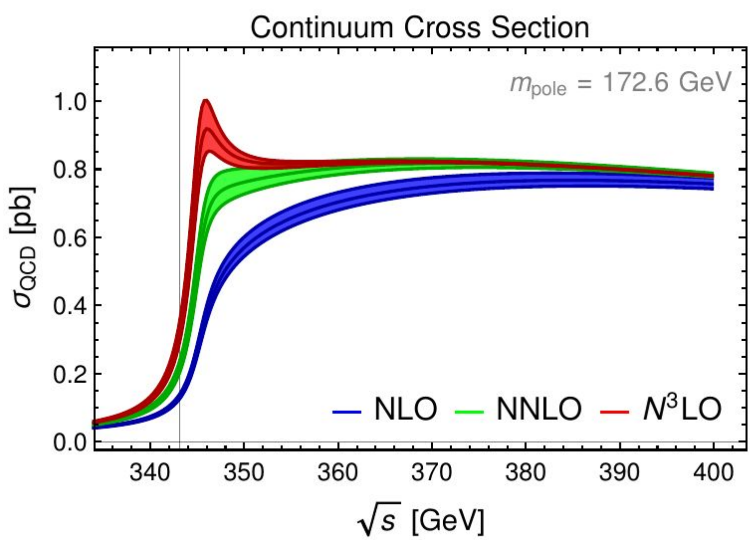


$$+ \ \ldots \Big]$$

$$= \frac{(4\pi\alpha)^2}{s}Q_t^2\,\mathrm{Im}\left[\Pi(\sqrt{s}+i\Gamma_t)\right]$$

In the continuum:



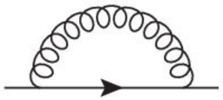
Inclusive Cross Section -

Mass Schemes

- pole mass scheme - renormalon
- 1S mass scheme - for the threshold
- MS mass scheme - for the continuum
- MSR mass scheme - for all regions

Mass Schemes - Pole Mass

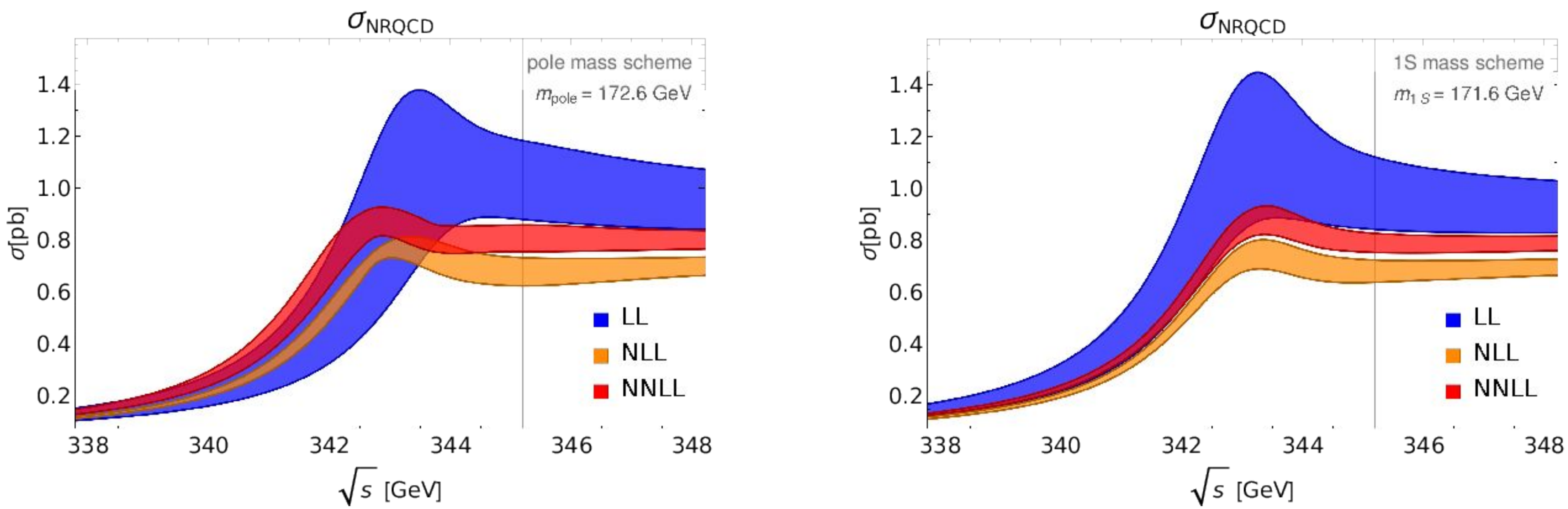
Full propagator: $$S_F^0 = \frac{i}{\not{p} - m_0 + \Sigma(\not{p},\, m_0)} \,,$$

$\Sigma(\not{p},\, m_0) =$  $+ \ldots$

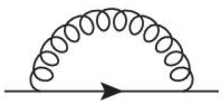
Pole mass: $$\not{p} - m_0 + \Sigma(\not{p},\, m_0)\big|_{p^2 = m^2_{\text{pole}}} = 0$$

→ pole mass renormalon leads to bad convergence of the cross section already at lower orders

Renormalon at threshold:



Mass Schemes - MS Mass

Full propagator: $$S_F = \frac{i}{\not{p} - \overline{m}(\mu) + \Sigma_{\text{finite}}(\not{p},\, \overline{m}(\mu))}\ , \qquad \Sigma(\not{p},\, m_0) =$$  $+ \ldots$

Conversion:

$$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n(n_l, n_h)\, \alpha_s(\overline{m})^n$$

$$= \overline{m} + \overline{m}\,\alpha_s\, a_1 + \ldots \qquad\qquad (\ \overline{m} = \overline{m}^{(n_l+1)}(\overline{m}^{(n_l+1)})\ )$$

$$\sim mv$$

➞ works in the continuum, but not at threshold

Breaking of non-relativistic power counting at threshold in the MS scheme:

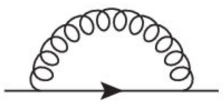
$$v_{\text{pole}} = \sqrt{\frac{\sqrt{s} - 2\, m_{\text{pole}}}{m_{\text{pole}}}}$$

$$= \sqrt{\frac{\sqrt{s} - 2\,(\overline{m} + \overline{m}\, a_1\, \alpha_s/4\pi)}{\overline{m} + \overline{m}\, a_1\, \alpha_s/4\pi}}$$

$$= v_{\overline{\text{MS}}} - a_1 \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{v_{\overline{\text{MS}}}}{2} + \frac{1}{v_{\overline{\text{MS}}}}\right) + a_1^2 \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{3\, v_{\overline{\text{MS}}}}{8} + \frac{1}{2\, v_{\overline{\text{MS}}}} - \frac{1}{2\, v_{\overline{\text{MS}}}^3}\right) + \mathcal{O}(\alpha_s^3)$$

$$\sim \alpha_s \qquad\qquad \sim \alpha_s^0 \qquad\qquad \sim \alpha_s^{-1}$$

Mass Schemes - MS Mass

Full propagator: $$S_F = \frac{i}{\not{p} - \overline{m}(\mu) + \Sigma_{\text{finite}}(\not{p},\, \overline{m}(\mu))}\,, \qquad \Sigma(\not{p},\, m_0) =$$  $+ \ldots$

Conversion:

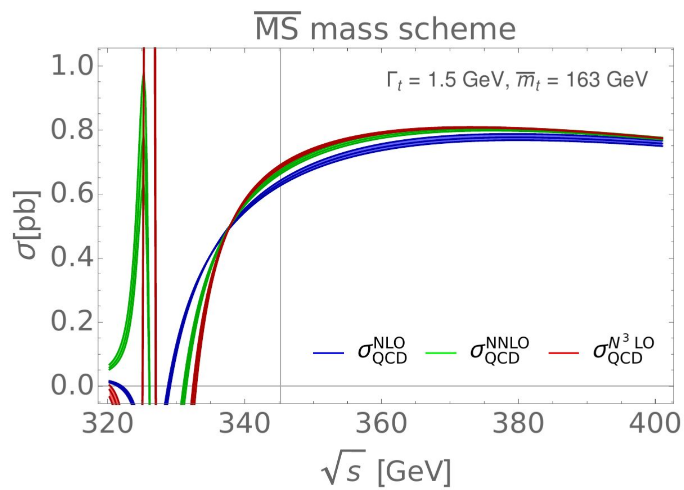
$$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n(n_l, n_h)\, \alpha_s(\overline{m})^n$$

$$= \overline{m} + \overline{m}\, \alpha_s\, a_1 + \ldots \qquad \left(\overline{m} = \overline{m}^{(n_l+1)}(\overline{m}^{(n_l+1)}) \right)$$

$$\sim mv$$

➞ works in the continuum, but not at threshold

Breaking of non-relativistic power counting at threshold in the MS scheme:

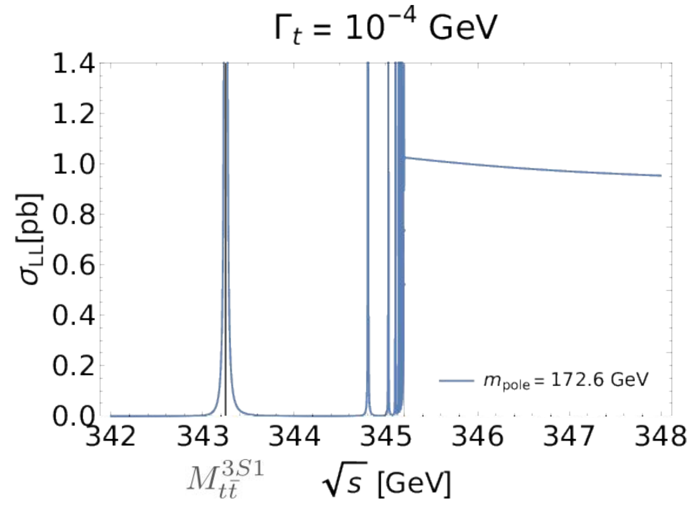


Mass Schemes - 1S Mass

[Hoang, Ligeti, Manohar 1998]

Mass of 1S resonance: $M_{t\bar{t}}^{3S1} = E_{\rm bin} + 2\, m_{\rm pole}$

1S mass: $m_{\rm 1S} = \frac{1}{2} M_{tt}^{3S1} = m_{\rm pole} + \frac{1}{2} E_{\rm bin}$



other low-scale short-distance mass schemes:
PS mass [Beneke 1998] , RS mass [Pineda 2001] , kinetic mass [Czarnecki, Melnikov, Uraltsev 1998]

Mass Schemes - 1S Mass

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Mass of 1S resonance: $M_{t\bar{t}}^{3S1} = E_{\text{bin}} + 2\, m_{\text{pole}}$

1S mass: $$m_{\text{1S}} = \frac{1}{2} M_{tt}^{3S1} = m_{\text{pole}} + \frac{1}{2} E_{\text{bin}}$$

Conversion: $$m_{1S} = m_{pole} + (C_F\, \alpha_s(\mu) m_{pole}) \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} c_{n,k} \alpha_s(\mu)^n \log\left(\frac{\mu}{C_F\, \alpha_s(\mu) m_{pole}}\right)$$
$$= m_{pole} - \frac{2}{9} \alpha_s^2\, m_{pole} + \ldots$$
$$\sim mv^2$$

no breaking of the non-relativistic power counting at threshold:

$$v_{\text{pole}} = \sqrt{\frac{\sqrt{s} - 2\, m_{\text{pole}}}{m_{\text{pole}}}}$$
$$= \sqrt{\frac{\sqrt{s} - 2\, (m_{\text{1S}} + m_{\text{1S}}\, a_1\, \alpha_s/4\pi)}{m_{\text{1S}} + m_{\text{1S}}\, a_1\, \alpha_s/4\pi}}$$
$$= v_{1\text{S}} - a_1 \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{v_{1\text{S}}}{2} + \frac{1}{v_{1\text{S}}}\right) + a_1^2 \left(\frac{\alpha_s}{4\pi}\right)^4 \left(\frac{3\, v_{1\text{S}}}{8} + \frac{1}{2\, v_{1\text{S}}} - \frac{1}{2\, v_{1\text{S}}^3}\right) + \mathcal{O}(\alpha_s^6)$$
$$\sim \alpha_s \qquad \sim \alpha_s \qquad \sim \alpha_s$$

Mass Schemes - MSR Mass

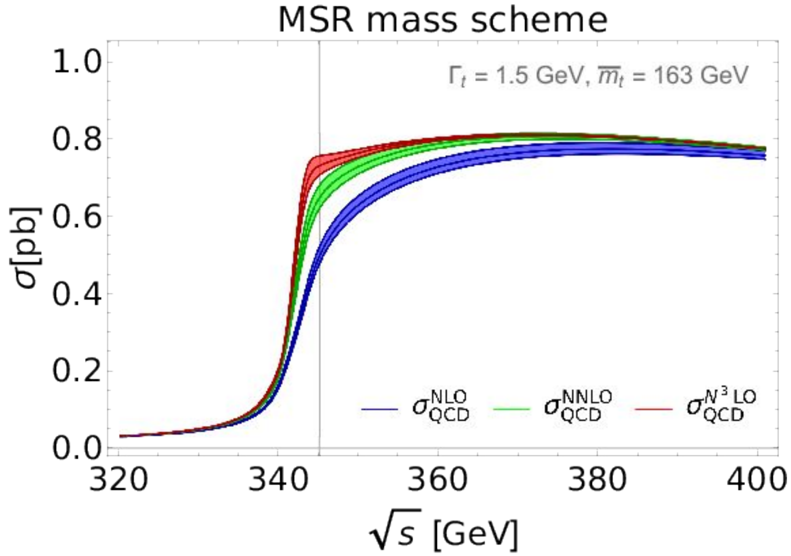
[Hoang, Jain, Scimemi, Stewart 2008], [Hoang, Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart 2017]

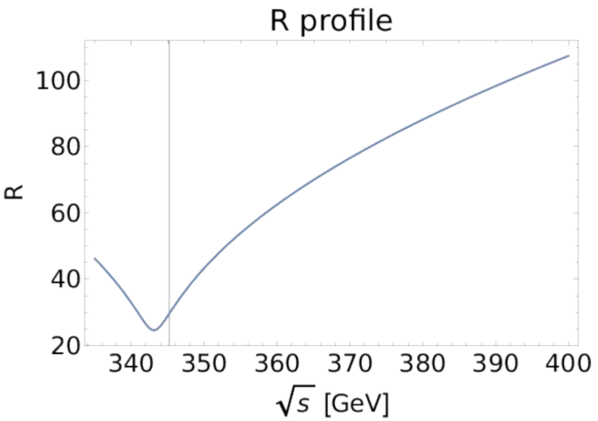
Conversion: $$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n \; \alpha_s(\overline{m})^n = \overline{m} + \overline{m} \, \alpha_s \, a_1 + \ldots$$

$$m_{pole} = m_{MSR}(R) + R \sum_{n=1}^{\infty} a_n \; \alpha_s(R)^n = m_{MSR}(R) + R \, \alpha_s \, a_1 + \ldots$$

→ no breakdown of the non-relativistic power counting at threshold

→ improves convergence of the continuum cross section in the intermediate region:





Mass Schemes - MSR Mass

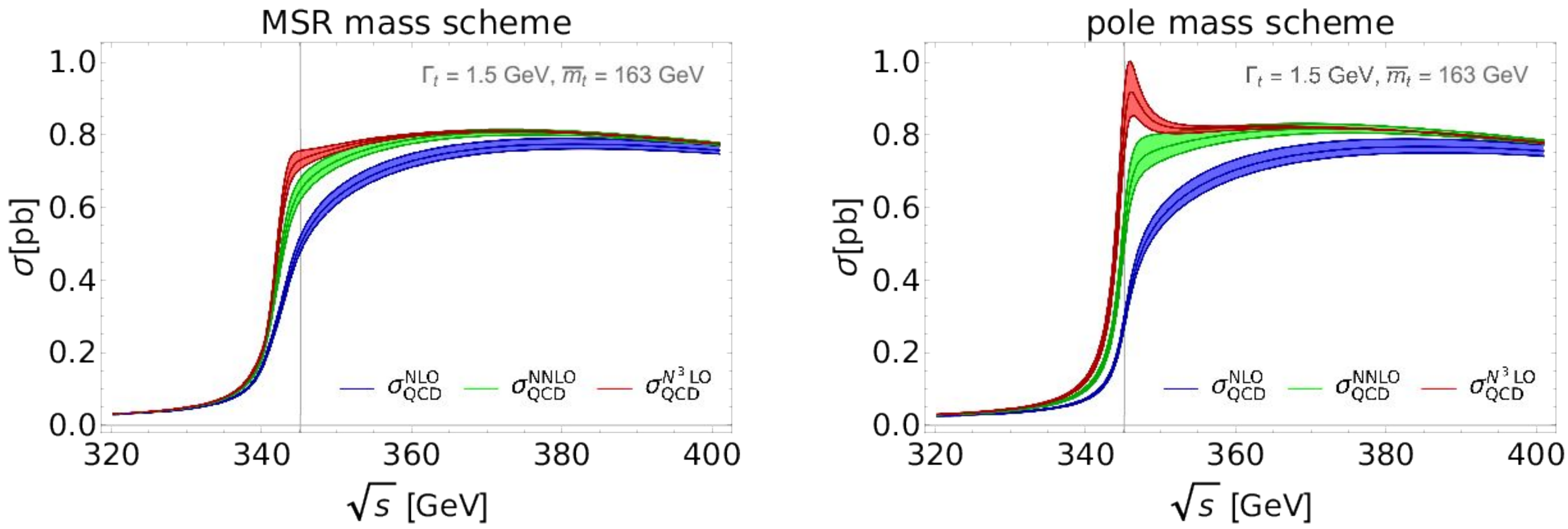
[Hoang, Jain, Scimemi, Stewart 2008], [Hoang, Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart 2017]

Conversion: $$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n \, \alpha_s(\overline{m})^n = \overline{m} + \overline{m}\, \alpha_s \, a_1 + \ldots$$

$$m_{pole} = m_{MSR}(R) + R \sum_{n=1}^{\infty} a_n \, \alpha_s(R)^n = m_{MSR}(R) + R\, \alpha_s \, a_1 + \ldots$$

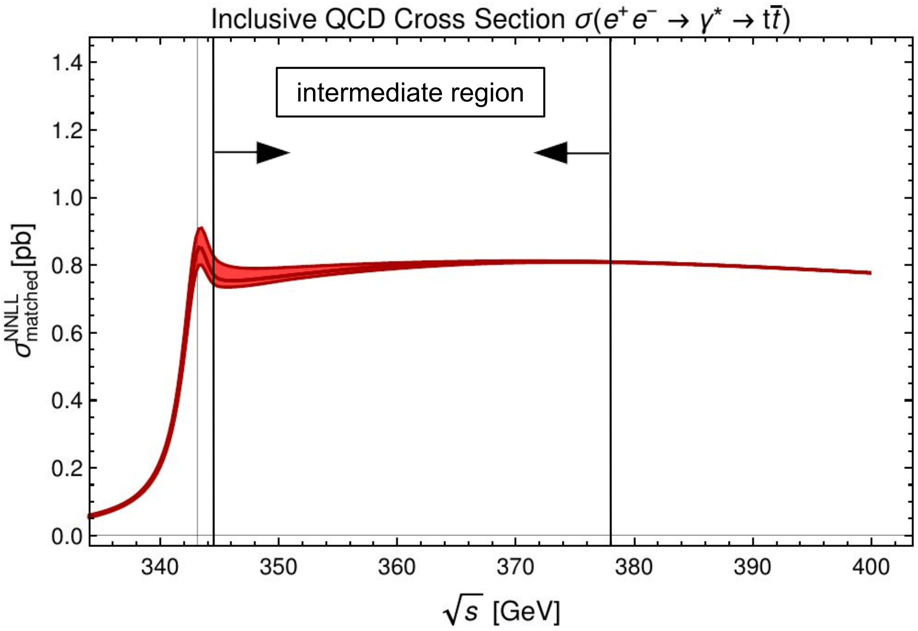
➞ no breakdown of the non-relativistic power counting at threshold

➞ improves convergence of the continuum cross section in the intermediate region:



Inclusive Cross Section -

Matching



Matching

$$\sigma_{\text{matched}} = \sigma_{\text{QCD}} + (\sigma_{\text{vNRQCD}} - \sigma_{\text{double-counted}}) \cdot f_s$$

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$$\begin{aligned}
\sigma^{\text{NNLO}}_{\text{vNRQCD}} = \quad & v \;+\; \alpha_s \;+\; \alpha_s^2/v + \alpha_s^3/v^2 + \alpha_s^4/v^3 + \;... \\
& + v^2 + \alpha_s\, v + \alpha_s^2 + \alpha_s^3/v + \alpha_s^4/v^2 + \;... \\
& + v^3 + \alpha_s\, v^2 + \alpha_s^2\, v + \alpha_s^3 + \alpha_s^4/v + \;...
\end{aligned}$$

$$\begin{aligned}
\sigma^{\text{N}^3\text{LO}}_{\text{QCD}} = \quad & v \;+ v^2 + v^3 + v^4 + \;... \\
& + \alpha_s + \alpha_s\, v + \alpha_s\, v^2 + \alpha_s\, v^3 + \;... \\
& + \alpha_s^2/v + \alpha_s^2 + \alpha_s^2\, v + \alpha_s^2\, v^2 + \;... \\
& + \alpha_s^3/v^2 + \alpha_s^3/v + \alpha_s^3 + \alpha_s^3 + \;...
\end{aligned}$$

Matching

$$\sigma_{\text{matched}} = \sigma_{\text{QCD}} + (\sigma_{\text{vNRQCD}} - \sigma_{\text{double-counted}}) \cdot f_s$$

$$\begin{aligned}
\sigma^{\text{NNLO}}_{\text{vNRQCD}} = \;& v + \alpha_s + \alpha_s^2/v + \alpha_s^3/v^2 + \alpha_s^4/v^3 + \ldots \\
& + v^2 + \alpha_s\, v + \alpha_s^2 + \alpha_s^3/v + \alpha_s^4/v^2 + \ldots \\
& + v^3 + \alpha_s\, v^2 + \alpha_s^2\, v + \alpha_s^3 + \alpha_s^4/v + \ldots
\end{aligned}$$

$$\begin{aligned}
\sigma^{\text{N}^3\text{LO}}_{\text{QCD}} = \;& v + v^2 + v^3 + v^4 + \ldots \\
& + \alpha_s + \alpha_s\, v + \alpha_s\, v^2 + \alpha_s\, v^3 + \ldots \\
& + \alpha_s^2/v + \alpha_s^2 + \alpha_s^2\, v + \alpha_s^2\, v^2 + \ldots \\
& + \alpha_s^3/v^2 + \alpha_s^3/v + \alpha_s^3 + \alpha_s^3 + \ldots
\end{aligned}$$

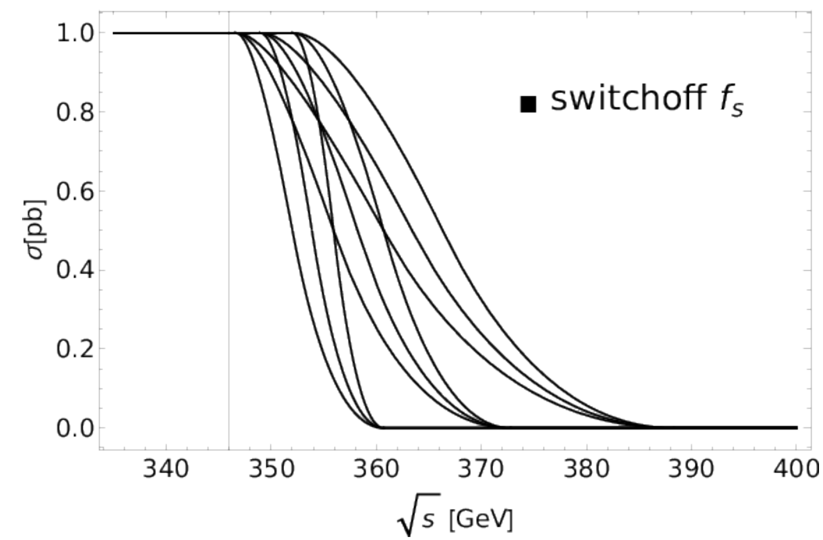
$\sigma_{\text{double-counted}}$

Matching

$$\sigma_{\text{matched}} = \sigma_{\text{QCD}} + (\sigma_{\text{vNRQCD}} - \sigma_{\text{double-counted}}) \cdot f_s$$

switch-off function:

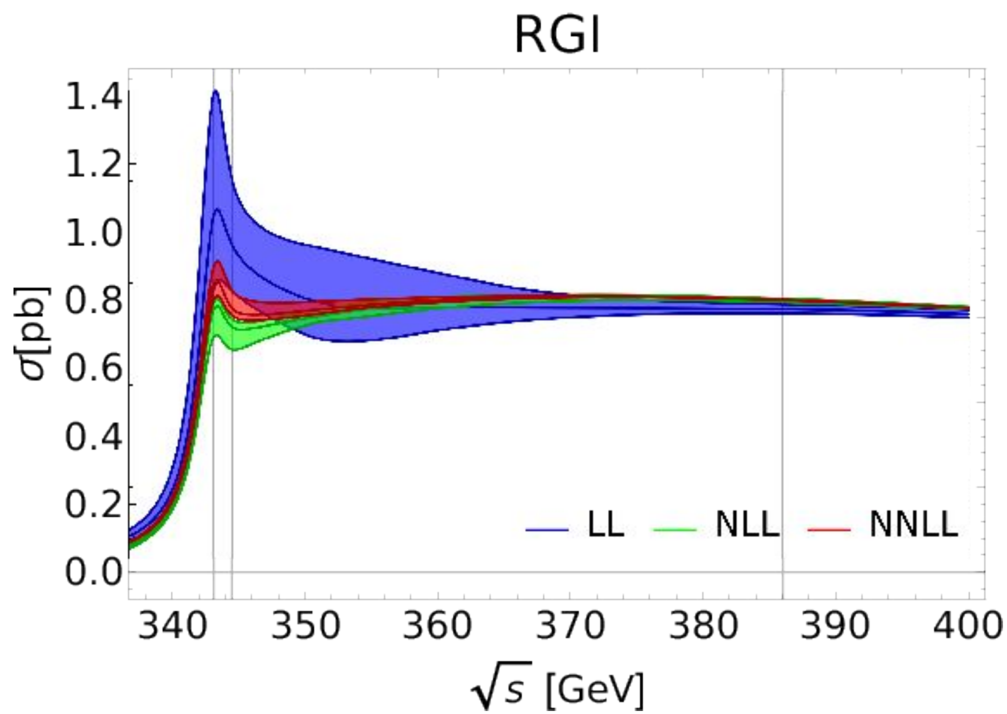
- variation gives an error estimate of the matching
- introduces scheme dependence
- do we get convergence when going to higher orders?



mass schemes:	σ_{vNRQCD}	1S mass scheme
	σ_{QCD}	MSR mass scheme
	$\sigma_{\text{double-counted}}$	MSR mass scheme

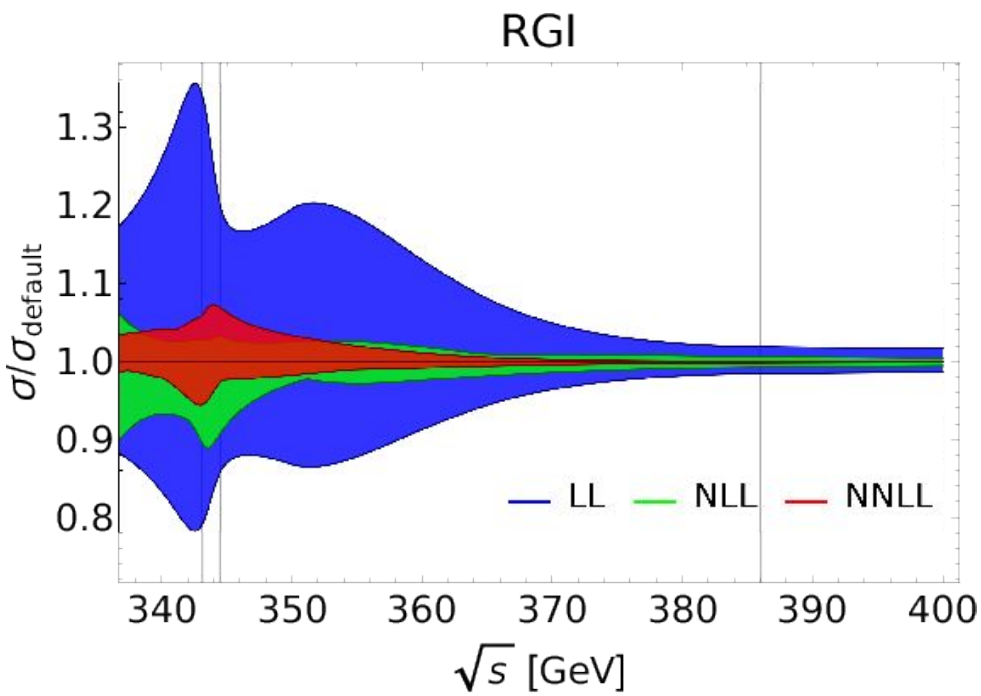
Matching

matched cross section from lowest to highest order:

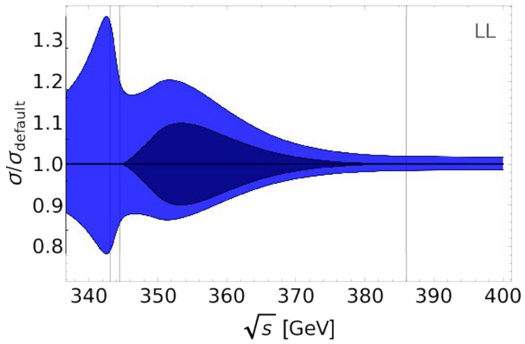


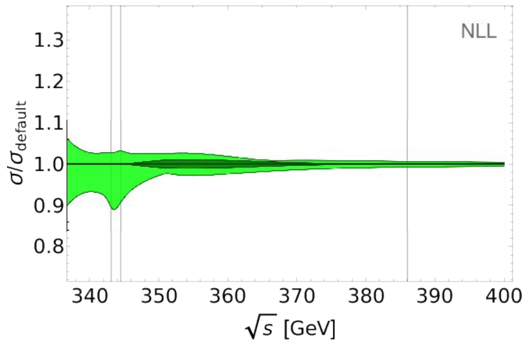
- error from variation of renormalization scales and the switch off function
- matching smoothly connects threshold with continuum
- overall error reduces from order to order

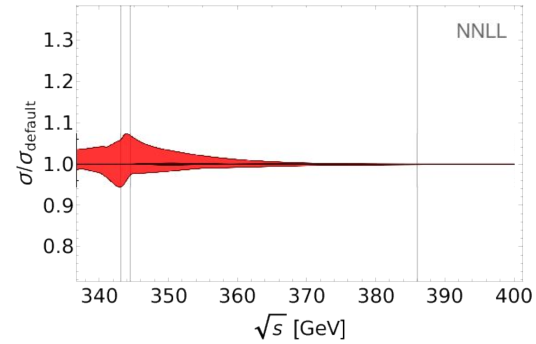
Matching



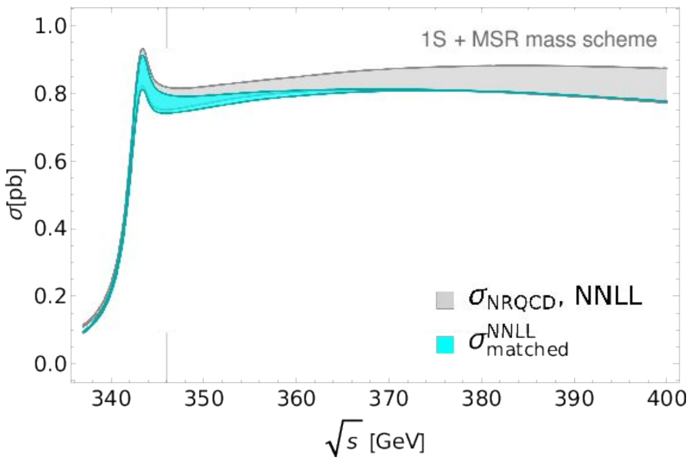
- good convergence from order to order
- matching error smaller than variation of renormalization scales
- matching error reduces from order to order







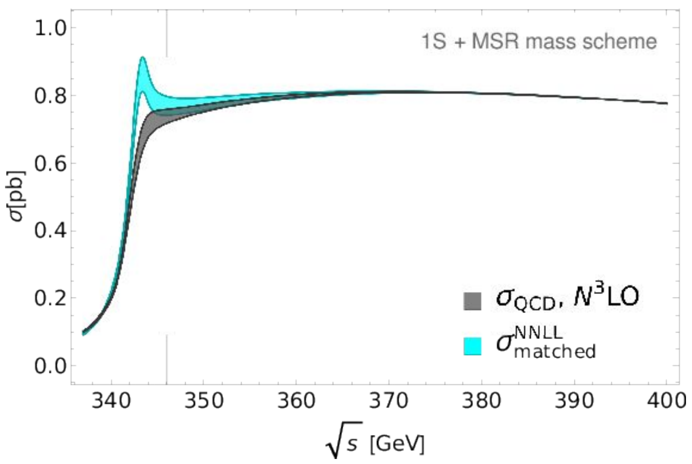
Matching



threshold cross section vs. matched cross section

→ matched cross section starts to differ from the threshold cross section immediately above the peak region

→ higher order corrections from continuum cross section give small shift at threshold



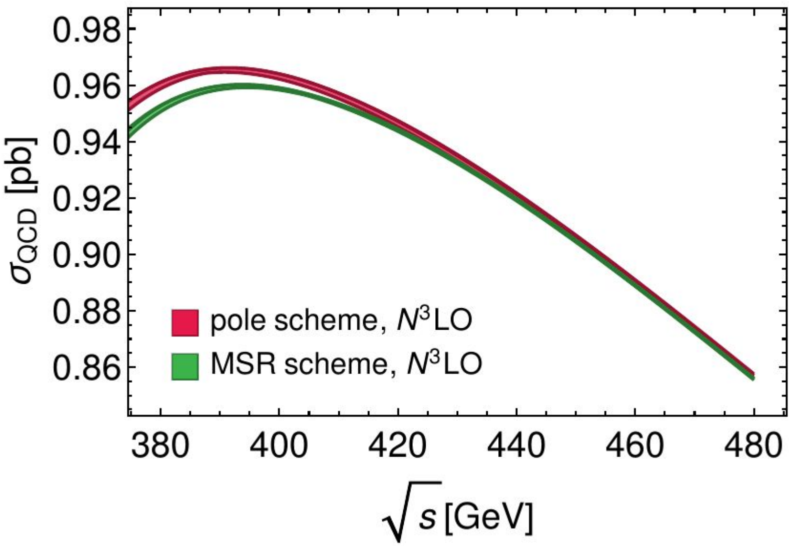
continuum cross section vs. matched cross section

→ matched cross section and continuum MSR cross section overlap above 365 GeV

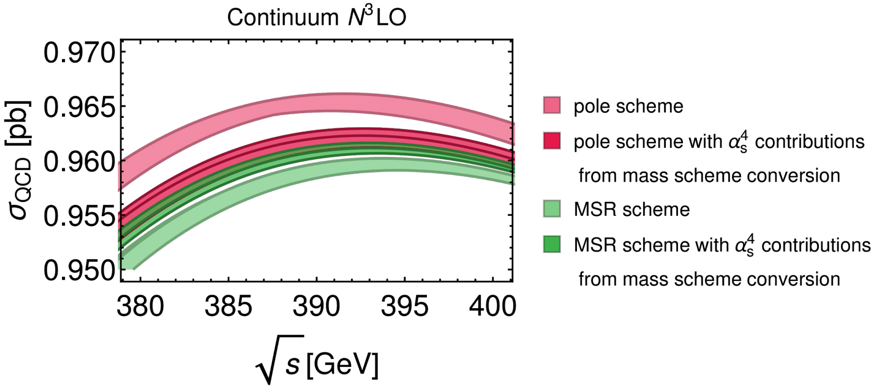
→ MSR mass scheme valid down to smaller center-of-mass energies than pole mass scheme and MS mass scheme

Theory Error for NNNLO$_{\text{continuum}}$

the cross section at NNNLO$_{\text{continuum}}$ shows a difference between the pole scheme and the MSR scheme:



- cross section in the pole scheme and the MSR scheme are incompatible (error bands do not overlap)
- scale variation seems to underestimate the error
- difference corresponds to 1 GeV difference in the top quark mass



- higher order mass corrections seem to favor MSR mass scheme

Conclusions

- The top quark pair production cross section at lepton colliders will provide high precision measurements for the top mass and width from a threshold scan and radiative events.
- We constructed a consistent matched cross section at QCD $NNLL_{threshold}$ + $NNNLO_{continuum}$ with LO electroweak corrections at threshold.
- The MSR mass provides a consistent mass scheme in all regions from threshold to the continuum.
- The cross section at $NNNLO_{continuum}$ in the pole and MSR scheme show a large difference
- Outlook:
    - study on the difference of the continuum cross section in different mass schemes
    - differential matched cross section at $NLL_{threshold}$ + $NLO_{continuum}$

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Thank you for your attention!