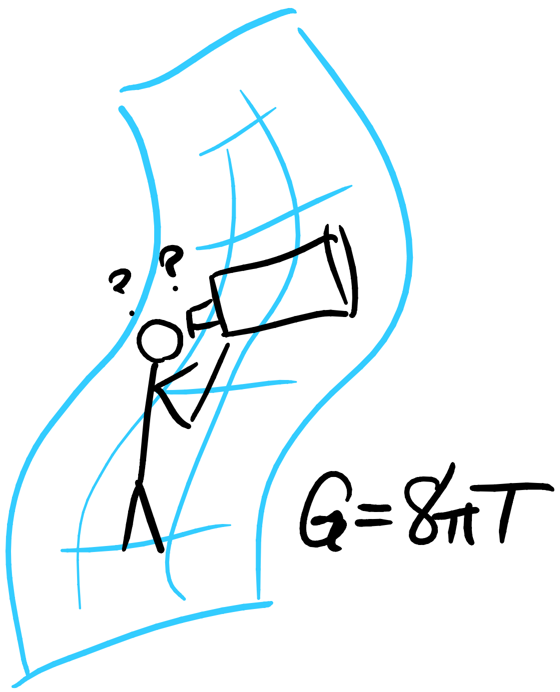
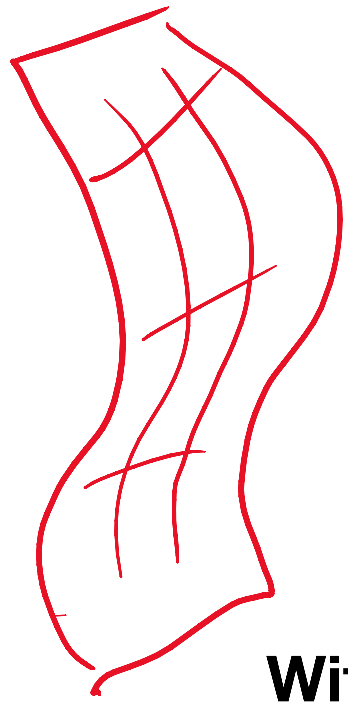
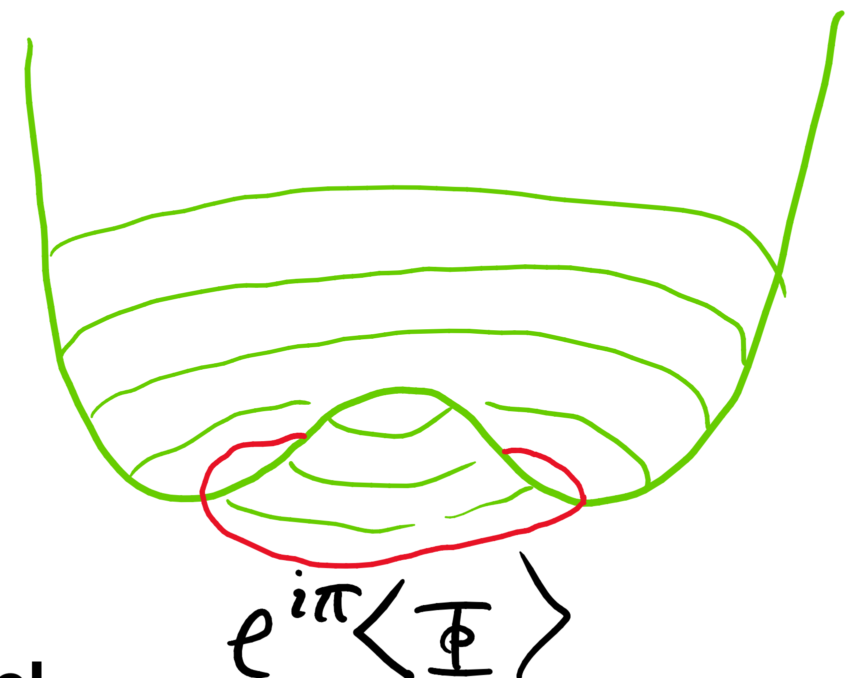
Brane World Cosmology From Symmetry Principles





Jay Hubisz
Syracuse University
Oct. 21 2019

work in progress
With Rigo, Csáki, Ismail, Eröncel



The Dark Universe
ICTP São Paulo

Dedicated to the memory of Eduardo Pontón

Brane Worlds and Problems of Particle Physics

- Extra dimensions large and small have proven to be rich troves for model building and phenomenology
  - many interesting ways to address
    - hierarchy problem
    - flavor
    - SUSY breaking
- New viewpoints on cosmological constant problem

Additionally, many intriguing cosmological behaviors in presence of instability

Examples: Matter/Radiation Cosmologies from Pure Tension

- The motion of branes in systems with no static or stable solution can have interesting consequences for localized observers

  - RS 2 brane model - mistuned tensions

    - solutions include 4D ρ_r, ρ_Λ cosmologies

E.g. Csáki, Graesser, Kolda, Terning hep-ph/9906513
Kaloper hep-th/9905210

  - Sen's "Tachyon condensation": $D - \bar{D}$ brane collapse/annihilation

e.g. Sen hep-th/0203265

    - rolling tachyon stress-energy approaches matter domination from

Darkness from Brane-worlds: Matter without Matter

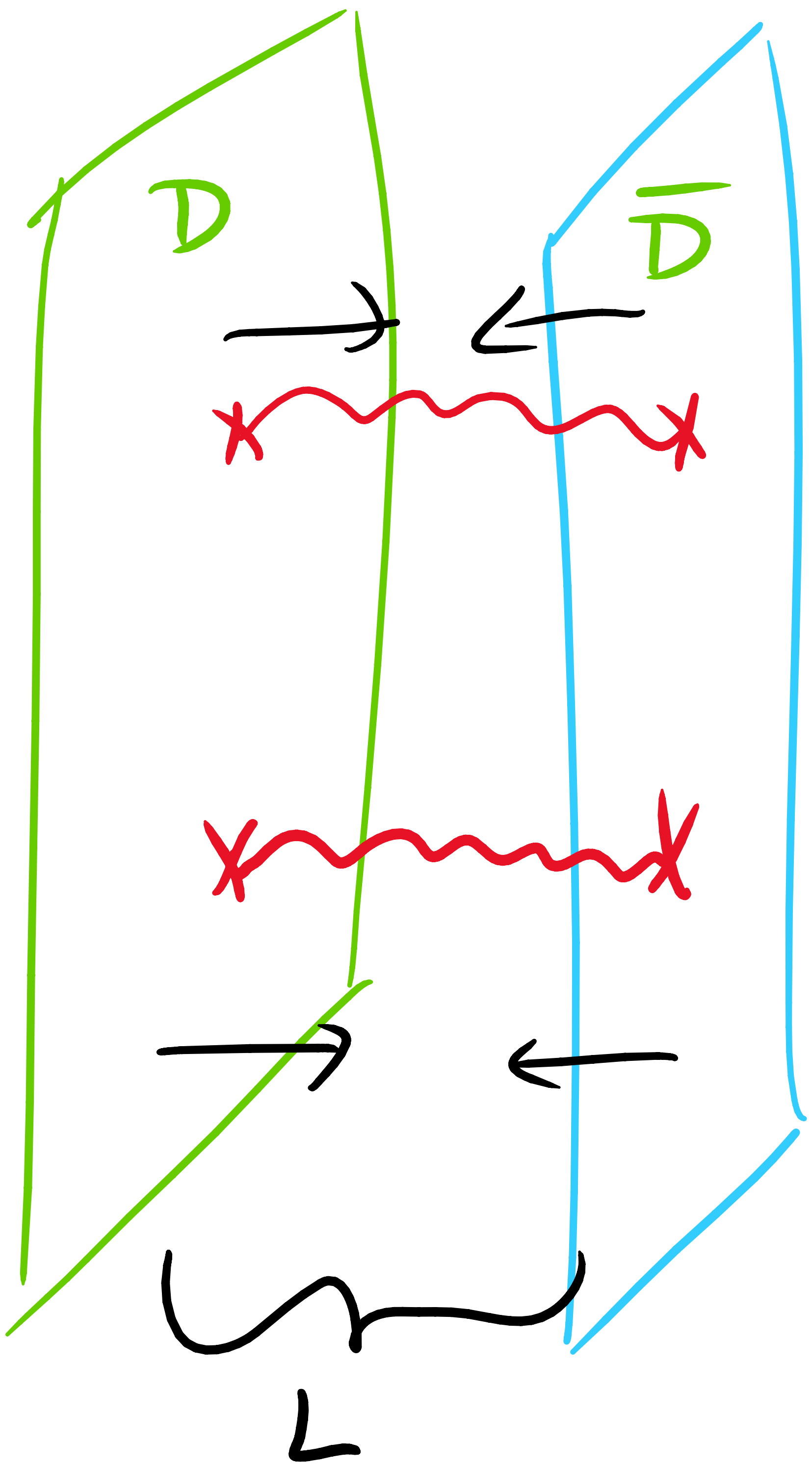
- In the early 2000's, Sen showed systems of unstable D-branes annihilate, while falling are described by an EFT:

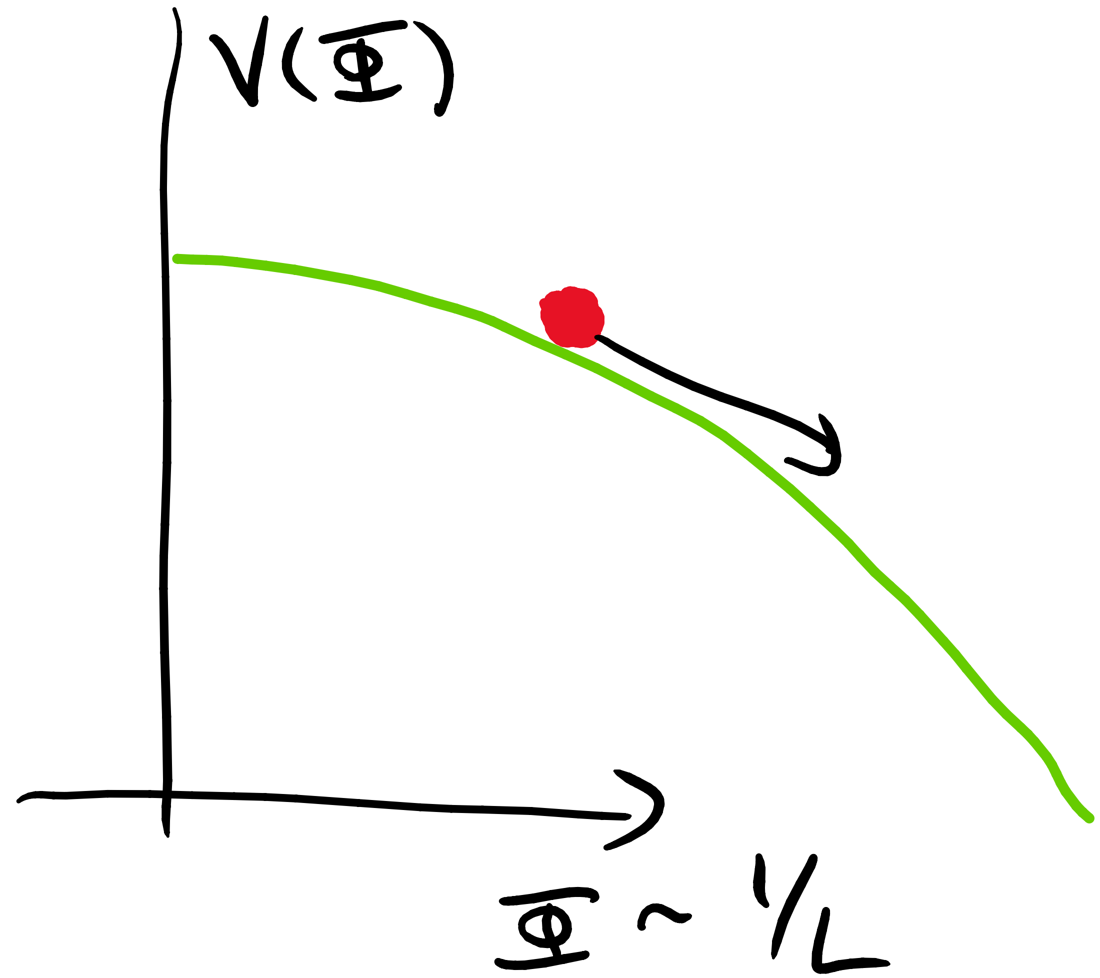
$$S_{\text{Tachyon}} = \int d^d x V(\Phi) \sqrt{\det(\eta_{\mu\nu} + \partial_\mu \Phi \partial_\nu \Phi)}$$

- Tachyon rolls, square root sets speed limit (c for branes)

- In epoch when near limit, stress tensor is that of pressure-less fluid…non-relativistic matter

Annihilating at c:





Higher derivative interactions
= speed limit

Stress-Tensor

Consider time-dependent brane separation

$$T_{00} \propto \frac{V(\Phi)}{\sqrt{1-\dot{\Phi}^2}} \qquad T_{ij} \propto -V(\Phi)\sqrt{1-\dot{\Phi}^2}\,\delta_{ij}$$

Tachyon rolls, picks up speed, asymptotically approaching $\dot{\Phi} \approx 1$

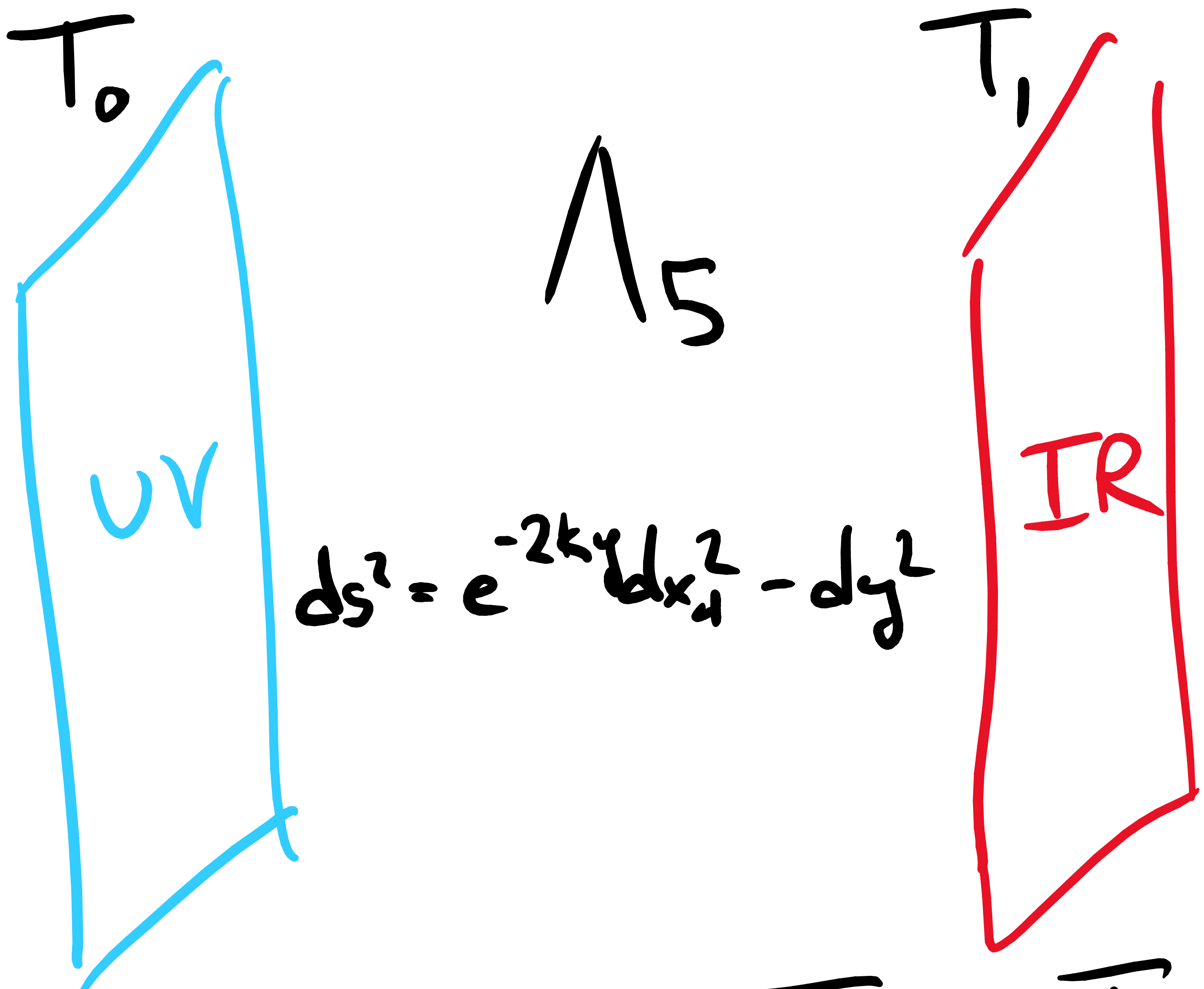
T_{00} remains constant (energy conserved)

Pressure drops to zero

Add weak gravity - cosmology tends from inflationary to being dominated by non-relativistic matter

(Eventually branes annihilate - not captured in this EFT)

Randall-Sundrum 2-Brane



Static solution if $T_1 = -T_0 = \frac{\Lambda_5}{k}$

FINE TUNED

Relaxing the tensions

The dynamics of this model have been well-studied, here we repeat

Using EE's, can simplify metric
depends only on one function, $a(y, t)$:

$$ds^2 = \left(\frac{\dot{a}}{\dot{a}_0}\right)^2 dt^2 - a^2 dx_3^2 - dy^2$$

$a_0(t)$ **is the scale factor observed on the UV brane,** $a(y = 0, t)$

t is proper time for UV brane observer

Can solve y-dependence:

$$a^2 = \Lambda_+(t)e^{2y} + \Lambda_-(t)e^{-2y} - \frac{1}{2}\dot{a}_0^2$$

And relate Λ's

$$\Lambda_+ = \left[\left(\frac{\dot{a}_0}{2}\right)^4 + \lambda\right]\frac{1}{\Lambda_-}$$

Integration constant (→ λ)

Impose UV brane metric junction condition and consistency relation

Radiation as an integration constant

Interesting result - Hubble on the UV brane:

$$H^2 = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{4\lambda}{a_0^4} + \delta_0(2+\delta_0)$$

$$\frac{\kappa^2}{6k}T_0 \equiv 1 + \delta_0$$

Without adding "stuff" beyond cosmological constants/tensions, system of branes has a big-bang

So far, we have ignored the IR brane its motion determined by IR brane metric junction conditions

$$\frac{\frac{\dot{R}}{n}\frac{\dot{a}}{an} + \frac{a'}{a}}{\sqrt{1-\left(\frac{\dot{R}}{n}\right)^2}} = \frac{\kappa^2}{6}T_1$$

$n = \dot{a}/\dot{a}_0$ **and metric functions evaluated at y=R(t)**

sqrt factor crucial - sets speed limit

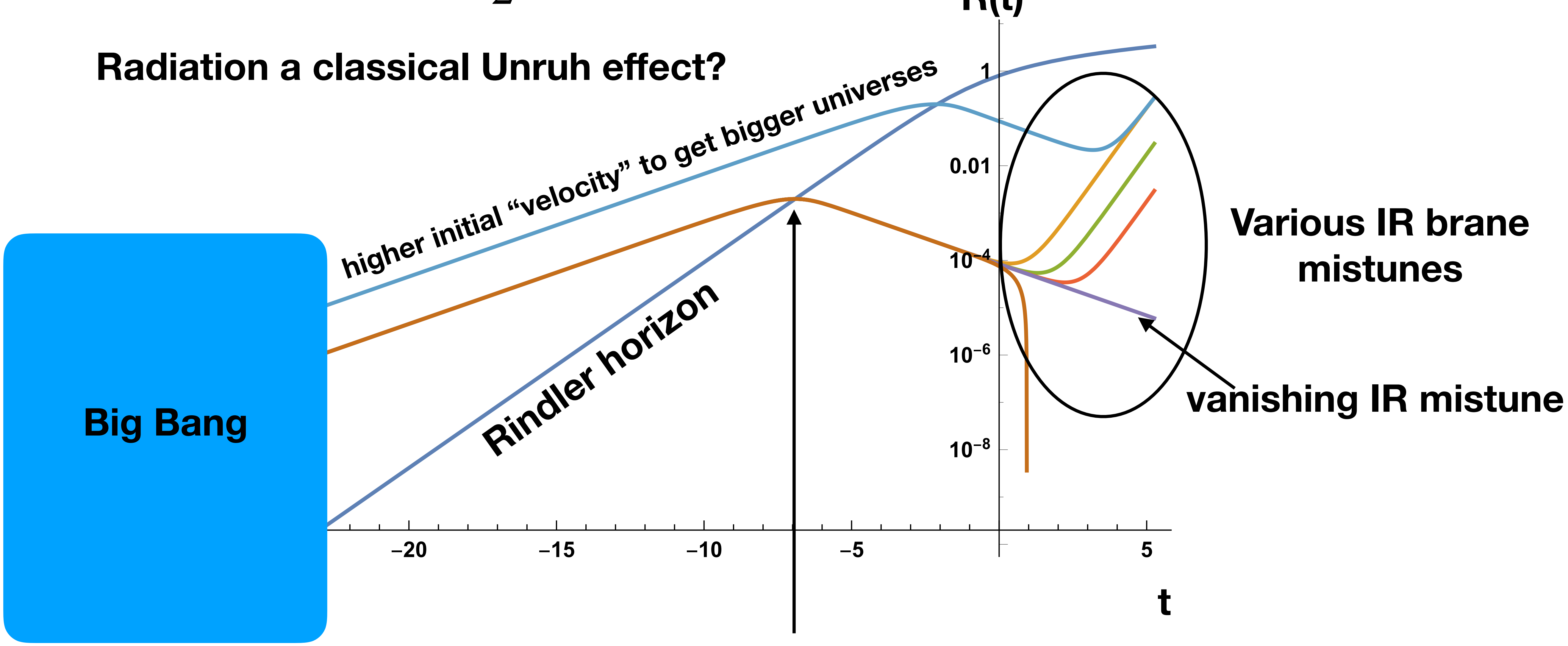
Big bang singularity = overlap of the branes, IR brane moving near the speed of light

Example behavior

Some R(t) solutions when $\delta_0 = 0$:

Rindler Horizon at $y = \frac{1}{2}\log(1 + 4kt)$

Radiation a classical Unruh effect?



IR brane “comes into view” for our UV brane observer

Darkness from Brane-worlds: Radiation without Radiation

- Dark Radiation From RS brane world:
  - Mis-tuning the UV or IR brane tensions can require big-bang cosmology without addition of matter (only cosmological constants)
  - radiation term in Hubble evolution appears as integration constant
  - Observer on UV brane (for example) sees Hubble law for radiation
- Radiation seems Unruh-like - Rindler style horizon in bulk recedes from UV brane with cosmological evolution
  - however it appears classically from bulk gravity

Instabilities of branes: Have we classified all phenomena?

- DGP gravity - interesting infrared modification of GR
  - plagued by ghosts (e.g. radion instability)
- Unstabilized (or unstabilizable) configurations
  - e.g. “Self-organized Higgs Criticality” JH, Rigo, Eröncel 2018
- What are the resolutions of generic brane-world instabilities?

Lessons learned from examples:
relativistic motion of branes is crucial ingredient
resulting phenomenology intriguing

The RS Radion

- Dynamics of the RS radion are well understood at the level of linear perturbation theory
  - expand RS metric to first order in perturbations, solve EE's and identify spectrum
    - 4D massless graviton, massless scalar
      (gains mass if radius is stabilized), massive tower of KK gravitons
- Ill-suited to macroscopic motion of branes
- Linearized metric expansion does not capture full physics of EFT
  - curvatures are small, full EFT can describe large scale motion (tilted branes)

Nambu-Goto Brane Action

- Action of a thin co-dim 1 brane in a spacetime parametrized by embedding functions $X^M(\xi^\mu)$, where ξ are coordinates on world-volume of brane:

$$S_{\text{brane}} = \int d^d\xi T \sqrt{-\det g^{\text{ind}}}$$

$$g^{\text{ind}}_{\mu\nu} = \eta_{MN} \frac{\partial X^M}{\partial \xi^\mu} \frac{\partial X^N}{\partial \xi^\nu}$$

Choose coordinates, $g^{\text{ind}}_{\mu\nu} \approx \eta_{MN} + \partial_M \phi \partial_N \phi$ **where** ϕ **captures motion of brane**

Sqrt enforces causality limit on classical motion of brane

Single massless degree of freedom living on the brane - Goldstone!

Radion action should be similar - IR brane can't move faster than c!

(Saw this in the dark radiation analysis)

Simplest Dilaton Theory

AdS/CFT relates the 5D radion to the dilaton of a spontaneously broken 4D CFT

4D conformally invariant actions:

$$g_{\mu\nu} = e^{2\tau(x)}\eta_{\mu\nu}$$

flat space rescaled by coordinate dependent conformal factor

Build actions consistent with general covariance:

E.g. $S = -\int d^4x\sqrt{g}\left(\Lambda + \frac{1}{12}R\right)$ **+ higher derivatives**

Expanding, performing a field redefinition, get:

$$S = \int d^4x \frac{1}{2}(\partial\phi)^2 - \lambda\phi^4$$

no higher derivatives = no "speed limit"

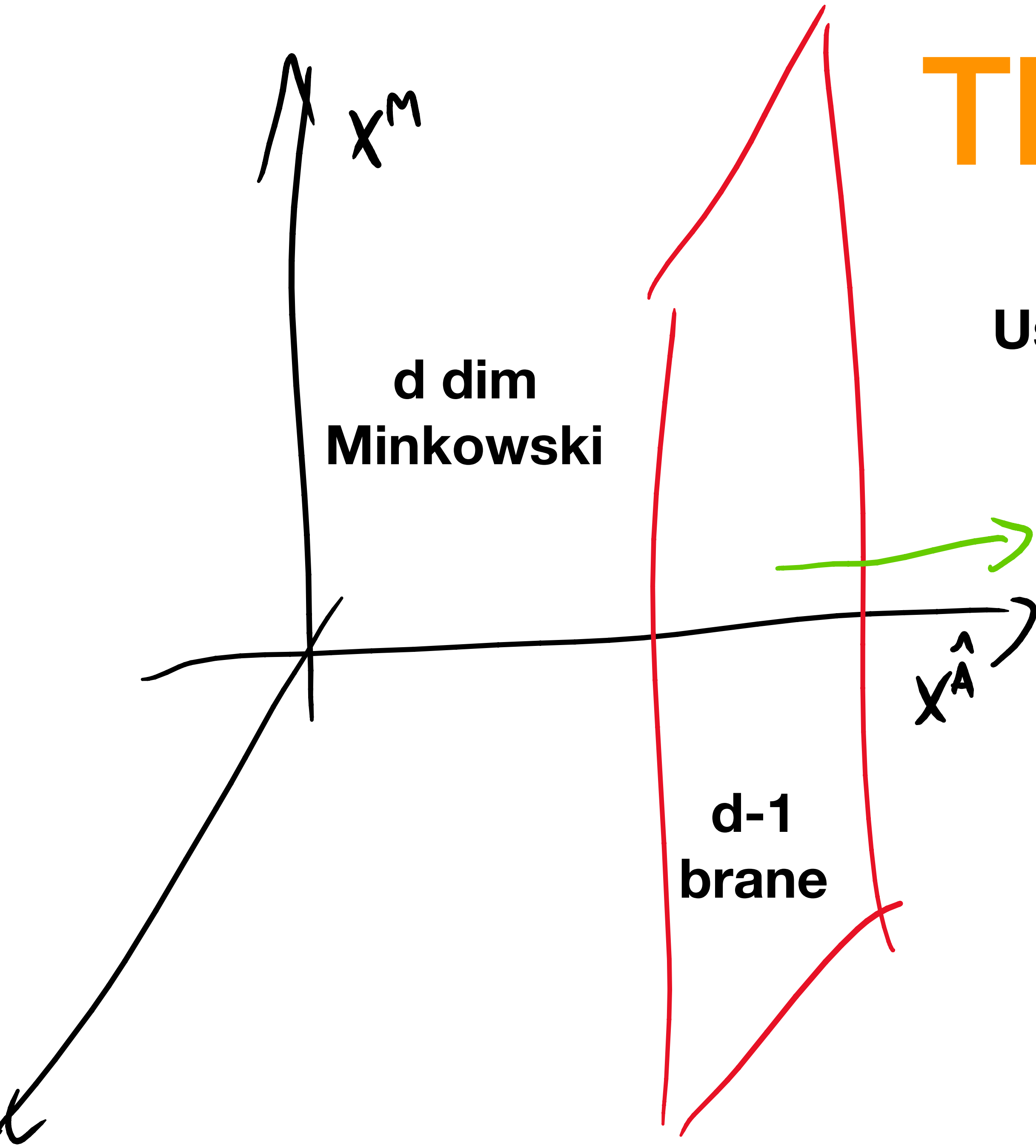
Radion is NOT a simple realization of the dilaton - 5D causality requires specific *pattern* of higher derivative operators (constraints on interacting CFTs)

Baumann, Green, and Hartman 1906.10226

Nambu-Goto From Symmetry Principles

- Embedding of a (d-1) dimensional brane into d-dimensional space endowed with Poincaré algebra, ISO(d-1,1)
  - brane spontaneously breaks subset of translations and rotations/boosts: ISO(d-1,1) → ISO(d-2,1)
  - low energy action should be described by coset construction, Goldstone bosons parametrize coset G/H

The Counting of The Goldstones



Usual counting rules don't apply in same way:

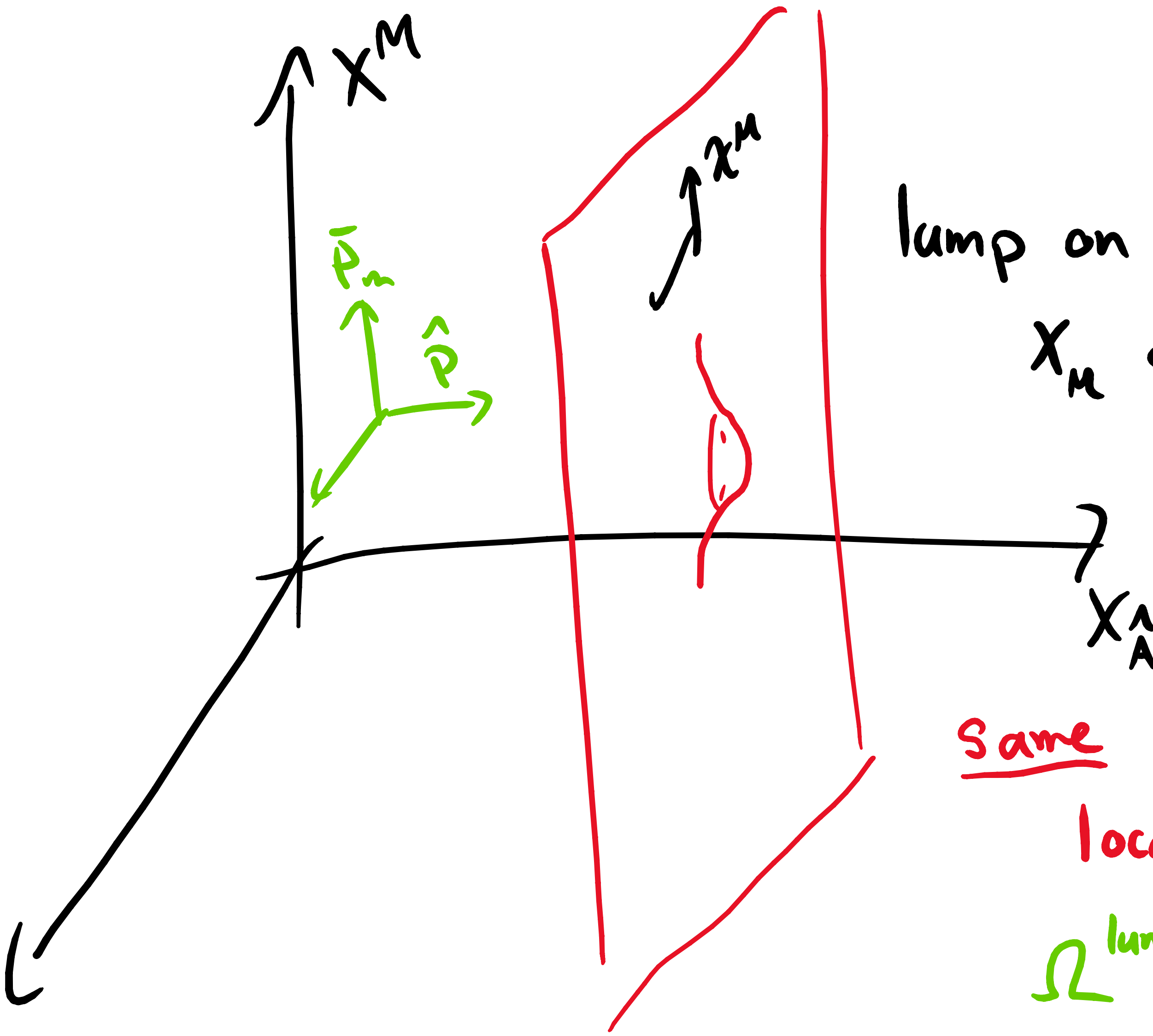
$\left\{ \begin{array}{l} P_{\hat{A}} \\ M_{\hat{A}N} \end{array} \right.$ spontaneously broken generators

Naively expect multiple Goldstone modes

$1 + (d-1) = d$ broken generators

(only 1 massless Goldstone boson)

Making brane bumps:



lump on the brane created by x_μ dependent translations

$$\Omega^{lump} = \exp[i\xi(x)P_{\hat{A}}]$$

Same lump can be created by local (broken) rotations / boosts

$$\Omega^{lump} = \exp[iY^m(x)\bar{P}_m]\exp[i\Phi^N(x^\mu)M_{\hat{A}N}]$$

We should thus be unsurprised that naïve counting of Goldstone bosons gets "wrong" answer

In spontaneous breaking of spacetime symmetries, counting (and spectrum) of Goldstone modes is modified

Aside: Not always a redundancy - brane can carry spin quantum numbers

Hikaka, Noumi, Shiu 1412.5601

CCWZ

Callan, Coleman, Wess, Zumino taught us how to build phenomenological lagrangians:

Low energy theory non-linearly realizes the broken symmetries via Goldstones

$$g = e^{i\Pi^{\hat{a}}T^{\hat{a}}}$$

Formalism to build G-invariant Lagrangians:

find objects that transform nicely under $g_0 \in G$, and make invariants

Maurer-Cartan form:

$$\Omega_\mu \equiv g^{-1}\partial_\mu g = \Omega_\mu^{\hat{a}}T^{\hat{a}} + \Omega_\mu^a T^a$$

Takes values on the algebra, break into components along broken/unbroken directions

Broken part transforms linearly, trace/contract

unbroken part transforms non-linearly, like gauge field:

invariant couplings of NGB's to light matter fields in reps of H

CCWZ and the brane

Delacrétaz, Endlich, Monin, Penco, Riva 1405.7384

A general (but with redundancies) representative of the coset

$$g = e^{iY^M(x)P_M} e^{i\Phi^N(x)M_{\hat{A}N}}$$

broken AND unbroken translations

Expand out the MC form: $\Omega_\mu = i\partial_\mu Y^A \Lambda_A^B P_B + \frac{i}{2}(\Lambda^{-1})_C^A \partial_\mu \Lambda^{CB} M_{AB}$

Φ dep. local lorentz transform

Can express MC form in manner where geometric interpretation clear:

$$\Omega_\mu = ie_\mu^N \left[\bar{P}_N + \nabla_N \xi \hat{P} + \nabla_N \Phi^M M_{\hat{A}M}\right] + iA_\mu^{MN} M_{MN}$$

Set to zero - removes redundancy

(relates Φ's and derivatives of ξ- "Inverse Higgs Constraints)

Can create invariant action using the "vielbein" $e \sim \partial Y$

Simplest Brane Action

Reproduces full Nambu-Goto action:

$$S = -T\int d^d x \det e = -T\int d^d x\sqrt{-\det \partial_\mu Y^M \partial_\nu Y_M}$$

Purely from symmetry considerations, have built up the action for a brane that automatically includes bulk symmetry constraints (Causality)

Relativistic constraints arose from structure of Maurer-Cartan form: vielbein and the Goldstone dependent volume element

Also: gives a formalism for adding higher derivative interactions (extrinsic curvature), couplings to matter fields on the brane, all with corresponding geometric structures

Would like to see this applied to realistic brane-worlds, RS type models know radion must be analog of this

Strategy/Prospects

Radion arises from two branes coupled to dynamical gravity

Formalism must give not just the brane, but also its mixing with bulk metric fluctuations (Radion interaction with both IR brane and UV brane degrees of freedom)

Gravity from cosets:

e.g. Goon, Hinterbichler, Joyce, Trodden 1412.6098

Gauge bosons and gravitons can be viewed as NGB's of infinite number of non-linearly realized space-time global symmetries

Try on KK theory (extra dimension compactified on S_1) - Graviton, Gauge Field, Dilaton

5D Kaluza Klein Theory:

$$P_A^{n\mu_1\ldots\mu_k} = e^{iny/R} x^{\mu_1}\ldots x^{\mu_k} P_A$$
$$Q_A^{\mu_1\ldots\mu_k} = y x^{\mu_1}\ldots x^{\mu_k} P_A$$
$$J_{AB}^{n\mu_1\ldots\mu_k} = e^{iny/R} x^{\mu_1}\ldots x^{\mu_k} J_{AB}$$
$$K_{AB}^{\mu_1\ldots\mu_k} = y x^{\mu_1}\ldots x^{\mu_k} J_{AB}.$$

Infinite goldstones before removing redundancies (inverse Higgs constraints)

$$\tilde{g} = e^{x^\mu P_\mu} e^{y P_y} e^{\phi_n^A P_A^n} e^{\phi_{n\mu}^A P_A^{n\mu}} e^{\phi_{n\mu\nu}^A P_A^{n\mu\nu}} e^{\zeta^A Q_A} e^{\zeta_\mu^A Q_A^\mu} e^{\zeta_{\mu\nu}^A Q_A^{\mu\nu}}$$
$$\times e^{\frac{1}{2}\theta_{n\mu}^{AB} J_{AB}^{n\mu}} e^{\frac{1}{2}\theta_{n\mu\nu}^{AB} J_{AB}^{n\mu\nu}} e^{\frac{1}{2}\tau^{AB} K_{AB}} e^{\frac{1}{2}\tau_\mu^{AB} K_{AB}^\mu} e^{\frac{1}{2}\tau_{\mu\nu}^{AB} K_{AB}^{\mu\nu}}$$

KK modes

Dilatations of S_1

Maurer-Cartan:

Lots of components - lots of redundancies, but easy to spot terms of interest:

$$\Omega_Q^A = \mathrm{d}\zeta^A - \mathrm{d}x^\mu\,\zeta_\mu^A + (\mathrm{d}\phi^B - \mathrm{d}x^\mu\,\phi_\mu^B)\tau_B^A + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta_\mu^B)\tau_B^A c_q - \mathrm{d}y\,\zeta^B\tau_B^A$$

$$\Omega_{Q\mu}^A = \mathrm{d}\zeta_\mu^A - 2\,\mathrm{d}x^\nu\,\zeta_{\mu\nu}^A + (\mathrm{d}\phi_\mu^B - 2\,\mathrm{d}x^\nu\,\phi_{\mu\nu}^B)\tau_B^A + (\mathrm{d}\phi^B - \mathrm{d}x^\mu\,\phi_\mu^B)\tau_{B\mu}^A + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta_\mu^B)\theta_{B\mu}^A$$
$$+ (\mathrm{d}\zeta_\mu^B - 2\,\mathrm{d}x^\nu\,\zeta_{\mu\nu}^B)\tau_B^A c_q + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta_\mu^B)\tau_{B\mu}^A c_q - \mathrm{d}y\,(\zeta^B\tau_{B\mu}^A + \zeta_\mu^B\tau_B^A)$$

Will be the vielbein:

The dilatation

$$\Omega^A = \mathrm{d}\phi^A - \mathrm{d}x^\mu\,\phi_\mu^A + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta_\mu^B)\tau_B^A c_0 - \mathrm{d}y\,\zeta^A$$

$$\Omega_\mu^A = \mathrm{d}\phi_\mu^A - 2\,\mathrm{d}x^\nu\,\phi_{\mu\nu}^A + (\mathrm{d}\phi^B - \mathrm{d}x^\mu\,\phi_\mu^B)\theta_{B\mu}^A + (\mathrm{d}\zeta_\mu^B - 2\,\mathrm{d}x^\nu\,\zeta_{\mu\nu}^B)\tau_B^A c_0$$
$$+ (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta_\mu^B)\tau_{B\mu}^A c_0 - \mathrm{d}y\,\zeta_\mu^A$$

The gauge field

$$\Omega_K^{AB} = \mathrm{d}\tau^{AB} - \mathrm{d}x^\mu\,\tau_\mu^{AB} - \mathrm{d}x^\mu\,\theta_\mu^{[A|C}\tau_C^{|B]} + \mathrm{d}\tau^{[A|C}\,\tau_C^{|B]}c_q - \mathrm{d}y\,\tau^{[A|C}\tau_C^{|B]}$$

$$\Omega_{K\mu}^{AB} = \mathrm{d}\tau_\mu^{AB} - 2\,\mathrm{d}x^\nu\,\tau_{\mu\nu}^{AB} + (\mathrm{d}\theta_\mu^{[A|C} - 2\,\mathrm{d}x^\nu\,\theta_{\mu\nu}^{[A|C)})\tau_C^{|B]} - \mathrm{d}x^\nu\,\theta_\nu^{[A|C}\tau_{C\mu}^{|B]}$$
$$+ (\mathrm{d}\tau^{[A|C} - \mathrm{d}x^\nu\,\tau_\nu^{[A|C)})\tau_{C\mu}^{|B]}c_q - \mathrm{d}y\,\tau^{[A|C}\tau_{C\mu}^{|B]}$$

$$\Omega^{AB} = -\,\mathrm{d}x^\mu\,\theta_\mu^{AB} + \mathrm{d}\tau^{[A|C}\,\tau_C^{|B]}c_0 - \mathrm{d}y\,\tau^{AB}$$

$$\Omega_\mu^{AB} = \mathrm{d}\theta_\mu^{AB} - 2\,\mathrm{d}x^\nu\,\theta_{\mu\nu}^{AB} - \mathrm{d}x^\nu\,\theta_\nu^{[A|C}\theta_{C\mu}^{|B} + (\mathrm{d}\tau^{[A|C} - \mathrm{d}x^\nu\,\tau_\nu^{[A|C)})\tau_{C\mu}^{|B]}c_0 - \mathrm{d}y\,\tau_\mu^{AB}.$$

Messy - but much of it can be ignored (e.g. gravitons, KK modes)

det e makes the sqrt

When the dust settles, will be surprised if the answer isn't analog of Dirac-Born-Infeld Action

Next..the radion

but not in this talk

Conclusions

- Brane worlds are fascinating models for cosmology
  - Extra dimensional brane dynamics can give matter/radiation cosmologies “for free”
    - to see it, require an EFT encoding relativistic constraints on the moduli (a la Nambu-Goto)
    - Radion action to all orders (highly non-trivial dilaton action encoding relativistic bulk causality constraints)
- Moduli are Goldstone bosons - bring the tools of CCWZ to go beyond linearized order
- Future: application to models with kinetic or tachyon instabilities
  - condense the tachyons and ghosts
  - Our Dark Universe as one animal in the zoo of braneworlds?