αs from R(s) (+ R(s) tests of related τ-based analysis strategies)

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• BGKMNPT: PRD98 (2018) 074030 [1805.08176]
### τ and electroproduction FESRs

- $\Pi(Q^2)$: kinematic-singularity-free scalar polarization ($\Pi_{EM}^J \Pi_{ud;V/A}^{J=0+1}$)
- $\rho(s)$: corresponding spectral function
- $w(s)$: here, analytic inside and on $|s|=s_0$
- $\Pi(Q^2) \equiv \Pi_{OPE}(Q^2) + \Pi_{DV}(Q^2)$
  ($\approx \Pi_{OPE}(Q^2)$ for spacelike $Q^2 \gg \Lambda_{QCD}^2$, up to exponentially suppressed corrections)
- Oscillatory (resonance) DV contributions in $\rho(s)$ (+near timelike axis) for $s, |Q^2| \sim$ a few GeV
  $\Rightarrow$ potential non-negligible RHS DV contributions (S. Peris talk)

**FESR relation (Cauchy’s theorem)**

$$\int_{s_{th}}^{s_0} ds \, w(s) \rho(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds \, w(s) \Pi(s)$$
- **OPE contributions**
  
  - D=0 (perturbative) known to 5-loop (O(\(\alpha_s^4\))) order
  
  - D=2 (mass-dependent perturbative): numerically negligible for I=1 \(\tau\) FESRs, small \(O(m_s^2)\), \(O(\alpha_{EM})\) contributions included for EM
  
  - higher D: \([\Pi(Q^2)]^{OPE}_{D\geq4}\) \(\equiv \sum_{D\geq4} [C_D/Q^D]\) with effective condensates \(C_D\)
  
  - for polynomial weights \(w(y) = w(s/s_0) = \sum_{k\geq0} b_k y^k\)
    
    \[
    \frac{-1}{2\pi i} \oint_{|s|=s_0} ds \ w(y) \ [\Pi(Q^2)]^{OPE}_{D\geq4} = \sum_{k\geq1} (-1)^k b_k \ C_{2k+2}/s_0^k
    \]
    
    up to \(\alpha_s\)-suppressed log corrections
  
  - degree \(N\) \(w(y) \leftrightarrow\) unsuppressed OPE contributions to D=2N+2
Qualitative aspects of $\tau$, EM FESR determinations

- Decreasing $\mu$ (with fixed precision at $\mu$) $\leftrightarrow$ increasing precision at $M_Z$
  \[
  \left[ \frac{\delta \alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} \right] \approx \left[ \frac{\alpha_s(M_Z^2)}{\alpha_s(\mu^2)} \right] \left[ \frac{\delta \alpha_s(\mu^2)}{\alpha_s(\mu^2)} \right]
  \]

- Advantage for low-scale $\tau$, EM analyses $\left[ \frac{\alpha_s(M_Z^2)}{\alpha_s(\mu^2)} \right] \approx 1/3$ for $\mu \approx m_\tau$

- BUT decreasing $\mu$ $\leftrightarrow$ increasing NP contributions: how large for $\mu \approx m_\tau$?

- Large $\alpha_s$-independent part of D=0 OPE integral, $c_w [1 + \alpha_s/\pi + w$-dependent h.o.],
  $\Rightarrow$ requirement for control of NP more stringent than naively expected
  e.g. NP to $\sim 0.5\%$ of corresponding spectral integral for $\alpha_s(m_\tau^2)$ to $\sim 3\%$
More re DV contributions

- Poggio, Quinn, Weinberg: DVs localized near timelike axis for intermediate $Q^2$

- With $\rho_{DV}(s) \equiv \frac{1}{\pi} Im \Pi_{DV}(s)$, theory side $\rightarrow$
  $$\frac{-1}{2\pi i} \oint_{|s|=s_0} ds \, w(s) \, \Pi_{OPE}(Q^2) - \int_{s_0}^{\infty} ds \, w(s) \, \rho_{DV}(s)$$

- (Channel-dependent) asymptotic form [2005 ansatz, Boito et al. PRD97 054007 [1711.10316] for theoretical basis]
  $$\rho_{DV}(s) = \kappa \, e^{-\gamma s} \sin(\alpha + \beta s)$$

- $s_0 \leq m_{\tau}^2$ kinematic restriction for $\tau$ FESRs, no such restriction for EM FESRs

- Exponential damping of $\rho_{DV}(s) \Rightarrow$ significant residual integrated DV reduction from modest $s_0$ increase (important advantage of EM c.f. $\tau$-based FESRs)
DV contributions in the $\tau$ and $e^+e^- \rightarrow$ hadrons spectra

- The $\tau$, $I=1$ V+A spectral function, showing “reduced” DVs above $s \sim 1.5$-2 GeV$^2$ (reduced c.f. those for V or A alone)

- In the literature: often used to argue for the neglect of DVs in this region

- However: assessment of relative roles of DV and $\alpha_s$-dependent perturbative contributions complicated by presence of $\alpha_s$-independent contribution (e.g. same figure with different (larger) $\alpha_s$-independent contribution)
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DV contributions in the τ and e^+e^- → hadrons spectra

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- C.f. the τ, I=1 V+A figure, now with the non-dynamical, α_s-independent parton model contribution removed
Evidence for the oscillatory, exponentially damped asymptotic DV behavior in the G-parity separated $I=1$ part of $R(s)$
\( \alpha_s \) from FESRs with KNT 2018 R(s) data

- \( \rho_{EM}(s) = \frac{1}{12\pi^2} R(s) \)
- Start with analyses neglecting DVs, \( s_0 \sim m_\tau^2 \) and above: fit parameters \( \alpha_s \) and relevant OPE condensates \( C_D \)
- Test stability of OPE parameters to inclusion of DVs (extended fits with \( I=1 \) DV parameters constrained from \( \tau \), new \( I=0 \) DV parameters \( \kappa_0, \alpha_0 \) fit with \( \beta_0 \approx \beta_1, \gamma_0 \approx \gamma_1 \) assumed)
More on the pure-OPE, no-DV fits

• OPE treatment
  - D=0 to 5 loops (O(\(\alpha_s^4\))), including O(\(\alpha_{EM}^\cdot\)) contributions
  - O(\(m_s^2\)) \(D = 2\) to \(3\) loops
  - avoid weights with term linear in \(s\) (convergence issues from Beneke, Boito, Jamin renormalon model studies [JHEP 1301 (2013) 125 [1210.8038]]

• Weight choices, \(w(y) = w(s/s_0)\)
  - \(w_0(y) = 1\) (no DV suppression near timelike point \(s=s_0\), fit parameter \(\alpha_s\))
  - \(w_2(y) = 1-y^2\) (single “pinch” DV suppression near \(s=s_0\), fit parameters \(\alpha_s, C_6\))
  - \(w_3(y) = 1 – 3y^2 + 2y^3\) (double “pinch” near \(s=s_0\), fit parameters \(\alpha_s, C_6, C_8\))
  - \(w_4(y) = 1 – 2y^2 + y^4\) (double “pinch” near \(s=s_0\), fit parameters \(\alpha_s, C_6, C_{10}\))
D=0 FOPT, no-DV fit results, \( w_3, w_4 \) FESRs, fit windows \( s_0^{\text{min}} \leq s_0 \leq 4 \, \text{GeV}^2 \)

<table>
<thead>
<tr>
<th>( s_0^{\text{min}} ) [GeV(^2)]</th>
<th>( \chi^2/\text{dof} ) ([w_3])</th>
<th>p-value ([w_3])</th>
<th>( \alpha_s(m_t^2) ) ([w_3])</th>
<th>( C_6 ) [GeV(^6)] ([w_3])</th>
<th>( \chi^2/\text{dof} ) ([w_4])</th>
<th>p-value ([w_4])</th>
<th>( \alpha_s(m_t^2) ) ([w_4])</th>
<th>( C_6 ) [GeV(^6)] ([w_4])</th>
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<td>0.0027(20)</td>
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<td>0.0070(25)</td>
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<td>0.302(16)</td>
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<td>0.301(16)</td>
<td>0.0081(29)</td>
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<td>0.292(18)</td>
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<td>0.32</td>
<td>0.289(19)</td>
<td>0.0036(39)</td>
<td>2.3/2</td>
<td>0.32</td>
<td>0.288(19)</td>
<td>0.0037(39)</td>
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</table>
Theory vs experiment matches, $s_0^{\text{min}} = 3.25 \text{ GeV}^2$, no-DV fits

• Left: $w_0$, right: $w_2$
  solid/dashed lines: FOPT/CIPT $D=0$ fits

• Left: $w_3$, right: $w_4$
  solid/dashed lines: FOPT/CIPT $D=0$ fits
\( \alpha_s(m_T^2) \) vs \( s_0^{min} \), various weights, with and without DVs

- **Blue**: \( w_0 \) FESR, no DVs
- **Red**: \( w_2 \) FESR, no DVs
- **Green**: \( w_3 \) FESR, no DVs
- **Black**: \( w_0 \) FESR, with DVs

**Addition of DVs stabilizes fits at lower \( s_0 \)**
Final averaged EM results for $\alpha_s$

**FOPT:** $\alpha_s^{(3)}(m_T^2) = 0.298(17) \leftrightarrow \alpha_s^{(5)}(M_Z^2) = 0.1158(22)$

**CIPT:** $\alpha_s^{(3)}(m_T^2) = 0.304(19) \leftrightarrow \alpha_s^{(5)}(M_Z^2) = 0.1166(25)$

- c.f. analogous ALEPH 2013 I=1, $\tau$-data-based analysis, including DVs
  

  **FOPT:** $\alpha_s^{(3)}(m_T^2) = 0.296(10) \leftrightarrow \alpha_s^{(5)}(M_Z^2) = 0.1155(14)$
  
  **CIPT:** $\alpha_s^{(3)}(m_T^2) = 0.310(14) \leftrightarrow \alpha_s^{(5)}(M_Z^2) = 0.1174(17)$

- EM errors currently data dominated

- *Note 0.014 $\rightarrow$ 0.006 reduction in FOPT-CIPT $\alpha_s(m_T^2)$ difference in higher-scale EM vs $\tau$ analysis (hence reduced theory uncertainty)*
PART II: R(s)-based tests of the “truncated OPE” (tOPE) approach (used for most results included in the PDG assessment of $\alpha_s$ from $\tau$)

- [E.g., Pich-Lediberder PLBB289, 165; ALEPH; OPAL; Pich, Rodriguez-Sanchez PRD94, 034027 [1605.06830]]
- $\tau$, $I=1$ V, A, V+A channel analyses using (at least) doubly pinched weights, neglecting DVs (with V+A argued safest)
- Final results from $s_0 = m_\tau^2$ only (minimizes residual DV contributions)
- Kinematic weight case $w_\tau(y) = 1-3y^2+2y^3$ (spectral integral from inclusive BFs)
  - insufficient as theory side involves 3 OPE parameters $\alpha_s$, $C_6$, $C_8$
- Additional (higher-degree-weight) FESRs to fit $C_6$, $C_8$
- Complication: new degree 4 $w(y)$ brings in the new OPE parameter $C_{10}$, new degree 5 $w(y)$ the new OPE parameter $C_{12}$, etc. ⇒ # of OPE parameters always exceeds # $s_0 = m_\tau^2$ spectral integrals without further assumptions/OPE truncation
With conventional Pich-Le Diberder spectral weights \( w_{km}(y) = y^m(1-y)^{2+k}(1+2y) \)

### D≥4 OPE contributions (dimensionless)

<table>
<thead>
<tr>
<th>Weight</th>
<th>D=4</th>
<th>D=6</th>
<th>D=8</th>
<th>D=10</th>
<th>D=12</th>
<th>D=14</th>
<th>D=16</th>
</tr>
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<tbody>
<tr>
<td>( w_{00} = w_\tau )</td>
<td>-3C_6/s_0^3</td>
<td>-2C_8/s_0^4</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_{10} )</td>
<td>C_4/s_0^2</td>
<td>-3C_6/s_0^3</td>
<td>-5C_8/s_0^4</td>
<td>-2C_{10}/s_0^5</td>
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</tr>
<tr>
<td>( w_{11} )</td>
<td>-C_4/s_0^2</td>
<td>-C_6/s_0^3</td>
<td>3C_8/s_0^4</td>
<td>5C_{10}/s_0^5</td>
<td>C_{12}/s_0^6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_{12} )</td>
<td>C_6/s_0^3</td>
<td>C_8/s_0^4</td>
<td>-3C_{10}/s_0^5</td>
<td>-5C_{12}/s_0^6</td>
<td>-C_{14}/s_0^7</td>
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</tr>
<tr>
<td>( w_{13} )</td>
<td></td>
<td>-C_8/s_0^4</td>
<td>-C_{10}/s_0^5</td>
<td>3C_{12}/s_0^6</td>
<td>5C_{14}/s_0^7</td>
<td>C_{16}/s_0^8</td>
<td></td>
</tr>
</tbody>
</table>

- 5 \( s_0 = m_\tau^2 \) spectral integrals; 4 OPE fit parameters: \( \alpha_s, C_4, C_6, C_8 \)
- D=10, 12, 14, 16 contributions dropped (the tOPE assumption) on grounds of assumed scaling with additional factors of \( \sim (\Lambda_{QCD}^2/m_\tau^2) \)
• With Pich, Rodriguez-Sanchez “optimal” weights $w_{2k}(y) = 1 - (k+2)y^{k+1} + (k+1)y^{k+2}$

**D ≥ 4 OPE contributions (dimensionless)**

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<th>Weight</th>
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<th>D=6</th>
<th>D=8</th>
<th>D=10</th>
<th>D=12</th>
<th>D=14</th>
<th>D=16</th>
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<tbody>
<tr>
<td>$w_{21} = w_\tau$</td>
<td>$-3C_6/s_0^3$</td>
<td>$-2C_8/s_0^4$</td>
<td></td>
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<td></td>
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<td></td>
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<tr>
<td>$W_{22}$</td>
<td>$4C_8/s_0^4$</td>
<td>$3C_{10}/s_0^5$</td>
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<tr>
<td>$W_{23}$</td>
<td></td>
<td>$-5C_{10}/s_0^5$</td>
<td>$-4C_{12}/s_0^6$</td>
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<tr>
<td>$W_{24}$</td>
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<td>$6C_{12}/s_0^6$</td>
<td>$5C_{14}/s_0^7$</td>
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<tr>
<td>$W_{25}$</td>
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<td></td>
<td>$-7C_{14}/s_0^7$</td>
<td>$-6C_{16}/s_0^8$</td>
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</tbody>
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- 5 $s_0 = m_\tau^2$ spectral integrals; 4 OPE fit parameters: $\alpha_s$, $C_6$, $C_8$, $C_{10}$
- D=12, 14, 16 contributions dropped (the tOPE assumption) on grounds of assumed scaling with additional factors of $\sim (\Lambda_{QCD}^2/m_\tau^2)$
tOPE assumptions, potential issues, and possible tests

• Basic tOPE assumptions
  - $s_0 = m_T^2$ large enough that residual integrated DVs negligible (at least for doubly pinched $w(y)$)
  - integrated OPE series behaves as if (rapidly) converging with $D$ for $s_0 = m_T^2$, out to at least $D=16$

• Potential tOPE issues
  - $s_0 = m_T^2$ only: precludes variable-$s_0$ tests of validity of assumed neglect of residual DVs
  - Even if residual DVs negligible, OPE asymptotic (at best) ⇒ assumed scaling with increasing $D$ (and related tOPE neglect of unsuppressed higher $D$ terms) certainly incorrect in general

• Potential tests of tOPE assumptions
  - exponential damping of $\rho_{DV}(s)$, decrease of higher $D$ non-perturbative contributions with increasing $s_0$ ⇒ if assumptions good for some $s_0^*$, should be even better for $s_0 > s_0^*$
  - Kinematic constraint $s_0 \leq m_T^2$ precludes test with $s_0 > m_T^2$ in $\tau$, but not EM case
An R(s)-based strategy for testing tOPE assumptions

- If residual integrated DVs not negligible, tOPE assumptions incorrect and tOPE ruled out, so assume DVs negligible for $s_0 \sim m_T^2$ and above and test OPE truncation assumption

- Find $s_0^* \geq m_T^2$ admitting a successful $s_0 = s_0^*$ tOPE optimal weight or $w_{km}$ spectral weight fit

- With resulting tOPE fit parameters, test theory predictions for the $s_0 > s_0^*$ spectral integrals

- Because of strong correlations between (i) spectral integrals for different $s_0$, (ii) theory integrals for different $s_0$, (iii) fitted OPE parameters and $\rho_{EM}(s)$ data (hence theory and spectral integrals) form single difference combinations

$$\Delta I_{th/exp}^{th/exp}(s_0; s_0^*) \equiv I_{th/exp}^{th/exp}(s_0) - I_{th/exp}^{th/exp}(s_0^*)$$

and display test results in double difference theory-minus-experiment form

$$\Delta^{(2)}(s_0; s_0^*) \equiv \Delta I_{th}(s_0; s_0^*) - \Delta I_{exp}(s_0; s_0^*)$$
tOPE test results

- $s_0^* = m_T^2$: very low correlated-fit p-values, incompatible correlated, diagonal fit $\alpha_s(m_T^2)$, incompatible $w_{km}$, optimal weight fit $\alpha_s(m_T^2)$
- $s_0^* > m_T^2$ for acceptable correlated EM fit (correlated, diagonal $\alpha_s(m_T^2)$ then compatible, but correlated $w_{km}$, optimal weight not)

<table>
<thead>
<tr>
<th>Weight type</th>
<th>$s_0^*$ [GeV$^2$]</th>
<th>p-value [corr fit]</th>
<th>$\alpha_s(m_T^2)$ [corr]</th>
<th>$\alpha_s(m_T^2)$ [diag]</th>
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<tbody>
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<tr>
<td>Optimal</td>
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<td>0.308(4)</td>
<td>0.245(10)</td>
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<td>$w_{km}$</td>
<td>3.7</td>
<td>0.16</td>
<td>0.277(5)</td>
<td>0.268(9)</td>
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<tr>
<td>Optimal</td>
<td>3.6</td>
<td>0.41</td>
<td>0.264(5)</td>
<td>0.256(12)</td>
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</table>
Correlated $s_0^* = 3.6$ GeV$^2$ optimal weight fit theory-experiment matches
Correlated $s_0^* = 3.7 \text{ GeV}^2$ $w_{km}$ fit theory-experiment matches
\( \Delta^{(2)}(s_0; s_0^*) \) correlated \( s_0^* = 3.6 \text{ GeV}^2 \) optimal weight fit results
$\Delta^{(2)}(s_0; s_0^*)$ correlated $s_0^* = 3.7 \text{ GeV}^2$ $w_{\text{km}}$ fit results
Conclusions of the R(s)-based tOPE strategy tests

• Good $\chi^2$ from single-$s_0$ tOPE fit demonstrably insufficient to ensure reliability of neglect of DVs and/or prematurely truncated OPE theory representation

$\Rightarrow$ even if integrated DVs negligible for conventional $\tau$ analyses (doubtful: see e.g. S. Peris talk), $\alpha_s$ results from tOPE implementations unreliable

• Strong correlations between different-$s_0$ spectral integrals, different-$s_0$ OPE integrals, and fitted OPE and spectral integrals make it easy to be misled re level of theory-experiment agreement: double-difference-type tests crucial
BACKUP SLIDES
$s_0^* = m_\tau^2$ diagonal tOPE optimal weight fit theory-experiment matches
Δ^{(2)}(s_0; s_0^* = m_T^2) diagonal optimal weight fit results