α_s from non-strange hadronic τ decays

SANTI PERIS (UAB + IFAE-BIST)

in Collaboration with D. Boito, M. Golterman, K. Maltman and J. Osborne.

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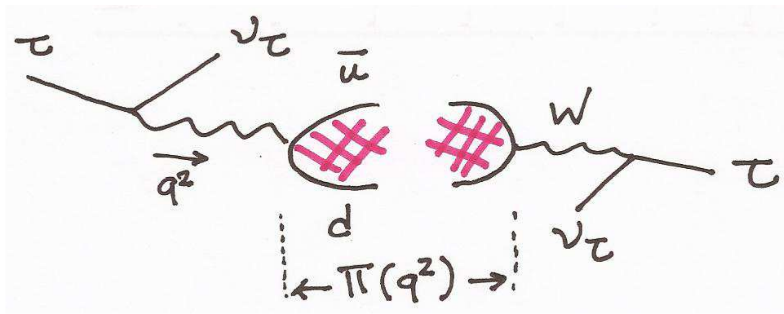
$$\delta\alpha_s(M_Z) \simeq \left(\frac{\alpha_s(M_Z)}{\alpha_s(m_\tau)}\right)^2 \delta\alpha(m_\tau)$$

QCD in τ decay



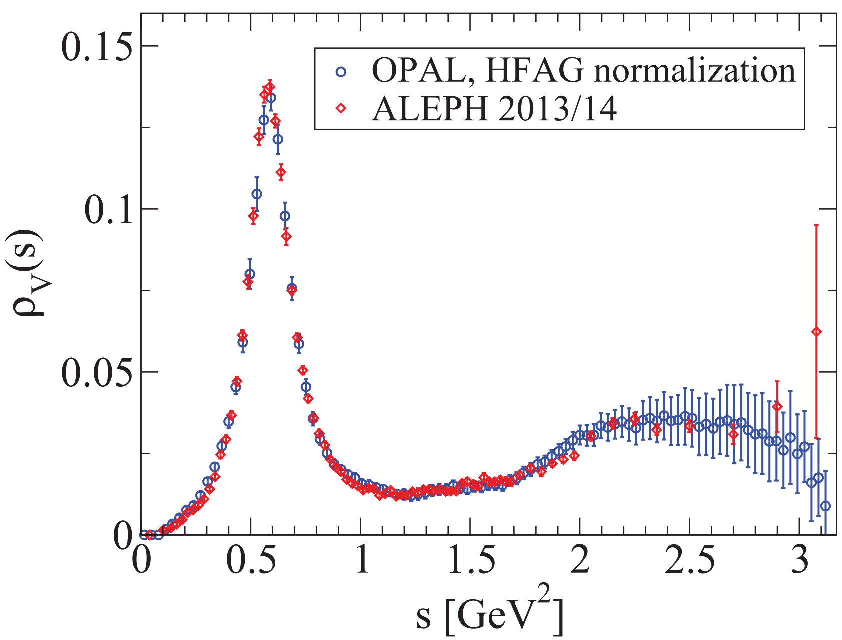
$$w_T(s;s_0) = \left(1 + 2\frac{s}{s_0}\right) \underbrace{\left(1 - \frac{s}{s_0}\right)^2}_{\text{doubly pinched}}$$

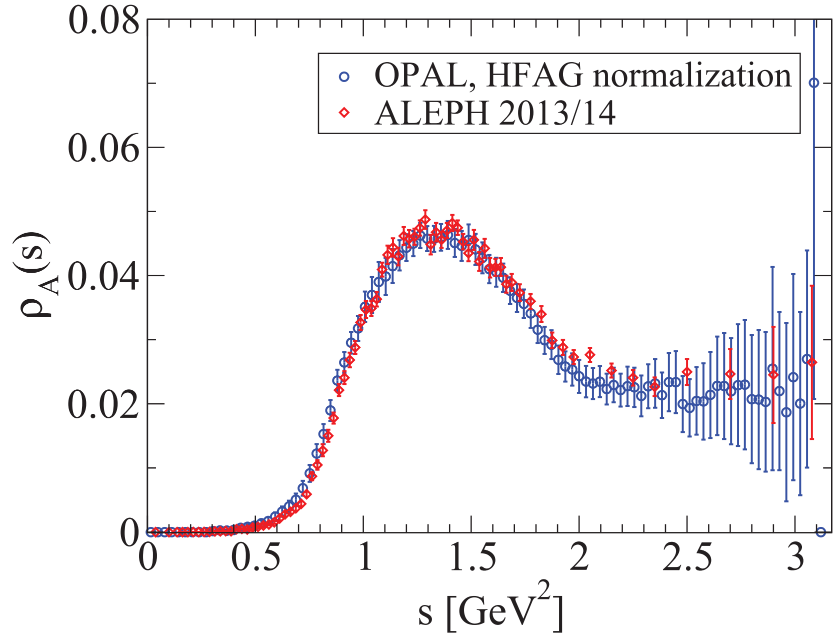
$$w_L(s;s_0) = 2\left(\frac{s}{s_0}\right) \underbrace{\left(1 - \frac{s}{s_0}\right)^2}_{\text{doubly pinched}}$$

$s_0 = m_\tau^2$

$\rho_{V,A} = \frac{1}{\pi}\mathrm{Im}\Pi_{V,A}$

$$\frac{\Gamma(\tau\to\nu_\tau \mathrm{H}_{ud}(\gamma))}{\Gamma[\tau\to\nu_\tau e\bar{\nu}_e(\gamma)]} = 12\pi^2|V_{ud}|^2 S_{EW}\int_0^{s_0}\frac{ds}{s_0}\left[w_T(s;s_0)\rho_{V+A}^{(1+0)}(s) - w_L(s;s_0)\rho_A^{(0)}(s)\right]$$

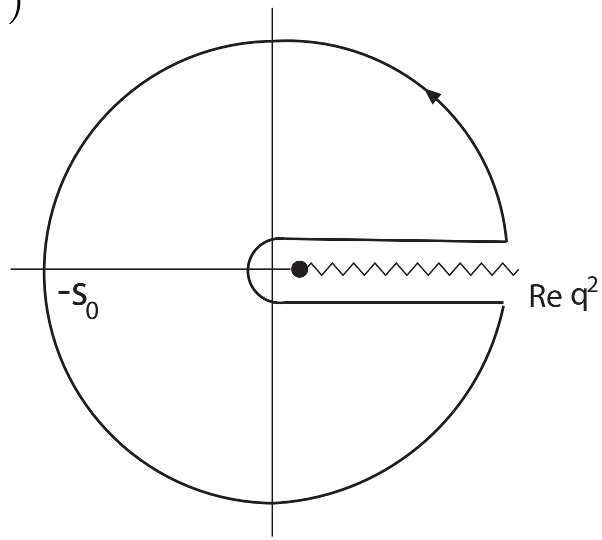




Theoretical Foundations: FESRs

Shankar '77; Braaten-Narison-Pich '92

★ $\Pi(q^2)$



★ "Cauchy's Theorem" $(z = q^2\ ;\ w(z) = \text{polynomial})$:

$$\int_0^{s_0} dt\ w(t)\ \underbrace{\frac{1}{\pi}\text{Im}\Pi(t)}_{exp.} = \frac{-1}{2i\pi}\oint_{|z|=s_0} dz\ w(z)\ \underbrace{\Pi(z)}_{\Pi_{\rm OPE}(z)+\Pi_{DV}(z)}$$

(Poggio,Quinn,Weinberg '76)

$$= \frac{-1}{2i\pi}\oint_{|z|=s_0} dz\ w(z)\ \underbrace{\Pi_{\rm OPE}(z)}_{\mathcal{O}(\alpha_s^5)+condensates}$$

$$-\int_{s_0}^{\infty} dt\ w(t)\ \frac{1}{\pi}\text{Im}\Pi_{DV}(t)$$

(Cata-Golterman-S.P. '05, '08, '09)

FESRs and the OPE

$$\int_0^{s_0} dt\, w(t)\, \underbrace{\frac{1}{\pi}\mathrm{Im}\Pi(t)}_{exp.} = \frac{-1}{2i\pi}\oint_{|z|=s_0} dz\, w(z)\, \underbrace{\Pi_{\rm OPE}(z)}_{\mathcal{O}(\alpha_s^5)+condensates} \quad - \int_{s_0}^{\infty} dt\, w(t)\, \frac{1}{\pi}\mathrm{Im}\Pi_{DV}(t)$$

Note,

- $w(t) \sim t^N \iff$ contribution from $\Pi_{OPE} \sim \frac{C_{2N+2}}{s_0^{N+1}}$ $\quad$ (C_N, condensates).
- $\Pi(z) = \Pi_{OPE}(z) + \Pi_{DV}(z)$. $\quad$ OPE convergent $\Leftrightarrow \Pi_{DV} = 0$.

  i.e. $\quad \frac{1}{\pi}\mathrm{Im}\Pi(t) = \mathrm{Pert.\ Theory} + \mathrm{DVs}$
- $\Pi_{\rm OPE}$ expected asymptotic : $\frac{1}{\pi}\mathrm{Im}\Pi_{DV}(t) \sim\ e^{-\gamma t} \times (oscillation), \quad t \to \infty$.

(Notice $\frac{1}{\pi}\mathrm{Im}\Pi_{DV}(t)$ is on the Minkowski axis.)

$\Rightarrow$ use polynomial with zero $w(s_0) = 0$ (“pinching”)

<u>WARNING</u>: more pinching $\Rightarrow$ higher degree polynomial $\Rightarrow$ higher condensates to consider.

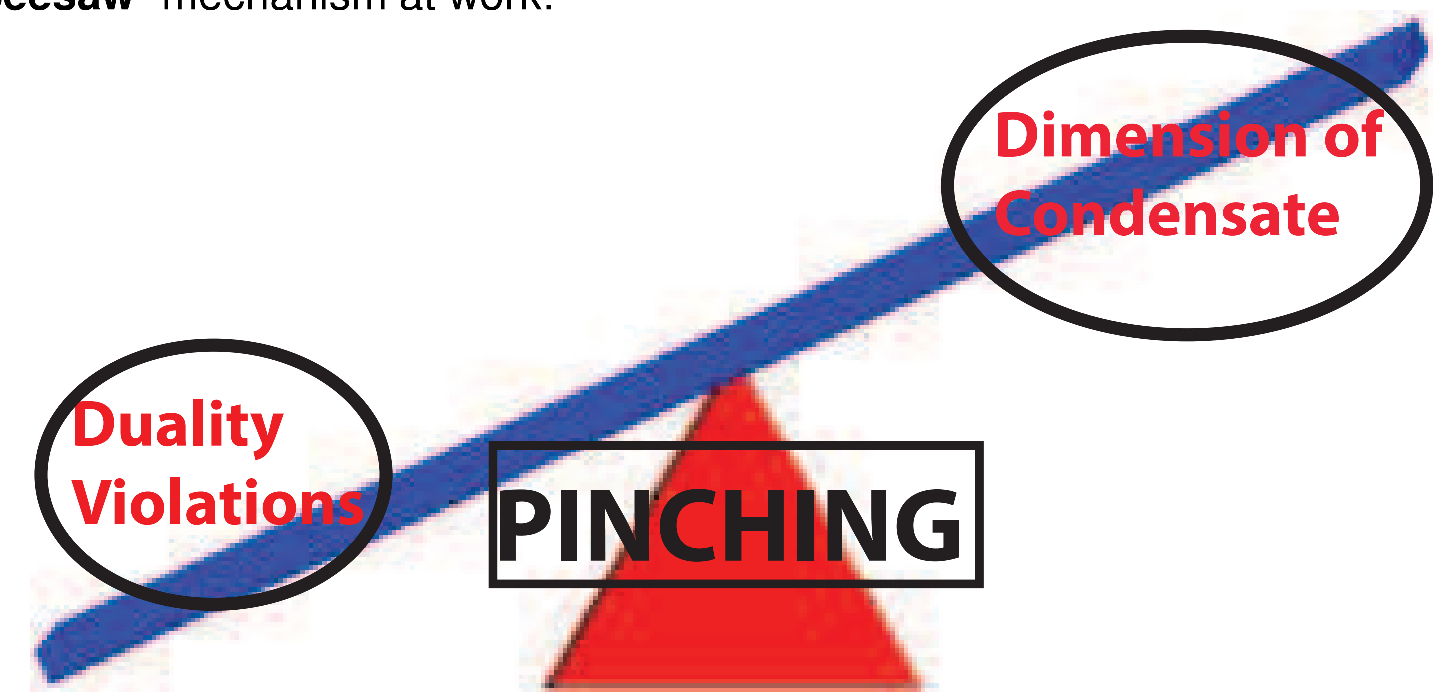
Main Theoretical Message:

(Maltman-Yavin '08, Boito et al. '11)

★ No free lunch. With pinching one has a **price to pay**:

It is not possible to simultaneously **suppress DVs** and **condensates**

★ "**Seesaw**" mechanism at work:



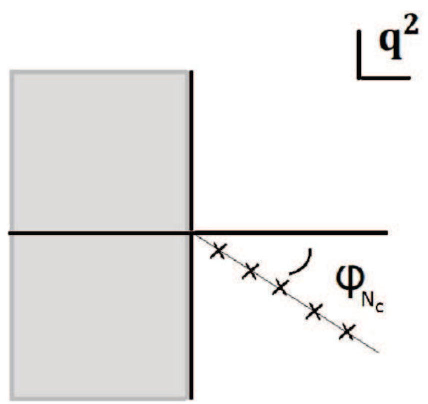
Theoretical Foundations: DVs

Boito, Caprini, Golterman, Maltman, SP '17

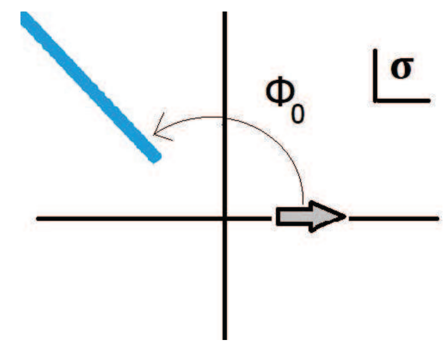
-A theory of DVs requires NP input.
-Let's start with Disp. Rel. of Adler's function ($q^2 < 0$, Euclidean):

$$\mathcal{A}(q^2) = -q^2 \int_0^\infty d\sigma \; \mathrm{e}^{\sigma q^2} \; \sigma \mathcal{B}(\sigma) \quad , \quad \mathcal{B}(\sigma) = \int_0^\infty dt \; \rho(t) \; \mathrm{e}^{-\sigma t} \qquad \text{(exact!!)}$$

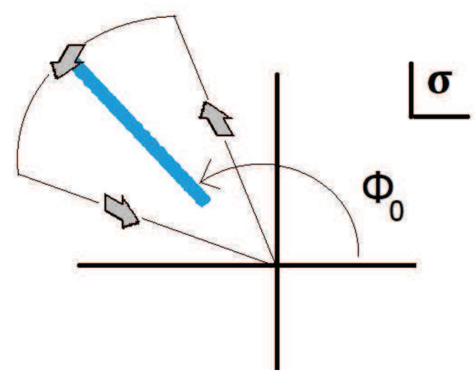
$\sigma q^2 = (\sigma > 0, q^2 < 0) = (\sigma < 0, q^2 > 0)$, i.e. rotate $\sigma \Leftrightarrow q^2$ analytic continuation.



$\Leftrightarrow$



$\Pi_{DV}(q^2) \quad \sim$



Towards a Theory of DVs

Boito et al. '17

- Asymptotic Regge-like spectrum for $N_c = \infty$ in chiral limit:

-Masses

$$M^2(n) = n\,\Lambda^2_{QCD} + b\log n + c + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_1}}, \frac{1}{n^{\lambda_1}(\log n)^{\nu_2}}\right) \quad , \quad n \gg 1\,.$$

-Decay constants

$$F(n) \propto 1 + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_3}}, \frac{1}{n^{\lambda_2}(\log n)^{\nu_4}}\right) \quad , \quad n \gg 1\,.$$

- When $N_c = 3,\ n \gg 1$ ($d = 2$ QCD, strings, phenomenology):

Blok et al. '98; Shifman et al. '08; Masjuan et al. '12

$$\frac{\Gamma}{M} \sim \frac{a}{N_c} + \ldots$$

leads to $(\mathrm{Re}\, q^2 > 0\ ,\ \mathrm{Im}\, q^2 > 0)$

$$\frac{1}{\pi}\mathrm{Im}\,\Pi_{DV}(q^2) \sim \mathrm{e}^{-2\pi\frac{a}{N_c}\ \frac{q^2}{\Lambda^2_{QCD}}} \sin\left[\frac{2\pi}{\Lambda^2_{QCD}}\left(q^2 - c - b\log\frac{q^2}{\Lambda^2_{QCD}}\right)\right]\left(1 + \mathcal{O}\left(\frac{1}{N_c}; \frac{1}{q^2}; \frac{1}{\log q^2}\right)\right)$$

$$\equiv e^{-\delta - \gamma q^2}\ \sin\left(\alpha + \beta\, q^2\right) \quad (\text{up to logs})$$

✓

Th. Foundations: CIPT vs. FOPT

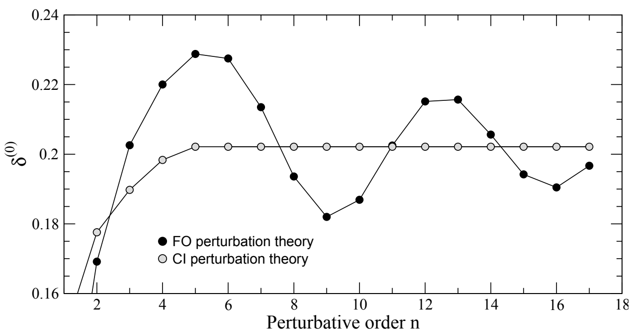
Caprini-Fischer '09; Menke '09; Descotes-Malaescu '09; Cvetic '10,...

★Partial integration: $\oint_{|z|=s_0} dz\, w(z)\, \Pi(z) = \oint_{|z|=s_0} dz\, \widetilde{w}(z)\, \underbrace{\left(-z\frac{d}{dz}\Pi(z)\right)}_{\text{Adler's D(z)}}$

★ $\mathrm{D}(z) = \frac{1}{4\pi^2}\sum_{n,k} c_{n,k}\ \underline{\alpha_s(\mu)^n\ \log^k\left(-z/\mu^2\right)}$, what μ ?

Known up to α_s^4 (Baikov et al. '08) with (educated) guess on $c_{5,1}$. (Beneke et al. '08; Boito et al. '18)

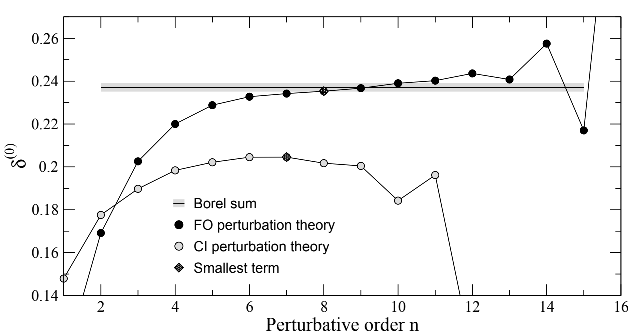
★ Since $z = s_0\ \mathrm{e}^{i\phi}$, two (∼ extreme) choices:



● CIPT: $\mu^2 = -z$.

Good *if* no important cancellations between Adler function series and running of α_s (e.g. if Adler function stops at finite order)

LeDiberder-Pich '92, Pivovarov '92



● FOPT: $\mu^2 = s_0$.

Good *if* there are cancellations between Adler function series and the running coupling (and leading IR renormalon dominates).

Beneke-Jamin '08, Beneke-Boito-Jamin '13

The Truncated-OPE Strategy

(LeDiberder, Pich '92; Davier et al. '14; Pich, Rguez-Sanchez '16)

★ Since τ width depends on α_s and $C_{6,8}$ $\Rightarrow$ more input needed .

- In order to try to suppress DVs, use polynomials $w(t)$ with double, triple pinching:

  E.g. $w_{kl}(y) = (1+2y)\,(1-y)^{2+k}\,y^l \quad , \quad y = \frac{s}{s_0}, \quad s_0 = m_\tau^2$ (only)

  with $(k,l) = \Big\{(0,0),(1,0),(1,1),(1,2),(1,3)\Big\}$. N.B.: $(0,0)$ is kin. weight.

  $\Longrightarrow$ polynomials up to $\mathcal{O}(y^7)$

  $\Longrightarrow$ $\exists$ contribution from condensates up to dimension 16.

- However, contribution from dimension ≥ 10 are set to zero, together with all DVs.

- Very dubious (if not murky) approximation to make with an asymptotic expansion, such as OPE. (See later and K. Maltman's talk.)

Safest to stay at low polynomials.

$\Longrightarrow$ **Errors underestimated** (Boito et al. '17)

The DV Strategy

(Boito et al. '11 and '12)

- Do not use $w(y)$ with a term linear in y. No gluon cond. contribution. (Beneke et al. '13)
- No neglect of any contributing condensate. Let the data speak.
- Do not assume that Duality Violations are zero. Let the data speak.

  For $s \geq s_{min}$ (Regge/asymp. series):

$$\frac{1}{\pi}\mathrm{Im}^{V,A}_{DV}(s) = \mathrm{e}^{-\delta_{V,A}-\gamma_{V,A}s}\sin\left(\alpha_{V,A}+\beta_{V,A}s\right)$$

  c.f. Truncated-OPE assumption $\Leftrightarrow$ $\mathrm{e}^{-\delta_{V,A}} = 0$.

- Fit to $\alpha_s, C_{D=6,8}$ and DV parameters with 3 weights (avoid high-orders in OPE):

$$w_0 = 1, w_2 = 1 - y^2 \quad \text{and} \quad w_3 = (1-y)^2(1+2y)$$

  Use all data for $s_0 \geq s_{min}$, (s_{min} to be determined by the fit as well).

- Use V and A.
- Make many checks: Weinberg sum rules, etc...

We did lots of other fits as well...

Fits :

- V channel, $w_0 = 1$.
- V and A channels, $w_0 = 1$.
- V channel, $w_0 = 1$ and $w_2 = 1 - y^2$.
- V and A channels, $w_0 = 1$ and $w_2 = 1 - y^2$.
- V channel, $w_0 = 1, w_2 = 1 - y^2$ and $w_3 = (1 - y)^2(1 + 2y)$.
- V and A channels, $w_0 = 1, w_2 = 1 - y^2$ and $w_3 = (1 - y)^2(1 + 2y)$.

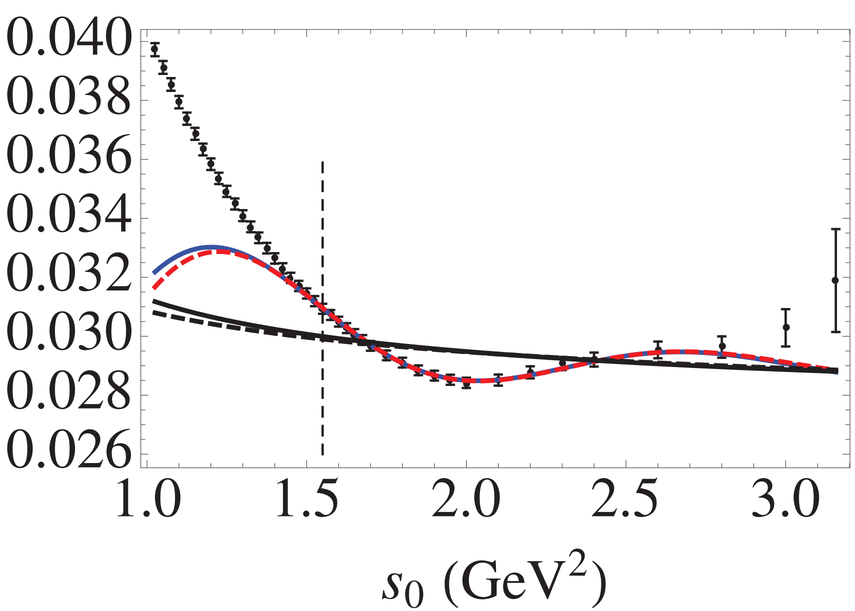
Consistent results in all cases

Example: Fit to $w_0 = 1$, V channel (I).

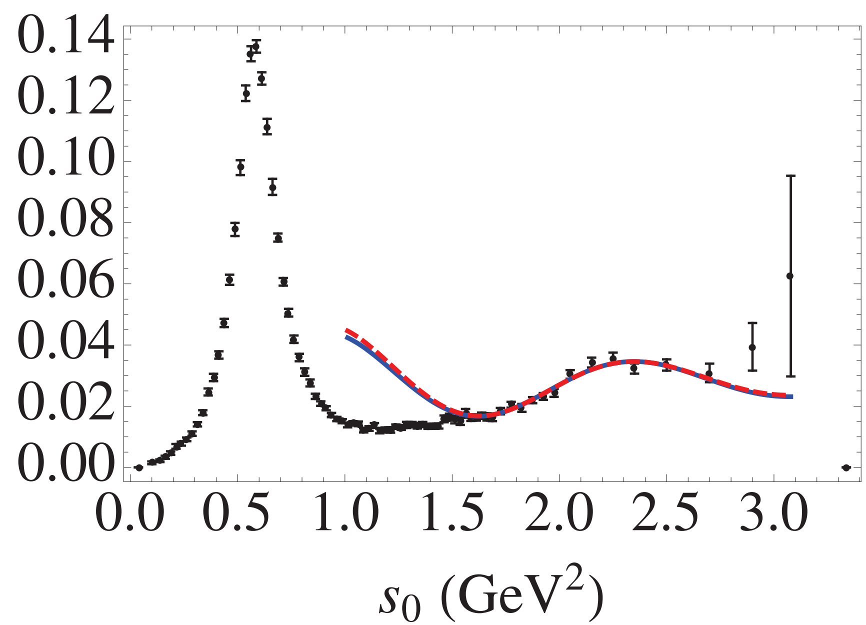
$s_{min} = 1.55 \text{ GeV}^2$, $\chi^2/dof = 24.5/16$ $(p = 8\%)$ (This is FOPT, CIPT similar)

curves: red=CIPT blue =FOPT black =no DV

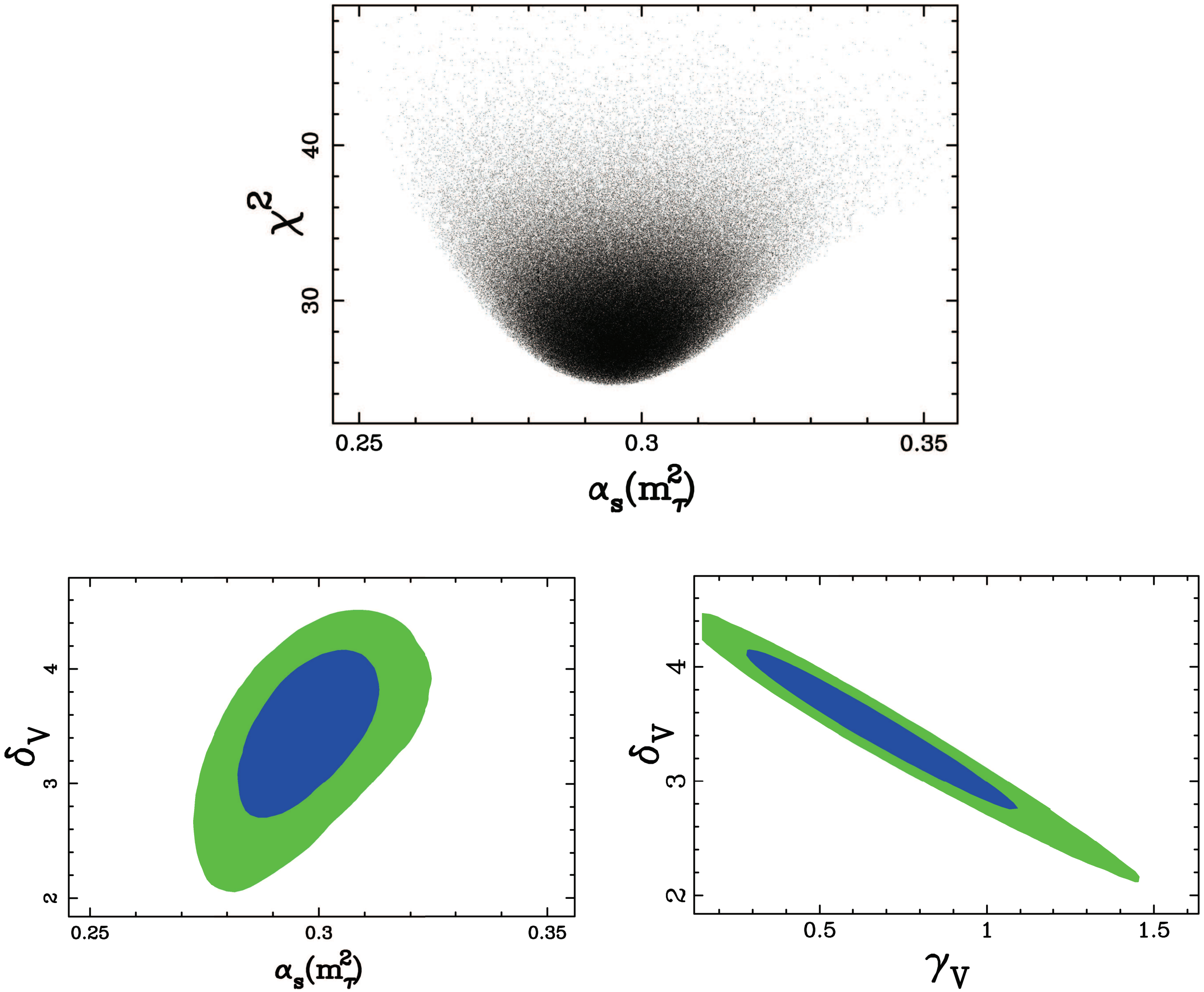
$w_0 = 1$ spectral integral



ALEPH's V spectrum



Example: Fit to $w_0 = 1$, V channel (II).



(68% and 95% contour plots), FOPT.

Clearly $DVs \neq 0$.

Comparison DVs and Radial Trajectories

Agreement DVs in tau decay $\Leftrightarrow$ Regge spectrum.

Fits of meson spectrum to radial trajectories:

Anisovich et al. '00; Klempt et al. '12; Masjuan et al. '12;

$$\Lambda^2 = 1.35(4)\ \mathrm{GeV}^2 \quad , \quad \frac{\Gamma}{M} = 0.12(8) \simeq \frac{a}{N_c}$$

leading to

$$\beta_V = \frac{2\pi}{\Lambda^2} = 4.7(2)\ \mathrm{GeV}^2 \quad , \quad \gamma_V = \frac{2\pi}{\Lambda^2}\frac{a}{N_c} = 0.6(4)\mathrm{GeV}^{-2}$$

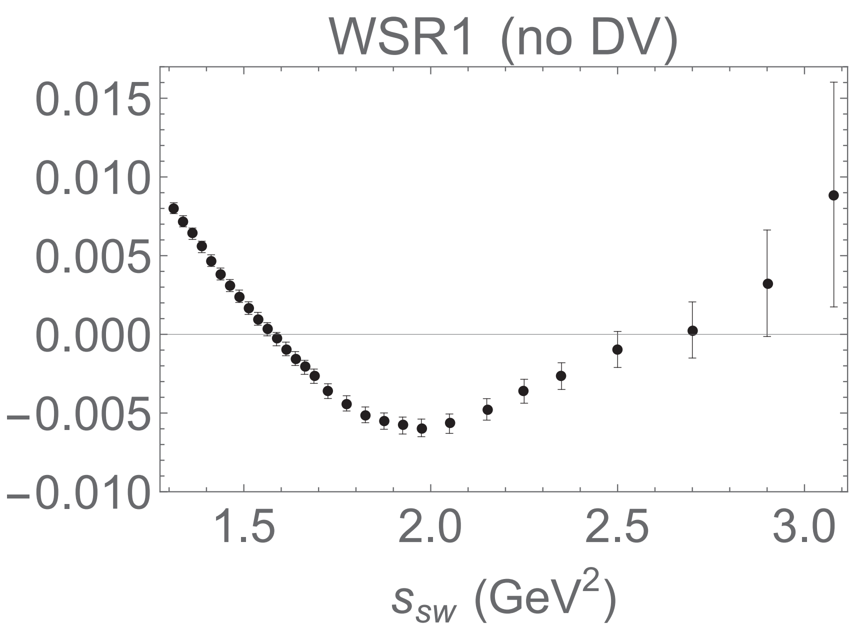
to be compared with previous results from fits to τ decay:

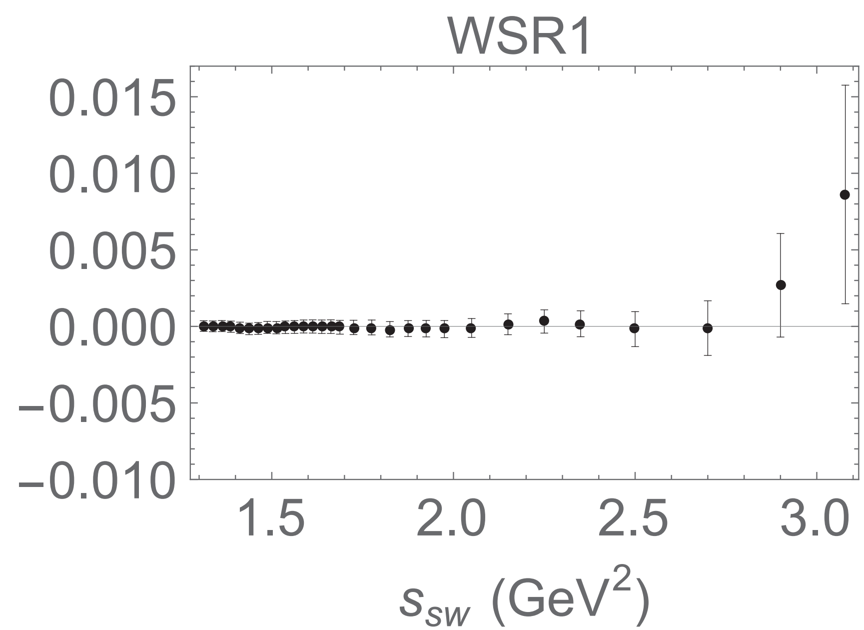
$$\beta_V = 4.2(5)\ \mathrm{GeV}^2 \quad , \quad \gamma_V = 0.7(3)\ \mathrm{GeV}^{-2}$$

coincidence ?,... Notice the 2π.

Classic Tests

Weinberg sum rule: $\int_0^\infty ds\ \left(\rho_V^{(1)}(s) - \rho_A^{(1)}(s)\right) - 2f_\pi^2 = 0$





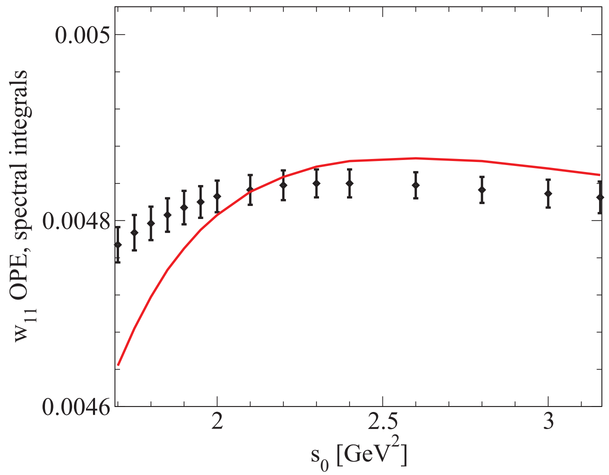
etc...

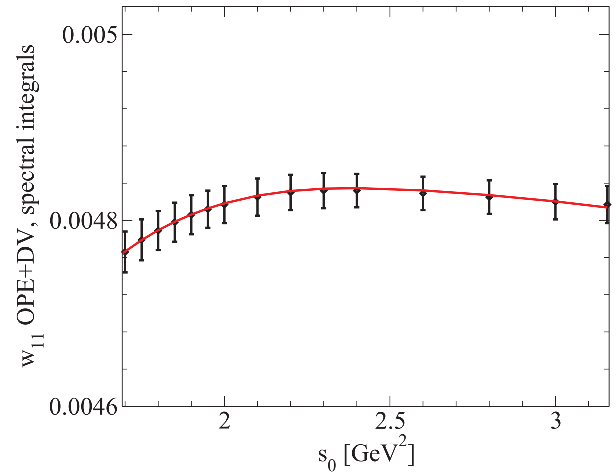
Further Tests (I)

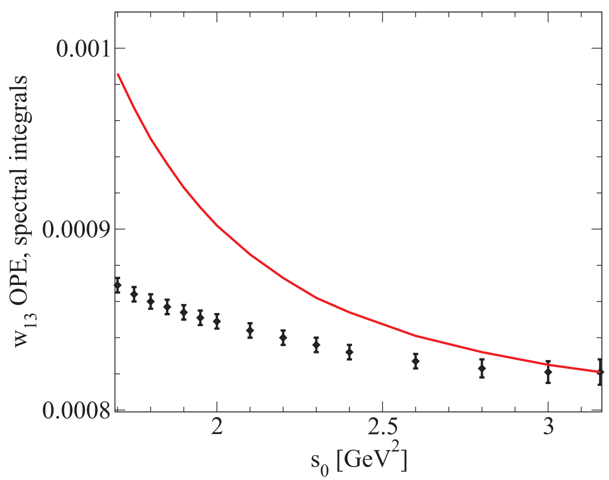
Truncated-OPE vs. DV strategies in FESRs ($V+A$, CIPT): w_{11}, w_{13}, etc... (many more)

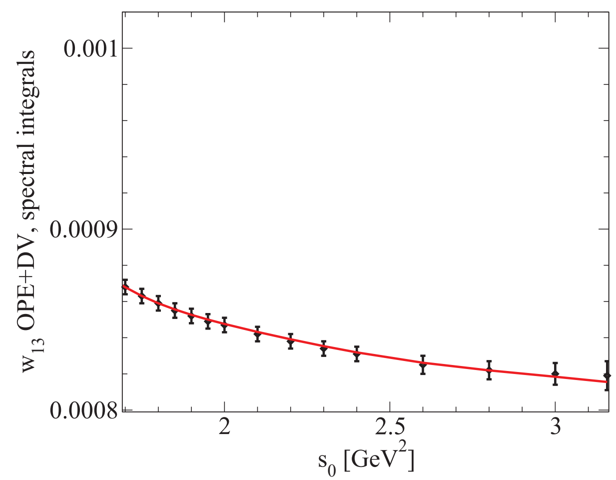
Truncated-OPE

with DV



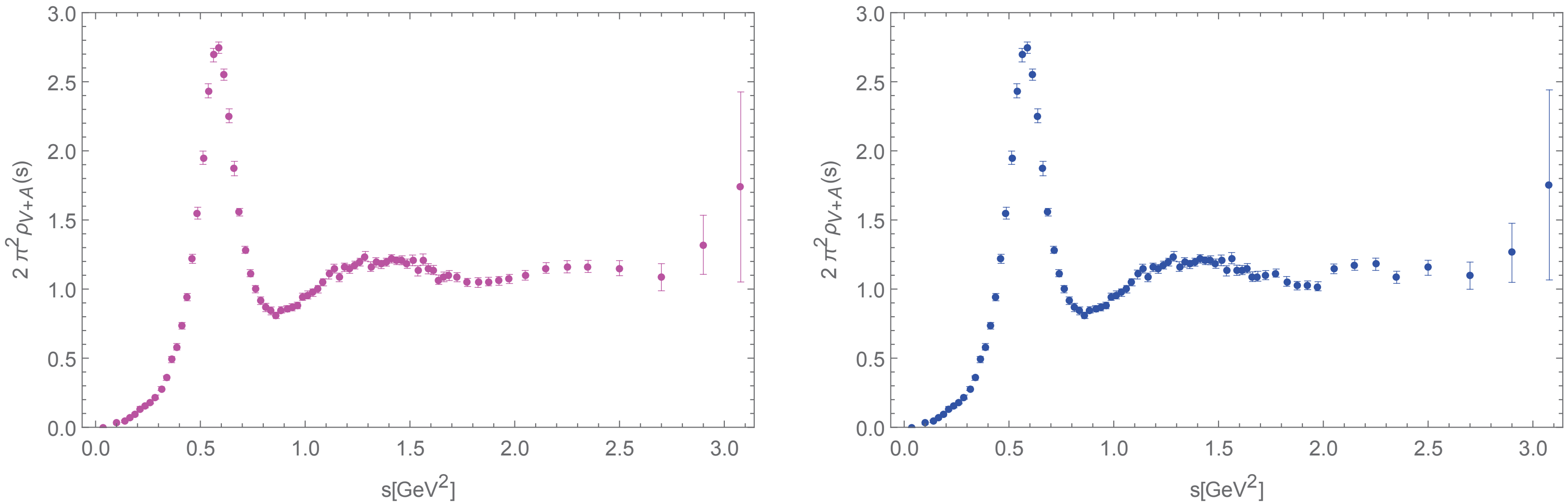






Further Tests (II)

(Boito et al. '17)



• Pseudo-data analysis in $V + A$:

input CIPT, $\alpha_s(m_\tau)^{fake} = 0.312$, DV parameters, and same Cov. matrix as experiment.

• Truncated-OPE fits are good and consistent (χ^2, p-value, etc...)

but

they produce a syst. error $\Delta\alpha_s(m_\tau) \sim +0.02$ with deviation from exact value $\sim 5 - 7\sigma$.

$\Rightarrow$ Conclusion: Truncated-OPE strategy unreliable

Results

DV strategy results with **(ALEPH data)**: $\delta\alpha_s(M_Z) \simeq \left(\frac{\alpha_s(M_Z)}{\alpha_s(m_\tau)}\right)^2 \delta\alpha(m_\tau)$

(FOPT)	$\alpha_s(m_\tau) = 0.296 \pm 0.010 \longrightarrow \alpha_s(m_Z) = 0.1155 \pm 0.0014$
(CIPT)	$\alpha_s(m_\tau) = 0.310 \pm 0.014 \longrightarrow \alpha_s(m_Z) = 0.1174 \pm 0.0019$

- N.B. Truncated-OPE strategy produces a shift, i.e. (Davier et al. '14; Pich-Rodriguez Sanchez '16)

$$\Delta\alpha_s(m_\tau) \sim +0.02 \qquad \underline{\text{higher}}$$

- DV strategy results with **ALEPH + OPAL** data:

$\alpha_s(m_Z) = 0.1165 \pm 0.0012$ (FOPT) $\qquad$ $\alpha_s(m_Z) = 0.1185 \pm 0.0015$ (CIPT)

(Current PDG 2018 world average: $\alpha_s(m_Z) = 0.1181 \pm 0.0011$)

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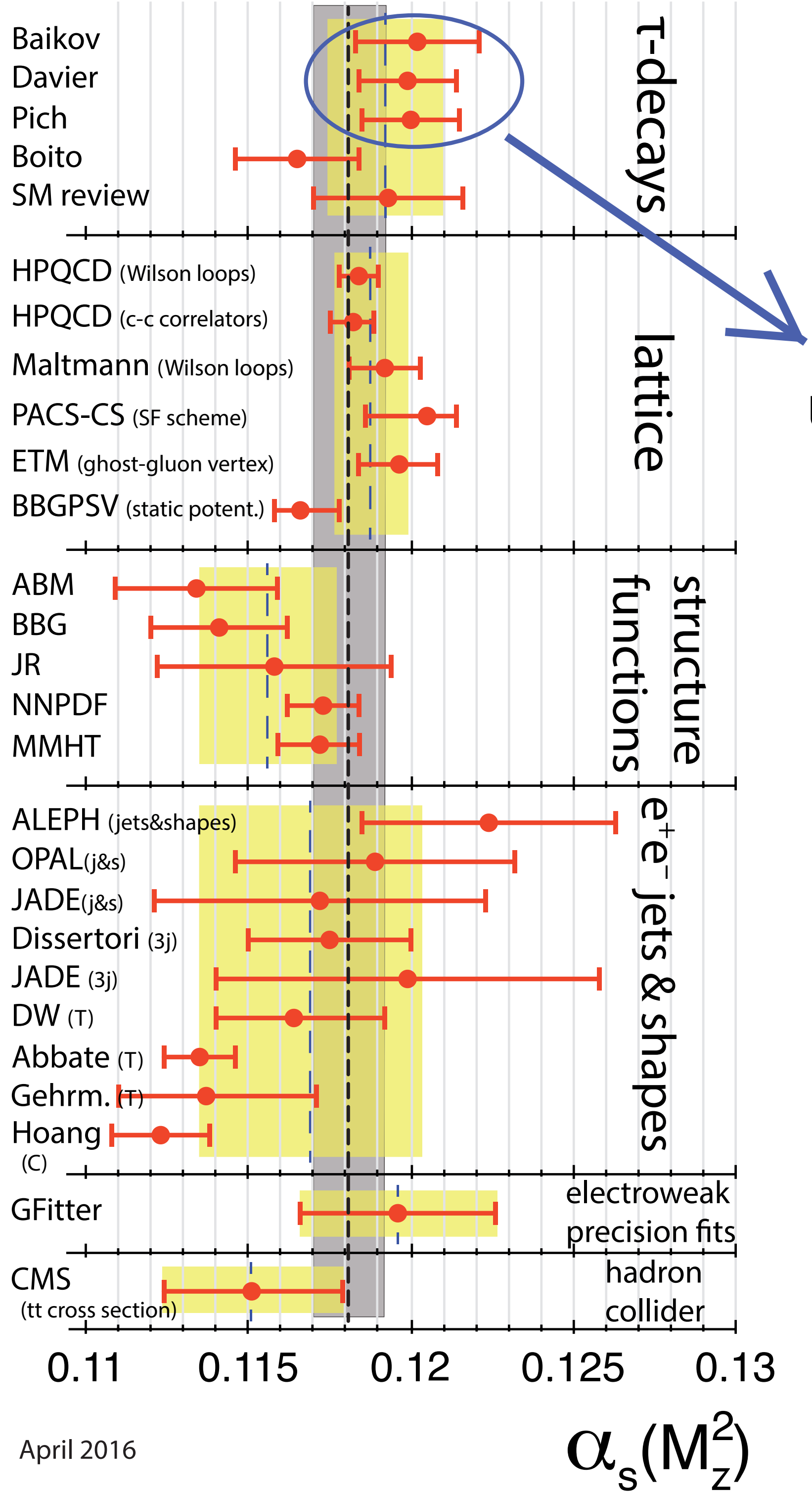
α_s Overview (PDG '18)



$\exists$ bias in α_s result.

Used same ALEPH non perturbative corrections:

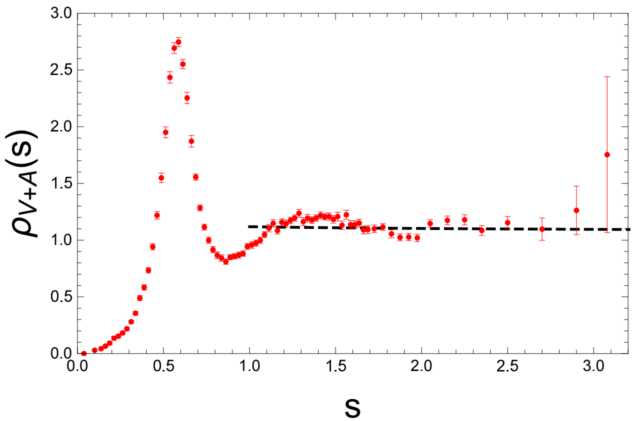
α_s *not* really independent.

Conclusions and Outlook

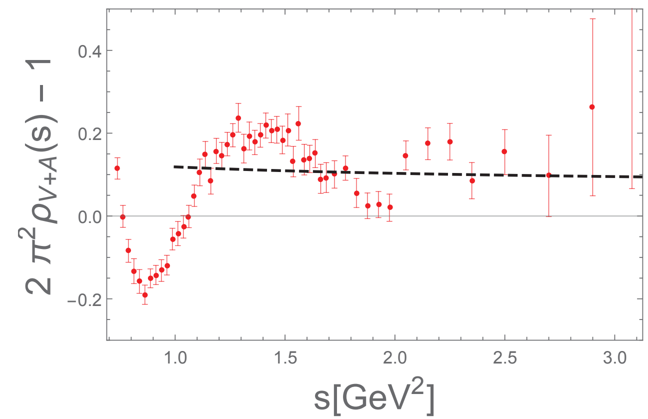
- Roughly, perturbative corrections are an order of magnitude larger than nonperturbative corrections, but the latter are important to determine α_s <u>even</u> in $V+A$.

$$\text{e.g.} \quad R_{V+A} = N_c S_{EW} |V_{ud}|^2 (1 + \underbrace{\delta_P}_{\simeq 0.18} + \underbrace{\delta_{NP}}_{\simeq 0.02})$$

- DVs are clearly visible in the data. (Dashed line= Pert. Theory)



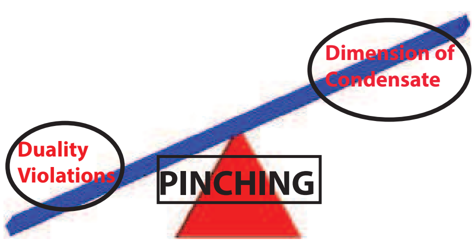
$\Rightarrow$



(DVs are not a question of principle, they exist in practice.)

- Pinching does not allow a simultaneous reduction of DVs **<u>and</u>** higher-dim condensates

(unlike what has been assumed in the Truncated-OPE method).



This introduces a significant systematic error.

Conclusions and Outlook (II)

• The new strategy using the asymptotic DV form passes all known tests, experimental and theoretical, performing better than the Truncated-OPE Strategy.

• New α_s determination from e^+e^- data in good agreement with DV-strategy result from τ data. However, because of higher energy, DVs in e^+e^- marginal.

($\rightarrow$ K. Maltman's talk)

•Better data (Babar and Belle ?) would help significantly.

SUMMARY

• Using **Aleph + Opal** data, we get:

$\alpha_s(m_Z) = 0.1165 \pm 0.0012$ (FOPT) $\alpha_s(m_Z) = 0.1185 \pm 0.0015$ (CIPT)

(Current PDG 2018 world average: $\alpha_s(m_Z) = 0.1181 \pm 0.0011$)

.

THANK YOU !

BACK-UP SLIDES

Papers

1) O. Cata, M. Golterman and S. Peris, "Duality violations and spectral sum rules," JHEP **0508**, 076 (2005).

2) O. Cata, M. Golterman and S. Peris, "Unraveling duality violations in hadronic tau decays," Phys. Rev.**D77**, 093006 (2008).

3) O. Cata, M. Golterman and S. Peris, "Possible duality violations in tau decay and their impact on the determination of alpha(s)," Phys. Rev.**D79**, 053002 (2009).

4) D. Boito, O. Cata, M. Golterman, M. Jamin, K. Maltman, J. Osborne and S. Peris, "A new determination of α_s from hadronic τ decays," Phys. Rev. **D84**, 113006 (2011).

5) D. Boito, M. Golterman, M. Jamin, A. Mahdavi, K. Maltman, J. Osborne and S. Peris, "An Updated determination of α_s from τ decays," Phys. Rev. **D85**, 093015 (2012).

6) D. Boito, M. Golterman, K. Maltman, J. Osborne and S. Peris, "Strong coupling from the revised ALEPH data for hadronic τ decays," Phys. Rev. **D91**, no. 3, 034003 (2015).

7) S. Peris, D. Boito, M. Golterman and K. Maltman, "The case for duality violations in the analysis of hadronic τ decays," Mod. Phys. Lett. **A31**, no. 30, 1630031 (2016).

8) D. Boito, M. Golterman, K. Maltman and S. Peris, "Strong coupling from hadronic τ decays: A critical appraisal," Phys. Rev. **D95**, no. 3, 034024 (2017).

9) D. Boito, I. Caprini, M. Golterman, K. Maltman and S. Peris, "Hyperasymptotics and quark-hadron duality violations in QCD," Phys. Rev. **D97**, no. 5, 054007 (2018).