

Infrared Parton Shower Dynamics and the Top Quark Mass

Daniel Samitz
(University of Vienna)

in collaboration with André Hoang and Simon Plätzer

based on *JHEP* **10** (2018) 200, [arXiv:1807.06617]

Workshop on Determination of Fundamental QCD Parameters, São Paulo
02 Oct 2019



universität
wien

FWF

Der Wissenschaftsfonds.

Motivation

- Parton showers as part of Monte Carlo (MC) event generators widely used in collider phenomenology

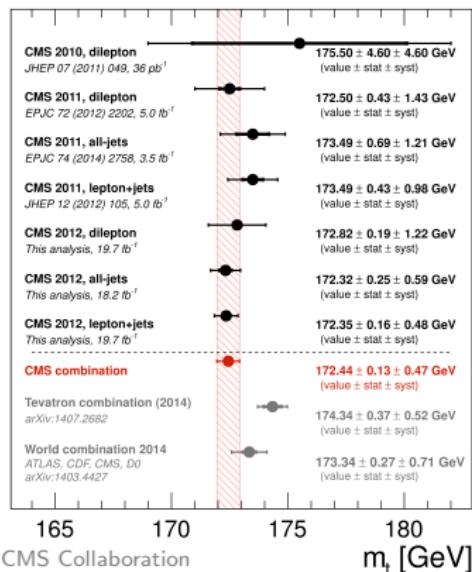
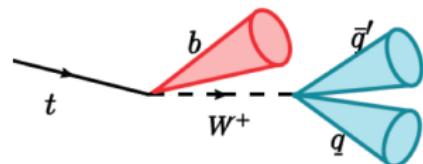
- Most precise top mass measurements based on direct reconstruction rely heavily on MCs

$$m_t^{\text{MC}} = 172.44 \pm 0.49 \text{ GeV (CMS)}$$

$$m_t^{\text{MC}} = 172.84 \pm 0.70 \text{ GeV (ATLAS)}$$

$$m_t^{\text{MC}} = 172.44 \pm 0.64 \text{ GeV (Tevatron)}$$

- Which mass scheme is determined in these measurements is still unsettled
- Shower cut is expected to have impact on the IR behavior of the parton shower and the MC \rightarrow mass scheme?



MC Top Quark Mass Parameter

Why is there a non-trivial issue in the interpretation of m_t^{MC} ?

- picture of “top quark particle” does not apply (non-zero color charge)
- m_t is a scheme-dependent parameter of a perturbative computation
→ in which scheme do MC event generators calculate?
- relation of m_t^{MC} to any field theory mass definition can be affected by different contributions (let's consider pole mass just for convention)

$$m_t^{\text{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\text{MC}}$$

Diagram illustrating the components of m_t^{MC} :

- pQCD contribution:**
 - perturbative corrections
 - depends on MC parton shower setup
- non-perturbative contribution:**
 - effects of hadronization model
 - may depend on parton shower setup
- Monte Carlo shift:**
 - contribution arising from systematic MC uncertainties
 - e.g. color reconnection, b-jet modelling, finite width,...
 - should be covered by “MC uncertainty” or better negligible

MC Top Quark Mass Parameter

Why is there a non-trivial issue in the interpretation of m_t^{MC} ?

- picture of “top quark particle” does not apply (non-zero color charge)
- m_t is a scheme-dependent parameter of a perturbative computation
→ in which scheme do MC event generators calculate?
- relation of m_t^{MC} to any field theory mass definition can be affected by different contributions (let's consider pole mass just for convention)

$$m_t^{\text{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\text{MC}}$$

pQCD contribution:

- perturbative corrections
- depends on MC parton shower setup

non-perturbative contribution:

- effects of hadronization model
- may depend on parton shower setup

Monte Carlo shift:

- contribution arising from systematic MC uncertainties
- e.g. color reconnection, b-jet modelling, finite width,...
- should be covered by “MC uncertainty” or better negligible

main part of this talk

MC Top Quark Mass Parameter

Why is there a non-trivial issue in the interpretation of m_t^{MC} ?

- picture of “top quark particle” does not apply (non-zero color charge)
- m_t is a scheme-dependent parameter of a perturbative computation
→ in which scheme do MC event generators calculate?
- relation of m_t^{MC} to any field theory mass definition can be affected by different contributions (let's consider pole mass just for convention)

$$m_t^{\text{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\text{MC}}$$

pQCD contribution:

- perturbative corrections
- depends on MC parton shower setup

non-perturbative contribution:

- effects of hadronization model
- may depend on parton shower setup

main part of this talk

outlook / work in progress

Monte Carlo shift:

- contribution arising from systematic MC uncertainties
- e.g. color reconnection, b-jet modelling, finite width,...
- should be covered by “MC uncertainty” or better negligible

Previous Quantitative Examinations of m_t^{MC}

- Butenschoen, Dehnadi, Hoang, Mateu, Preisser, Stewart (2017), arxiv:1608.01318

- ▶ **numerical** relation between Pythia MC top mass and MSR mass using 2-jettiness in e^+e^- in the resonance region from calibration fits
- ▶ “MC top mass calibration”

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(1 \text{ GeV}) + (0.18 \pm 0.22) \text{ GeV}$$

$$m_t^{\text{MC}} = m_t^{\text{pole}} + (0.57 \pm 0.28) \text{ GeV}$$

- ▶ universality conjectured but not proven

$$m_t^{\text{MC}} = m_t^{\text{pole}} + \underbrace{\Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\text{MC}}}_{\text{numerical calibration cannot distinguish the three contributions}}$$

Recent work on related issues (selection)

- Ravasio, Jezo, Nason, Oleari (2018), arxiv:1801.03944
 - ▶ POWHEG study: NLO corrections in various approximations
- Corcella, Franceschini, Kim (2017), arxiv:1712.05801
 - ▶ Dependence of m_t^{MC} from kinematic decay distributions on fragmentation parameters
- Heinrich, Maier, Nisius, Schlenk, Winter (2017), arxiv:1709.08615
 - ▶ Effects of off-shell top production compared to narrow width approximation

Aim of our work

- want to examine theoretical properties of **parton showers** (Δ_m^{pert}) with respect to dependence of shower cut Q_0
 - to avoid infrared singularities every parton shower has to terminate at infrared cutoff here: cutoff on transverse momentum in splitting $q_\perp > Q_0$
- want to understand mass of the top quark state (= top + gluons around) that is produced in the hard interaction
 - do not address issues related to **decay** (1. restriction)
- adopt narrow width approximation as used in state of the art MCs
 - we do not address **finite lifetime** issues (2. restriction)
(factorization of production and decay)
- parton showers for top quarks only conceptually valid in the quasi-coll. limit
 - consider only **boosted** tops (3. restriction)

What is the effect of the shower cutoff on the generator mass scheme?

Overview of our work

- study coherent branching (CB) - basis of the Herwig 7 angular ordered parton shower
- shower cut on the transverse momentum in the splitting $q_\perp > Q_0$
- study 2-jettiness distribution in the peak region for $e^+ e^-$ for boosted tops
- can be calculated analytically in QCD factorization (SCET+bHQET) and CB
- model hadronization by convolution with non-perturbative shape function

$$\frac{d\sigma}{d\tau}(\tau, Q, m) = \int_0^{Q\tau} d\ell \frac{d\hat{\sigma}}{d\tau}\left(\tau - \frac{\ell}{Q}, Q, m\right) S_{\text{mod}}(\ell - \Delta)$$

$Q_0 = 0$ (strict perturbative expansion in α_s)

- SCET+bHQET: in the resonance region the partonic cross section is factorized into hard, (bHQET-) jet and soft functions

$$\frac{1}{\sigma_0} \frac{d\hat{\sigma}}{d\tau}(\tau, Q, m) = H_Q(Q, \mu_H) \times U_H(\mu_H, \mu_m) U_m(\mu_m, \mu_H) H_m(\mu_m) \\ \times \left[\color{blue} U_{J_B}(\mu_H, \mu_{J_B}) \otimes J_B(\mu_{J_B}) \otimes \color{orange} U_S(\mu_H, \mu_S) \otimes S(\mu_S) \right](\tau)$$

ultra-collinear radiation wide angle soft radiation

[Fleming, Hoang, Mantry, Stewart (2008)])

$$\mu_H^2 \sim Q^2 \quad \mu_m^2 \sim m^2 \quad \mu_{J_B}^2 \sim \frac{Q^4 \tau^2}{m^2} \quad \mu_S^2 \sim Q^2 \tau^2$$

mass scheme fixed in the jet function

- coherent branching: analytic solution in the resonance region in Laplace space

[Catani, Marchesini, Webber (1991); Gieseke, Stephens, Webber (2003)]

$$\mathcal{L} \left[\frac{1}{\sigma_0} \frac{d\hat{\sigma}}{d\tau}(\tau, Q, m) \right](\nu) = \exp \left[2 \int_{m^2}^{Q^2} \frac{d\tilde{q}^2}{\tilde{q}^2} \int_{\frac{m}{\tilde{q}}}^1 dz P_{qq} \left[\alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2} \right] \left(e^{\frac{-\nu(1-z)\tilde{q}^2}{Q^2}} - 1 \right) \right] \\ \underset{\text{NLL}}{\approx} \mathcal{L} \left[U_H(\mu_H, \mu_m) \times U_m(\mu_m, \mu_H) \times \color{blue} U_{J_B}(\mu_H, \mu_{J_B}) \otimes \color{orange} U_S(\mu_H, \mu_S) \right](\nu)$$

- coherent branching with $Q_0 = 0$ and SCET+bHQET with $m = m^{\text{pole}}$ equivalent at NLL
(known for the massless case, new for the massive case)

NLO precision in the peak region

- partonic cross section in SCET at NLO (massless)

$$\frac{d\hat{\sigma}}{d\tau} = \delta(\tau) + \frac{\alpha_s C_F}{4\pi} \left\{ \underbrace{-8 \left[\frac{\ln \tau}{\tau} \right]_+}_{\text{LL}} - \underbrace{6 \left[\frac{1}{\tau} \right]_+}_{\text{NLL}} + \underbrace{\delta(\tau) \left(\frac{2\pi^2}{3} - 2 \right)}_{\text{N}^2\text{LL}} \right\} + \mathcal{O}(\alpha_s^2)$$

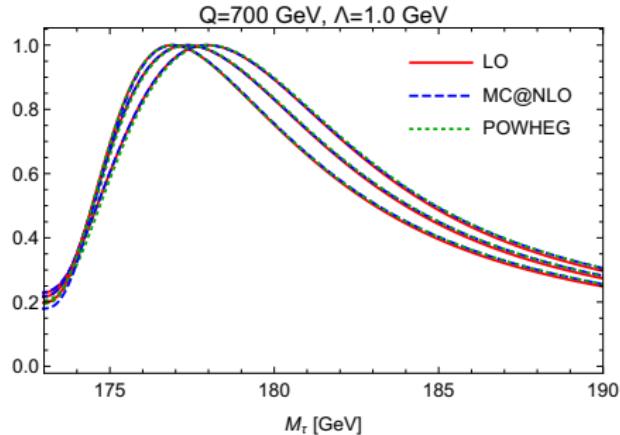
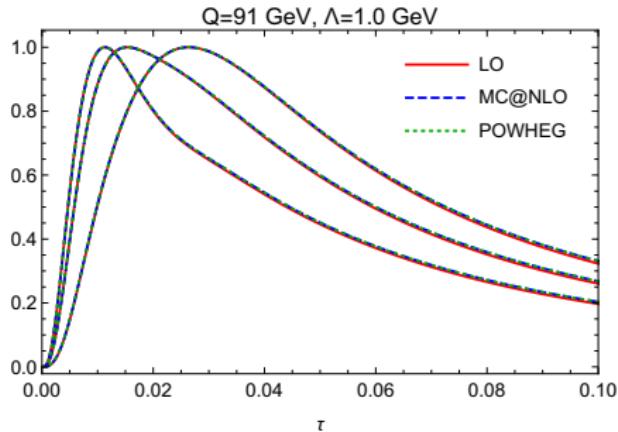
this piece is not correctly reproduced by CB

- N^2LL term at NLO is proportional to LO order cross section
- contributes only at higher orders to the position of the resonance peak τ_{peak}

$$\frac{d^2\sigma^{\text{NLO}}(\tau)}{d\tau^2} \Big|_{\tau=\tau_{\text{peak}}^{\text{NLO}}} = 0$$

- $\tau_{\text{peak}}^{\text{NLO}}$ fully determined by NLL terms
- NLL sufficient for full NLO information in the peak
- mass scheme of coherent branching without shower cut is pole mass**

NLO matched shower



$Q_0 = 1.0$ GeV right bunch of curves

$Q_0 = 1.5$ GeV middle bunch of curves

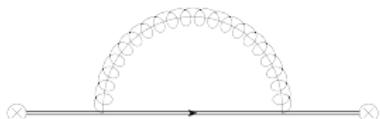
$Q_0 = 2.0$ GeV left bunch of curves

NLO matching improves events with radiation of a hard gluon \rightarrow larger τ values
NLO matching does not increase precision for simulations in the resonance region

Effects of a cutoff $Q_0 > 0$

- pole of the top quark propagator = $m_t^{\text{CB}}(Q_0) \neq m_t^{\text{pole}}$ (**coherent branching mass**)

$$m_t^{\text{CB}}(Q_0) = m_t^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$



- In the presence of the shower cut the **ultra-collinear radiation** generated by CB produces exactly the mass scheme change correction that is required so that the generator mass is exactly the coherent branching mass $m_t^{\text{CB}}(Q_0)$

$$\sigma(m_1, Q, \dots) = \sigma(m_2, Q, \dots) + \delta m \times \left. \frac{d}{dm} \sigma(m, Q, \dots) \right|_{m=m_1} + \dots$$

$$\delta m = m_2 - m_1$$

- CB mass is a **short-distance mass**: free of the pole mass renormalon
- The shower cut also affects **large-angle soft radiation**. The corresponding effects are directly tied to the amount of hadronization effects that are supposed to be fixed by tuning.
- All conclusions explicitly cross checked by correspondence between analytic QCD factorization calculations and analytic solutions of the CB algorithm
- All results checked directly by comparing with Herwig 7 event generator.

$Q_0 > 0$: coherent branching (angular ordered parton shower)

- we can now work out the leading effects of introducing a shower cut $q_\perp > Q_0$
- keep only terms linear in Q_0 and m , only NLO in α_s
- with these expansions the difference of the distributions with and without cutoff can be calculated analytically
- leading effect of Q_0 is a shift in the partonic cross section with contributions coming from the **soft** and **ultra-coll.** regions

$$\frac{d\sigma^{cb}}{d\tau}(\tau, Q, m, \textcolor{red}{Q}_0) = \frac{d\sigma^{cb}}{d\tau}\left(\tau + \frac{\alpha_s(Q_0)}{4\pi} \left[\textcolor{red}{16C_F} \frac{Q_0}{Q} - 8\pi C_F \frac{Q_0 m}{Q^2} \right], Q, m, \textcolor{red}{Q}_0 = 0 \right)$$

$Q_0 > 0$: QCD factorization theorem

- introduce the cutoff $q_\perp > Q_0$ in the one-loop diagrams for the soft and jet functions in the QCD factorization theorem
- keep only terms linear in Q_0 and m , multipole expansion for real radiation terms
- SCET soft function at one-loop with q_\perp cut:

$$S(\ell, Q_0) = S(\ell) + \frac{\alpha_s(Q_0)}{4\pi} 16C_F Q_0 S'(\ell) + \mathcal{O}(\alpha_s^2)$$

extra term needs to be absorbed into a change in the non-pert. shape function

- bHQET jet function at one-loop with q_\perp cut: **off-shell**

$$J_B^{\text{off}}(\hat{s}, m^{\text{pole}}, Q_0) = J_B^{\text{off}}(\hat{s}, m^{\text{pole}}) - \frac{\alpha_s(Q_0)}{4\pi} 8\pi C_F m Q_0 J_B^{\text{off}\prime}(\hat{s}, m^{\text{pole}}) + \mathcal{O}(\alpha_s^2)$$

extra term gets absorbed by change of mass scheme

- bHQET jet function at one-loop with q_\perp cut: **on-shell self energy**

$$\begin{aligned} J_B^{\text{os}}(\hat{s}, m^{\text{pole}}, Q_0) &= J_B^{\text{os}}(\hat{s}, m^{\text{pole}}) + \frac{\alpha_s(Q_0)}{4\pi} 8\pi C_F m Q_0 J_B^{\text{os}\prime}(\hat{s}, m^{\text{pole}}) + \mathcal{O}(\alpha_s^2) \\ &= J_B^{\text{os}}(\hat{s}, m^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0)) + \mathcal{O}(\alpha_s^2) \end{aligned}$$

change of mass scheme: $m^{\text{pole}} \rightarrow m^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0)$

Comparison with CB

- if NOT compensated by **retuning of hadronization model** and **redefinition of mass scheme**, change of cutoff leads to shift that gets contributions from **soft** and **ultra-collinear** radiation

$$\tau_{\text{peak}}(Q_0) = \tau_{\text{peak}}(Q'_0) - \frac{1}{Q} \left(16C_F - 8\pi C_F \frac{m}{Q} \right) \int_{Q'_0}^{Q_0} dR \frac{\alpha_s(R)}{4\pi}$$

- agrees with result for shift of peak position obtained from coherent branching
- mass reduces leading coefficient of R-evolution
- dependence of the peak position on shower cut Q_0 can be compared to actual angular ordered parton shower in MC

Comparison with Herwig

- compare our predictions for peak shift with real parton shower
- used Herwig 7 with angular ordered shower for $e^+e^- \rightarrow t\bar{t}$
- modifications:
 - ▶ set all constituent masses of light quarks and gluon to zero
 - ▶ unrestricted kinematics in evolution of CB
 - ▶ on-shell top production
 - ▶ only leptonic W-decays
 - ▶ switched off: QED radiation, hadronization
- only partonic distribution from Herwig
“hadronization”: convolution with model function \rightarrow disentangle parton shower and hadronization model

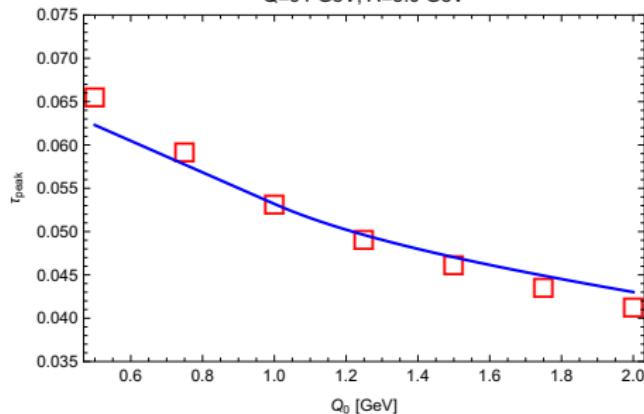
$$S_{\text{mod}}(k, \lambda) = \frac{128 k^3 e^{-\frac{4k}{\lambda}}}{3\tilde{\lambda}^4} \quad \tilde{\lambda} = \lambda + \frac{4m_t \Gamma_t}{Q}$$

- $m_t = 173 \text{ GeV}$ $\Gamma_t = 1.5 \text{ GeV}$
- run for different values of Q_0 , Q and λ .
- use rescaled τ variable $M_\tau = \frac{Q^2 \tau}{m_t}$ (partonic threshold at m_t)
- compare cutoff dependence of peak position with R-evolution

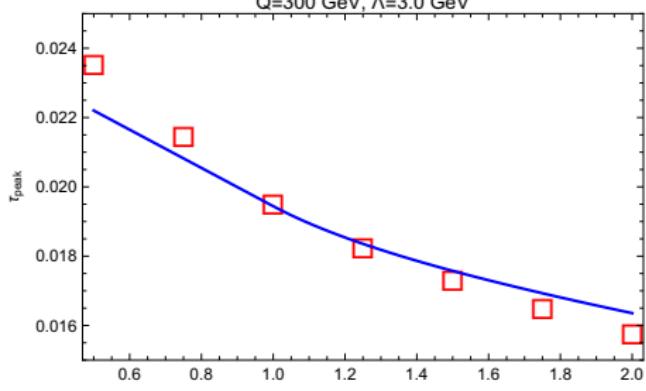
$$M_{\tau, \text{peak}}(Q_0) = M_{\tau, \text{peak}}(Q'_0) - \left(8C_F \frac{Q}{m_t} - 4\pi C_F \right) \int_{Q'_0}^{Q_0} dR \frac{\alpha_s(R)}{4\pi}$$

Comparison with Herwig

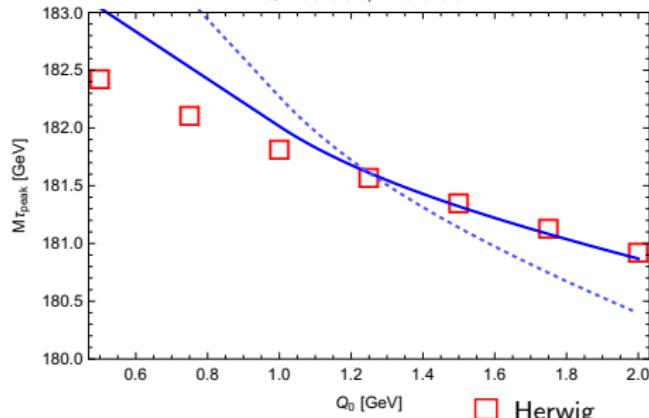
$Q=91 \text{ GeV}, \Lambda=3.0 \text{ GeV}$



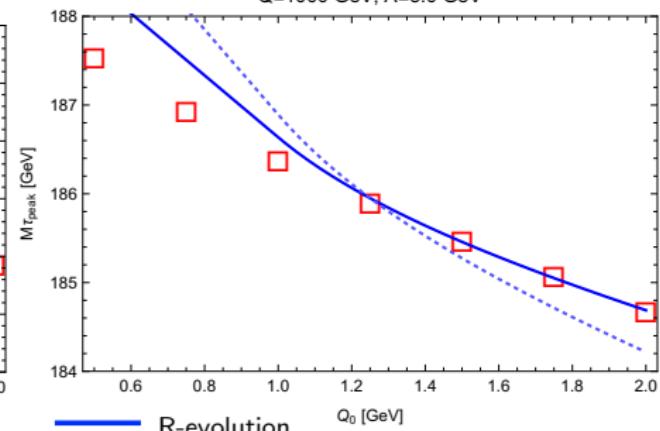
$Q=300 \text{ GeV}, \Lambda=3.0 \text{ GeV}$



$Q=700 \text{ GeV}, \Lambda=3.0 \text{ GeV}$



$Q=1000 \text{ GeV}, \Lambda=3.0 \text{ GeV}$



□ Herwig

— R-evolution

Relation of m_t^{CB} to other Masses

$$\text{Herwig 7: } Q_0 = 1.25 \text{ GeV} \quad \rightarrow \quad m_t^{\text{Herwig}} = m_t^{\text{CB}}(1.25 \text{ GeV})$$

MSR Mass:

$$m_t^{\text{MSR}}(Q_0) = m_t^{\text{CB}}(Q_0) + 0.24 Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$\Rightarrow m_t^{\text{MSR}}(Q_0) = m_t^{\text{CB}}(Q_0) + (0.190 \pm 0.070) \text{ GeV}$$

- CB and MSR masses do not suffer from $\mathcal{O}(\Lambda_{\text{QCD}})$ renormalon (due to IR cut)
→ good convergence
- uncertainty estimated from difference between α_s in $\overline{\text{MS}}$ and MC schemes
- precision sufficient for all possible applications at the LHC!
(recall restriction 1-3)
- more precision may be needed for a future e^+e^- collider

Relation of m_t^{CB} to other Masses

Herwig 7: $Q_0 = 1.25 \text{ GeV}$ \rightarrow $m_t^{\text{Herwig}} = m_t^{\text{CB}}(1.25 \text{ GeV})$

Pole Mass:

$$m_t^{\text{pole}} = m_t^{\text{MSR}}(Q_0) + (0.350 \pm 0.250) \text{ GeV}$$

[Hoang, Lepenik, Preisser (2017)]

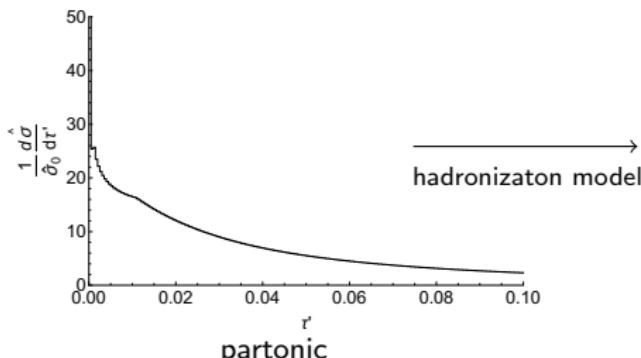
$[\pm 110 \text{ MeV}]: \text{Beneke, Marquard, Nason, Steinhauser (2017)}$

$$\Rightarrow m_t^{\text{pole}} = m_t^{\text{CB}}(Q_0) + (0.540 \pm 0.260) \text{ GeV}$$

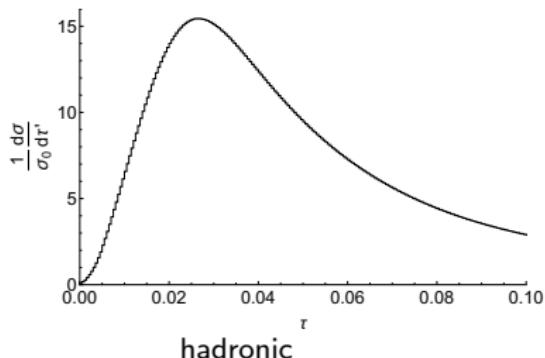
- pole mass suffers from $\mathcal{O}(\Lambda_{\text{QCD}})$ renormalon \rightarrow irreducible ambiguity of 250 MeV
- difference between m_t^{pole} and $m_t^{\text{Herwig}} \sim 500 \text{ MeV} > \text{ambiguity!}$
- \Rightarrow important to study Δ_m^{pert} beyond current set up (lift restrictions 1-3)
- shift as large as current experimental uncertainty from direct methods

Hadronization Effects (work in progress)

- so far considered only perturbative contributions (Δ_m^{pert})
no hadronization effects
- found agreement between Herwig's (angular ordered) parton shower and analytic QCD predictions
- question: how well does Herwig's hadronization model match the analytic predictions?
- goal: understand possible non-pert. contributions to generator mass ($\Delta_m^{\text{non-pert}}$)
- start again with thrust distribution for massless quarks



hadronization model



hadronic

Hadronization Effects: QCD Factroization Theorem

- hadronization modeled by convolution with non-pert. function S_{mod}

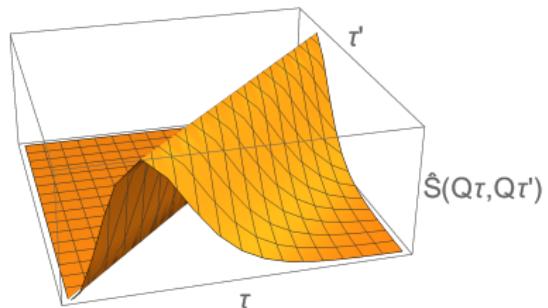
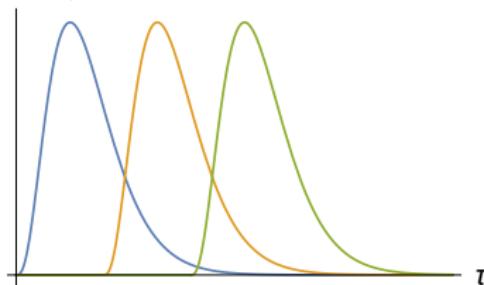
$$\frac{d\sigma}{d\tau}(\tau, Q, Q_0) = \int_0^\tau d\tau' \frac{d\hat{\sigma}}{d\tau'}(\tau', Q, Q_0) S_{\text{mod}}(Q(\tau - \tau') + \Delta_{\text{soft}}(Q_0))$$

- peak region: change of cutoff Q_0 is compensated by shift in soft function

$$\Delta_{\text{soft}}(Q_0) = \Delta_{\text{soft}}(Q'_0) + 16 \int_{Q'_0}^{Q_0} dR \left[\frac{\alpha_s(R) C_F}{4\pi} + \mathcal{O}(\alpha_s^2) \right]$$

- convolution above implies that each bin τ' in the partonic distribution gets smeared with a function $\hat{S}(Q\tau, Q\tau') = S_{\text{mod}}(Q(\tau - \tau'))$

$\hat{S}(Q\tau, Q\tau')$



- linear dependence of peak of \hat{S} on the partonic value τ'
 $\tau_{\text{peak}}(\tau') = \tau_{\text{peak}}(0) + \tau'$

Herwig Cluster Model

- standard hadronization model of Herwig:
cluster hadronization model
[Webber (1984)]
- final state gluons split into $q\bar{q}$
- color-connected quarks combined into preconfined clusters
- for heavy clusters: fission along string axis (repeat until light enough)
- final clusters decay isotropically into hadrons
- various tuning parameters, specifying e.g. mass spectrum of daughter clusters, maximum mass of final clusters, constituent masses, ...

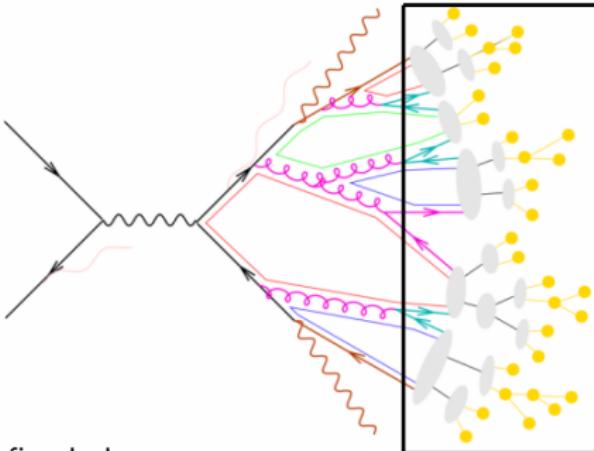


Figure from D. Zeppenfeld

Hadronization Effects: Herwig

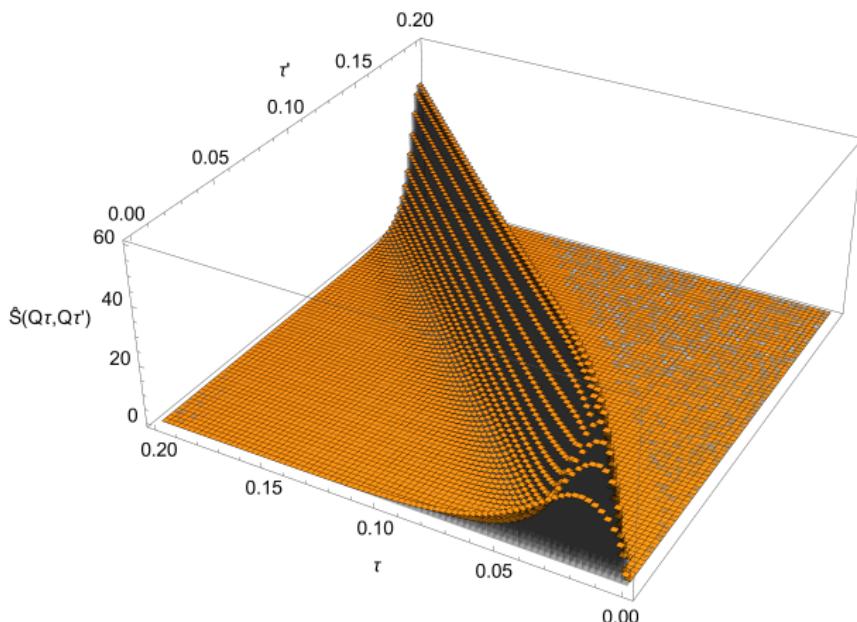
- want to understand Herwig's cluster hadronization model in more detail
- define (observable dependent) effective hadronization function \hat{S}

$$\frac{d\sigma}{d\tau}(\tau, Q) = \int d\tau' \frac{d\hat{\sigma}}{d\tau'}(\tau', Q) \hat{S}(Q\tau, Q\tau')$$

- interpretation of $\hat{S}(Q\tau, Q\tau')$: probability distribution that an event with partonic thrust value τ' has the thrust value τ after hadronization.
- first step: want to check if the behavior of \hat{S} is compatible with analytic QCD, i.e. $\hat{S}(Q\tau, Q\tau') = S(Q(\tau - \tau'))$.
- second step: compare behavior of \hat{S} after a retuning when the shower cut Q_0 is changed with our predictions.

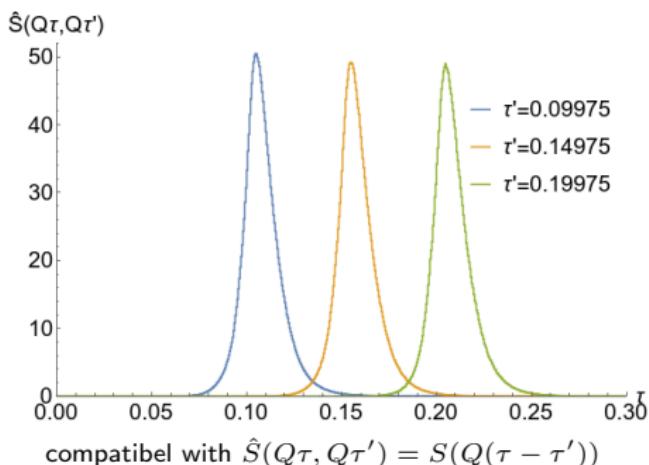
Hadronization Effects: Herwig

- modified Herwig to give partonic and hadronic thrust value for each event
- results can be filled in a 2D-Histogram that shows how each partonic bin is migrated into the hadronic bins
- can be used to extract the effective hadronization function \hat{S}

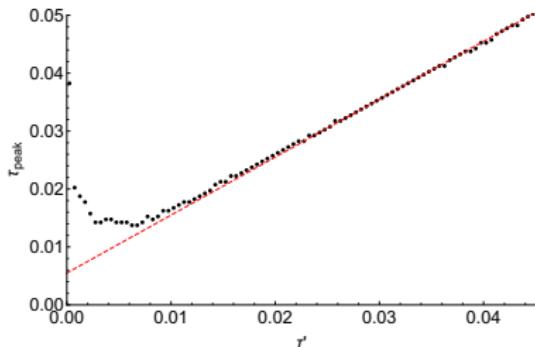
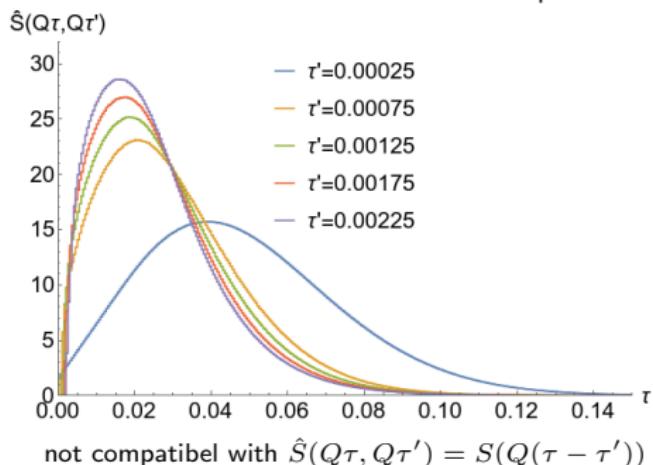


Hadronization Effects: Herwig

effective hadronization function in the tail



effective hadronization function in the peak



For very small τ' the effective hadronization function of Herwig does not match the behavior predicted by the analytic QCD factorization theorem.

Why is this the case?

Hadronization Effects: Herwig

- if there are only few perturbative emissions the color-connected partons can be far apart in phase-space
⇒ clusters with large invariant mass
- extreme case: event with no additional radiation, only two back-to-back light quarks
⇒ one cluster at rest in lab frame with mass $M = E_{\text{cm}}$.
- seems that in this case fission process does not produce sufficiently light and fast clusters to map a partonic event with $\tau' = 0$ to a hadronic event with extremely small τ .
- is this a problem? if yes, how can it be cured?
⇒ more studies needed

Conclusions

- for angular ordered parton showers (Herwig) one can derive the **perturbative contributions** between generator mass and pole masse (Δ_m^{pert})

$$m^{\text{CB}}(Q_0) = m^{\text{pole}} - \frac{2}{3}\alpha_s(Q_0)Q_0 + \mathcal{O}(\alpha_s^2)$$

this corresponds to the **pole of the quark propagator** in presence of a shower cut

- current restrictions:
 - ▶ boosted top quarks
 - ▶ narrow width approximation
 - ▶ top production (2-jettiness)needed to remove restriction:
 - parton shower algorithm for slow tops
 - parton shower for unstable tops
 - factorized predictions including top decay
- for all three new conceptual developments are required (w.i.p.)
- numerical calibration still important tool for consistency checks
- ongoing work to better understand MC **hadronization models** and eventually **non-perturbative contributions** $\Delta_m^{\text{non-pert}}$ to the relation between generator mass and pole mass (w.i.p.)

Backup

Can CB describe peak position at NLO?

peak position with NLL

partonic cross section in SCET at NLO:

$$\frac{d\hat{\sigma}}{d\tau} = \delta(\tau) + \frac{\alpha_s C_F}{4\pi} \left\{ -\underbrace{8 \left[\frac{\ln \tau}{\tau} \right]_+}_{\text{LL}} - \underbrace{6 \left[\frac{1}{\tau} \right]_+}_{\text{NLL}} + \underbrace{\delta(\tau) \left(\frac{2\pi^2}{3} - 2 \right)}_{\text{N}^2\text{LL}} \right\} + \mathcal{O}(\alpha_s^2)$$

can use that N^2LL @NLO part is proportional to LO. $f(\tau) = \frac{d\hat{\sigma}}{d\tau} \otimes S_{\text{mod}}$

$$f_{\text{NLO}}(\tau) = \tilde{f}_{\text{NLO}}(\tau) + \alpha f_{\text{LO}}(\tau)$$

$$f(\tau) = f_{\text{LO}}(\tau) + f_{\text{NLO}}(\tau) = (1 + \alpha) f_{\text{LO}}(\tau) + \tilde{f}_{\text{NLO}}(\tau) + \mathcal{O}(\alpha_s^2)$$

LO peak position:

$$f'_{\text{LO}}(\tau_0) = 0$$

NLO peak position: $\delta\tau \sim \alpha_s$

$$f'(\tau_0 + \delta\tau) \stackrel{!}{=} 0 + \mathcal{O}(\alpha_s^2)$$

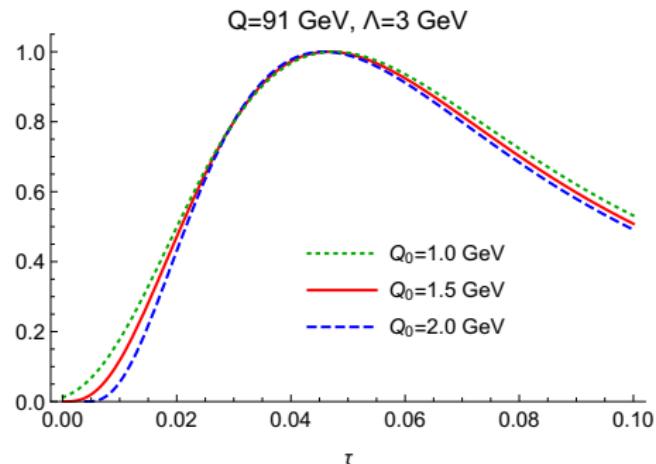
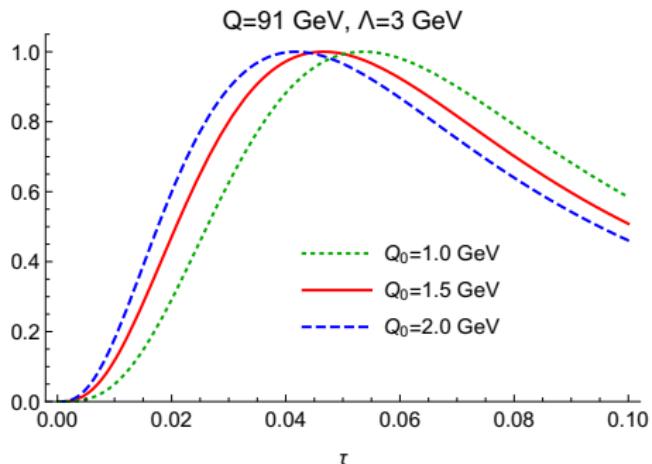
$$= (1 + \alpha) f'_{\text{LO}}(\tau_0 + \delta\tau) + \tilde{f}'_{\text{NLO}}(\tau_0 + \delta\tau) + \mathcal{O}(\alpha_s^2)$$

$$= (1 + \alpha) \underbrace{f'_{\text{LO}}(\tau_0)}_{=0} + \delta\tau f''_{\text{LO}}(\tau_0) + \tilde{f}'_{\text{NLO}}(\tau_0) + \mathcal{O}(\alpha_s^2)$$

$$\Rightarrow \delta\tau = \frac{-\tilde{f}'_{\text{NLO}}(\tau_0)}{f''_{\text{LO}}(\tau_0)} + \mathcal{O}(\alpha_s^2)$$

CB at **NLL** has full NLO information on peak position.

Massless τ Distributions

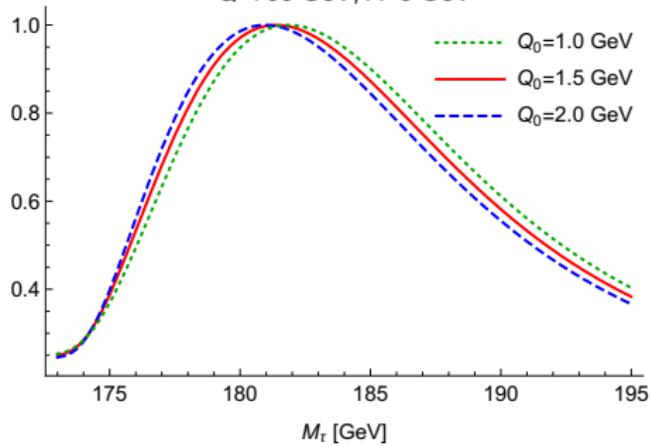


no gap

$$\Delta(Q_0) = \frac{16}{3\pi} \int_{1.5\text{GeV}}^{Q_0} dR \alpha_s(R)$$

Massive M_τ Distributions

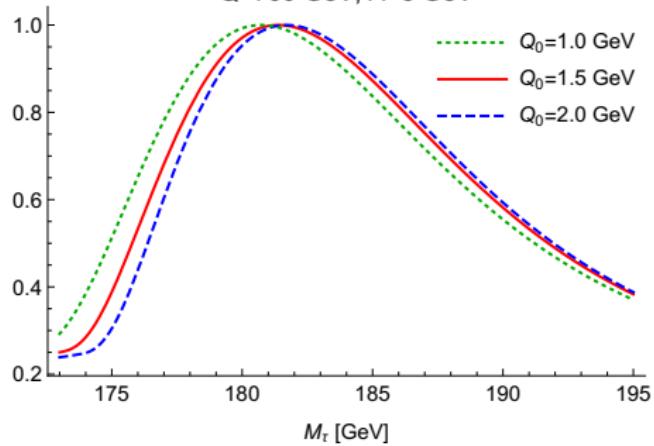
$Q=700 \text{ GeV}, \Lambda=3 \text{ GeV}$



no gap

$m = 173 \text{ GeV}$

$Q=700 \text{ GeV}, \Lambda=3 \text{ GeV}$

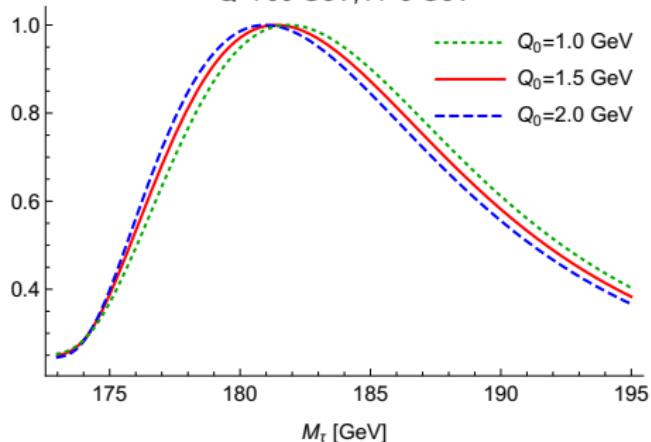


$$\Delta(Q_0) = \frac{16}{3\pi} \int_{1.5 \text{ GeV}}^{Q_0} dR \alpha_s(R)$$

$m = 173 \text{ GeV}$

Massive M_τ Distributions

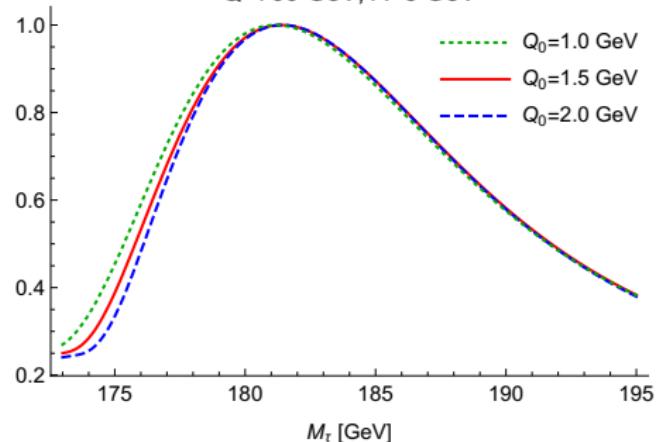
$Q=700 \text{ GeV}, \Lambda=3 \text{ GeV}$



no gap

$m = 173 \text{ GeV}$

$Q=700 \text{ GeV}, \Lambda=3 \text{ GeV}$



M_τ [GeV]

175 180 185 190 195

175

M_τ [GeV]

175 180 185 190 195

175

$$\Delta(Q_0) = \frac{16}{3\pi} \int_{1.5 \text{ GeV}}^{Q_0} dR \alpha_s(R)$$

$$m(Q_0) = 173 \text{ GeV} - \frac{2}{3} \int_{1.5 \text{ GeV}}^{Q_0} dR \alpha_s(R)$$

$$m(1.0 \text{ GeV}) = 173.22 \text{ GeV}$$

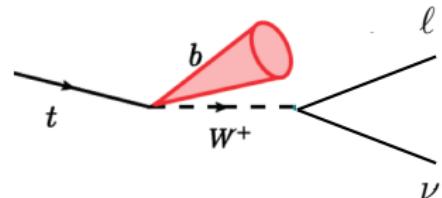
$$m(2.0 \text{ GeV}) = 172.86 \text{ GeV}$$

Reconstructed Observables: m_{bjl} and m_{bjW}

- studied two different observables (for boosted tops)

$$m_{bjl} = \sqrt{(p_{bj} + p_\ell)^2}$$

$$m_{bjW} = \sqrt{(p_{bj} + p_\ell + p_\nu)^2}$$



- jet distance measures

$$d_{ij} = \min(E_i^{2p}, E_j^{2p}) \frac{1 - \cos \theta_{ij}}{1 - \cos R} \quad d_{iB} = E_i^{2p} \quad p = \{-1, 0, 1\}$$

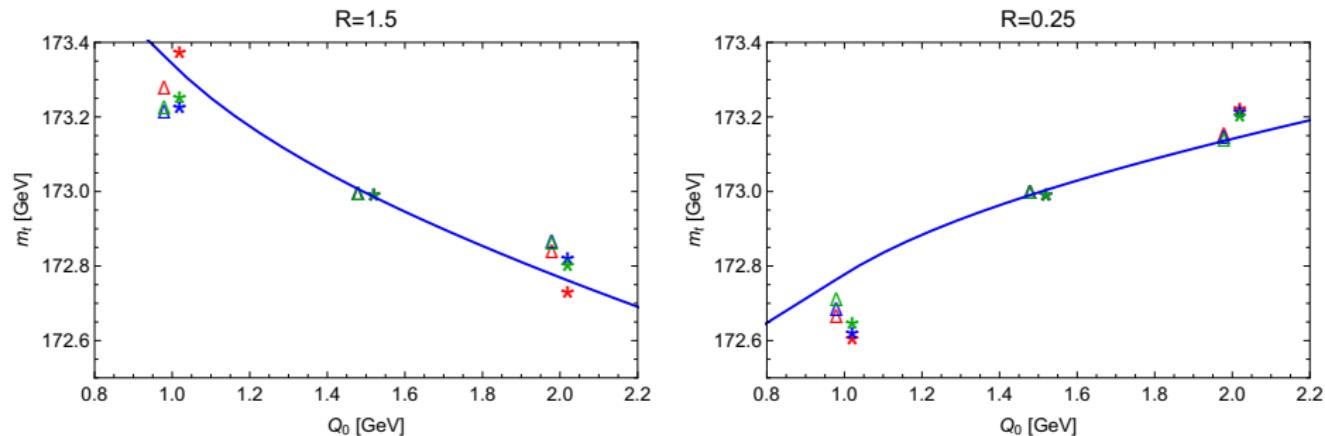
- for $R = \pi/2$: recover full hemisphere mass

$$m_{t,\text{fit}}^{R=\pi/2}(Q_0) = m_{t,\text{fit}}^{R=\pi/2}(Q'_0) - \left[4C_F \frac{Q}{m_t} - 2\pi C_F \right] \int_{Q'_0}^{Q_0} dR \frac{\alpha_s(R)}{4\pi}$$

- for $R \sim m_t/Q$: wide angle soft radiation is cut away, only ultra-collinear (boosted with top quark) radiation inside the cone

$$m_{t,\text{fit}}^{R \sim m_t/Q}(Q_0) = m_{t,\text{fit}}^{R \sim m_t/Q}(Q'_0) + 2\pi C_F \int_{Q'_0}^{Q_0} dR \frac{\alpha_s(R)}{4\pi}$$

Reconstructed Observables: m_{bjl} and m_{bjW} : comparison with Herwig



generate distributions for given Q_0 for $m_t = 173$ GeV

fit top mass with distribution for reference cutoff $Q_0 = 1.5$ GeV

$$Q = 700 \text{ GeV} \Rightarrow m_t/Q \sim 0.25$$

m_{bjW}

△ $p = -1$

△ $p = 0$

△ $p = +1$

m_{bjl}

* $p = -1$

* $p = 0$

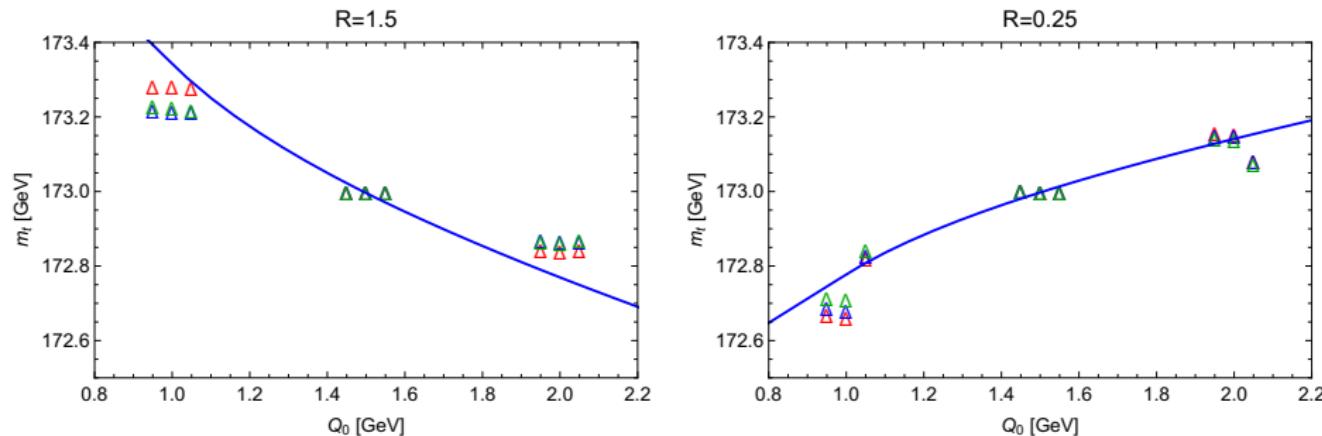
* $p = +1$

anti- k_t type

Cambridge-Aachen type

k_t type

Reconstructed Observables: m_{bjl} and m_{bjW} : NLO matched shower



for each cutoff:

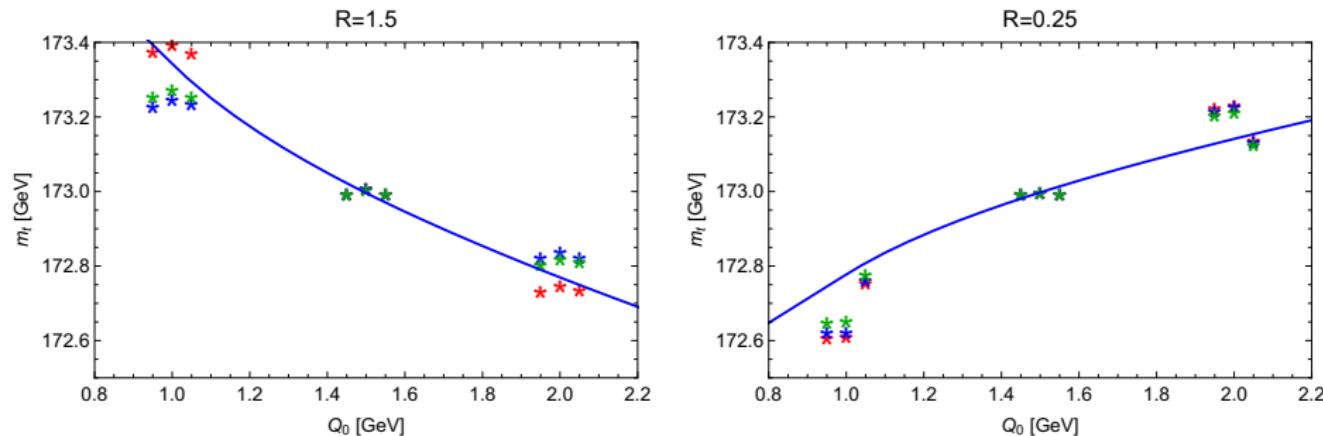
left: LO

middle: MC@NLO type

right: POWHEG type

symbols and colors as in previous slide

Reconstructed Observables: m_{bjl} and m_{bjW} : NLO matched shower



for each cutoff:

left: LO

middle: MC@NLO type

right: POWHEG type

symbols and colors as in previous slide