Infrared Parton Shower Dynamics and the Top Quark Mass

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Motivation

- Parton showers as part of Monte Carlo (MC) event generators widely used in collider phenomenology
- Most precise top mass measurements based on direct reconstruction rely heavily on MCs

\[ m_t^{\text{MC}} = 172.44 \pm 0.49 \text{ GeV (CMS)} \]
\[ m_t^{\text{MC}} = 172.84 \pm 0.70 \text{ GeV (ATLAS)} \]
\[ m_t^{\text{MC}} = 172.44 \pm 0.64 \text{ GeV (Tevatron)} \]

- Which mass scheme is determined in these measurements is still unsettled
- Shower cut is expected to have impact on the IR behavior of the parton shower and the MC \( \rightarrow \) mass scheme?
Why is there a non-trivial issue in the interpretation of $m_{t}^{MC}$?

- picture of “top quark particle” does not apply (non-zero color charge)
- $m_{t}$ is a scheme-dependent parameter of a perturbative computation → in which scheme do MC event generators calculate?
- relation of $m_{t}^{MC}$ to any field theory mass definition can be affected by different contributions (let’s consider pole mass just for convention)

\[
 m_{t}^{MC} = m_{t}^{\text{pole}} + \Delta_{m}^{\text{pert}} + \Delta_{m}^{\text{non-pert}} + \Delta_{m}^{\text{MC}}
\]

- pQCD contribution:
  - perturbative corrections
  - depends on MC parton shower setup
- non-perturbative contribution:
  - effects of hadronization model
  - may depend on parton shower setup
- Monte Carlo shift:
  - contribution arising from systematic MC uncertainties
  - e.g. color reconnection, b-jet modelling, finite width,...
  - should be covered by “MC uncertainty” or better negligible
Why is there a non-trivial issue in the interpretation of $m_t^{MC}$?

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\[
m_t^{MC} = m_t^{pole} + \Delta m_{pert} + \Delta m_{non-pert} + \Delta m_{MC}
\]

- **pQCD contribution:**
  - perturbative corrections
  - depends on MC parton shower setup

- **non-perturbative contribution:**
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  - may depend on parton shower setup

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MC Top Quark Mass Parameter

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- pQCD contribution:
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  - depends on MC parton shower setup
  main part of this talk

- non-perturbative contribution:
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- Monte Carlo shift:
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  - e.g. color reconnection, b-jet modelling, finite width,…
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outlook / work in progress
Previous Quantitative Examinations of $m_t^{\text{MC}}$

- Butenschoen, Dehnadi, Hoang, Mateu, Preisser, Stewart (2017), arxiv:1608.01318
  
  ▶ **numerical** relation between Pythia MC top mass and MSR mass using 2-jettiness in $e^+e^-$ in the resonance region from calibration fits

  ▶ “MC top mass calibration”

  ▶ $m_t^{\text{MC}} = m_t^{\text{MSR}}(1 \text{ GeV}) + (0.18 \pm 0.22) \text{ GeV}$

  $m_t^{\text{MC}} = m_t^{\text{pole}} + (0.57 \pm 0.28) \text{ GeV}$

  ▶ universality conjectured but not proven

  $m_t^{\text{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\text{MC}}$

  **numerical calibration cannot distinguish the three contributions**
Recent work on related issues (selection)

- Ravasio, Jezo, Nason, Oleari (2018), arxiv:1801.03944
  - POWHEG study: NLO corrections in various approximations

- Corcella, Franceschini, Kim (2017), arxiv:1712.05801
  - Dependence of $m_t^{MC}$ from kinematic decay distributions on fragmentation parameters

- Heinrich, Maier, Nisius, Schlenk, Winter (2017), arxiv:1709.08615
  - Effects of off-shell top production compared to narrow width approximation
Aim of our work

• want to examine theoretical properties of parton showers \( \Delta^\text{pert} \) with respect to dependence of shower cut \( Q_0 \)

to avoid infrared singularities every parton shower has to terminate at infrared cutoff here: cutoff on transverse momentum in splitting \( q_\perp > Q_0 \)

• want to understand mass of the top quark state (= top + gluons around) that is produced in the hard interaction

do not address issues related to decay (1. restriction)

• adopt narrow width approximation as used in state of the art MCs

we do not address finite lifetime issues (2. restriction)
(factorization of production and decay)

• parton showers for top quarks only conceptually valid in the quasi-coll. limit

consider only boosted tops (3. restriction)

What is the effect of the shower cutoff on the generator mass scheme?
Overview of our work

• study coherent branching (CB) - basis of the Herwig 7 angular ordered parton shower

• shower cut on the transverse momentum in the splitting \( q_\perp > Q_0 \)

• study 2-jettiness distribution in the peak region for \( e^+e^- \) for boosted tops

• can be calculated analytically in QCD factorization (SCET+bHQET) and CB

• model hadronization by convolution with non-perturbative shape function

\[
\frac{d\sigma}{d\tau}(\tau, Q, m) = \int_0^{Q\tau} d\ell \frac{d\hat{\sigma}}{d\tau}(\tau - \frac{\ell}{Q}, Q, m) S_{\text{mod}}(\ell - \Delta)
\]
$Q_0 = 0$ (strict perturbative expansion in $\alpha_s$)

- **SCET+bHQET**: in the resonance region the partonic cross section is factorized into hard, (bHQET-) jet and soft functions

$$
\frac{1}{\sigma_0} \frac{d\hat{\sigma}}{d\tau} (\tau, Q, m) = H_Q(Q, \mu_H) \times U_H(\mu_H, \mu_m) U_m(\mu_m, \mu_H) H_m(\mu_m) 
$$

$$
\times \left[ U_{JB}(\mu_H, \mu_{JB}) \otimes J_B(\mu_{JB}) \otimes U_S(\mu_H, \mu_S) \otimes S(\mu_S) \right] (\tau)
$$

- **mass scheme fixed in the jet function**

- **coherent branching**: analytic solution in the resonance region in Laplace space

$$
\mathcal{L} \left[ \frac{1}{\sigma_0} \frac{d\hat{\sigma}}{d\tau} (\tau, Q, m) \right] (\nu) = \exp \left[ 2 \int_{m^2}^{Q^2} \frac{d\tilde{q}^2}{\tilde{q}^2} \int_{\frac{m}{\tilde{q}}}^{1} dz \alpha_s((1-z)\tilde{q}, z, \frac{m^2}{\tilde{q}^2}) \left( e^{-\nu \frac{(1-z)\tilde{q}^2}{Q^2}} - 1 \right) \right]
$$

$$
\approx \mathcal{L}_{\text{NLL}} \left[ U_H(\mu_H, \mu_m) \times U_m(\mu_m, \mu_H) \times U_{JB}(\mu_H, \mu_{JB}) \otimes U_S(\mu_H, \mu_S) \right] (\nu)
$$

- **coherent branching with** $Q_0 = 0$ and **SCET+bHQET with** $m = m^{\text{pole}}$ equivalent at NLL

  (known for the massless case, new for the massive case)
NLO precision in the peak region

- partonic cross section in SCET at NLO (massless)
  \[
  \frac{d\hat{\sigma}}{d\tau} = \delta(\tau) + \frac{\alpha_s C_F}{4\pi} \left\{ -8 \left[ \frac{\ln \tau}{\tau} \right] + 6 \left[ \frac{1}{\tau} \right] + \delta(\tau) \left( \frac{2\pi^2}{3} - 2 \right) \right\} + O(\alpha_s^2)
  \]

- $N^2LL$ term at NLO is proportional to LO order cross section

- contributes only at higher orders to the position of the resonance peak $\tau_{\text{peak}}$
  \[
  \left. \frac{d^2\sigma^{\text{NLO}}}{d\tau^2} \right|_{\tau=\tau_{\text{NLO peak}}} = 0
  \]

- $\tau_{\text{peak}}^{\text{NLO}}$ fully determined by NLL terms

- NLL sufficient for full NLO information in the peak

- mass scheme of coherent branching without shower cut is pole mass

this piece is not correctly reproduced by CB
NLO matched shower

\[ Q = 91 \text{ GeV}, \Lambda = 1.0 \text{ GeV} \]

\[ Q = 700 \text{ GeV}, \Lambda = 1.0 \text{ GeV} \]

massless
\[ Q_0 = 1.0 \text{ GeV} \quad \text{right bunch of curves} \]
\[ Q_0 = 1.5 \text{ GeV} \quad \text{middle bunch of curves} \]
\[ Q_0 = 2.0 \text{ GeV} \quad \text{left bunch of curves} \]

NLO matching improves events with radiation of a hard gluon → larger \( \tau \) values
NLO matching does not increase precision for simulations in the resonance region
Effects of a cutoff $Q_0 > 0$

- pole of the top quark propagator $= m_{t}^{CB}(Q_0) \neq m_{t}^{pole}$ (coherent branching mass)

\[ m_{t}^{CB}(Q_0) = m_{t}^{pole} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2) \]

- In the presence of the shower cut the ultra-collinear radiation generated by CB produces exactly the mass scheme change correction that is required so that the generator mass is exactly the coherent branching mass $m_{t}^{CB}(Q_0)$

\[ \sigma(m_1, Q, ...) = \sigma(m_2, Q, ...) + \delta m \times \frac{d}{dm} \sigma(m, Q, ...) \bigg|_{m=m_1} + \ldots \]

\[ \delta m = m_2 - m_1 \]

- CB mass is a short-distance mass: free of the pole mass renormalon
- The shower cut also affects large-angle soft radiation. The corresponding effects are directly tied to the amount of hadronization effects that are supposed to be fixed by tuning.
- All conclusions explicitly cross checked by correspondence between analytic QCD factorization calculations and analytic solutions of the CB algorithm
- All results checked directly by comparing with Herwig 7 event generator.
$Q_0 > 0$: coherent branching (angular ordered parton shower)

- we can now work out the leading effects of introducing a shower cut $q_\perp > Q_0$

- keep only terms linear in $Q_0$ and $m$, only NLO in $\alpha_s$

- with these expansions the difference of the distributions with and without cutoff can be calculated analytically

- leading effect of $Q_0$ is a shift in the partonic cross section with contributions coming from the soft and ultra-coll. regions

$$\frac{d\sigma^{cb}}{d\tau}(\tau, Q, m, Q_0) = \frac{d\sigma^{cb}}{d\tau}(\tau + \frac{\alpha_s(Q_0)}{4\pi} \left[ 16C_F \frac{Q_0}{Q} - 8\pi C_F \frac{Q_0 m}{Q^2} \right], Q, m, Q_0 = 0)$$
$Q_0 > 0$: QCD factorization theorem

- introduce the cutoff $q_\perp > Q_0$ in the one-loop diagrams for the soft and jet functions in the QCD factorization theorem

- keep only terms linear in $Q_0$ and $m$, multipole expansion for real radiation terms

- SCET soft function at one-loop with $q_\perp$ cut:

$$S(\ell, Q_0) = S(\ell) + \frac{\alpha_s(Q_0)}{4\pi} 16 C_F Q_0 S''(\ell) + \mathcal{O}(\alpha_s^2)$$

extra term needs to be absorbed into a change in the non-pert. shape function

- bHQET jet function at one-loop with $q_\perp$ cut: off-shell

$$J_B^{\text{off}}(s, m_{\text{pole}}, Q_0) = J_B^{\text{off}}(s, m_{\text{pole}}) - \frac{\alpha_s(Q_0)}{4\pi} 8\pi C_F m Q_0 J_B^{\text{off'}}(s, m_{\text{pole}}) + \mathcal{O}(\alpha_s^2)$$

extra term gets absorbed by change of mass scheme

- bHQET jet function at one-loop with $q_\perp$ cut: on-shell self energy

$$J_B^{\text{os}}(s, m_{\text{pole}}, Q_0) = J_B^{\text{os}}(s, m_{\text{pole}}) + \frac{\alpha_s(Q_0)}{4\pi} 8\pi C_F m Q_0 J_B^{\text{os'}}(s, m_{\text{pole}}) + \mathcal{O}(\alpha_s^2)$$

$$= J_B^{\text{os}}(s, m_{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0)) + \mathcal{O}(\alpha_s^2)$$

change of mass scheme: $m_{\text{pole}} \rightarrow m_{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0)$
Comparison with CB

- if NOT compensated by retuning of hadronization model and redefinition of mass scheme, change of cutoff leads to shift that gets contributions from soft and ultra-collinear radiation

\[
\tau_{\text{peak}}(Q_0) = \tau_{\text{peak}}(Q'_0) - \frac{1}{Q} \left(16C_F - 8\pi C_F \frac{m}{Q}\right) \int_{Q'_0}^{Q_0} dR \frac{\alpha_s(R)}{4\pi}
\]

- agrees with result for shift of peak position obtained from coherent branching

- mass reduces leading coefficient of R-evolution

- dependence of the peak position on shower cut \(Q_0\) can be compared to actual angular ordered parton shower in MC
Comparison with Herwig

- compare our predictions for peak shift with real parton shower
- used Herwig 7 with angular ordered shower for $e^+e^- \rightarrow t\bar{t}$

- modifications:
  - set all constituent masses of light quarks and gluon to zero
  - unrestricted kinematics in evolution of CB
  - on-shell top production
  - only leptonic W-decays
  - switched off: QED radiation, hadronization

- only partonic distribution from Herwig
  “hadronization”: convolution with model function $\rightarrow$ disentangle parton shower and hadronization model

\[
S_{\text{mod}}(k, \lambda) = \frac{128}{3} \frac{k^3 e^{-4k}}{\lambda^4} \quad \tilde{\lambda} = \lambda + \frac{4m_t \Gamma_t}{Q}
\]

- $m_t = 173$ GeV $\quad \Gamma_t = 1.5$ GeV
- run for different values of $Q_0$, $Q$ and $\lambda$.
- use rescaled $\tau$ variable $M_\tau = \frac{Q^2 \tau}{m_t}$ (partonic threshold at $m_t$)
- compare cutoff dependence of peak position with R-evolution

\[
M_{\tau,\text{peak}}(Q_0) = M_{\tau,\text{peak}}(Q'_0) - \left( 8C_F \frac{Q}{m_t} - 4\pi C_F \right) \int_{Q'_0}^{Q_0} dR \frac{\alpha_s(R)}{4\pi}
\]
Comparison with Herwig

$Q = 91 \text{ GeV}, \Lambda = 3.0 \text{ GeV}$

$Q = 300 \text{ GeV}, \Lambda = 3.0 \text{ GeV}$

$Q = 700 \text{ GeV}, \Lambda = 3.0 \text{ GeV}$

$Q = 1000 \text{ GeV}, \Lambda = 3.0 \text{ GeV}$
Relation of $m_t^{\text{CB}}$ to other Masses

Herwig 7: $Q_0 = 1.25 \text{ GeV} \quad \Rightarrow \quad m_t^{\text{Herwig}} = m_t^{\text{CB}}(1.25 \text{ GeV})$

MSR Mass:

$$m_t^{\text{MSR}}(Q_0) = m_t^{\text{CB}}(Q_0) + 0.24 Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$\Rightarrow m_t^{\text{MSR}}(Q_0) = m_t^{\text{CB}}(Q_0) + (0.190 \pm 0.070) \text{ GeV}$$

- CB and MSR masses do not suffer from $\mathcal{O}(\Lambda_{\text{QCD}})$ renormalon (due to IR cut) \quad \rightarrow \quad \text{good convergence}

- uncertainty estimated from difference between $\alpha_s$ in $\overline{\text{MS}}$ and MC schemes

- precision sufficient for all possible applications at the LHC! (recall restriction 1-3)

- more precision may be needed for a future $e^+ e^-$ collider
Relation of $m_t^{\text{CB}}$ to other Masses

Herwig 7: $Q_0 = 1.25 \text{ GeV} \quad \Rightarrow \quad m_t^{\text{Herwig}} = m_t^{\text{CB}}(1.25 \text{ GeV})$

Pole Mass:

$$m_t^{\text{pole}} = m_t^{\text{MSR}}(Q_0) + (0.350 \pm 0.250) \text{ GeV}$$

$$\Rightarrow m_t^{\text{pole}} = m_t^{\text{CB}}(Q_0) + (0.540 \pm 0.260) \text{ GeV}$$

• pole mass suffers from $\mathcal{O}(\Lambda_{\text{QCD}})$ renormalon $\rightarrow$ irreducible ambiguity of 250 MeV

• difference between $m_t^{\text{pole}}$ and $m_t^{\text{Herwig}} \sim 500 \text{ MeV} >$ ambiguity!

$\Rightarrow$ important to study $\Delta_m^{\text{pert}}$ beyond current set up (lift restrictions 1-3)

• shift as large as current experimental uncertainty from direct methods

[Hoang, Lepenik, Preisser (2017)]

[±110 MeV: Beneke, Marquard, Nason, Steinhauser (2017)]
Hadronization Effects (work in progress)

• so far considered only perturbative contributions ($\Delta_{m}^{\text{pert}}$)
  no hadronization effects

• found agreement between Herwig’s (angular ordered) parton shower and analytic QCD predictions

• question: how well does Herwig’s hadronization model match the analytic predictions?

• goal: understand possible non-pert. contributions to generator mass ($\Delta_{m}^{\text{non-pert}}$)

• start again with thrust distribution for massless quarks

\[
\frac{1}{\sigma_0} \frac{d\sigma}{d\tau'}
\]

hadronization model

\[
\frac{1}{\sigma_0} \frac{d\sigma}{d\tau}
\]

partonic

hadronic
Hadronization Effects: QCD Factroization Theorem

- hadronization modeled by convolution with non-pert. function $S_{\text{mod}}$

\[
\frac{d\sigma}{d\tau}(\tau, Q, Q_0) = \int_{Q_0}^{\tau} d\tau' \frac{d\hat{\sigma}}{d\tau}(\tau', Q, Q_0) S_{\text{mod}}(Q(\tau - \tau') + \Delta_{\text{soft}}(Q_0))
\]

- peak region: change of cutoff $Q_0$ is compensated by shift in soft function

\[
\Delta_{\text{soft}}(Q_0) = \Delta_{\text{soft}}(Q_0') + 16 \int_{Q_0'}^{Q_0} dR \left[ \frac{\alpha_s(R)C_F}{4\pi} + \mathcal{O}(\alpha_s^2) \right]
\]

- convolution above implies that each bin $\tau'$ in the partonic distribution gets smeared with a function $\hat{S}(Q\tau, Q\tau') = S_{\text{mod}}(Q(\tau - \tau'))$

$\hat{S}(Q\tau, Q\tau')$

- linear dependence of peak of $\hat{S}$ on the partonic value $\tau'$

$\tau_{\text{peak}}(\tau') = \tau_{\text{peak}}(0) + \tau'$
Herwig Cluster Model

- standard hadronization model of Herwig: cluster hadronization model
  [Webber (1984)]

- final state gluons split into $q\bar{q}$

- color-connected quarks combined into preconfined clusters

- for heavy clusters: fission along string axis (repeat until light enough)

- final clusters decay isotropically into hadrons

- various tuning parameters, specifying e.g. mass spectrum of daughter clusters, maximum mass of final clusters, constituent masses, ...
Hadronization Effects: Herwig

- want to understand Herwig’s cluster hadronization model in more detail

- define (observable dependent) effective hadronization function $\hat{S}$

$$
\frac{d\sigma}{d\tau}(\tau, Q) = \int d\tau' \frac{d\hat{\sigma}}{d\tau}(\tau', Q) \hat{S}(Q\tau, Q\tau')
$$

- interpretation of $\hat{S}(Q\tau, Q\tau')$: probability distribution that an event with partonic thrust value $\tau'$ has the thrust value $\tau$ after hadronization.

- first step: want to check if the behavior of $\hat{S}$ is compatible with analytic QCD, i.e. $\hat{S}(Q\tau, Q\tau') = S(Q(\tau - \tau'))$.

- second step: compare behavior of $\hat{S}$ after a retuning when the shower cut $Q_0$ is changed with our predictions.
Hadronization Effects: Herwig

- modified Herwig to give partonic and hadronic thrust value for each event

- results can be filled in a 2D-Histogram that shows how each partonic bin is migrated into the hadronic bins

- can be used to extract the effective hadronization function $\hat{S}$
For very small $\tau'$ the effective hadronization function of Herwig does not match the behavior predicted by the analytic QCD factorization theorem.

Why is this the case?
Hadronization Effects: Herwig

• if there are only few perturbative emissions the color-connected partons can be far apart in phase-space
  ⇒ clusters with large invariant mass

• extreme case: event with no additional radiation, only two back-to-back light quarks
  ⇒ one cluster at rest in lab frame with mass $M = E_{\text{cm}}$.

• seems that in this case fission process does not produce sufficiently light and fast clusters to map a partonic event with $\tau' = 0$ to a hadronic event with extremely small $\tau$.

• is this a problem? if yes, how can it be cured?
  ⇒ more studies needed
Conclusions

- for angular ordered parton showers (Herwig) one can derive the **perturbative contributions** between generator mass and pole masse ($\Delta^\text{pert}_m$)

\[
m^{\text{CB}}(Q_0) = m^{\text{pole}} - \frac{2}{3} \alpha_s(Q_0) Q_0 + \mathcal{O}(\alpha_s^2)
\]

this corresponds to the **pole of the quark propagator** in presence of a shower cut

- current restrictions:
  - boosted top quarks
  - narrow width approximation
  - top production (2-jettiness)

- needed to remove restriction:
  - parton shower algorithm for slow tops
  - parton shower for unstable tops
  - factorized predictions including top decay

- for all three new conceptual developments are required (w.i.p.)

- numerical calibration still important tool for consistency checks

- ongoing work to better understand MC **hadronization models** and eventually **non-perturbative contributions** $\Delta^\text{non-pert}_m$ to the relation between generator mass and pole mass (w.i.p.)
Backup
Can CB describe peak position at NLO?

peak position with NLL

partonic cross section in SCET at NLO:

\[
\frac{d\hat{\sigma}}{d\tau} = \delta(\tau) + \frac{\alpha_s C_F}{4\pi} \left\{ -8 \left[ \frac{\ln \frac{\tau}{\tau_0}}{\tau_0} \right]_{\text{LL}} + 6 \left[ \frac{1}{\tau} \right]_{\text{NLL}} + \delta(\tau) \left( \frac{2\pi^2}{3} - 2 \right) \right\} + \mathcal{O}(\alpha_s^2) 
\]

can use that N\textsuperscript{2}LL@NLO part is proportional to LO. \( f(\tau) = \frac{d\hat{\sigma}}{d\tau} \otimes S_{\text{mod}} \)

\[ f_{\text{NLO}}(\tau) = \tilde{f}_{\text{NLO}}(\tau) + \alpha f_{\text{LO}}(\tau) \]

\[ f(\tau) = f_{\text{LO}}(\tau) + f_{\text{NLO}}(\tau) = (1 + \alpha) f_{\text{LO}}(\tau) + \tilde{f}_{\text{NLO}}(\tau) + \mathcal{O}(\alpha_s^2) \]

LO peak position:

\[ f'_{\text{LO}}(\tau_0) = 0 \]

NLO peak position: \( \delta \tau \sim \alpha_s \)

\[ f'(\tau_0 + \delta \tau) \overset{!}{=} 0 + \mathcal{O}(\alpha_s^2) \]

\[ = (1 + \alpha) f'_{\text{LO}}(\tau_0 + \delta \tau) + \tilde{f}'_{\text{NLO}}(\tau_0 + \delta \tau) + \mathcal{O}(\alpha_s^2) \]

\[ = (1 + \alpha) f'_{\text{LO}}(\tau_0) + \delta \tau f''_{\text{LO}}(\tau_0) + \tilde{f}'_{\text{NLO}}(\tau_0) + \mathcal{O}(\alpha_s^2) \]

\[ \Rightarrow \delta \tau = \frac{-\tilde{f}'_{\text{NLO}}(\tau_0)}{f''_{\text{LO}}(\tau_0)} + \mathcal{O}(\alpha_s^2) \]

CB at NLL has full NLO information on peak position.
Massless $\tau$ Distributions

\[ Q = 91 \text{ GeV}, \Lambda = 3 \text{ GeV} \]

\[ \Delta(Q_0) = \frac{16}{3\pi} \int_{1.5 \text{ GeV}}^{Q_0} dR \alpha_s(R) \]

no gap
Massive $M_\tau$ Distributions

\[ \Delta(Q_0) = \frac{16}{3\pi} \int_{1.5\text{GeV}}^{Q_0} dR \alpha_s(R) \]

\[ m = 173 \text{ GeV} \]
Massive $M_\tau$ Distributions

Q=700 GeV, $\Lambda=3$ GeV

$\Delta(Q_0) = \frac{16}{3\pi} \int_{1.5\text{GeV}}^{Q_0} dR \alpha_s(R)$

$m(Q_0) = 173 \text{ GeV} - \frac{2}{3} \int_{1.5\text{GeV}}^{Q_0} dR \alpha_s(R)$

$m(1.0 \text{ GeV}) = 173.22 \text{ GeV}$

$m(2.0 \text{ GeV}) = 172.86 \text{ GeV}$

no gap

$m = 173 \text{ GeV}$
Reconstructed Observables: $m_{bj\ell}$ and $m_{bjW}$

- studied two different observables (for boosted tops)

\[
m_{bj\ell} = \sqrt{(p_{bj} + p_\ell)^2}
\]
\[
m_{bjW} = \sqrt{(p_{bj} + p_\ell + p_\nu)^2}
\]

- jet distance measures

\[
d_{ij} = \min(E_i^{2p}, E_j^{2p}) \frac{1 - \cos \theta_{ij}}{1 - \cos R}
\]
\[
d_{iB} = E_i^{2p} \quad p = \{-1, 0, 1\}
\]

- for $R = \pi/2$: recover full hemisphere mass

\[
m^{R=\pi/2}_{t,\text{fit}}(Q_0) = m^{R=\pi/2}_{t,\text{fit}}(Q'_0) - \left[4C_F \frac{Q}{m_t} - 2\pi C_F\right] \int_{Q'_0}^{Q_0} dR \frac{\alpha_s(R)}{4\pi}
\]

- for $R \sim m_t/Q$: wide angle soft radiation is cut away, only ultra-collinear (boosted with top quark) radiation inside the cone

\[
m^{R\sim m_t/Q}_{t,\text{fit}}(Q_0) = m^{R\sim m_t/Q}_{t,\text{fit}}(Q'_0) + 2\pi C_F \int_{Q'_0}^{Q_0} dR \frac{\alpha_s(R)}{4\pi}
\]
Reconstructed Observables: $m_{bj \ell}$ and $m_{bj W}$: comparison with Herwig

![Graphs showing comparison of reconstructed observables $m_{bj \ell}$ and $m_{bj W}$ with Herwig.]

- Generate distributions for given $Q_0$ for $m_t = 173$ GeV
- Fit top mass with distribution for reference cutoff $Q_0 = 1.5$ GeV

$Q = 700$ GeV $\Rightarrow m_t/Q \sim 0.25$

**Observables:***
- $m_{bj W}$
  - $p = -1$ (△)
  - $p = 0$ (△)
  - $p = +1$ (△)
- $m_{bj \ell}$
  - $p = -1$ (★)
  - $p = 0$ (★)
  - $p = +1$ (★)

- $m_{bj W}$: anti-$k_t$ type
- $m_{bj \ell}$: Cambridge-Aachen type
- $k_t$ type
Reconstructed Observables: $m_{bjl}$ and $m_{bjW}$: NLO matched shower

for each cutoff:

left: LO
middle: MC@NLO type
right: POWHEG type

symbols and colors as in previous slide
Reconstructed Observables: $m_{bjl}$ and $m_{bjW}$: NLO matched shower

for each cutoff:

left: LO
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symbols and colors as in previous slide