α_s determination from static QCD potential with lattice data

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Based on collaboration:
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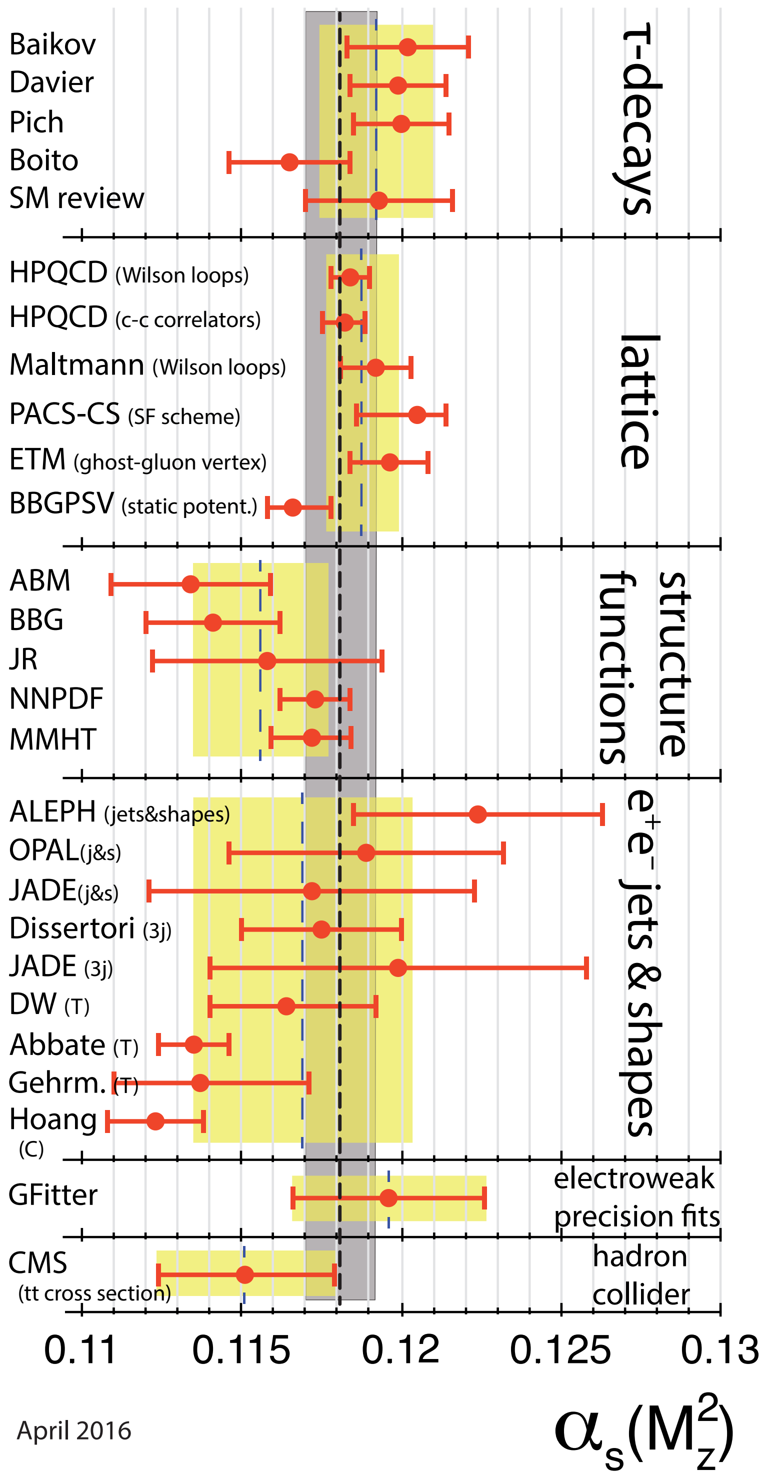
QCD coupling constant



Lattice QCD has advantages

- small error
- First principle, nonperturbative
- Further improvement can be expected

Static QCD potential is a good quantity for QCD coupling measurement

- Intuitively clear: interquark force
- Tools available: EFT and OPE, NNNLO perturbation
- Data available from lattice simulations

We extract α_s by enlarging matching region of OPE and lattice QCD for static potential.

Matching at shorter distance
→ Johannes's talk 1907.117474[hep-lat]

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Static QCD potential

Static potential in multipole expansion

Brambilla-Vairo-Pineda-Soto (2000)

$$V_{QCD}(r) = 2\delta m + V_S(r) + \delta E_{US}(r) + \cdots$$

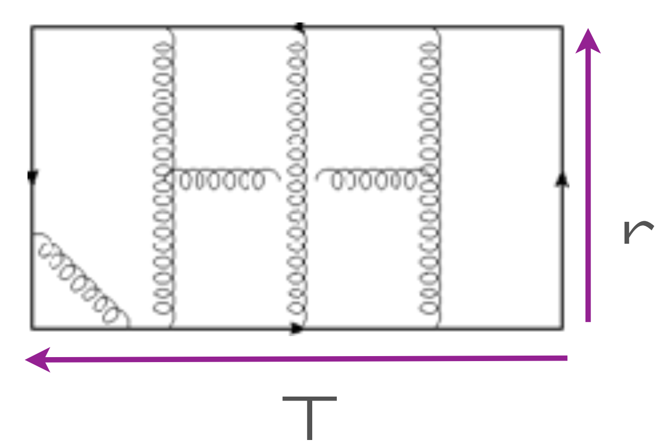
$$-\frac{C_F\alpha_s}{r} + \cdots \qquad \mathcal{O}(r^2)$$

Pole mass renormalon ⟷ Coulom + pert. corr. ⟷ Non-local gluon condensate

Renormalon cancellation | Renormalon cancellation

Lattice QCD simulates Wilson loop:

$$W[C] = \langle \mathrm{Tr}\,\mathrm{P}e^{ig\int_C dx\cdot A(x)}\rangle \overset{T\to\infty}{\to} e^{-TV_{QCD}(r)}$$



$V_s(r)$ and RG

We use momentum space RG improved Vs(r):

$$V_S(r) = -4\pi C_F \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{\alpha_V(q)}{q^2}$$

$$\alpha_V(q^2) = \alpha_s(q^2)\left\{ a_0 + a_1\left(\frac{\alpha_s(q^2)}{4\pi}\right) + a_2\left(\frac{\alpha_s(q^2)}{4\pi}\right)^2 + a_3\left(\frac{\alpha_s(q^2)}{4\pi}\right)^3 \right\}$$

W.Fischler
A.Billoire

M.Peter
Y.Schroeder

Smirnov-Smirnov-Steinhauser
Anzai-YK-Sumino, Lee-
Smirnov-Smirnov-Steinhauser

NNNLL: 4-loop RG running

$$\frac{d}{d\ln q^2}\alpha_s(q^2) = \beta(\alpha_s(q^2)) = -\alpha_s(q^2)\sum_{i=0}^{3}\beta_i\left(\frac{\alpha_s(q^2)}{4\pi}\right)^{i+1}$$

Renormalon subtraction

Renormalons in the static potential

$$[V_S]_{IR} = -4\pi C_F \int_{q \le \mu_f} \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{\alpha_V(q)}{q^2} \sim \tilde{C}_0 \Lambda + \tilde{C}_2 \Lambda^3 r^2 + \mathcal{O}(r^3)$$

IR Renormalons in Vs

IR subtracted potential (μ_f - dependent)

Contour deformation prescription by Sumino 05

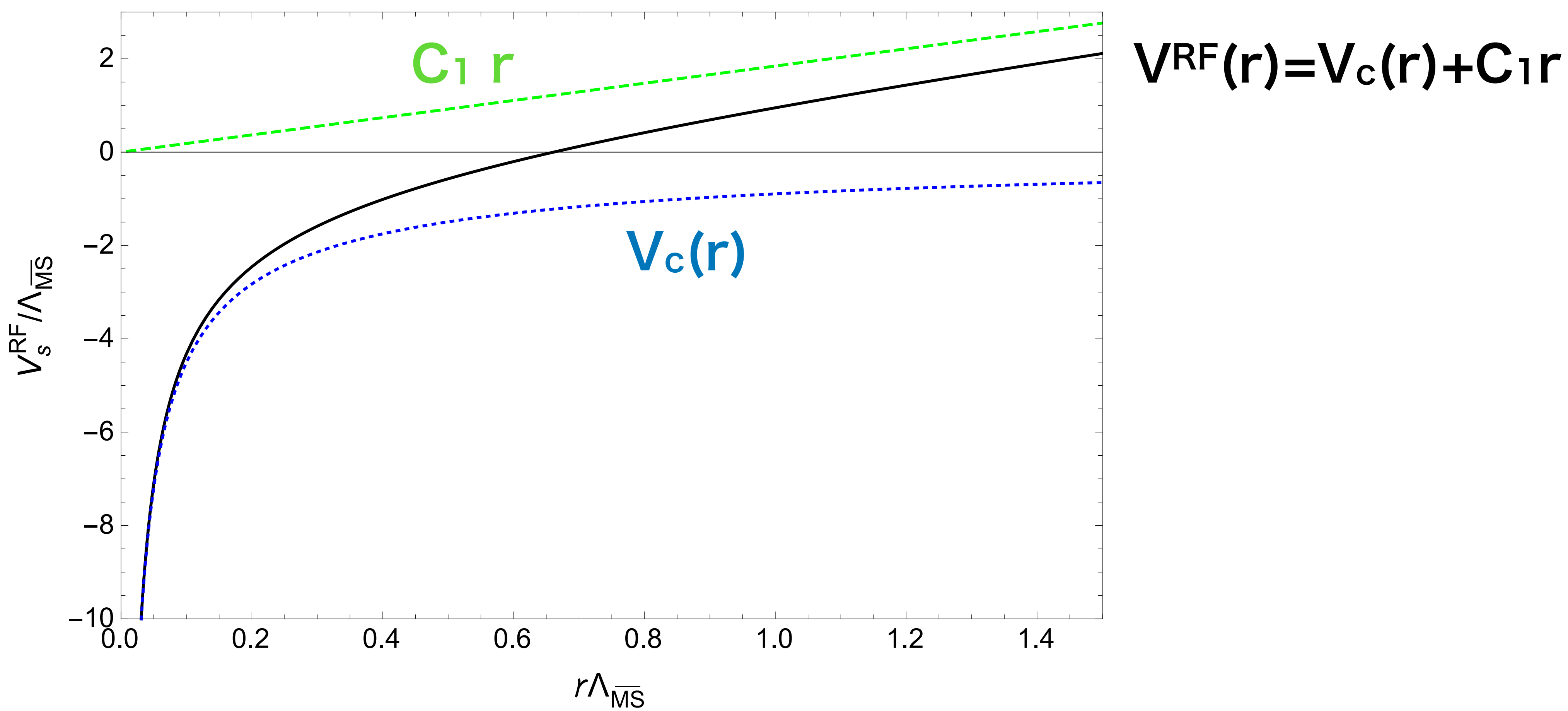
$$V_S(\mu_f, r) = -4\pi C_F \int_{q > \mu_f} \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{\alpha_V(q)}{q^2}$$
$$= V_C(\mu_f, r) + C_0(\mu_f) + C_1(\mu_f) r + C_2(\mu_f) r^2 + \mathcal{O}(r^3)$$

- ✦ Lim $_{\mu_f \to 0}$ $C_0(\mu_f)$, $C_2(\mu_f)$ are divergent
- ✦ $V_C(\mu_f, r)$, $C_1(\mu_f)$ are actually μ_f independent

Renormalon subtracted potential:

$$V_S^{RF}(r) = V_C(r) + C_1 r \qquad (C_1 = 1.8444 \Lambda_{\overline{\rm MS}}^2 \ \textbf{for } n_f = 3 \ \ \textbf{at N}^3\textbf{LL})$$

RF potential



- NNNLL $\alpha_V(q)$ is used
- Free from $O(\Lambda)$, $O(r^2\Lambda^2)$ renormalons
- Conventional RG improvements show unphysical fall-off around r∼ Λ but RF potential nicely rises up
- Extend the OPE validity region→ soften the window problem

OPE and Renormalon subtraction

We call the pNRQCD multipole expansion formula as OPE for static QCD potential

$$V_{\rm QCD}(r) = 2\delta m(\mu_f) + V_S(\mu_f, r) + \delta E_{US}(\mu_f, r)$$

$$= V_S^{RF}(r) + \{2\delta m(\mu_f) + C_0(\mu_f)\} + \{C_2(\mu_f) r^2 + \delta E_{US}(\mu_f, r)\}$$

renormalon free $\quad \equiv A_0 \quad \equiv \delta E_{US}^{RF}(r)$

We fit the OPE formula with lattice data: fit.param(A0, A2) and $\Lambda_{\rm QCD}$

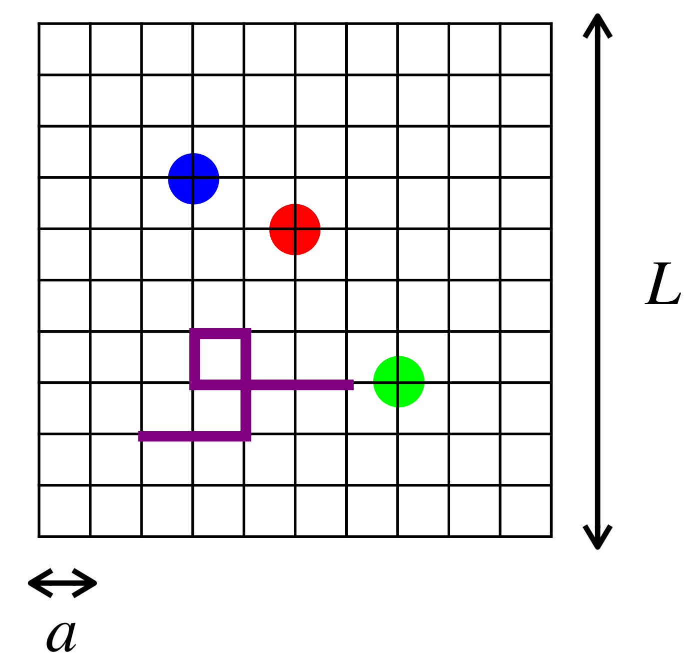
$$\Lambda_{\overline{\rm MS}}^{-1} V_S^{\rm OPE}(r) = \Lambda_{\overline{\rm MS}}^{-1}\left[V_S^{RF}(r) + A_0 + A_2 r^2\right]$$

Lattice simulation

JLQCD T. Kaneko

0.08, 0.05, 0.04 fm

- Lattice Size: 32^3x64, 48^3x96, 64^3x128
  - $a_1^{-1} = 2.453(4), \;\; a_2^{-1} = 3.610(9), \;\; a_3^{-1} = 4.496(9)\text{GeV}$
- Fermion: 2(u,d)+1(s) Domain-wall fermion
  - $M_\pi \sim 300\text{MeV} \quad (M_\pi^{phys} = 140\text{MeV})$
  - $M_K \sim 520\text{MeV} \quad (M_K^{phs} = 500\text{MeV})$
- Action: O(a)-improved action
  - discretization error～O(a^2)



Data analysis

- Statistics;
  - χ^2 -fit, covariance matrix, in Jackknife method
- Continuum limit:
  - extrapolation with a^2-fit
  - tree-level improvement
- Systematic error
  - mass effect, finite a, higher order, US, matching range, factorization scale

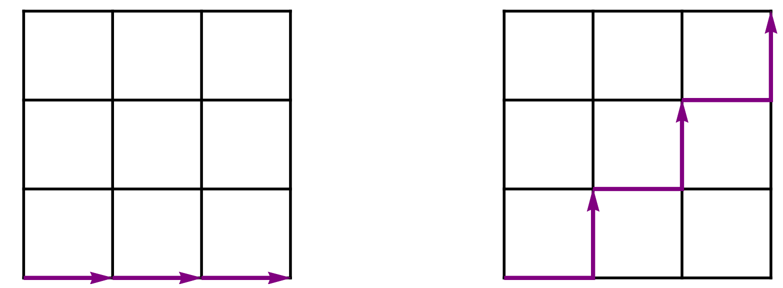
Analysis

- Analysis(I)
  - Continuum extrapolation
  - Matching with OPE
    - Check $O(a^2)$ discretization error
    - Check OPE validity; $V_S^{RF}(r) - V_{latt}(r) = O(r^2)$
- Analysis(II)
  - Continuum limit +Matching with OPE at once via a global fit
    - First principle analysis
    - tree-level improvement taken into account

A(I): cont. limit

- Continuum limit for direction=1,2

$$X_{\text{latt}} = r_1[V_{\text{latt}}(r) - V_{\text{latt}}(r_1)] \qquad \left(r_1^2 \frac{dV}{dr} = 1\right)$$



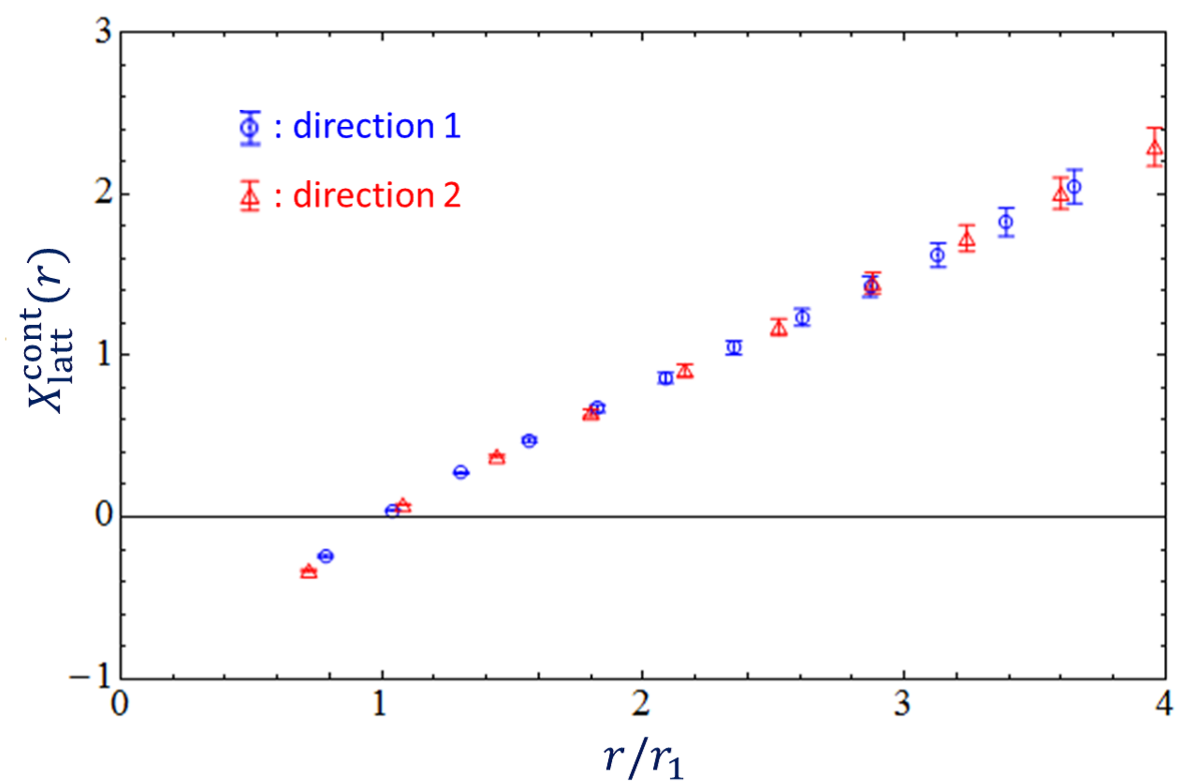
direction1 direction2

Extrapolation: $X_{\text{latt}}(r, a_1) \to X_{\text{latt}}(r, a_2) \to X_{\text{latt}}(r, a_3) \longrightarrow X_{\text{latt}}(r, a = 0)$

＊Model-like interpolations needed in analysis(I)

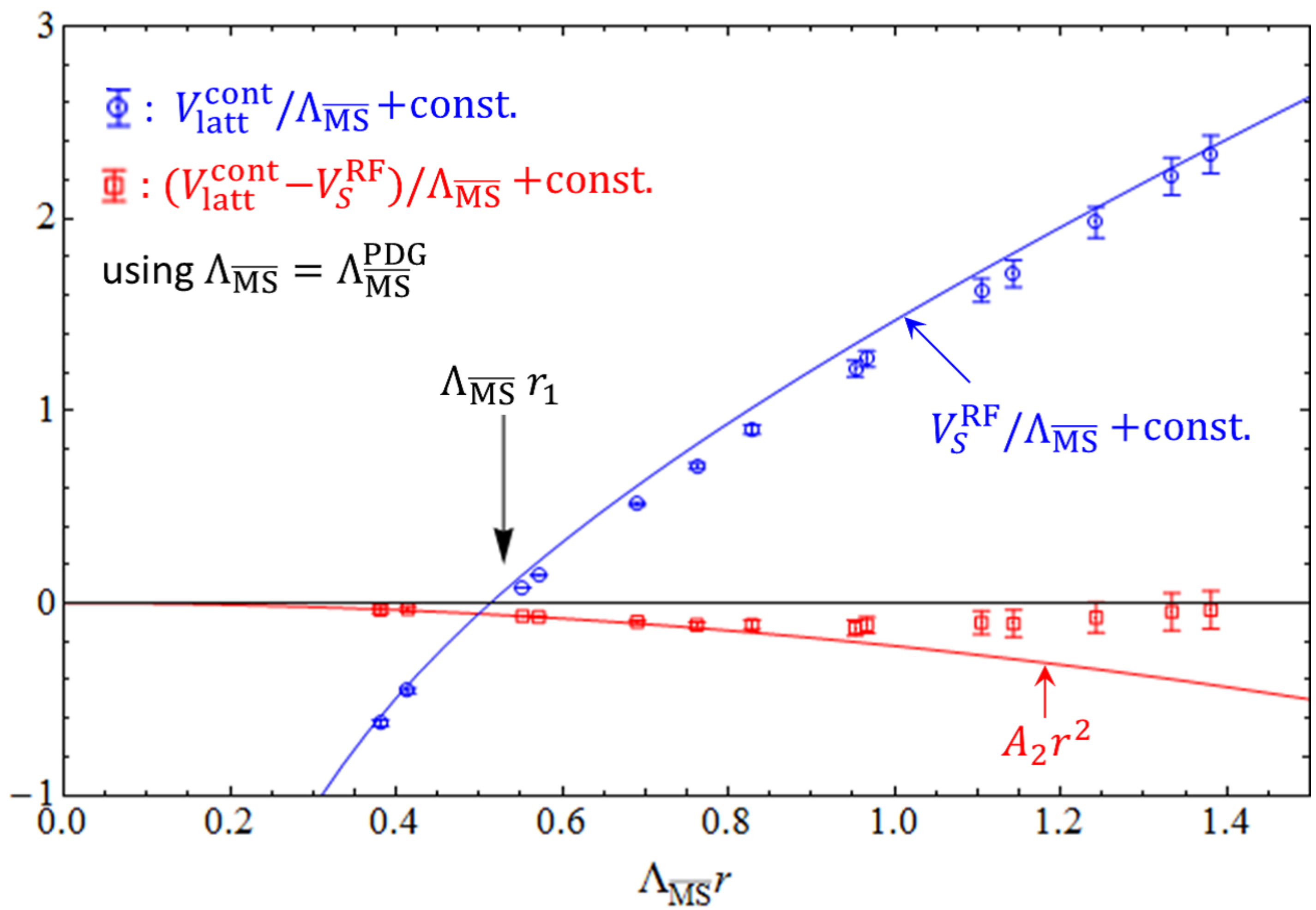
Controlling finite a and L effects, we used lattice data in the range *2a<r<L/2*

To avoid severe discretization error in the shortest distance,
And to avoid finite size effect in long distance



A(I): OPE validity range

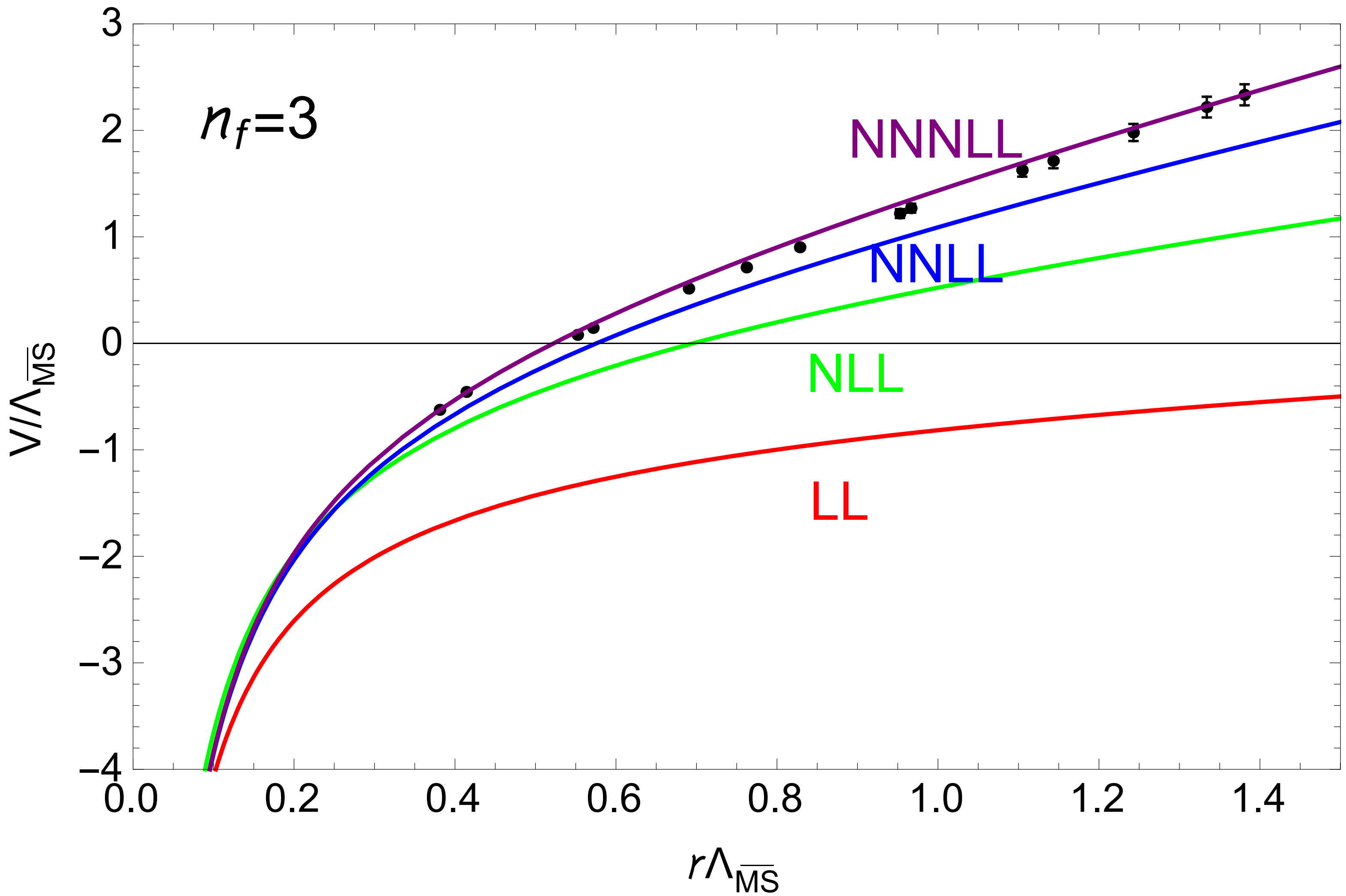
Check of OPE formula $V_{latt}^{cont} - V_S^{RF} = O(r^2)$ using $\Lambda_{\overline{\rm MS}}^{\rm PDG} = 336{\rm MeV}$



- Validity range of OPE $\Lambda r \lesssim 0.8 \quad (r \lesssim 0.5{\rm fm})$
- Conventional perturbation $\Lambda r \lesssim 0.3 \quad (r \lesssim 0.2{\rm fm})$

A(I): pert. convergence of $V_S^{\mathrm{RF}}(r)$

Check of perturbative convergence using $\Lambda_{\overline{\mathrm{MS}}}^{\mathrm{PDG}} = 336\mathrm{MeV}$



- As an input for $V_S^{RF}(r)$, $[\Lambda_{\overline{\mathrm{MS}}}^{\mathrm{N^3LL}} = \Lambda_{\overline{\mathrm{MS}}}^{\mathrm{PDG}} = 336\mathrm{MeV},\ \alpha_s(Q^2) = 0.2]$ is used
- convergence: $\Lambda_{\overline{\mathrm{MS}}} = 160^{\mathrm{LL}} \rightarrow 359^{\mathrm{NLL}} \rightarrow 339^{\mathrm{N^2LL}} \rightarrow 336^{\mathrm{N^3LL}}$ MeV

A(I): α_s determination

Maching OPE with continuum lattice data($r_1 = 0.311(2)\mathrm{fm}$)

$$\Lambda_{\overline{MS}}^{-1} V^{\mathrm{OPE}}(r) = \Lambda_{\overline{MS}}^{-1}\left[V_S^{RF}(r) + A_0 + A_2 r^2\right] \quad \longleftrightarrow \quad (\Lambda_{\overline{MS}} r_1)^{-1}\left[X_{\mathrm{latt}}^{\mathrm{cont}}\right](r/r_1)$$

$$\longrightarrow \quad \Lambda_{\overline{MS}}^{(n_f=3)} = 315 \pm 15(\mathrm{stat})\mathrm{MeV}$$

RG run from nf=3 to nf=5 is performed up to Mz scale

$$\alpha_s(M_z) = 0.1166^{+0.0010}_{-0.0011}(\mathrm{stat})^{+0.0018}_{-0.0017}(\mathrm{sys})$$

	finite a	interpol. fn.	subt. point	h.o.	US	range	r_1
Obtained value	-4	$+4$	-8	$+14$ $(t{=}-1)$ -12 $(t{=}1)$	$+1$ $(3\Lambda_{\overline{MS}})$ -0 $(4\Lambda_{\overline{MS}})$	$+5$ (0.7) -8 (0.9)	± 1
Assigned error	± 4	± 4	± 8	$^{+14}_{-12}$	± 1	$^{+5}_{-8}$	± 1

Table 5. Estimates of systematic errors in Analysis (I) from variations of the central value of $\alpha_s(M_Z^2)$ in units of 10^{-4} when varying the analysis conditions. In the upper row, variations are shown. (Detailed conditions are shown inside brackets). Assigned systematic errors are shown in the lower row.

Analysis(II):

Global fit: continuum limit and matching at once

$$V^{\text{cont}}_{\text{latt}}(r) = V_{\text{latt},d,i}(r) - \kappa_{d,i}\left(\frac{1}{r} - \left[\frac{1}{r}\right]_{d,i}\right) + f_d\frac{a_i^2}{r^3} - c_{0,d,i}$$

Matching

Tree-level improvement

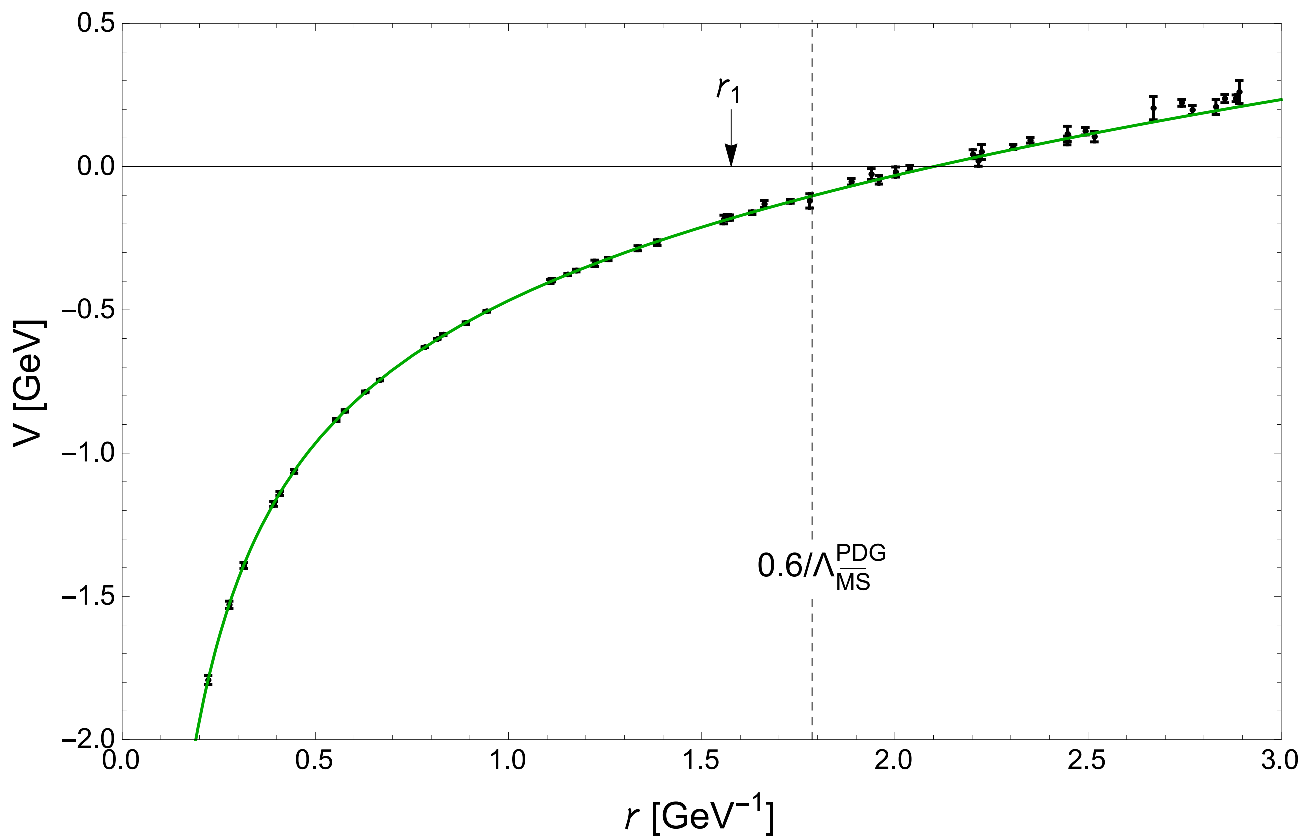
Remove disc. error of O(a²)

$$V_{\text{OPE}}(r) = V_S^{\text{RF}}(r) + A_2 r^2$$

- i=1,2,3 for lattice setup a_1, a_2, a_3
- d=1, 2 direction

- conversion from lattice a-unit(Wilson-flow scale) to Λ-unit
- 16 fitting parameters: $\{\Lambda_{\overline{\text{MS}}}, A_2, \kappa_{d,i}, f_d, c_{0,d,i}\}$
- Fit range $0.07 \le \Lambda^{\text{PDG}}_{\overline{\text{MS}}} r \le 0.6$ $(a \sim 0.05\ \text{fm} \le r \le 0.35\ \text{fm})$

A(II): α_s determination



Lattice in cont. limit(black) and OPE fit(green)

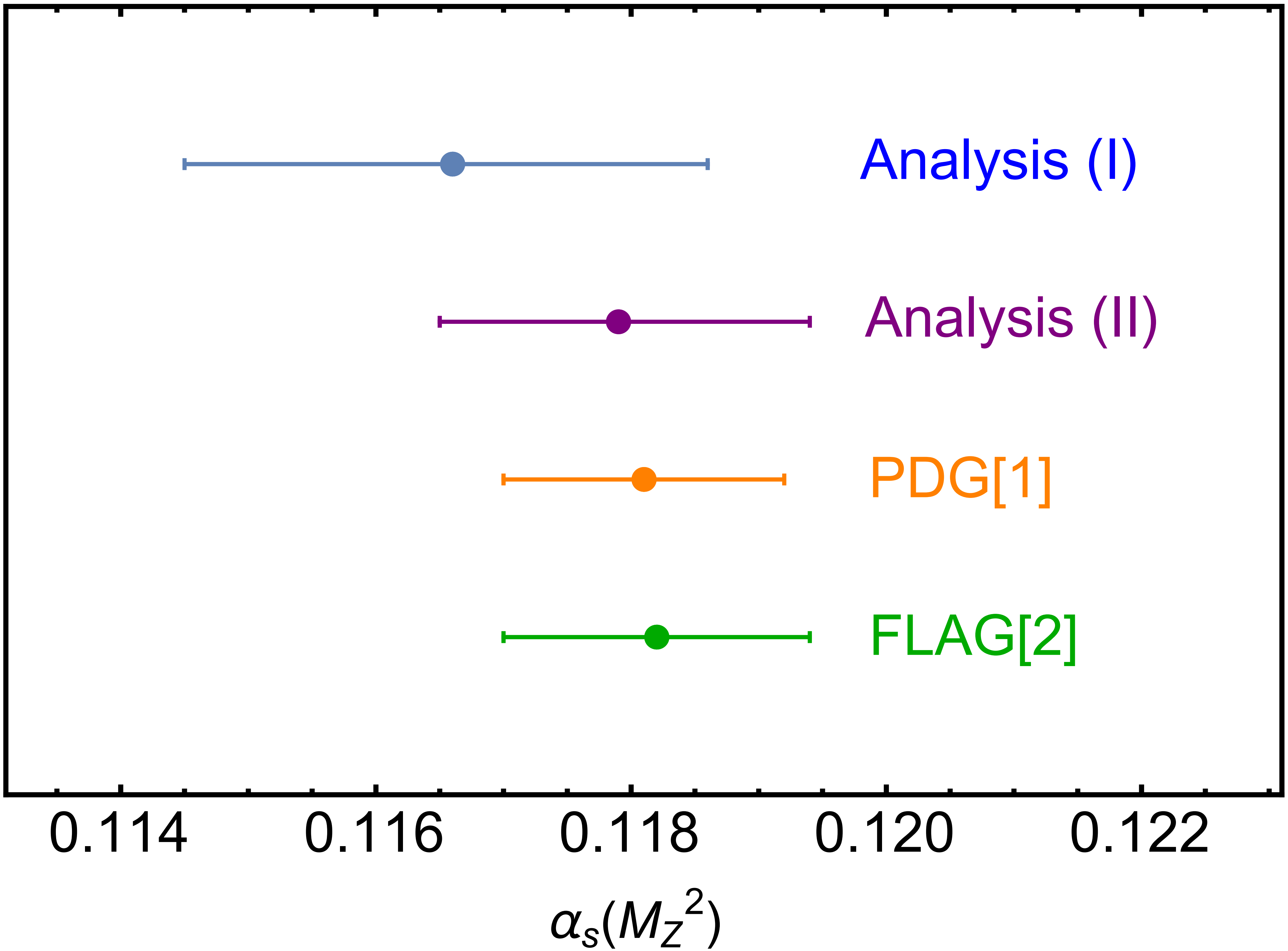
$$\Lambda_{\overline{\rm MS}}^{(n_f=3)} = 334 \pm 10({\rm stat})\ {\rm MeV}$$

$$\alpha_s(M_z) = 0.1179 \pm 0.0007({\rm stat})$$

$$\alpha_s(M_z) = 0.1179 \pm 0.0007({\rm stat})^{+0.0014}_{-0.0012}({\rm sys})$$

	finite a	h.o.	US	Mass	range	fact. scheme	latt. spacing
Obtained value	-2	$+12$ $(t=-1)$ -10 $(t=1)$	$+2$ $(3\Lambda_{\overline{\rm MS}})$ $+0$ $(4\Lambda_{\overline{\rm MS}})$	$-0(\overline{\rm MS}$mass / Constituent mass$)$	-3 (0.5) -4 (0.8)	$+3$	± 4
Assigned error	± 2	$^{+12}_{-10}$	± 2	± 0	± 4	± 3	± 4

Table 7. Estimates of systematic errors in Analysis (II) from variations of the central value of $\alpha_s(M_Z^2)$ in units of 10^{-4} when varying the analysis conditions. In the upper row, variations are shown. (Detailed conditions are shown inside brackets). Mass effects are negligibly small in both cases. Assigned systematic errors are shown in the lower row.


Analysis (I)
Analysis (II)
PDG[1]
FLAG[2]
0.114
0.116
0.118
0.120
0.122
$\alpha_s(M_Z^2)$

Summary

We extracted QCD strong coupling constant from static QCD potential computed in lattice QCD simulation:

- analysis (I): continuum limit of lattice and matching with OPE
  - Matching range: $3a \leq r < 0.8/\Lambda_{\overline{\rm MS}}^{\rm PDG}$
  - $\Lambda_{\overline{\rm MS}}^{n_f=3} = 315 \pm 15({\rm stat})^{+26}_{-25}({\rm sys})\ {\rm MeV}$
  - $\alpha_s(M_z) = 0.1166^{+0.0010}_{-0.0011}({\rm stat})^{+0.0018}_{-0.0017}({\rm sys})$
- Analysis (II): global fit for continuum limit and matching simultaneously
  - Matching range: $a \leq r < 0.6/\Lambda_{\overline{\rm MS}}^{\rm PDG}$
  - $\Lambda_{\overline{\rm MS}}^{n_f=3} = 334 \pm 10({\rm stat})^{+22}_{-18}({\rm sys})\ {\rm MeV}$
  - $\alpha_s(M_z) = 0.1179 \pm 0.0007({\rm stat})^{+0.0014}_{-0.0012}({\rm sys})$

Global fit

- Controlling finite a effects: The data at $r \geq a$ are used combined with the tree-level correction.

- Singlet potential: $V_S^{\mathrm{RF}}(r)$ defined by Eq. (2.10), which has N^{3}LL accuracy

- Regularization of US divergence: Prescription I [Eq. (2.20)]

- Quark masses: We use the lattice data obtained with unphysical quark mass inputs and V_S^{RF} in the massless quark approximation.

- Matching range: $\Lambda_{\overline{\mathrm{MS}}}^{\mathrm{PDG}} r < 0.6$

Systematic error

- *Finite a effects*: We use the lattice data at $r \geq 2a$. In this case, we omit the tree-level correction by setting κ's to zero. This is because the role of the tree-level correction is similar to that of the a^2/r^3-term under the current hierarchy $a/r \leq 1/2$, where the tree-level correction is well approximated in expansion in a/r.[27]

- *Higher order uncertainty*: We replace $V_S^{\rm RF}$ in matching as

$$V_S^{\rm RF} + t\delta V_S^{\rm RF} \tag{3.20}$$

  with $t = -1$ or 1 in order to estimate higher order uncertainty; see Eq. (2.23) for $\delta V_S^{\rm RF}$.

- *US regularization*: We adopt the regularization method II, given by Eq. (2.21). We have chosen $\mu_{\rm US}$ as $3\Lambda_{\overline{\rm MS}}$ and $4\Lambda_{\overline{\rm MS}}$.

- *Mass effects*: Lattice data are obtained with the unphysical mass inputs. We include an estimation of this mass difference effect as a systematic error, since we do not know the true correction. We estimate the lattice data on the physical point as

$$V_{{\rm latt},d,i}(r;m^{{\rm latt},i}) \to V_{{\rm latt},d,i}(r;\overline{m}) = V_{{\rm latt},d,i}(r;m^{{\rm latt},i}) + [V_{{\rm pt},i}(r;\overline{m}) - V_{{\rm pt},i}(r;m^{{\rm latt},i})]\,, \tag{3.21}$$

  where $\overline{m}$ is the $\overline{\rm MS}$ masses for the light quarks (u,d,s); $V_{\rm pt}$ is the finite mass correction evaluated in perturbative QCD at N^{2}LO [41–43]. More precisely, it is a function of $\{r,m,\mu\}$ of the form

$$V_{\rm pt}(r;m) = c_1(r,m)\alpha_s^2 + c_2(r,m,\mu)\alpha_s^3\,, \tag{3.22}$$

- *Matching range*: We vary the range of the lattice result used in the matching as

$$\Lambda_{\overline{\rm MS}}^{\rm PDG} r < 0.5 \text{ or } 0.8 \tag{3.24}$$

  to examine the stability of the OPE truncated at $\mathcal{O}(r^2)$.

- *Factorization scheme*: In extracting the renormalon free part $V_S^{\rm RF}$, we rewrite the integrand of V_S by a complex function; see (B.2) in Appendix B. In general, there can be other choices for this function, and in this regard, we have chosen a certain scheme. A different scheme practically causes an $\mathcal{O}(r^3)$ difference in the OPE prediction truncated at $\mathcal{O}(r^2)$; see Ref. [16] for details.[28] To see an effect of this scheme dependence, we add an $A_3 r^3$-term in the fit so that this scheme dependence is absorbed. (Note that, in order to determine coefficients up to higher orders in r, a wider fitting range is required. We choose the range in this analysis as $\Lambda_{\overline{\rm MS}}^{\rm PDG} r < 0.8$, where A_2 and A_3 are stable against variation of the range.[29])

- *Lattice spacing*: The lattice spacing a, used to convert r and $V_{\rm latt}$ into physical units, has an error as shown in table 1, and has an additional error of 1.7 % due to the uncertainty of the physical value of the Wilson-flow scale [44]. For the former one, the error is etimated by the largest deviation detected from a set of six data, $\{\{a_1 \pm \delta a_1, a_2, a_3\}, \{a_1, a_2 \pm \delta a_2, a_3\}, \{a_1, a_2, a_3 \pm \delta a_3\}\}$, where δa_i denotes the error shown in table 1. The error associated with the latter is estimated by shifting all the a's simultaneously by its uncertainty. By combining these two errors in $\alpha_s(M_Z^2)$ in quadrature, the uncertainty from the lattice spacing is estimated.

Fitting parameters

$$V_{\rm latt}^{\rm cont}(r) = V_{{\rm latt},d,i}(r) - \kappa_{d,i}\left(\frac{1}{r} - \left[\frac{1}{r}\right]_{d,i}\right) + f_d \frac{a_i^2}{r^3} - c_{0,d,i}$$

$$V_{\rm OPE}(r) = V_S^{\rm RF}(r) + A_2 r^2$$

- i=1,2,3 for lattice setup a_1, a_2, a_3
- d=1, 2 direction

i (size)	$i=1$ $(32^3\times 64)$		$i=2$ $(48^3\times 96)$		$i=3$ $(64^3\times 128)$	
d $(N_{i,d})$	$d=1$ (4)	$d=2$ (3)	$d=1$ (6)	$d=2$ (4)	$d=1$ (8)	$d=2$ (5)
κ	0.19(15)	−0.26(85)	0.27(12)	−0.53(88)	0.27(11)	−0.57(91)
c_0 [GeV]	2.245(11)	2.300(87)	3.012(11)	3.099(89)	3.546(10)	3.631(86)
χ^2	χ^2/d.o.f. $= 8.7/(30-16)$ (global fit)					
f_d	$f_1 = 0.0004(18)$, $f_2 = -0.025(32)$ (common to all i)					
A_2	$A_2 = -0.0091(54)$ GeV^3 (common to all i, d)					

Table 6. Fitting parameters in Analysis (II). Only statistic errors are shown. $N_{i,d}$ expresses the number of data points for direction d of the i-th lattice.