Determination of the strong coupling constant and heavy quark masses from moments of quarkonium correlators

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Determination of the strong coupling constant and heavy quark masses from the 2+1 flavor lattice QCD calculations of the moments of pseudo-scalar quarkonium correlators

PP, J. Weber, PRD 100 (2019) 034519
PP, arXiv:1902.02381, PoS (Confinement2018) 023

Extension of the previous work
Maezawa, PP, PRD PRD 94 (2016) 034504

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Moments of quarkonium correlators

We use moments method pioneered by HPQCD and Karlsruhe group:

$$G_n = \sum_t t^n G(t),\ \ G(t) = a^6 \sum_{\mathbf{x}} (am_{h0})^2 \langle j_5(\mathbf{x},t) j_5(0,0)\rangle \qquad j_5 = \bar{\psi}\gamma_5\psi$$

Calculated continuum perturbation theory to order α_s^3

$$G_n = \frac{g_n(\alpha_s(\mu), \mu/m_h)}{am_h^{n-4}(\mu_m)}$$

To cancel lattice effects consider the reduced moments

Note: $G_n \sim M_{2m+2},\ m = 1, 2, ...$

$$R_n = \left(\frac{G_n}{G_n^0}\right)^{1/(n-4)}$$

Allison et al, PRD78 (2008) 054513

and similarly on the weak coupling side:

$$R_n = \begin{cases} r_4 & (n=4) \\ r_n \cdot (m_{h0}/m_h(\mu)) & (n \geq 6) \end{cases},$$

$$r_n = 1 + \sum_{j=1}^{3} r_{nj}(\mu/m_h)\left(\frac{\alpha_s(\mu)}{\pi}\right)^j$$

+ contribution from condensate

$$\sim \frac{1}{m_h^4}\langle \frac{\alpha_s}{\pi} G^2 \rangle$$

$R_4,\ R_6/R_8, R_8/R_{10} \Rightarrow \alpha_s(\mu)$

$R_6,\ R_8\ R_{10} \Rightarrow m_h(\mu_m)$

Some lattice details

Highly improved Staggered Quark (HISQ) action and tree-level improved gauge action

HotQCD gauge configurations : 2+1 flavor QCD
physical m_s , $m_l=m_s/20$: $m_K=504$ MeV, $m_\pi=161$ MeV
Bazavov et al, PRD 90 (2014) 094503

Lattice spacing set by the r_1 scale

$$\left(r^2 \frac{dE_0(r)}{dr}\right)_{r=r_1} = 1$$

$r_1=0.3106(14)(8)(4)$ fm (pion decay constant)

Temperature is varied by the lattice spacing a

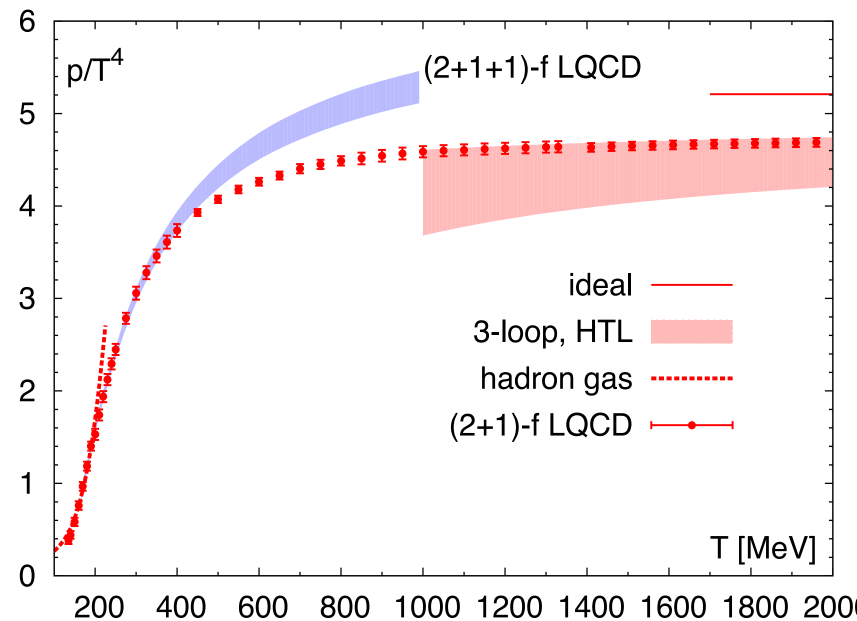
$T = (1/N_\tau a)$ →
Many lattice spacings available, $a_{min}=0.041$fm

Additional gauge configurations for
$m_l=m_s/5$ on 64^4 lattices with $a=0.035$ fm,
0.029 fm and 0.025 fm to obtain the static
quark potential at shorter distances
Bazavov, PP, Weber, PRD 97 (2018)

statistics for the $T=0$ runs:

24^3x32:	4-8K TU
32^4, 32^3x64:	7-40K TU
48^4:	8-16K TU
48^3x64:	8-9K TU
64^4 :	9K TU

in molecular dynamic time units (TU)



am_c^0 from spin averaged 1S charmonium mass or from $\eta_c(1S)$ mass

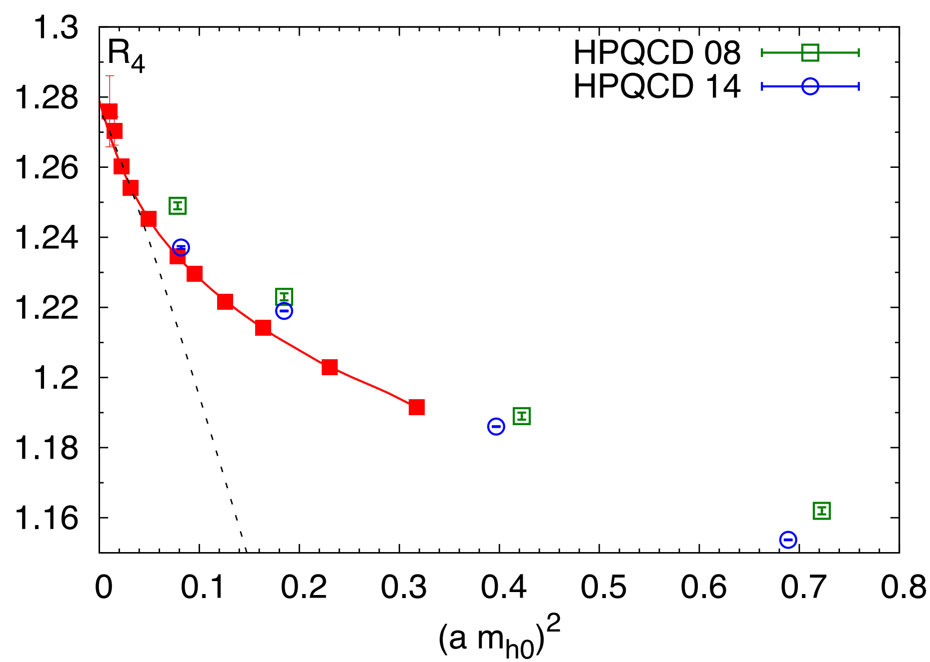
Lattice results on the moments of quarkonium correlators

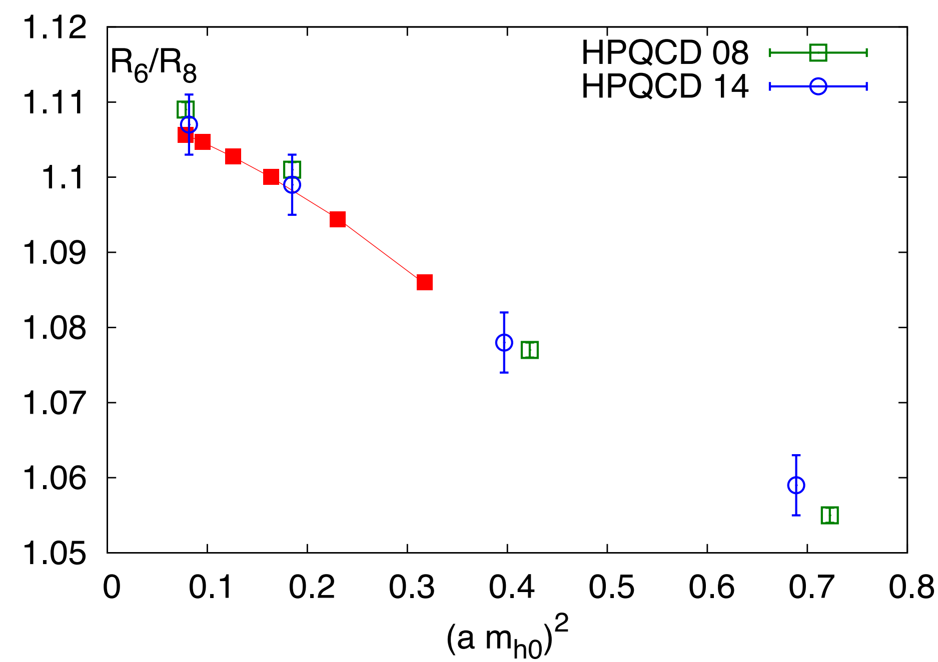
$$m_h = m_c,\ 1.5m_c,\ 2m_c,\ 3m_c,\ 4m_c,\ m_b$$

Random color wall sources => statistical errors are negligible; Dominant errors are the finite volume errors and errors due to mistuning of the heavy quark mass.

Volume errors are estimated from the free theory calculations (upper bound), are largest at small a

Sea quark mass effects are smaller than statistical errors $\Rightarrow$ combine $m_s/20$ and $m_s/5$ data

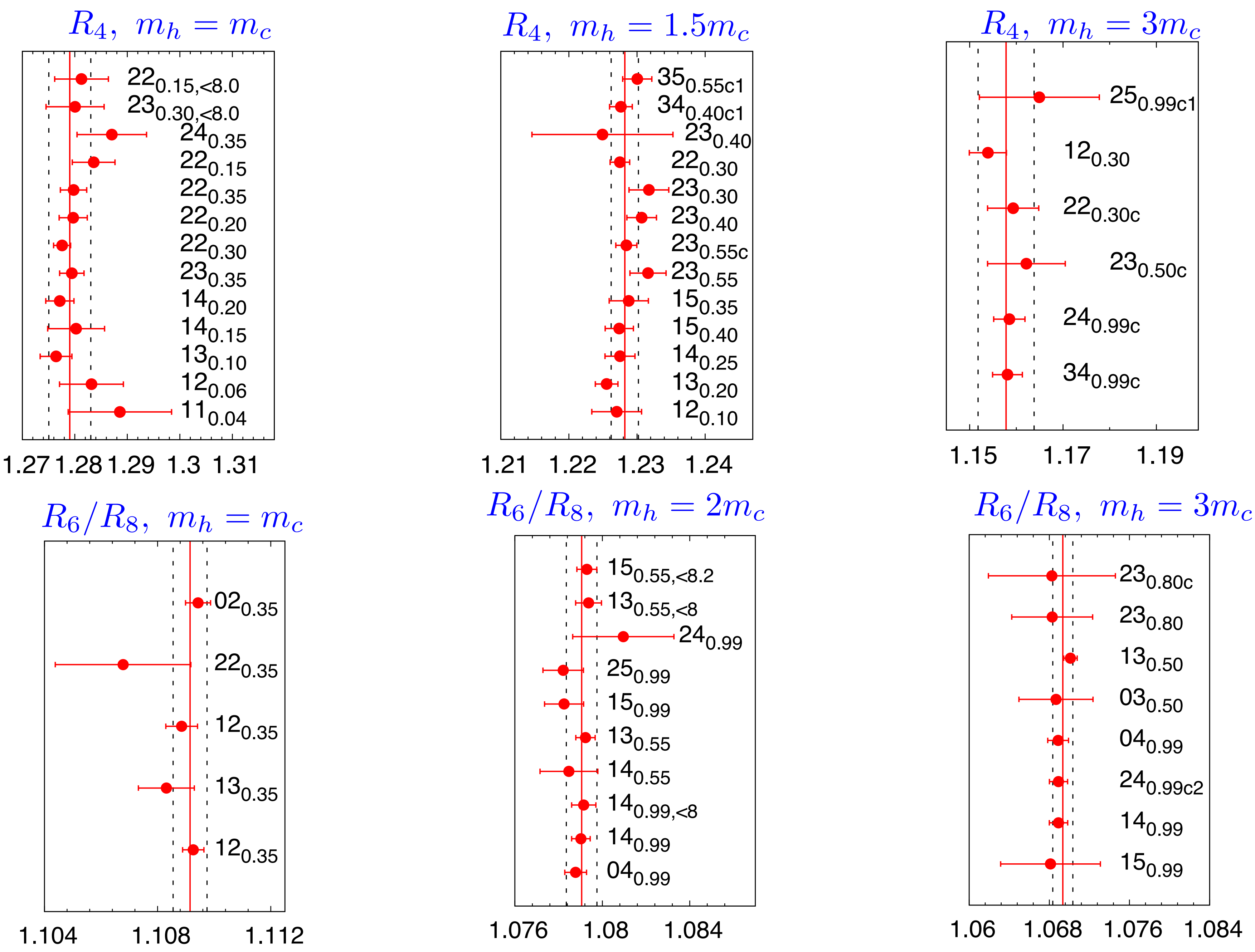




Continuum results are needed but there is a significant dependence on the lattice spacing

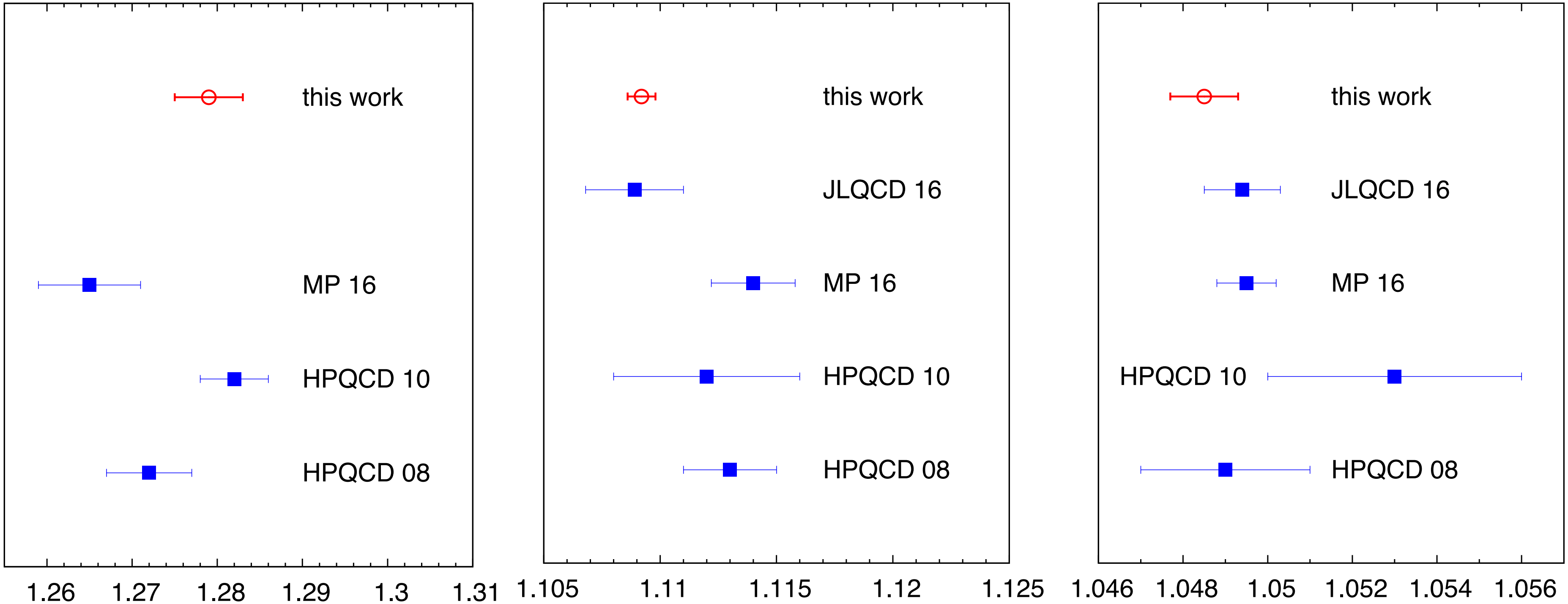
Lattice cutoff effects: $\sim \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_s^i (a m_{h0})^{2j} \Rightarrow$ use fits with $I = 2,\ J = 5$ and $\alpha_s = \frac{g_0^2}{4\pi u_0^4}$

Systematics of continuum extrapolations



Continuum results on the moments

$R_4,\ m_h = m_c$ $R_6/R_8,\ m_h = m_c$ $R_8/R_{10},\ m_h = m_c$



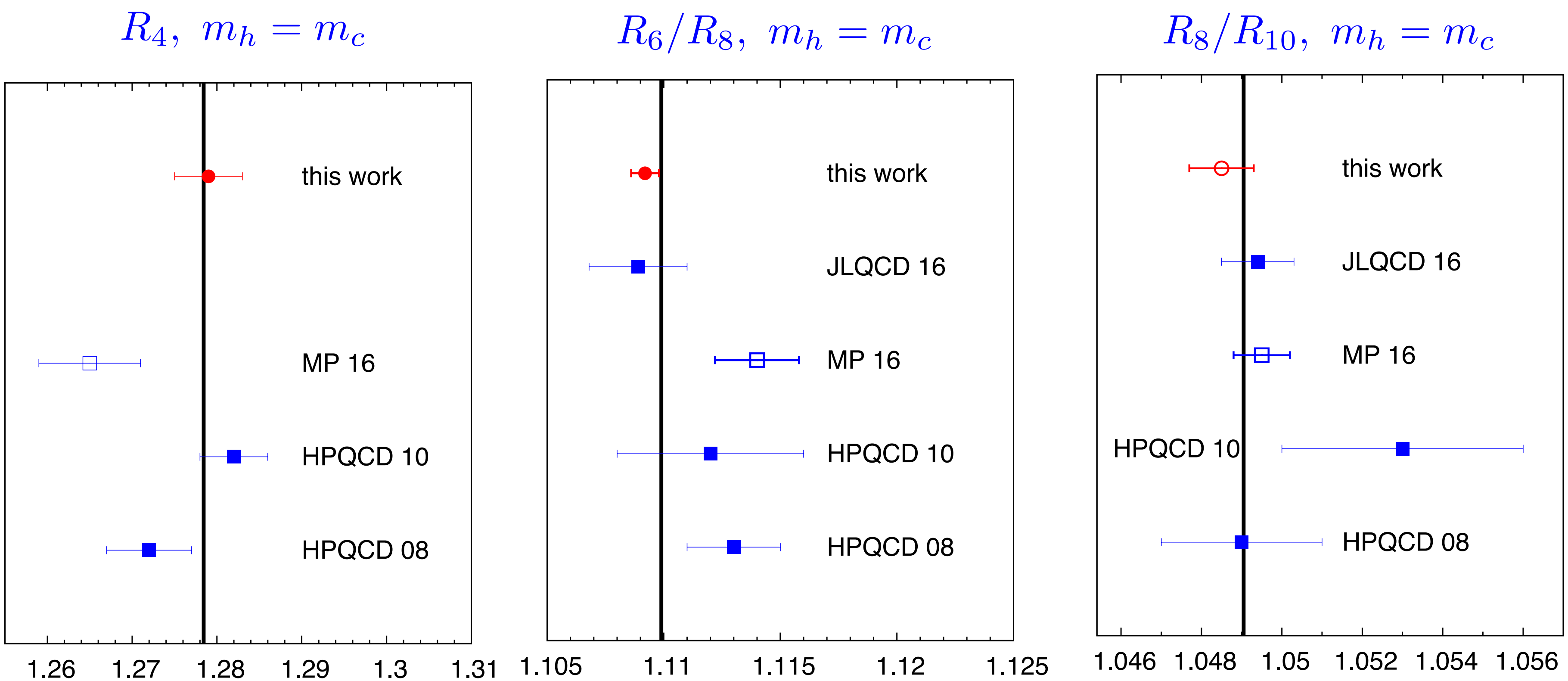
HPQCD: 2+1 flavor improved staggered (asqtad) sea + valence HISQ,
Allison et al, PRD 78 (2008) 054513; McNeile, PRD 82 (2010) 034512

JLQCD: 2+1 flavor Domain-Wall Fermions,
Nakayama, Fahy, Hashimoto, PRD 94 (2016) 054507

MP 16: 2+1 flavor HISQ (sea and valence sectors)
Maezawa, PP, PRD PRD 94 (2016) 034504

Discrepancies are understood to be due simple $a^2 + a^4$ extrapolations

Continuum results on the moments (cont'd)



Weighted average over independent lattice determinations:

$$R_4 = 1.2784 \pm 0.0027,\ \ \chi^2/df = 1.235$$

$$R_6/R_8 = 1.1099 \pm 0.0008,\ \ \chi^2/df = 2.364$$

$$R_8/R_{10} = 1.04905 \pm 0.00050,\ \ \chi^2/df = 0.786$$

There are some tension between different lattice determination (underestimated error, continuum extrapolation ?) But no major discrepancies

Extracting the strong coupling constant in 3f QCD

Natural choice: $\mu = \mu_m = m_h(m_h)$

$$R_4,\ R_6/R_8,\ R_8/R_{10} \to \alpha_s(m_h)$$

perturbative error: $\pm 5 \times r_{n3}\alpha_s^4$

condensate error: $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (-0.006 \pm 0.012)\ \mathrm{GeV}^4$

m_h	R_4	R_6/R_8	R_8/R_{10}	av.	$\Lambda_{\overline{MS}}^{n_f=3}$ MeV
$1.0m_c$	0.3815(55)(30)(22)	0.3837(25)(180)(40)	0.3550(63)(140)(88)	0.3782(65)	314(10)
$1.5m_c$	0.3119(28)(4)(4)	0.3073(42)(63)(7)	0.2954(75)(60)(17)	0.3099(48)	310(10)
$2.0m_c$	0.2651(28)(7)(1)	0.2689(26)(35)(2)	0.2587(37)(34)(6)	0.2648(29)	284(8)
$3.0m_c$	0.2155(83)(3)(1)	0.2338(35)(19)(1)	0.2215(367)(17)(1)	0.2303(150)	284(48)

perturbative and condensate errors decrease with increasing m_h/m_c

R_8/R_{10} gives systematically lower values of $\alpha_s(m_h)$ except for $m_h = 2m_c$

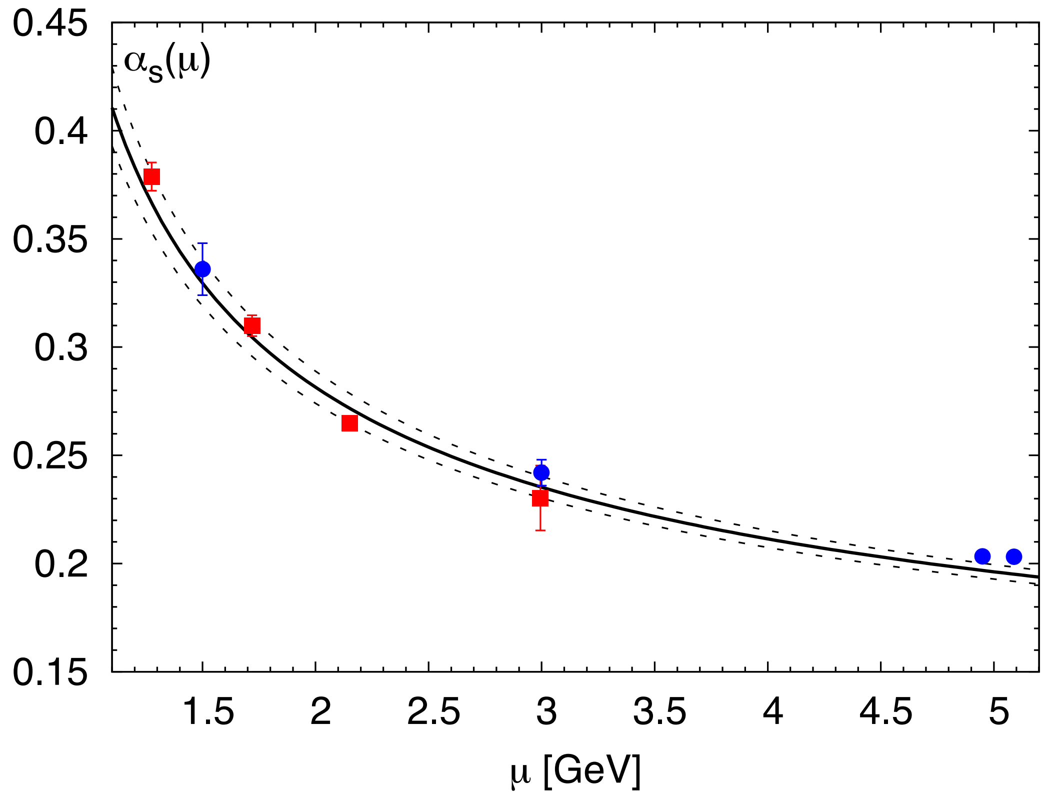
Take weighted average of R_4, R_6/R_8, R_8/R_{10} results to get the final $\alpha_s(m_h)$

$$\alpha_s(m_h),\ R_6,\ R_8,\ R_{10} \to m_h(m_h) \to \Lambda_{\overline{MS}}^{n_f=3}$$

$m_h = 2m_c$ result is an outlier; weighted average + spread :

$$\Lambda_{\overline{MS}}^{n_f=3} = 298 \pm 16\ \mathrm{MeV}$$

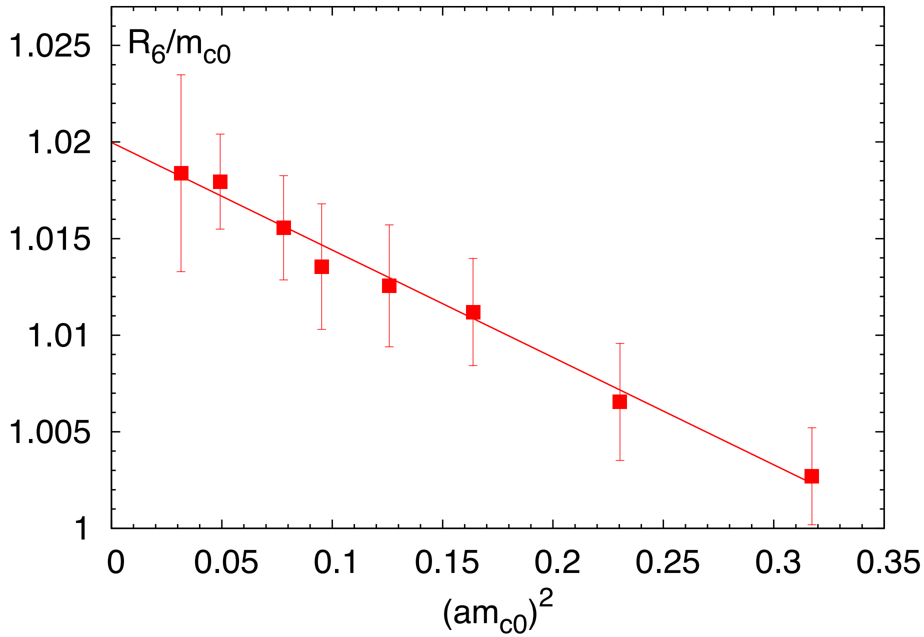
The lattice results on the running coupling constant

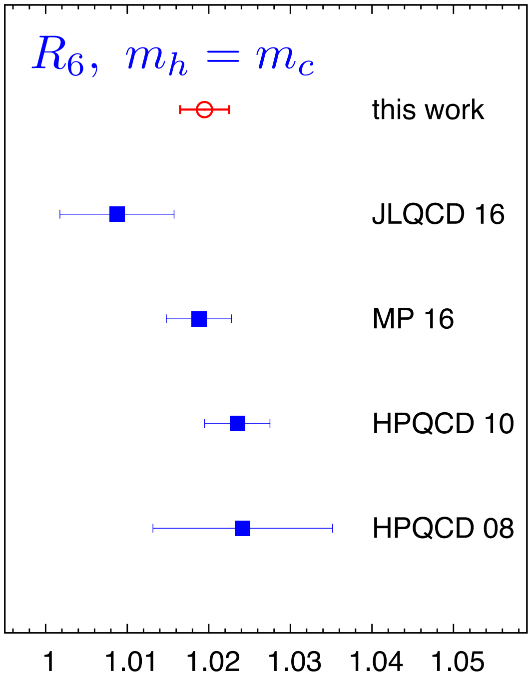


Our result on α_s from the moments agrees with the α_s from the static potential
Bazavov et al, PRD 90 (2014) 074038

but is lower than HPQCD result from the moments
Allison et al, PRD 78 (2008) 054513; McNeile et al, PRD 82 (2010) 034512;
Chakraborty et al, PRD 91 (2015) 054508

Determination of the quark masses

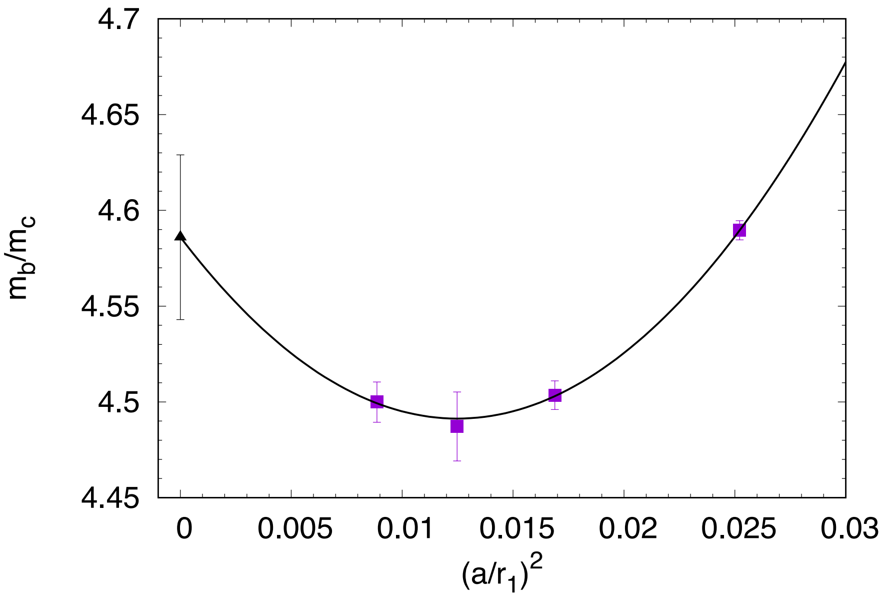




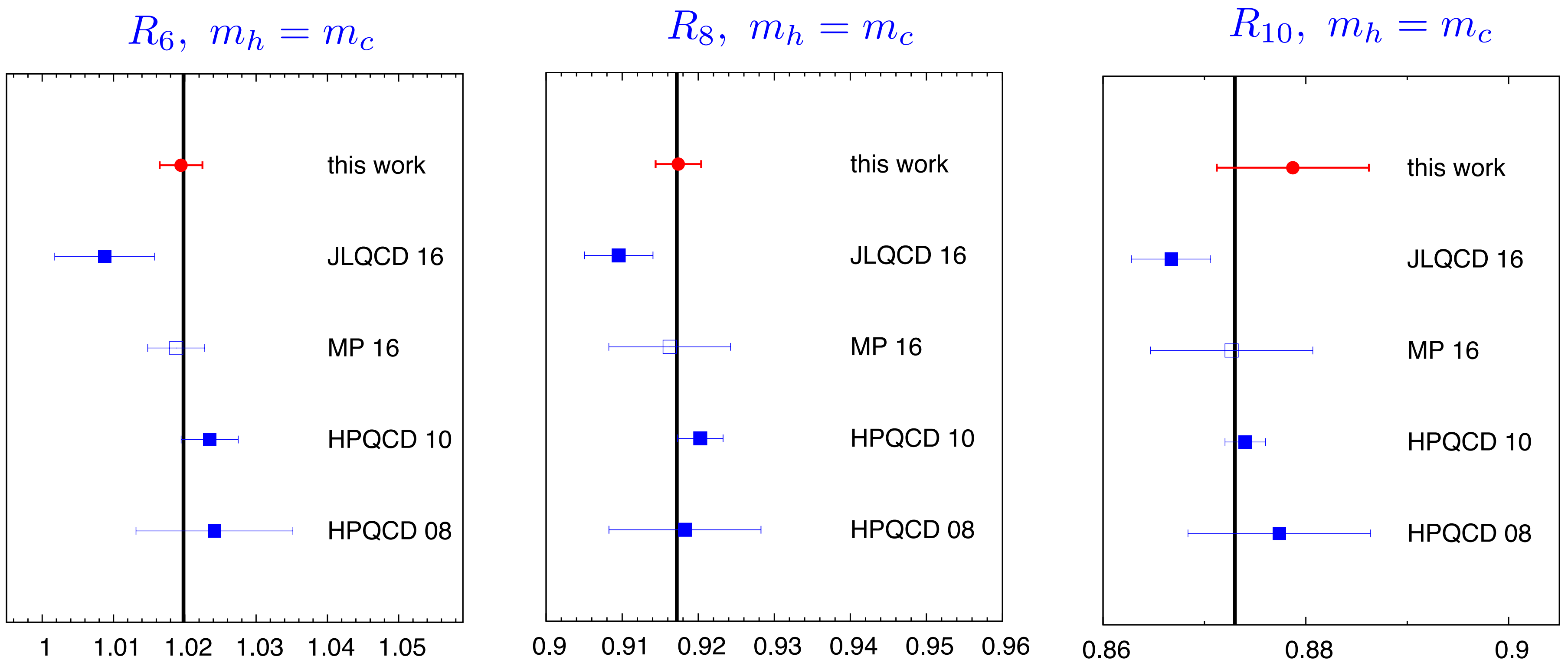
Determine am_b^0 by fixing the $\eta_b(1S)$ mass to its physical value from PDG

$$m_b/m_c = 4.586(43)$$

agrees with other lattice determinations



Continuum results on the moments (cont'd)



Weighted average over independent lattice determinations:

$$R_6 = 1.0198 \pm 0.0024,\ \chi^2/df = 2.60786$$

$$R_8 = 0.9172 \pm 0.0022,\ \chi^2/df = 2.93482$$

$$R_{10} = 0.8730 \pm 0.0019, \chi^2/df = 2.72583$$

There are some tension between different lattice determination (underestimated error, continuum extrapolation ?) But no major discrepancies

Determination of the quark masses

m_h	R_6	R_8	R_{10}	av.
$1.0m_c$	1.2740(25)(17)(11)(61)	1.2783(28)(23)(00)(43)	1.2700(72)(46)(13)(33)	1.2754(39)
$1.5m_c$	1.7147(83)(11)(03)(60)	1.7204(42)(14)(00)(40)	1.7192(35)(29)(04)(30)	1.7191(38)
$2.0m_c$	2.1412(134)(07)(01)(44)	2.1512(71)(10)(00)(29)	2.1531(74)(19)(02)(21)	2.1507(52)
$3.0m_c$	2.9788(175)(06)(00)(319)	2.9940(156)(08)(00)(201)	3.0016(170)(16)(00)(143)	2.9949(153)
$4.0m_c$	3.7770(284)(06)(00)(109)	3.7934(159)(08)(00)(68)	3.8025(152)(15)(00)(47)	3.7956(110)
m_b	4.1888(260)(05)(00)(111)	4.2045(280)(07)(00)(69)	4.2023(270)(14)(00)(47)	4.1985(163)

R_6, R_8 and R_{10} give consistent results for $m_h(m_h)$

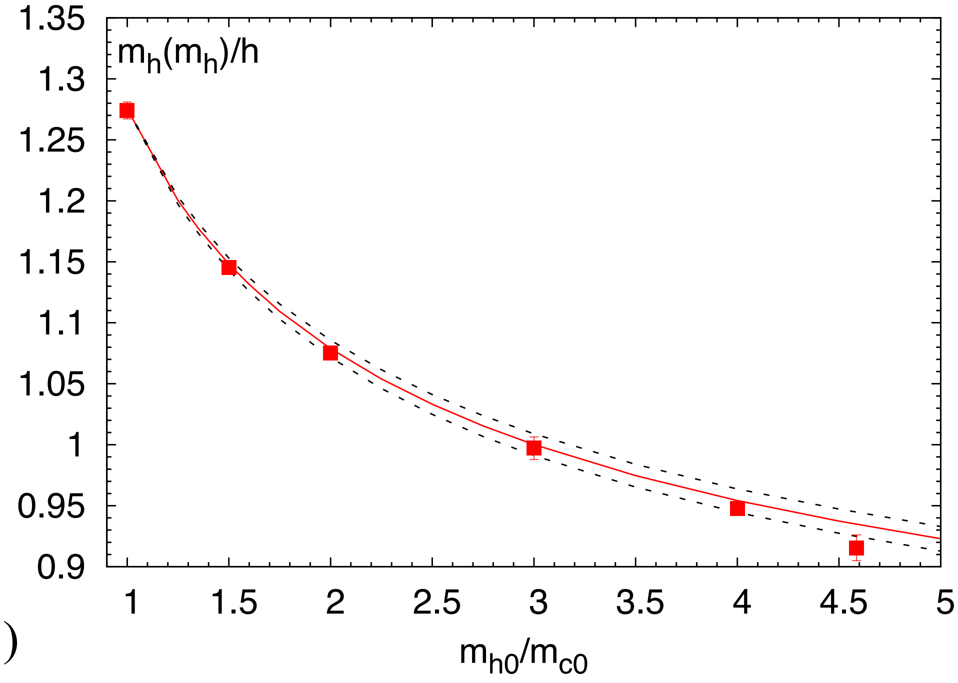
$m_h(m_h)/h$, $h = m_h/m_c$ is consistent with expected $m_c(\mu)$

Final results after running with RUNDeC

$m_c(\mu = m_c, n_f = 4) = 1.265(10)$ GeV

$m_b(\mu = m_b, n_f = 5) = 4.188(37)$ GeV

The error is dominated by the error of
The lattice spacing determination (~0.6%)



The strong coupling constant in 5f QCD from moments

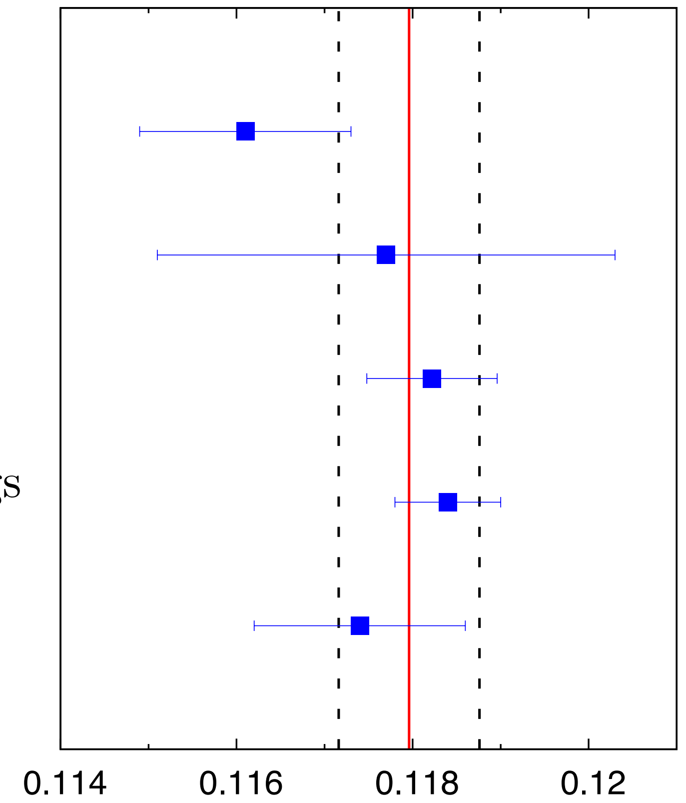
Matching to 5 flavor theory using RunDeC
at $m_{charm} = 1.5$ GeV and $m_{bot} = 4.7$ GeV

$$\alpha_s(M_Z, n_f = 5) = 0.1159(12)$$

Is compatible with the previous 3 flavor HISQ
determination $\alpha_s(M_Z, n_f = 5) = 0.11622(84)$

MP '16: Maezawa, PP, PRD 94 (2016) 034507

but has larger errors despite smaller lattice spacings
and drastically reduced statistical errors



Weighted average of all the lattice α_s
determinations from the moments, $\chi^2/df = 0.8$

$$\alpha_s(M_z, n_f = 5) = 0.11796(40)$$

Increase the error by two to account the spread
and correlated systematics

$$\rightarrow \alpha_s(M_z, n_f = 5) = 0.11796(80)$$

Summary

- Precise determination of the strong coupling constant from the moments of quarkonium correlators is challenging because:

1) Large lattice cutoff dependence of the lowest moment and the ratios of the moments
2) Uncertainties in the weak coupling expansion

- The most recent determination gives

$$\alpha_s(M_Z, n_f = 5) = 0.1159(12)$$

which is smaller than earlier HPQCD results

- The heavy quark masses can be well determined from the moments of quarkonium correlators

$$m_c(\mu = m_c, n_f = 4) = 1.265(10)\ \text{GeV} \qquad m_b(\mu = m_b, n_f = 5) = 4.188(37)\ \text{GeV}$$

- Comparison of different lattice data from the moments suggests a more conservative result

$$\alpha_s(M_z, n_f = 5) = 0.11796(80)$$

- There are some tension in the continuum extrapolated values of the moments from different groups, but the main cause of discrepancy could be due to the perturbative error

➡ 5-loop calculations will certainly help.

Back-up:

n	r_{n1}	r_{n2}	r_{n3}
4	2.3333	-0.5690	1.8325
6	1.9352	4.7048	-1.6350
8	0.9940	3.4012	1.9655
10	0.5847	2.6607	3.8387