Describing Warm Dark Matter with the Reduced Relativistic Gas

Based on
arxiv:1904.09904
G. Pordeus-da-Silva, R. C. Batista, L. G. Medeiros

Ronaldo Carlotto Batista

Escola de Ciências e Tecnologia
Universidade Federal do Rio Grande do Norte

Dark Universe Workshop

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Outline

- **History and basics of RRG**
- **RRG as perfect fluid**
- **Einstein-Botzmann eqs. for RRG**
- **Effects of Warm Dark Matter**
- **Conclusions**

History of RRG

- First proposed in 1966 by Sakharov to interpolate between radiation a dust eras.

- Revived in 2005:
  - In Cosmology: Berredo-Peixoto, I. Shapiro & F. Sobreira
  - Relativistic Fluid Dynamics: Mignone, Plewa & Bode
  - Similar ideas implemented by Macorra, 2010, “Bound Dark Matter”

- Recently applied for studies of:
  - Warm Dark Matter: 1706.08595
  - Photon-Barion Fluid: 1312.1937
  - Neutrinos: 1710.01785
  - Relativistic Fluid Dynamics

What is RRG?

RRG is a classic ideal gas composed by particles that all have the same momentum magnitude.

Relativistic Energy

$$E = \sqrt{m^2c^4 + p^2c^2}$$

Equation of State

$$P = \frac{\rho}{3}\left[1 - \left(\frac{mc^2}{E}\right)^2\right] = \frac{\rho}{3}\left[1 - \left(\frac{\rho_d}{\rho}\right)^2\right]$$

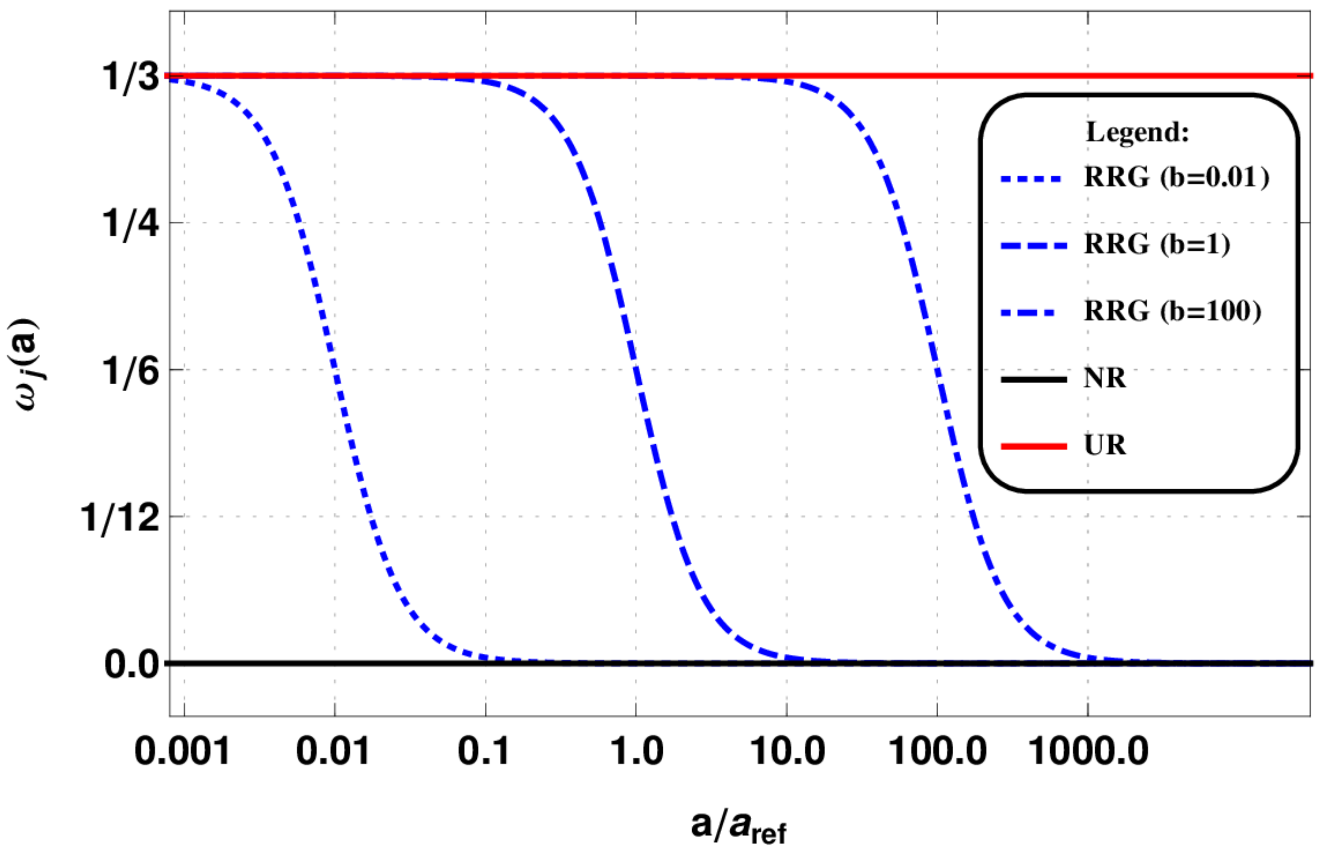
Cosmological evolution

$$\rho(a) = \rho_d(a_{\rm ref})\sqrt{\left(\frac{a}{a_{\rm ref}}\right)^{-6} + b^2\left(\frac{a}{a_{\rm ref}}\right)^{-8}}$$

What is RRG?

Evolution of EoS

$$w = \frac{1}{3}\left[\frac{b^2}{(a/a_{\rm ref})^2 + b^2}\right] = \frac{1}{3}v_{\rm th}^2$$



Transition from UR to NR occurs when

$$b \sim \frac{a}{a_{\rm ref}}$$

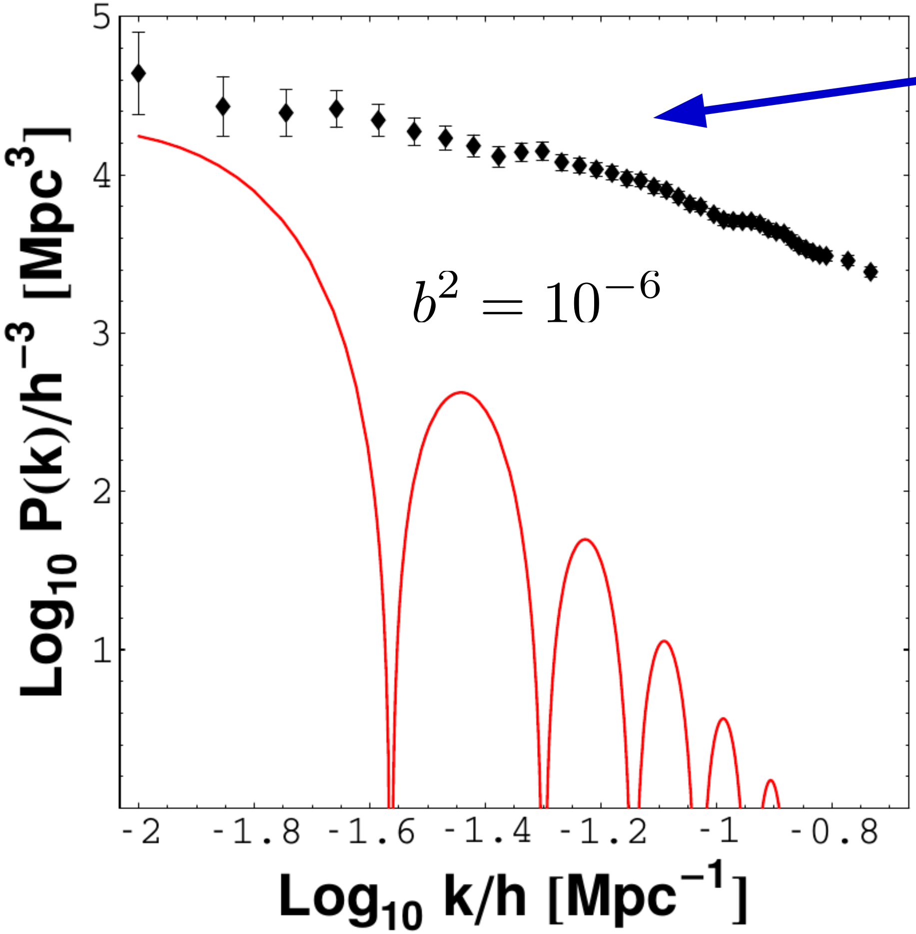
RRG Linear Pertubations

Fabris, Shapiro, Sobreira 2009

$$\delta P = \frac{1}{3}\delta\rho(1-r)$$

pressure perturbation

$a_{\rm ref} = 1$ assumed



2dF Matter Power Spectrum data

Upper bound

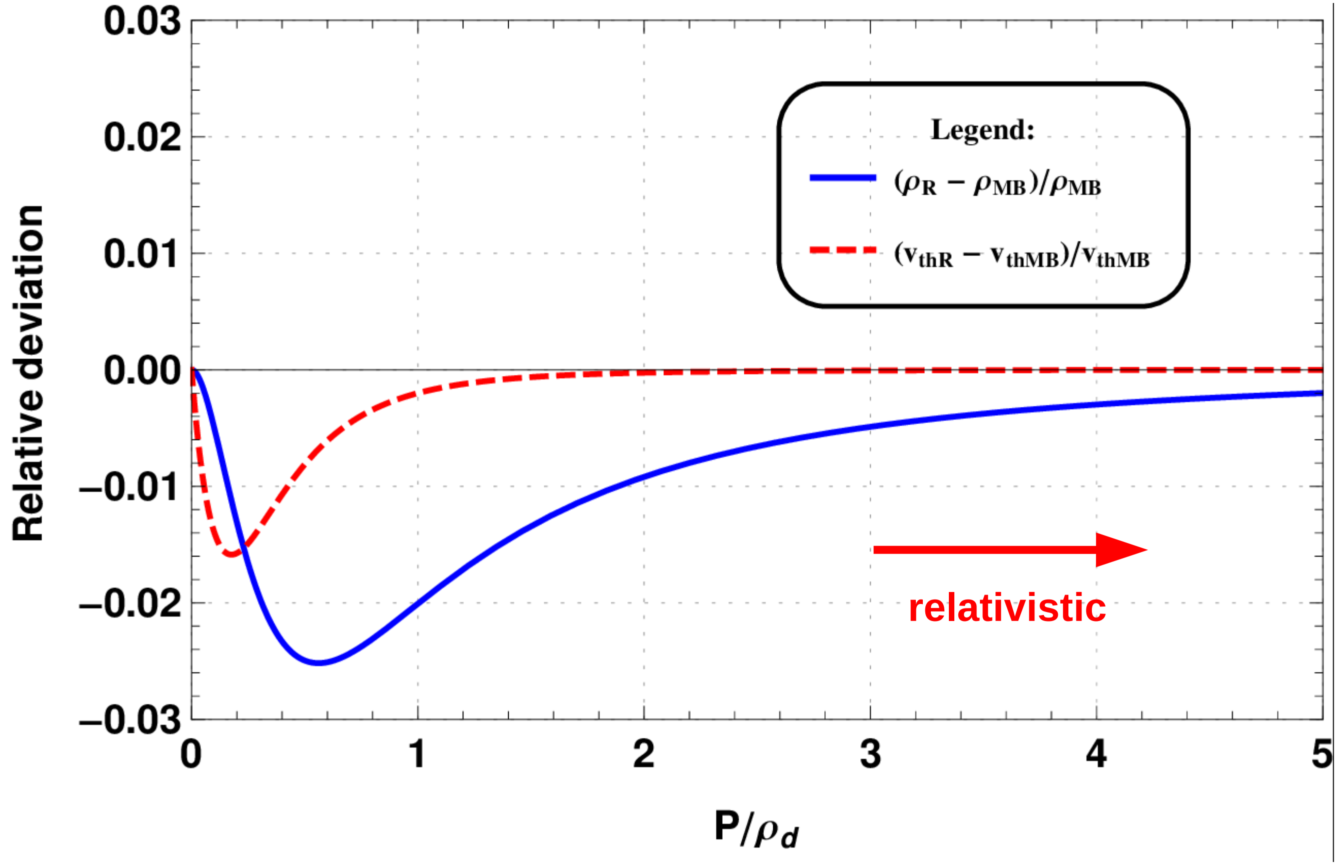
$b^2 < 10^{-9}$

RRG x Maxwell-Boltzman

Relativistic MB
Jüttner 1911

$$P = nT \text{ and } \rho = 3nT + nm\frac{K_1(m/T)}{K_2(m/T)}$$

RRG Error



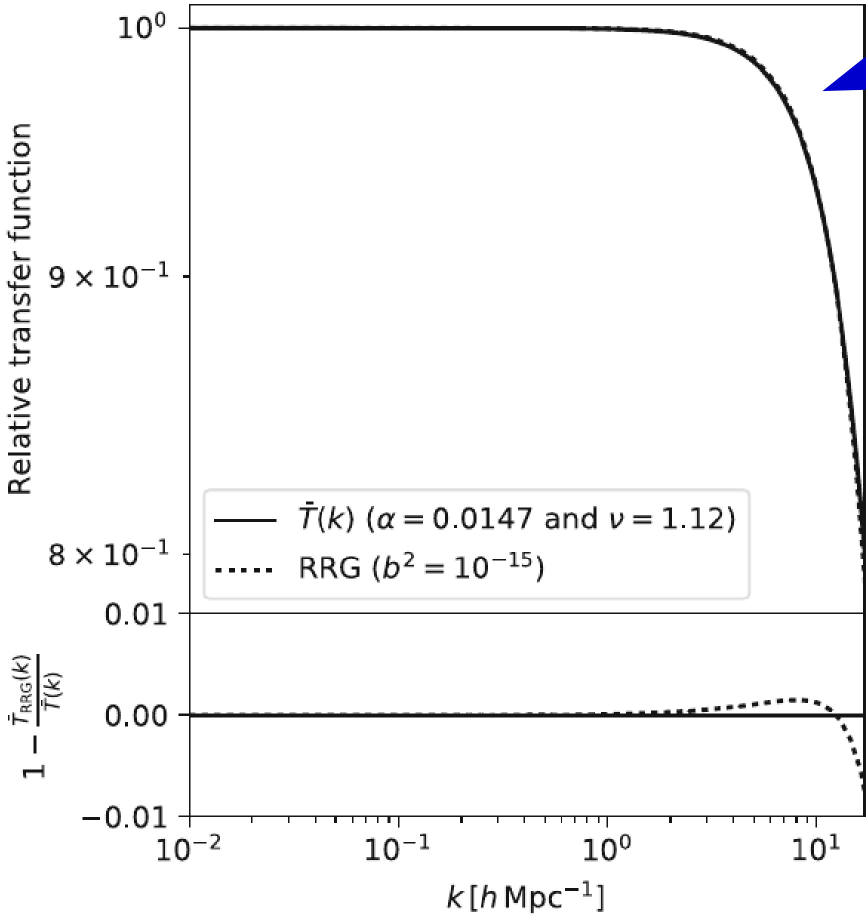
$$\rho_d = nm$$

RRG Linear Pertubations

Hipólito-Ricaldi et al.
1706.08595

$$\delta P = c_{\rm a}^2 \delta\rho$$

Numerical transfer function from CLASS



Bode, Ostriker,Turok
Transfer function for
Thermal Relics
Full Einstein-Boltzmann Eqs.

$$T(k) = [1 + (\alpha k)^{2\nu}]^{-5/\nu}$$

$$\alpha = 0.049 \left(\frac{m}{\rm keV}\right)^{-1.11} \left(\frac{\Omega_{\rm DM}}{0.25}\right)^{0.11} \left(\frac{h}{0.7}\right)^{1.22}$$

Upper bound including CMB

$$b^2 < 10^{-10}$$

RRG b-mass relation

$$m = \frac{4.65}{10^6 b^{4/5}} \rm keV$$

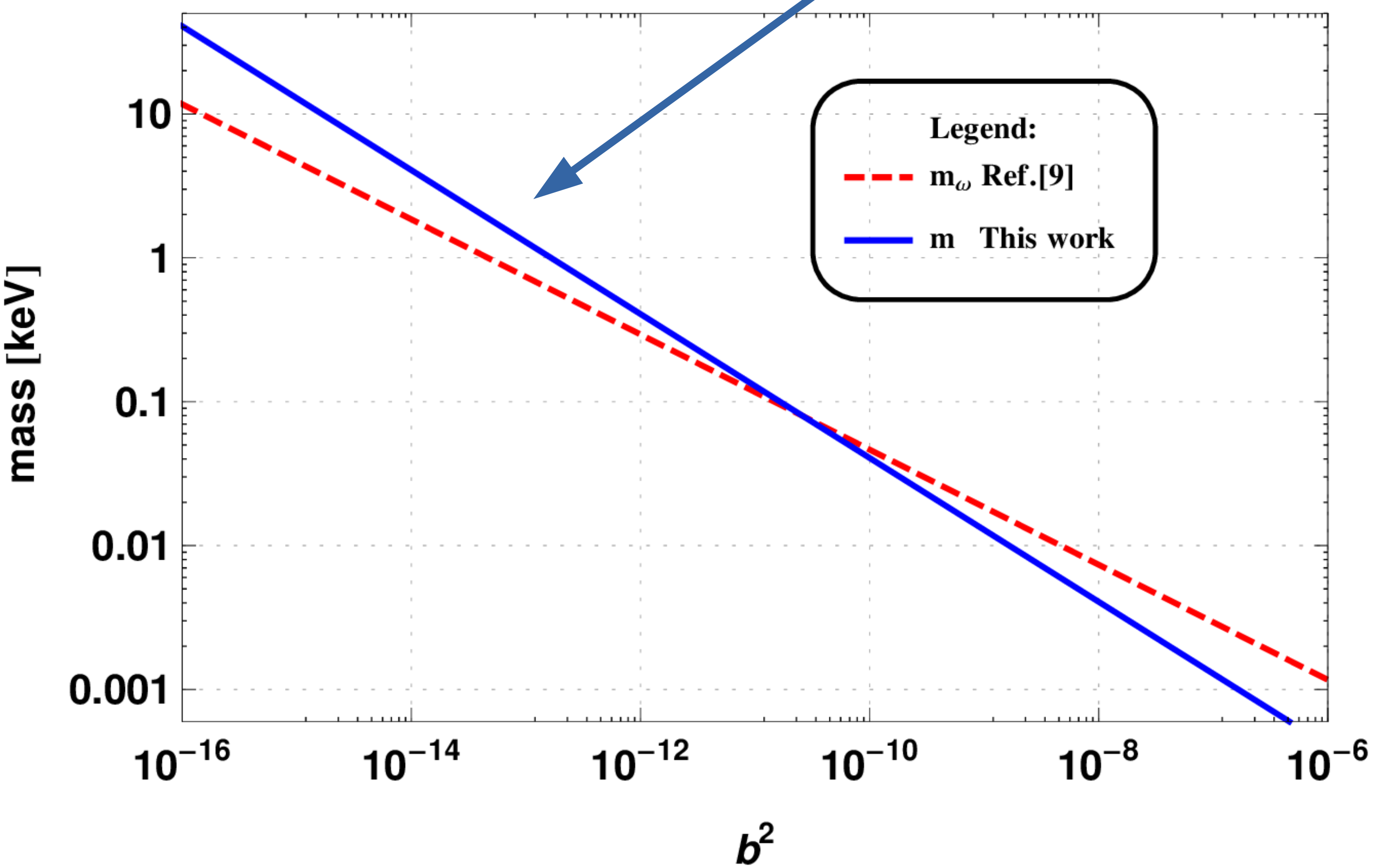
Another b-mass relation

P.B.G 2019

Assumptions:

- **RRG and MB with same energy now.**
- **MB with sharp transition from UR to NR at T=m**
- **MB in equilibrium with photons at a_nuc**
- **RRG with** $m \gg T_\gamma$

$$m = \frac{4.07}{10^7 b}\text{keV}$$



Distribution Function for RRG

Background DF

$$\bar{f}(p,t) = Cp\delta_{\rm D}(p-\bar{p})$$

RRG: all particle with same momemtum magnitude

First order Momentum perturbation

$$p(\vec{x},\hat{p},t) = \bar{p}(t) + \delta p(\vec{x},\hat{p},t)$$

First order DF

$$f(\vec{x},p,\hat{p},t) \simeq \bar{f} + \frac{\partial \bar{f}}{\partial p}\delta p = \bar{f} - p\frac{\partial \bar{f}}{\partial p}M(\vec{x},\hat{p},t)$$

Momemtum Constrast

EMT

$$T^{\mu}_{\nu} = \int \frac{d\vec{P}^3}{(2\pi)^3\sqrt{-g}}\frac{P^{\mu}P^{\nu}}{P^0}f$$

Reproduces bck. EoS, gives the sound speed and anisotropic stress

$$\delta P = \frac{w}{3}\frac{(5-3w)}{1+w}\delta\rho = c_a^2\delta\rho \qquad \sigma = -6c_a^2 M_2$$

Distribution Function for RRG

Newtonian gauge
(Ma&Bertschinger notation)

$$ds^2 = a^2 \left[-(1+2\psi) d\eta^2 + (1+2\phi) d\vec{x}^2 \right]$$

Multipolar expansion for M

$$M_l(k,t) = \frac{1}{2(-1)^l} \int_{-1}^{1} d\mu P_l(\mu) M(k,\mu,t)$$

Boltzmann equations:

$$M_0' + \frac{k}{4} (5-3w) \sqrt{3w} M_1 + \phi' = 0$$

$$M_1' + \frac{1}{3} \frac{k}{4} \sqrt{3w} (5-3w) (2M_2 - M_0) - \frac{1}{4} \frac{(1+w)}{\sqrt{3w}} k\psi = 0$$

$$M_l' + \frac{k}{4(2l+1)} \sqrt{3w} (5-3w) \left[(l+1) M_{l+1} - l M_{l-1} \right] = 0$$

Warm Approximation

$$\frac{\rho}{\rho_d} \simeq 1 + x$$

$$\delta\rho/\bar{\rho} = (3 + 2x + \mathcal{O}(x^2))M_0$$

COLD

$$\delta p/\bar{\rho} = \left(\frac{10}{3}x - \frac{19}{3}x^2 + \mathcal{O}(x^3)\right) M_0$$

$$v = -i\left(4\sqrt{2x} - \frac{17}{3}\sqrt{2}x^{3/2} + \mathcal{O}(x^{5/2})\right) M_1$$

$$\sigma = \left(-\frac{20}{3}x + \frac{154}{9}x^2 + \mathcal{O}(x^3)\right) M_2$$

WARM

Warm Sound Speed $c_s = \frac{\sqrt{5}}{3}\frac{b}{a}$

Warm Approximation

Boltzmann equations in warm approximation

$$M_0' + \frac{3\sqrt{5}}{4} k c_s M_1 + \phi' = 0$$

$$M_1' + \frac{\sqrt{5}}{4} c_s k \left(2M_2 - M_0\right) - \frac{\sqrt{5}}{12} \frac{k}{c_s} \psi = 0$$

$$M_l' + \frac{\sqrt{5}}{12\left(2l+1\right)} c_s k \left[\left(l+1\right) M_{l+1} - l M_{l-1}\right] = 0$$

Higher multipoles become important on scales $c_s \eta k > 1$

Perfect Fluid Scale

Higher multipoles become important inside free-stream scale

$$k_{\rm FS} = \frac{3a}{\sqrt{5}b}\frac{1}{\eta}$$

For keV mass scale

$$b^2 \sim 10^{-14}$$

$$k_{\rm FS} \sim \frac{10^7}{3000{\rm Mpc/h}} \simeq 10^3 \frac{h}{\rm Mpc}$$

More conservative estimate: sound horizon scale
(ΛCDM backgound)

$$k_s = 2\pi H_0 c^{-1} / \int_{a_i}^{1} \frac{c_s da}{a^2 E(a)}$$

$$k_s(b^2 = 10^{-14}) = 12.3h/{\rm Mpc}$$

Is it safe to use perfect fluid RRG?

1306.2314
WDM mass constraint:

$$m > 3\text{keV}$$

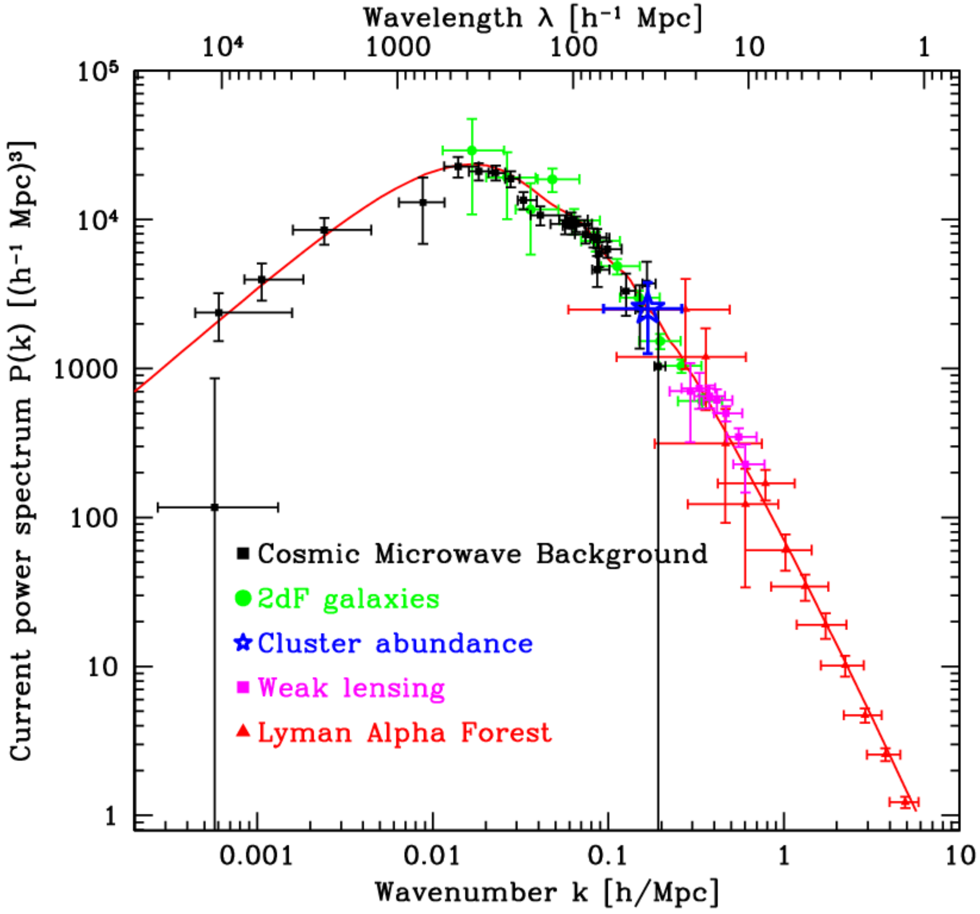


Fig. by M. Tegmark

$$k_s = \left(\frac{10^{-14}}{b^2}\right) 12.3h/\text{Mpc} = \left(\frac{m}{\text{keV}}\right) 3.6h/\text{Mpc}$$

Initial Conditions

Warm approximation breaks $a \sim b$

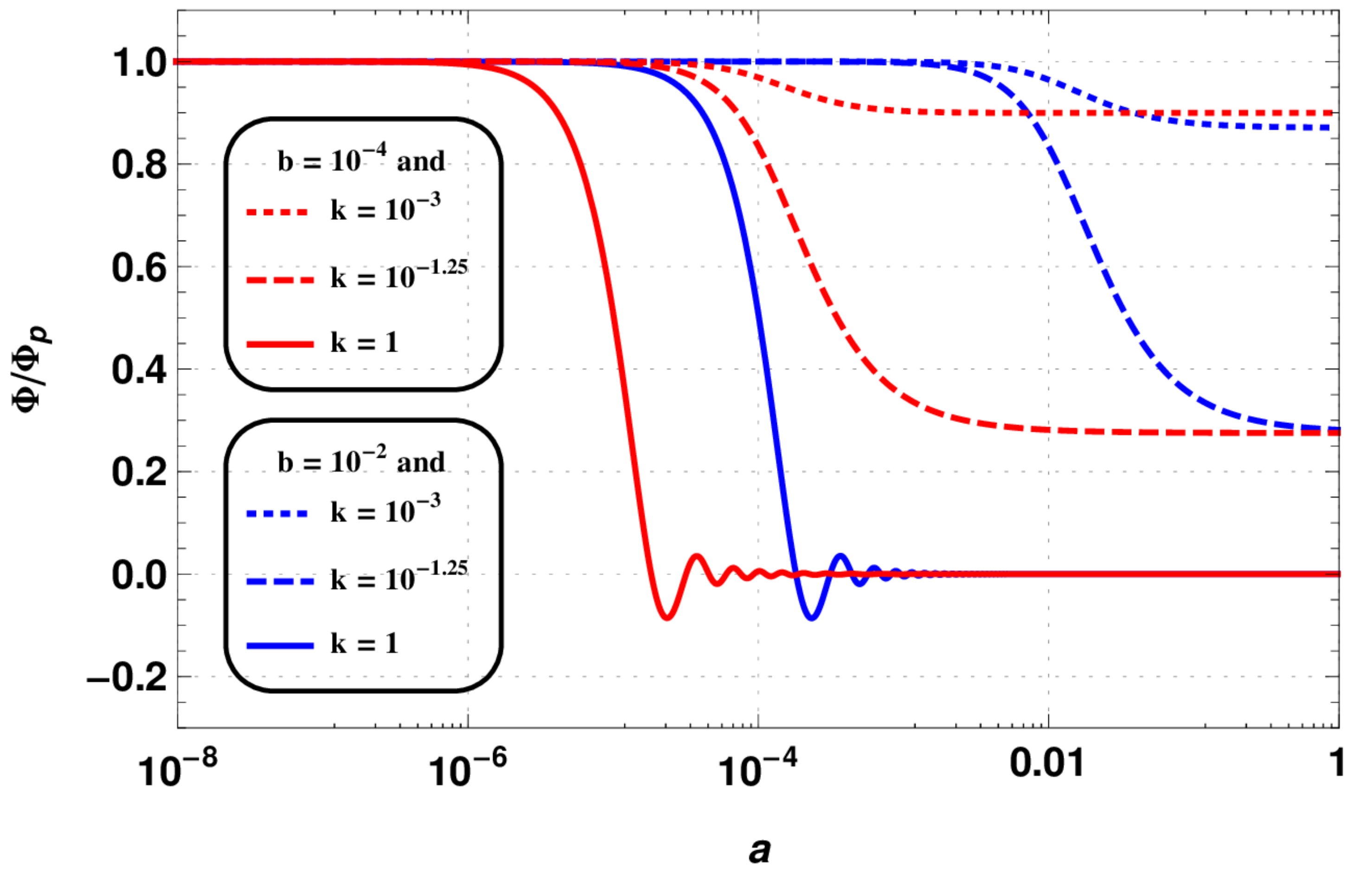
For keV mass scale $b^2 \sim 10^{-14}$ RRG is relativistic at $a < 10^{-7}$

Use relativistic IC (e.g. massless neutrinos)

At 1 MeV scale ($a > 10^{-10}$) RRG is non relativistic for $m > 400\mathrm{keV}$

Pedagogical Model

Universe with RRG only



Conclusions

- **RRG is a simple and precise model to interpolate between UR and NR regimes of a gas.**

- **For WDM application RRG framwork shows dissipative effects are very small, perfect fluid can be used.**

- **Better to use RRG than w=const. to constraint general DM models.**

Thanks!