The Strong Coupling from Low Energy Physics

Tomasz Korzec

BERGISCHE UNIVERSITÄT WUPPERTAL

ALPHA Collaboration

SFB TR55 Hadron Physics from Lattice QCD

- Strong Coupling by the Alpha Collaboration (2017)
- Perturbative vs. Non-perturbative Decoupling
- Strong Coupling from Decoupling

Strong Coupling by the Alpha Collaboration (2017)

- SF-Coupling

  [M.Dalla Brida, P.Fritzsch, T.K., A.Ramos, S.Sint, R.Sommer,Eur.Phys.J. C78 (2018) & PRL 117 (2016)]

- GF-Coupling

  [M.Dalla Brida, P.Fritzsch, T.K., A.Ramos, S.Sint, R.Sommer, PRD 95 (2017)]

- Large Volume (CLS)

  [M.Bruno et al,JHEP 1502 (2015)]

- Scale Setting

  [M.Bruno, T.K., S.Schaefer, PRD 95 (2017)]

- Final Result

  [M.Bruno, M.Dalla Brida, P.Fritzsch, T.K., A.Ramos, S.Schaefer, H.Simma, S.Sint, R.Sommer,PRL 119 (2017)]

Running Couplings

- Renormalized couplings $\alpha(\mu) \equiv \frac{\overline{g}^2(\mu)}{4\pi}$ depend on renormalization scale μ
- Dependence is described by β function
  $\mu \frac{\partial \overline{g}}{\partial \mu} = \beta(\overline{g})$
- In perturbation theory
  $\beta(g) \sim -g^3(b_0 + b_1 g^2 + b_2 g^4 + \dots)$
- Integration of RG equation introduces the dimensionful Λ parameter

$$\Lambda/\mu = \underbrace{(b_0\overline{g}^2)^{-b_1/(2{b_0}^2)}\, e^{-1/(2b_0\overline{g}^2)}\, \exp\left\{-\int_0^{\overline{g}} \left[\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{{b_0}^2 x}\right] dx\right\}}_{\equiv \phi(\overline{g})}$$

- Two values of the coupling, $\overline{g}_1$, $\overline{g}_2$ correspond to scale ratio
  $\frac{\mu_1}{\mu_2} = \frac{\phi(\overline{g}_2)}{\phi(\overline{g}_1)}$
- Special case: scale ratio of $\mu_1/\mu_2 = 2 \Rightarrow$ step-scaling function
  $\sigma(\overline{g}_1^2) = \overline{g}_2^2$
  Contains the same information as $\beta(g)$, but better suited for numerical methods

Strong Coupling from Lattice QCD

Goal: obtain value of $\alpha^{(5)}_{\overline{\rm MS}}(M_Z)$
Different approaches are being used

- Determine some "high energy" quantity Φ both in lattice QCD and in PT
  $\Phi = p_0 + p_1\alpha_{\overline{\rm MS}}(\mu) + p_2\alpha^2_{\overline{\rm MS}}(\mu) + \dots$
  Solve for $\alpha_{\overline{\rm MS}}(\mu)$, use PT to evolve to $\mu = M_Z$
  - Energy scale of Φ: $E \approx \mu$ for PT to work well
  - μ high enough for PT to be reliable
  - Lattice spacing fine enough to resolve Φ: $a \ll 1/E$
  - Volume large enough to be effectively in infinite volume: $Lm_\pi > 4$
- Define a non-perturbative coupling, e.g. $\alpha_{qq}(\mu = 1/r) = \frac{3}{4}r^2F(r)$
  Obtain Λ in this scheme, translate to $\Lambda_{\overline{\rm MS}}$
  - β-function can be computed and compared to PT
  - Needs large volume simulations at $\overline{m} = 0$ (impossible!)
  - Again window problem: $Lm_\pi > 4, r \gg a$

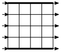
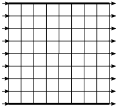
Finite Volume Renormalization Schemes

Solution: define couplings with $\mu \equiv 1/L$
$\Rightarrow L/a \gg 1$ is enough to control systematics

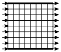
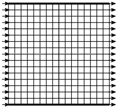
- Simulations are cheap, but new set for each μ necessary
  Large volume still necessary for scale setting
- Boundary conditions are part of the definition of the scheme
- E.g. Schrödinger Functional (periodic in space, Dirichlet in time)
- Simulations at $\overline{m} = 0$ possible (with SF boundaries)
- Many possibilities to define couplings, we use
  - Schrödinger-Functional Coupling $\overline{g}^2_{\mathrm{SF}}$
    [M.Lüscher, R.Sommer, P.Weisz, U.Wolff, Nucl.Phys. B413 (1994)]
  - Gradient-Flow Coupling $\overline{g}^2_{\mathrm{GF}}$
    [P.Fritzsch, A.Ramos, JHEP 1310 (2013)]

Computing Step Scaling Functions

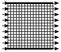
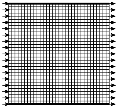
Instead of $\beta(\bar{g})$, compute: $\sigma(u) = \bar{g}^2(\mu/2)|_{u=\bar{g}^2(\mu)}$

$m_0^{(1)}, g_0^{(1)}$:  same $a^{(1)}$ $\leftrightarrow$  $= \Sigma(u, \frac{a^{(1)}}{L})$

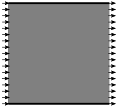
$\updownarrow$ same L, $\bar{g}^2(L^{-1})$

$m_0^{(2)}, g_0^{(2)}$:  same $a^{(2)}$ $\leftrightarrow$  $= \Sigma(u, \frac{a^{(2)}}{L})$

$\updownarrow$ same L, $\bar{g}^2(L^{-1})$

$m_0^{(3)}, g_0^{(3)}$:  same $a^{(3)}$ $\leftrightarrow$  $= \Sigma(u, \frac{a^{(3)}}{L})$

$\downarrow$cont. limit

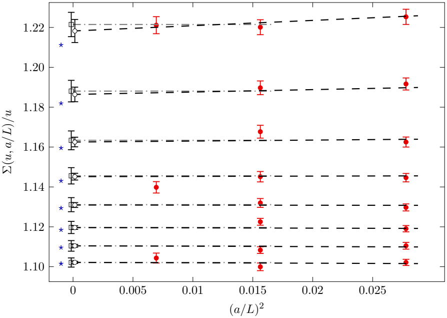
$\bar{g}^2 = u, \ \overline{m} = 0$  $= \sigma(u)$

Step Scaling

SF-coupling

[M. Dalla Brida, P. Fritzsch, T. K., A. Ramos, S. Sint, R. Sommer,

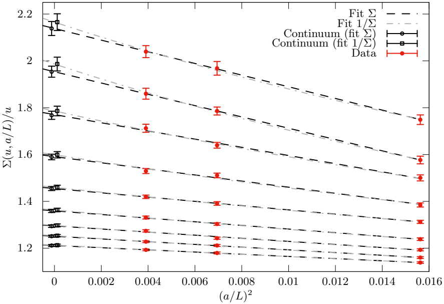
PRL 117 (2016)]



GF-coupling

[M. Dalla Brida, P. Fritzsch, T. K., A. Ramos, S. Sint, R. Sommer,

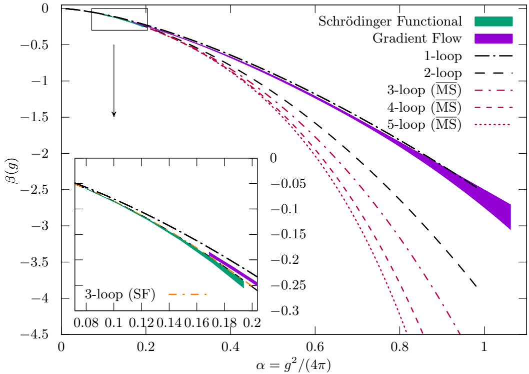
PRD 95 (2017)]



Step Scaling

$\sigma(g)$ and $\beta(g)$ contain the same information

$$\ln(\mu_1/\mu_2) = -\int_{\overline{g}(\mu_1)}^{\overline{g}(\mu_2)} \frac{dg}{\beta(g)}$$



Final Result

- From non-perturbative β-function: $L^{\rm had}\Lambda^{(3)}_{\overline{\rm MS}} = 1.729(57)$
  in units of $L^{\rm had}$, defined by $\bar{g}^2_{\rm GF}(1/L^{\rm had}) = 11.31$
- Missing piece: scale setting, i.e. $L^{\rm had}$ in fm
  Needs large volume Simulations at (close to) physical masses
  Result: $L^{\rm had}\left[\frac{2}{3}f_K + \frac{1}{3}f_\pi\right]^{\rm phys} = 0.750(13) \Rightarrow L^{\rm had} = 1.002(16)$ fm

[M.Bruno, T.K., S.Schaefer, PRD 95 (2017)]

[M.Bruno, M.Dalla Brida, P.Fritzsch, T.K., A.Ramos, S.Schaefer, H.Simma, S.Sint, R.Sommer,PRL 119 (2017)]

Result 2017

$$\Lambda^{(3)}_{\overline{\rm MS}} = 341(12)\ \text{MeV} \qquad (3.5\%\ \text{error})$$

$$\Lambda^{(5)}_{\overline{\rm MS}} = 215(10)(03)\ \text{MeV} \qquad (4.7\%\ \text{error}), \qquad \text{pert. decoupling}$$

$$\alpha_{\overline{\rm MS}}(M_Z) = 0.1185(8)(3)$$

$$0.1174(16) \qquad \text{PDG non-lattice}$$

Perturbative vs Non-Perturbative Decoupling

- Power Corrections

  [F.Knechtli , T.K., B.Leder, G.Moir, Phys.Lett. B774 (2017)]

- (Non-) Perturbative Decoupling

  [A.Athenodorou, J.Finkenrath, F.Knechtli, T.K., B.Leder, M.Krstić Marinković, R.Sommer, Nucl.Phys. B943 (2019)]

Decoupling

Effective theory

Situation: QCD with N_f flavors out of which N_ℓ are light. Heavy quarks have RGI mass M.

- Fundamental theory: $\mathcal{L}_{\mathrm{QCD}_{N_\mathrm{f}}}$
  Parameters: coupling and masses
- Effective low energy theory: $\mathcal{L}_\mathrm{dec} = \mathcal{L}_{\mathrm{QCD}_{N_\ell}} + \frac{1}{M^2}\sum_i \omega_i \Phi_i + \ldots$
  Parameters: coupling, light masses, ω_i

[T.Appelquist, J.Carazzone, PRD11 (1975)], [S.Weinberg, Phys.Lett b91 (1980)]

Tuning of parameters: adjust parameters of $\mathcal{L}_\mathrm{dec}$ such that a set of low-energy quantities $\mathcal{S}$ of mass-dimension 1 are identical in the two theories. Then

$$\mathcal{S}^\mathrm{f} = \mathcal{S}^\ell \qquad + \qquad \underbrace{O(M^{-x})}_{x=2 \text{ for L.O. eff. theory}}$$

for all $\mathcal{S}$.

Model

Simplified setup

- Fundamental theory: QCD with two heavy quarks and no light quarks $N_{\mathrm{f}} = 2$, $N_{\ell} = 0$.
- Leading order effective theory: Yang-Mills theory

Advantages:

- No light pions $\Rightarrow$ no finite volume effects $\propto e^{-m_\pi L}$.
  $\Rightarrow$ comparatively small volumes are sufficient $L \approx 1.5$ fm
- Allows in turn to reach very fine lattice spacings
  $\Rightarrow$ small cutoff effects $\propto (aM)^2$
- Simulations in effective theory: almost for free (pure gauge)
- Two charm quarks $\Rightarrow$ $O(M^{-2})$ effects better visible

Disadvantage:

- Not the real world

Power corrections

$$\mathcal{S}^{\mathrm{f}} = \mathcal{S}^{\ell} \qquad + \qquad O(M^{-2})$$

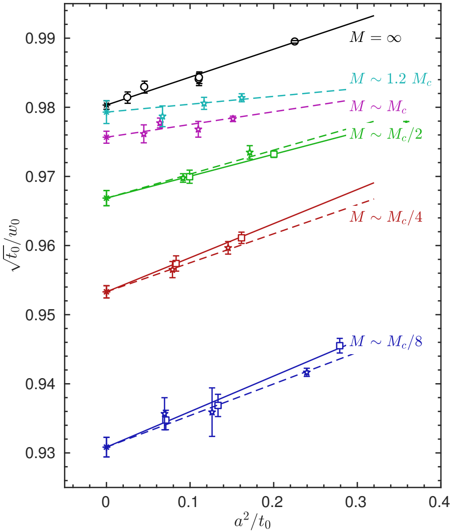
But in our model: $\mathcal{S}^{\ell} = \text{number} \times \Lambda_{\ell}$
$\Rightarrow$ ratios of scales give access to power corrections without matching

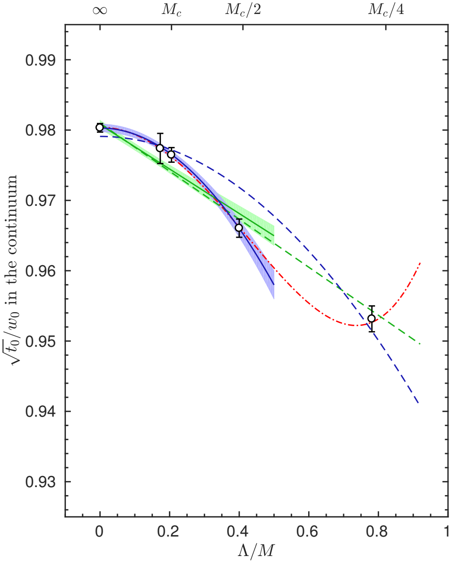
$$\frac{\mathcal{S}_1^{\mathrm{f}}}{\mathcal{S}_2^{\mathrm{f}}} - \underbrace{\frac{\mathcal{S}_1^{\ell}}{\mathcal{S}_2^{\ell}}}_{\text{indep. of } \Lambda_{\ell}} = O(M^{-2})$$

In pure gauge theory: Not many possibilities to choose $\mathcal{S}$ from. E.g. $1/\sqrt{t_0}$, $1/\sqrt{t_c}$, $1/w_0$, $1/r_0$, static force, glue-ball masses

[M.Lüscher, JHEP 1008 (2010)], [S.Borsanyi et al. JHEP 09 (2012)], [R.Sommer, Nucl.Phys. B411 (1994)]

Power corrections





This and other ratios $\Rightarrow$ power corrections $\approx$ 0.2% with one charm quark

[F.Knechtli , T.K., B.Leder, G.Moir, Phys.Lett. B774 (2017)]

Perturbative decoupling

$$\overline{g}_\ell^2(\mu/\Lambda_\ell) = \overline{g}_\mathrm{f}^2(\mu/\Lambda_\mathrm{f}) + c_1(\mu/\overline{m}(\mu))\, \overline{g}_\mathrm{f}^4(\mu/\Lambda_\mathrm{f}) + \ldots$$

- $\overline{m}(\mu)$ renormalized heavy quark mass $\Leftrightarrow M$
- Convenient choice of scheme and scale:
  $\overline{\mathrm{MS}}$-scheme with $\mu = m_*$ such that $\overline{m}(m_*) = m_*$
  - $c_1 = 0$
  - $log(\mu/\overline{m})$ vanish
    $\Rightarrow c_2, \ldots, c_4$ are pure numbers
  - $c_2, \ldots, c_3$ known for arbitrary $N_\mathrm{f} - N_\ell$
    c_4 known for $N_\mathrm{f} - N_\ell = 1$

    [K.Chetyrkin, J.H.Kühn, C.Sturm, Nucl.Phys.B744 (2006)]

    [B.A.Kniehl, A.V.Kotikov, A.I.Onishchenko, O.L.Veretin, PRL 97 (2006)]

    [A.G.Grozin, M.Hoeschele, J.Hoff, M.Steinhauser, JHEP 1109 (2011)]

Equivalently: relation between Λ parameters

$$P_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f}) \equiv \Lambda_\ell/\Lambda_\mathrm{f} = \frac{\phi^{(N_\ell)}(\overline{g}_\ell)}{\phi^{(N_\mathrm{f})}(\overline{g}_\mathrm{f})}$$

But $\mu = \overline{m}_c$ is not a high scale. Does perturbation theory work well?

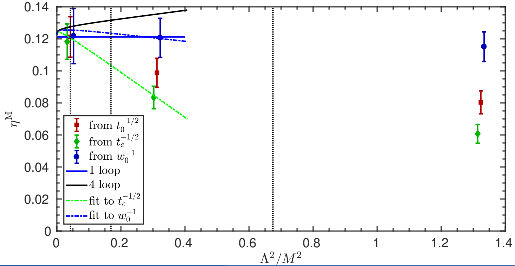
Mass scaling function

$\eta^{\mathrm{M}}(M) \equiv \frac{M}{P}\frac{\partial P}{\partial M} = \eta_0 + \eta_1 \bar{g}^2 + \ldots$ is accessible to NP study because

$$\frac{\mathcal{S}_\mathrm{f}(M)}{\mathcal{S}_\mathrm{f}(0)} = \underbrace{\frac{\mathcal{S}_\ell/\Lambda_\ell}{\mathcal{S}_\mathrm{f}(0)/\Lambda_\mathrm{f}}}_{\equiv Q^{\mathcal{S}}_{\ell,\mathrm{f}},\ \text{indep. of } M} \times \underbrace{P_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f})}_{\text{indep. of } \mathcal{S}} + O(M^{-2})$$

$$\Rightarrow \frac{M}{\mathcal{S}_\mathrm{f}}\frac{\partial \mathcal{S}_\mathrm{f}}{\partial M} = \eta^{\mathrm{M}} + O(M^{-2})$$

For any low energy scale $\mathcal{S}$!



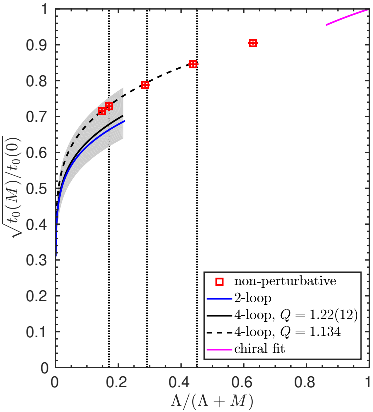
$\Rightarrow \Lambda_3/\Lambda_4$ from PT accurate to 1.5%

[A.Athenodoru et al. Nucl.Phys. B943 (2019)]

Factorization formula

$$\frac{\mathcal{S}_\mathrm{f}(M)}{\mathcal{S}_\mathrm{f}(0)} = Q^{\mathcal{S}}_{\ell,\mathrm{f}} \times P_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f}) \qquad + \qquad O(M^{-2})$$

- Apply to $\mathcal{S} = t_0^{-1/2}$
- P: perturbatively
- $Q = \frac{[\sqrt{t_0(0)}\Lambda]_{N_\mathrm{f}=2}}{[\sqrt{t_0}\Lambda]_{N_\mathrm{f}=0}}$
- Error dominated by Λ_2
- Determine Λ from mass dependence of $\mathcal{S}$?



[A.Athenodoru et al. Nucl.Phys. B943 (2019)]

Strong Coupling from Decoupling

Mattia Dalla Brida, Roman Höllwieser, Francesco Knechtli, Tomasz Korzec, Alberto Ramos, Stefan Sint, Rainer Sommer

PRELIMINARY

Strategy

$$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = \frac{\Lambda^{(3)}_{\overline{\mathrm{MS}}}}{\Lambda^{(0)}_{\overline{\mathrm{MS}}}} \quad \times \quad \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\mathcal{S}} \quad \times \quad \mathcal{S}$$

- Use $N_\mathrm{f} = 0$ QCD as effective theory for $N_\mathrm{f} = 3$ QCD with 3 degenerate quarks of mass M.
  $M \in \{1.6, 3.2, 4.7, 6.3\}$ GeV
- $\frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\Lambda^{(3)}_{\overline{\mathrm{MS}}}} \equiv P_{0,3}(M/\Lambda^{(3)}_{\overline{\mathrm{MS}}})$
  computed perturbatively
- $\mathcal{S}$ low energy scale, for which decoupling applies
  $\mathcal{S}^{(3)} = \mathcal{S}^{(0)} + O((\Lambda/M)^2, (\mathcal{S}/M)^2)$
- $\frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\mathcal{S}}$ from non-perturbative β-function of the $N_\mathrm{f} = 0$ theory

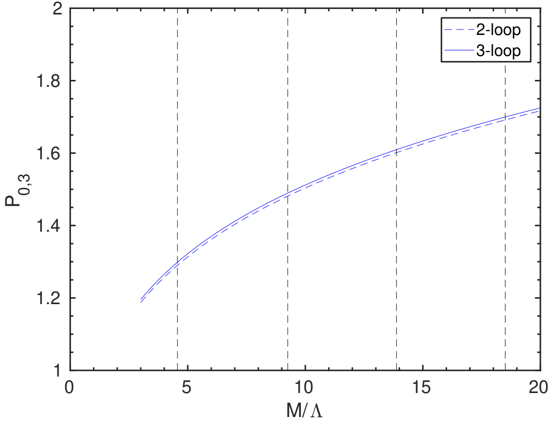
Strategy

$$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = \underbrace{\frac{\Lambda^{(3)}_{\overline{\mathrm{MS}}}}{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}}_{P^{-1}_{0,3}(M/\Lambda^{(3)}_{\overline{\mathrm{MS}}})} \times \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{(0)}} \times \mathcal{S}^{(3)} + O((\Lambda/M)^2, (\mathcal{S}/M)^2)$$

- Use $N_\mathrm{f} = 0$ QCD as effective theory for $N_\mathrm{f} = 3$ QCD with 3 degenerate quarks of mass M.
  $M \in \{1.6, 3.2, 4.7, 6.3\}$ GeV
- $\frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\Lambda^{(3)}_{\overline{\mathrm{MS}}}} \equiv P_{0,3}(M/\Lambda^{(3)}_{\overline{\mathrm{MS}}})$
  computed perturbatively
- $\mathcal{S}$ low energy scale, for which decoupling applies
  $\mathcal{S}^{(3)} = \mathcal{S}^{(0)} + O((\Lambda/M)^2, (\mathcal{S}/M)^2)$
- $\frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\mathcal{S}}$ from non-perturbative β-function of the $N_\mathrm{f} = 0$ theory

P

For the case of simultaneous decoupling of three quarks $P_{0,3}$ is known to 3-loops



Little caveat: it depends on $M/\Lambda^{(3)}_{\overline{\mathrm{MS}}}$, but $\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ only known at the end

Low Energy Quantity

We need

- $M \gg \mathcal{S}$ and $M \gg \Lambda$, for decoupling to apply
- $aM \ll 1$, for reliable continuum limits
- Nothing speaks against a finite volume quantity
  $\Rightarrow$ small a can be reached at moderate L/a

$\mathcal{S}$

- We define a gradient-flow coupling in a Schrödinger Functional
  $T = 2L$, mass M
- $\bar{g}^{(3)}_{\mathrm{GF}}(\mu^{(3)}, M) = \bar{g}^{(0)}_{\mathrm{GF}}(\mu^{(0)})$
  $\Rightarrow \mu^{(0)} = \mu^{(3)} + O(M^{-2})$
- So: we choose $\mathcal{S} \equiv \mu$ at some relatively low energy

Massive Coupling

To determine $\overline{g}^{(3)}_{\mathrm{GF}}(\mu^{(3)}, M)$

- From previous work we know $\overline{g}^2_{\mathrm{GF}}(\mu)\Big|_{T=L,M=0} = 3.95$
  $\Leftrightarrow \mu = 789(15)$ MeV $\equiv \mathcal{S}$

  [M.Dalla Brida, P.Fritzsch, T.K., A.Ramos, S.Sint, R.Sommer, PRD 95 (2017)]

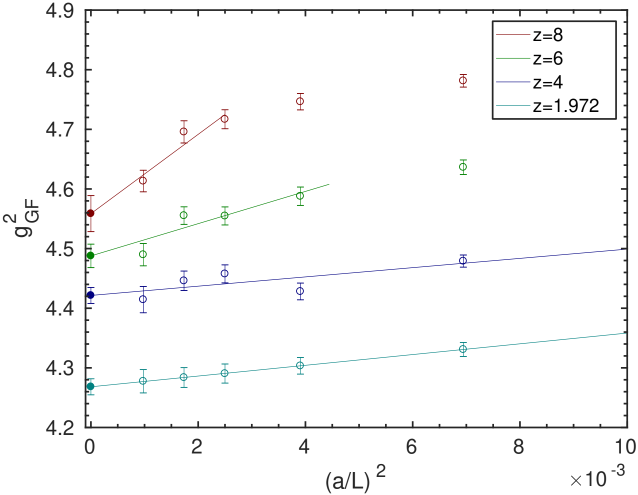
  [M.Bruno, M.Dalla Brida, P.Fritzsch, T.K., A.Ramos, S.Schaefer, H.Simma, S.Sint, R.Sommer,PRL 119 (2017)]
- We look for "lines of constant physics", i.e. simulation parameters $\{L/a, g_0, am_0\}$ such that $M = 0$, $\overline{g}^2_{\mathrm{GF}} = 3.95$
  For $L/a \in \{12, 16, 20, 24, 32\}$
- From these, simulation parameters of massive simulations with $T = 2L$ can be deduced: $\{L/a, \tilde{g}_0, a\tilde{m}_0\}$ such that
  - Lattice spacing unchanged up to $O(a)$: $\tilde{g}_0^2 = g_0^2$
  - $z \equiv ML$ constant up to $O(a)$: $z = \frac{L}{a}\frac{M}{\overline{m}(\mu)} Z_m(\mu)[a\tilde{m}_0 - am_{\mathrm{crit}}]$
    Needs $M/\overline{m}$ etc.

    [I.Campos, P.Fritzsch, C.Pena, D.Preti, A.Ramos, A.Vladikas, Eur.Phys.J. C78 (2018)]
  - $O(a)$ effects can be canceled (and are) $\Rightarrow$ modified formulae

Continuum extrapolation yields $\overline{g}^{(3)}_{\mathrm{GF}}(\mathcal{S}, M)$

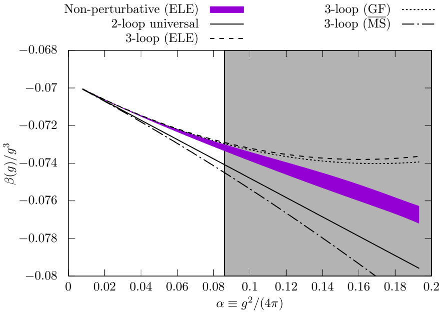
Continuum Extrapolations (preliminary)

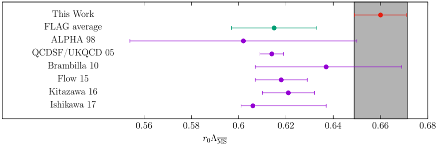
Cut on $aM \Rightarrow$ some L/a are neglected for large M. Here: $aM < 0.4$



Recent result based on finite volume GF schemes:

[Eur.Phys.J. C79 (2019), M.Dalla Brida, A.Ramos]





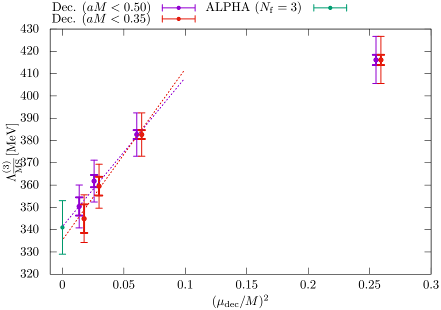
Given a coupling $\bar{g}^2_{\text{GF}}(\mathcal{S})$ we can obtain $\Lambda^{(0)}_{\overline{\text{MS}}}/\mathcal{S}$

E.g. at $z = 6$: $\bar{g}^2_{\text{GF}}(\mathcal{S}) = 4.466(37) \Rightarrow \Lambda^{(0)}_{\overline{\text{MS}}}/\mathcal{S} = 0.741(12)$

$$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = P^{-1}_{0,3}(M/\Lambda^{(3)}_{\overline{\mathrm{MS}}}) \quad \times \quad \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{(0)}} \quad \times \quad \mathcal{S}^{(3)} \qquad + \qquad O(M^{-2})$$

Errors, e.g. $z = 6$, $M = 4.7$ GeV:

- $P_{0,3}$: negligible
- $\mathcal{S}^{(3)}$: 1.9 %
  Scale-Setting + $N_{\mathrm{f}} = 3$ running up to 789 MeV
- $\Lambda^{(0)}_{\overline{\mathrm{MS}}}/\mathcal{S}^{(0)}$: 1.6 %
  $N_{\mathrm{f}} = 0$ running from 789 MeV to ∞



Conclusions and outlook

Conclusions

- α_S from hadronic inputs with fully controlled systematics
- Perturbative decoupling of the charm quark is OK
- New strategy to determine $\Lambda^{(3)}_{\overline{\mathrm{MS}}}$
  - It Works
  - Systematics can be controlled (in the future)
  - Errors dominated by scale-setting

Outlook

- Finer lattices for $\bar{g}^2_{\mathrm{GF}}(\mu, M)$ continuum extrapolations
- Scale-setting update
- Improvements of $N_{\mathrm{f}} = 3$ running
  benefits for traditional and new method