The warped dark sector

Sylvain Fichet
Caltech & ICTP/SAIFR

Based on

1905.05779 [PRD]

1906.02199 [PLB] (with P. Brax, F. Tanedo)

1910.02972 (with F. Tanedo, A. Costantino)
+ upcoming works

IFT 10/25/2019

The warped dark sector

1. Motivations and context
2. EFT, opacity and all that
3. Some phenomenology
4. Outlook

1. Motivations and context

The Universe content

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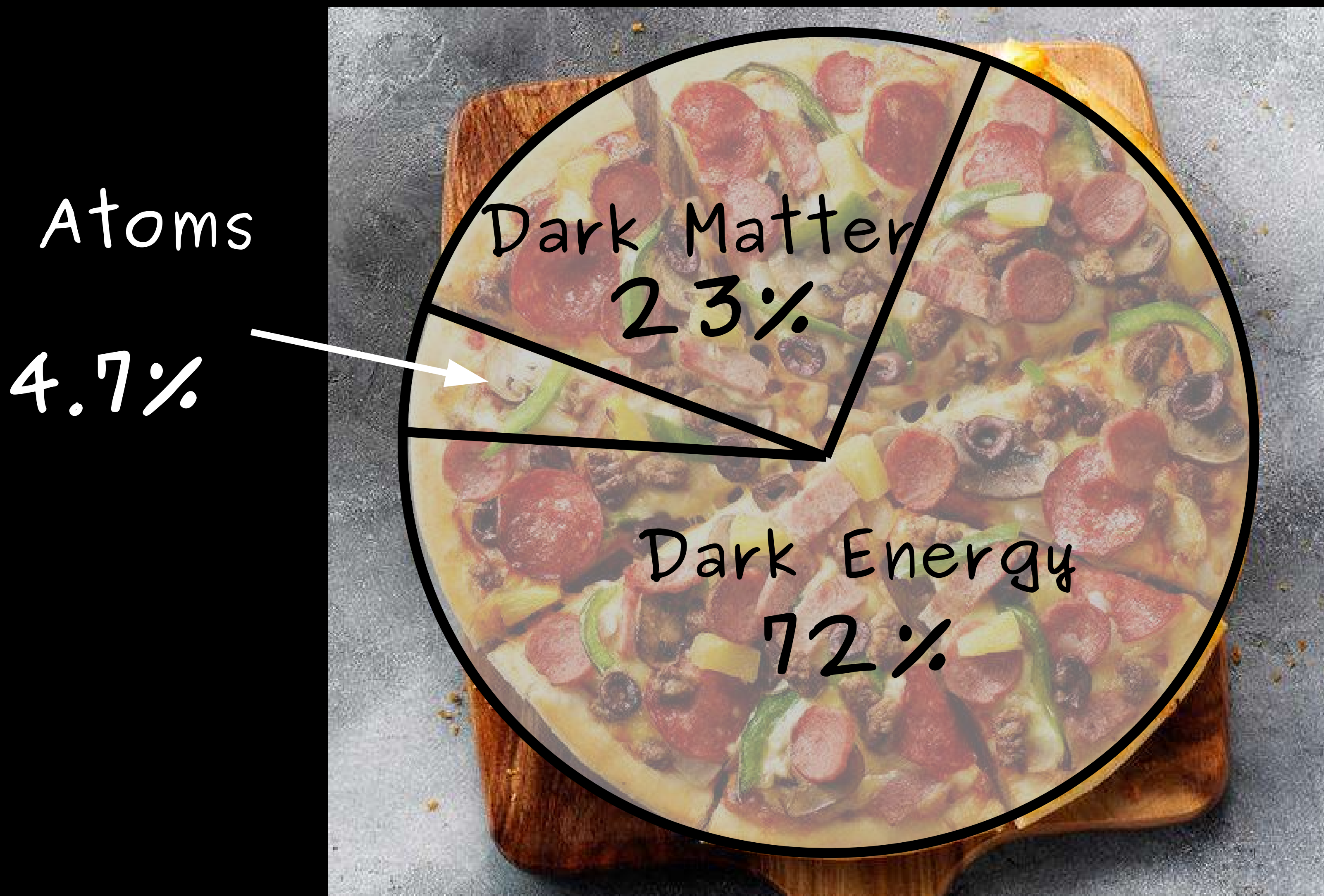
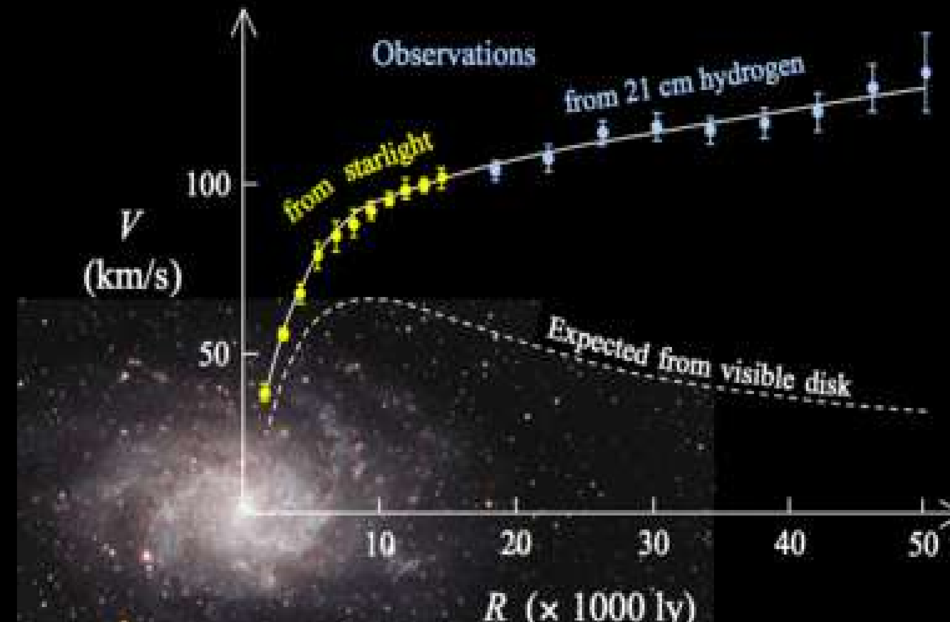
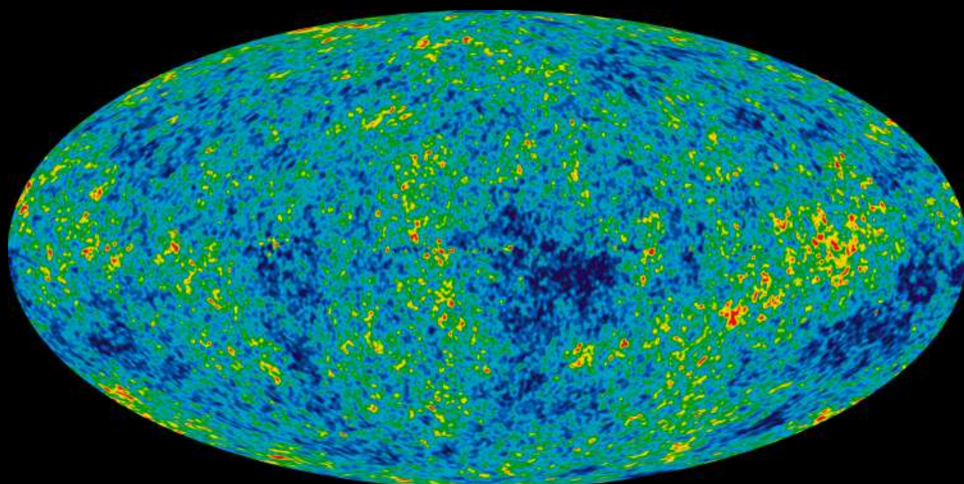


How do we know?

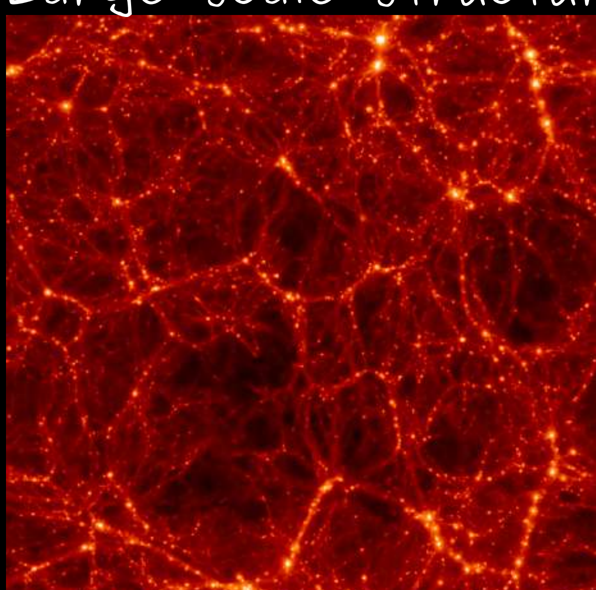
Galactic rotation curves

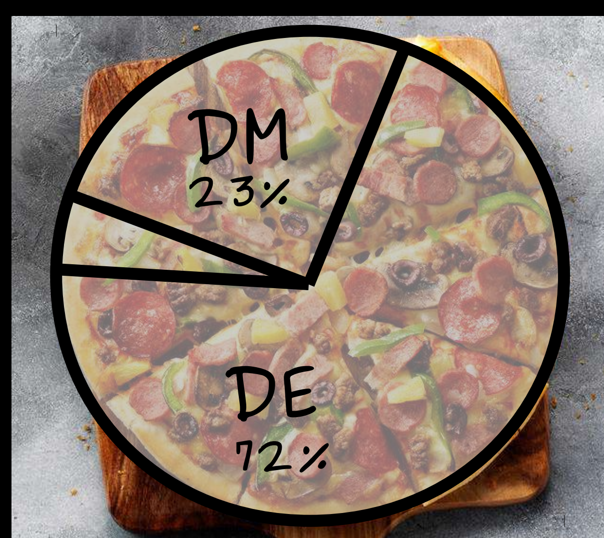


CMB



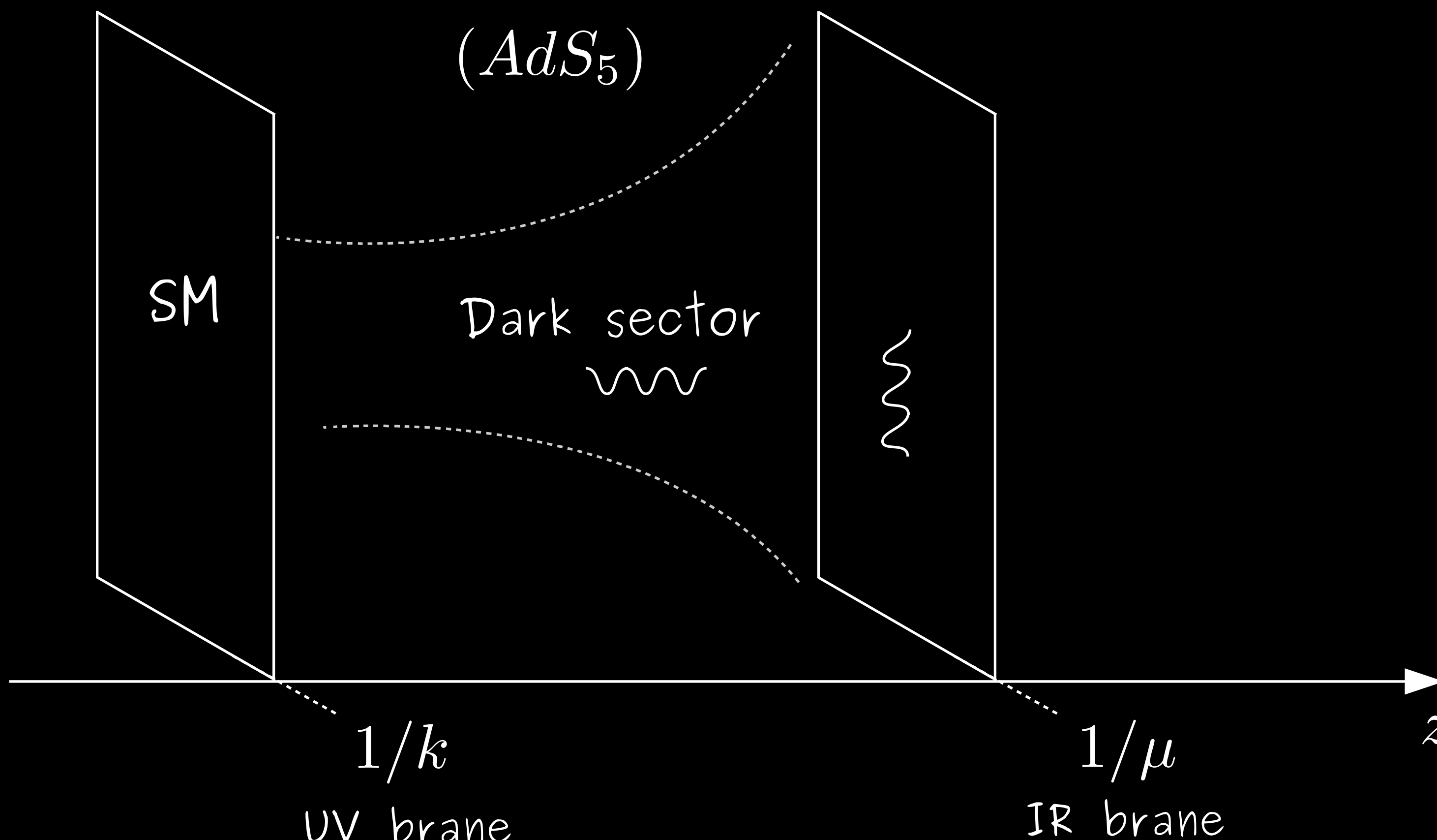
Large scale structures





Cluster collisions

The warped dark sector

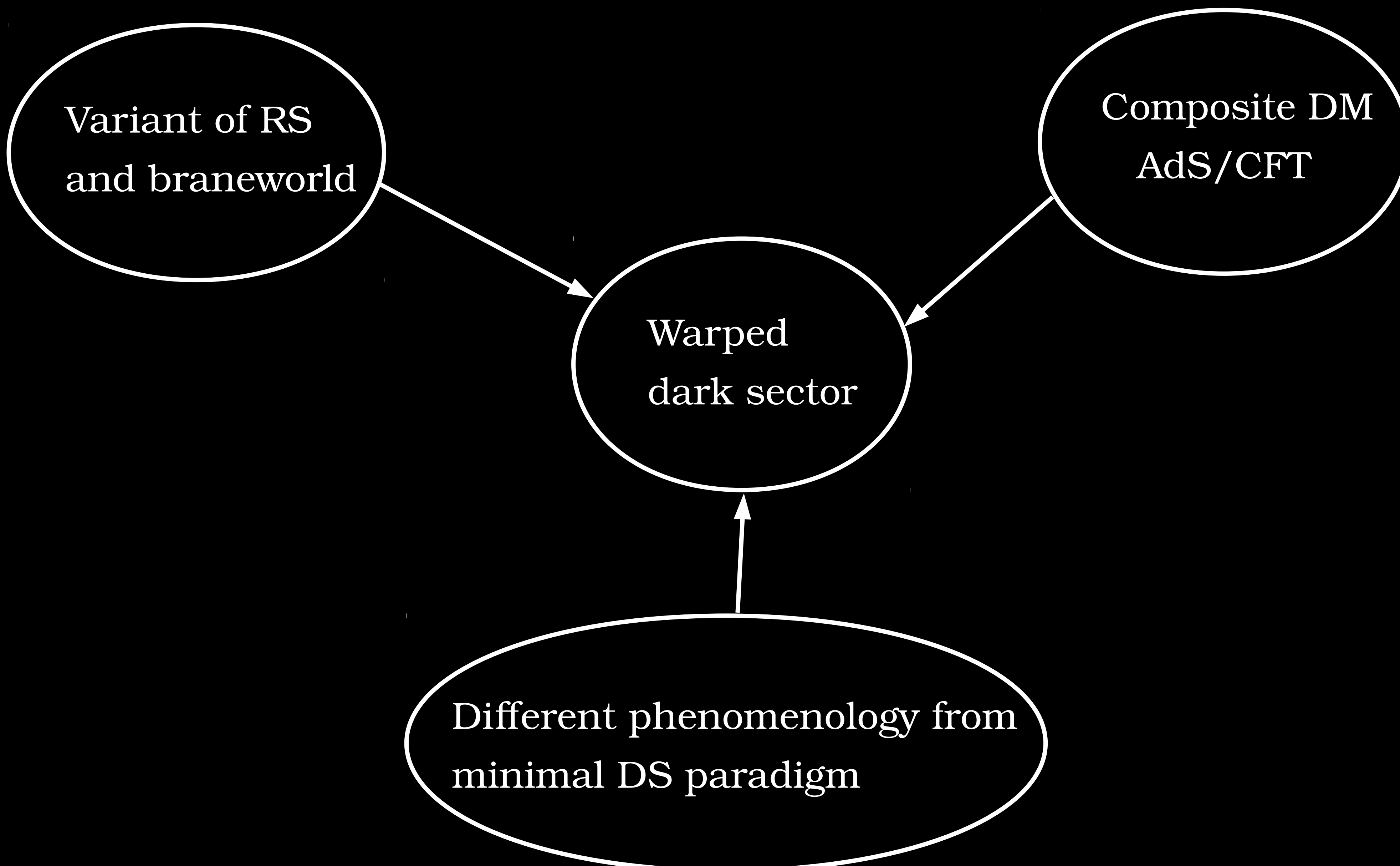


Metric (conformal coordinates): $ds^2 = (kz)^{-2}(\eta_{\mu\nu}x^\mu x^\nu - dz^2)$

Some context

- Not like a Randall-Sundrum model
- Reminiscent of braneworlds (RSII), but here with bulk matter and an IR brane.
- Recent dark sector works with flat extra dimensions [Rizzo '18, '19]...
- Only a few works exist in warped extra dimensions
  [von Harling/McDonald '12, McDonald '12, McDonald/Morrisey '11, '12]
- Extra motivation: the model is the AdS dual of a 4d strongly interacting dark sector

Motivations



2. EFT, opacity and all that

[SF 1905.05779]

Some basics

We assume the AdS background is stabilized. Our focus is on the 5D effective QFT (EFT) living on it.
Use conformal coordinates, $ds^2 = (kz)^{-2}(\eta_{\mu\nu}x^\mu x^\nu - dz^2)$ and work in position-momentum space (p^μ, z) .

Free propagator is denoted $\Delta_p(z,z')$, with $p = \sqrt{p_\mu p^\mu}$. p is real for timelike four-momentum and is imaginary for spacelike four-momentum.

$$\Delta(p;z,z') = i\frac{\pi k^3 (zz')^2}{2}\frac{\left[\tilde{Y}_\alpha^{\rm UV} J_\alpha(pz_<) - \tilde{J}_\alpha^{\rm UV} Y_\alpha(pz_<)\right]\left[\tilde{Y}_\alpha^{\rm IR} J_\alpha(pz_>) - \tilde{J}_\alpha^{\rm IR} Y_\alpha(pz_>)\right]}{\tilde{J}_\alpha^{\rm UV}\tilde{Y}_\alpha^{\rm IR} - \tilde{Y}_\alpha^{\rm UV}\tilde{J}_\alpha^{\rm IR}}$$

(with $z_> = \max(z,z')$, $z_< = \min(z,z')$)

It can also always be formally written in the Kaluza-Klein (KK) representation
(however there is a caveat, cf next slide)

$$\Delta_p(z,z') = i\sum_n \frac{f_n(z) f_n(z')}{p^2 - m_n^2}$$

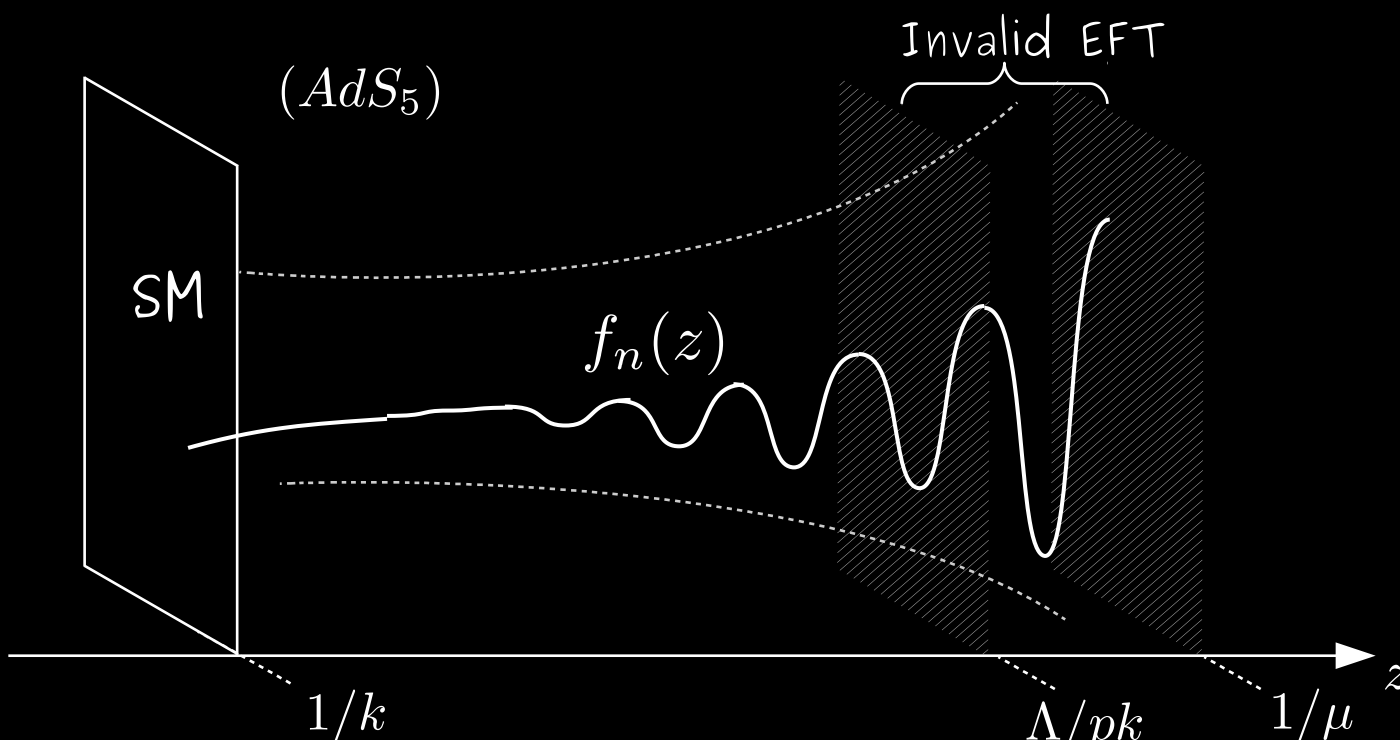
KK masses are $m_n \sim n\pi\mu$. μ is the KK scale, set by the IR brane position.

Effective field theory

The cutoff of the 5D EFT is z-dependent. The EFT breaks down in the IR region.

To see it explicitly, simply consider a higher-dimensional operator $\mathcal{L} \supset \frac{1}{\Lambda^2}\Box^2\Phi^2$
It dresses the propagator and dominates when $|p|z_> \sim \Lambda/k$

(with $z_> = \max(z, z')$)



$$\Lambda \sim 24^{1/3}\pi M_5$$
$$\sim 10(M_{\rm Pl}^2 k)^{1/3}$$
$$\gtrsim 10k$$

For example, one can check that any KK mode profile is invalid in that region.

[Goldberger/Rothstein '02]

In the $z_> > 1/|p|$ region of position/momentum space, the propagator has the asymptotic form

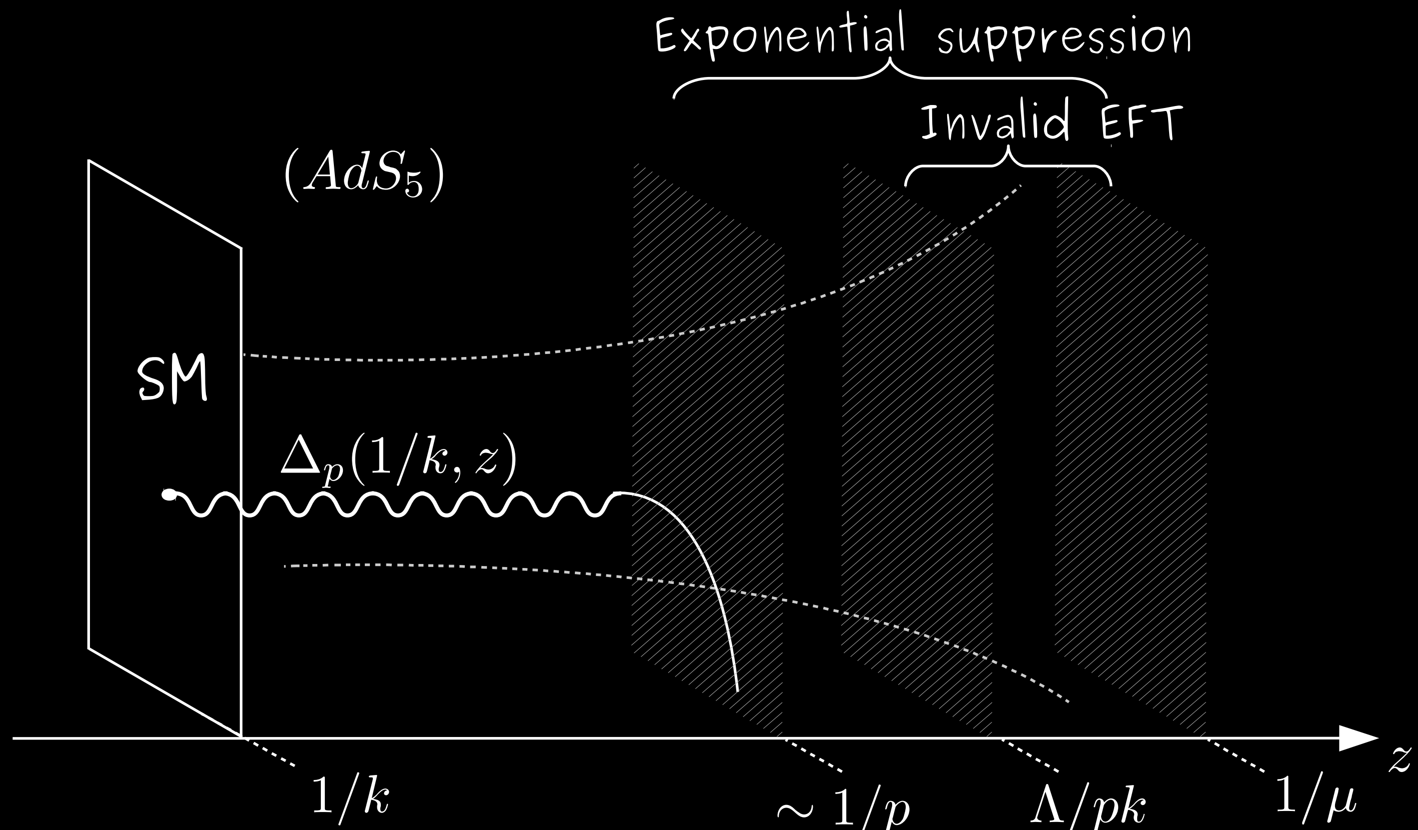
$$\Delta_p(z, z') \propto \frac{\cos\left(\frac{p}{\mu} - pz_>\right)}{\cos\left(\frac{p}{\mu} + \frac{\pi}{4}(1-2\alpha)\right)}$$

(with $z_> = \max(z, z')$)

(In the KK picture, the KK modes nontrivially conspire to give this expression)

Opacity

For spacelike momentum $ip \in \mathbb{R}$, propagator is exponentially suppressed in a bigger IR region, $\Delta_p(z,z') \sim e^{-pz_>}$ when $|p|z_> \sim 1$ [Gherghetta/Pomarol '03, SF '19]



Inspiring because:

- In a sense, the region of invalid EFT gets censored.
- Any field/operator localized near the IR brane is effectively "emergent" from the viewpoint of the UV brane. No similar effect in flat space

Opacity

What happens for timelike momentum?
(Relevant both for phenomenology and for a full theoretical understanding)
For timelike momentum $p \in \mathbb{R}$, the free propagator has a series of poles

$$\Delta_p\,(z, z') \propto \frac{1}{\cos\left(\frac{p}{\mu} + \frac{\pi}{4}(1-2\alpha)\right)} \qquad \text{in } z_> > \frac{1}{|p|} \text{ region}$$

and no suppression occurs.

However interactions should distort $\Delta_p(z, z')$ with imaginary contributions.
May this change the picture?

(In 4d: $\dfrac{i}{p^2 - m_{\rm ren}^2 + i\Sigma(p)}$)

Brane-localized dressing amounts to change the boundary conditions and therefore does not affect the above behaviour.

To understand the propagator's behaviour for timelike momentum, let us therefore investigate the propagator dressed by bulk interactions.
(Dressing always occurs from 5d gravity -which is strong in the bulk)

Dressed propagator

The dressed propagator satisfies the dressed EOM:

$$\frac{1}{\sqrt{g}}\partial_M\left(g^{MN}\sqrt{g}\partial_N\Delta(X,X')\right)+\int dY\,\Pi(X,Y)\Delta(Y,X')=\frac{-i}{\sqrt{g}}\delta^{(d)}(X,X')\,.$$

The 1PI subdiagram.
Contains the bulk mass $\Pi(X,X')\supset m_\Phi^2\delta^{(d)}(X-X')$

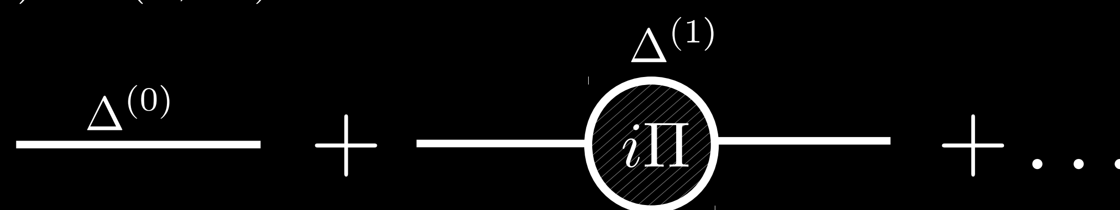
Note the geometric series appears when treating the dressing perturbatively,

$$\mathcal{D}\Delta^{(0)}(X,X')=-i\frac{1}{\sqrt{g}}\delta^{(d)}(X)$$

$$\mathcal{D}\Delta^{(1)}(X,X')=-\int dY\,\Pi(X,Y)\Delta^{(0)}(Y,X')$$

$$\ldots$$

$$\mathcal{D}=\frac{1}{\sqrt{g}}\partial_M\left(g^{MN}\sqrt{g}\partial_N\right)$$

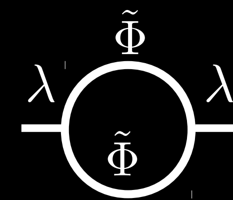
$\Delta(X,X')=$ 

Our interest is in the imaginary part of Π induced by bulk interactions, which is finite.

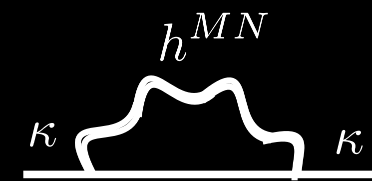
In the KK representation, the KK modes are mixed by the self energies which is highly unconvenient. Hence we rather use the closed-form expressions

Dressed propagator

Cubic scalar interaction: $\mathcal{L} \supset \lambda \Phi \tilde{\Phi}^2\,, \quad i\Pi =$



5d gravity: $\mathcal{L} = \sqrt{g}\frac{1}{\sqrt{2M^3}} h^{MN} T_{MN}\,, \quad i\Pi =$



$\kappa = k/M_{\rm Pl}$

[Dudas/Gersdorff '13]

• • •

[Details available upon request]

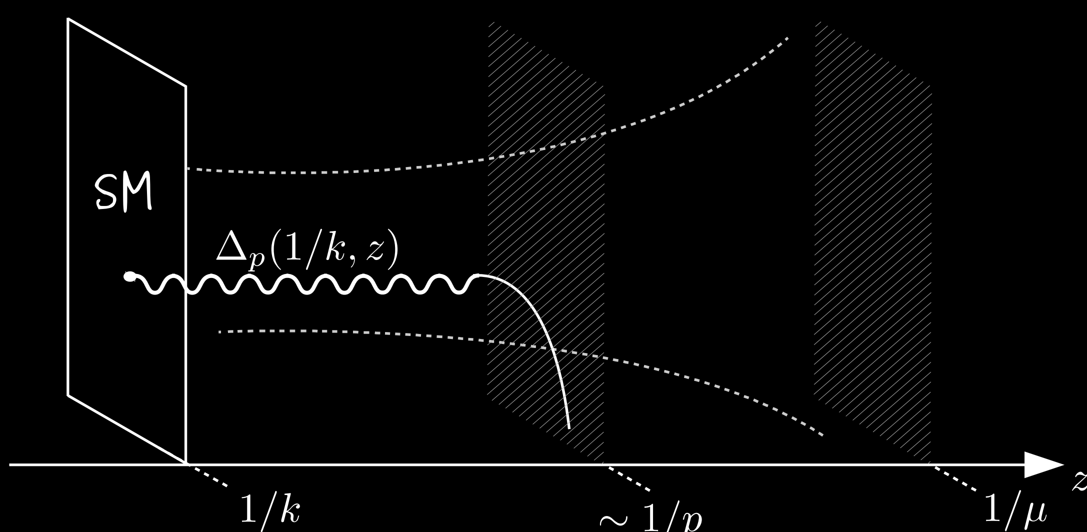
Opacity

...

Conclusion (near strong coupling, $\kappa \sim 1\,,\ \lambda \sim (24)^{1/3}\pi$) :

$$z_> > 1/p \qquad \Delta_p(z,z') \sim \begin{cases} e^{-Cpz_>} & \text{if } p^\mu \text{ timelike} \\ e^{-pz_>} & \text{if } p^\mu \text{ spacelike} \end{cases} \qquad C = O(0.1-1)$$

(At weaker coupling, similar results are expected but a numerical solving of the dressed EOM would be needed.)

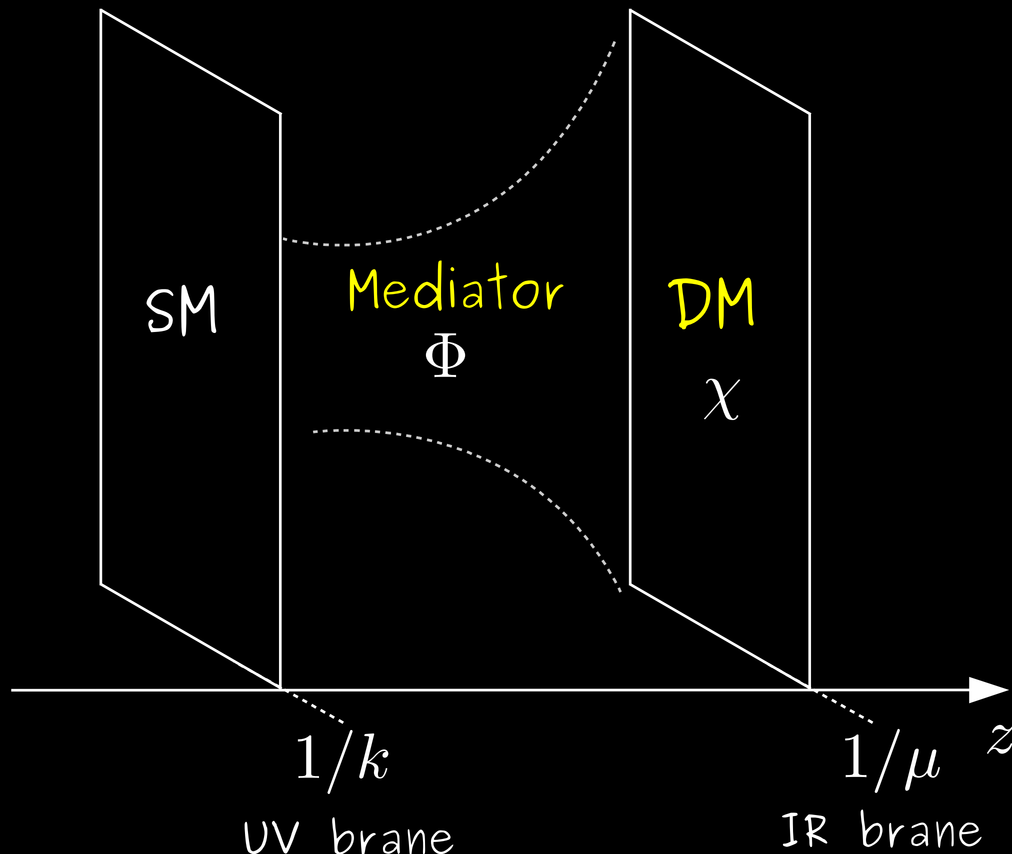


Any field/operator localized near the IR brane is effectively "emergent" for a UV-brane observer. There is no similar effect in flat space.

3. Some phenomenology

[Brax/SF/Tanedo 1906.02199] + upcoming works

A dark sector model



To be concrete:

SM = quarks

Φ = scalar

χ = Dirac

$\mathcal{O}_{\rm SM} = \bar{N}N(\bar{q}q)$

$\mathcal{O}_{\rm D} = \bar{\chi}\chi$

$m_\chi \sim 4\pi\mu$

$k \sim M_{\rm Pl}$

$$S \supset \int_{\rm bulk} d^5X\sqrt{|g|}\left(\frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{m_\Phi^2}{2}\Phi^2\right) + \int_{\rm UV} d^4X\sqrt{|\gamma|}\left(\mathcal{L}_{\rm SM} + \frac{\lambda}{\sqrt{k}}\mathcal{O}_{\rm SM}\Phi - \frac{m_{\rm UV}}{2}\Phi^2\right)$$
$$+ \int_{\rm IR} d^4X\sqrt{|\gamma|}\left(\mathcal{L}_{\rm IR} + \frac{\kappa}{\sqrt{k}}\mathcal{O}_{\rm D}\Phi - \frac{m_{\rm IR}}{2}\Phi^2\right)$$

Parameters

$$S \supset \int_{\text{bulk}} d^5X\sqrt{|g|}\left(\frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{m_\Phi^2}{2}\Phi^2\right) + \int_{\text{UV}} d^4X\sqrt{|\gamma|}\left(\mathcal{L}_{\text{SM}} + \frac{\lambda}{\sqrt{k}}\mathcal{O}_{\text{SM}}\Phi - \frac{m_{\text{UV}}}{2}\Phi^2\right)$$
$$+ \int_{\text{IR}} d^4X\sqrt{|\gamma|}\left(\mathcal{L}_{\text{IR}} + \frac{\kappa}{\sqrt{k}}\mathcal{O}_{\text{D}}\Phi - \frac{m_{\text{IR}}}{2}\Phi^2\right)$$

μ

IR scale

Sets dark sector mass scale

(Not O(TeV) !)

$\alpha = \sqrt{4 + m_\Phi^2/k^2}$

Controls localization of bulk field and thus how strong it couples to SM

m_{UV}

UV brane mass is set to the special value $(2-\alpha)k$

Implies an exponentially light mode for $\alpha > 1$

No light mode for $\alpha < 1$ (our focus today)

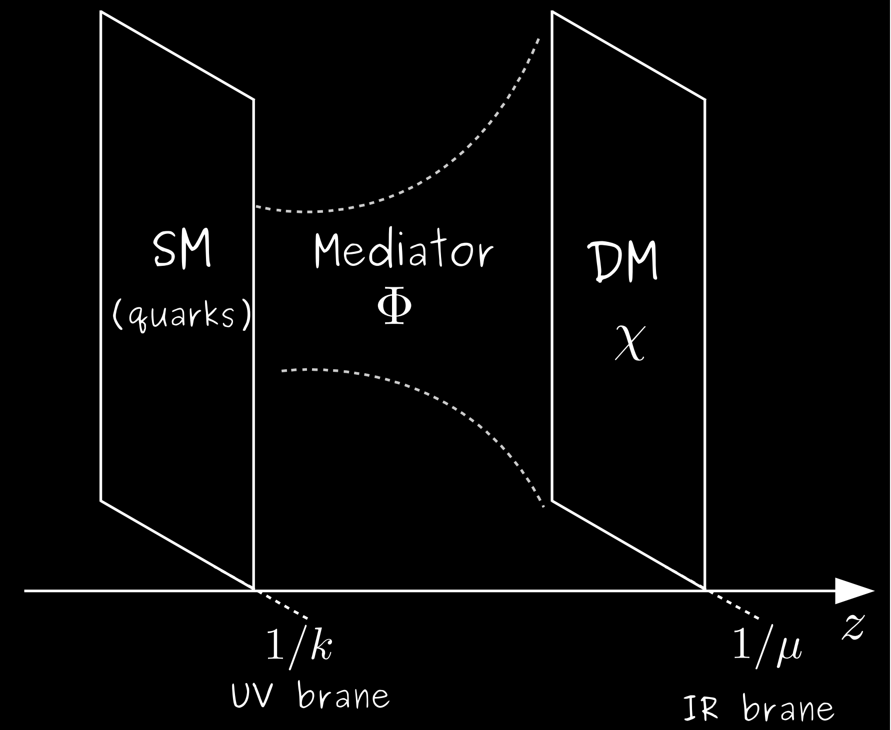
m_{IR}

IR brane mass is left free

(and is mostly irrelevant)

4D regime

- Low-energy regime $|p| < \mu$



KK modes of Φ are integrated out, giving a familiar 4d DM effective theory, with $O(\mu)$ cutoff.

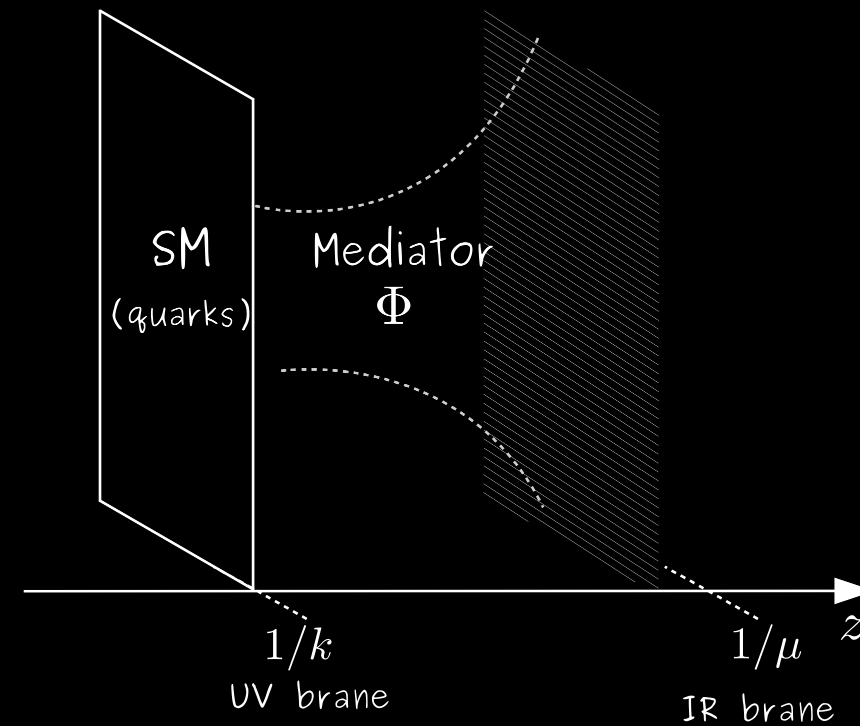
$$\mathcal{L}_{4d} \sim \kappa^2 \frac{1}{\mu^2} (\mathcal{O}_{\rm D})^2 + \lambda\kappa \frac{\varepsilon^{1-\alpha}}{\mu^2} \mathcal{O}_{\rm SM}\mathcal{O}_{\rm D} + \lambda^2 \frac{\varepsilon^{2-2\alpha}}{\mu^2} (\mathcal{O}_{\rm SM})^2 \dots$$

Strong self-interactions

Exponentially small coupling
Ideal for DM freeze-in

$$\mathcal{O}_{\rm SM} = \bar{N} N \,, \dots$$
$$\mathcal{O}_{\rm D} = \bar{\chi}\chi \,, \dots$$
$$\alpha \in [0,1]$$
$$\lambda, \kappa = O(1)$$
$$\varepsilon \equiv \mu/k \ll 1$$
(warp factor)

5D regime



- High-energy regime $|p| > \mu$

DM vanishes from the amplitudes. Amplitudes can be described by pure AdS with only mediator and SM as dofs,

E.g. $\mathcal{A}(N\chi \to N\chi) \sim e^{-|p|/\mu}$

for $|p| > \mu$

Exact holographic dual: $$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm CFT} + \frac{1}{M^{\Delta_{CFT}-1}} \mathcal{O}_{\rm SM}\mathcal{O}_{\rm CFT}$$

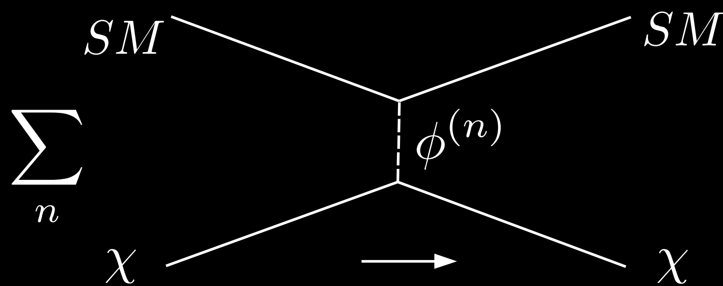
with $\Delta_{\rm CFT} = 2 - \alpha$

(Δ^- branch, valid for $\alpha \leq 1$)

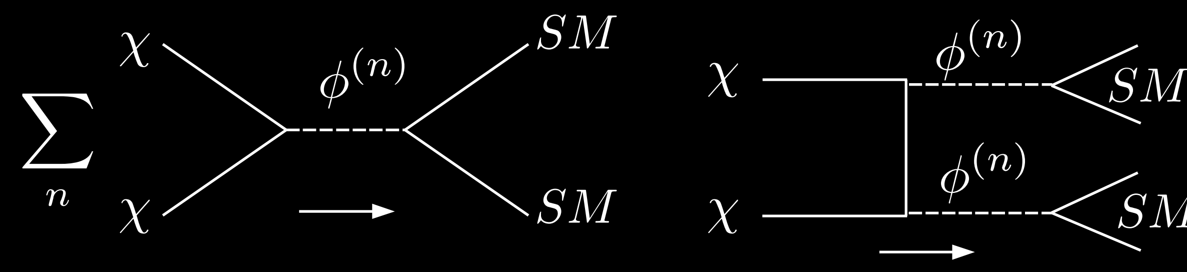
Dark matter complementarity

Dark matter is effectively emergent. DM complementarity gets modified.

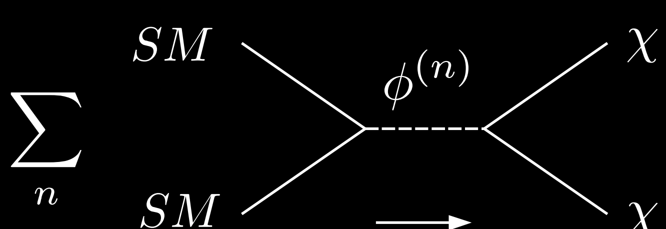
Direct detection $\sqrt{|t|} < \mu$



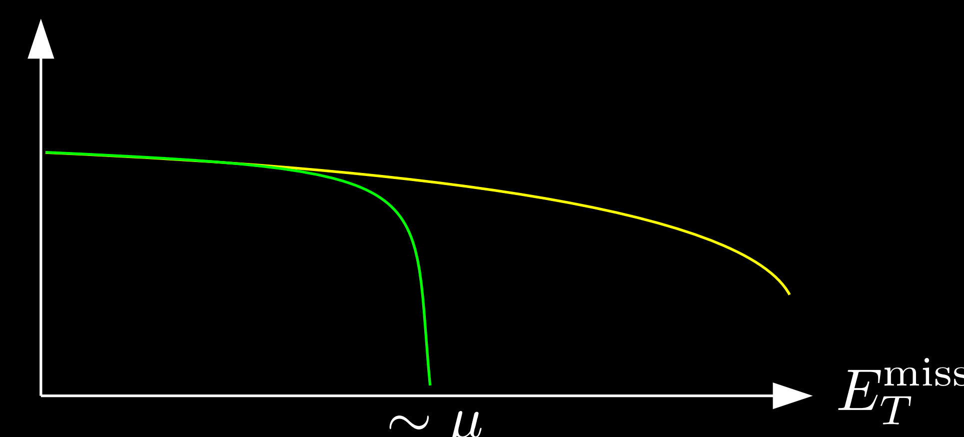
Indirect detection / relic density $\sqrt{s} \sim \mu$



DM production (colliders)



For $\sqrt{s} > \mu$, the mediators conspire such that the full amplitude is exponentially suppressed. Hence one expects suppression of missing energy above the IR scale μ



Some generic signatures

Phenomenology of the warped model is rich and only partly familiar.

Some features:

TODAY:

- Exotic fifth force
- Non-standard momentum losses (meson decays, star cooling)

} Quantitative

- Periodic signals at colliders
- Dark radiation

} Qualitative

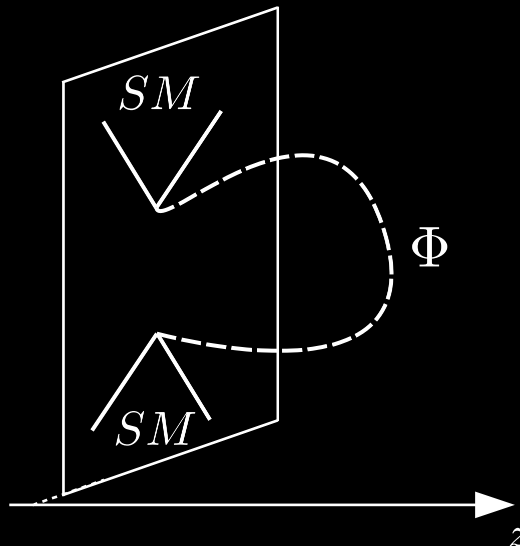
- Soft spherical events ("soft bombs")
- Dark phase transition around $T \sim \mu$

} Not discussed

Exotic fifth force

Exchange of a bulk field in the 5d regime leads to a fifth force with non-integer behaviour

$$V(r) \propto \int d^3q e^{iqr} \Delta_{i|q|}(z_0, z_0) \propto \int d^3q |q|^{-2\alpha}$$
$$\propto \frac{1}{r} \frac{1}{(kr)^{2\alpha-2}}$$



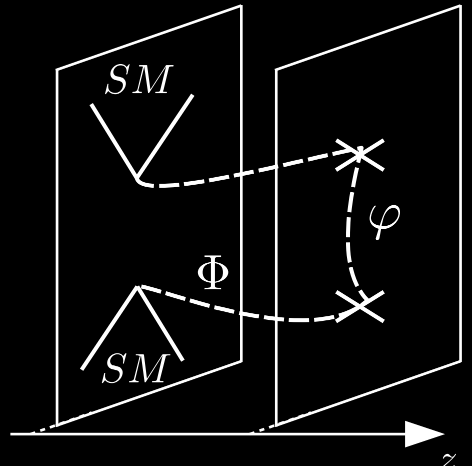
(Behaviour can be exactly reproduced by CFT model)

Bounds from torsion pendulum, Casimir, molecules, neutrons experiments...

Can interpolate with other behaviour in 4d regime.
Gives an “emergent” force. [Costantino/SF/Tanedo ‘19, soon]

E.g. mixing with IR brane scalar φ ,

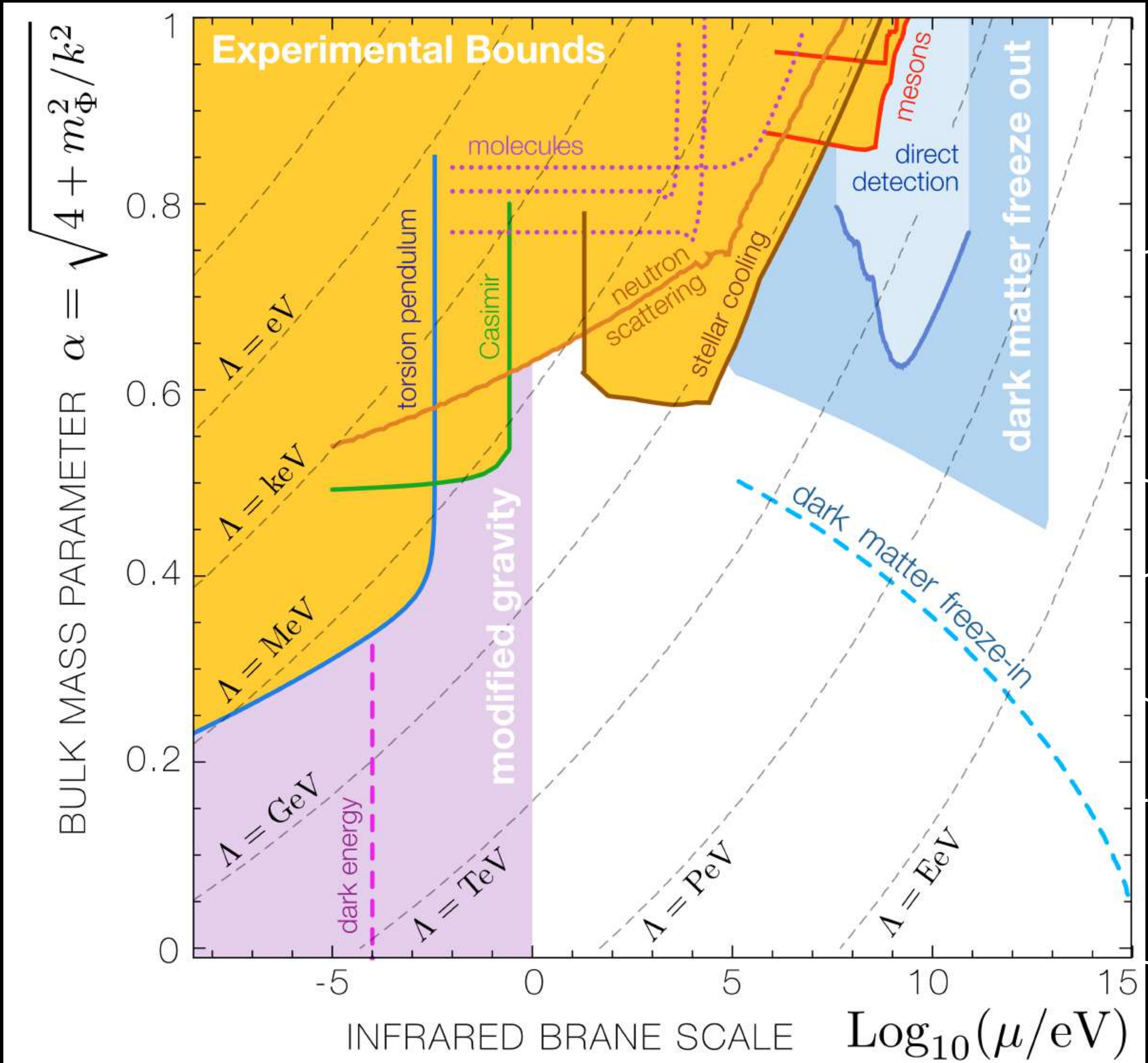
$$\mathcal{L} \supset \delta(z - z_1) \left(\mathcal{L}_\varphi + \omega \Phi \varphi\right)$$



(Exact result using dressing)

$$V(r) \propto \int d^3q e^{iqr} \left[\Delta_{i|q|}(z_0, z_0) + \left(\Delta_{i|q|}(z_0, z_1)\right)^2 \frac{i\omega^2}{p^2 - m_\varphi^2 + i\omega^2 \Delta_{i|q|}(z_1, z_1)} \right]$$

Parameter space



Λ^{-2} is the low-energy SM-DS effective coupling

$$\mathcal{L} \supset \frac{1}{\Lambda^2}\bar{N}N\bar{\chi}\chi$$

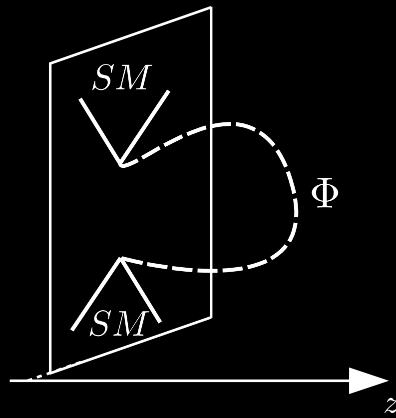
μ and Λ can be very low and still evade bounds

Can UV-complete a very low-energy 4d EFT (cutoff ~ μ)

Periodic signals at colliders

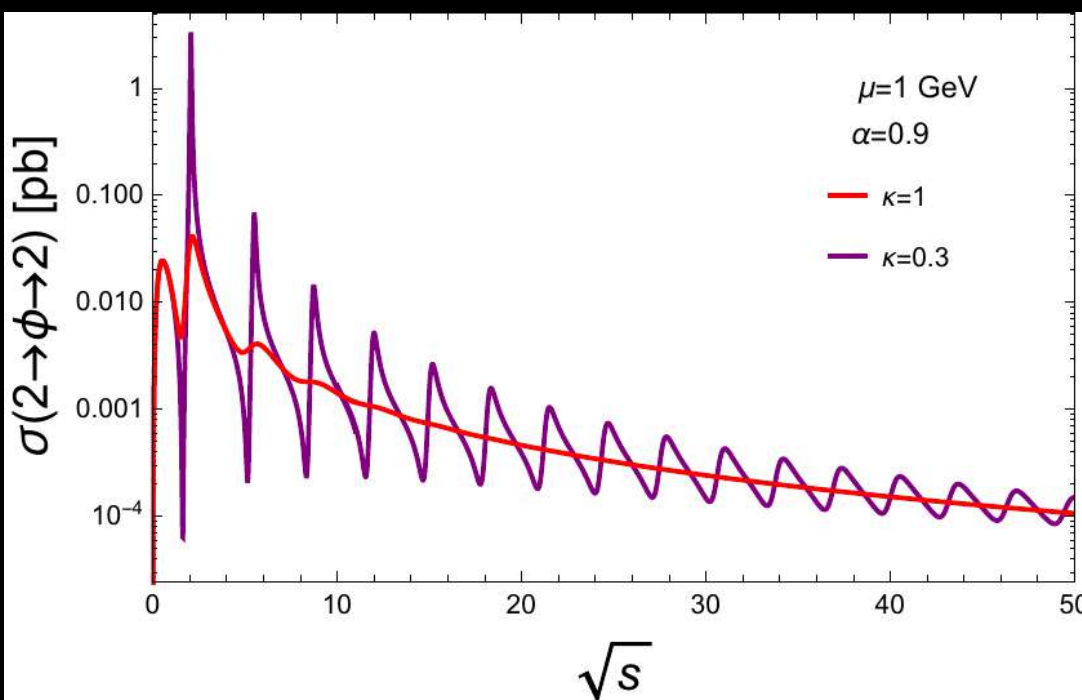
- Collider signals with periodic bumps and dips

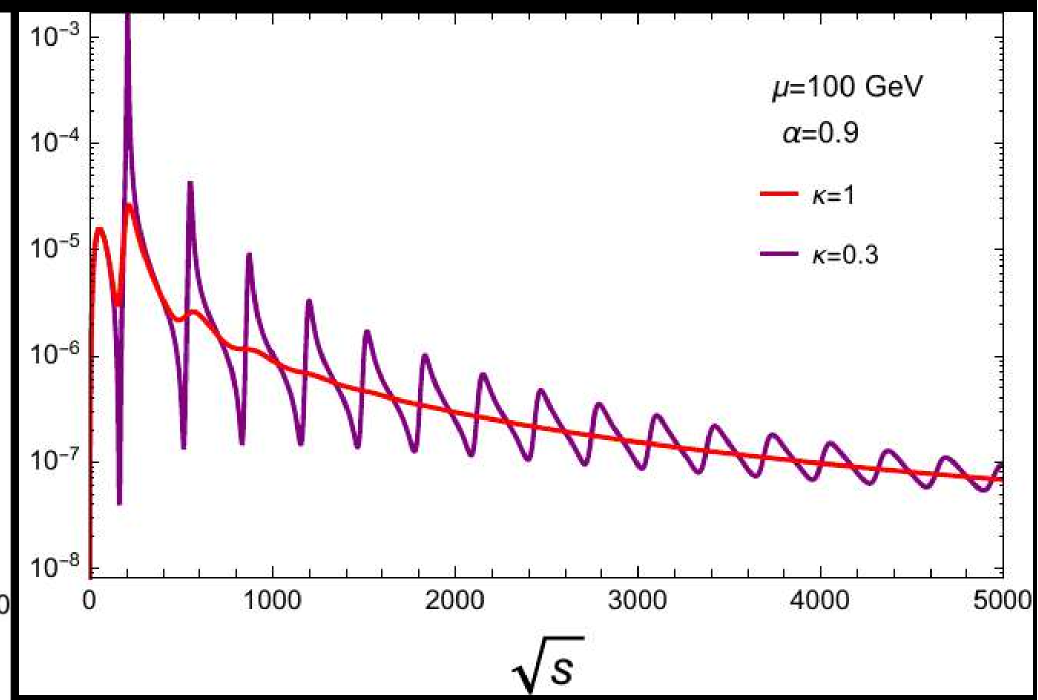
  (Similar signal pointed out in linear dilaton model [Giudice et al '18])



- Smearing depends on details of bulk and brane dressing
- Here only the cross section for $|\mathcal{M}_{\rm BSM}|^2$ is shown
- Search for signal by taking the Fourier transform of the lineshape

  (recent technical developments in [Beauchesne/Kats '19] and [Lillard/Plehn/Romero/Tait '19])

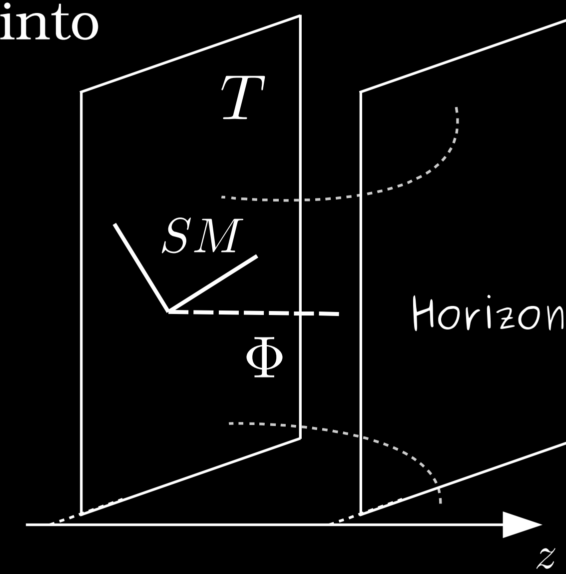




Dark radiation

At finite temperature, metric is AdS-Sch, energy leaks into the bulk and equilibrate with the bulk black hole

$$\frac{d\rho}{dt} + 3H(\rho + p) = -\sigma \equiv -\sum_n \sigma_n$$



Calculation has been made for gravitons in

[Hebecker/March-Russel '02], [Langlois/Sorbo/Rodriguez '02]

giving $\sigma \propto T^8/M_5^3$

But approximation of the KK mode profiles was not so appropriate. Using KK continuum trick, calculation doable with exact profiles, giving $\sigma \propto T^9/kM_5^3$

$$\sigma = \frac{1}{32\pi^3}\int dE_1 \int dE_2 (E_1 + E_2)\, f_1 f_2 \sum_{n=0}^{\tilde{n}} \overline{|\mathcal{M}_n(m_n^2)|^2}$$

$$\sum_{n=0}^{\tilde{n}} \overline{|\mathcal{M}_n(m_n^2)|^2} = \sum_{n=0}^{\tilde{n}} \frac{1}{M_5^3} f_n^2(z_0) A \frac{m_n^4}{8} = \frac{A}{8M_5^3}\int_{\mathcal{C}[\tilde{n}]} \frac{d\rho}{-2\pi} \Delta_{\sqrt{\rho}}(z_0, z_0)\rho^2 \approx A\frac{E_1^3 E_2^3}{3M_5^3}\,.$$

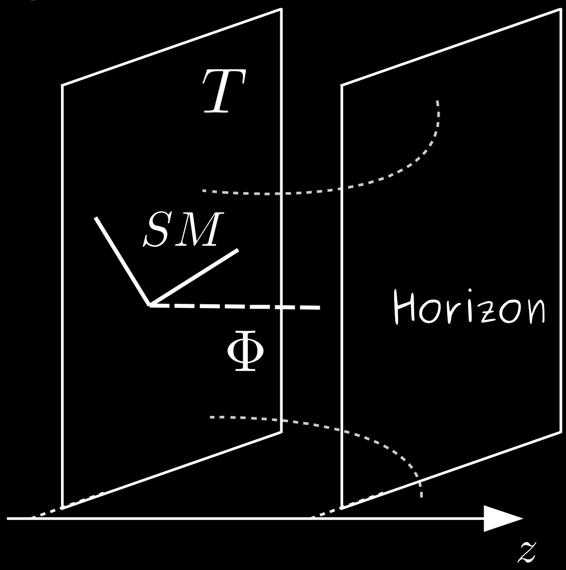
$$\sigma = A\frac{\pi\zeta(5)\, T^9}{60k}$$

Dark radiation

Technique can be applied similarly for leak into Φ, giving

$$\sigma \propto T^6 \left(\frac{T}{k}\right)^{1-2\alpha} .$$

(and more generally into any bulk matter fields)



For gravitons and for Φ with $\alpha < 1/2$, an estimate

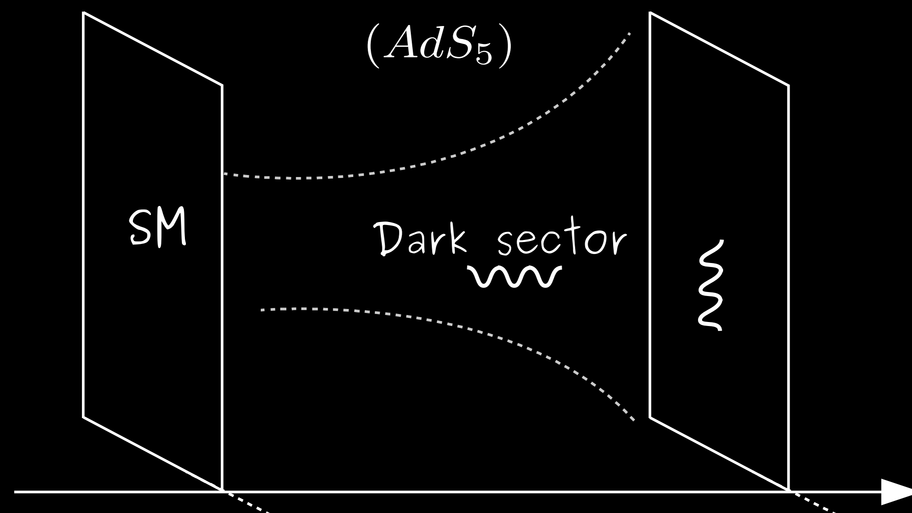
$$\Omega_{\rm D} = \int_{\tau_{\rm in}}^{\infty} d\tau \frac{\sigma}{\rho_{\rm tot}}$$ (à la HMR)

typically gives $\Omega_{\rm D} \lesssim 1\%$ at BBN time.

Hence: - Too small to be constrained by present measurement.
- Might be probed in future

- This property of the old braneworld is very nice for recent sub-GeV dark sector model-building, to alleviate dark radiation constraints
- Many cases to analyze, numerical study probably needed.
- Topic of future work

Summary and outlook

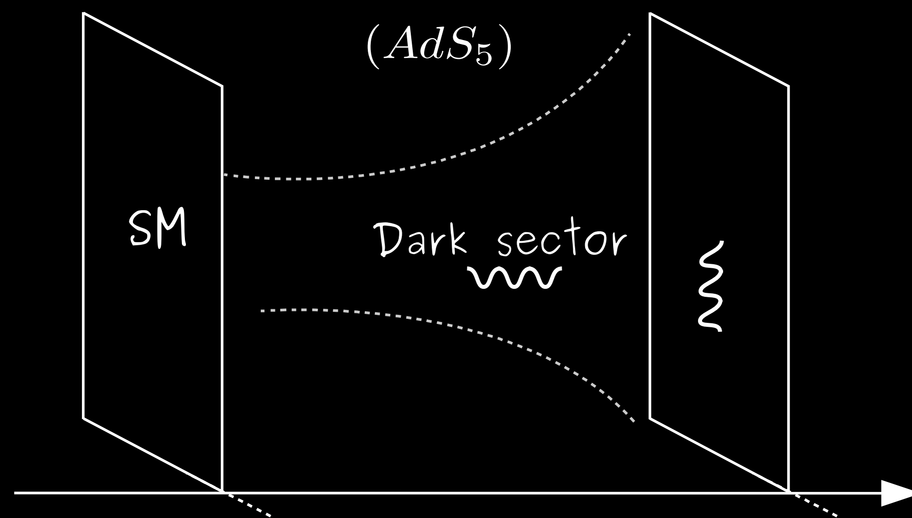


- A warped extradimension naturally gives rise to dark sector physics. A conceptually simple possibility, which is further motivated as the AdS dual of a composite dark sector.

- If DM is on IR brane, it is effectively "emergent": At high-energy it vanishes from all amplitudes as a result of IR opacity. This implies non-standard DM complementarity.

- Model features a variety of "exotic" signatures. Fifth forces, non-standard stellar cooling and invisibe meson decays, etc. At the LHC, expects periodic signals and vanishing of $E_T^{\rm miss}$ above IR scale

Summary and outlook



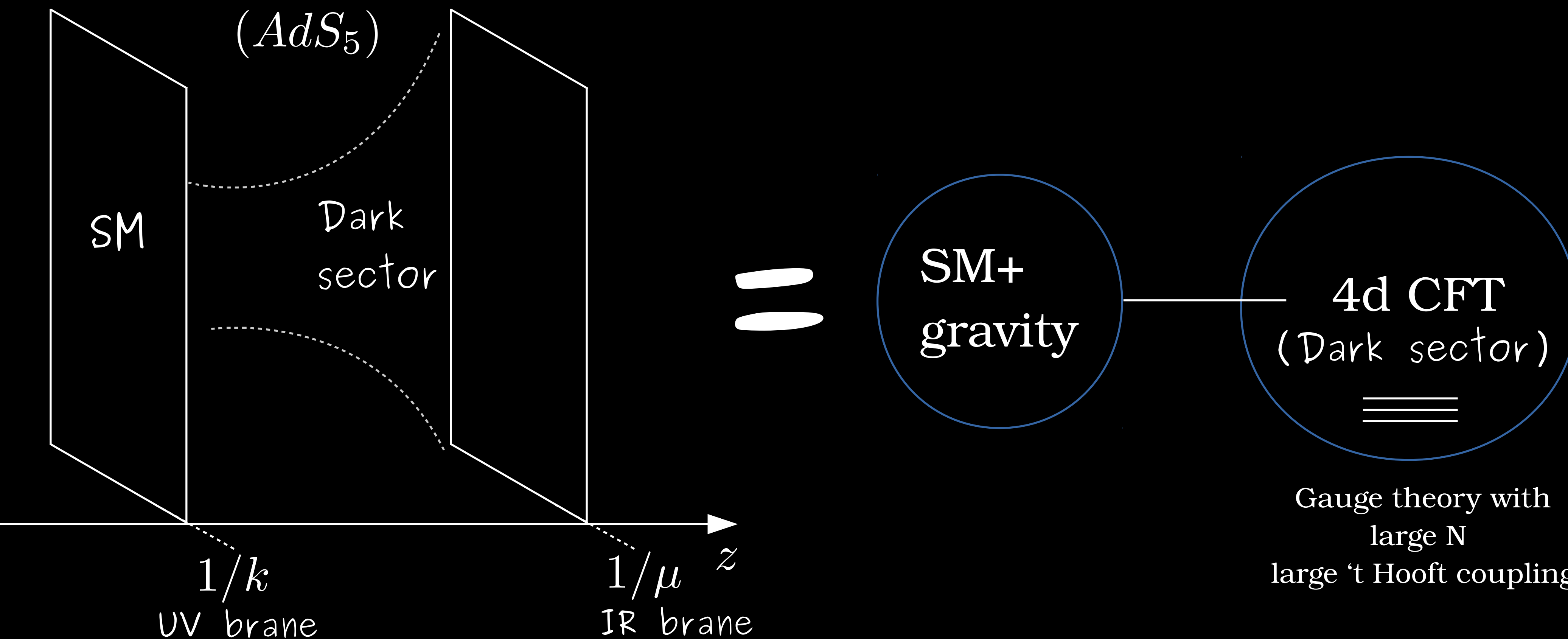
- Many important developments remain to be done, both on theoretical and phenomenological sides. The next papers on our to-do list are about
  - Bulk cascade decay rates
  - Dark radiation
  - Screened modified gravity

- Collaborations welcome, e.g. for a cosmological study of emergent DM or for a collider-oriented study
  Let me know if you are interested!

THANKS

More

Duality



AdS model is the holographic description of a strongly interacting dark sector with IR confinement scale at $\sim \mu$.

Some formalism

5d action (an exact slice of AdS):

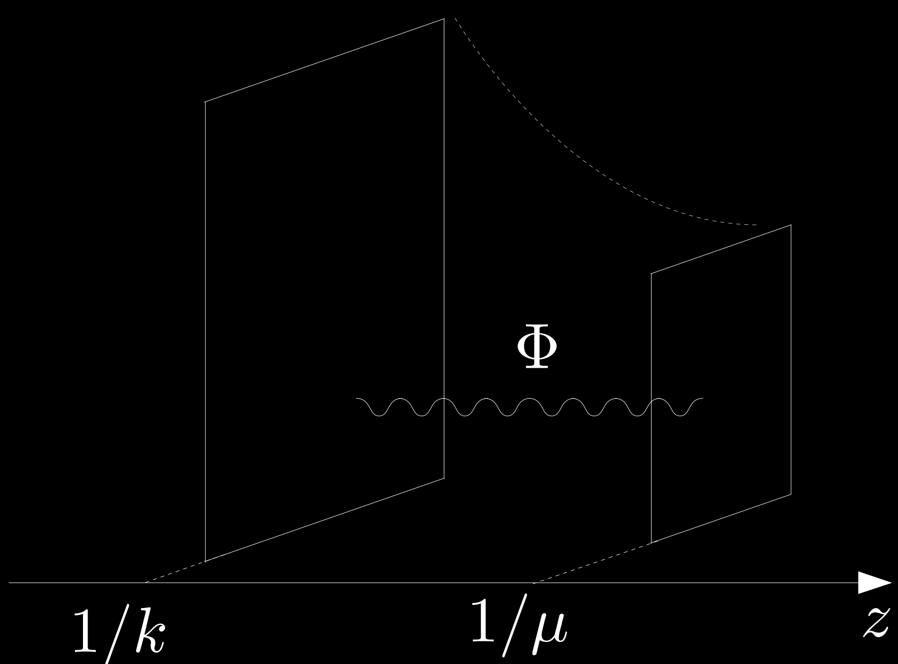
$$S_{\text{AdS}} = \int d^5x^M \sqrt{g} \left(\frac{1}{2} \nabla_M \Phi \nabla^M \Phi - \frac{1}{2} m_\Phi^2 \Phi^2 \right) + S_{\mathcal{B}} + S_{\text{int}}$$

Metric (conformal coordinates):

$$ds^2 = g_{MN} dx^M dx^N = (kz)^{-2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

Brane mass terms:

$$S_{\mathcal{B}} = \int d^5x^M \sqrt{\bar{g}}\, \frac{1}{2} \left(\delta(z - z_0)\, M_{\text{UV}} - \delta(z - z_1)\, M_{\text{IR}} \right) \Phi^2$$



Some definitions:

$z_0 = 1/k, \quad z_1 = 1/\mu$

$M_{\text{UV}} = (\alpha - 2)k - b_{\text{UV}} k \,,$

$M_{\text{IR}} = (\alpha - 2)k + b_{\text{IR}} k \,.$

Spectrum has a massless mode for

$b_{\text{UV}} = b_{\text{IR}} = 0, \; 2\alpha$

Derivative expansion

$$\frac{1}{\sqrt{g}}\partial_M\left(g^{MN}\sqrt{g}\partial_N\Delta(X,X')\right)+\int dY\,\Pi(X,Y)\Delta(Y,X')=-i\frac{1}{\sqrt{g}}\delta^{(d)}(X)\,.$$

In the EOM, Π is a distribution convoluted with the EOM solution.

Π can formally be expanded as

$$\Pi(z,z')=F_0(z)\delta(z,z')-F_1(z)\delta^{(1)}(z,z')+\frac{1}{2}F_2(z)\delta^{(2)}(z,z')+\dots$$

where $F_i(z)=\int dz' z^i\Pi(z,z')$. Note $F_i(z)/F_0(z)$ are the moments of Π

(Note the Laplace transform amounts to the moment-generating function)

Series can be truncated when distribution is narrow wrpto test function

(when keeping leading term, it's just usual approximation as a Dirac peak)

In the case of the AdS propagator, truncation goes as $O(pz >)$

$$F_0(z)=\lambda^2 kC_0\,\frac{1}{(kz)^5}\,,\quad F_1(z)=\lambda^2 C_1\,\frac{1}{(kz)^4}\,,\quad F_2(z)=\frac{\lambda^2}{k}C_2\,\frac{1}{(kz)^3}$$

Contributes to imaginary bulk mass

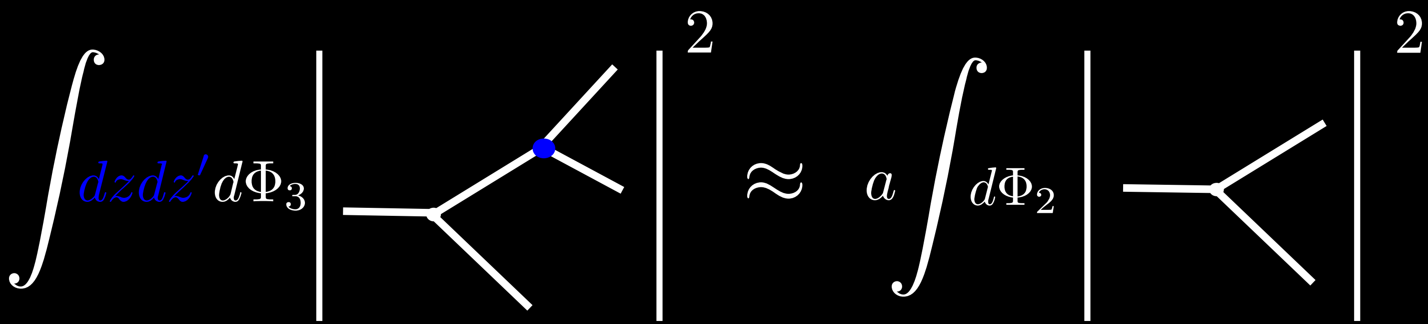
Contributes to imaginary bulk mass + harmless phases

Contributes to imaginary 4-momentum

Cascade decays

Even though the propagators with timelike momentum cannot access the deep IR, another possibility may be the cascade decay of the continuum.
As the field fragments, p reduces and the daughters progressively reach further in the IR.

One finds an approximate recursion relation



$$a = \frac{\lambda^2}{k} \frac{\Gamma(-\alpha)^2}{(\Gamma(\alpha+1)\Gamma(\alpha+2))^2} \frac{1}{16\pi^2} \frac{1}{(3\alpha+1)^2} \frac{1}{4^{5\alpha+4}} \ll 1$$

λ is cubic scalar coupling
Uncertainty is likely $O(1)$

This can be used to estimate the rate of a cascade decay:

$$|\mathcal{M}|^2(1 \to n) \sim a^{2^n - 1}$$

Cascade decays

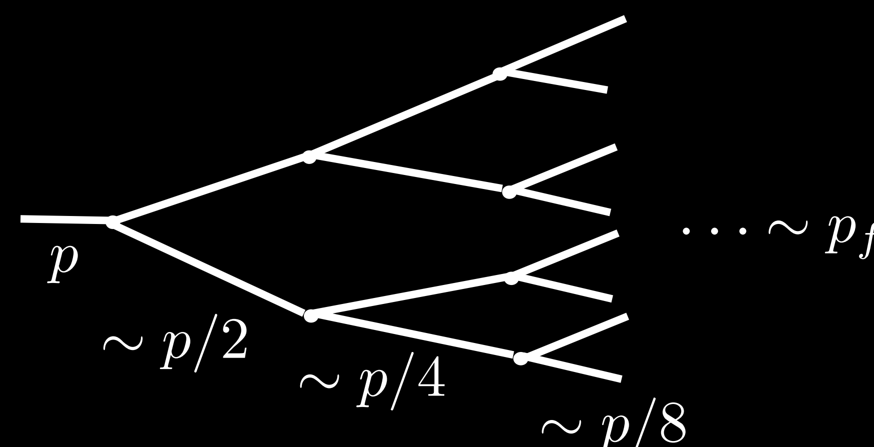
Moreover, from the KK continuum trick, one has

$$\frac{1}{2i\pi}\int_{\mathcal{C}[n_{\rm th}]} d\rho\Gamma(\sqrt{\rho})\underbrace{G_{\sqrt{\rho}}(z,z')}_{\propto\,\rho^{\alpha}}$$

which means that decays into heaviest modes are preferred.

(Implies that events will tend to be spherical and soft, same conclusion as [Csaki/Reece/Terning '08])

To make a gross estimate, let us consider the most likely phase space configuration,



For final states with momentum $p_f = p/2^n$ we have thus

$$|\mathcal{M}|^2 \sim a^{p/p_f - 1}$$

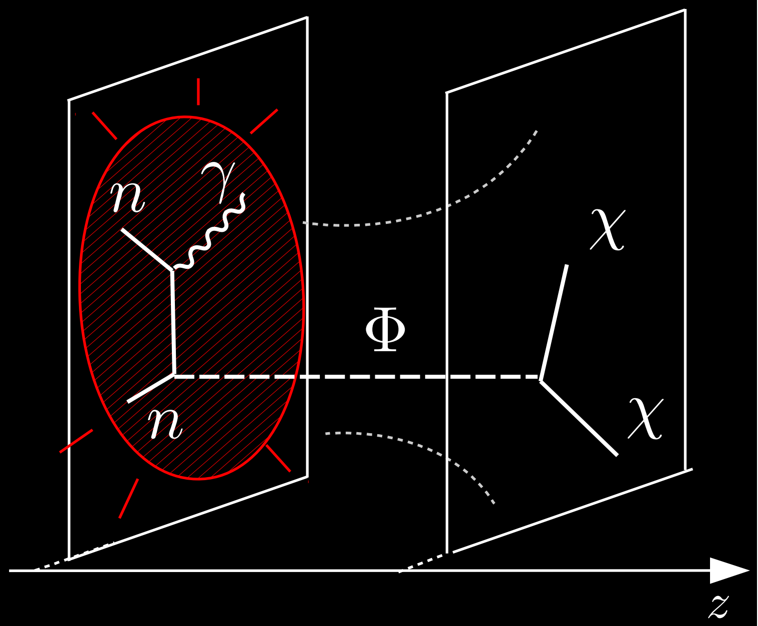
Since $a \ll 1$, this is a strong exponential suppression.

(The value chosen for p_f depends on the other scales of the object considered (detector, star...), because lifetime of final states depends on p_f)

Stellar cooling

Stellar energy can leak into the IR brane

Known 4d results can be recast using phase space recursion and KK representation,



$$R^{(n)}(m_n^2) = \frac{1}{2m_N}\int_0^\infty \frac{dq q^2}{2\pi^2}\,\frac{1}{e^{q/T}-1}\,\sigma^{(n)}_{\rm Compton}(q^2, m_n^2)\, q$$

$$L^{(n)}(m_n^2) = \int_0^\infty \frac{dq^2}{\pi} R^{(n)}(q^2)\left|\frac{i}{q^2 - m_n^2 + iq\Gamma^{(n)}_{\rm tot}(q)}\right|^2 q\,\Gamma^{(n)}(q^2)$$

$$L = \int_0^\infty \frac{dq^2}{\pi} R(q^2)\,|\Delta(q; z_0, z_1)|^2\, q\,\Gamma_{\Phi\to\chi\bar\chi}(q^2)$$

IR opacity implies that for $\mu \ll T_{\rm star}$ the process vanishes because IR states are not reached anymore. Such behaviour differs from a simple 4d mediator.

Bound from red giants $(T \sim 10^8\,{\rm K})$: $L^{\rm an}_{\rm RG} < 100$ erg ${\rm s}^{-1}{\rm g}^{-1}$

Meson invisible decays: Same spirit