Representations of Lestrygonian Nichols algebra

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Workshop on Quantum Symmetries

Sketch

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- The natural numbers are denoted by $\mathbb{N}$ and $\mathbb{N}_0 = \mathbb{N} \cup 0$.
- If $k < t \in \mathbb{N}_0$, then $\mathbb{I}_{k,t} = \{n \in \mathbb{N}_0 : k \leq n \leq t\}$, and $\mathbb{I}_t := \{1, \dots, t\}$.

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Particularly, if V is an object in ${}^H_H\mathcal{YD}$ then $(V, c_{V,V})$ is a braided vector space.

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The Nichols algebra $\mathscr{B}(V)$ associates to (V, c) is given by

$$\mathscr{B}(V, c) = \Bbbk \oplus V \oplus_{n\geq 2} V^{\otimes n} / \ker \mathfrak{S}_n$$

where $\mathfrak{S}_n$ is the quantum symmetrizer.

Let Γ an abelian group. Consider $V = V_1 \oplus V_2 \in {}^{\Bbbk\Gamma}_{\Bbbk\Gamma}\mathcal{YD}$, where V_1 has a basis $x_1,\ x_2$ and V_2 has a basis x_3 such that the braid is given by:

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$$(c(x_i \otimes x_j))_{i,j\in\mathbb{I}_3} = \begin{pmatrix} \varepsilon x_1 \otimes x_1 & (\varepsilon x_2 + x_1) \otimes x_1 & q x_3 \otimes x_1 \\ \varepsilon x_1 \otimes x_2 & (\varepsilon x_2 + x_1) \otimes x_2 & q x_3 \otimes x_2 \\ q^{-1} x_1 \otimes x_3 & q^{-1}(x_2 + a x_1) \otimes x_3 & q_{22} x_3 \otimes x_3 \end{pmatrix}$$

where $a,\ q_{22},\ q,\ \varepsilon \in \Bbbk$.

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Braided vector space $\mathfrak{L}_q(1, \mathscr{G})$

If $\mathscr{G} := -2a \in \mathbb{N}$ and $q_{22} = \varepsilon = 1$ then the braided vector space above will be denoted by $\mathfrak{L}_q(1, \mathscr{G})$.

The Nichols algebra $\mathscr{B}(\mathfrak{L}_q(1,\mathscr{G}))$ was calculated in [1].

Generators and relations

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$$x_2x_1 - x_1x_2 + \frac{1}{2}x_1^2, \tag{1}$$

$$x_1 z_0 - q\, z_0 x_1, \tag{2}$$

$$z_n z_{n+1} - q^{-1} z_{n+1} z_n, \qquad 0 \leq n < \mathscr{G}. \tag{3}$$

$$x_2 z_n - q z_n x_2 - z_{n+1}, \qquad 0 \leq n < \mathscr{G}, \tag{4}$$

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$$B = \{x_1^{m_1} x_2^{m_2} z_{\mathscr{G}}^{n_{\mathscr{G}}} \dots z_1^{n_1} z_0^{n_0} : m_i, n_j \in \mathbb{N}_0\};$$

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We define for each $a \in \Bbbk^\times$ the following one-dimensional modules of $\Bbbk_q[X,Y]$:

$$\Bbbk_a^X = \Bbbk : \qquad X \cdot 1 = a, \quad Y \cdot 1 = 0$$

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- $\diamond$ $\dim V = 1 \Rightarrow V \simeq \Bbbk_a^X$ or $V \simeq \Bbbk_a^Y$.
- $\diamond$ $\dim V > 1 \Rightarrow \operatorname{ord} q =: N < \infty$ and $V \simeq \mathcal{U}_{a,b}$.

Simple modules of the $\mathscr{B}(\mathfrak{L}_q(1, \mathscr{G}))$

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$$\operatorname{Irrep} \mathscr{B}(\mathfrak{L}_q(1,\mathscr{G})) \simeq \operatorname{Irrep} \mathbb{k}_q[X, Y].$$

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Definition

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- V is ciclic and generated in degree 0, i. e., $V = A \cdot V^0$,
- V has Hilbert series $h_V(t) = 1/(1-t)$, in other words $\dim_{\Bbbk} V^n = 1$, for all $n \geq 0$.

We denote a point of the projective space $\mathbb{P}^n$ over $\Bbbk$ by $(a_0 : a_1 : \cdots : a_n)$.

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Point modules of free algebras

Let $A = \mathbb{k}\langle x_0, x_1, \ldots, x_n\rangle$ be the free associative algebra. The isomorphism classes of point modules over A are in bijective correspondence with points in the infinite product $\mathbb{P}^n \times \mathbb{P}^n \times \cdots = \prod_{i=0}^{\infty} \mathbb{P}^n$.

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$$V = \oplus_{j\in\mathbb{N}_0}\langle v_j\rangle \;\mapsto (a_{0,j} : \cdots : a_{n,j}), \text{ where } x_i v_j = a_{i,j} v_{j+1}.$$

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$$x_1 v_j = a_j v_{j+1}, \quad x_2 v_j = b_j v_{j+1}, \quad z_0 v_j = c_j v_{j+1}, \quad a_j, b_j, c_j \in \mathbb{k}.$$

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Then V is completely determined by $(P_0, P_1, \dots) \in \prod_{i=0}^{\infty}\mathbb{P}^2$, where $P_j = (a_j : b_j : c_j)$.

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ii) If $b_k = 0$ for some $k \in \mathbb{N}_0$ then $b_j = 0$ for all $j \in \mathbb{N}_0$ and V is determined by $P_j = (0 : 0 : 1)$.

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Theorem 2

The isomorphism classes of point modules over $\mathscr{B}(\mathfrak{L}_q(1, \mathscr{G}))$ are parametrized by $Y \cup Z \cup \{(0 \ : \ 0 \ : \ 1)\}$.

References

References

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