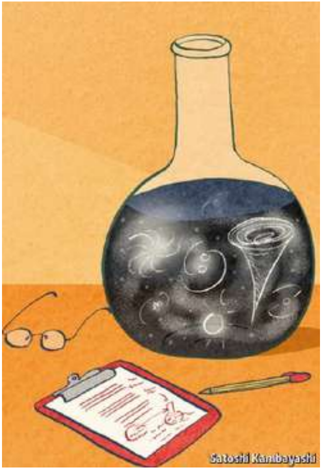
Analogue Gravity: an overview of recent experiments

Maurício Richartz (mauricio.richartz@ufabc.edu.br)
Universidade Federal do ABC (UFABC), Brasil

(Collaborators: S. Weinfurtner, A. Geelmuyden, ...)

November 21, 2019

Motivation



- Some processes in the Universe (specially when gravity and quantum physics are both important) are impossible (or extremely hard) to observe.
- Can we set up experiments to rebuild the Universe in the laboratory?
- Can we study fundamental processes (like Hawking radiation and cosmic inflation) using analogue classical or quantum simulators?

UFABC FAPESP

Experimental black hole evaporation?

PHYSICAL REVIEW LETTERS

VOLUME 46 | 25 MAY 1981 | NUMBER 21

Experimental Black-Hole Evaporation?

W. G. Unruh
Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada
(Received 8 December 1980)

It is shown that the same arguments which lead to black-hole evaporation also predict that a thermal spectrum of sound waves should be given out from the sonic horizon in transsonic fluid flow.

What are analogue models of gravity?

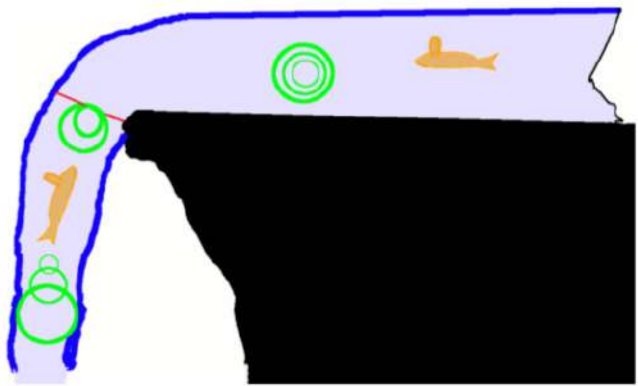
– Short answer: analogue models are systems that can mimick curved spacetime effects (both classical and quantum), like Hawking radiation and superradiance, leading naturally to the question: "Can we see black hole evaporation and black hole superradiance in a laboratory?"

$$\text{Analogue model} \leftrightarrow \text{Gravity}$$

– They were first proposed by Unruh in 1981. In 1980, however, Moncrief had already used similar ideas to study accretion processes in black holes. In 1993, unaware of Unruh's paper, Matt Visser rediscovered the analogy.

Simplest case: sound waves

Bill Unruh, when trying to explain for a broad audience what a black hole is, came up with the following pictorial analogy:



Ref.: Unruh (Found Phys, 2014)

In 1980, when teaching a Fluid Mechanics course, he realized how good his analogy was by showing that sound waves propagate as if they were in a curved spacetime. Using the basic equations for an inviscid, barotropic, and irrotational flow he showed that pressure perturbations (i.e. sound waves) propagate as a massless field in a curved spacetime.

In other words: velocity perturbations (which are related to a scalar field ψ) satisfy the Klein-Gordon equation in a curved spacetime whose metric is

$$ds^2 \propto -\left(c^2 - v^2\right) dt^2 + 2v dr dt + r^2 d\Omega^2,$$

where v is the velocity of the fluid and c denotes the speed of sound.

Analogue black holes

Compare the acoustic metric

$$ds^2 \propto -\left(c^2 - v^2\right) dt^2 + 2v dr dt + dr^2 + r^2 d\Omega^2.$$

with the Schwarzschild black hole metric in the Painlevé-Gullstrand coordinates:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 \pm 2\sqrt{\frac{2M}{r}} dr dt + dr^2 + r^2 d\Omega^2.$$

Recall that the coordinate singularity $r = 2M$ is the event horizon of a black hole (or of a white hole). Following the same steps, one can show that the points where $|v| = c$ represent an analogue event horizon.

Following Hawking's derivation of black hole evaporation step-by-step, one is able to conclude that acoustic black holes should emit Hawking-like radiation with a temperature of

$$T = \frac{\hbar}{2\pi c k_B} \frac{\partial}{\partial r} \left(c^2 - v^2\right)\Big|_{|v|=c}$$

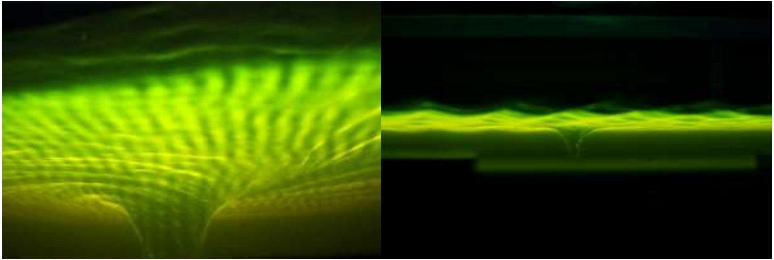
There is, however, a problem with Hawking's original derivation: when traced back in time, outgoing Hawking radiation modes have infinitely high frequencies (which is inconsistent with the semiclassical approximation). The first success of analogue models was to provide a possible solution to this problem.

Recent experiments (Nottingham)

UFABC FAPESP

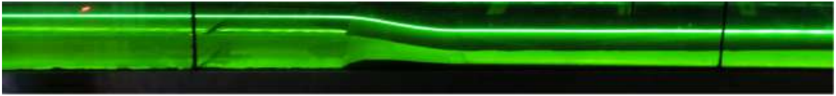
Recent experiments in Nottingham

- Superradiant scattering
- Characteristic oscillations
- Hawking radiation (ongoing)



Recent experiments in Nottingham

Based on the propagation of surface waves on the air-water interface of a rectangular tank.



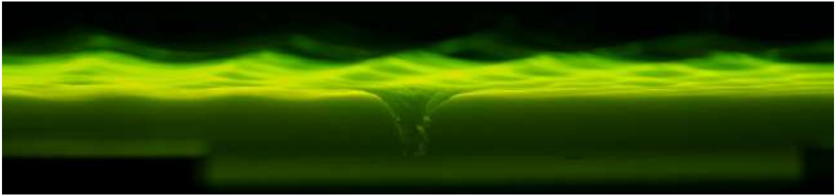
Ref.: Euvé et al (1806.05539, 2018)

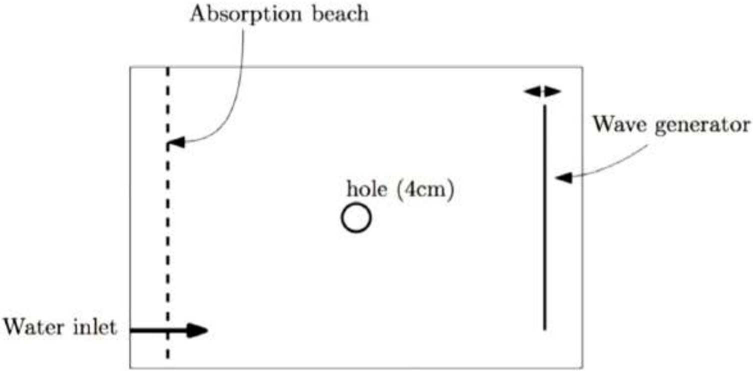
The main advantage of using surface waves (in comparison with sound waves) is that the propagation speed of the waves can be easily controlled by changing the depth of the fluid since $c_{\mathrm{waves}} = \sqrt{gh}$. These velocities can be made much smaller than the sound speed in water ($\sim$ 1400 m/s), thus requiring slower flows to set up an event horizon.

UFABC FAPESP

Recent experiments in Nottingham

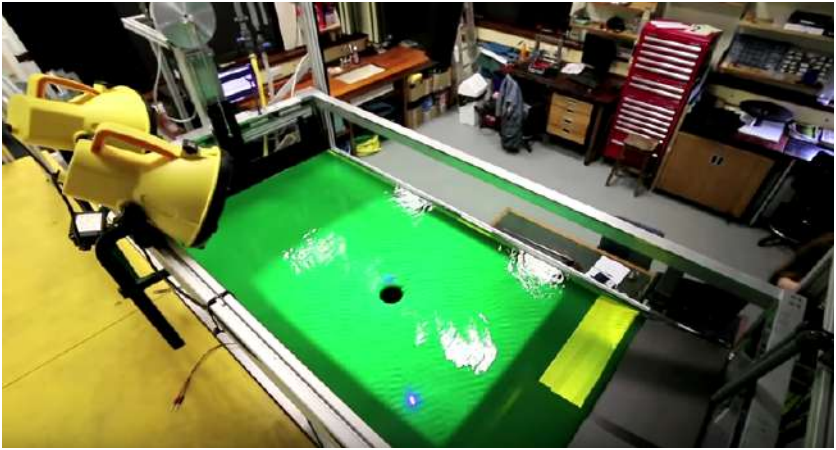
Set up an analogue rotating black hole





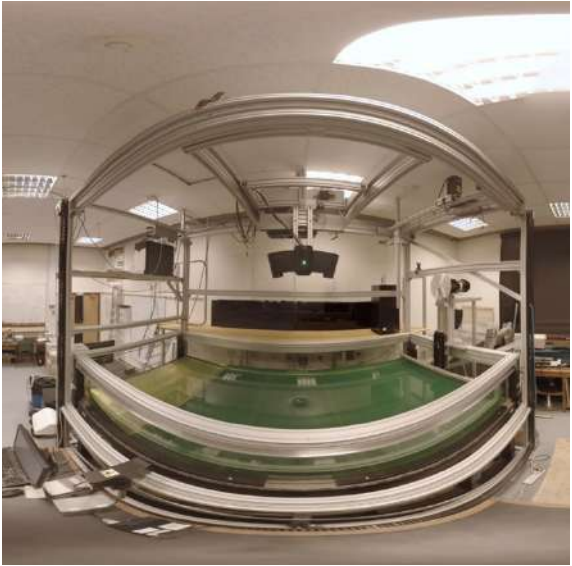
- Water tank: $\sim$ 1,5 m $\times$ 3,0 m.
- Closed circuit, water pumped from auxilliary tank (below main tank). Stationary rotating flow obtained.
- Wave generator on one side, absorption beach on another.
- Cameras and detector placed above the tank (close to the ceiling).

Recent experiments in Nottingham



UFABC FAPESP

Recent experiments in Nottingham



Background flow

- Flow: constant density, inviscid, irrotational, and axisymmetric.

$$\nabla \cdot \vec{v} = 0, \quad \rho(\partial_t + \vec{v} \cdot \nabla)\vec{v} = -\nabla P, \quad \nabla \times \vec{v} = 0$$

- Free boundary problem: depth h is unknown a priori.

- The simplest solution is the **Irrotational vortex ("Draining bathtub"):** $\vec{v} = -\frac{D}{r}\hat{\mathbf{r}} + \frac{C}{r}\hat{\phi}$

- If surface waves propagate with velocity c(r), an analogue event horizon will be present where $c(r) = D/r$.

Wave propagation

- Surface waves with frequency ω and wavenumber k satisfy the dispersion relation

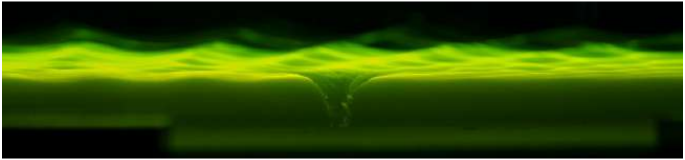
$$\left(\omega - \vec{v}\cdot\vec{k}\right)^2 = \left(gk + \frac{\sigma}{\rho}k^3\right)\tanh(kh).$$

- For long wavelengths ($kh << 1$), no surface tension ($\sigma = 0$) and zero flow ($\vec{v} = 0$), the dispersion relation is linear $\omega = \sqrt{gh}k$, and we can see that $c = \sqrt{gh}$ is the wave speed.

Analogy (linear dispersion):

$$(\partial_t + \vec{v}\cdot\nabla)^2\,\delta h = gh\nabla^2\delta h \;\leftrightarrow\; \partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\partial\psi\right) = 0$$

Detection methods



How do we measure the flow velocity?

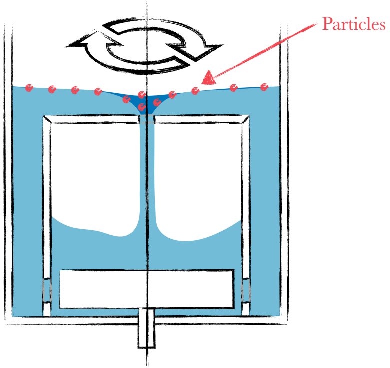
– Particle Image Velocimetry (PIV)

How do we measure the height perturbations (i.e. waves on the surface)?

– Depending on the experiment, we can use two different methods: "Enshape sensor" and Fast Checkerboard Demodulation (FCD).

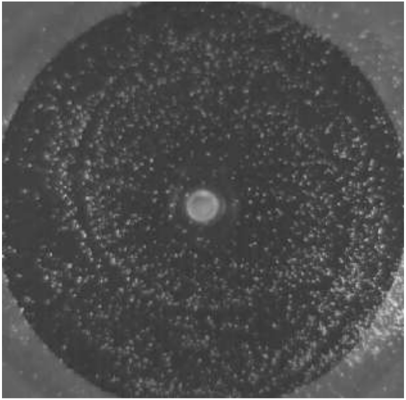
The velocity field can be measured by monitoring particles on the surface

Particle Image Velocimetry (PIV)

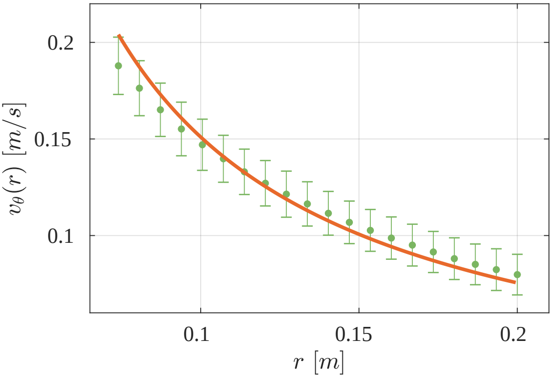


The velocity field can be measured by monitoring particles on the surface

Particle Image Velocimetry (PIV)

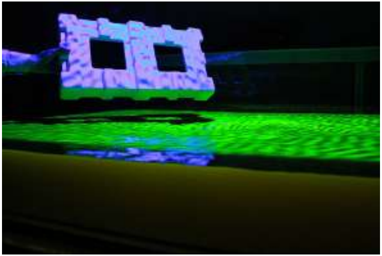


E.g. of background flow (from PIV)

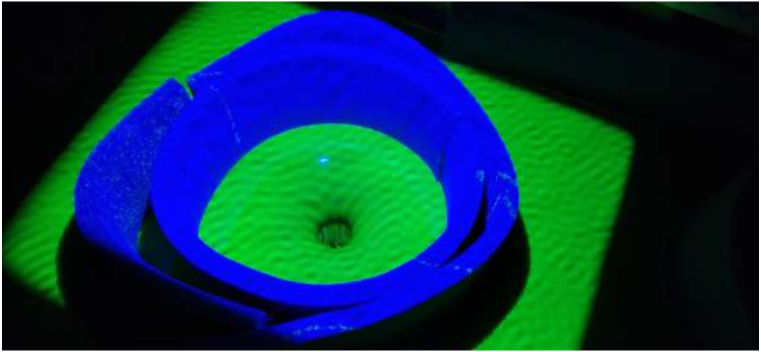


Green dots correpond to the averaged (over time, angle and experiments) angular velocity.

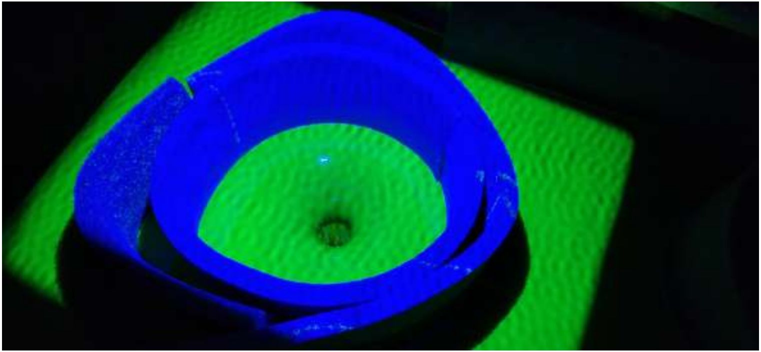
Detection methods: Enshape



Detection methods: Enshape

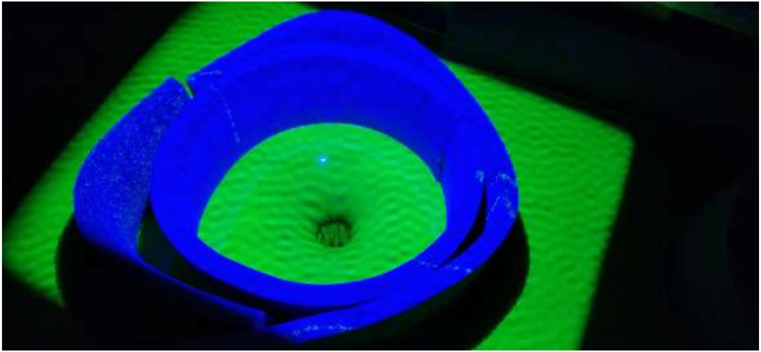


Detection methods: Enshape



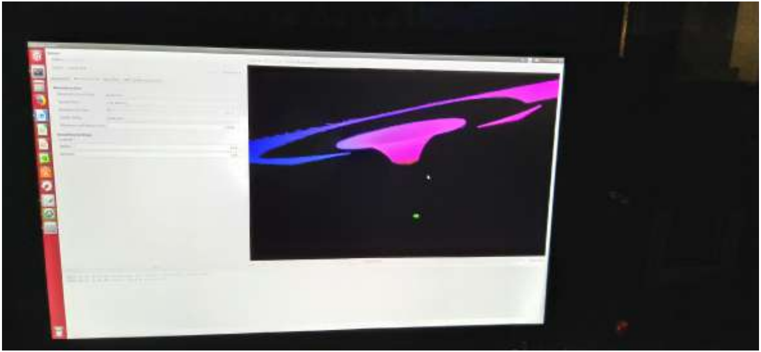
UFABC FAPESP

Detection methods: Enshape



Detection methods: Enshape

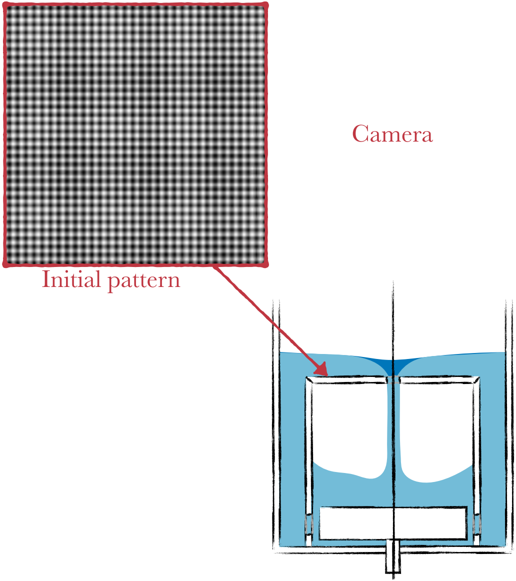
Reconstructed vortex:



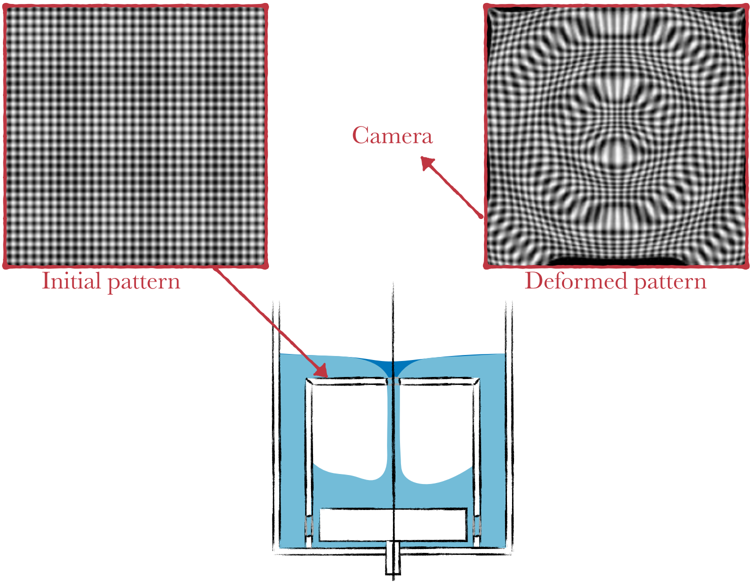
Detection methods: FCD

A periodic pattern composed of two orthogonal sinusoidal waves is placed at the bottom of the tank. Deformations of this pattern due to free surface fluctuations are recorded using a high speed camera (usually at a frame rate of 40 fps–200 fps) for a pre-defined amount of time (usually 10s–30s). For each of the pictures of the deformed pattern, we reconstruct the free surface in the form of an array $h(t_k, x_i, y_j)$ giving the height of the water at the 1600×1600 points on the free surface (x_i, y_j) at every time step t_k. We choose the centre of our coordinate system to be the centre of the hole and convert the signal from cartesian to polar coordinates. We end up with $h(t_k, r_i, \theta_j)$.

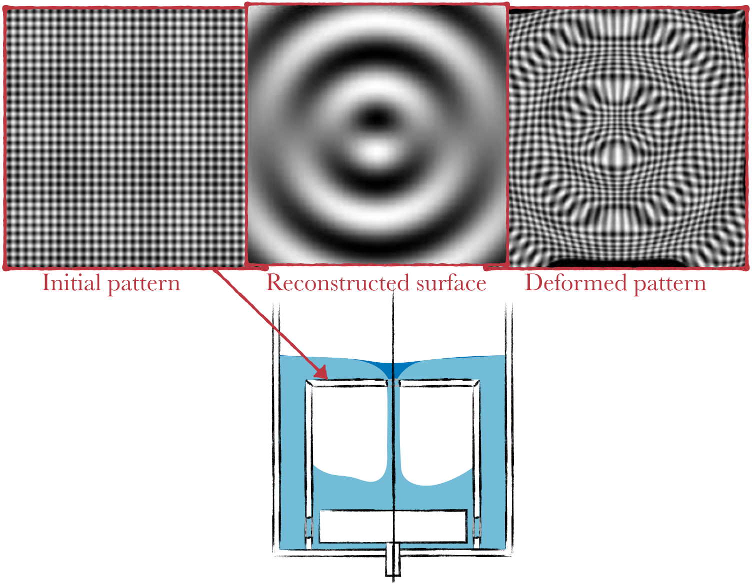
The Fast Checkerboard Demodulation (FCD) method



The Fast Checkerboard Demodulation (FCD) method



The Fast Checkerboard Demodulation (FCD) method



Experiment 1: superradiance

Superradiance is a special type of scattering in which the reflected wave has larger amplitude than the incident wave. We call it rotational superradiance when the energy extracted is rotational energy.

Occurs when $0 < \omega < m\Omega$. Was discovered by Zel'dovich (1971). **Only waves corotating with the cylinder (**$m > 0$**) are amplified**.

Experiment 1: superradiance

Zel'dovich (1971)

The foregoing pertains to a body made of a material that absorbs waves when at rest; the conditions for amplification and generation are obtained after transforming the equations to the moving system. A similar situation can apparently arise also when considering a rotating body in the state of gravitational relativistic collapse.

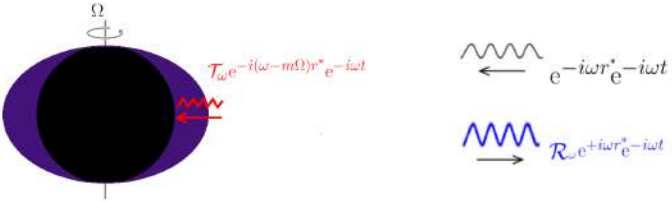
The metric near such a body is described by the well-known Kerr solution. The gravitational capture of the particles and the waves by the so-called trapping surface replaces absorption; the trapping surface ("the horizon of events") is located inside the surface $g_{00} = 0$. Finally, in a quantum analysis of the wave field one should expect spontaneous radiation of energy and momentum by the rotating body. The effect, however, is negligibly small, less than $\hbar\omega^4/c^3$ for power and $\hbar\omega^3/c^3$ for the decelerating moment of the force (for a rest mass m = 0, in addition, we have omitted the dimensionless function β).

ZhETF Pis. Red. 14, No. 4, 270 - 272 (20 August 1971)

Experiment 1: superradiance

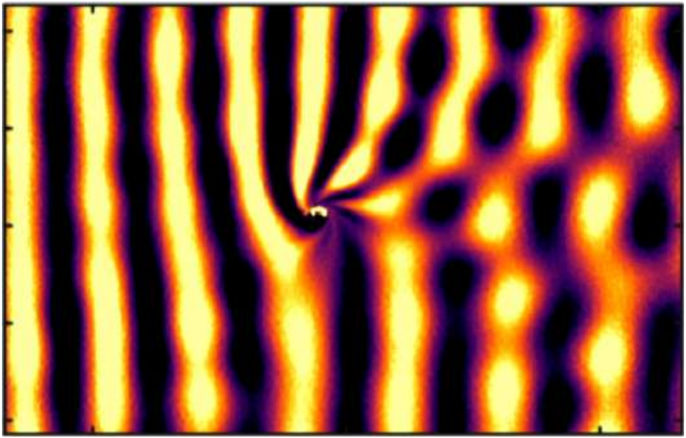
Solve the wave equation near the event horizon and far away from it. Impose the appropriate boundary condition at the event horizon. Calculate the reflection coefficient:

$$|\mathcal{R}_\omega|^2 = 1 - \left(\frac{\omega - m\Omega}{\omega}\right)|\mathcal{T}_\omega|^2$$



No evidence/measurements of superradiance in Astrophysics. Can we see the effect in analogue models? Basak, Majumdar (2002).

Experiment 1: superradiance



- Plane wave (created by the wave generator) propagates and scatters on the vortex. We work in the frequency range 2,9 Hz - 4,1 Hz. The fluid is 6.25 cm deep.
- The observed pattern is a mixture: incident + scattered waves.

Data analysis for superradiance

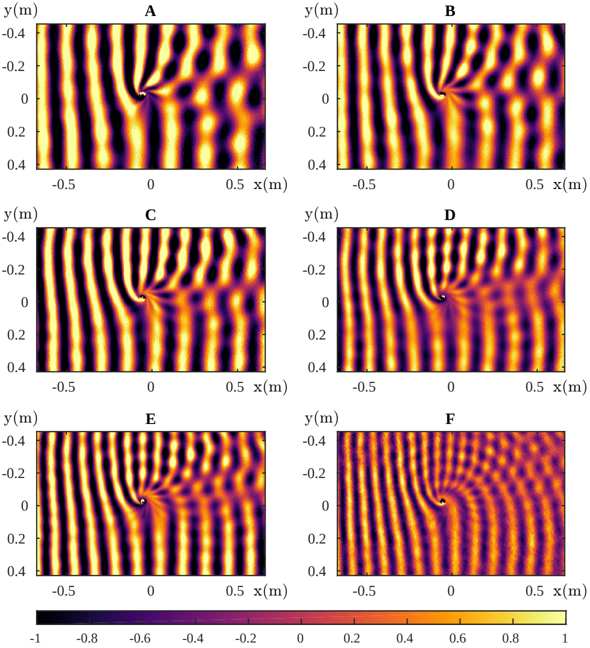
- The data acquired by the detector gives us the surface of the fluid as a function of time and position. We fourier transform the signal in time and filter out the excitation frequency f_0. Higher harmonics ($2f_0$, $3f_0$, ...) and background flow ($f = 0$) are eliminated.

- We decompose the signal into azimuthal waves:

$$\varphi_m(r) = \frac{\sqrt{r}}{2\pi} \int_0^{2\pi} \phi(r, \theta) e^{im\theta} d\theta$$

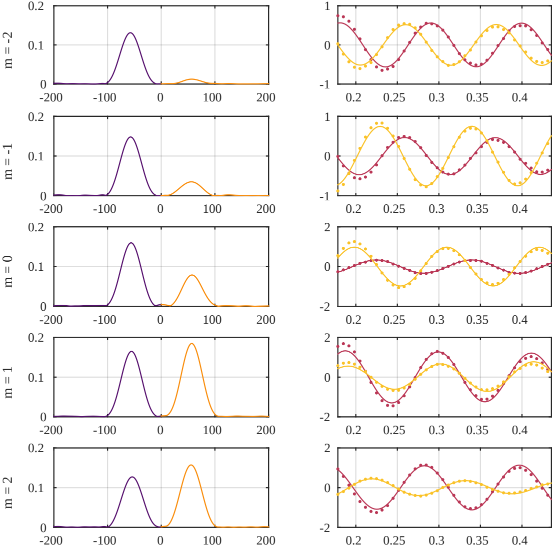
  For each m, we write radial profile as: $\varphi_m = A_m^{in} e^{-ikr} + A_m^{out} e^{+ikr}$

- By comparing the ingoing and outgoing parts, we extract the reflection coefficients.



A
B
C
D
E
F
y(m)
x(m)
-0.4
-0.2
0
0.2
0.4
-0.5
0
0.5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Radial profiles



Right side

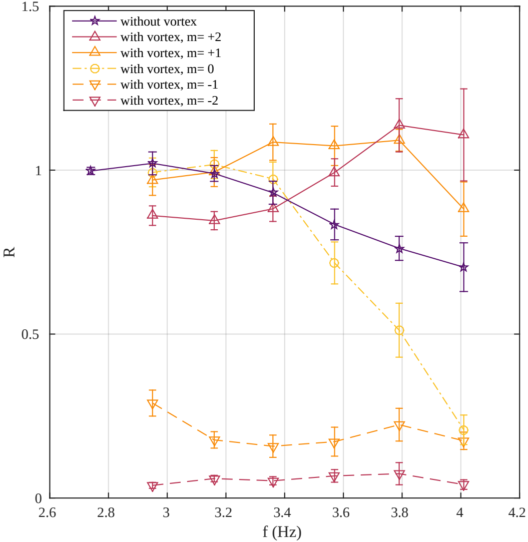
- Dots: signal
- Lines: fit to

$$A_{\text{in}} e^{-ikr} + A_{out} e^{ikr}$$

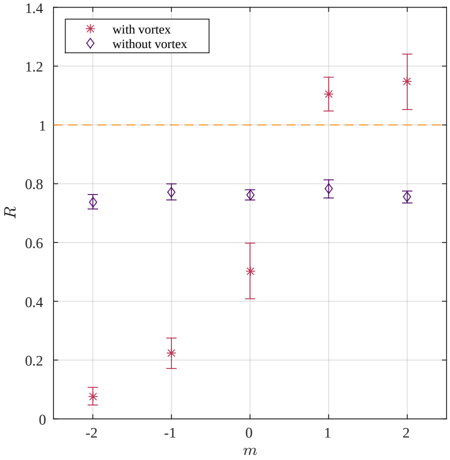
Left side

- Fourier profiles

Observed spectrum



Reflection coefficient for $f = 3{,}7$ Hz



Vortex flow:

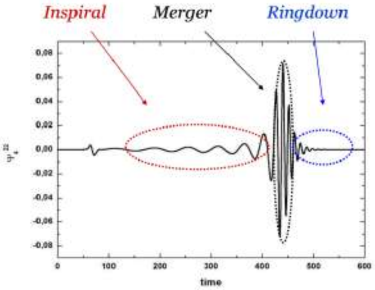
- $m \leq 0$ modes are absorbed ($|R| < 1$);
- $m = 1, 2$ modes are amplified ($|R| > 1$).
- Amplification for $m = 2$ modes was $14\% \pm 8\%$: **superradiance**, arXiv:1612.06180

Without vortex:

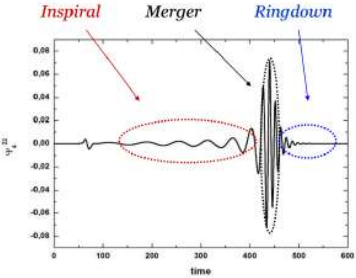
- No scattering: $|R| < 1$.

Experiment 2: characteristic modes

- Rotating BHs are simple objects (in the sense that they depend only on two parameters: mass M and angular momentum J).
- Once perturbed, a BH will relax through the emission of characteristic waves which are independent of the initial perturbation.



Experiment 2: characteristic modes



– BH spectroscopy: to identify a black hole by observing the spectrum of its characteristic waves (their oscillation frequencies and decay times).

– Vortex spectroscopy: can we identify a hydrodynamical vortex by observing the spectrum of its characteristic waves?

Theoretical predictions for the characteristic modes

- The characteristic spectrum can be approximated using the properties of lightrings (LRs):

$$\omega_{\mathrm{QNM}}(m) \approx \omega_\star(m) - i\Lambda(m)\left(n + \frac{1}{2}\right),$$

  where $\omega_\star(m) = 2\pi f_\star(m)$ is the angular frequency of an m-mode orbiting on the LR, $\Lambda(m)$ is the *Lyapunov exponent* of the orbit for this specific m-mode, and n is the overtone number.

- The LR properties can be deduced from the hamiltonian

$$\mathcal{H} = \omega - \vec{v}\cdot\vec{k} = \pm\sqrt{\left(gk + \frac{\sigma}{\rho}k^3\right)\tanh(kh)}$$

  by looking for critical points: $\mathcal{H} = 0$, $\frac{\partial\mathcal{H}}{\partial r} = 0$, $\frac{\partial\mathcal{H}}{\partial k_r} = 0$.

Experiment 2: characteristic modes

- We set up a vortex flow out of equilibrium to observe the emission of characteristic modes during its relaxation. Our experiment was conducted in a 3 m long and 1.5 m wide rectangular tank with a 2 cm-radius sink hole at the centre. Water is pumped continuously from one corner at a flow rate of 15 ± 1 ℓ/min.

- The sink-hole is covered until the water raises to a height of 10.00 ± 0.05 cm. Water is then allowed to drain, leading to the formation of a vortex. We recorded the perturbations of the free surface when the flow was in a quasi-stationary state at a water depth of 5.55 ± 0.05 cm. The entire procedure was repeated 25 times.

- The data acquired by the detector gives us the surface of the fluid as a function of time and position.

Data Analysis

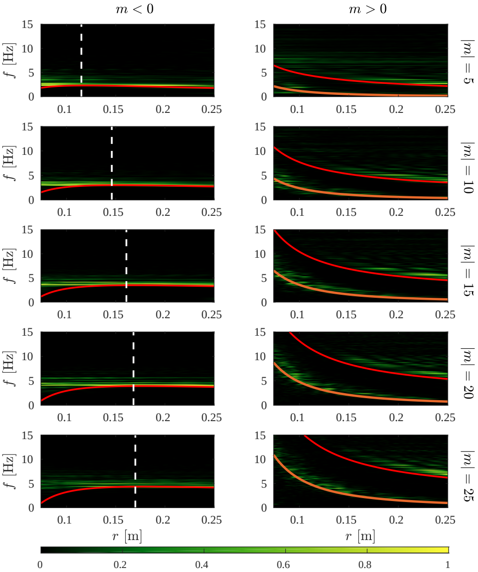
- The resulting vortex is axisymmetric to a good approximation, allowing us to perform an azimuthal decomposition to study the characteristic modes:

$$\delta h(t, r, \theta) = \mathrm{Re}\left[\sum_{m\in\mathbb{Z}} \delta h_m(t, r) e^{im\theta}\right]$$

- We select specific azimuthal modes by performing a polar Fourier transform and we extract the associated radial profiles $\delta h_m(t, r)$. Azimuthal modes with $m > 0$ are co-rotating with the flow while modes with $m < 0$ are counter-rotating with the flow.

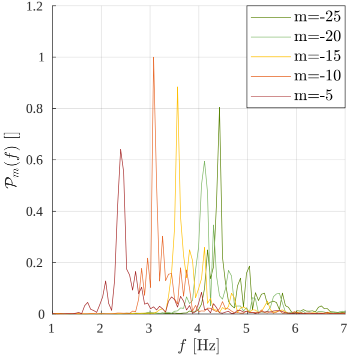
- By calculating the time Fourier transform of $\delta h_m(t, r)$, we estimate the *Power Spectral Density* (PSD) of each m-mode for $r \in [7.4\ \mathrm{cm}, 25\ \mathrm{cm}]$.

PSDs in a single experiment

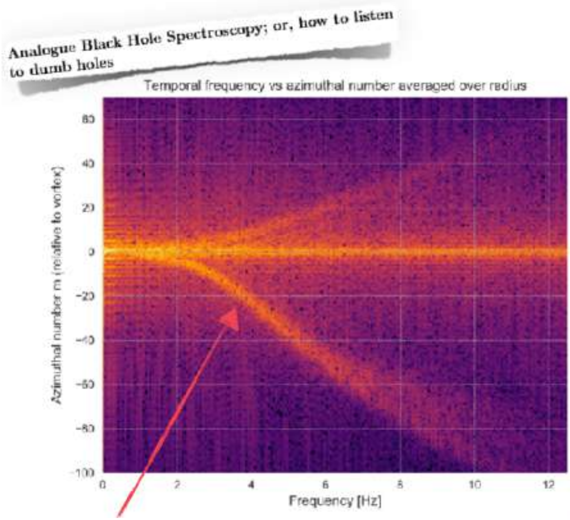


Averaged PSDs

The PSDs are finally averaged over the radius in order to look at the r-independent frequency content, i.e. the oscillation frequency of the LR modes. For each averaged PSD, corresponding to a different m, the location of the peak, $f_{\mathrm{peak}}(m)$, is obtained.

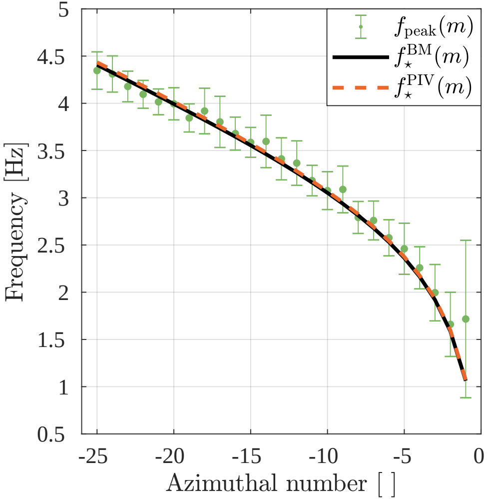


Characteristic spectrum

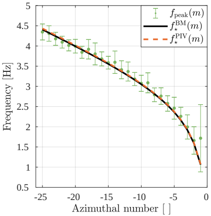


UFABC FAPESP

Characteristic spectrum

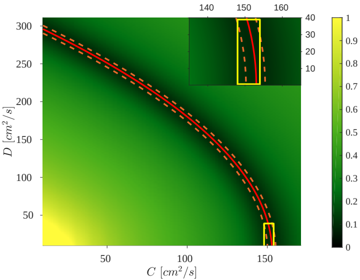


UFABC FAPESP



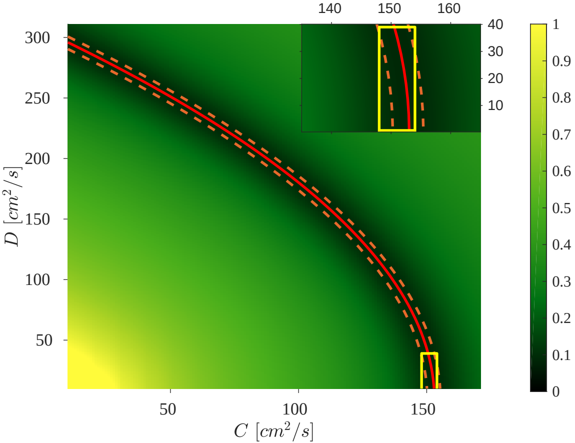
The frequency spectrum $f_{\rm peak}(m)$, extracted from the experimental data and represented by green dots, is compared with the theoretical prediction for the light-ring frequencies, $f_\star(m)$. The error bars indicate the standard deviation over 25 experiments. The dashed orange curve is the predicted spectrum, $f_\star^{\rm PIV}(m)$, computed for $\rm C = 151\ cm^2/s$ and $\rm D = 0\ cm^2/s$. These flow parameters were obtained via an independent flow measurement, in our case Particle Imaging Velocimetry (PIV). The two spectra agree, confirming the detection of light-ring mode oscillations. The solid black curve, $f_\star^{\rm BM}(m)$, is the non-linear regression of the experimental data to the draining bathtub vortex model - **arXiv:1811.07858 and 1905.00356**.

Vortex spectroscopy



The intensity of the background image represents the normalised weighted sum of squared residuals between the experimental spectrum, $f_{\text{peak}}(m)$, and the theoretical prediction for the light-ring frequencies, $f_\star(m)$, as a function of the flow parameters. The red curve represents the family of possible values for C and D that best match the experimental data.

Vortex spectroscopy



Conclusions

- Analogue models of gravity have been around for more than 35 years, but only in the past 10 years people have started doing experiments with them. Most experiments are based on 1D systems, so we want to improve and explore more possibilities in 2D systems.
- We have been able to observe superradiant scattering and characteristic oscillations of a vortex.
- Next steps: match superradiance data with theoretical prediction, observe the characteristic decay times of the vortex, better visualization of the core, measurement of Hawking radiation.