

# Whispers from space

## Remarkable implications of the double neutron star merger GW170817

John Friedman



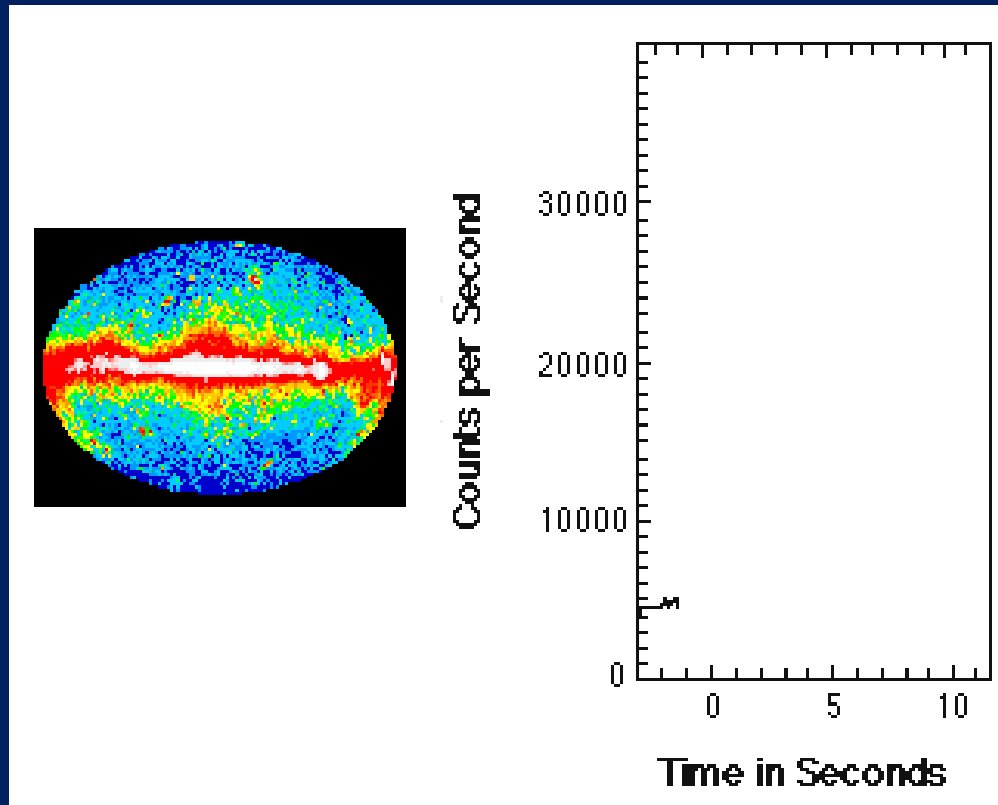
Leonard E. Parker Center for  
Gravitation, Cosmology, and Astrophysics

- I. Short  $\gamma$ -ray bursts and NS-NS mergers
- II. r-process elements
- III. The equation of state (EOS) above nuclear density –the NS EOS.
- IV. Tidal imprint on inspiral waveform
- V. Implications of post-merger observations
- VI. Hubble constant

Downloaded animations that were embedded in the powerpoint presentation are replaced in this pdf file by links to their locations on the web

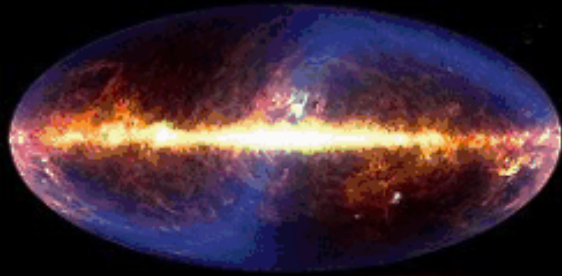
# I. $\gamma$ -ray bursts and NS-NS mergers

In late 60s, the Vela satellites monitoring the recently signed nuclear test-ban treaty, saw occasional bursts of  $\gamma$ -rays, and their direction ruled out Sun and Earth as sources.

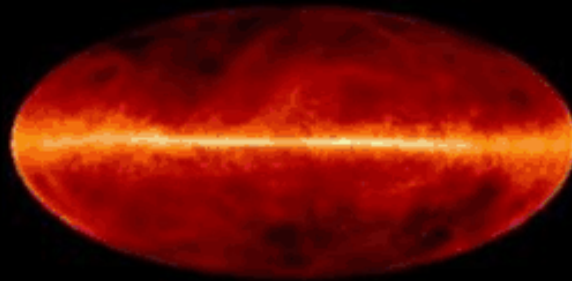


Pogosyan slide

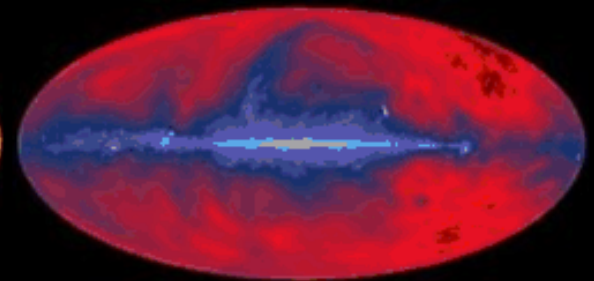
Had they come from the Milky Way,  
their distribution would look like this



Infrared (DIRBE Team, COBE, NASA)



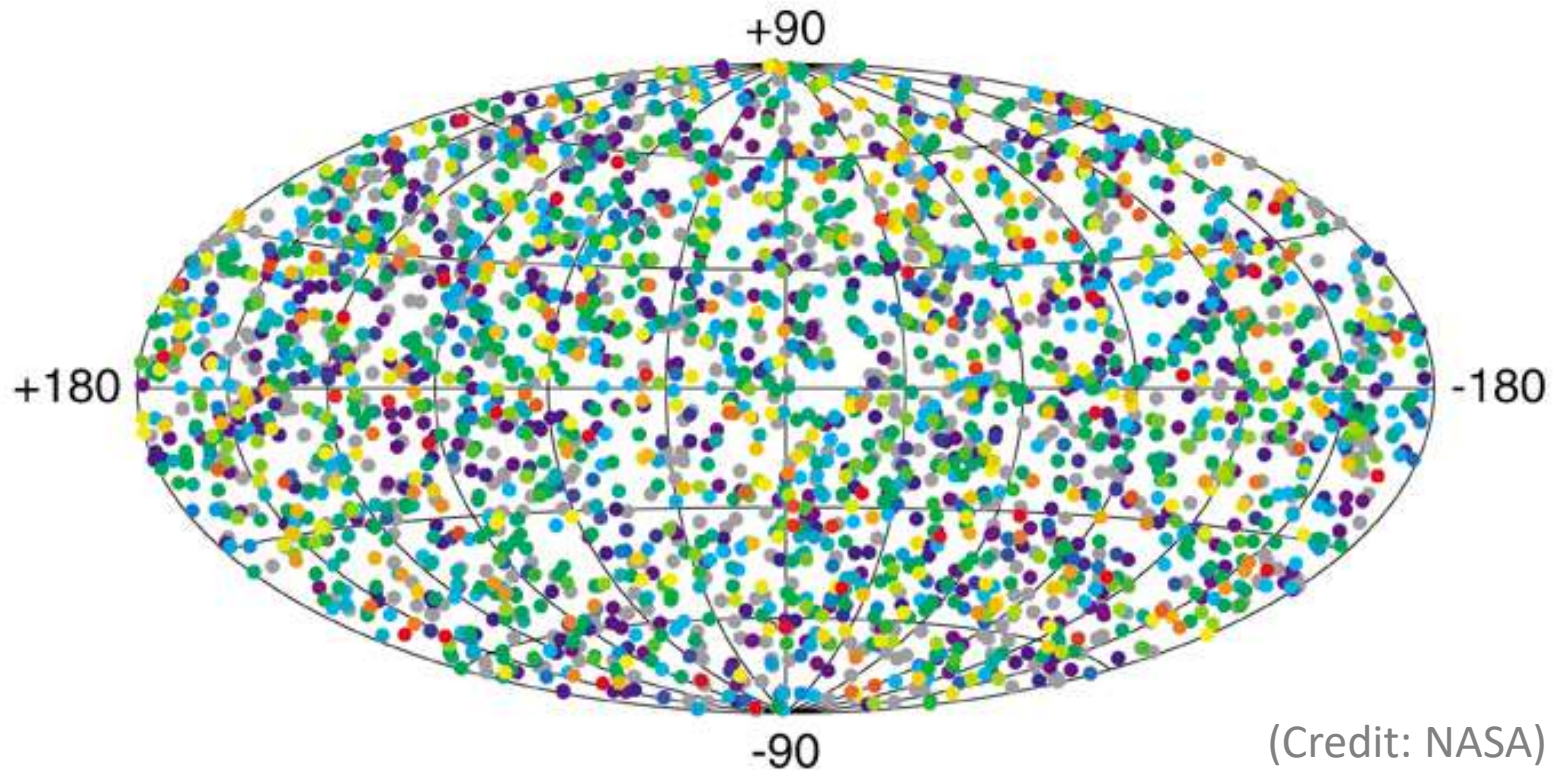
Radio 1420MHz (J. Dickey et.al. UMn.  
NRAO SkyView)



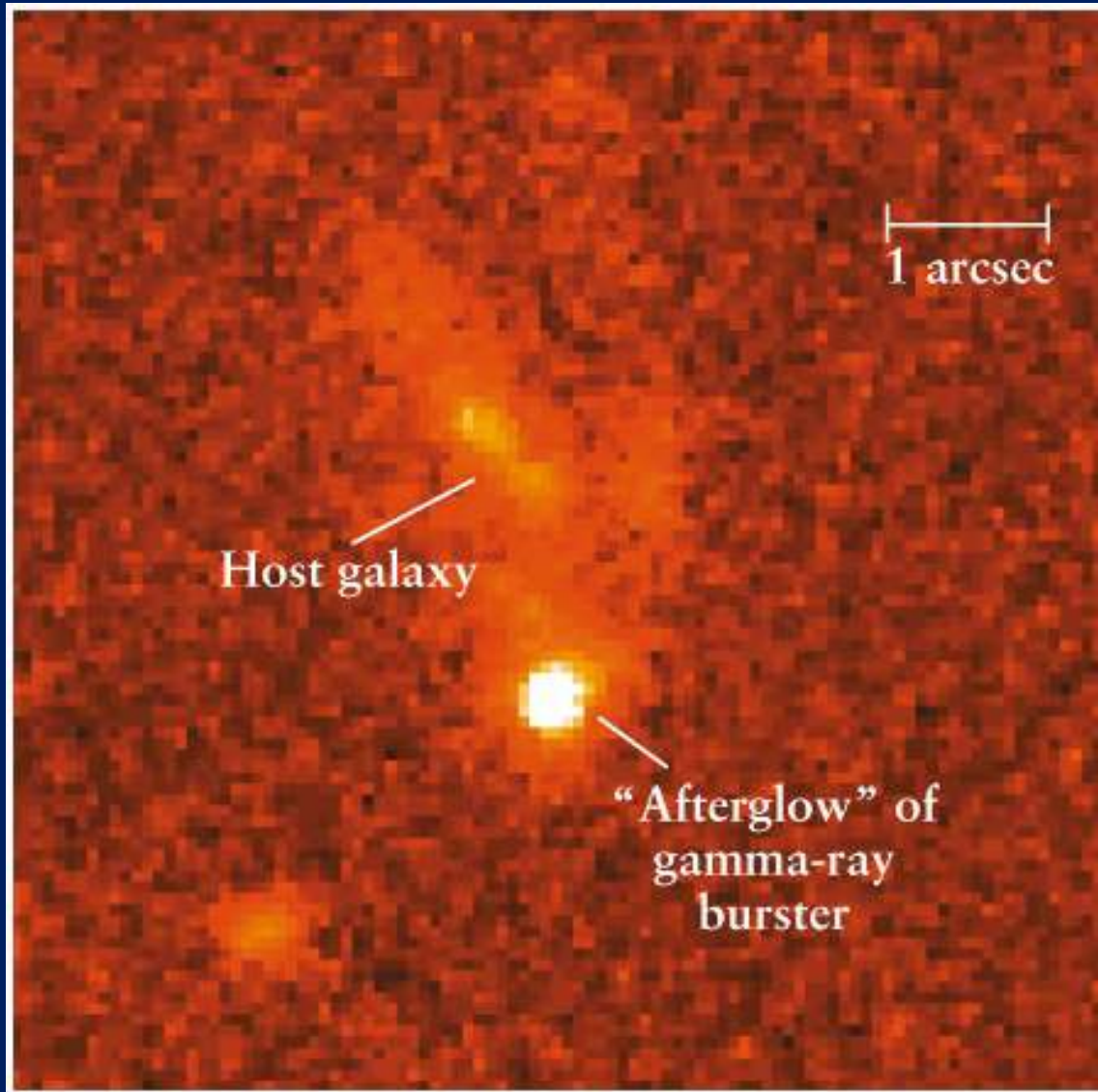
Radio 408MHz (C. Haslam et al., MPIfR,  
SkyView)

Instead, they are distributed isotropically across the sky

## 2704 BATSE Gamma-Ray Bursts



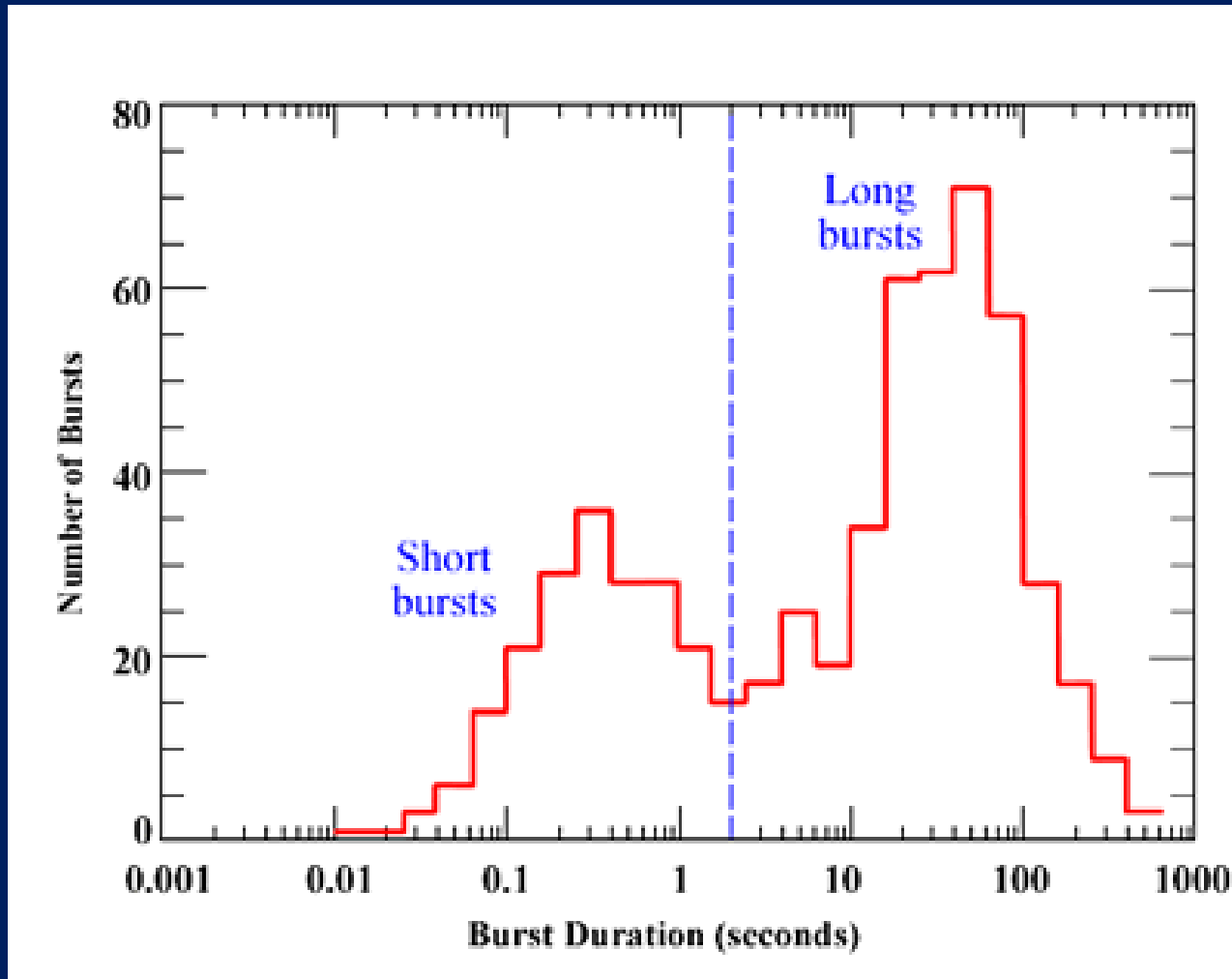
# Cosmological distance confirmed when found host galaxies



implying these are the brightest  
electromagnetically observed events in the  
universe, up to  $10^{54}$  erg,  
1000 times the energy emitted by the Sun  
over its 10-billion year lifetime

# Two classes

Short  $0.1 - 1$  s    Long  $> 1$  s





Long gamma-ray bursts seen only in galaxies that have recently formed stars. This is consistent with their identification with the most luminous supernovae:

Only stars with large mass end as supernovae, and large mass stars burn their fuel quickly, living for less than 50 million years. So supernovae occur in star-forming galaxies.

Short gamma-ray bursts are generally associated with galaxies that have *no young stars or with no obvious galaxy at all.*

They are events with energy comparable to the energy of collapse to a NS, where no stars massive enough to collapse are present. That fact made binary coalescence of dead stars – of NS-NS or NS-BH binaries – the leading candidate.

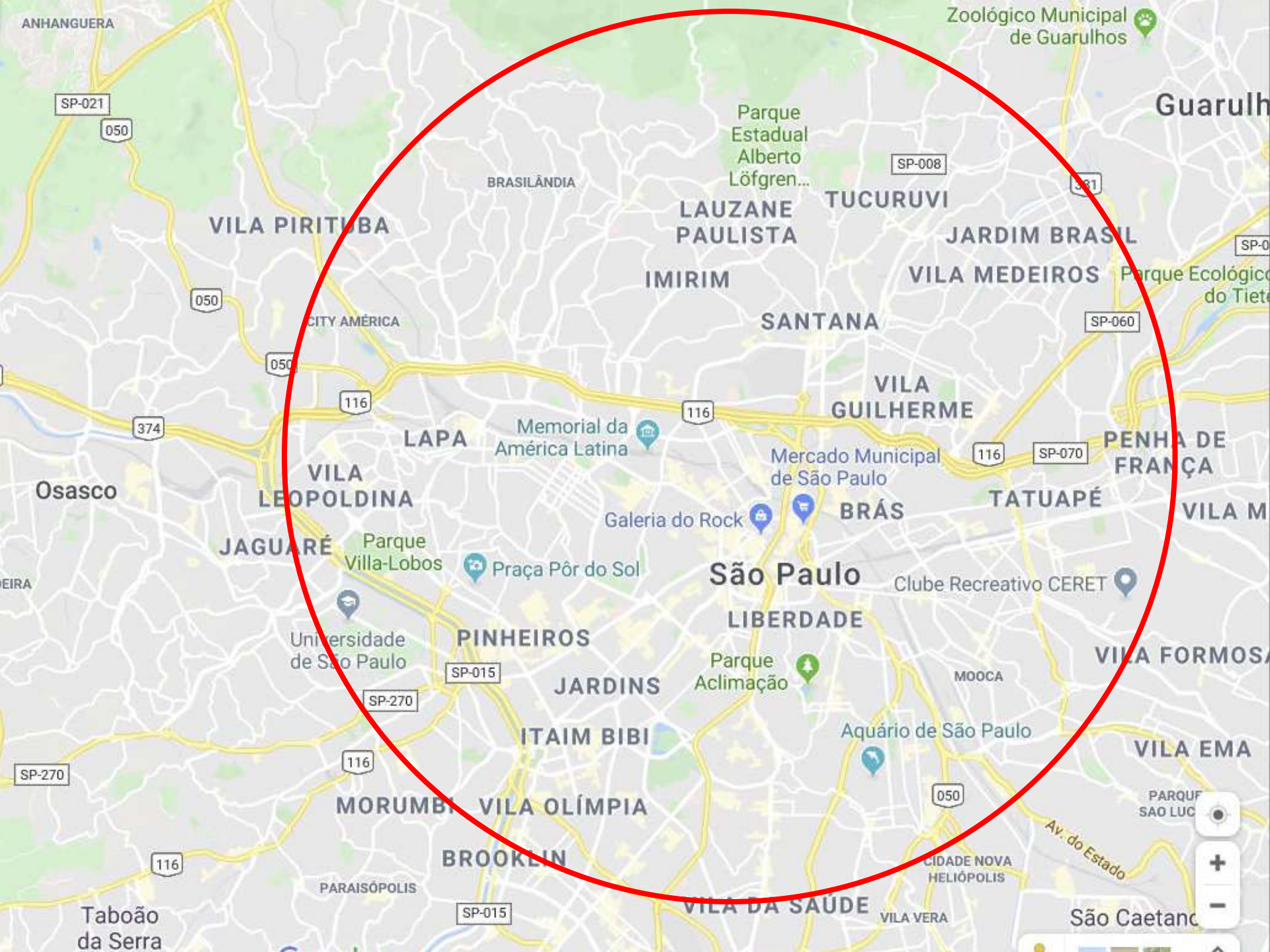
## Radius of a neutron star

neutron star of mass  $1.4 M_{\odot} = 2.8 \times 10^{33} \text{g}$

nuclear density  $\rho_n = \frac{M}{(4\pi/3) R^3} = 2.3 \times 10^{14} \text{g/cm}^3$

$$R = \left( \frac{1.4 M_{\odot}}{4\pi/3 \rho_n} \right)^{1/3} = 14 \text{ km}$$

Actual: average density somewhat  
higher than nuclear density;  
radius between 9 and 14 km



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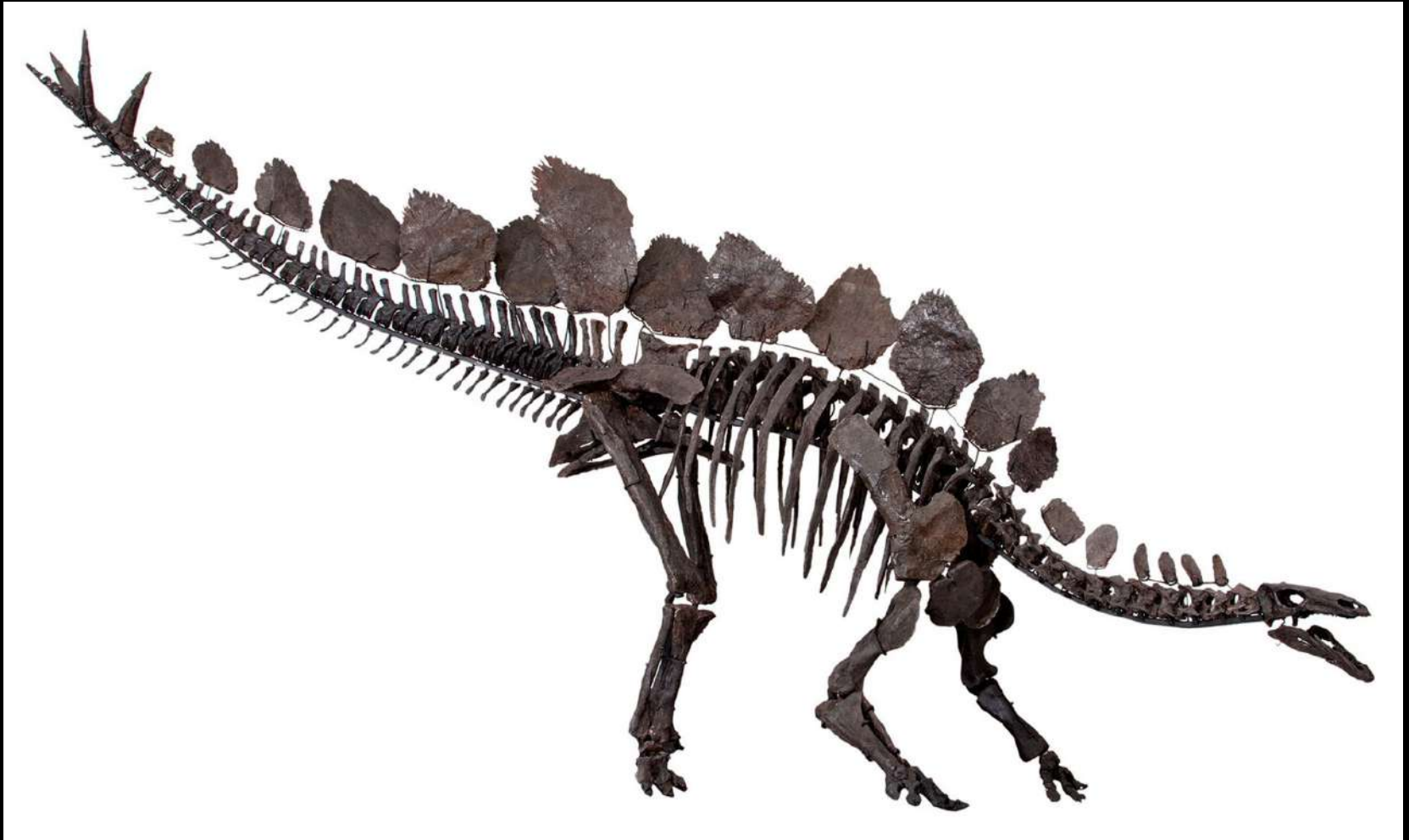
São Caetano

Energy from collapse to neutron star or collision of two neutron stars

$$\frac{GM^2}{R_{\text{neutron star}}} = \frac{G(1.4M_{\odot})^2}{(11\text{km})^2} = 4 \times 10^{53} \text{ erg}$$

Energy of  $\gamma$ -ray bursts =  $10^{50}$  to  $10^{54}$  erg in light,  
rest in neutrinos and gravitational waves

# 130 million years ago



Susannah Maidment et al. & Natural History Museum, London - Maidment SCR, Brassey C, Barrett PM (2015) <https://commons.wikimedia.org/w/index.php?curid=45275061>

in a galaxy far away



Neutron star inspiral and merger by Dana Berry, produced by Erica Drezek:

[https://www.youtube.com/watch?v=tjd\\_eyHe6PQ](https://www.youtube.com/watch?v=tjd_eyHe6PQ)

GRMHD simulation by Ruiz, Tsokaros, Paschalidis, Shapiro at [https://www.youtube.com/embed/d\\_Q7Yr5U\\_g?rel=0](https://www.youtube.com/embed/d_Q7Yr5U_g?rel=0)





# FERMI



Fermi and LIGO/Virgo signals from  
GRB 170817A and  
GW 170817A at  
<https://svs.gsfc.nasa.gov/12740>

# LIGO



Credit: NASA's Goddard Space Flight Center, Caltech/MIT/LIGO Lab

The gravitational waves match the waveform from two neutron stars, each with mass approximately  $1 \frac{1}{2} M_{\text{sun}}$

# From the glow hours after the collision



Discovery Image  
August 17, 2017

# a galaxy was identified



NGC4993

distance

130 million light-years

# Speed of gravitational waves

Time from end of GW chirp to  $\gamma$ -ray burst  $< 1.8$  s  
( $1.74 \pm .05$  s)

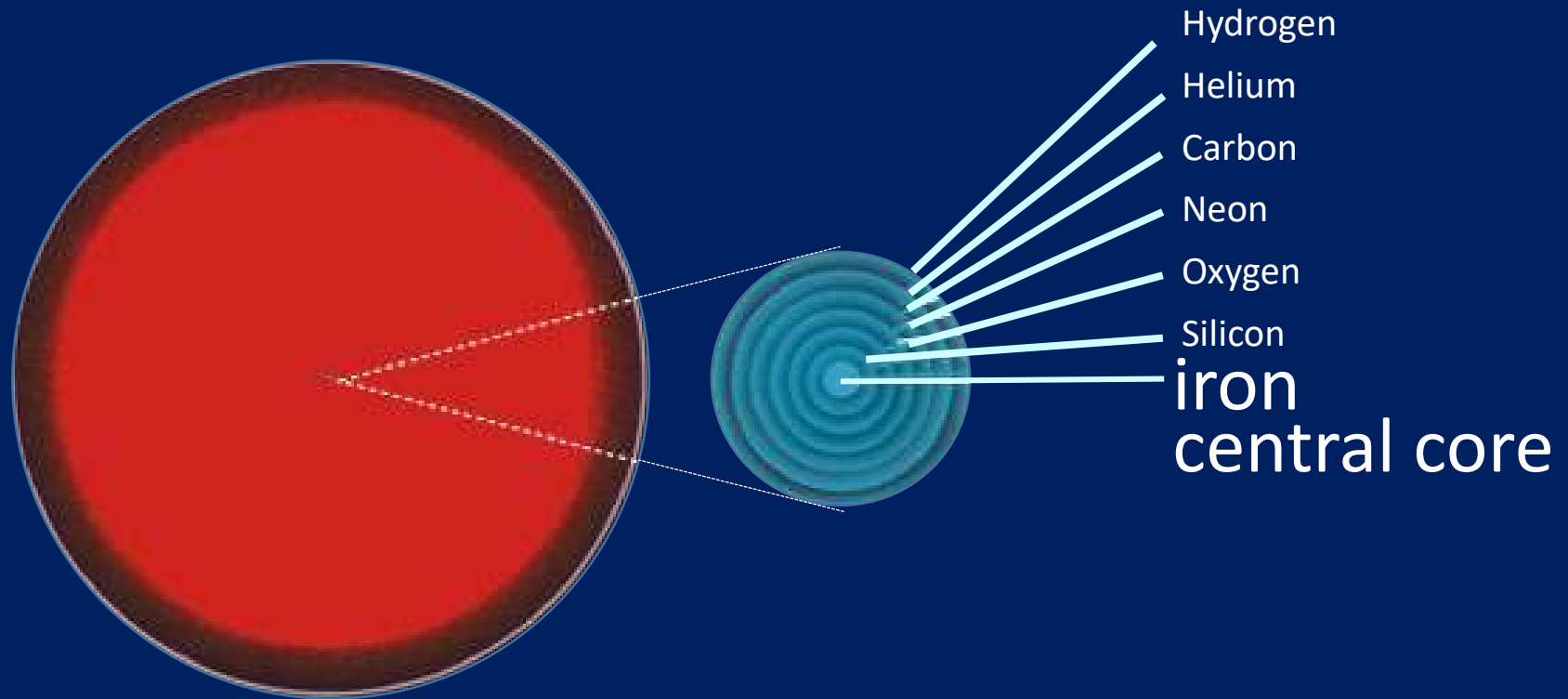
Because burst launched after merger - after end of chirp, if GW slower than light

$$(c - v_{\text{GW}})/c < 1.8 \text{ s} / 130 \text{ Myr} < 5 \times 10^{-16}$$

If light slower and emitted (conservatively) up to 10 s after burst,  $(v_{\text{GW}} - c)/c < 3 \times 10^{-15}$

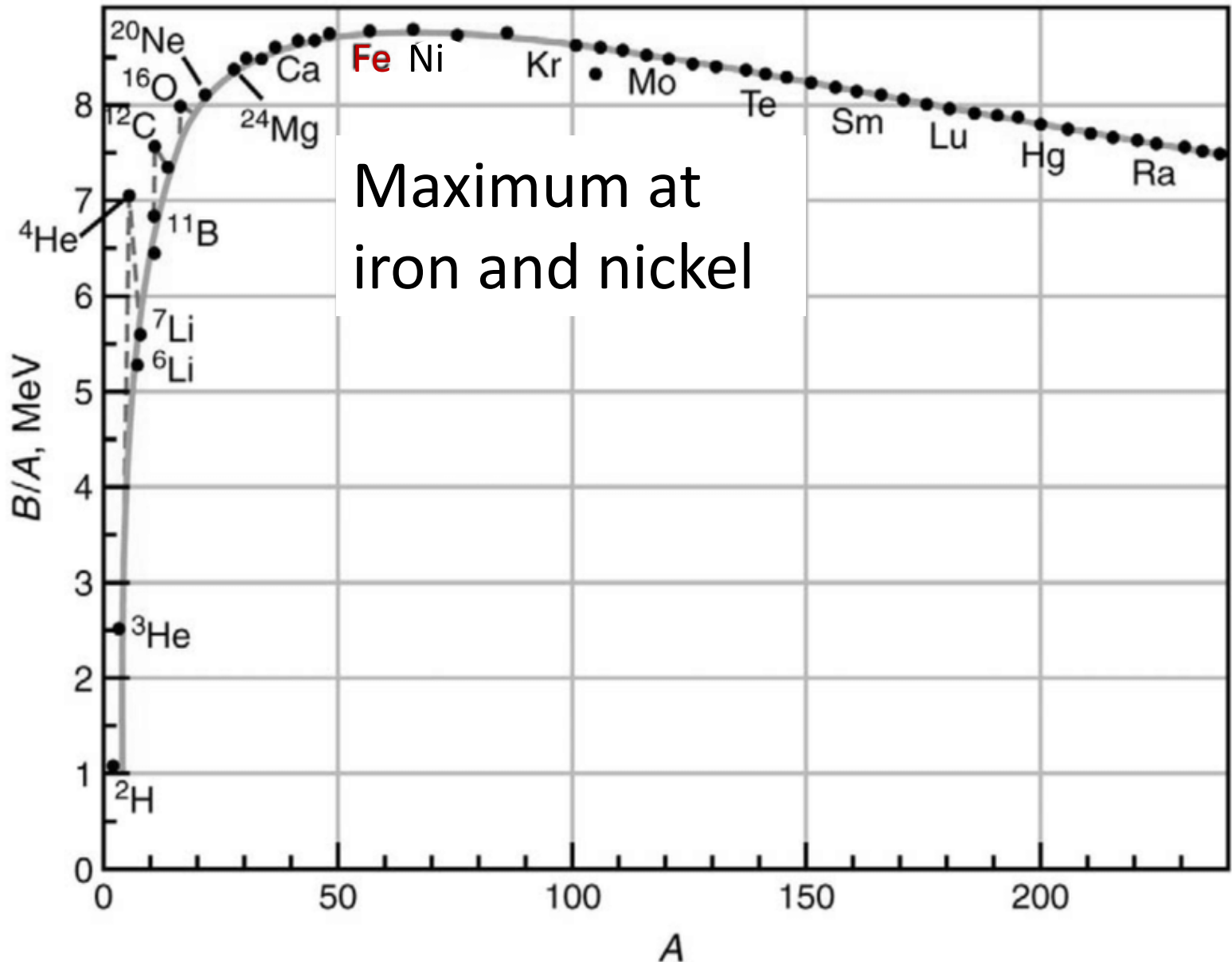


# The most massive stars fuse elements up to iron in their cores





# Binding energy per nucleon



No elements heavier than  
iron group

Heavier elements, with  $A > 90-100$  have lower energy/baryon than iron.



They form by successive capture of neutrons.

Hoyle and Fowler, with help from Margaret and Geoffrey Burbidge, elucidate pathways to form the heavier elements.



Burbidge Fowler

Hoyle

Burbidge

# Fowler gets a Nobel prize



Not Hoyle



I have no idea how the Swedes decided to make an award to Chandrasekhar and Fowler but not to Hoyle. However, I think it would be widely accepted that it was an unfair misjudgment.

Sir Martin Rees

He was so critical of the committee that I imagine someone there just took a large pen and crossed his name off the list of those being considered for future prizes

Simon Mitton



s-process: slow bombardment.

Time between captures longer than time to decay (typically tens to thousands of years).

r-process: rapid bombardment.

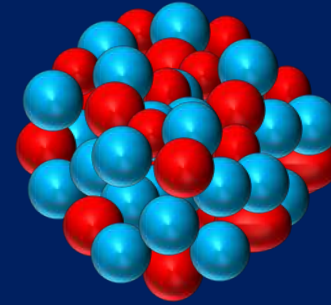
Time between captures shorter than decay time (typically much shorter than a second)

## s-process

Neutrons are produced in cores of red giants by helium nuclei hitting carbon and neon:



# Neutrons fly from the core



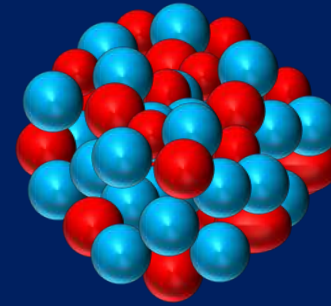
Iron

26 protons

30 neutrons



# Neutrons fly from the core

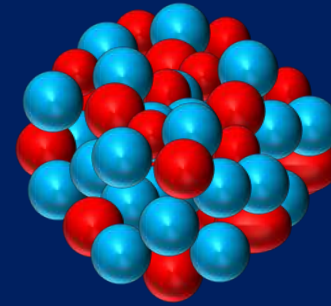


Iron

26 protons

31 neutrons

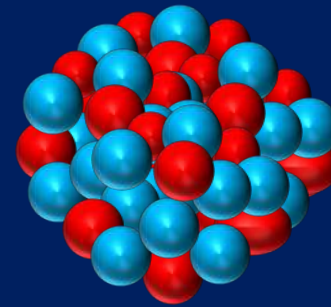
100,000 years later . . .



Iron

26 protons

31 neutrons



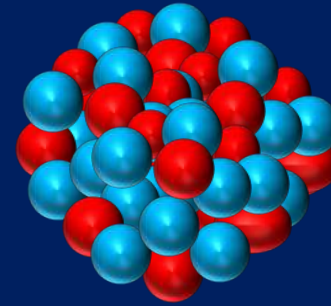
Iron

26 protons

32 neutrons

100,000 years later . . .

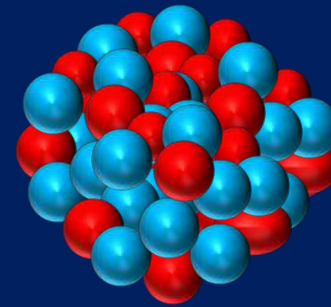




Iron

26 protons

32 neutrons

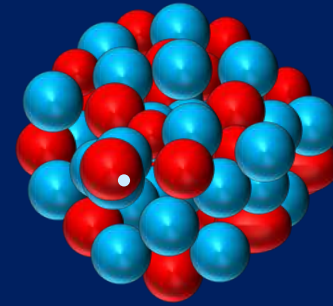


Iron

26 protons

33 neutrons

45 days later . . .

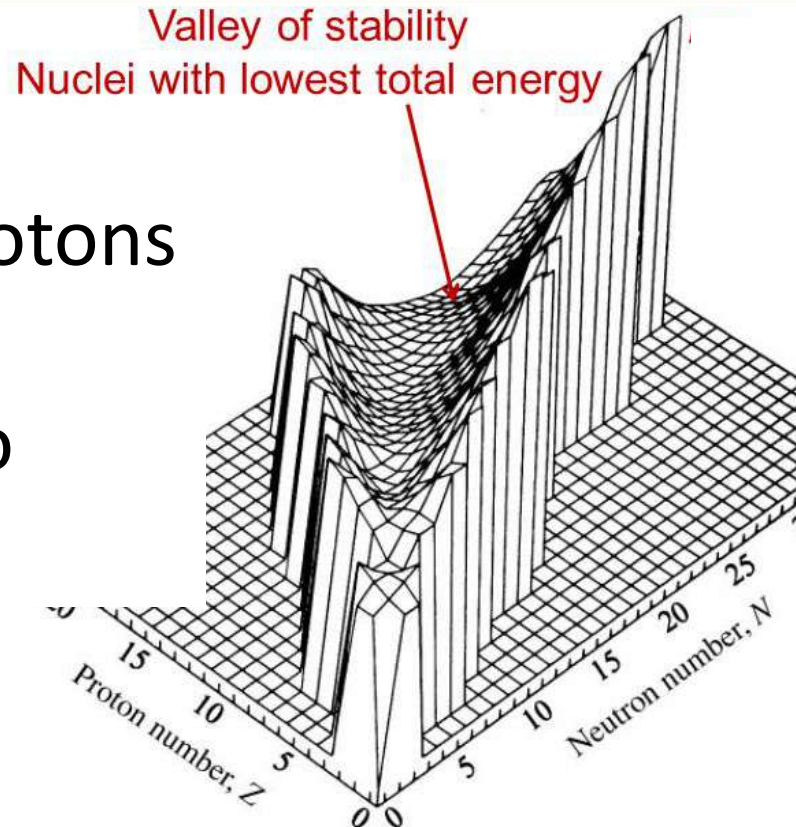


Cobalt

27 protons

32 neutrons

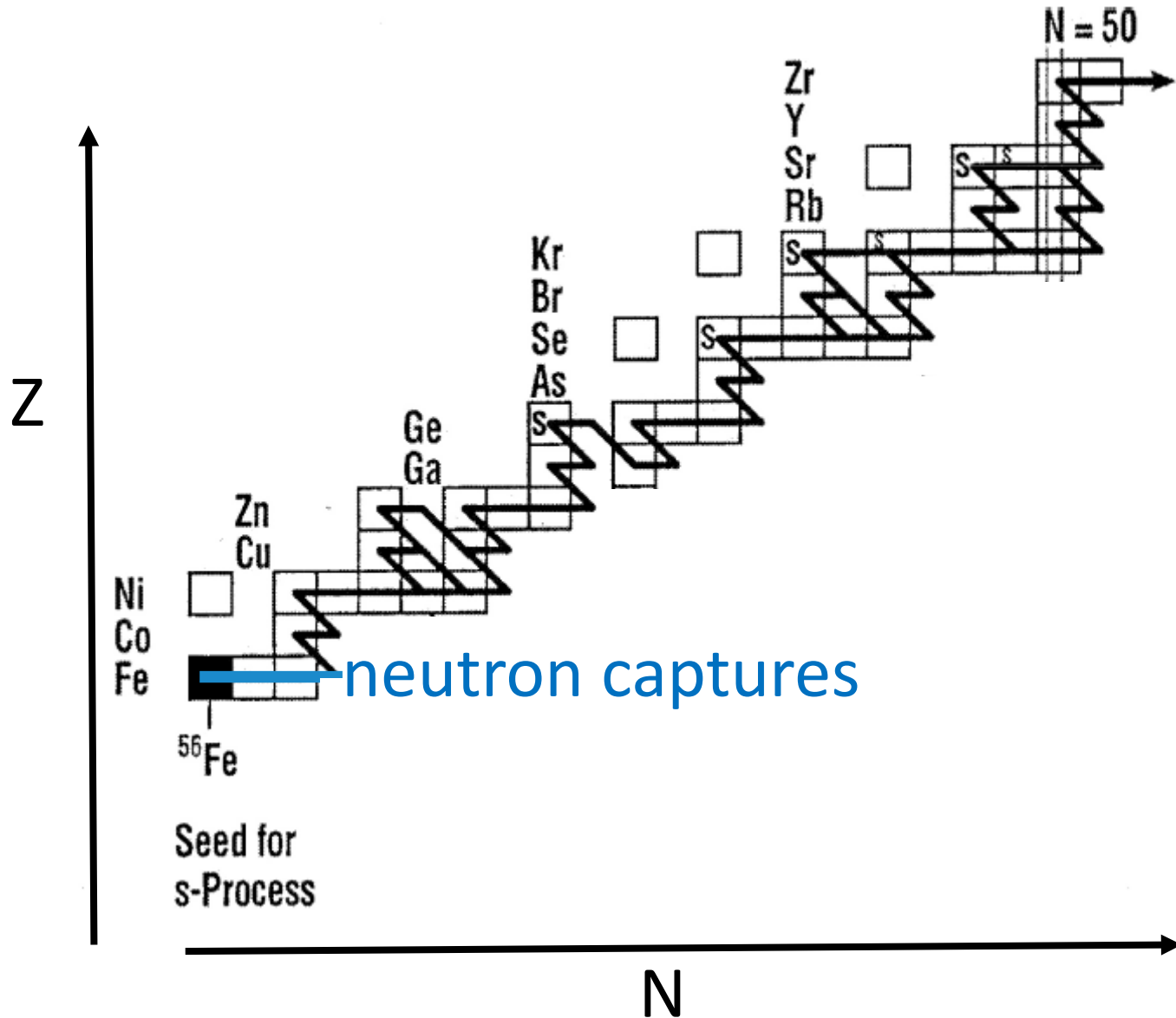
Slow bombardment moves you through stable nuclei until you reach an unstable nucleus that then decays back to the “valley of stability.”



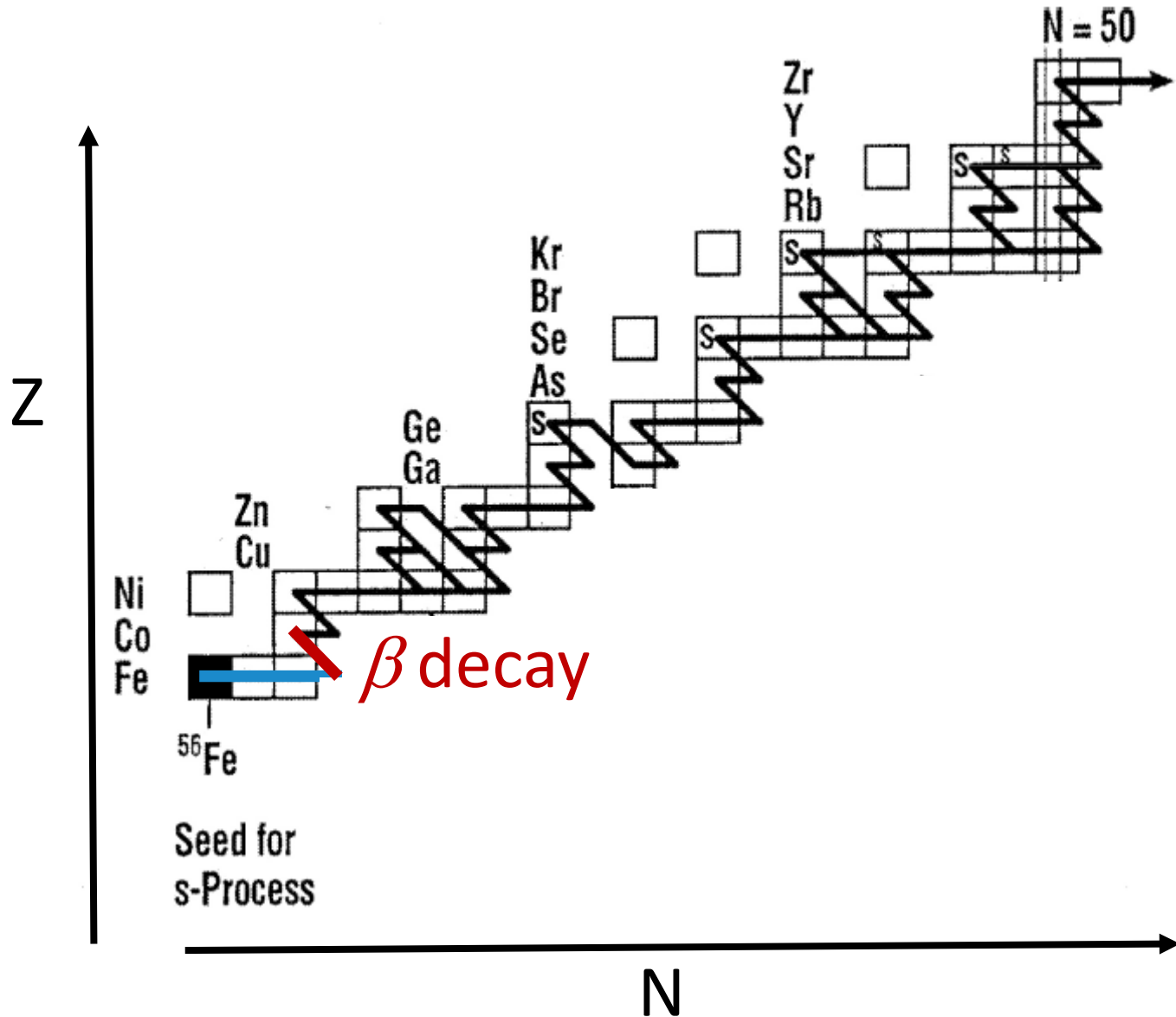
Too many protons  
Coulomb  
repulsion too  
large

Too many  
neutrons-  
neutron fermi  
energy too high

# s-process



# s-process



There's a problem:  
Too many large nuclei are **unstable**.

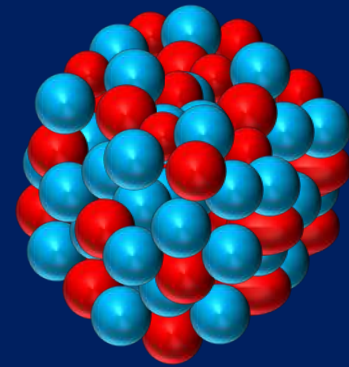


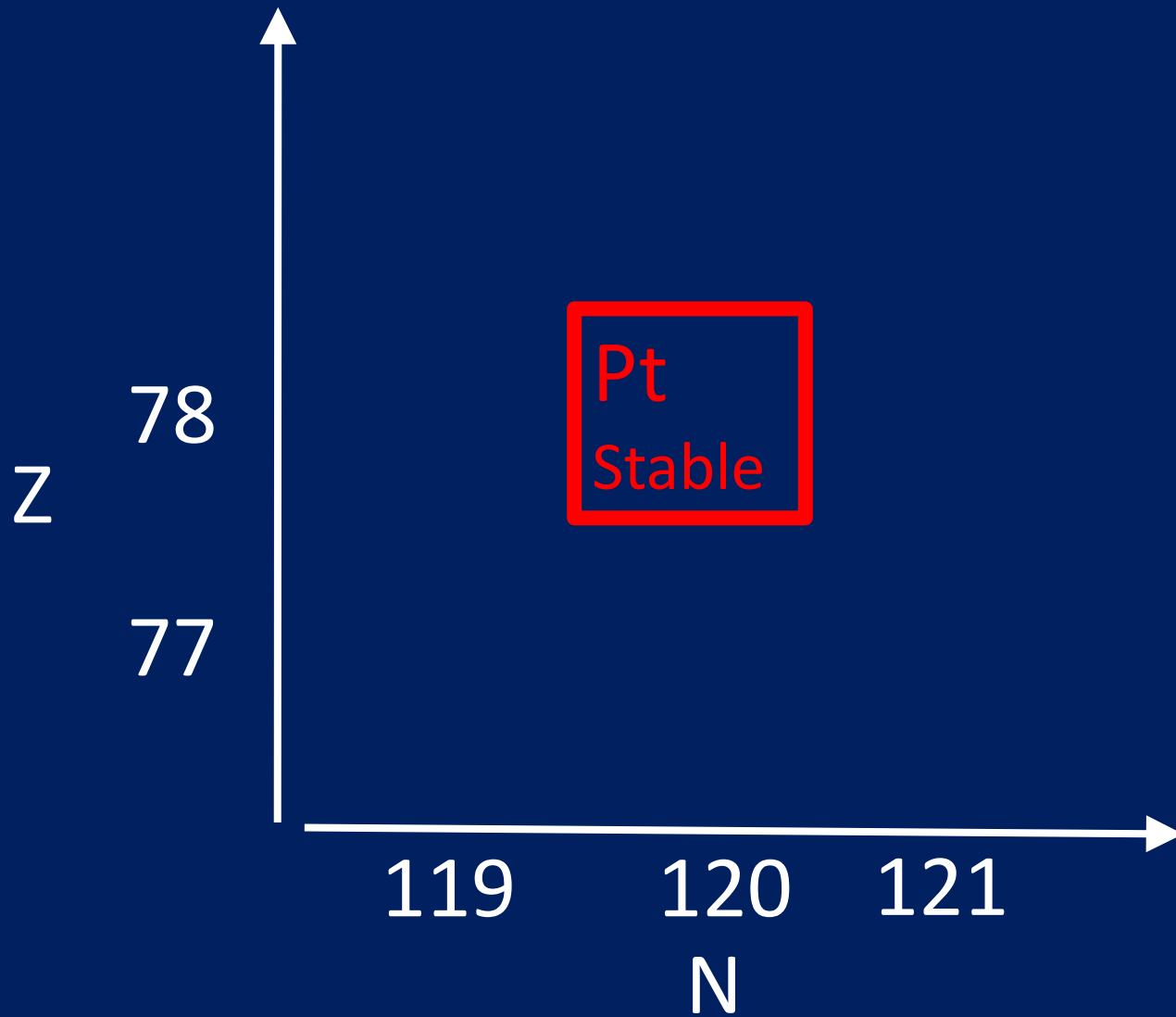
Large stable nuclei may have no stable guy with an extra neutron that can reach them by decay.

Platinum

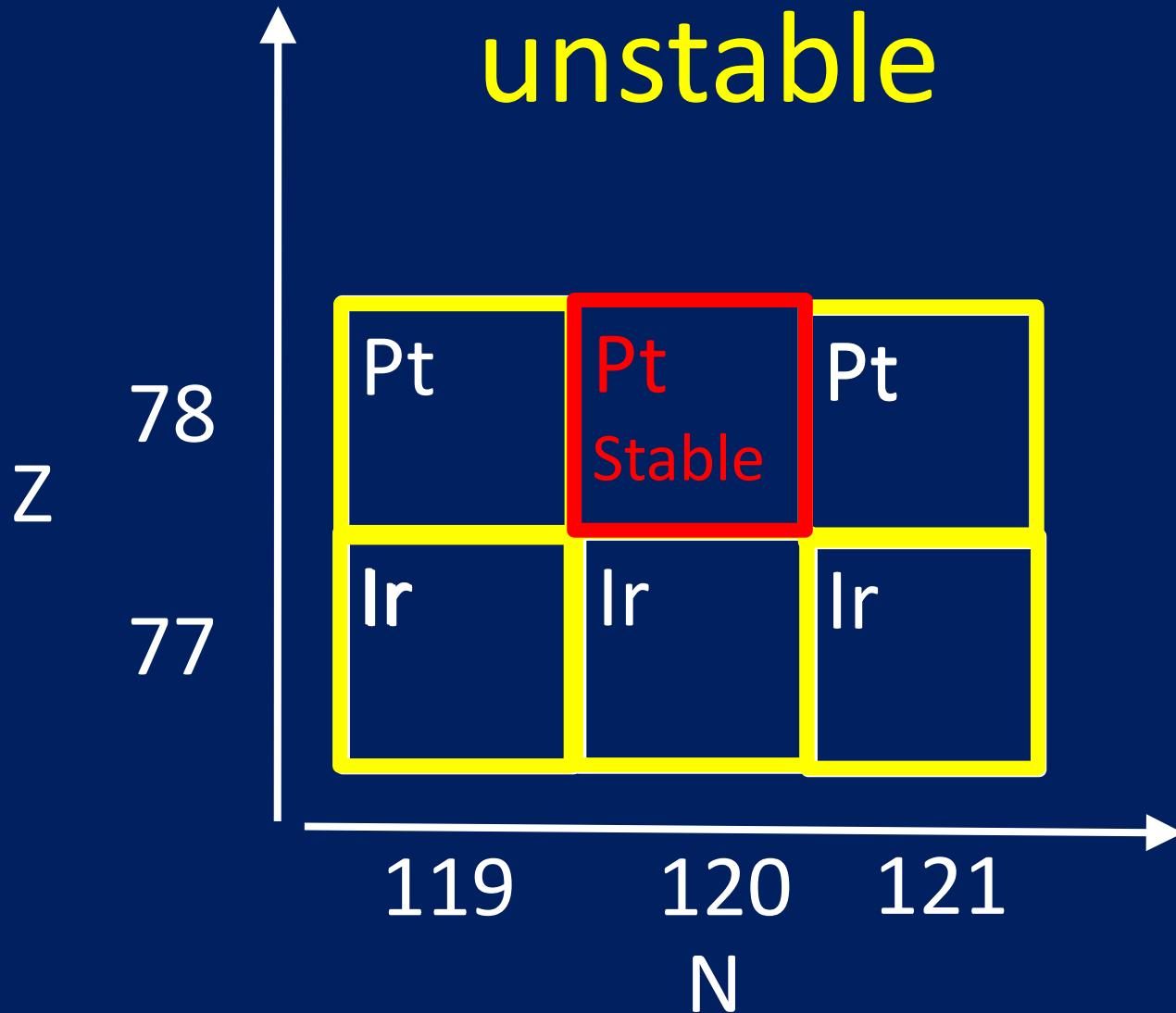


78 protons  
120 neutrons





unstable

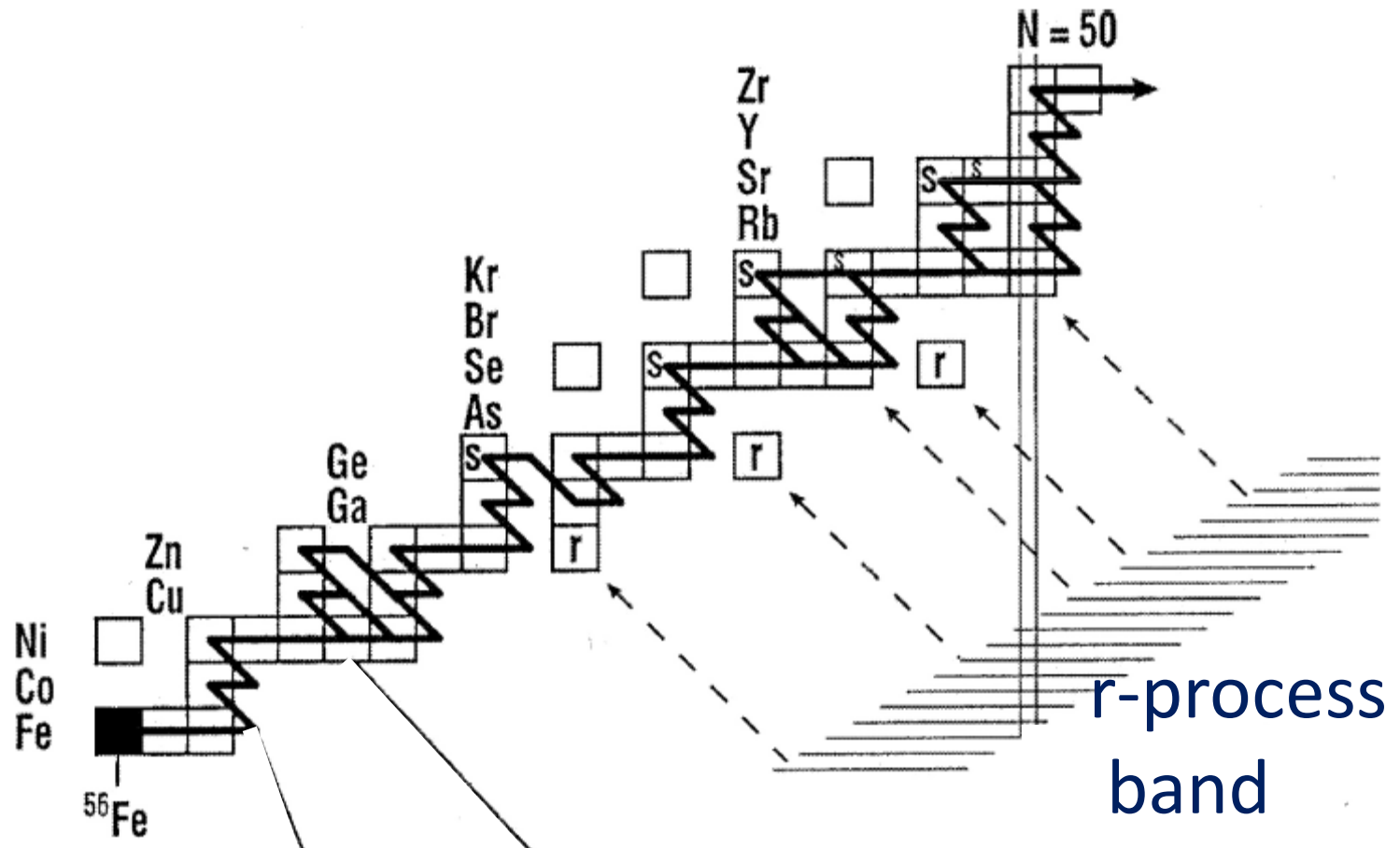


decay in less than a day

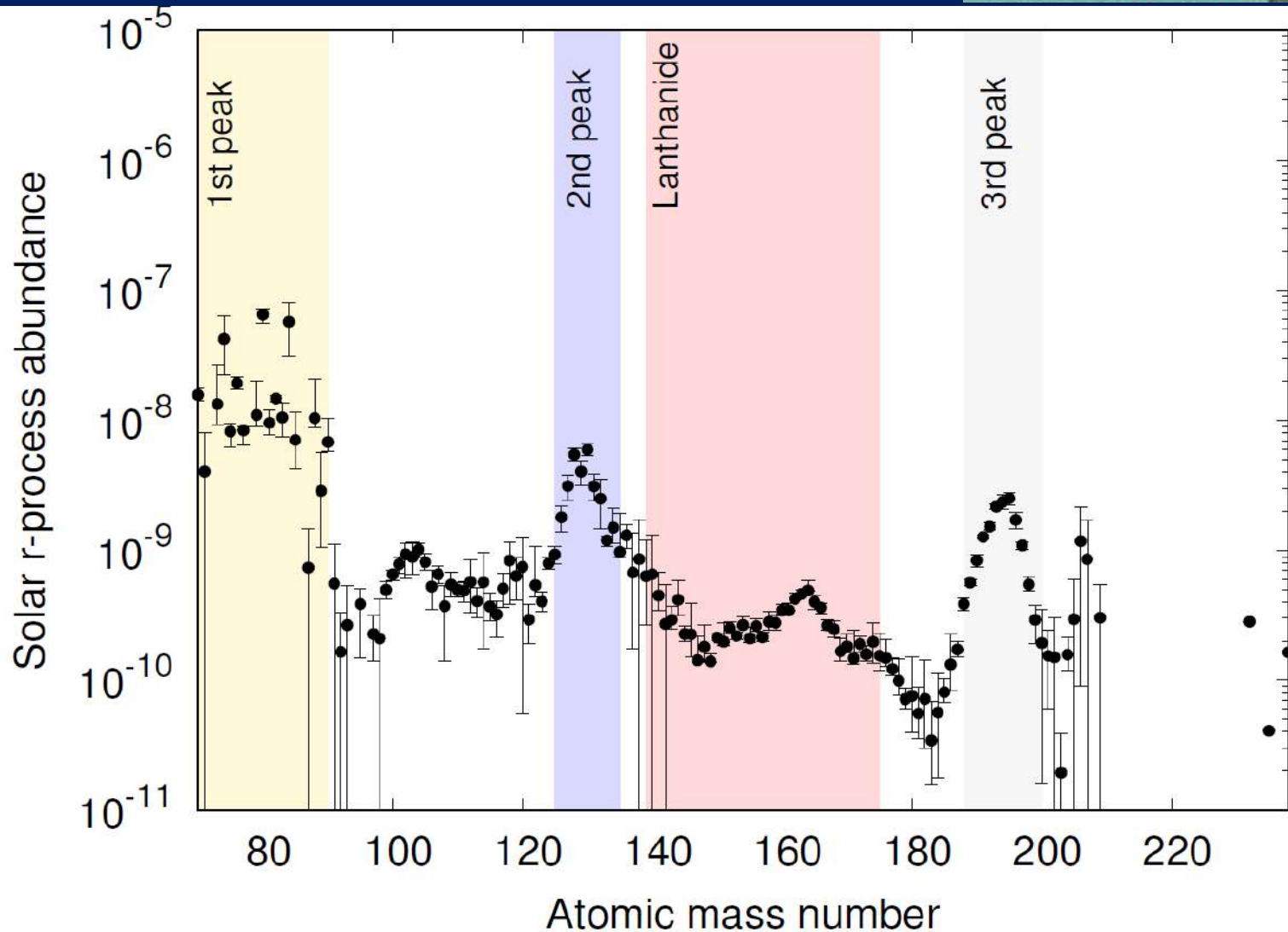
To get to platinum, neutrons need to bombard nuclei before they can decay.

After the bombardment stops, unstable nuclei with too many neutrons are left.

The extra neutrons decay to protons, leaving a final stable nucleus.



# Abundance peaks at *magic numbers*, at filled neutron shells in nucleus





r-process simulation Wanajo et al.

<http://www.ph.sophia.ac.jp/~shinya/research/research.htm>



# r-process elements: elements built primarily or exclusively by rapid bombardment

The periodic table below highlights r-process elements in yellow. Labels with arrows point to Silver (Ag), Platinum (Pt), Gold (Au), Rare Earths (La-Lu), and Uranium (U).

1 H																	2 He									
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne									
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar									
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr									
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe									
55 Cs	56 Ba											72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra																	89 Ac	90 Th	91 Pa	92 U					
Rare Earths		57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu										

*The tidal breakup of a neutron star near a black hole is examined. . . the estimated quantity of ejected material is found to be roughly comparable to the abundance of r-process material.*

Lattimer, Schramm 1974 . . .

*A natural consequence of the binary pulsar's evolution is a neutron star collision. . . . Taking reasonable estimates for the number of such events over the history of the galaxy, it may be that they account for all of the [remaining] nuclei.*

Symbalisty, Schramm 1982

# Galactic mass of r-process elements

$$= 10^{-7} M_{\text{galaxy}}$$

$$= 10^4 M_{\odot}$$

Can NS-NS mergers produce almost all of this?

Simulations of neutron star collisions  
(before GW1701817 and then afterwards with  
observations at all wavelengths to match) give

10 times the mass of the earth  
in platinum and gold



$> 10^{-2} M_{\odot}$   
in heavy elements

## Rate of neutron-star mergers:

Closest seen is GW170817 at 40 Mpc (130 Mly)  
about 1 merger every (.5 – 10) years within this

distance,

$$V = \frac{4}{3}\pi(40\text{Mpc})^3 = 2.7 \times 10^5 \text{Mpc}^3$$

With, say, 2 years per merger within this distance,  
merger rate is

$$\frac{1}{2(2.7 \times 10^5 \text{Mpc}^3)} \approx 4 \times 10^{-6} \text{Mpc}^{-3} \text{yr}^{-1}$$

Per galaxy: About  $100 \text{Mpc}^3$  per

Milky-Way equivalent galaxy, giving

merger rate  $\approx 10^{-4} \text{MWEG}^{-1} \text{yr}^{-1}$



galactic mass of r-process elements

$$= 10^4 M_{\odot}$$

r-process mass per merger

$$\gtrsim 10^{-2} M_{\odot} / \text{merger}$$

r-process mass in Galaxy produced in age of universe

$$= (10^{-4} \text{ mergers/yr})(10^{-2} M_{\odot} / \text{merger})(10^{10} \text{ yr})$$
$$\approx 10^4 M_{\odot}$$

with uncertainties in amount per merger and in merger rate giving range

$$7 \times 10^3 M_{\odot} \text{ to } 10^5 M_{\odot}$$

# Prediction

Radioactive decay powers an afterglow, *a kilonova*, that dims as the decays lead to stable nuclei

# The glow hours after the collision



Discovery Image  
August 17, 2017

The rate at which the glow faded and the shift to lower wavelengths, agreed with predictions from decay of r-process elements.



Discovery Image  
August 17, 2017

The rate at which the glow faded and the shift to lower wavelengths, agreed with predictions from decay of r-process elements.



4 Days Later  
August 21, 2017



OPINION  
Once Bitten, Beyoncé  
Turns the Other Cheek



OPINION  
Maybe the Drop in  
Student Visas Isn't All  
That Bad



U.S. NEWS  
NASA Delays James  
Webb Space Telescope  
By About a Year



U.S. NEWS  
The Decision That Hurts  
Your Chances of Getting  
Into Harvard



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EXPLORE

## A Clash of Neutron Stars Forges Gold

Researchers say this is the first time a cosmic event has been seen both with gravitational waves and with the full electromagnetic spectrum

By *Robert Lee Hotz*

Oct. 16, 2017 10:00 a.m. ET

THE WEALTH OF

# Gravitational waves just led us to the incredible origin of gold in the universe

LIGO kick-started an astronomical treasure hunt that ended with colliding neutron stars and gold.

By Brian Resnick | @B\_resnick | brian@vox.com | Updated Oct 16, 2017, 6:28pm EDT

PhysOrg

Gold origin confirmed with first ever gravitational wave sighting

# III. Equation of state (EOS) above nuclear density

Because neutron stars are cold  
( $kT \ll$  Fermi energy of neutrons and protons)  
the EOS is essentially the zero-temperature EOS,  
depending on only one parameter:

$$p = p(\rho)$$

$$\epsilon = \epsilon(\rho)$$

$\rho$  = rest mass density

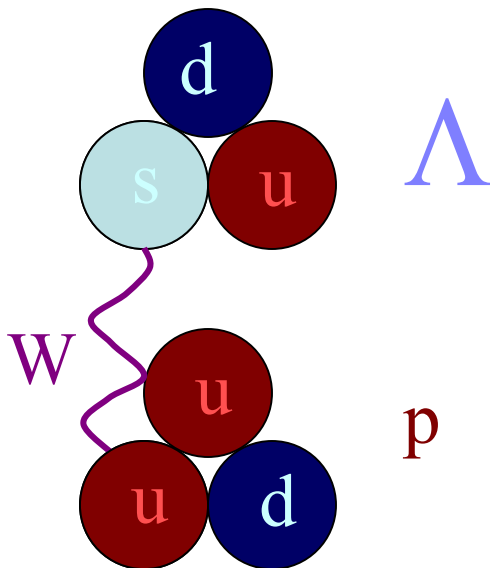
$\epsilon$  = energy density

$p$  = pressure



Is the core dense enough to create strange quarks  
–to push the Fermi energy of some down quarks above the  
threshold to convert to strange quarks

e.g., Fermi energy of neutron pushed above mass of  $\Lambda$ :

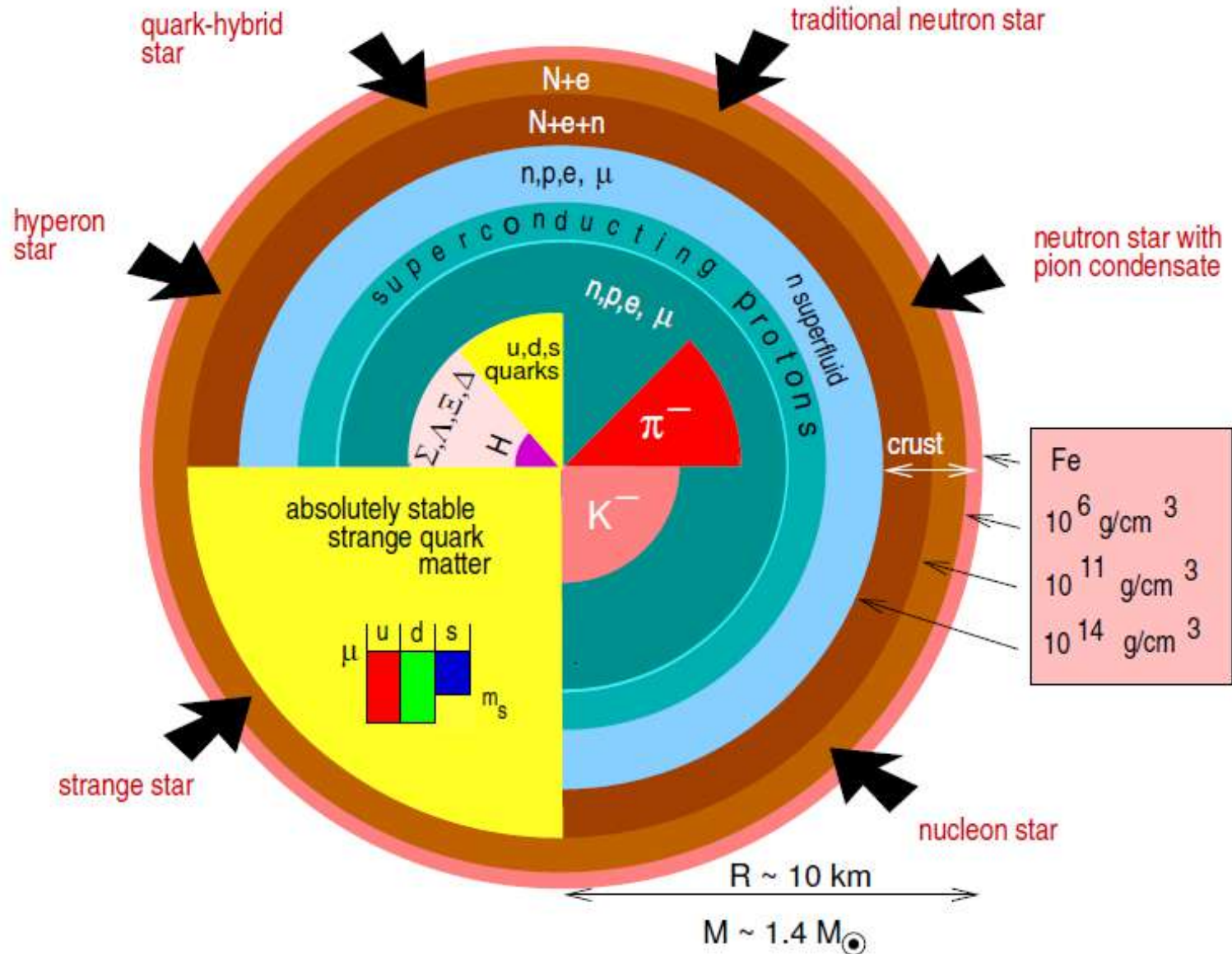


If so, do the quarks group as hyperons in core?

Is the core dense enough to dissolve nucleons into strange quark matter: a large bag of free up, down and strange quarks?

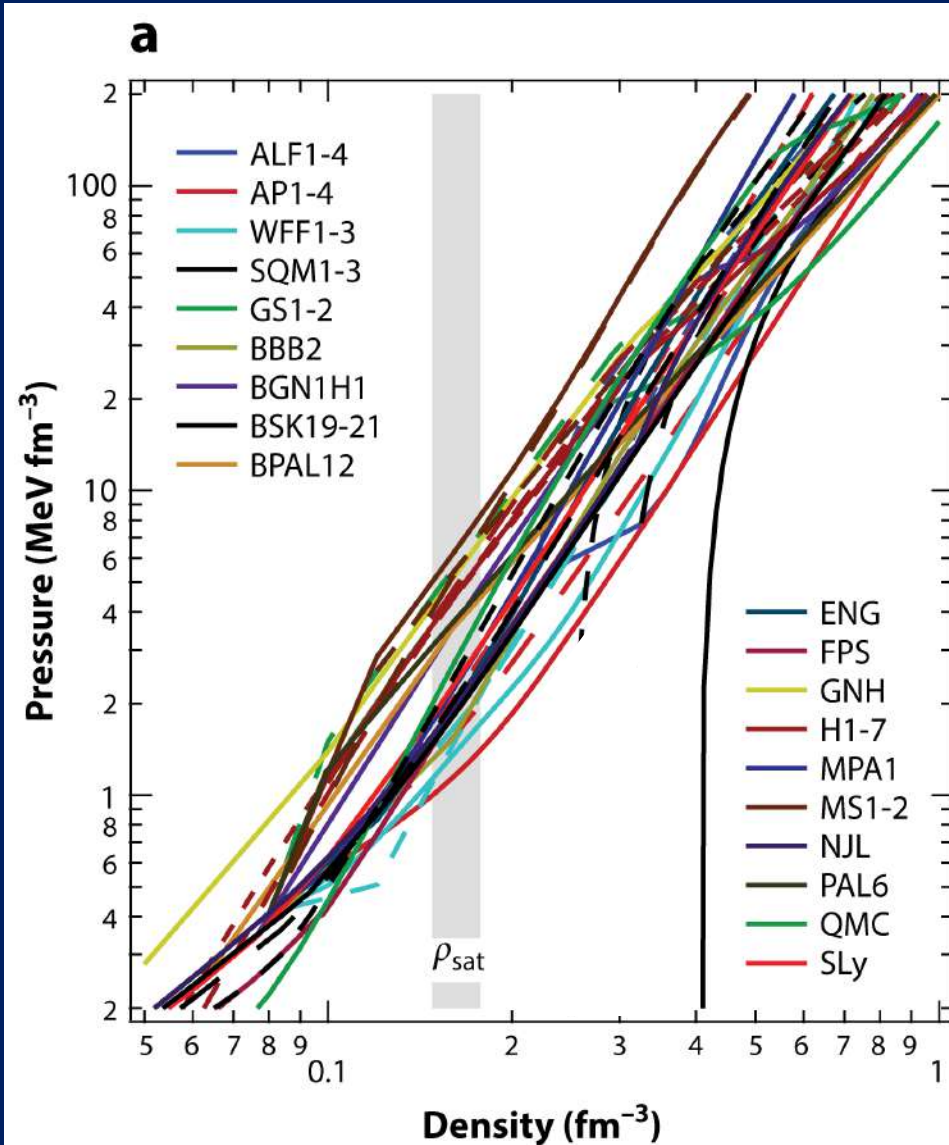
Is the true ground state of cold matter strange quark matter, for any collection larger than a few hundred quarks (Bodmer, Witten)?

# Internal structure of compact stars



[Weber, J. Phys. G 27, 465 (2001)]

# EOS based on the different alternatives



Ozel & Freire '16  
Ann Rev A & Ap

Can approximate  $\log p$  ( $\log r$ ) for universe of candidate EOS by piecewise linear curves to about 3%

a

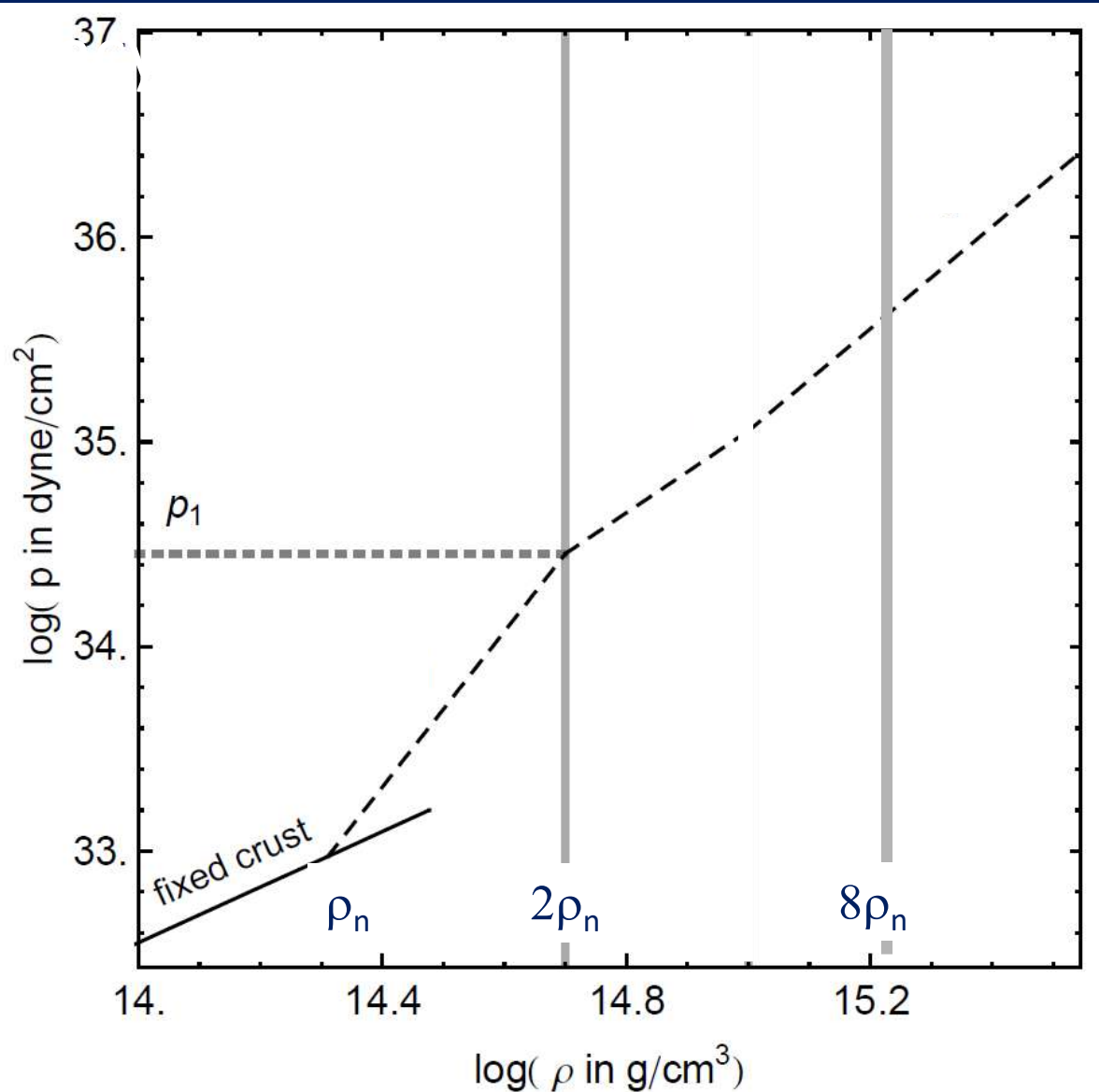
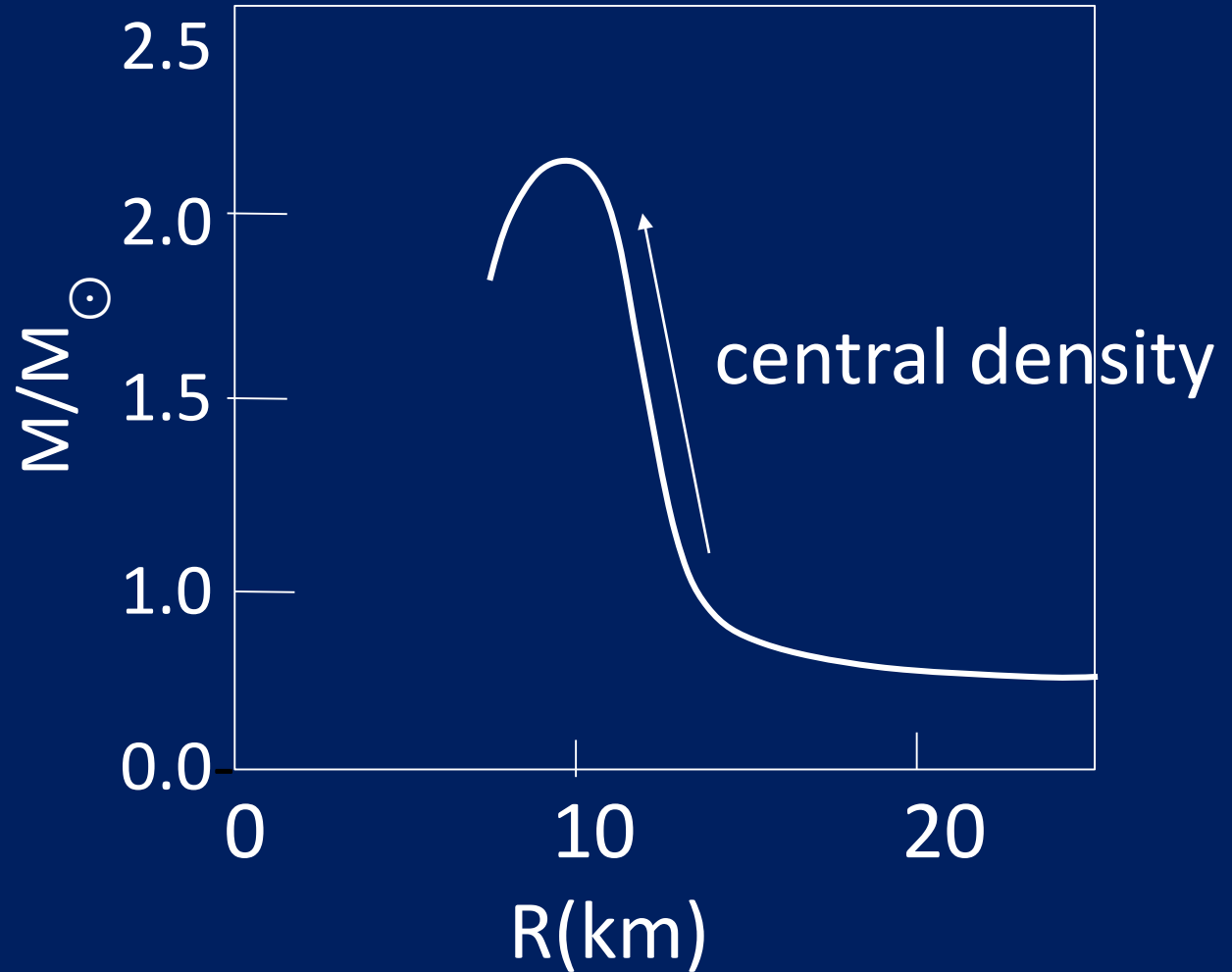
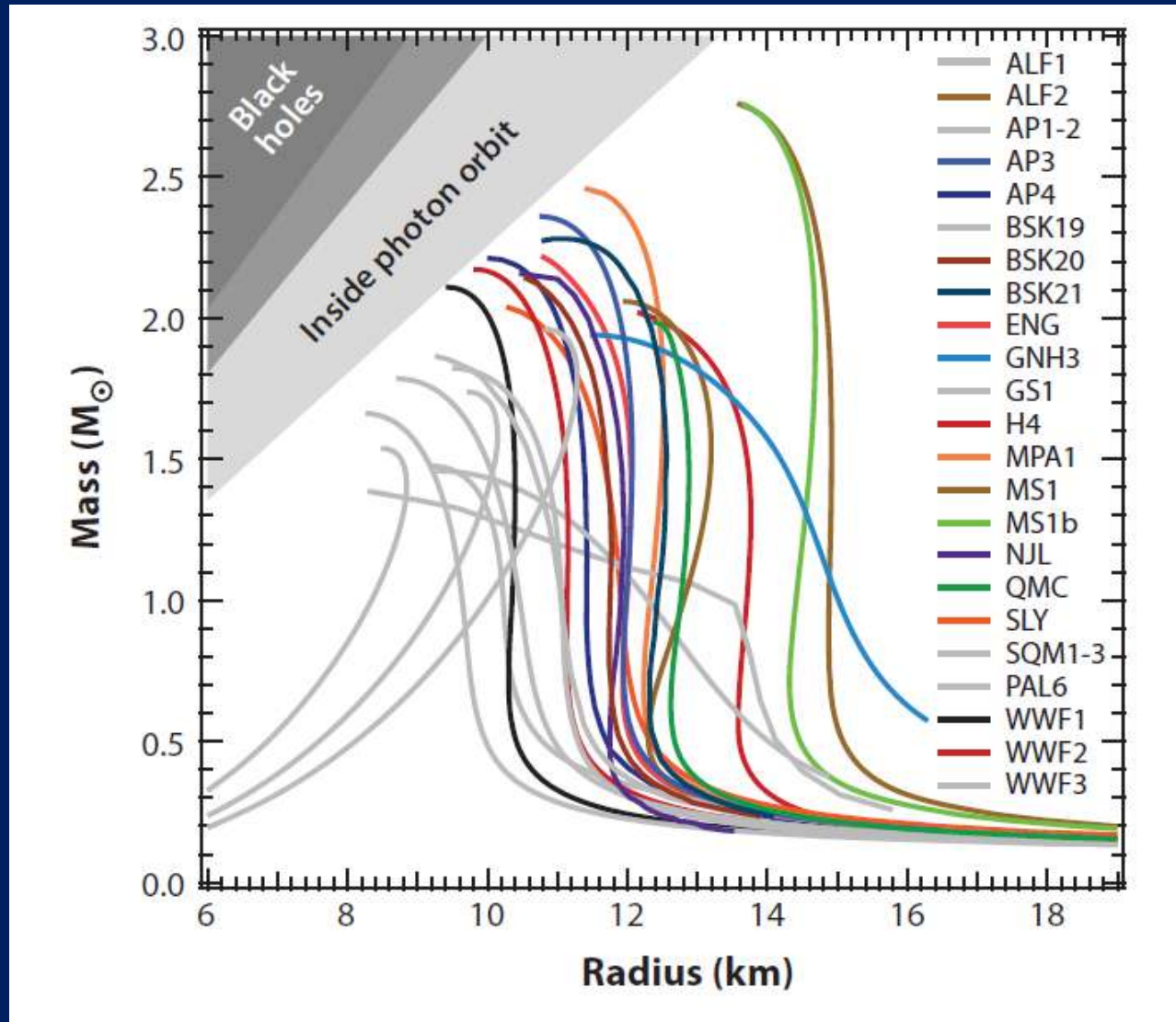


Fig modified from Read, Lackey, Owen, JF

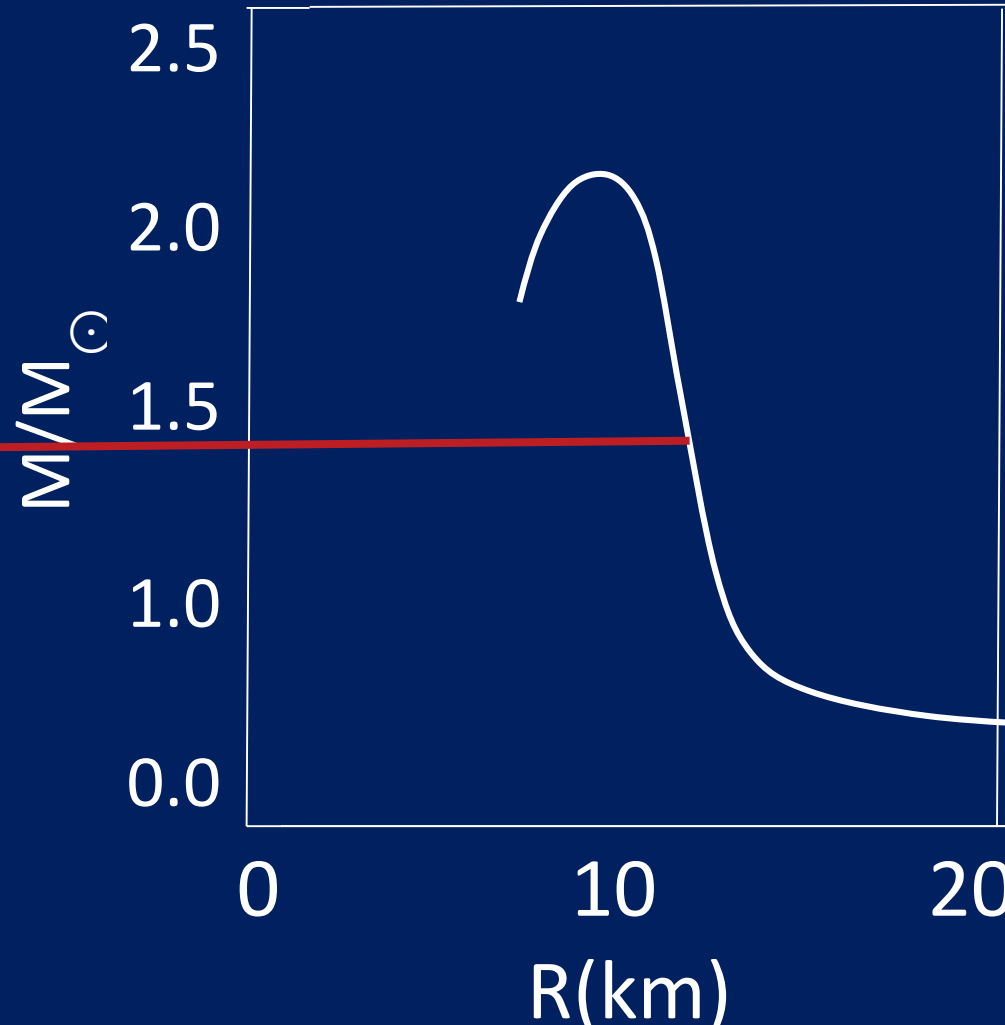
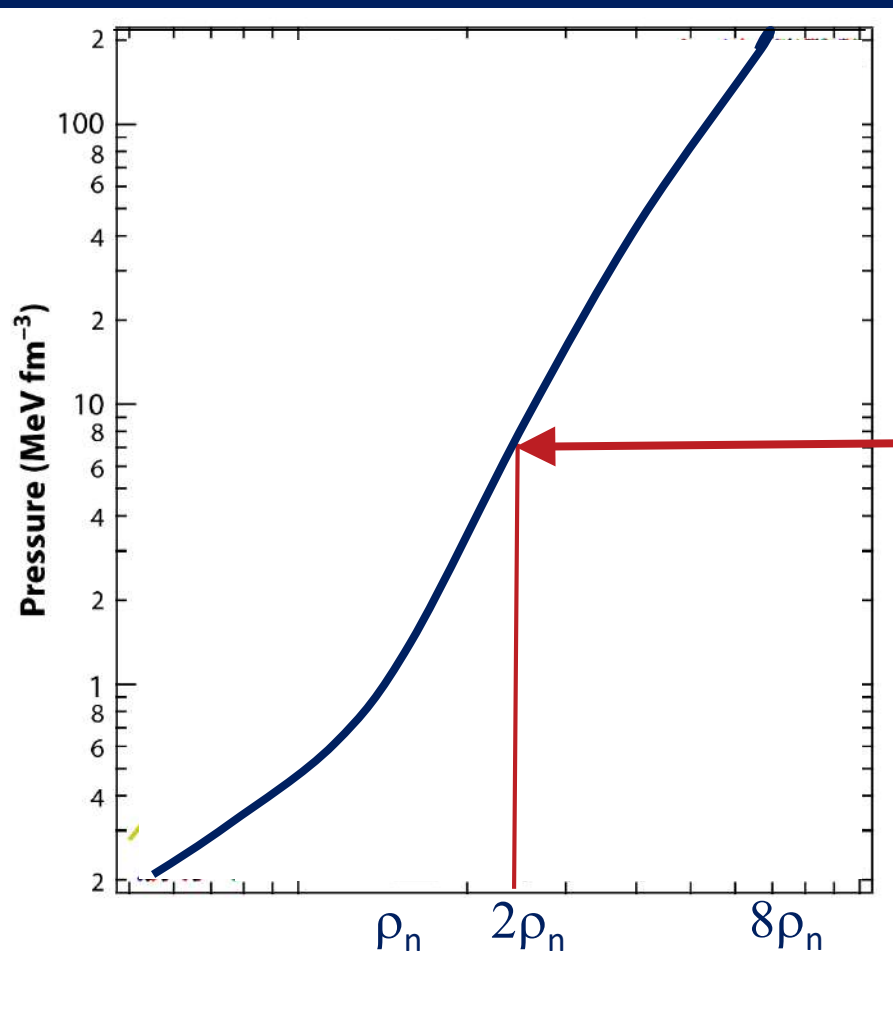


The 1-parameter family of neutron stars portrayed as a curve of mass vs radius

To each candidate EOS corresponds a M vs R curve.



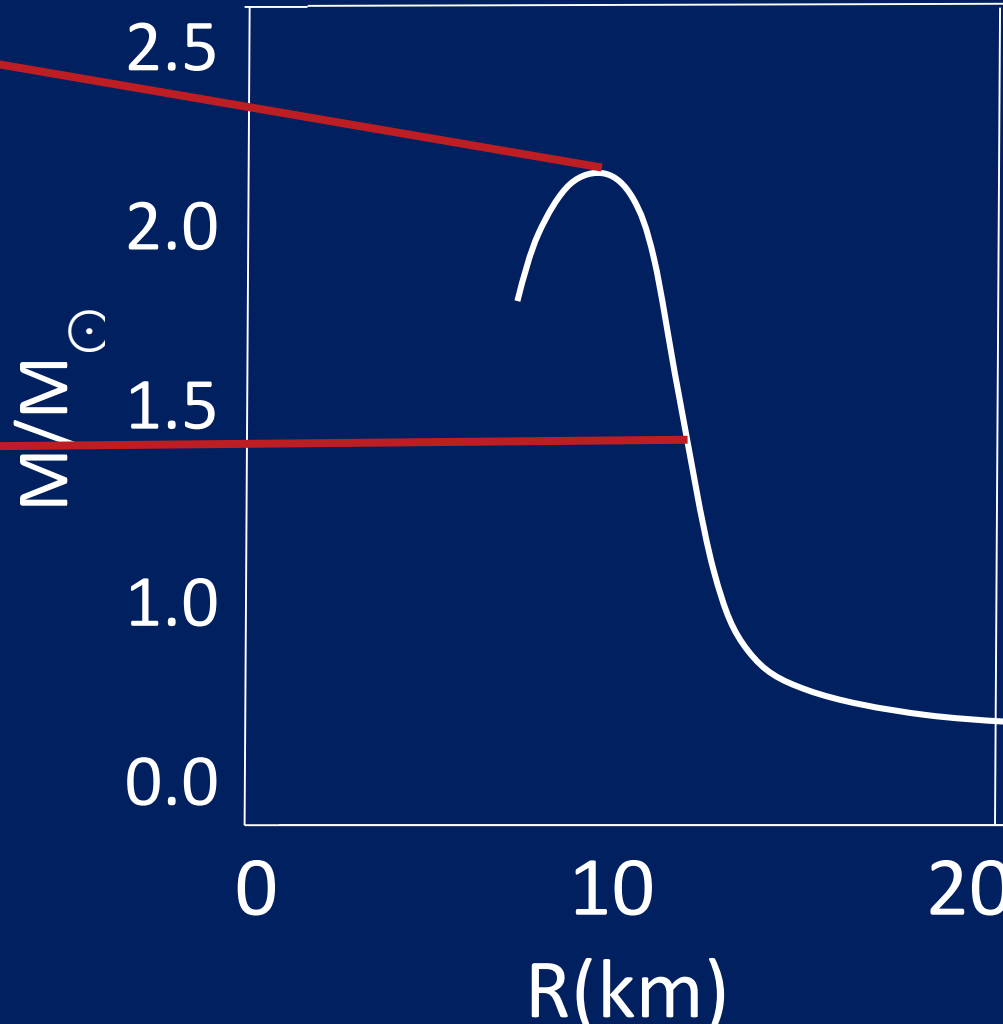
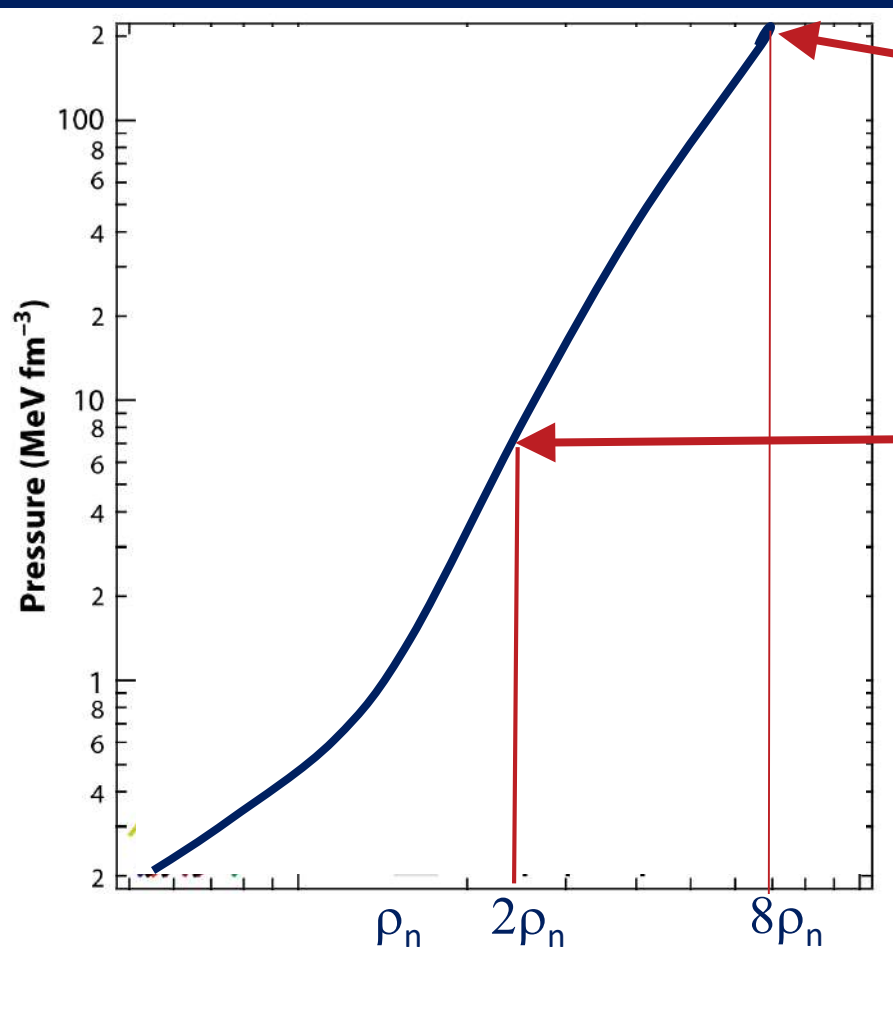
Infer EOS from measurements of mass and radius.  
Radius at  $1.4 M_{\odot}$  gives pressure at  $\rho \sim 1.8 \rho_n$



(inverse construction by Lindblom; Fig. idea from Lattimer)



Maximum mass gives pressure at  $\rho \sim 8 \rho_n$



(inverse construction by Lindblom; Fig. idea from Lattimer)

If the EOS of cold matter above nuclear density is soft  
the radius of a  $1.4 M_{\odot}$  neutron star will be small

Soft EOS:  $p$  small for  $\rho \sim 2\rho_{\text{nuclear}}$  implies  
star more centrally condensed,  
 $R$  small



If the EOS of cold matter above nuclear density is stiff  
the radius of a  $1.4 M_{\odot}$  neutron star will be large

Stiff EOS:  $p$  large for  $\rho \sim 2\rho_{\text{nuclear}}$  implies  
star less centrally condensed,  
 $R$  large



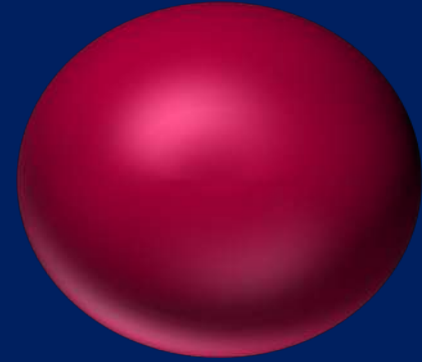
## IV. Tidal imprint on the inspiral waveform

With no tide, distance  $d$  between stars decreases at rate determined by energy loss to gravitational waves.

$$E_{\text{orbit}} = -\frac{M^2}{d} \implies \frac{\dot{E}_{\text{orbit}}}{E_{\text{orbit}}} = \frac{\dot{d}}{d}$$

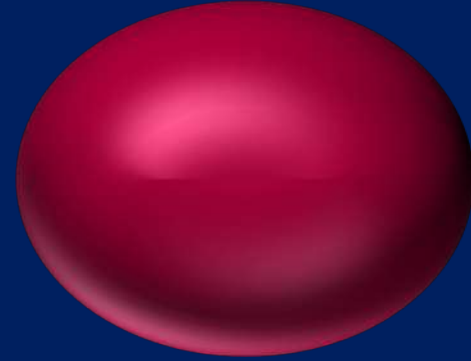
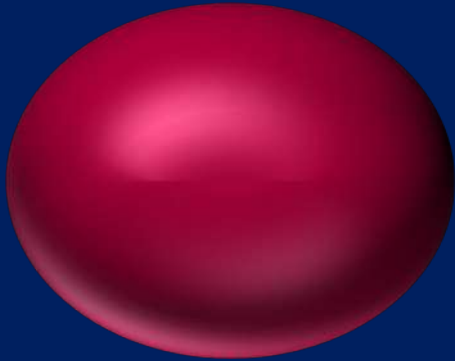
# Tidal imprint on the inspiral waveform

As  $d$  decreases, the height of the tides increases



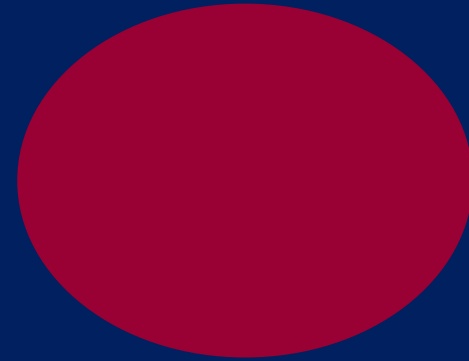
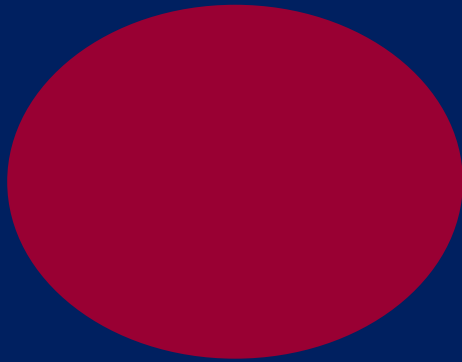
# Tidal imprint on the inspiral waveform

As  $d$  decreases, the height of the tides increases




# Tidal imprint on the inspiral waveform

The tidal distortion of each star increases the total quadrupole moment  $Q$  of the orbiting stars



Tides increase the rate at which the orbit loses energy in two ways:

orbital energy  stellar deformation  
enhanced GW from larger  $Q$

the orbit shrinks faster

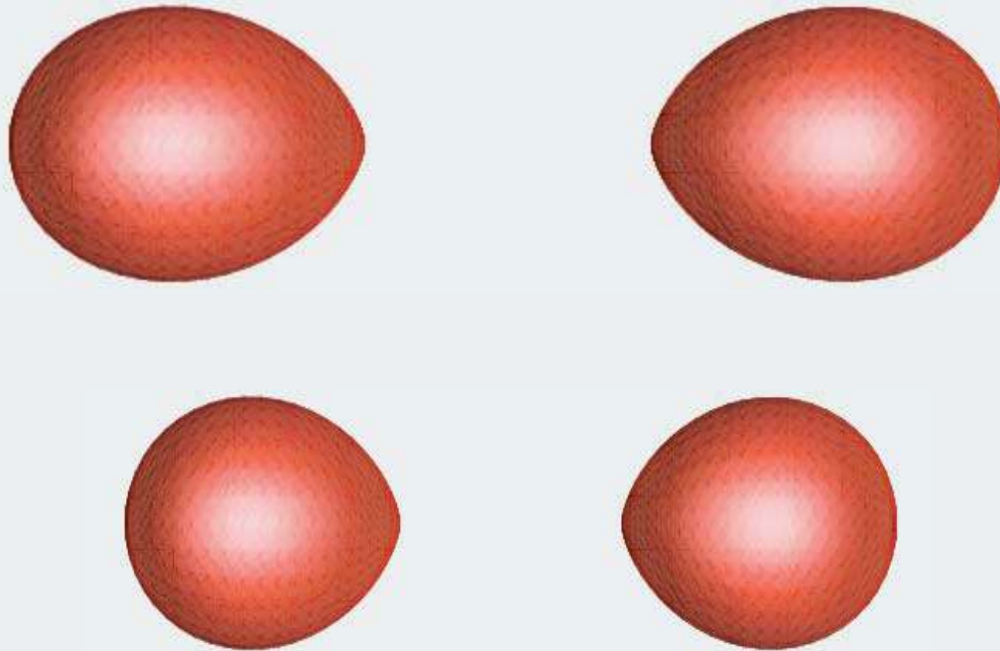
the frequency  $f$  increases more quickly.

and tidal disruption ends the inspiral sooner – the cutoff frequency is lower.



In a binary system, the tides raised on each star depend on the deformability of that star:

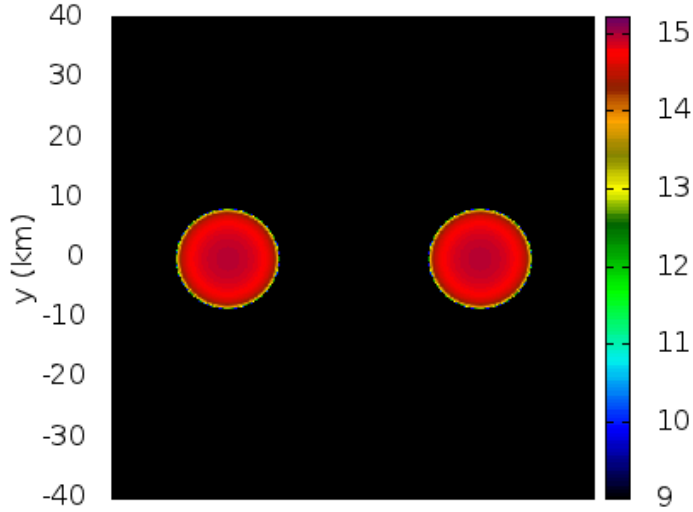
Tides are larger for large radii and so larger for stiffer EOS.



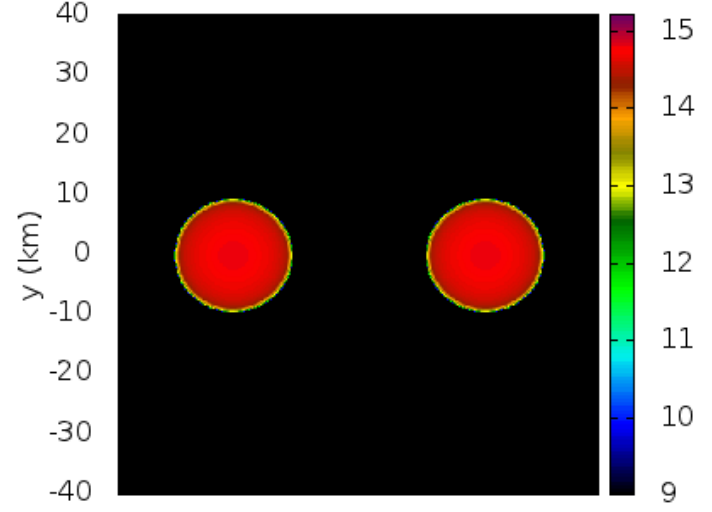
# Merger of $1.35-1.35M_{\odot}$ NS with four EOSs

$t=0$  ms

$t=0$  ms



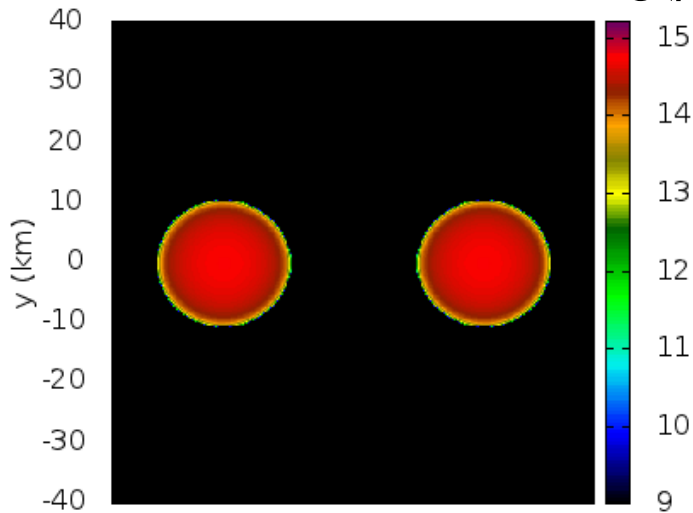
APR4:  $R=11.1$ km



ALF2:  $R=12.4$ km

$t=0$  ms

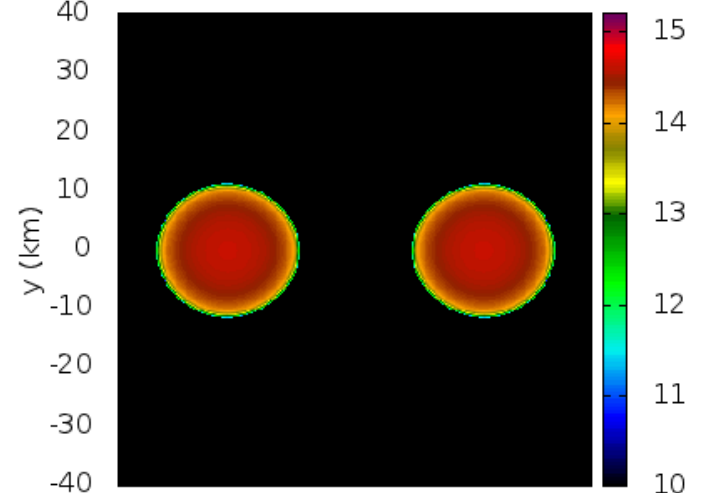
Log( $\rho$  g/cc)



H4:  $R=13.6$ km

$t=0$  ms

Log( $\rho$  g/cc)



MS1:  $R=14.5$ km

Formally, define deformability  $\lambda$



For an imposed external quadrupole field  $E$ , a star acquires a quadrupole moment  $Q$

$$Q = \Lambda E$$

We'll see that  $\Lambda$  is roughly proportional to  $R^5$

The tidal effect on the inspiral waveform  
measures the neutron-star radius

# Estimating the tidal imprint

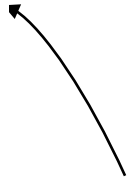
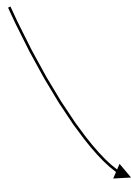
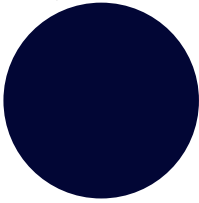
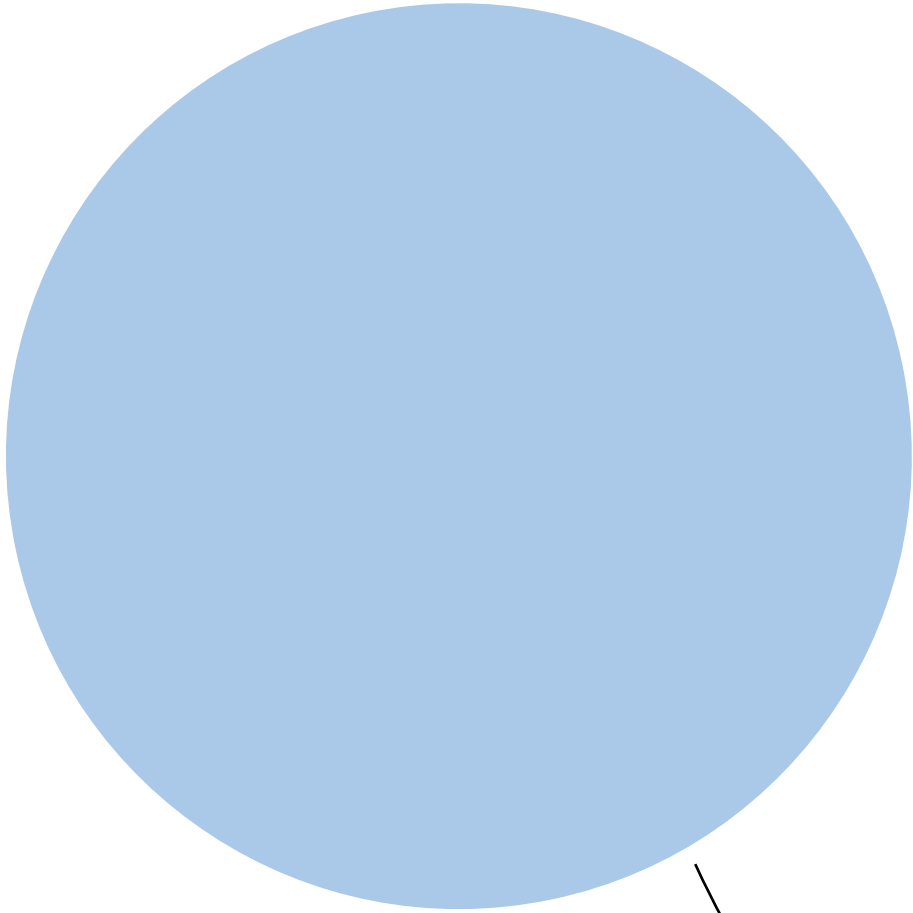
- Height of the tide

- $\frac{\delta \dot{E}_{\text{GW}}}{\dot{E}_{\text{GW}}} \sim \frac{R^5}{d^5}$  (change in radiated power due to increased quadrupole moment)

- $\frac{\dot{E} \text{ to raise tides}}{\dot{E}_{\text{GW}}} \sim \frac{R^5}{d^5}$

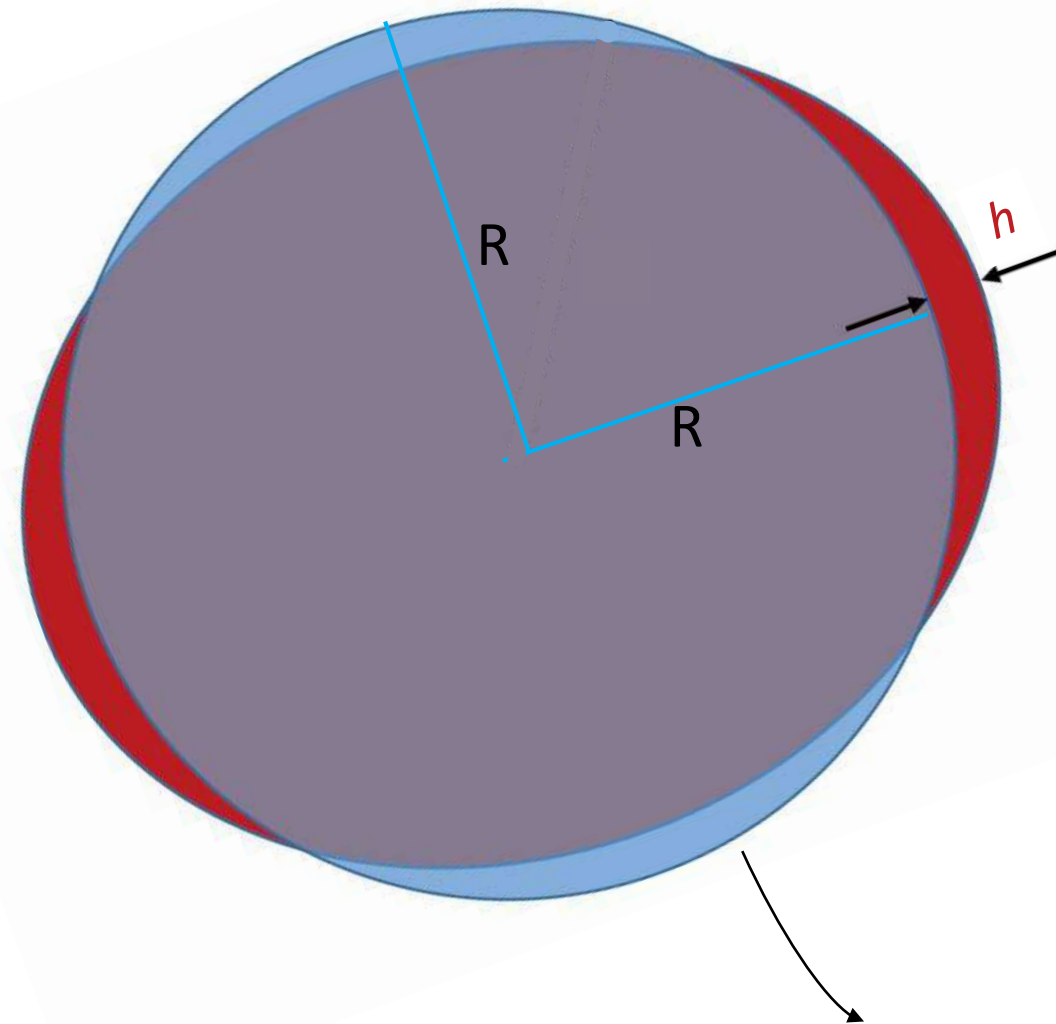
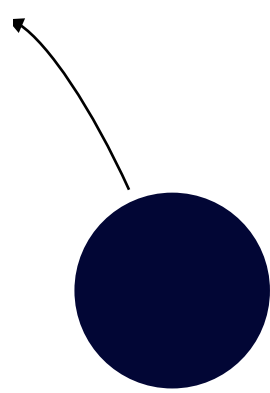
Height  $h$  of tide

from conservation of energy in rotating frame



Height  $h$  of tide

from conservation of energy in rotating frame



Tidal deformation is equivalent to moving blue arcs, mass  $\delta M$ , to position of red arcs:

- Each arc
- 1) rises by height  $h$  in parent star's field
  - 2) falls distance  $\sim R$  in tidal field of companion

energy gained by rising a height  $h$  in parent star's field  
=  
energy lost falling distance  $R$  in other star's tidal field

force of parent star  $\times h \sim$  tidal force  $\times R$

$$\frac{\cancel{M\delta M}}{R^2} h \sim \frac{\cancel{M\delta M} R}{d^3} R$$

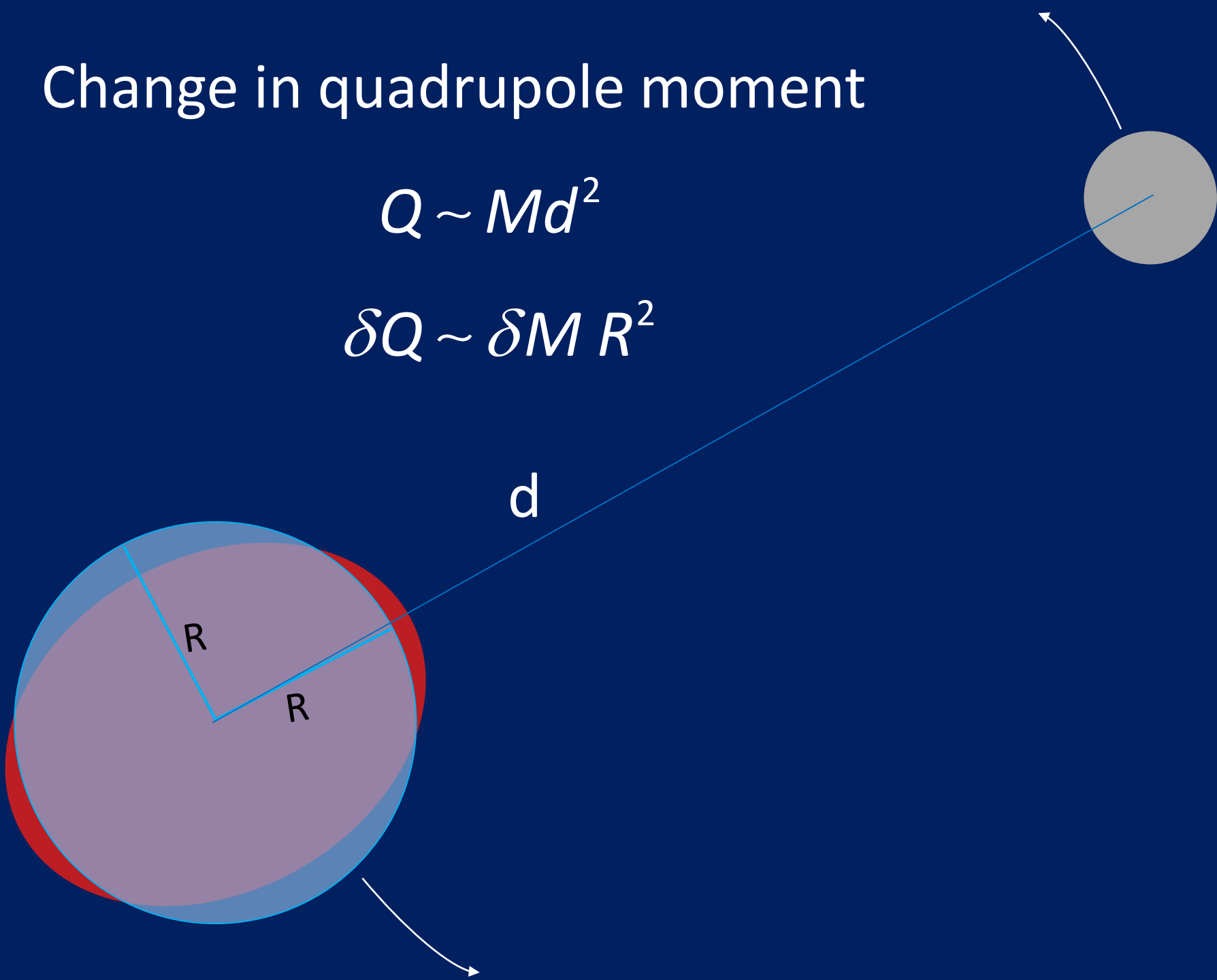
Then

$$h \sim \frac{R^4}{d^3} \quad \delta M \sim M \frac{h}{R} \sim M \frac{R^3}{d^3}$$

# Change in quadrupole moment

$$Q \sim M d^2$$

$$\delta Q \sim \delta M R^2$$





# Enhanced gravitational waves from larger Q

$$\delta Q \sim \delta M R^2$$

$$\frac{\delta Q}{Q} \sim \frac{\delta M}{M} \frac{R^2}{d^2} \sim \frac{R^5}{d^5}$$

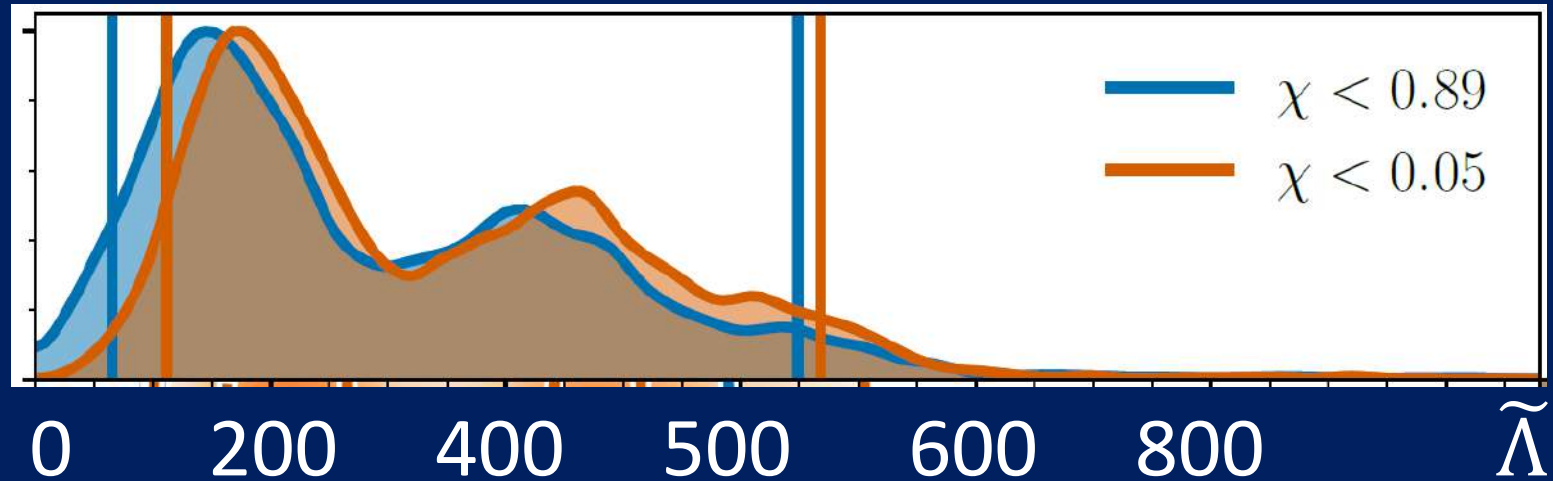
$$\frac{\delta M}{M} \sim \frac{R^3}{d^3}$$

$$\frac{\delta \dot{E}_{\text{GW}}}{\dot{E}_{\text{GW}}} \sim \frac{\delta Q}{Q} \sim \frac{R^5}{d^5}$$

Measuring tidal deformability is equivalent to measuring  $R$  within current GW observational error.

# LIGO/VIRGO analysis of GW170817

Probability density of an average value  $\tilde{\Lambda}$  of the two stars' deformability



9 km



14 km

At 90% probability, neutron stars' radii in the range  
 $8.7 \text{ km} \leq R \leq 14.1 \text{ km}$

Bonus:

Ejected mass is roughly  $\delta M$  near merger,  
say  $d \sim 3R$   $\delta M \sim M/3^3$ , a few percent.

For NS-NS mergers, ejecta from shock  
after merger give a somewhat larger  
contribution, but of the same magnitude.

## V. Implications of post-merger observations:

Maximum mass of neutron star and constraints on the highest-density part of the EOS

# Neutron star masses

Largest seen  
electromagnetically:

$$1.97 \pm 0.04 M_{\odot}$$

Demorest *et al.* '10

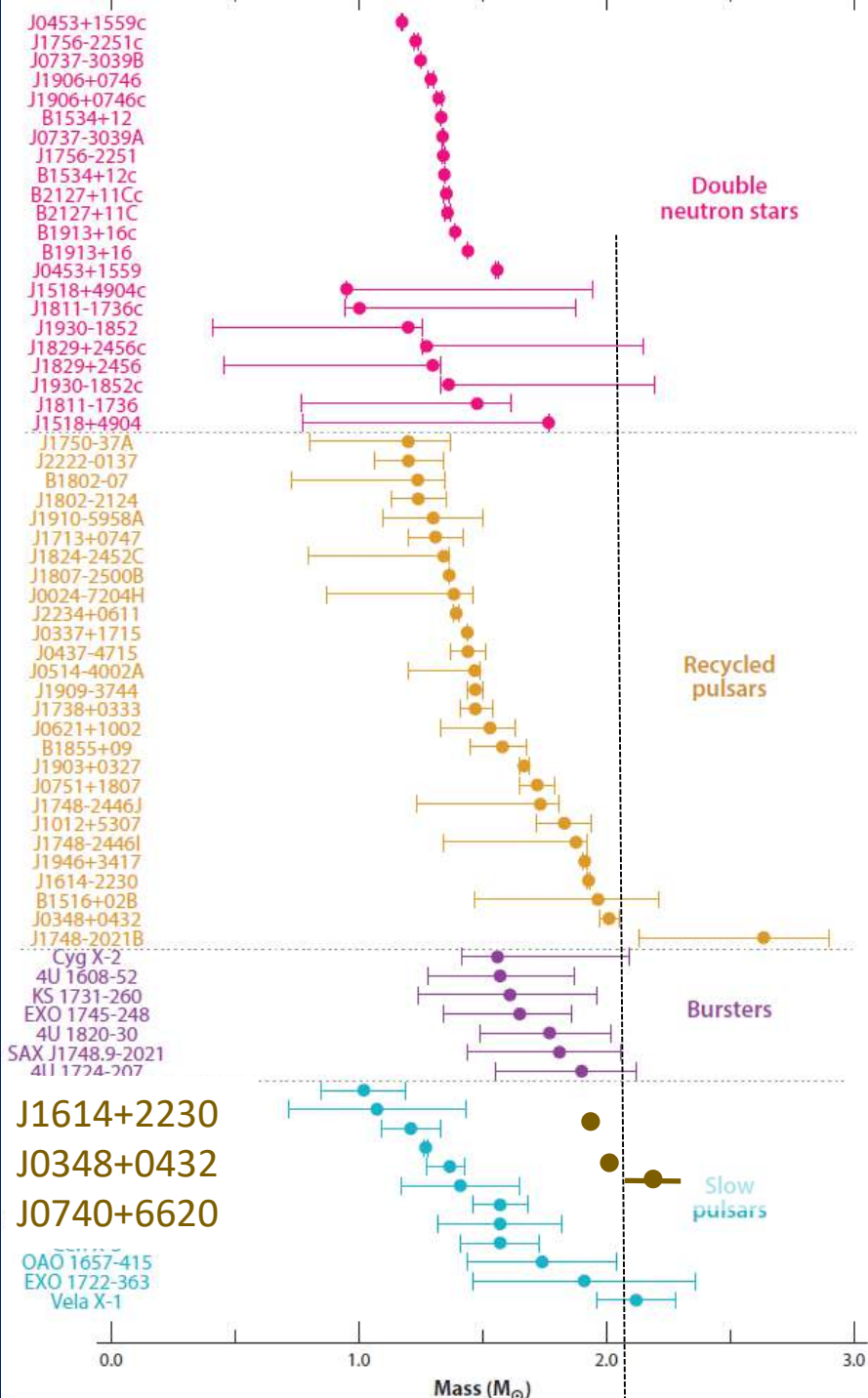
$$2.01 \pm 0.04 M_{\odot}$$

Antoniadis *et al.* '13

$$2.14 \pm 0.1 M_{\odot}$$

Cromatie *et al.* '19

Ozel, Freire, '16



# Maximum mass from post-merger observations

Prompt collapse: Mass above upper limit for hot, differentially rotating star

$$M > M_{\text{max, hot, differential rotation}}$$

Delayed collapse:

$$M_{\text{max, hot, differential rotation}} > M > M_{\text{max, uniform rotation}}$$

No collapse :  $M < M_{\text{max, uniform rotation}}$

# Maximum mass from post-merger observations

Total mass  $\geq 2.73 M_{\odot}$  (LIGO observation)

Mass of ejected material  $\leq 0.08 M_{\odot}$   
(e-m observations+simulations)

No prompt collapse implies

$$M_{\text{max, hot, differential rotation}} \geq 2.73 - 0.08 = 2.65 M_{\odot}$$

Rotating stellar models with a threshold mass this large have

$$M_{\text{max, spherical}} > 2.15 M_{\odot}$$



# Maximum mass from post-merger observations

Initially remnant differentially rotating and hot.  
It collapsed before the differential rotation ended and it reached uniform rotation. The maximum mass of a uniformly rotating cold neutron star is

$$M_{\text{max, uniform rotation}} = 1.2 M_{\text{max, spherical}}$$

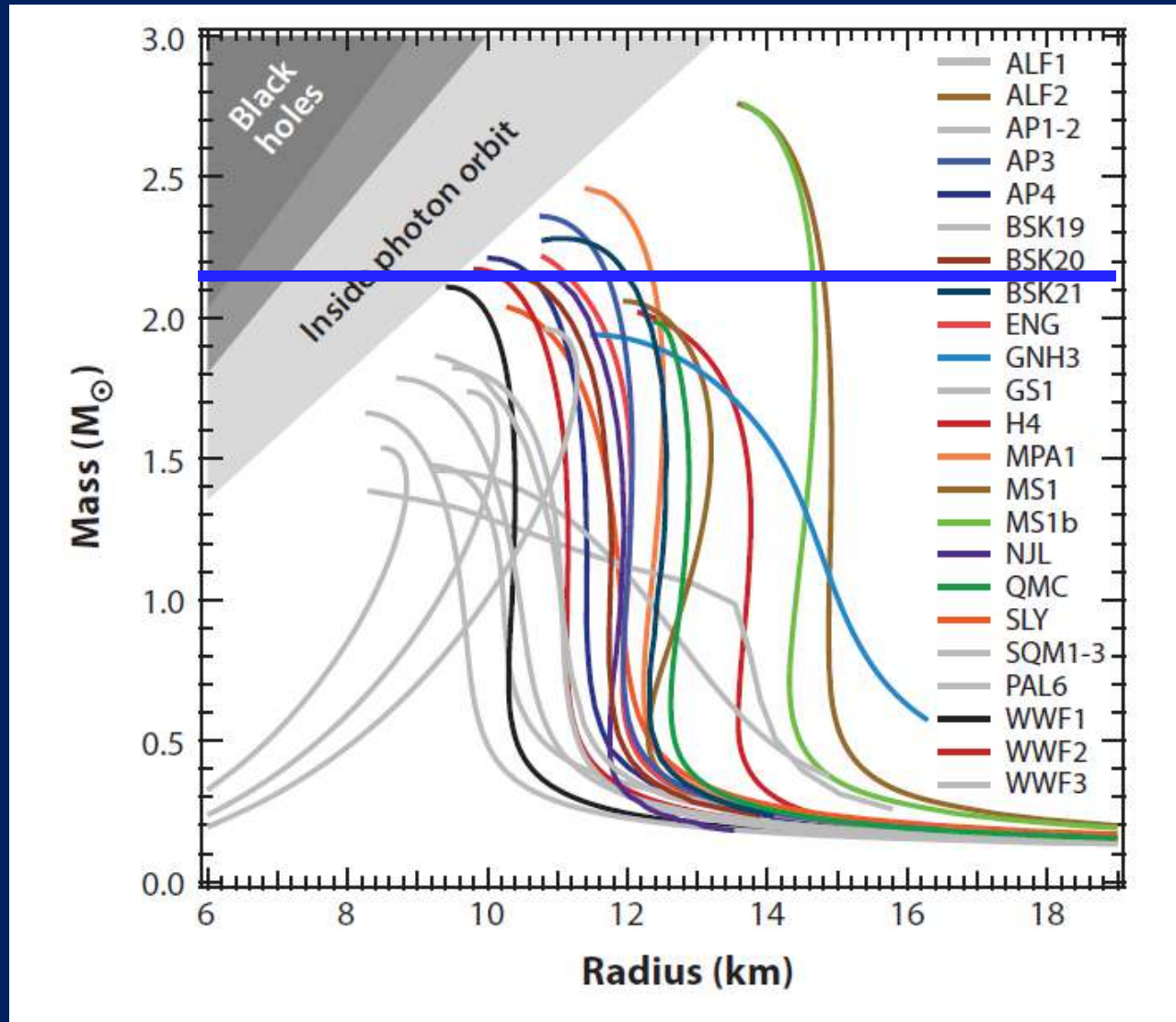
$$M_{\text{max, spherical}} < 2.73 M_{\odot} / 1.2 = 2.28 M_{\odot}.$$

Combined,

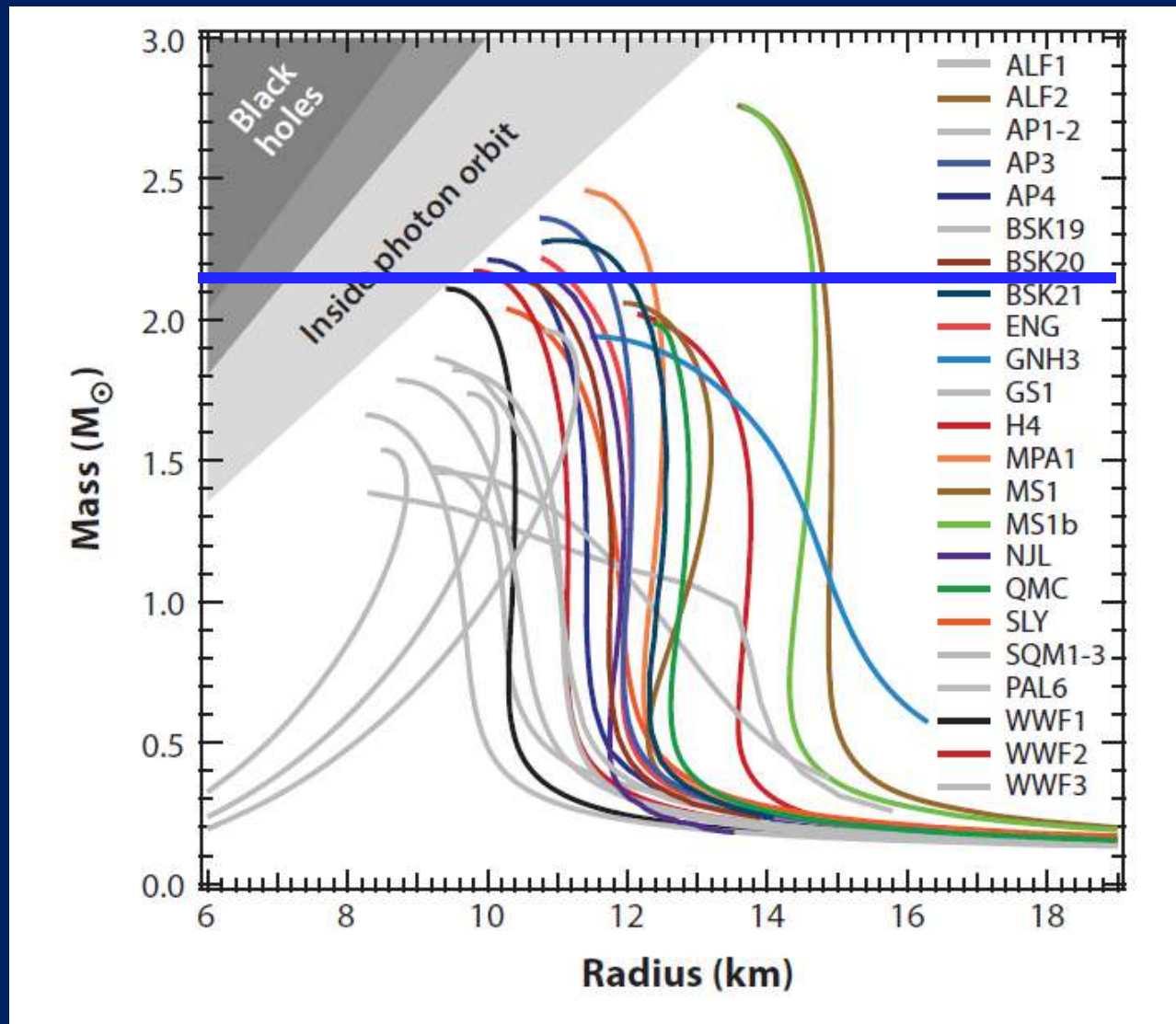
$$2.15 M_{\odot} < M_{\text{max, spherical}} < 2.28 M_{\odot}$$

More stringent claims by various authors:  
e.g., Margalit & Metzger find  $M_{\text{max}} < 2.17 M_{\text{sun}}$ .  
These rely on modeling the ejecta and so have more  
systematic uncertainty.

Equations of state that are soft at high density are ruled out by a  $2.15 M_{\odot}$  neutron star



Cores with hyperons, quark cores, pion or kaon condensates (gray curves) all increasingly unlikely.



## VI. The Hubble Constant

The Hubble constant  $H_0$  relates the velocity at which galaxies recede from us to their distance.

$$H_0 = \frac{v}{d}, \quad d = \text{proper distance to galaxy}$$

If can identify the galaxy of an observed binary inspiral, have  $v$  from the galaxy's redshift.

Find the distance  $d$  from the inspiral waveform as follows

For quadrupole radiation,  
GW frequency = 2 x orbital frequency

$$\omega = 2 \Omega$$

$E =$  Newtonian energy of binary  $\propto -\mu a^2 \Omega^2$   
 $\propto \omega^{3/2}$ , (by Kepler's law  $a \propto \Omega^{-2/3}$ )  $\Rightarrow$

$$\frac{\dot{E}}{E} = \frac{3 \dot{\omega}}{2 \omega}$$

For quadrupole radiation,

$$\text{Amplitude } h \propto \frac{\ddot{Q}}{D} \propto \frac{\mu a^2 \Omega^2}{D} \propto \frac{E}{D}$$

D = luminosity distance  
=  $d(1+z)$

Finally,

$$\dot{E} \propto \hbar^2 D^2 = \omega^2 \hbar^2 D^2$$

$$\text{Then } \frac{\dot{\omega}}{\omega} = \frac{3 \dot{E}}{2 E} \propto \frac{\omega^2 \hbar^2 D^2}{\hbar D}$$

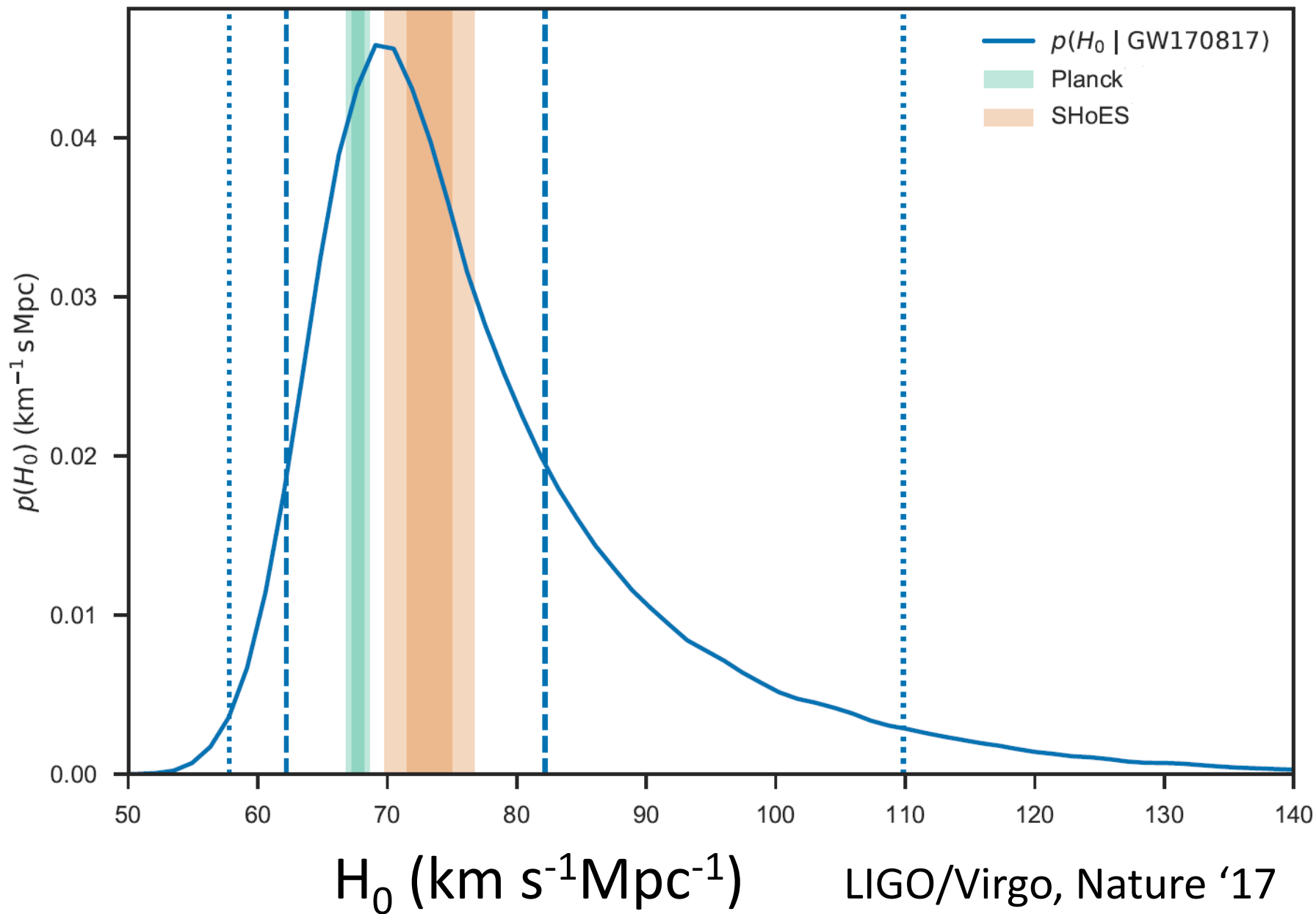
$$\text{implying } D \propto \frac{\dot{\omega}}{\omega^3 \hbar}$$

With numerical constants kept,

$$D \propto 780 \frac{\dot{f}_{100}}{f_{100}^3 (h / 10^{-23})} \text{Mpc}, \quad f_{100} := \frac{f}{100 \text{Hz}}$$



# GW710817:

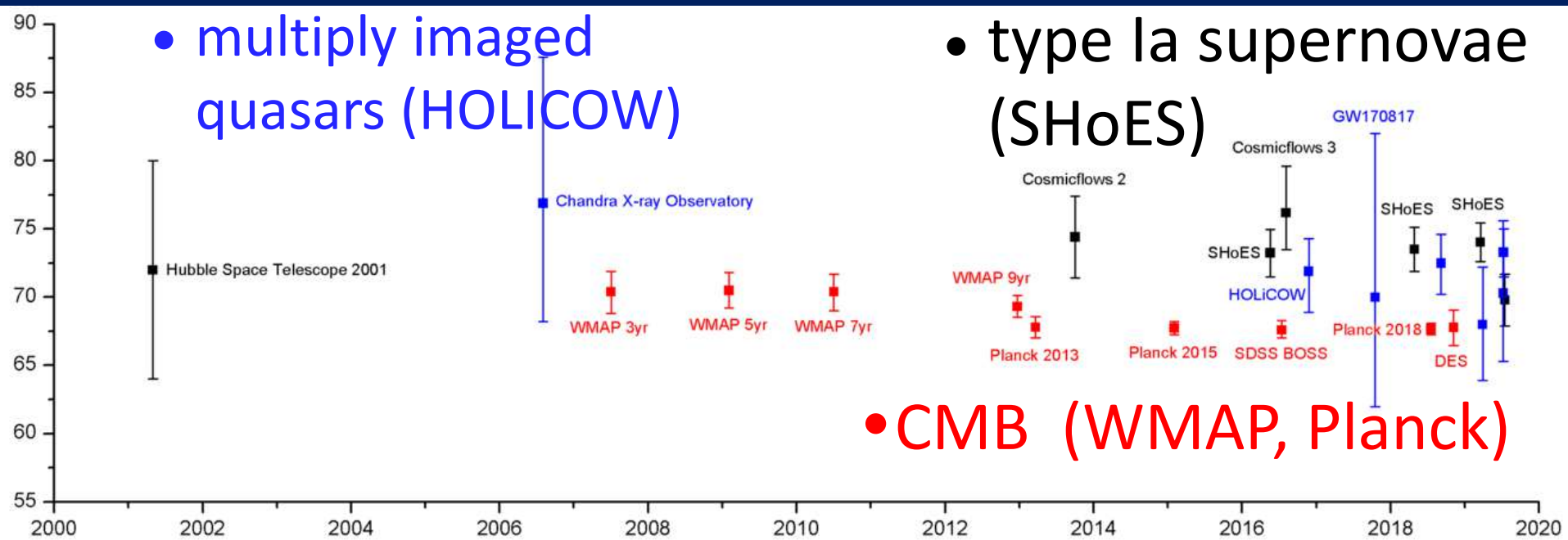


Need  $\sim 50$  similar detections to resolve the issue

- multiply imaged quasars (HOLICOW)

- type Ia supernovae (SHoES)

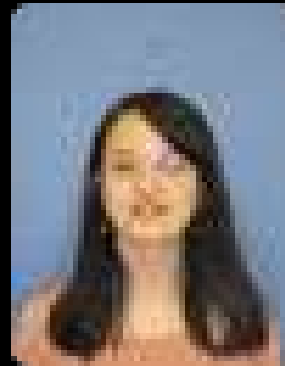
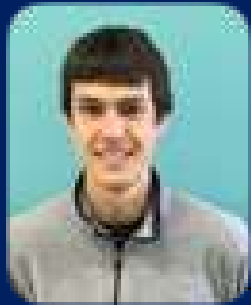
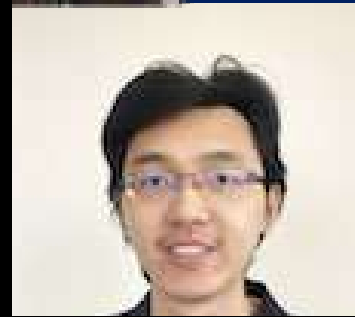
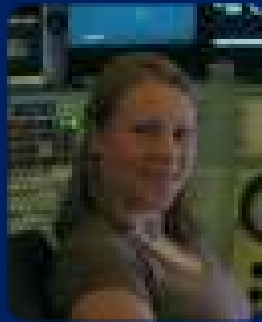
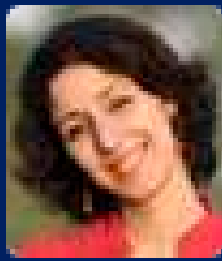
- CMB (WMAP, Planck)



Wikipedia compendium



Postdocs, faculty, and research scientists  
at UWM in gravitational wave astronomy  
and at the Leonard Parker Center for  
Gravitation, Cosmology and Astrophysics



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