Unruh meets Larmor:

acceleration, radiation, and the surprising role of zero-energy Rindler modes

André G. S. Landulfo

UFABC-Brazil

(in collaboration with S. Fulling and G. Matsas)

FAPESP

Radiation emitted by an accelerated charge

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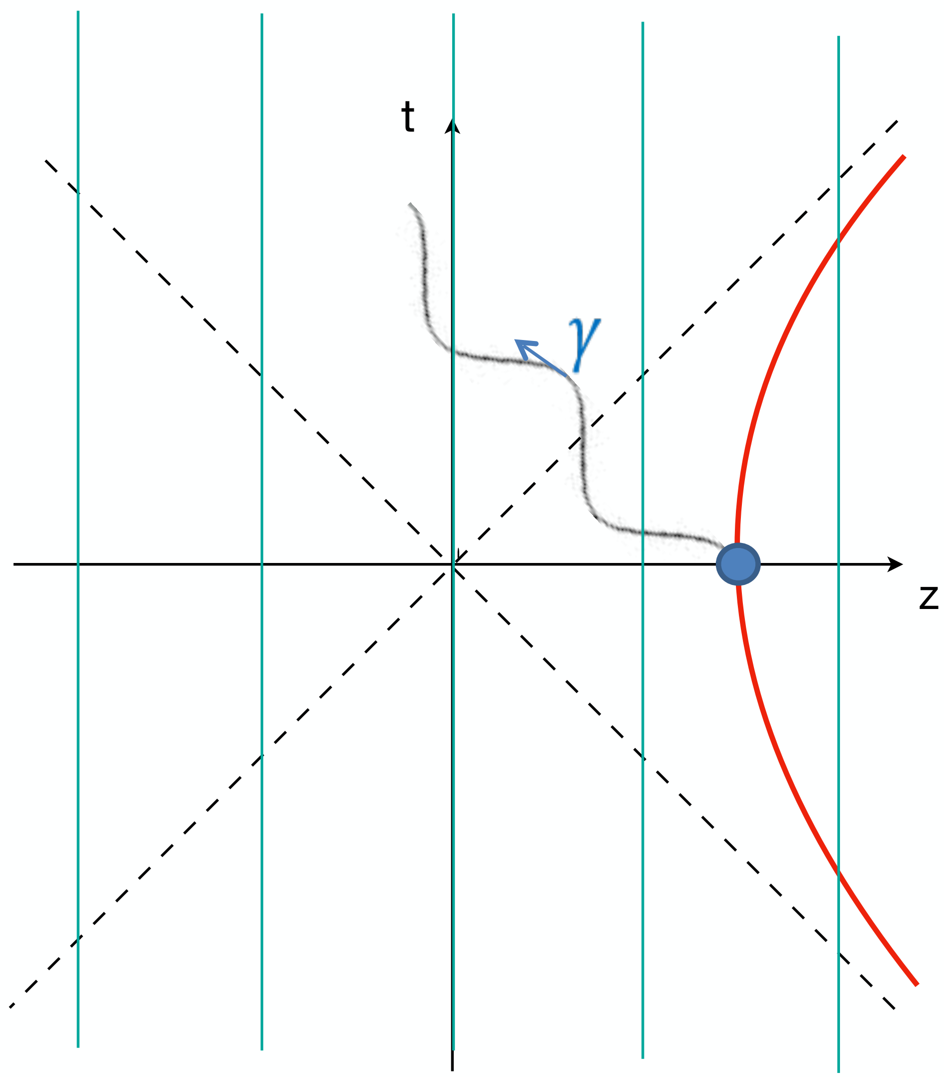

Uniformly accelerated charge
[proper acceleration a]

Acceleration and radiation:

➡Uniformly accelerated charge radiates (w.r.t. inertial observers) with power $P = \dfrac{2e^2a^2}{3c^3}$ (Larmor, 1897)

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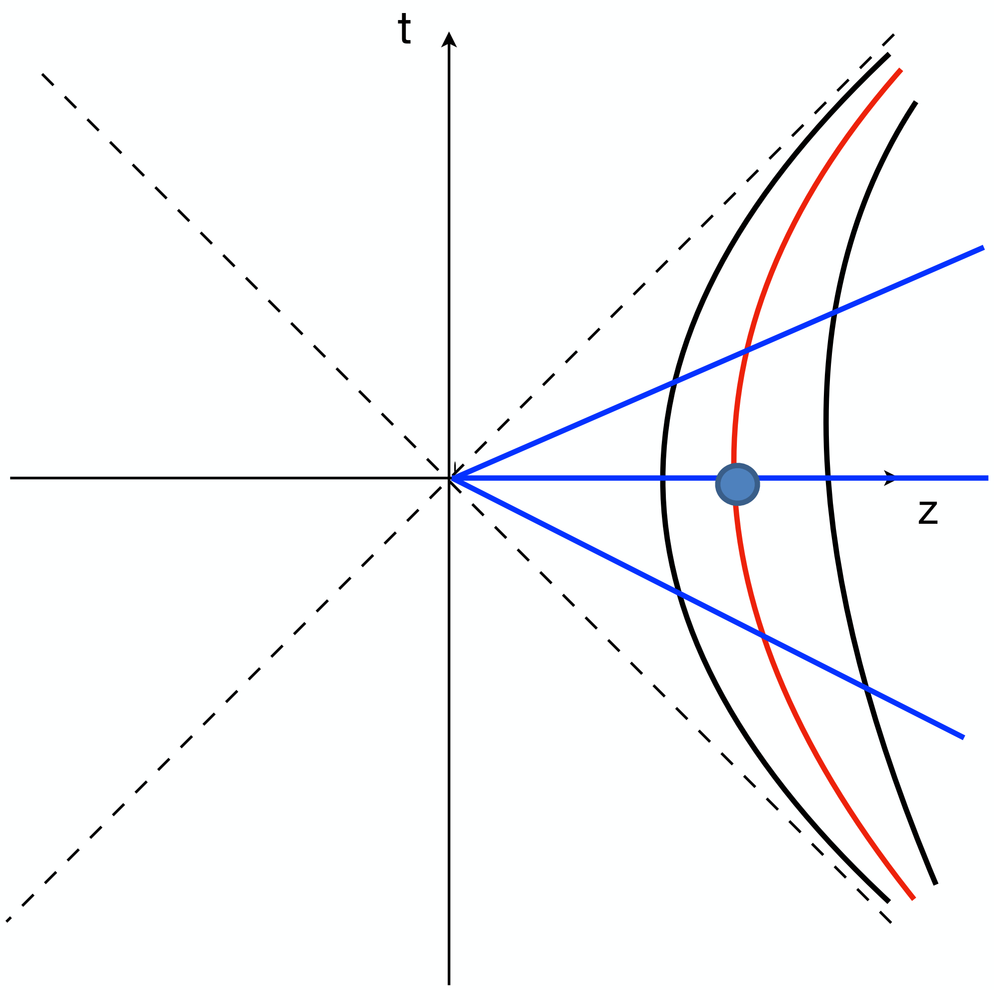


Uniformly accelerated charge
[proper acceleration a]

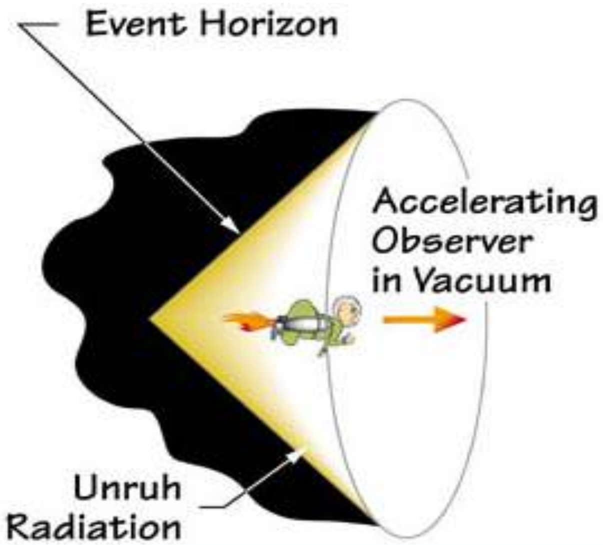
Acceleration and radiation:

- Uniformly accelerated charge radiates (w.r.t. inertial observers) with power $P = \frac{2e^2a^2}{3c^3}$ (Larmor, 1897)
- Radiation concept is observer-dependent: co-accelerating observers see no acceleration (Rohrlich, 1961 and Boulware, 1980)

The Unruh Effect

Bill Unruh, "Notes on black hole evaporation", Phys. Rev. D, 14, 870 (1976).

Uniformly accelerating observers in Minkowski vacuum experience a thermal bath of elementary particles at a temperature proportional to their proper acceleration.



$$T_{\rm U} = \frac{\hbar a}{2\pi c k_{\rm B}} = a/(10^{21} cm/s^2)\, K$$

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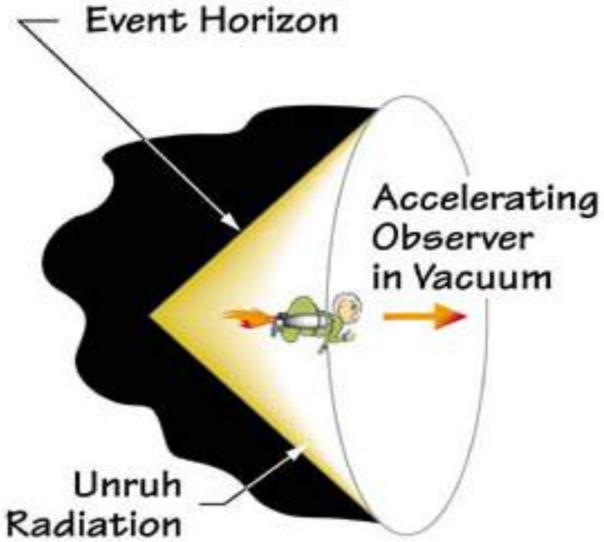

$$T_{\mathrm{U}} = \frac{\hbar a}{2\pi c k_{\mathrm{B}}} = a/(10^{21} cm/s^2)\, K$$

The Unruh effect vindicated Fulling's previous discovery that the particle concept in QFT is observer dependent.

Steve Fulling, "Nonuniqueness canonical field quantization in Riemannian space-time", Phys. Rev. D, 7, 2850 (1973).

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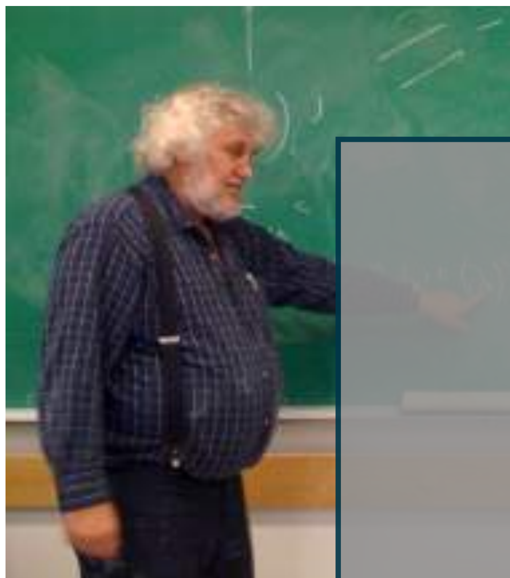

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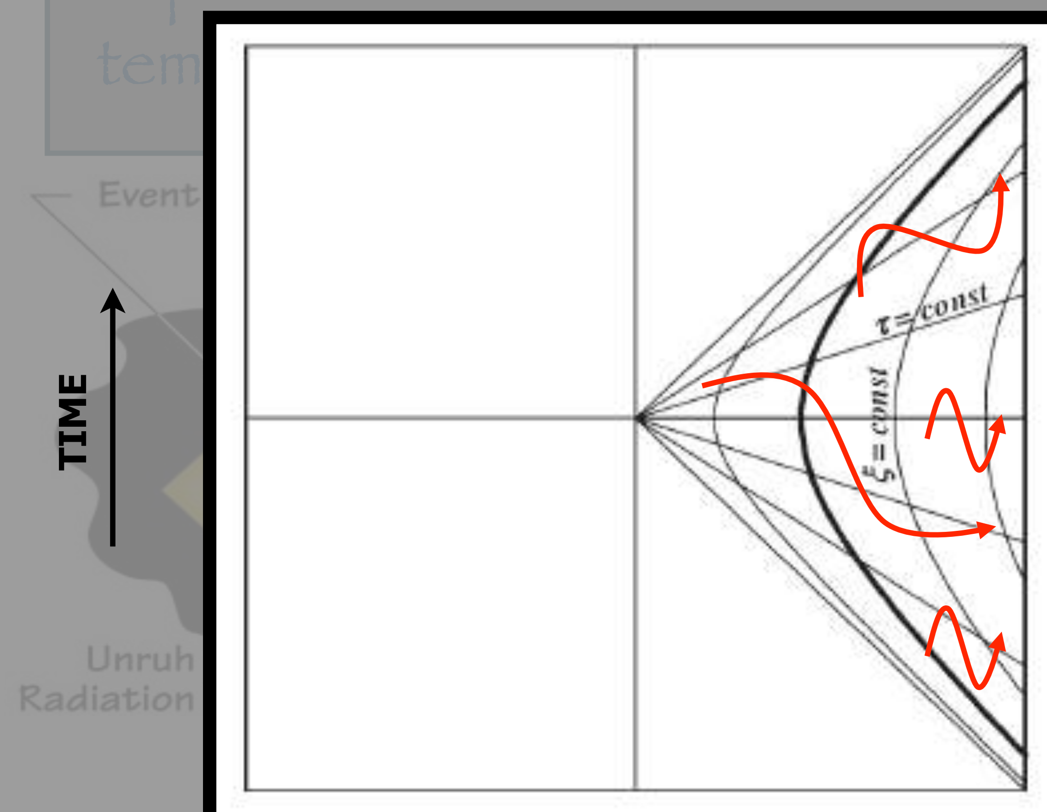


$a/(10^{21}\, cm/s^2)\, K$

The Unruh effect vindicated Fulling's previous discovery that the particle concept in QFT is observer dependent.

$$\rho = \prod_i (C_i^2 \sum_{n_i} e^{\frac{-2\pi n_i \omega_i}{a}} |n_{iR}\rangle\langle n_{iR}|)$$

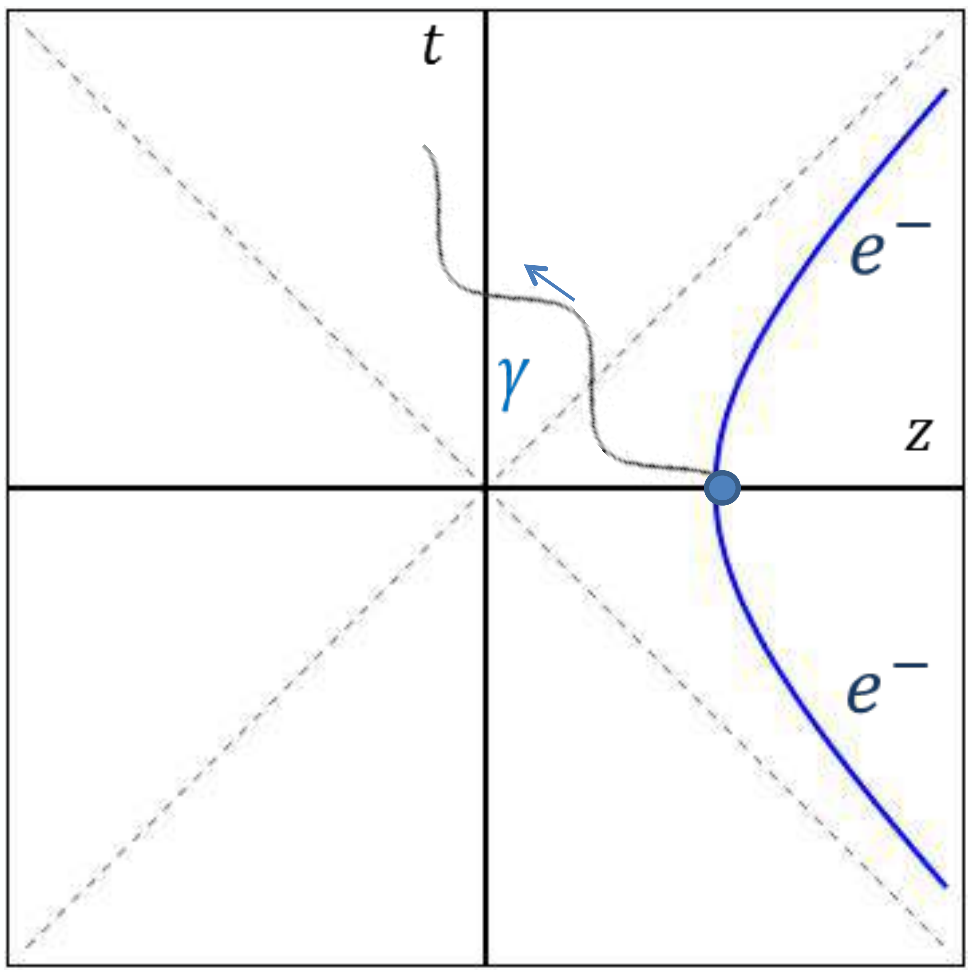
Steve Fulling, "Nonuniqueness canonical field quantization in Riemannian space-time", Phys. Rev. D, 7, 2850 (1973).

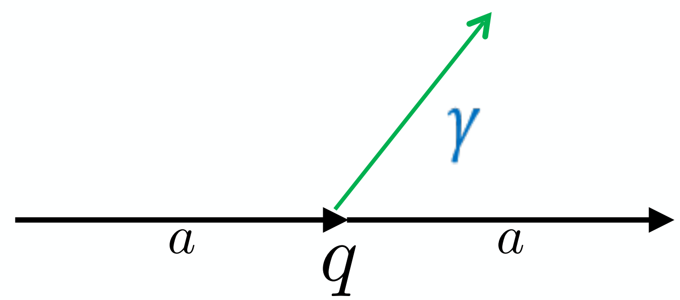
The Unruh Effect and bremsstrahlung

Higuchi, Matsas, and Sudarsky, PRD 46 3450

Inertial Calculation

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$





$$\hat{S}_I = \int d^4x \sqrt{-g}\hat{A}^\mu(x) j_\mu(x)$$

$$\hat{A}^\mu = \sum_\sigma \int d^3\mathbf{k}\, [\hat{a}_{\sigma\mathbf{k}} A^\mu_{\sigma\mathbf{k}} + H.c.]$$

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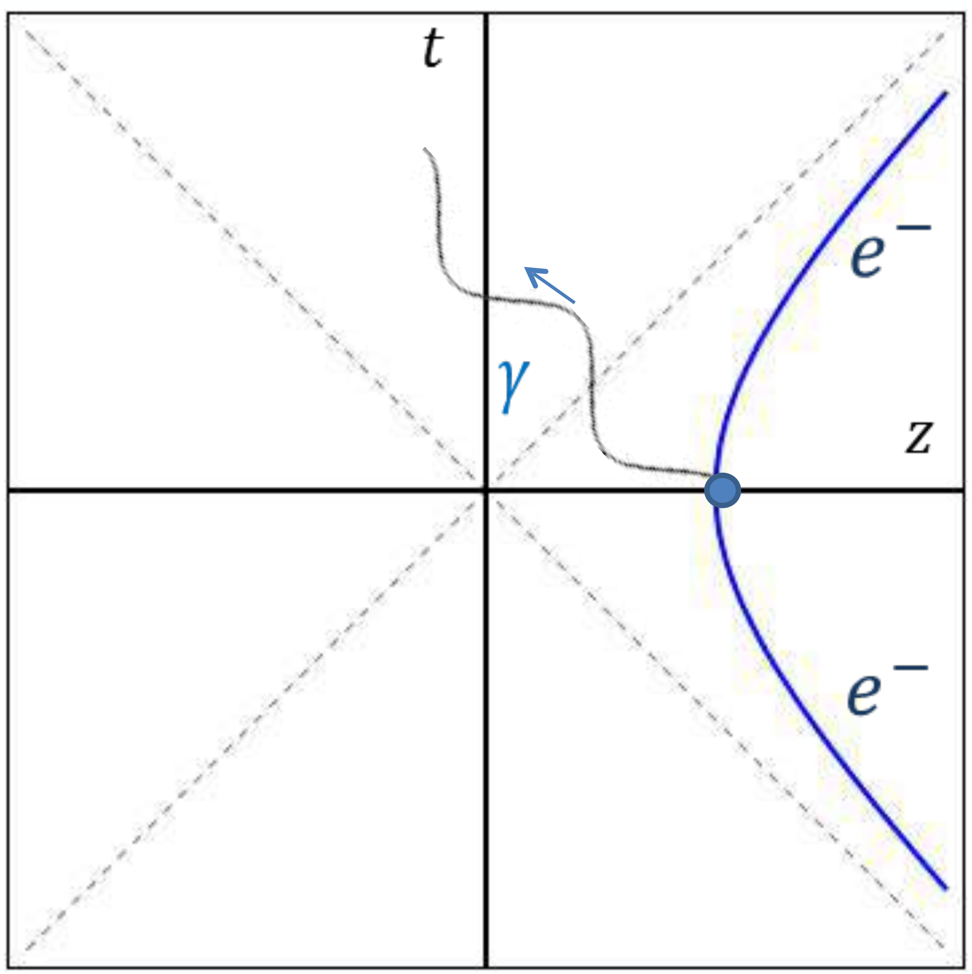

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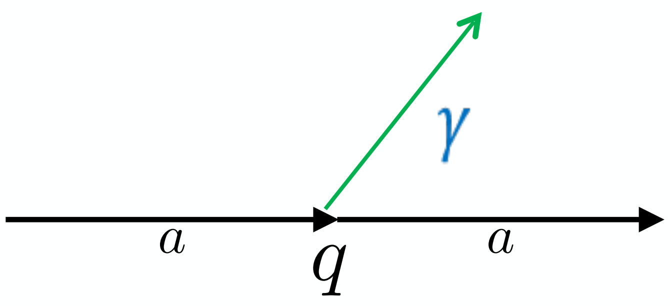

$$\hat{S}_I = \int d^4x \sqrt{-g}\hat{A}^\mu(x) j_\mu(x)$$

Emission rate per transverse momentum:

$$\frac{d\Gamma}{d^2\mathbf{k}_\perp} = \frac{q^2}{4\pi^3 a} |K_1(k_\perp/a)|^2$$

The Unruh Effect and bremsstrahlung

Higuchi, Matsas, and Sudarsky, PRD 46 3450 (1992)

Uniformly Accelerated Frame

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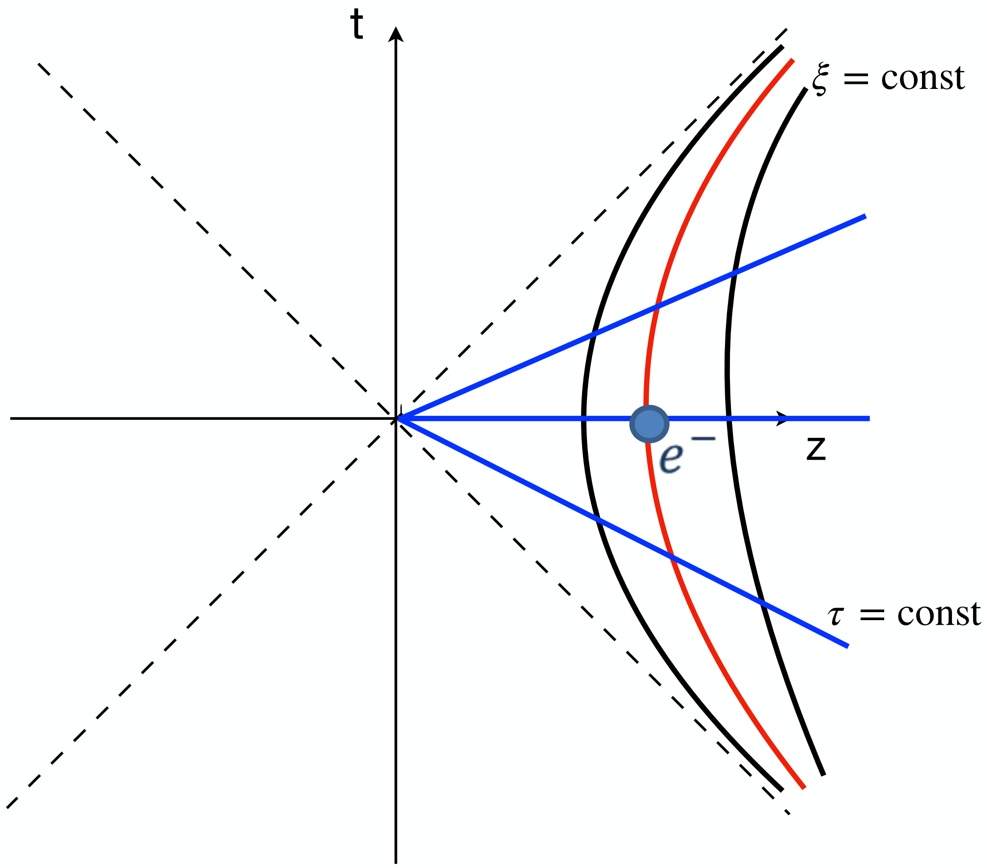
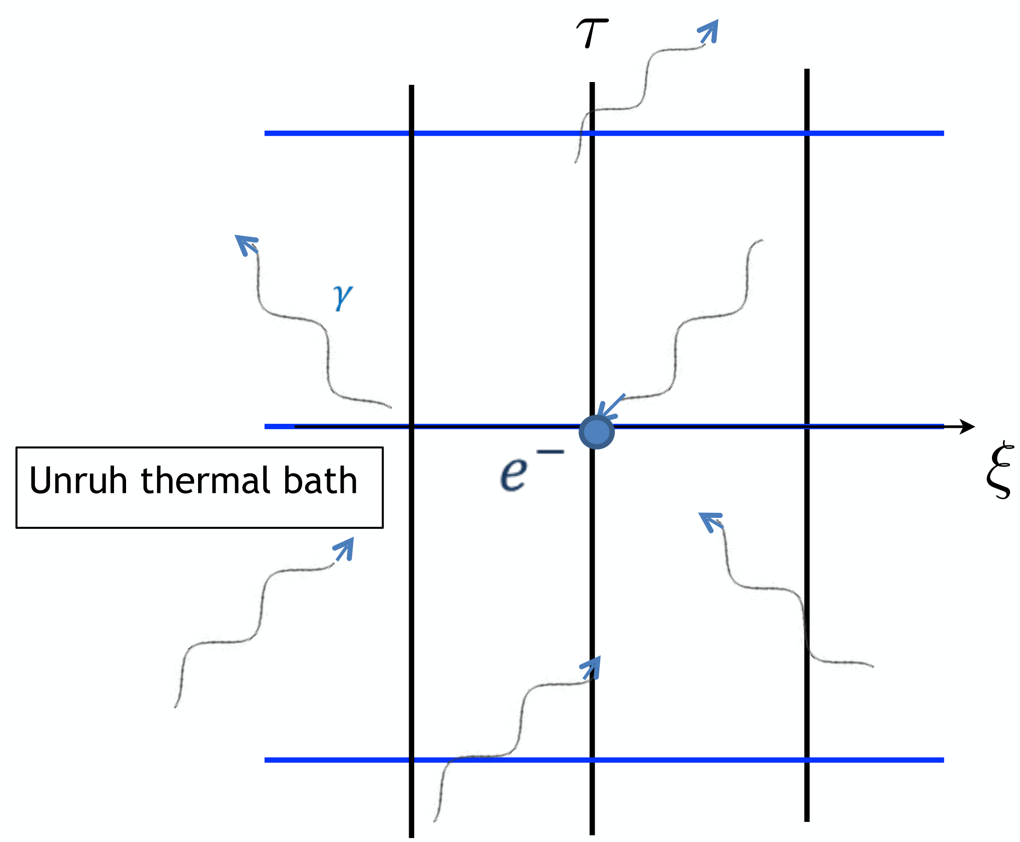


$$ds^2 = e^{2a\xi}\left(-d\tau^2 + d\xi^2\right) dy^2 + dz^2$$



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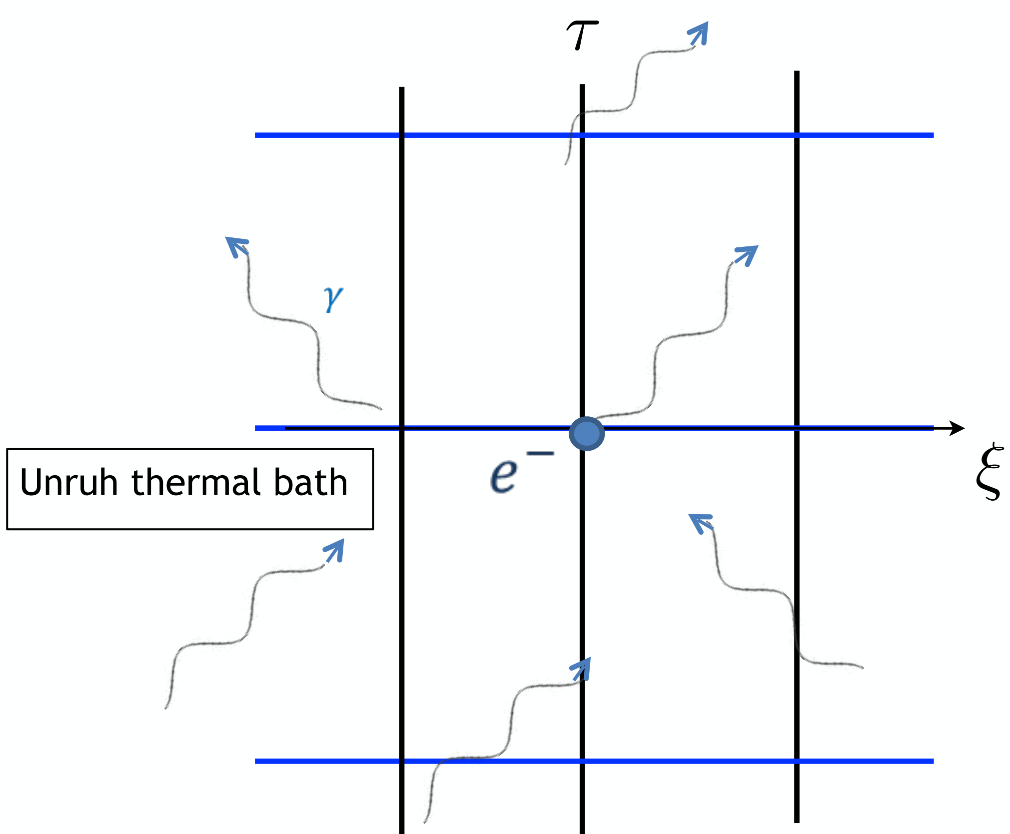

Emit zero-energy Rindler photon

$$\frac{d\Gamma^{em}}{d^2\mathbf{k}_\perp} = \frac{q^2}{8\pi^3 a}|K_1(k_\perp/a)|^2$$

$$\hat{A}^\mu = \sum_\sigma \int_0^\infty d\omega \int d^2\mathbf{k}_\perp \left[\hat{b}_{\sigma\omega\mathbf{k}_\perp} A^\mu_{\sigma\omega\mathbf{k}_\perp} + H.c.\right]$$

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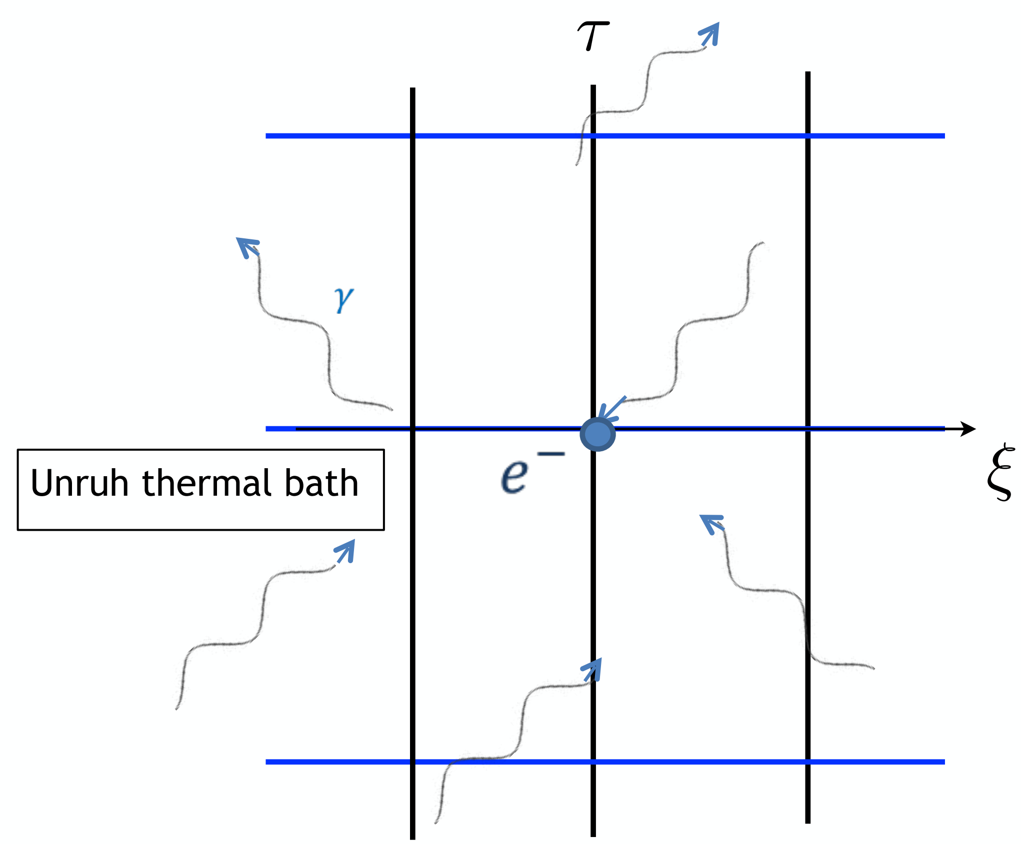

Absorb zero-energy Rindler photon

$$\frac{d\Gamma^{abs}}{d^2\mathbf{k}_\perp} = \frac{q^2}{8\pi^3 a}|K_1(k_\perp/a)|^2$$

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The Unruh Effect and bremsstrahlung

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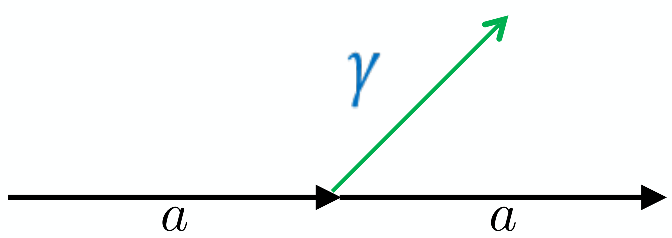
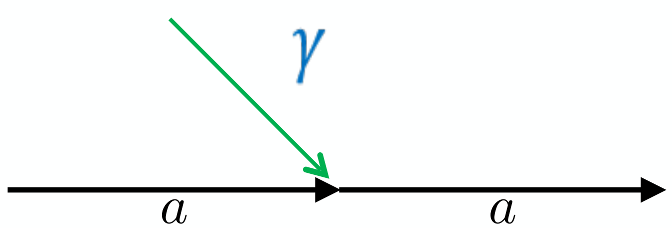
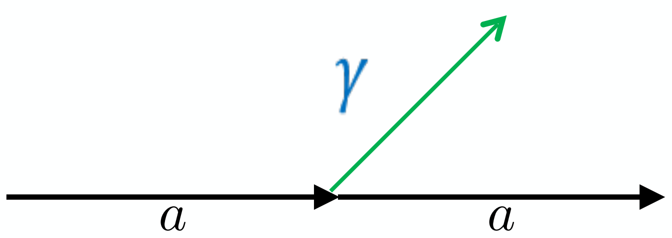
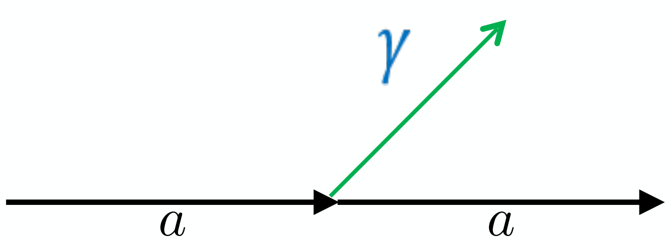




Inertial Frame





The Unruh Effect and bremsstrahlung

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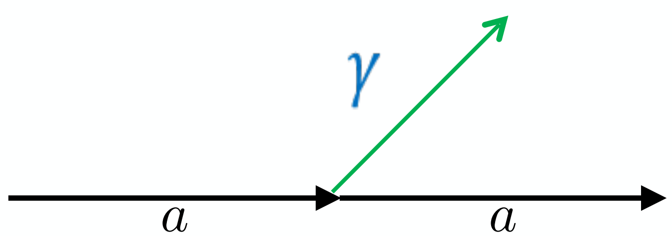

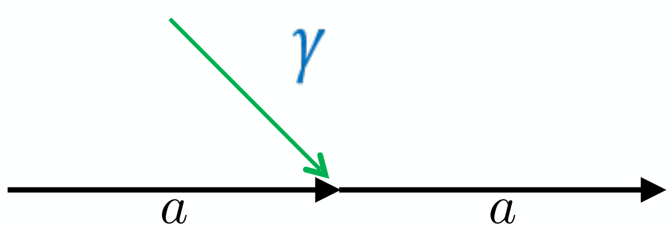

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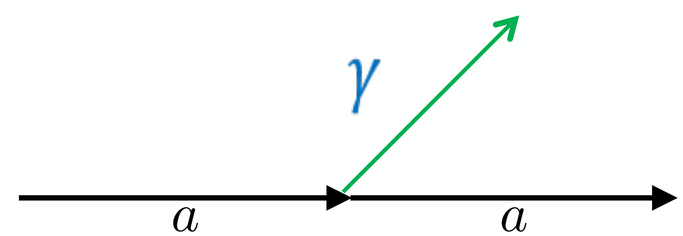
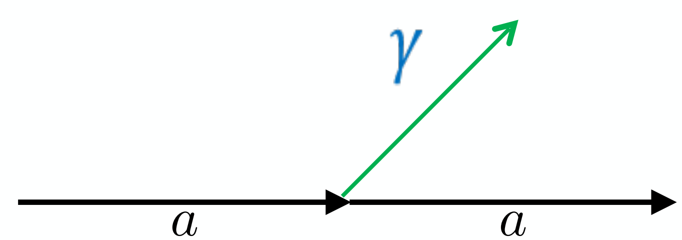
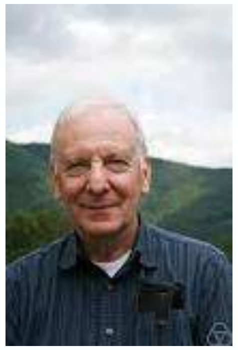




Bill Unruh and Bob Wald, "What happens when an accelerating observer detects a Rindler particle", Phys. Rev. D 29, 1047 (1984).



The Unruh Effect and bremsstrahlung

Higuchi, Matsas, and Sudarsky, PRD 46 3450 (1992)

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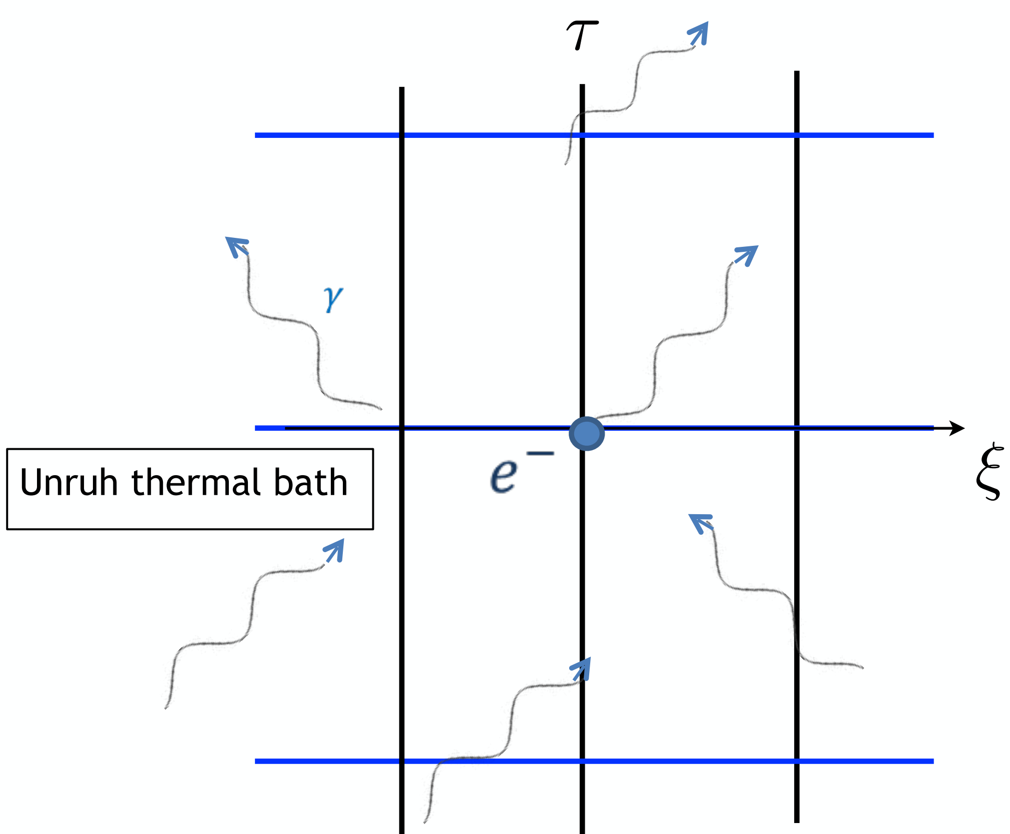

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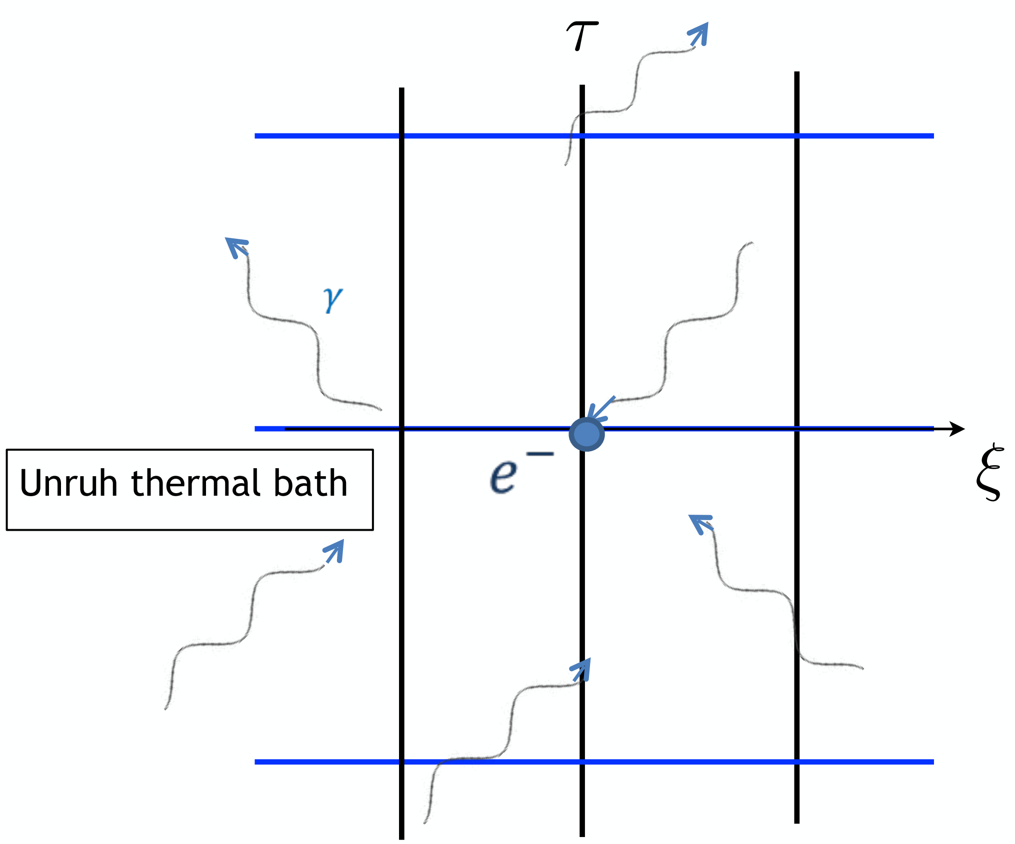

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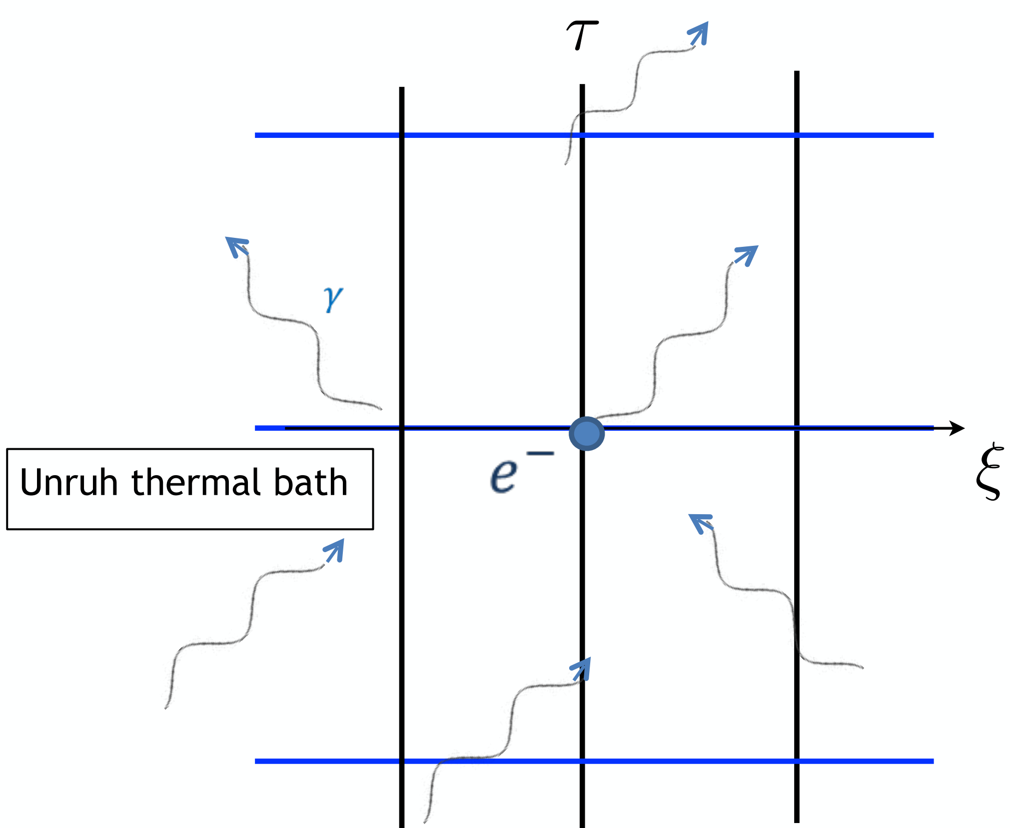

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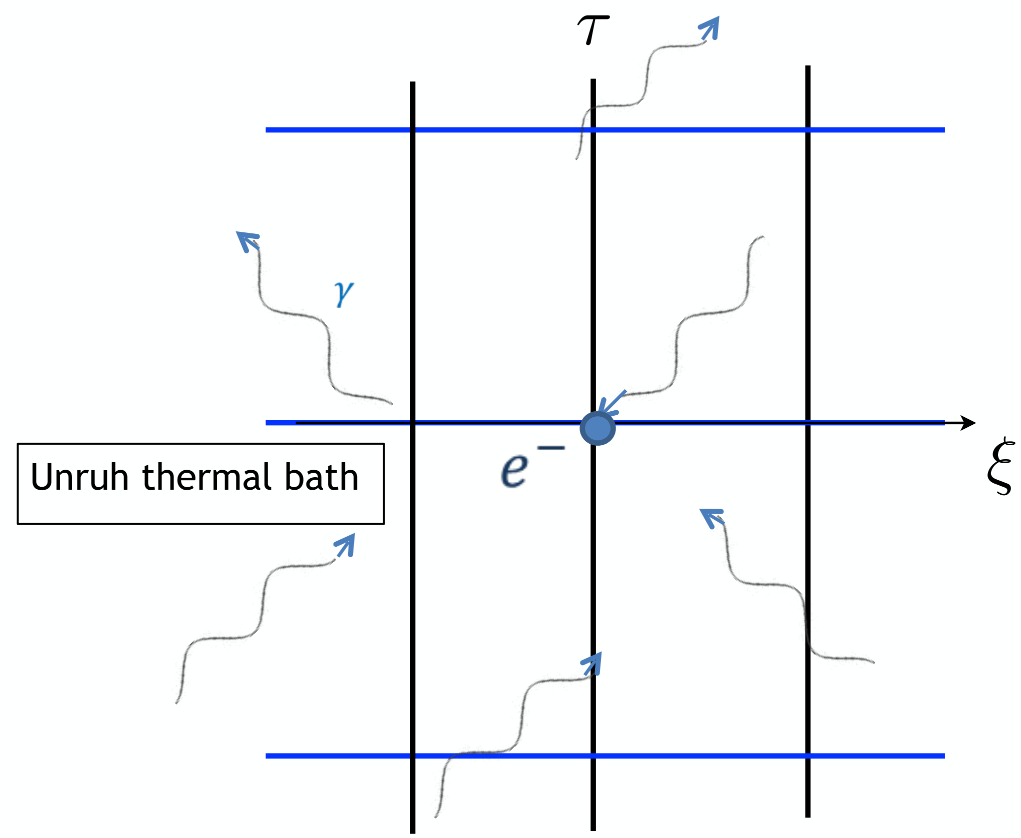

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The Unruh Effect and bremsstrahlung

PRL **118**, 161102 (2017) PHYSICAL REVIEW LETTERS week ending 21 APRIL 2017

Proposal for Observing the Unruh Effect using Classical Electrodynamics

Gabriel Cozzella,[1,*] André G. S. Landulfo,[2,†] George E. A. Matsas,[3,‡] and Daniel A. T. Vanzella[4,§]

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The Unruh Effect and bremsstrahlung

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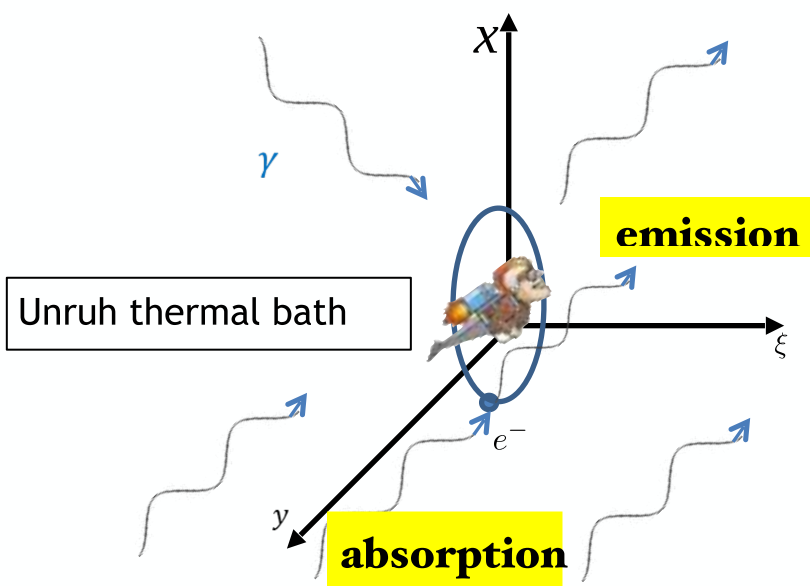
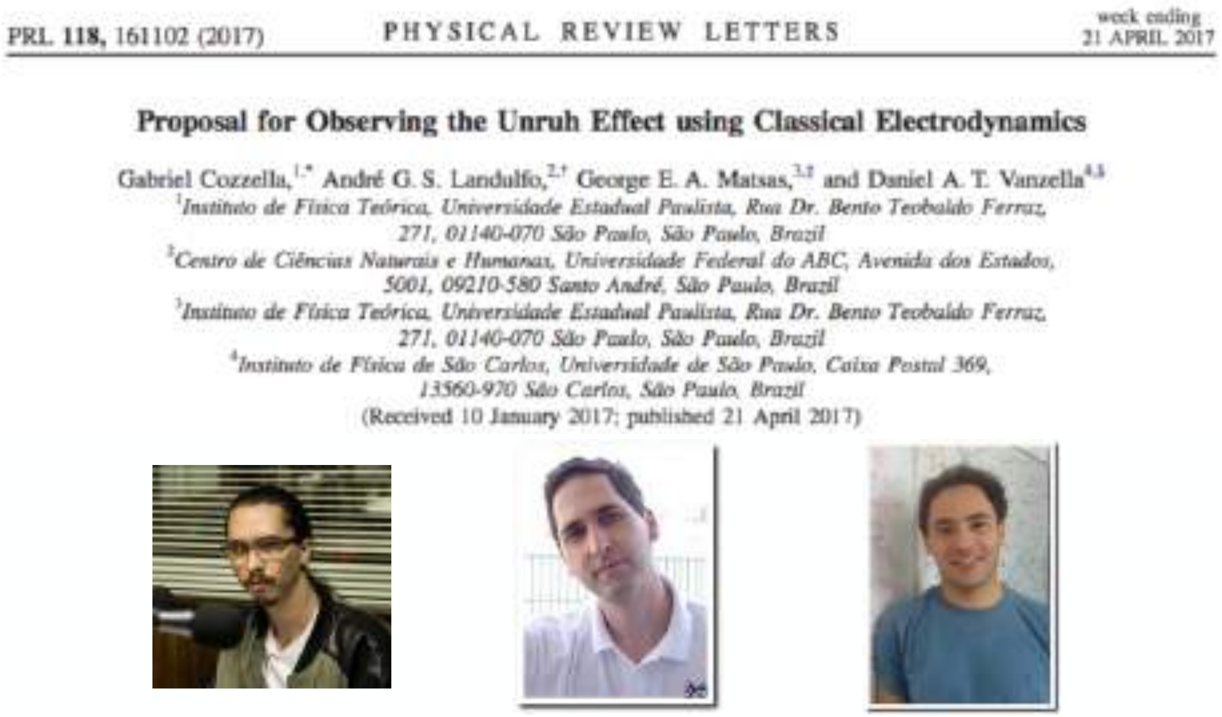
Accelerated Observer



Accelerated observer assumes the existence of the Unruh thermal bath

The Unruh Effect and bremsstrahlung



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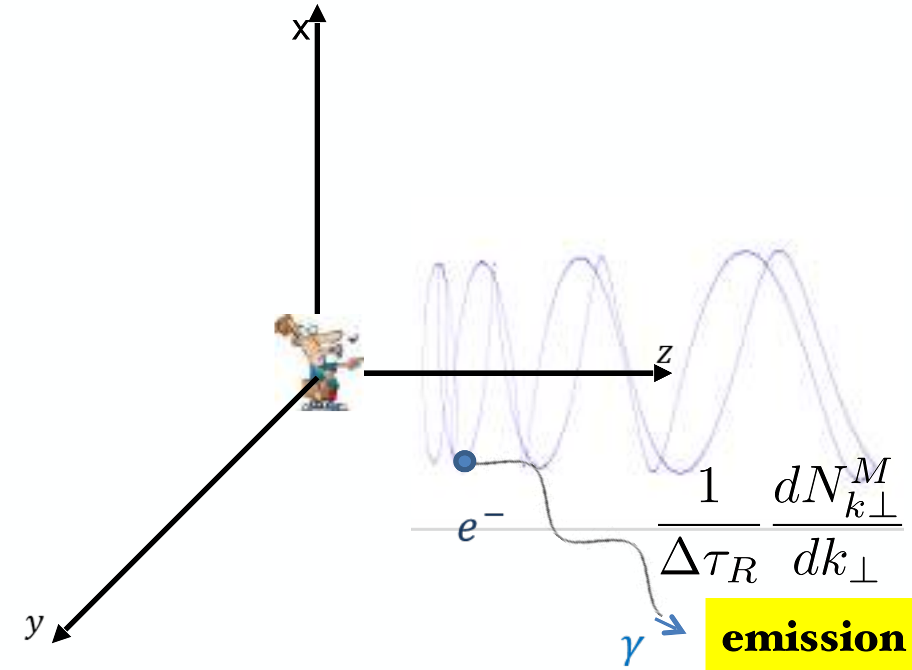
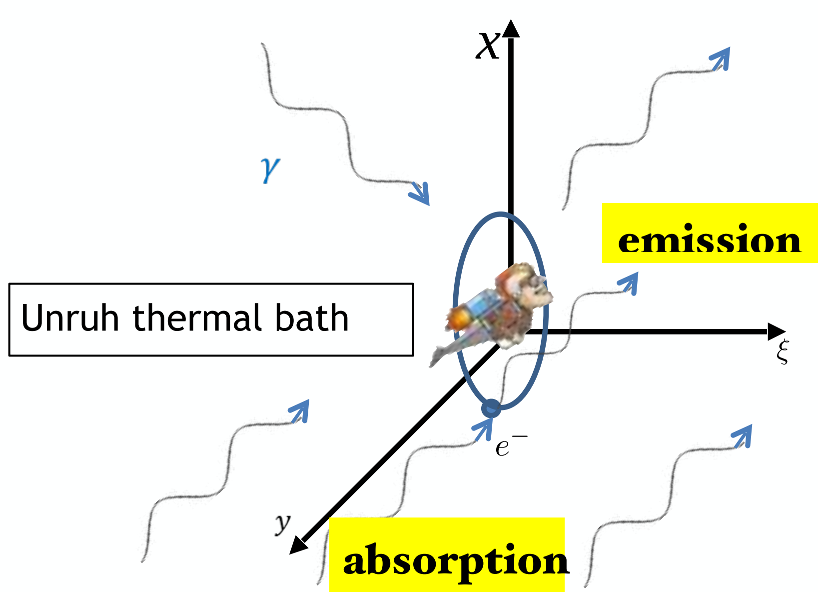
Inertial Observer



and predicts what an inertial experimentalist should observe

Accelerated Observer



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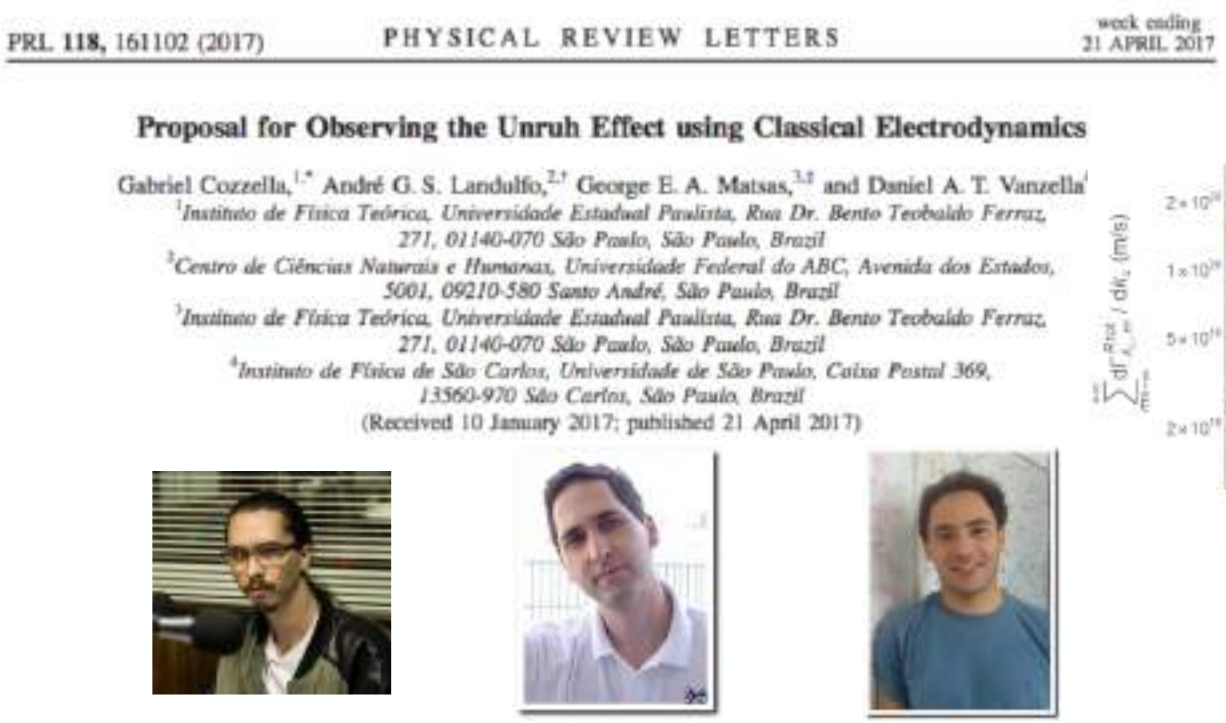
The Unruh Effect and bremsstrahlung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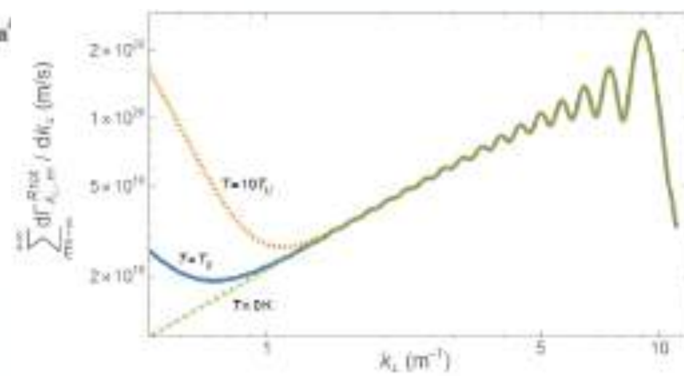

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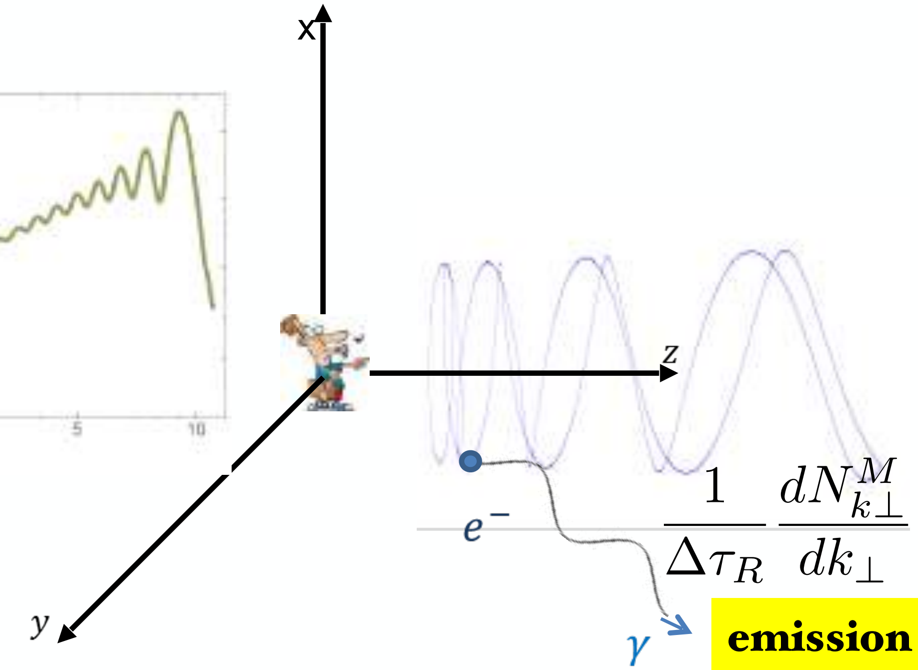

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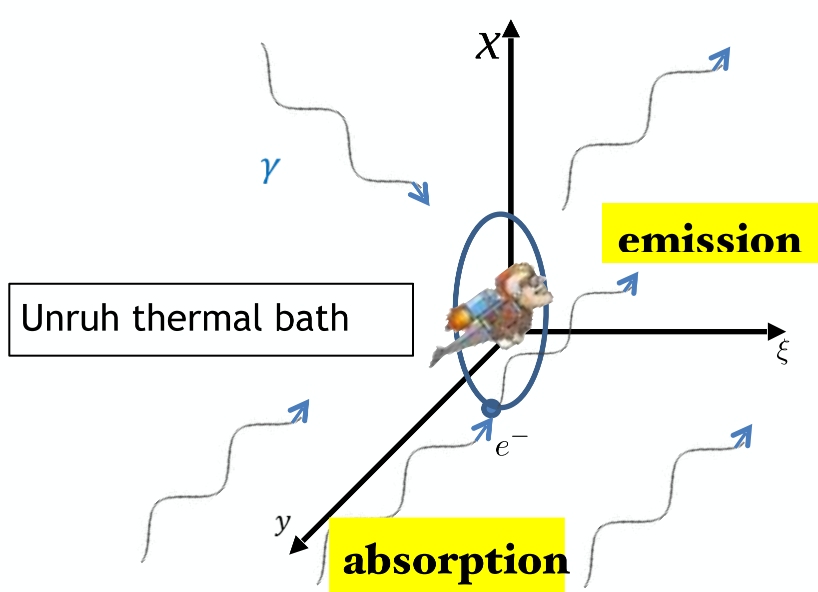
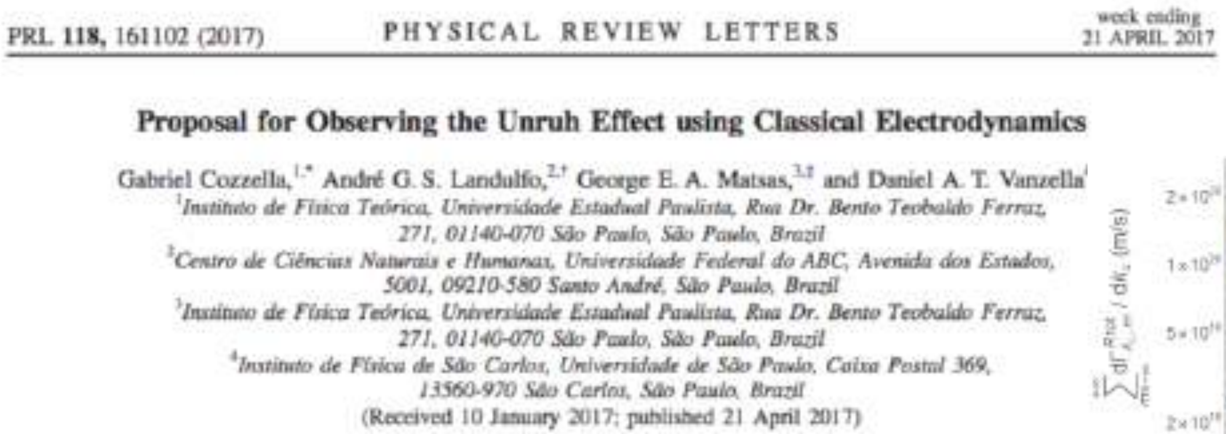


Accelerated observer assumes the existence of the Unruh thermal bath

and predicts what an inertial experimentalist should observe

The Unruh Effect and bremsstrahlung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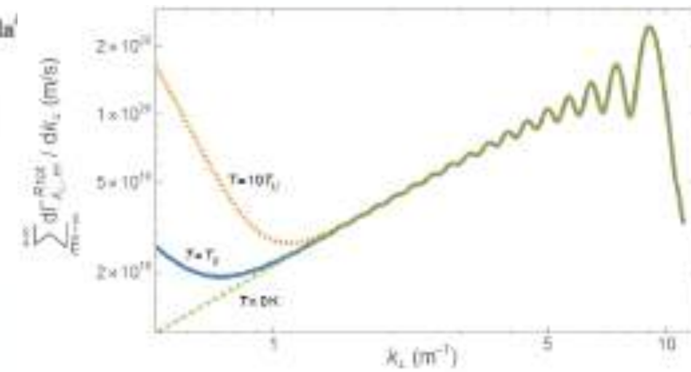

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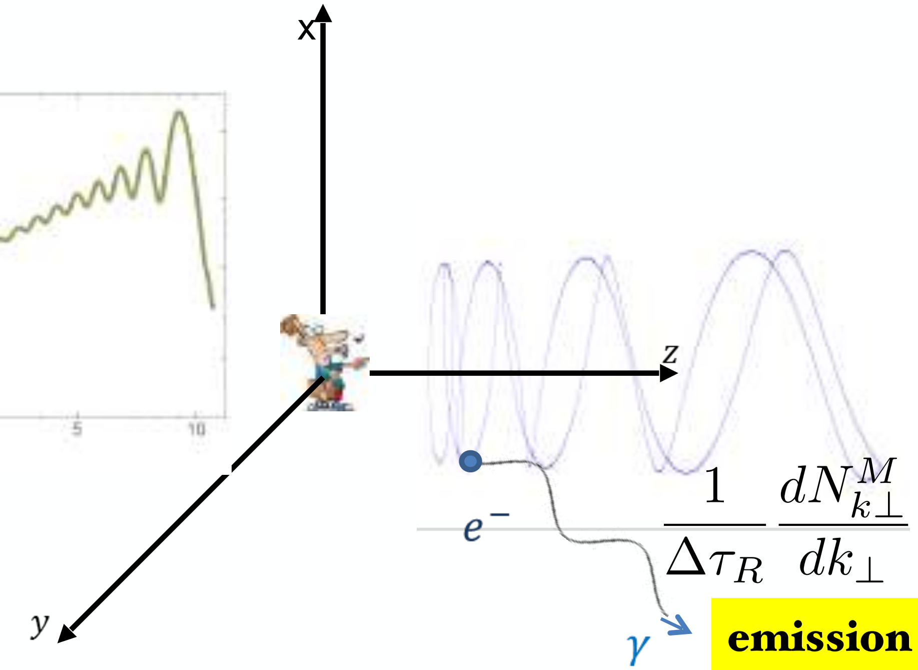

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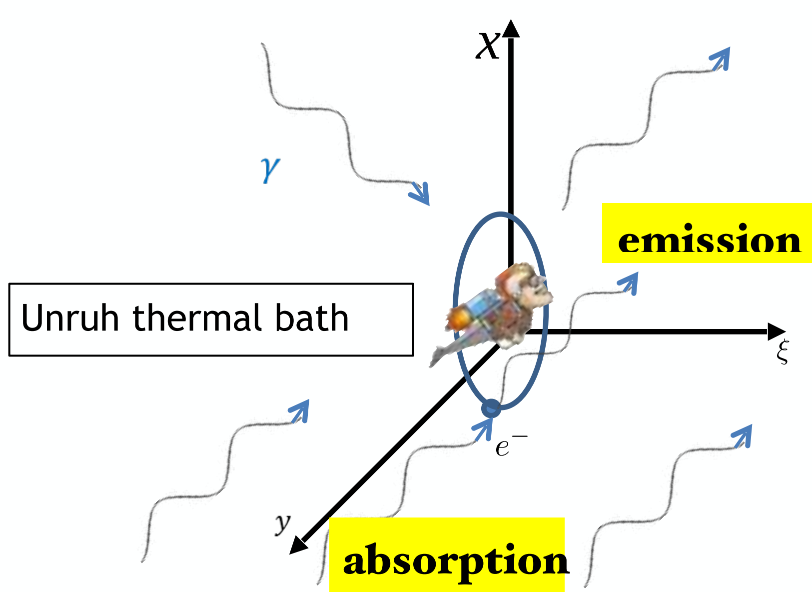


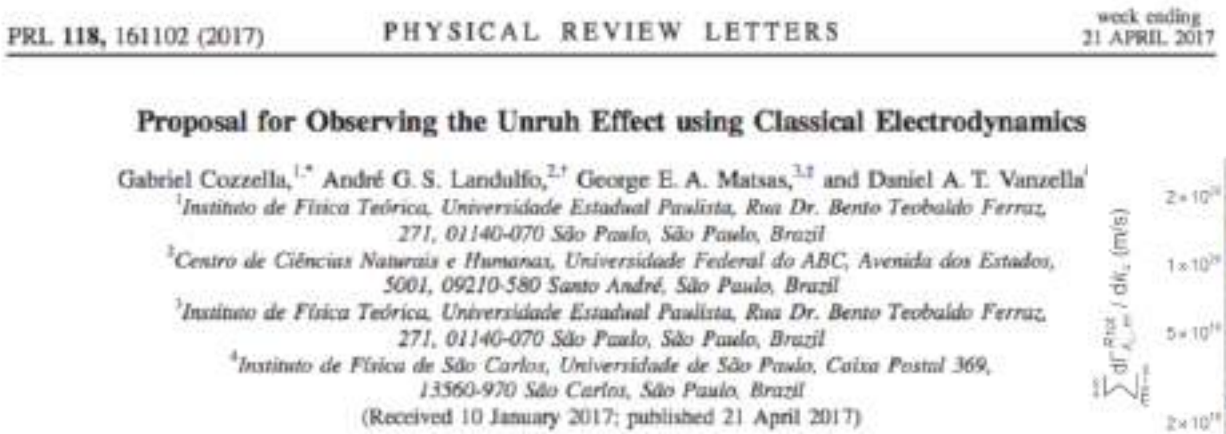
Accelerated observer assumes the existence of the Unruh thermal bath

and predicts what an inertial experimentalist should observe

spectral-angular distribution (from classical electrodynamics)

$$I(\omega,\theta,\phi) \equiv \frac{d\mathcal{E}(\omega,\theta,\phi)}{d\omega\, d(\cos\theta)\, d\phi}$$

The Unruh Effect and bremsstrahlung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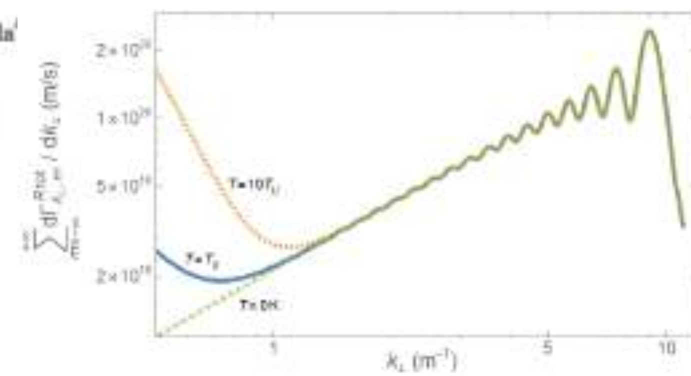
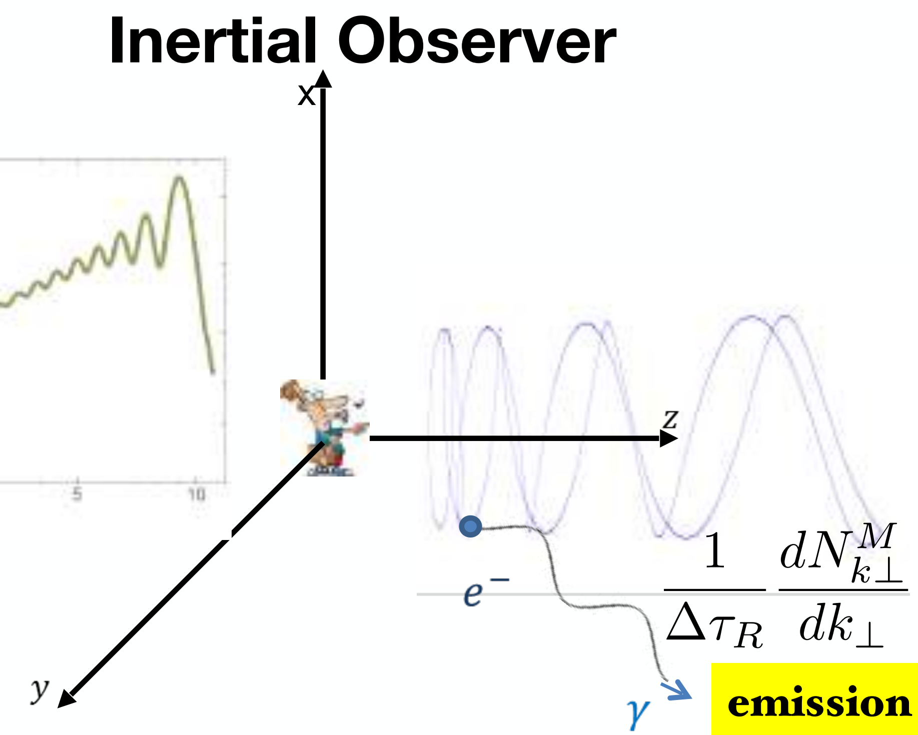
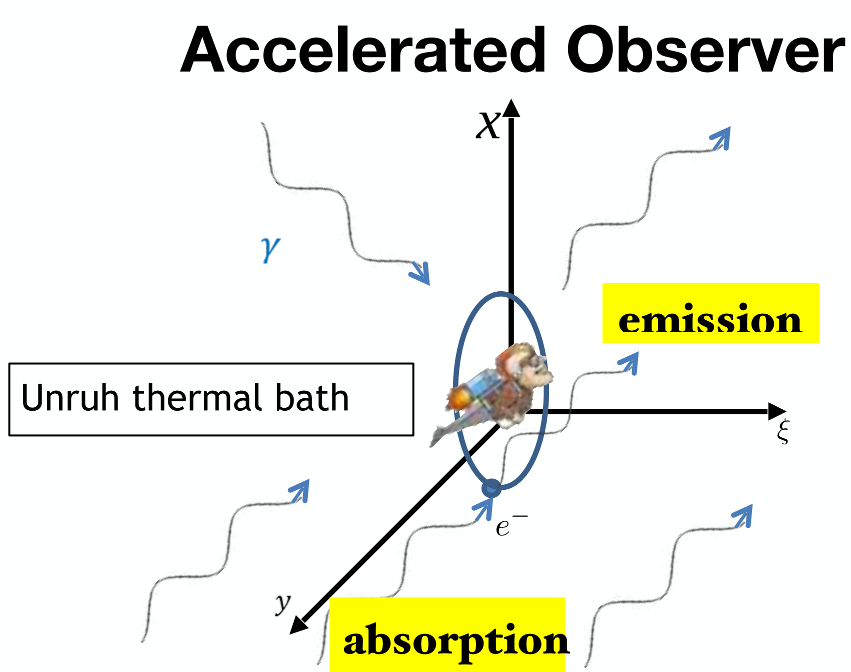






Accelerated observer assumes the existence of the Unruh thermal bath

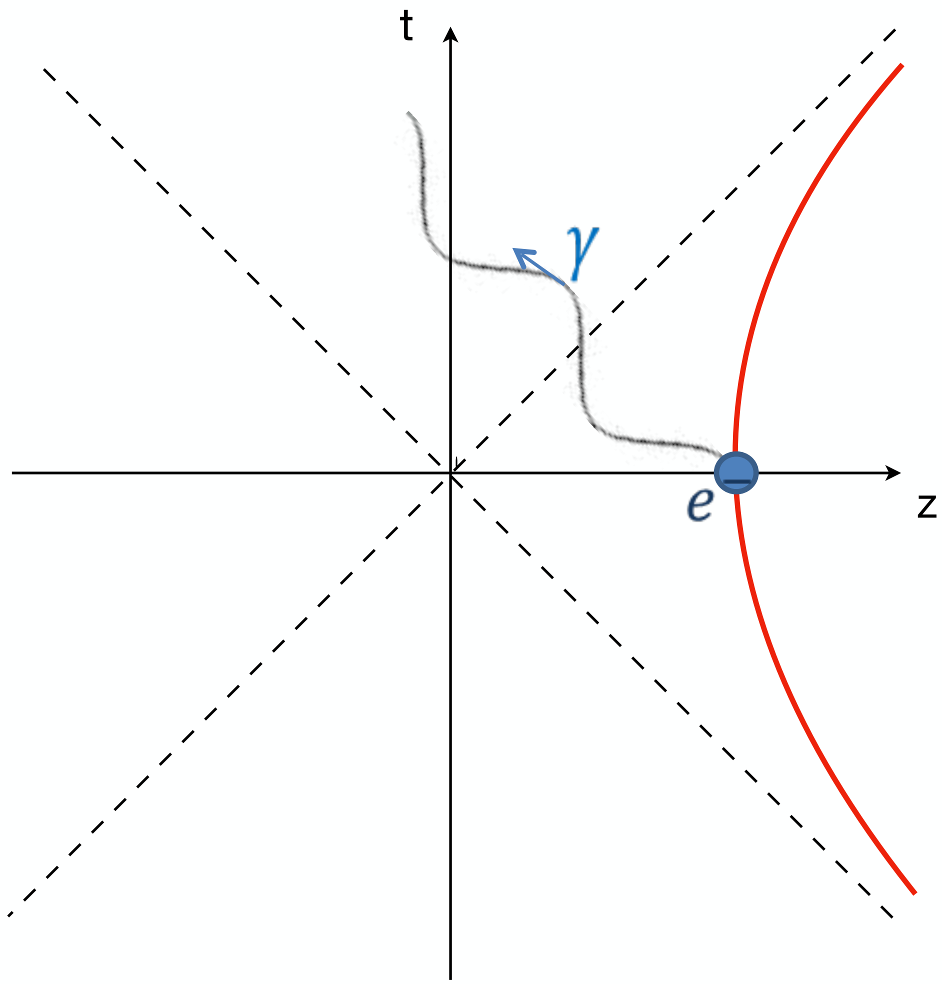
and predicts what an inertial experimentalist should observe

spectral-angular distribution (from classical electrodynamics)

$$I(\omega,\theta,\phi) \equiv \frac{d\mathcal{E}(\omega,\theta,\phi)}{d\omega\, d(\cos\theta)\, d\phi}$$

$$I(\omega,\theta,\phi) \underset{\mathcal{E}=\hbar\omega}{\rightarrow} \frac{1}{\Delta\tau_R}\frac{dN^M_{k\perp}}{dk_\perp}$$

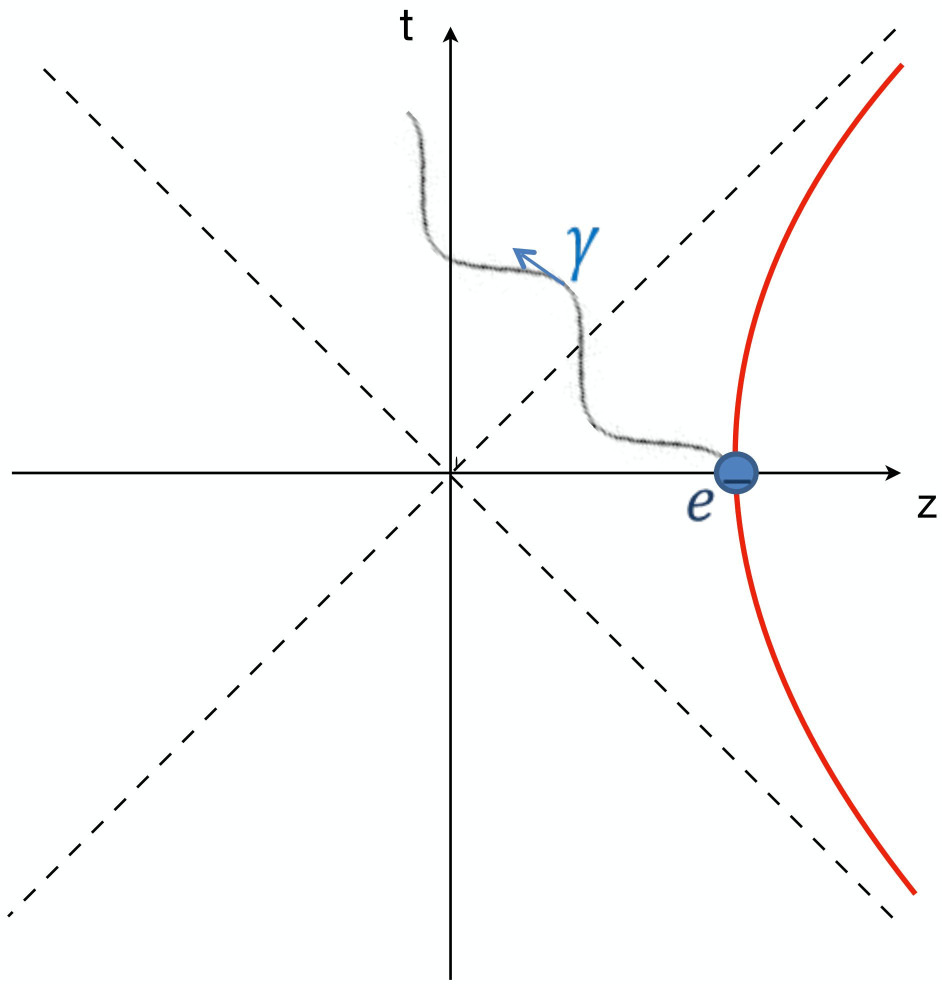
The Unruh Effect and bremsstrahlung



Uniformly accelerated charge
[proper acceleration a]

Acceleration, radiation, and the Unruh effect— Two puzzling aspects:

The Unruh Effect and bremsstrahlung



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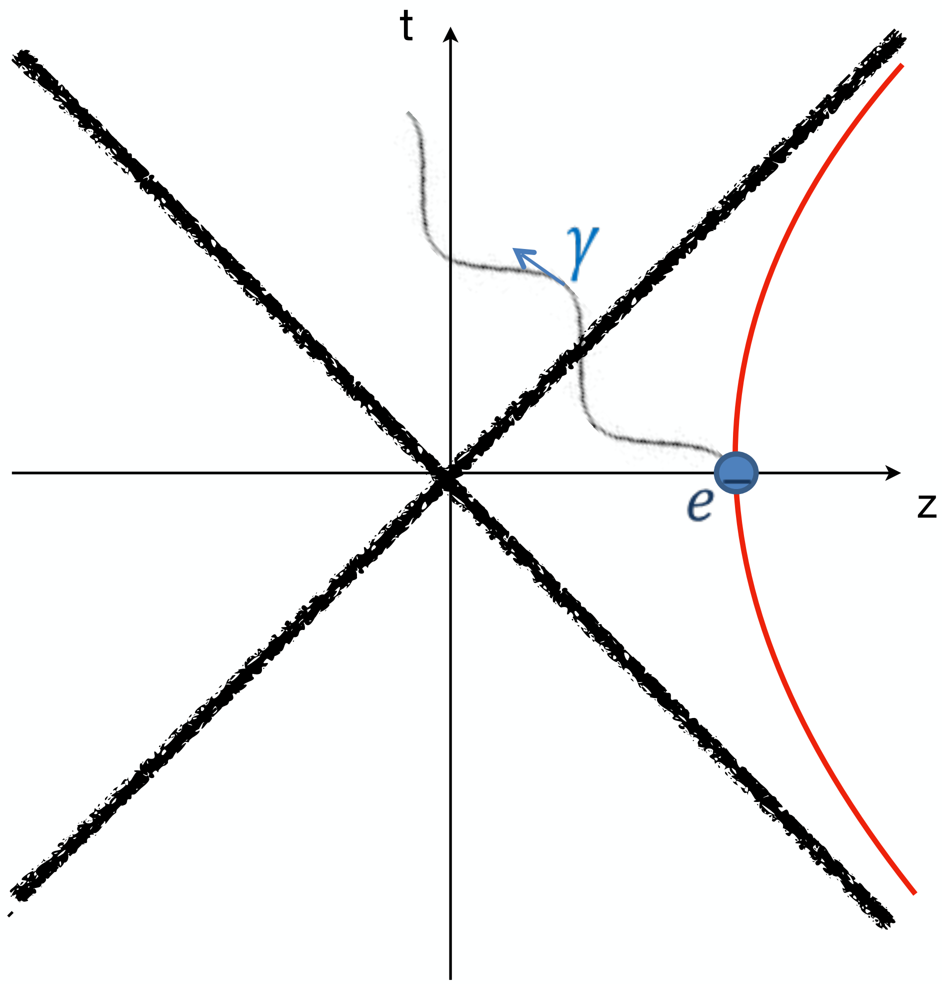
Acceleration, radiation, and the Unruh effect— Two puzzling aspects:

➡ The Unruh effect is a strictly quantum effect while Larmor radiation is a classical one

The Unruh Effect and bremsstrahlung



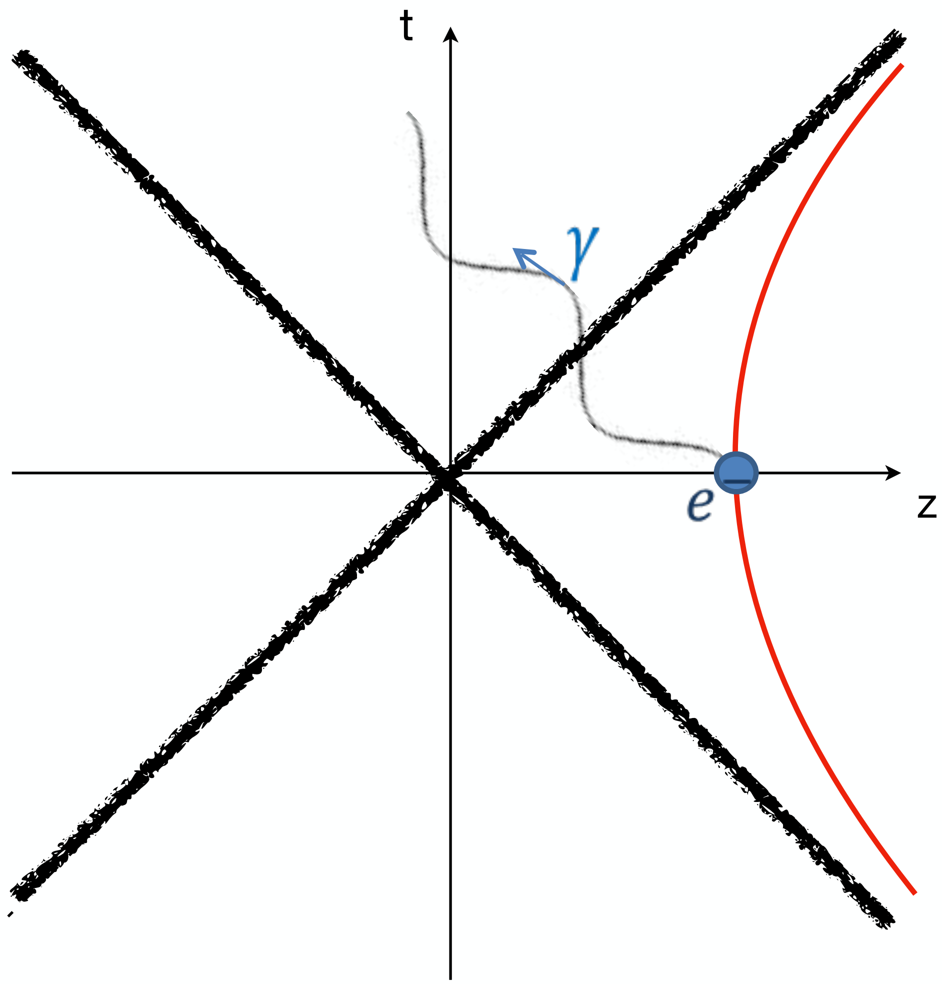
Uniformly accelerated charge
[proper acceleration a]

Acceleration, radiation, and the Unruh effect— Two puzzling aspects:

- ➡The Unruh effect is a strictly quantum effect while Larmor radiation is a classical one
- ➡The crucial role played by zero-energy Rindler photons in this context

The Unruh Effect and bremsstrahlung

Landulfo, Fulling, and Matsas, PRD 100 042020 (2019)



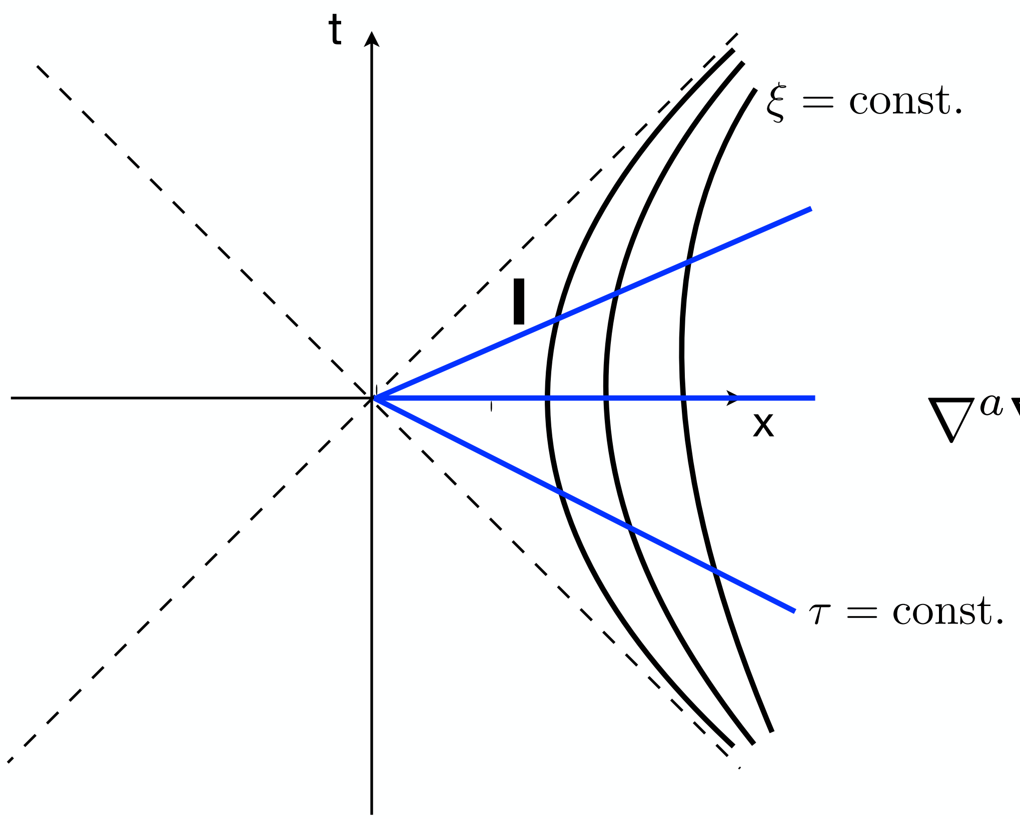
Uniformly accelerated charge
[proper acceleration a]

Acceleration, radiation, and the Unruh effect— Two puzzling aspects:

- The Unruh effect is a strictly quantum effect while Larmor radiation is a classical one
- The crucial role played by zero-energy Rindler photons in this context

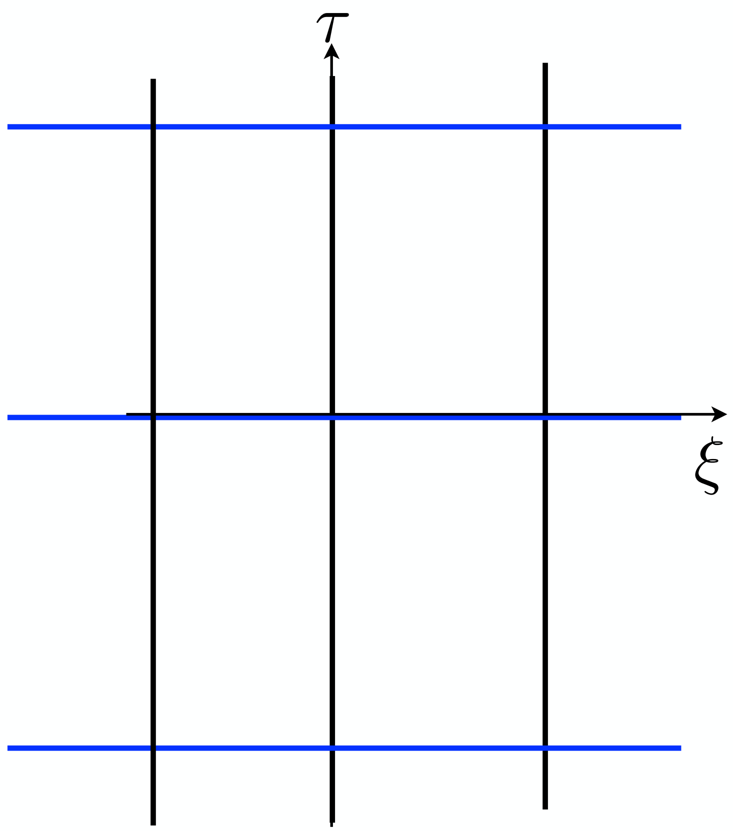
Unruh modes

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



$$\nabla^a \nabla_a \phi = 0$$

$$ds^2 = e^{2a\xi}\left(-d\tau^2 + d\xi^2\right) + dy^2 + dz^2$$



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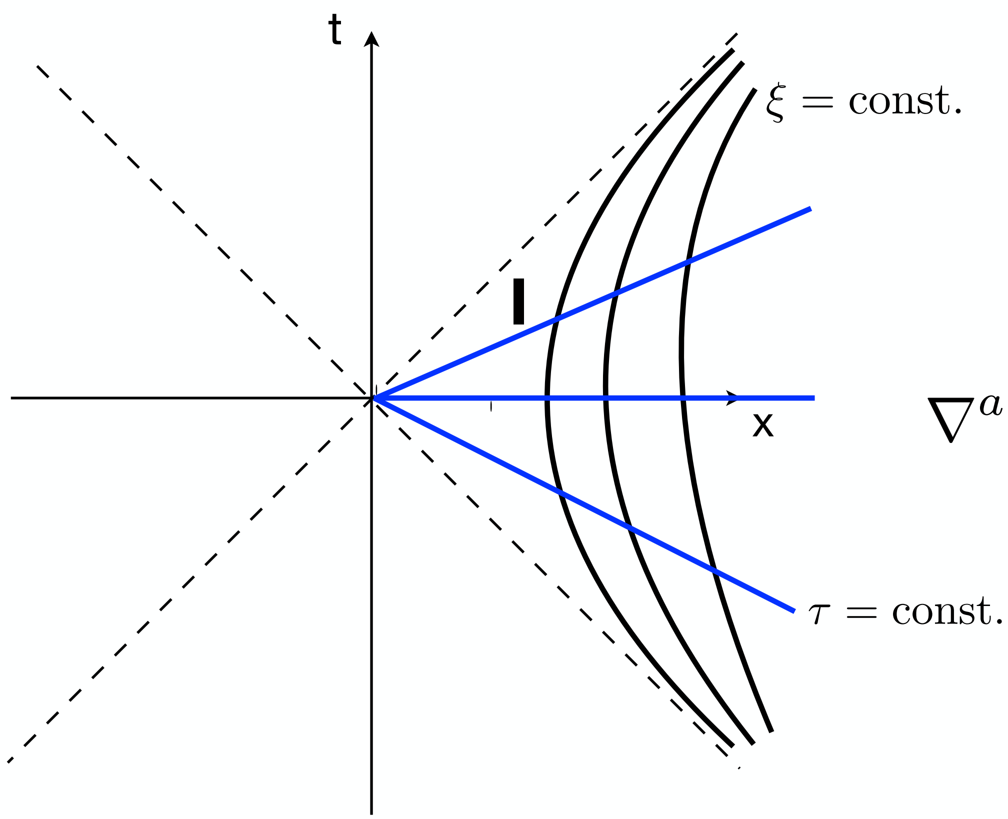

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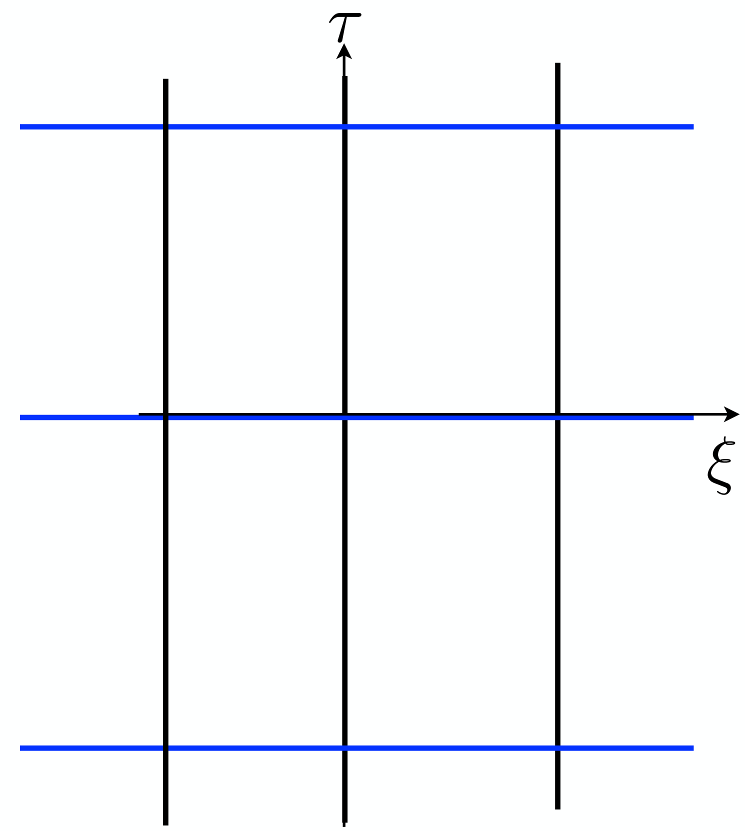
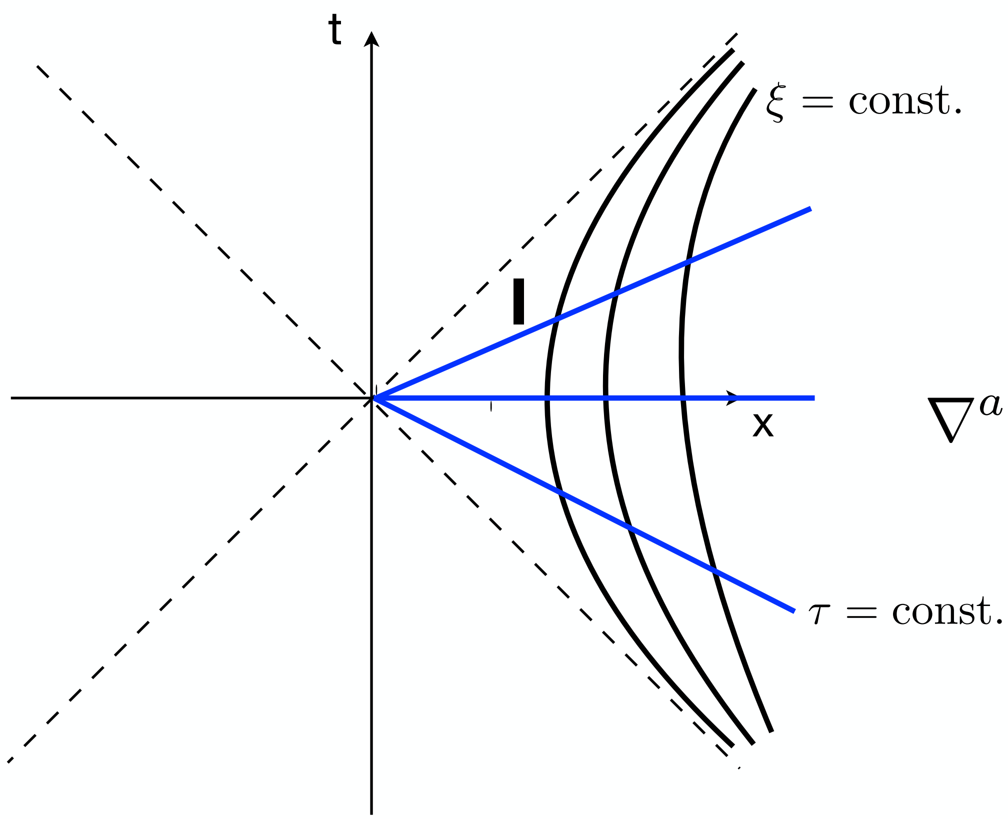


(Right) Rindler Modes (positive-frequency wrt τ)

Unruh modes

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



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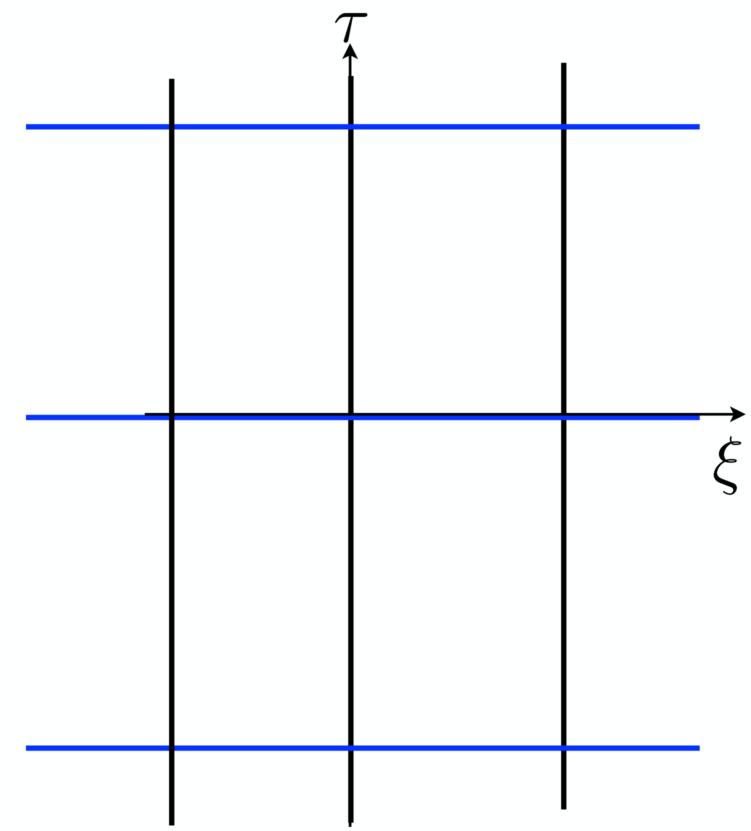
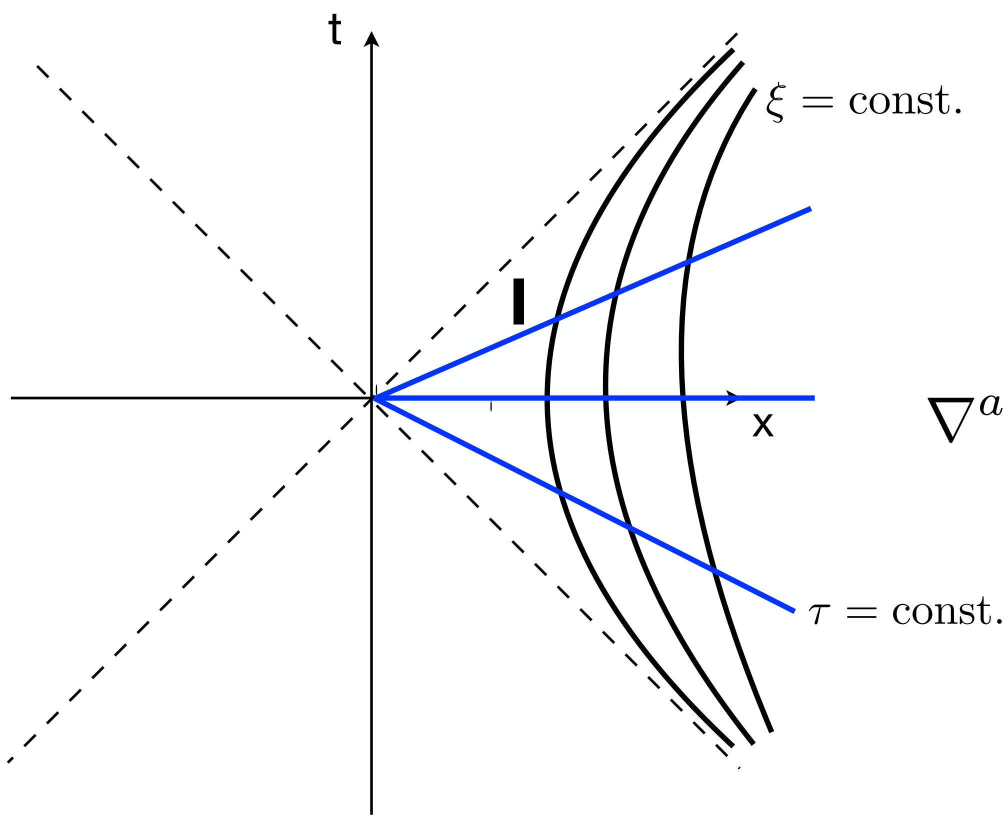


(Right) Rindler Modes (positive-frequency wrt τ)

$$\begin{cases} v^R_{\omega \mathbf{k}_\perp} = e^{-i\omega\tau} F_{\omega \mathbf{k}_\perp}(\xi, \mathbf{x}_\perp) & I \\ 0 & II \end{cases}$$

Unruh modes

$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$



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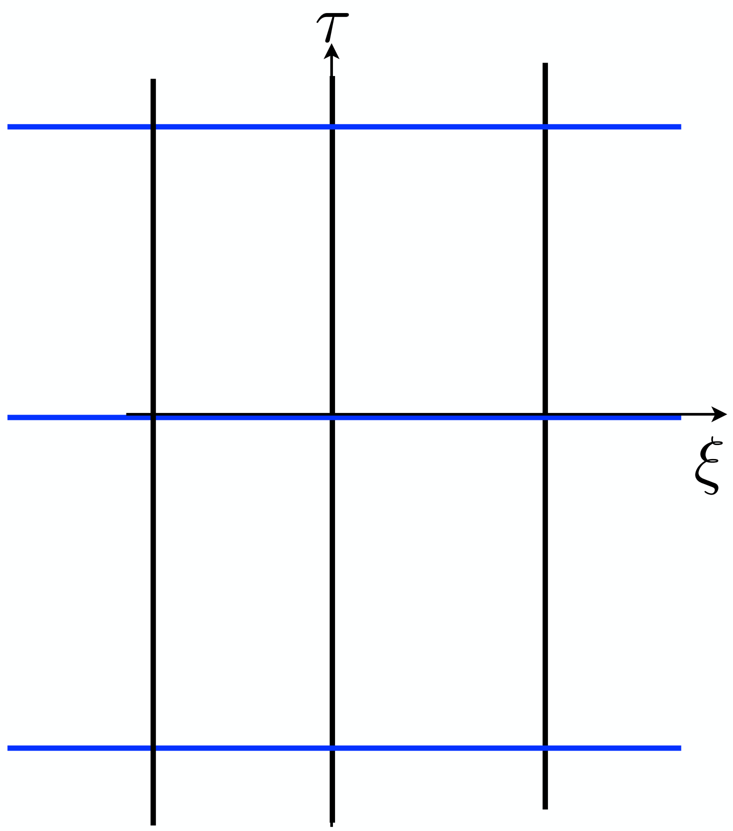

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$$\begin{cases} v^R_{\omega \mathbf{k}_\perp} = e^{-i\omega\tau} F_{\omega \mathbf{k}_\perp}(\xi, \mathbf{x}_\perp) & I \\ 0 & II \end{cases}$$

$$F_{\omega \mathbf{k}_\perp}(\xi, \mathbf{x}_\perp) = \left[\frac{\sinh(\pi\omega/a)}{4\pi^4 a}\right]^{1/2} K_{i\omega/a}\left(\frac{k_\perp}{a} e^{a\xi}\right) e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}$$

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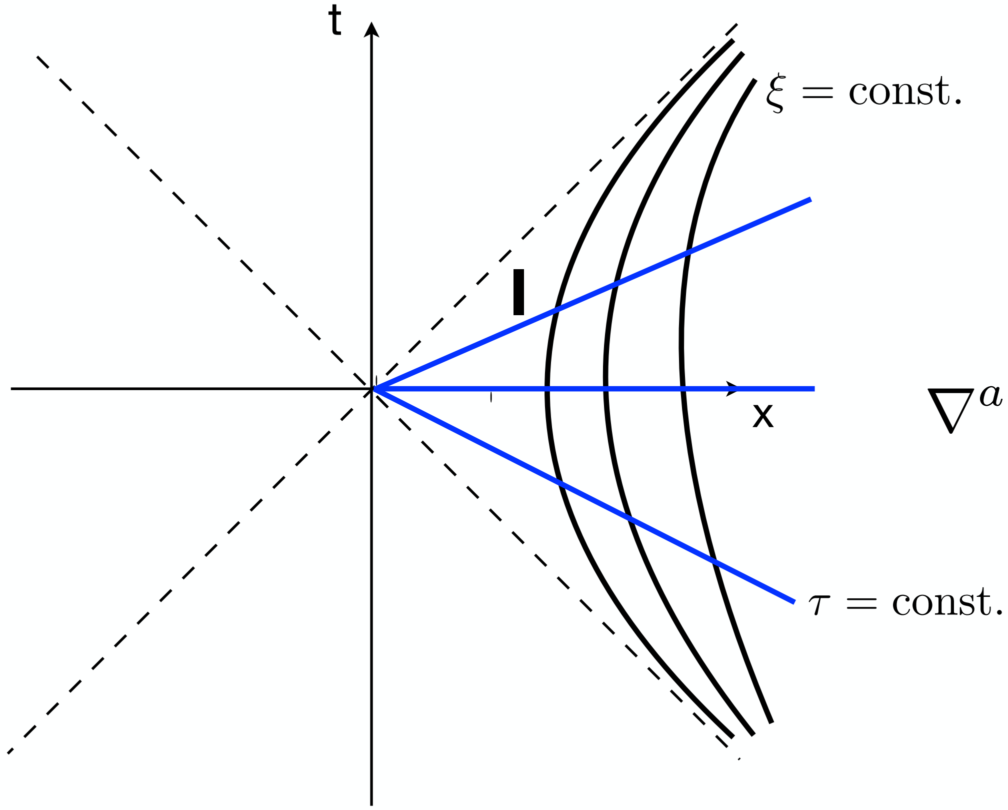

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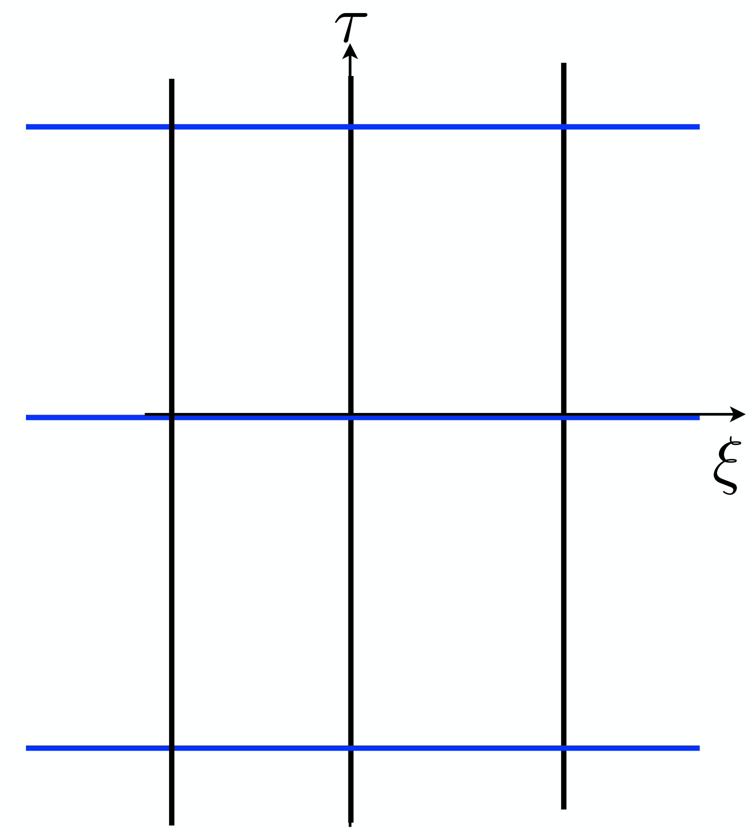

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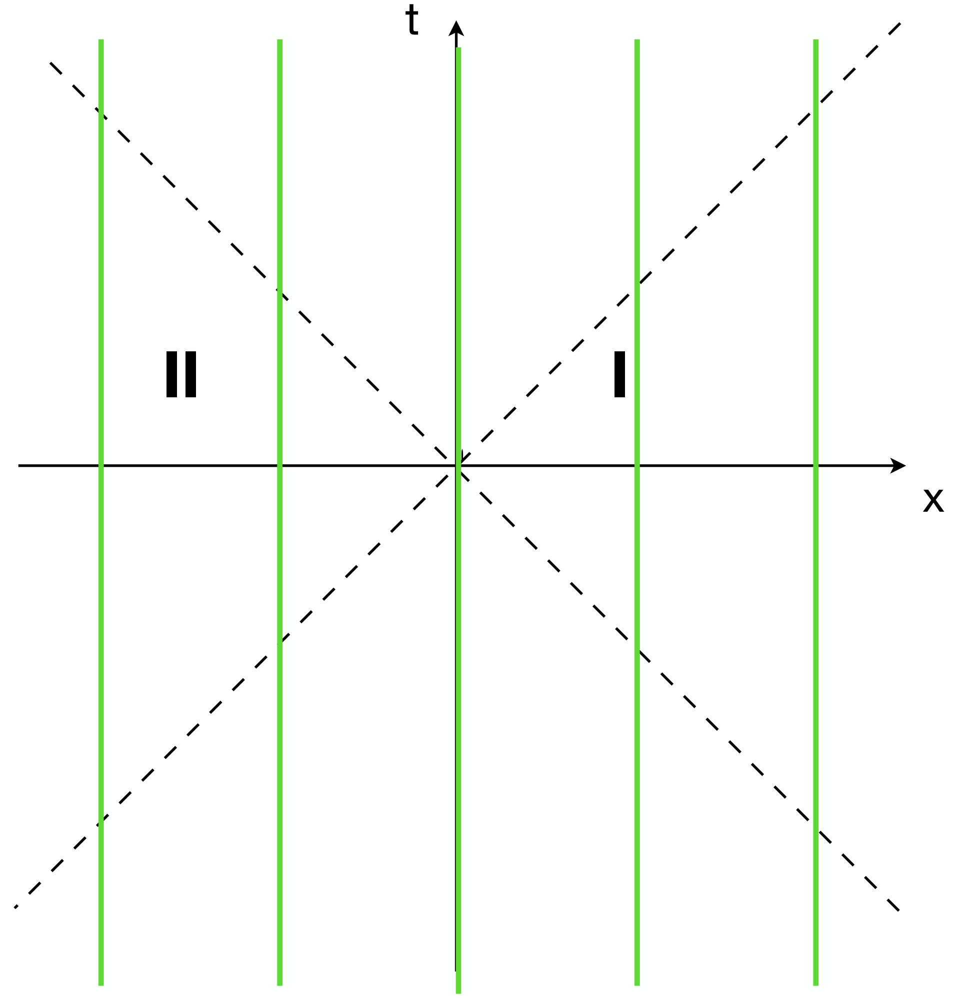
(left) Rindler Modes are defined analogously by interchanging the roles of I and II in the above definition

Unruh modes

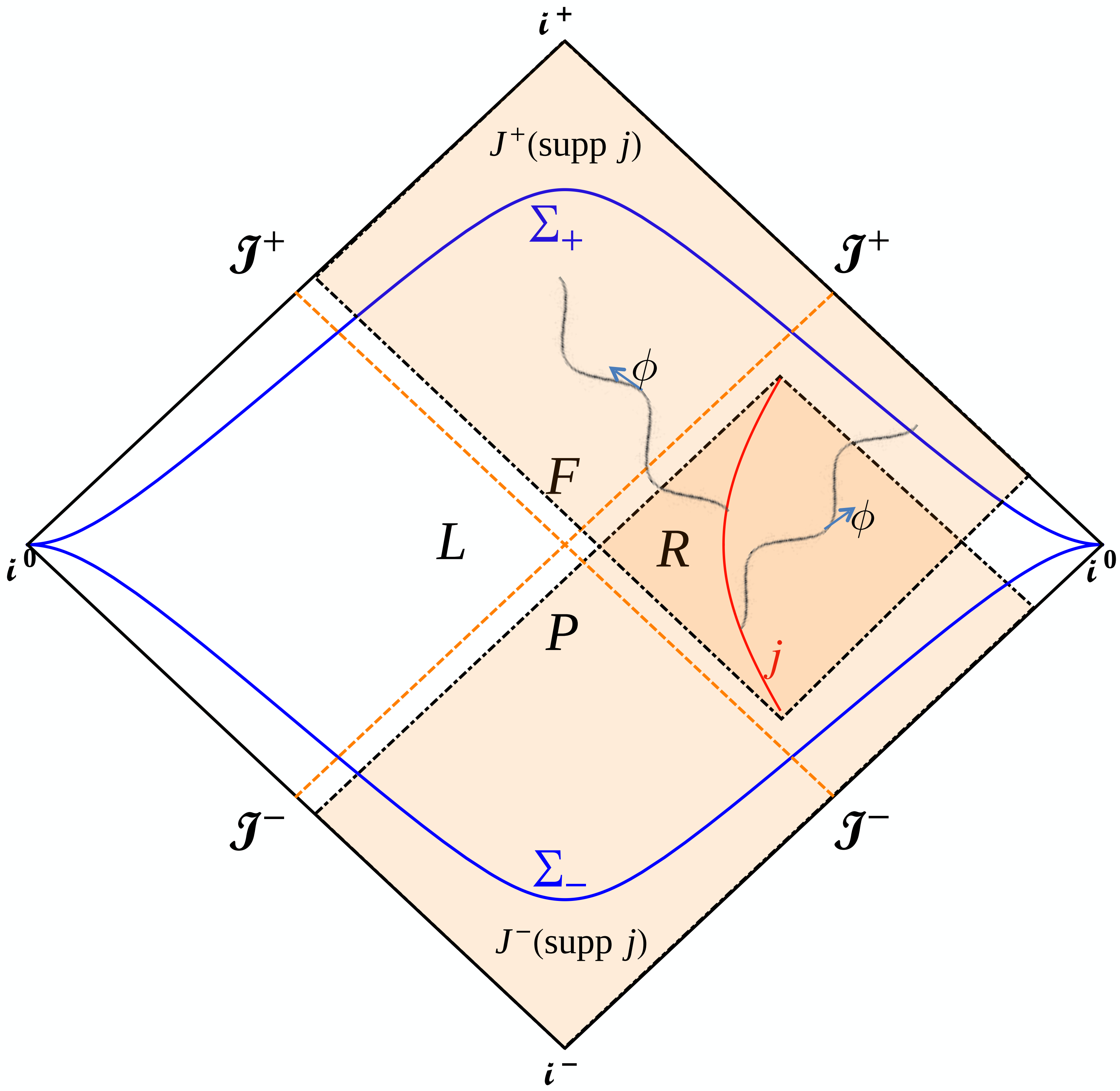
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



Unruh Modes (positive-frequency wrt the inertial time t)

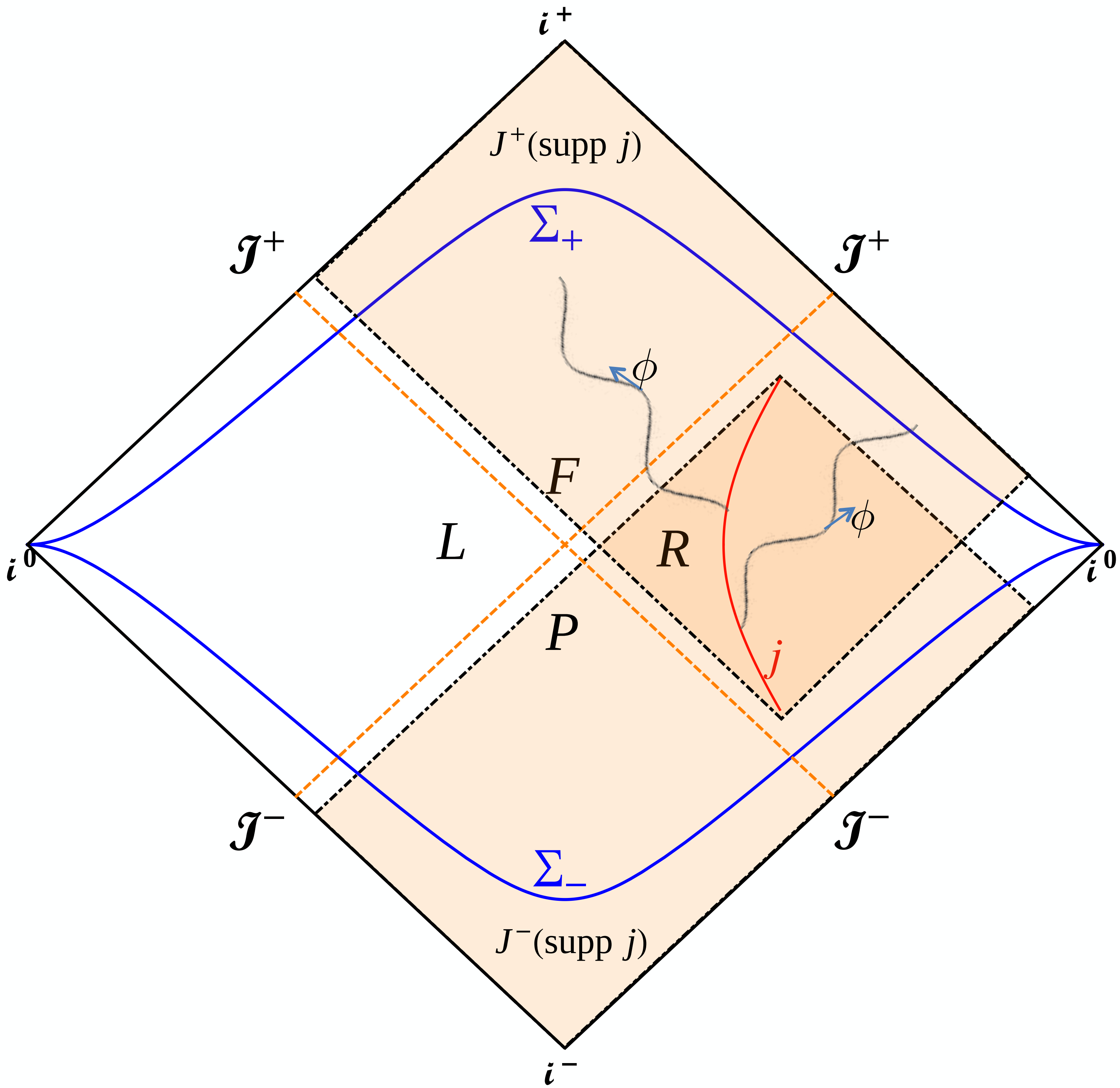
$$w^1_{\omega \mathbf{k}_\perp} \equiv \frac{v^R_{\omega \mathbf{k}_\perp} + e^{-\pi\omega/a} v^{L*}_{\omega\, -\mathbf{k}_\perp}}{\sqrt{1 - e^{-2\pi\omega/a}}}, \qquad w^2_{\omega \mathbf{k}_\perp} \equiv \frac{v^L_{\omega \mathbf{k}_\perp} + e^{-\pi\omega/a} v^{R*}_{\omega\, -\mathbf{k}_\perp}}{\sqrt{1 - e^{-2\pi\omega/a}}}$$

Classical radiation and zero-energy modes



$$j = \begin{cases} q\delta(\xi)\delta^2(\mathbf{x}_\perp) & -T < \tau < T \\ 0 & |\tau| > T \end{cases}$$

Classical radiation and zero-energy modes



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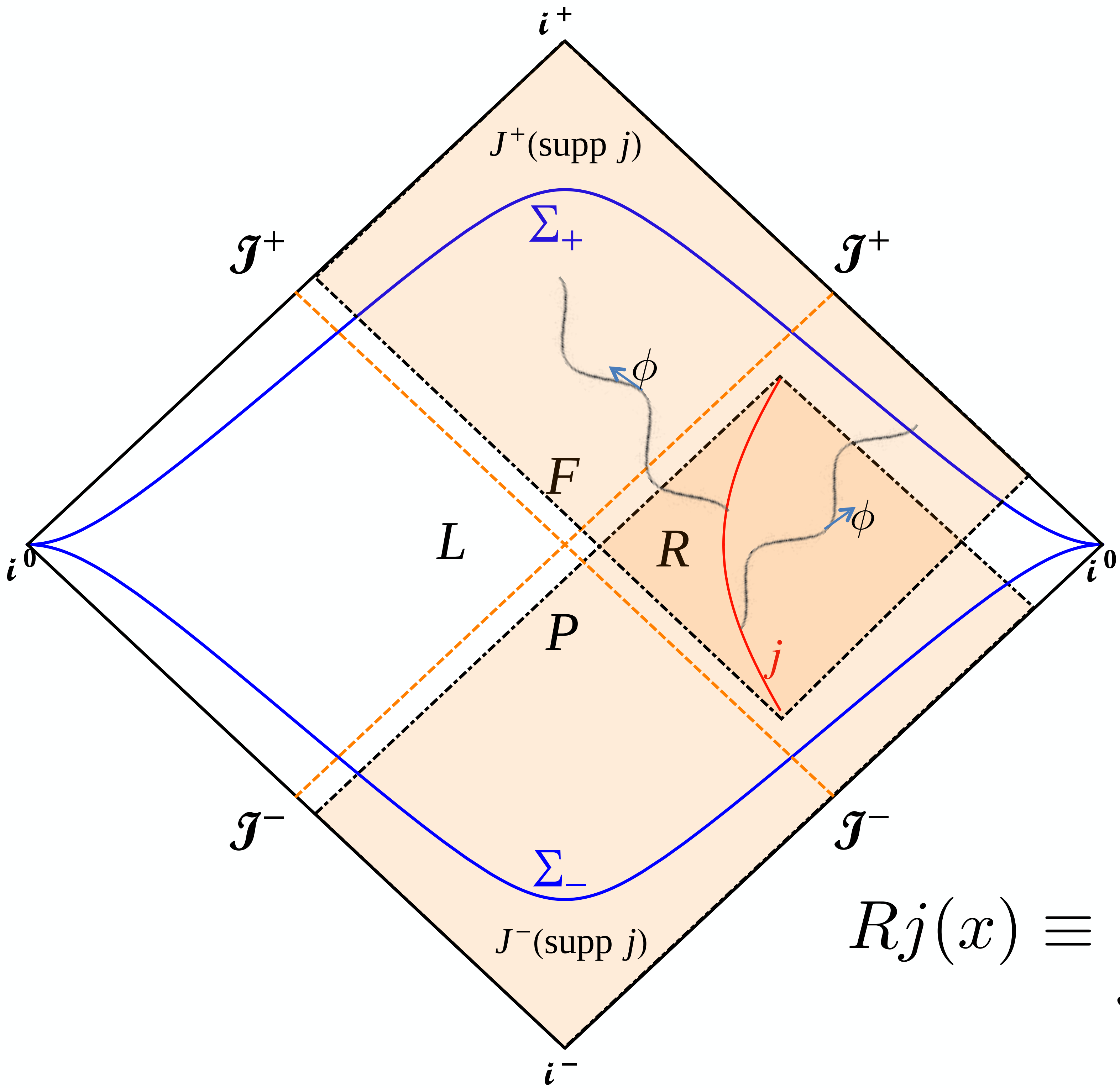

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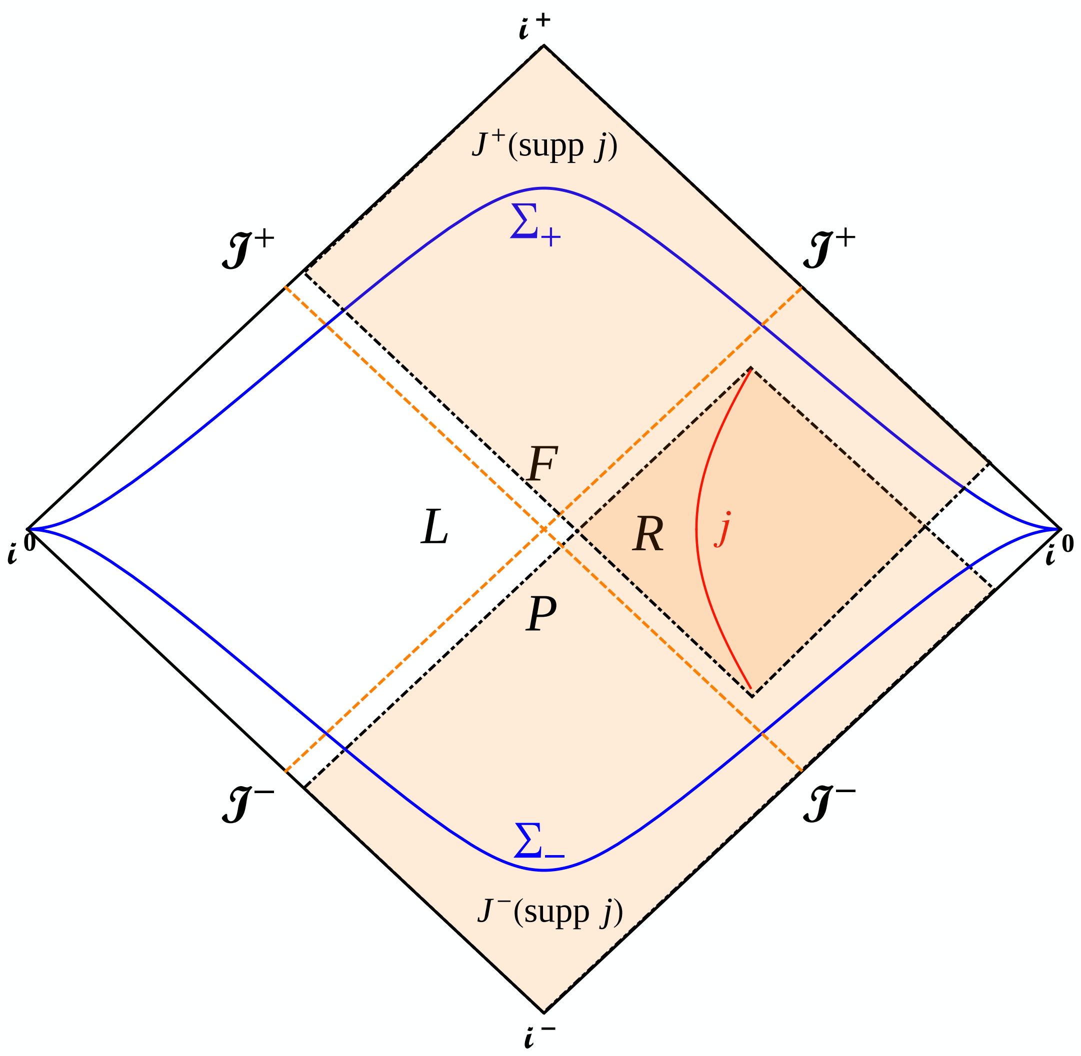
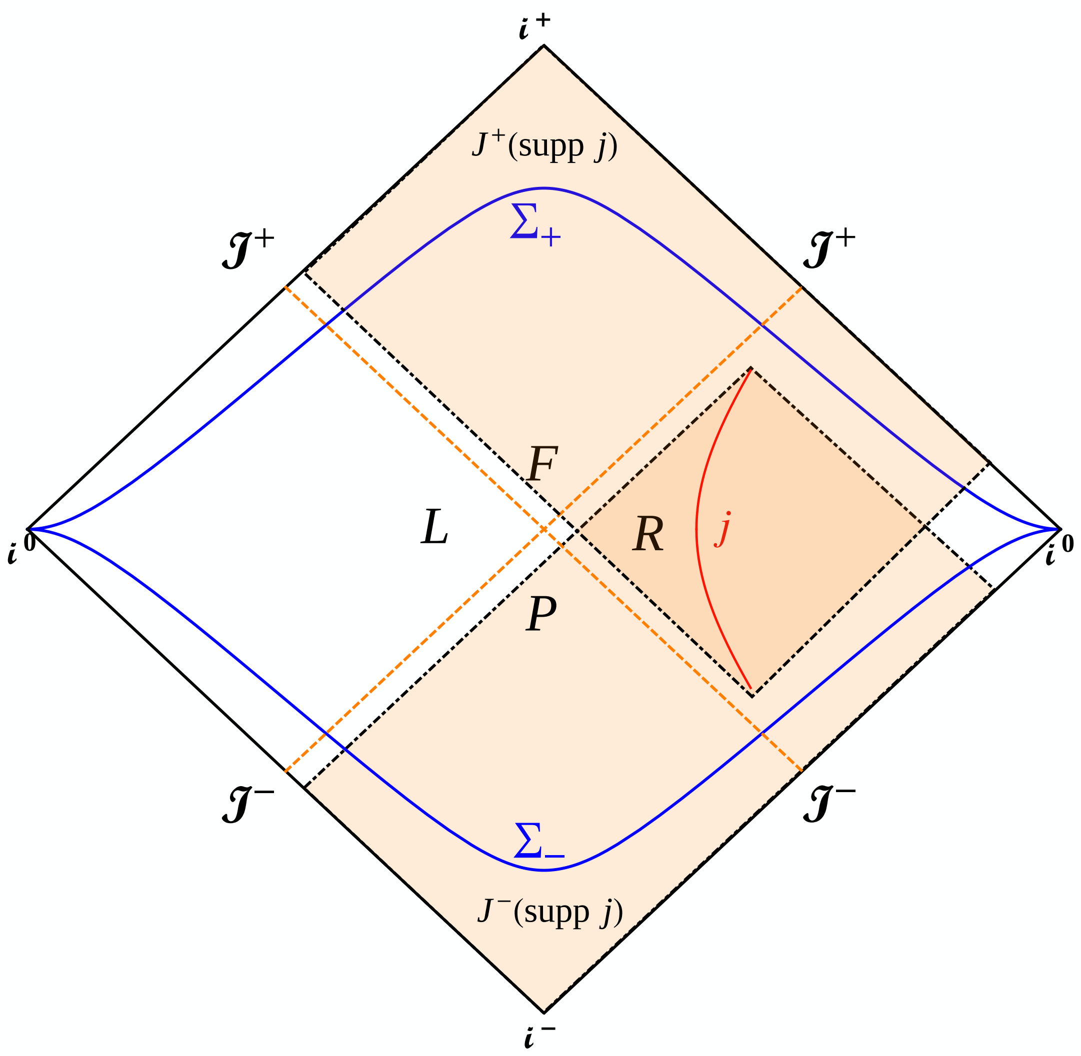
Retarded solution:

$$Rj(x) \equiv \int_{\mathbb{R}^4} dx' \sqrt{-g} G_{\text{ret}}(x, x') j(x')$$

Classical radiation and zero-energy modes



Classical radiation and zero-energy modes

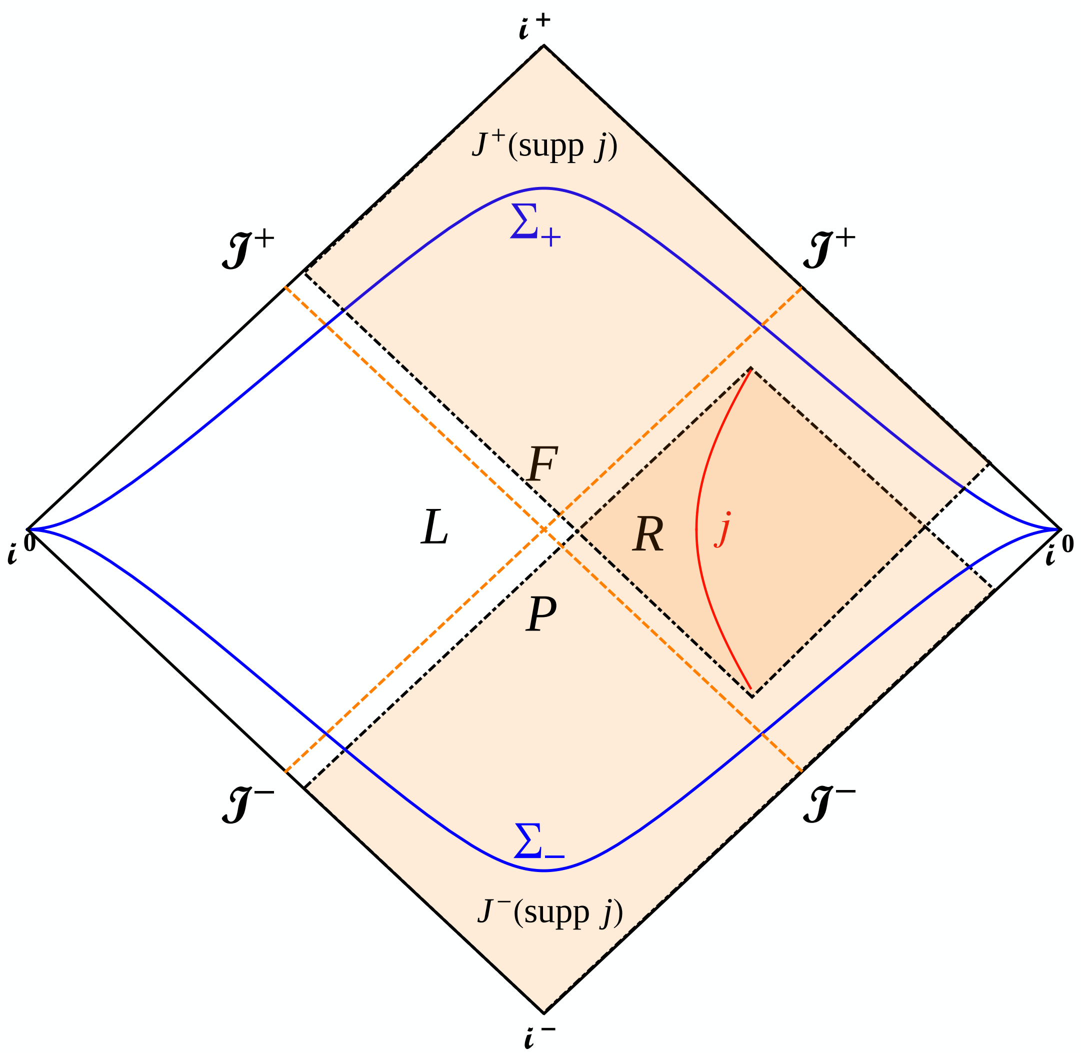


In the asymptotic future (Σ_+)

$$Rj = -\sum_{\sigma=1}^{2}\int_0^\infty d\omega \int d^2\mathbf{k}_\perp \langle w^\sigma_{\omega\mathbf{k}_\perp}, Ej\rangle_{\mathrm{KG}}\, w^\sigma_{\omega\mathbf{k}_\perp} + \text{H.c.},$$

$$Ej = Aj - Rj$$

Classical radiation and zero-energy modes



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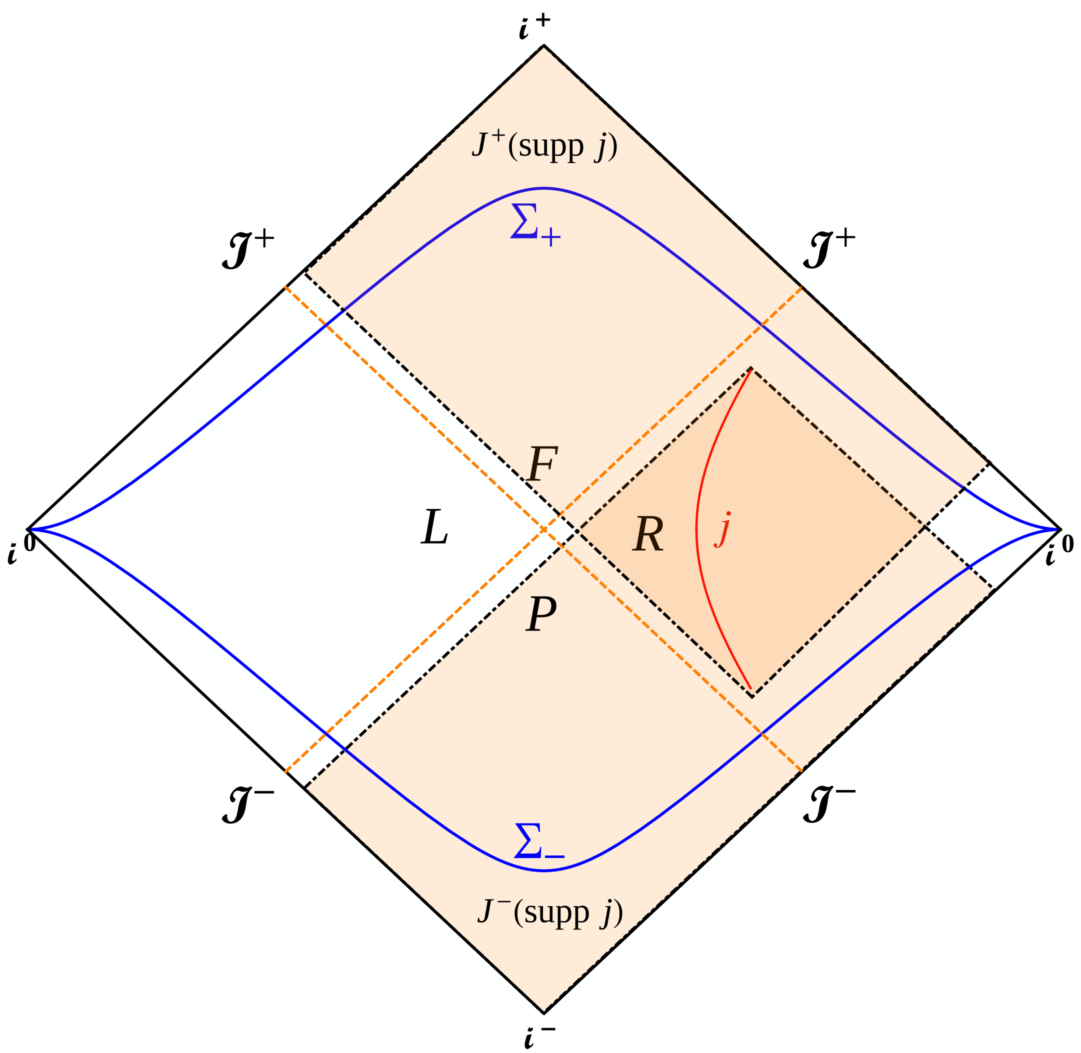
The coefficients can be written in terms of a spacetime integral

$$\langle w^1_{\omega\mathbf{k}_\perp}, Ej\rangle_{\text{KG}} = \frac{i}{\sqrt{1-e^{-2\pi\omega/a}}} \int_{\mathbb{R}^4} d^4x \sqrt{-g}\, v^{R*}_{\omega\mathbf{k}_\perp}(x) j(x)$$

$$\langle w^2_{\omega\mathbf{k}_\perp}, Ej\rangle_{\text{KG}} = \frac{i e^{-\pi\omega/a}}{\sqrt{1-e^{-2\pi\omega/a}}} \int_{\mathbb{R}^4} d^4x \sqrt{-g}\, v^{R}_{\omega-\mathbf{k}_\perp}(x) j(x)$$

Classical radiation and zero-energy modes



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And computed exactly yielding

$$\langle w^1_{\omega\mathbf{k}_\perp}, Ej\rangle_{\text{KG}} = \frac{2iq}{\sqrt{1-e^{-2\pi\omega/a}}} F^*_{\omega\mathbf{k}_\perp}(0)\frac{\sin\omega T}{\omega} \xrightarrow[T\to\infty]{} \frac{iqK_0(k_\perp/a)}{\sqrt{2\pi^2 a}}\,\delta(\omega)$$

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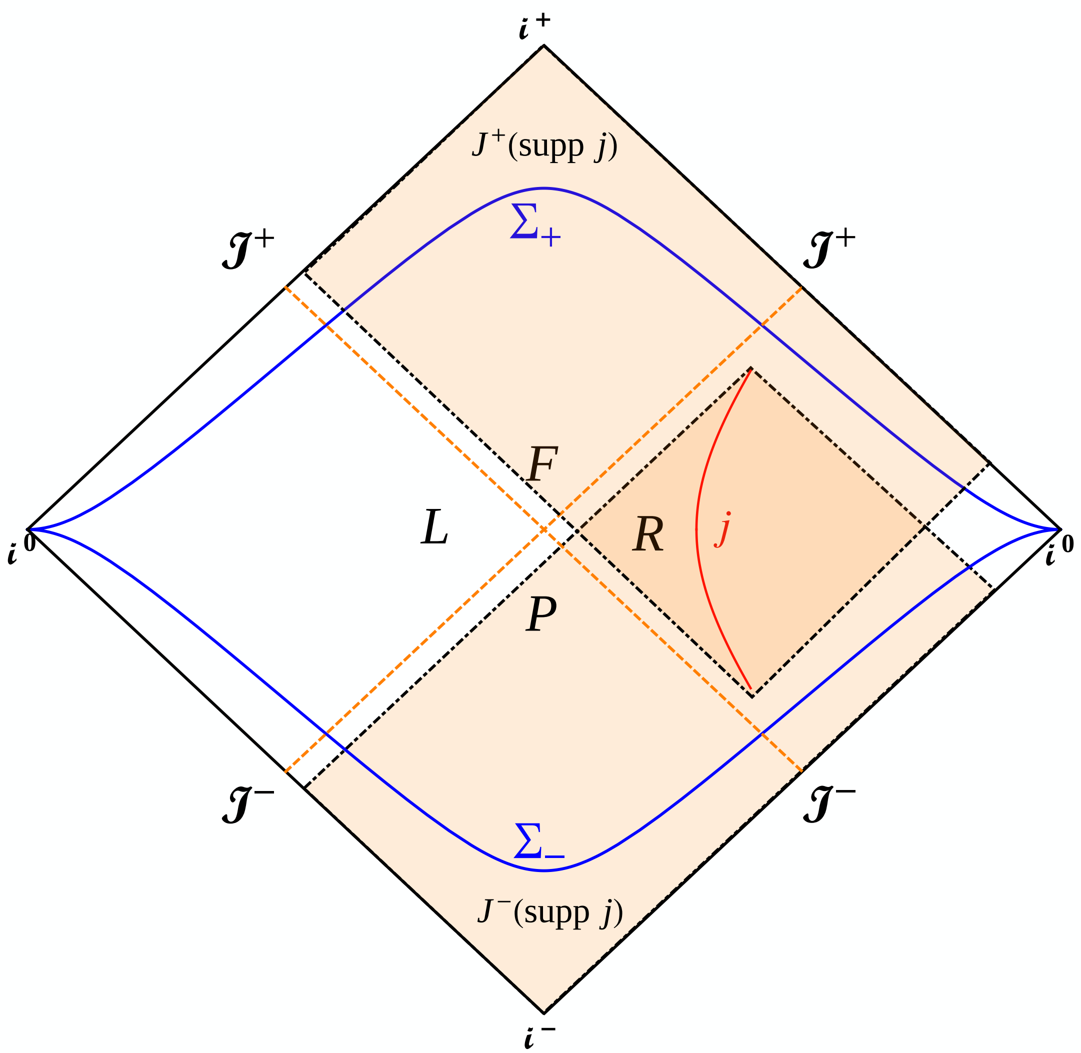

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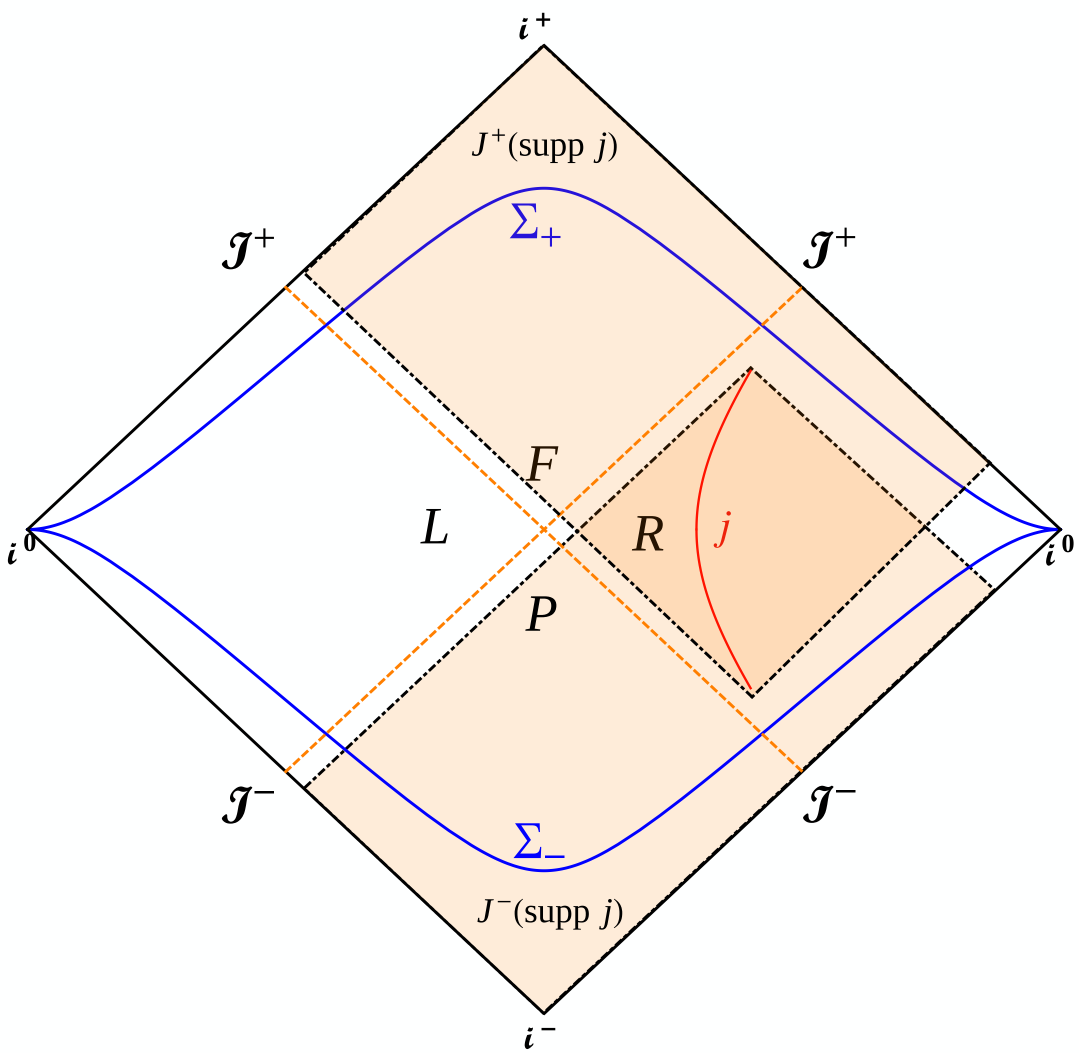

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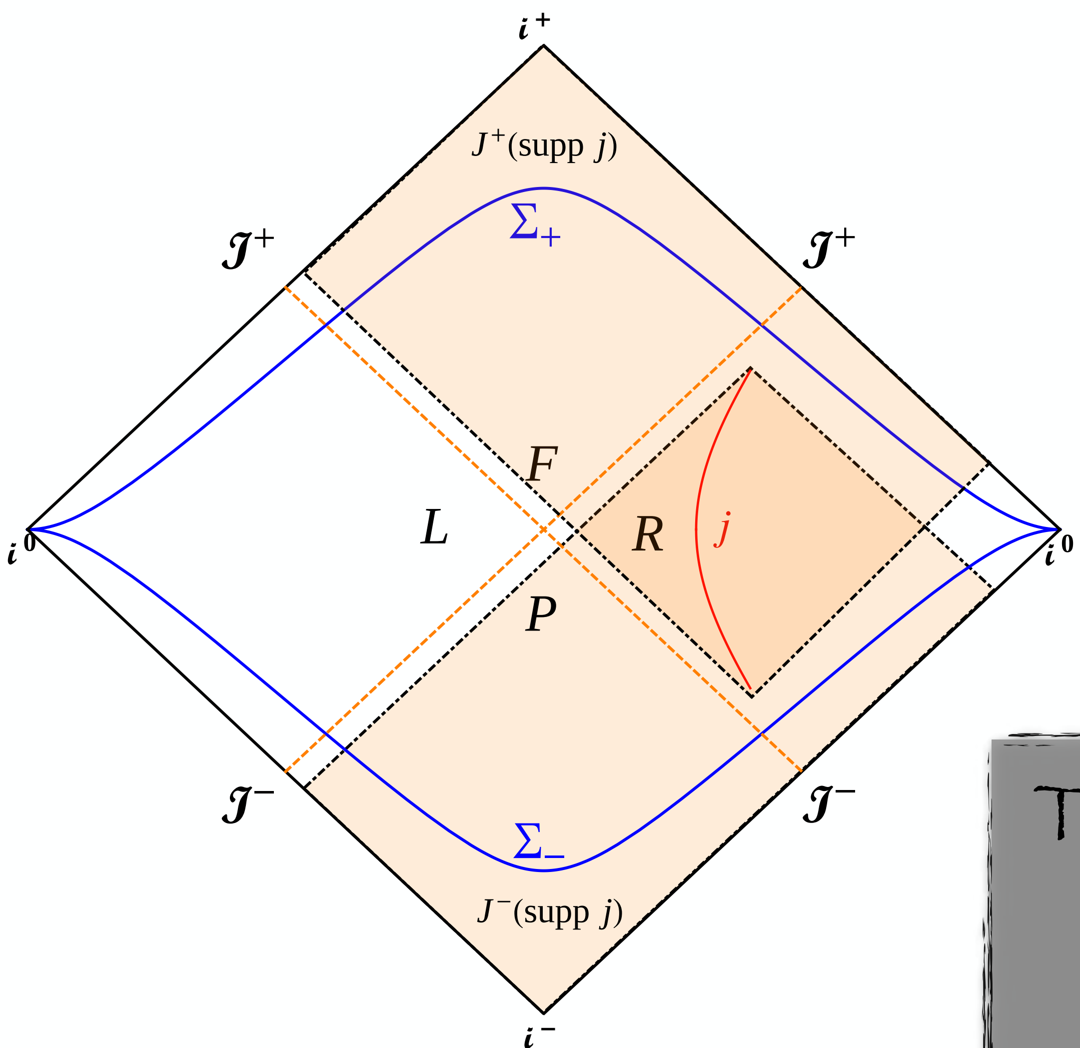
Classical radiation and zero-energy modes



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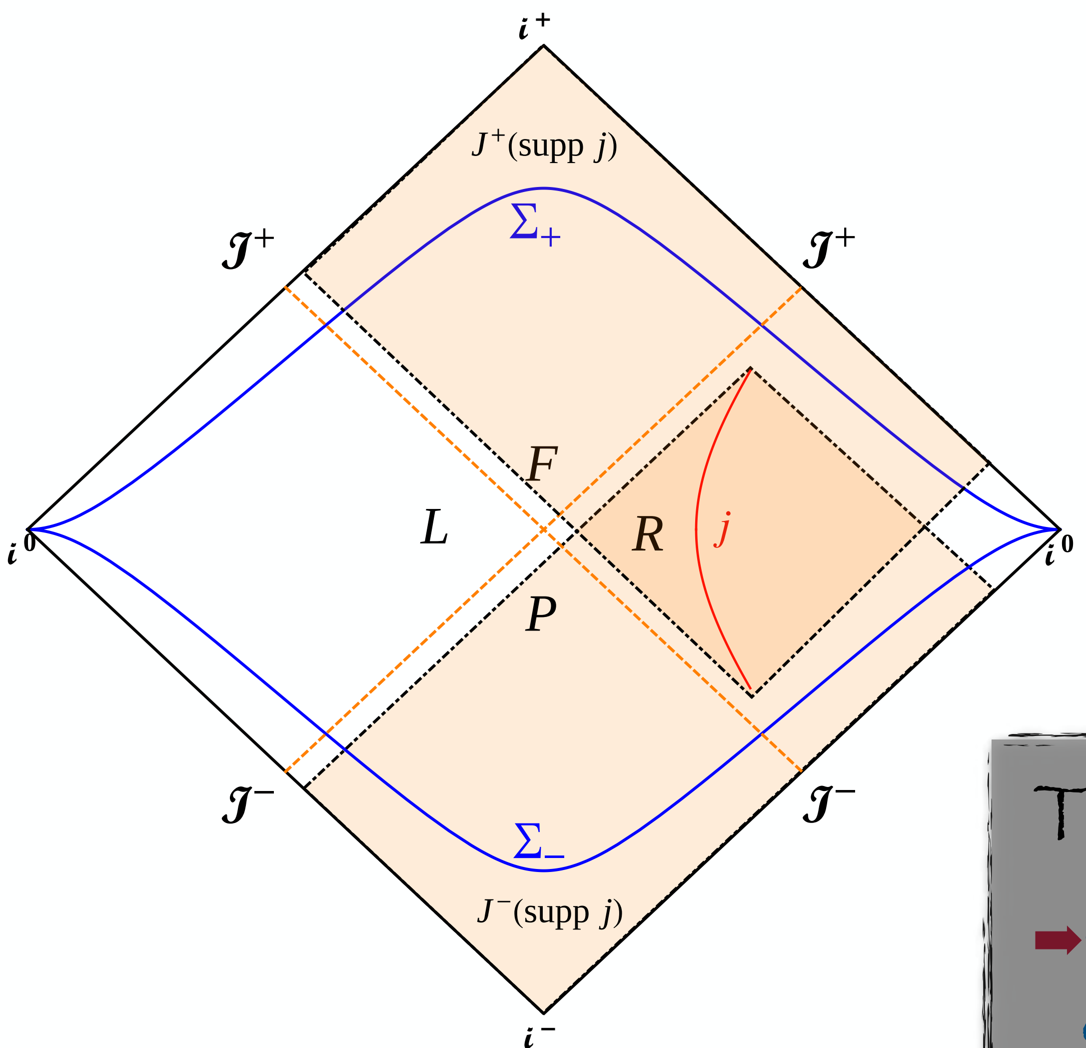
The classical (retarded) solution:

Classical radiation and zero-energy modes



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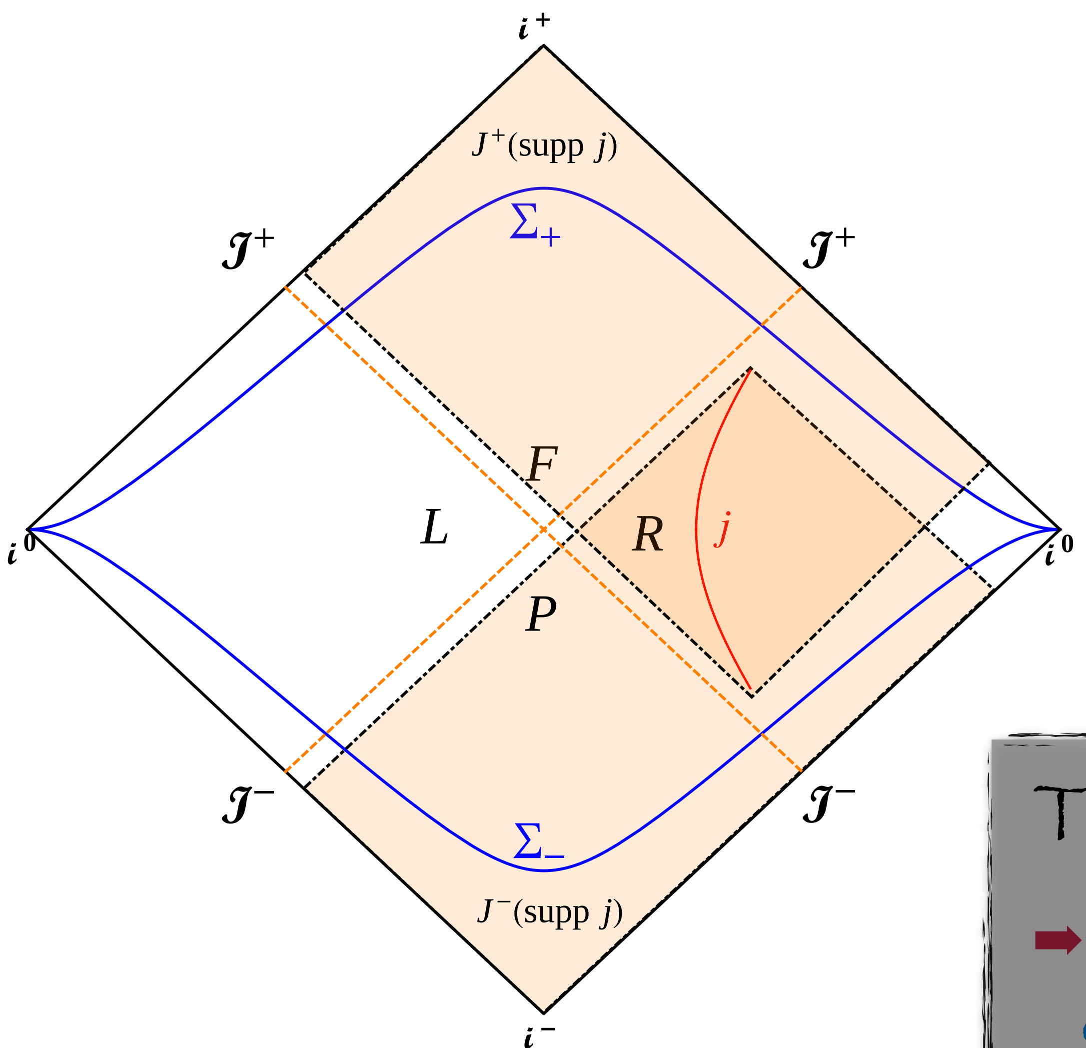
➡ Only zero-energy Unruh modes contribute to the classical radiation seen by inertial observers in the asymptotic future.

Classical radiation and zero-energy modes



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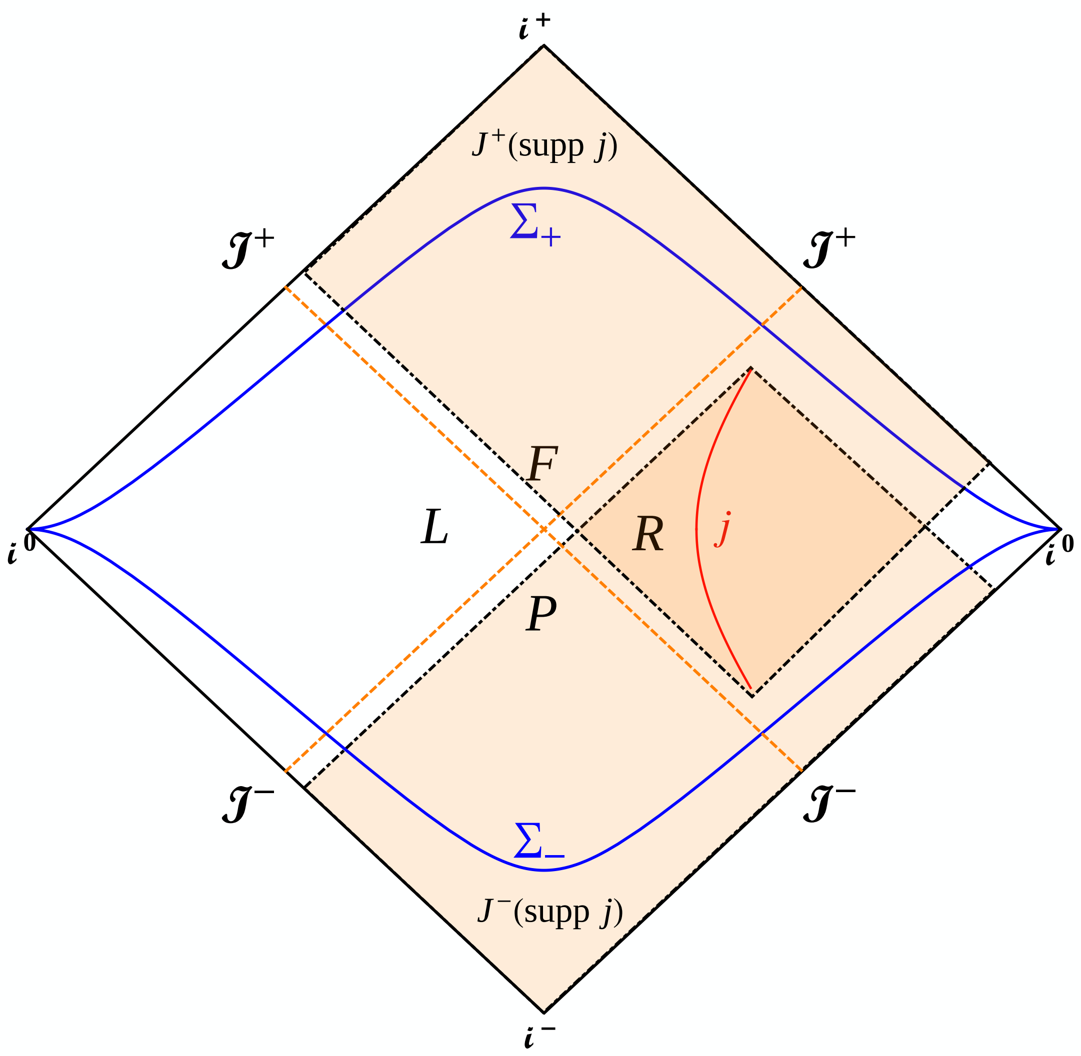
The classical (retarded) solution:

- Only zero-energy Unruh modes contribute to the classical radiation seen by inertial observers in the asymptotic future.
- The expansion amplitudes are built entirely from zero-energy Rindler modes in the right wedge

Classical radiation and zero-energy modes



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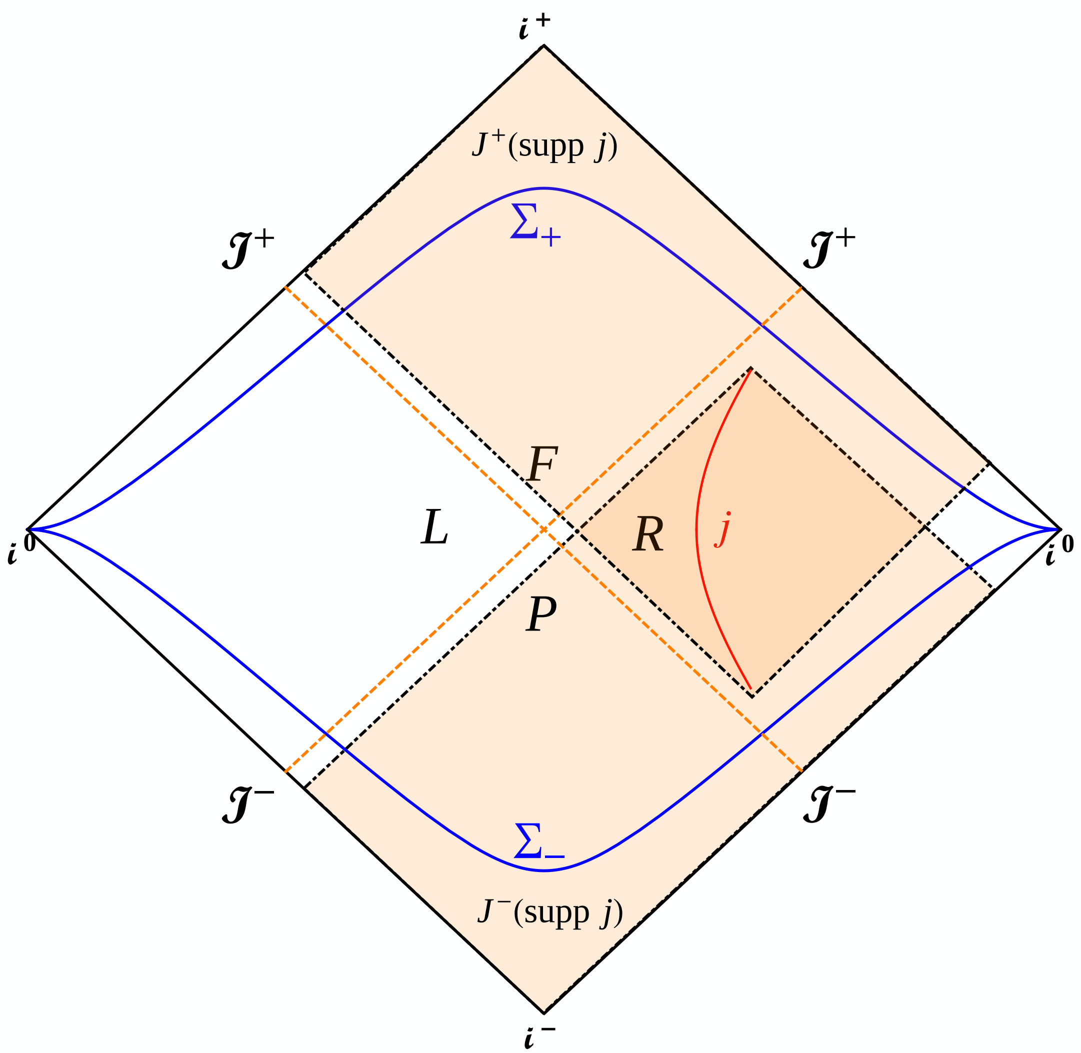

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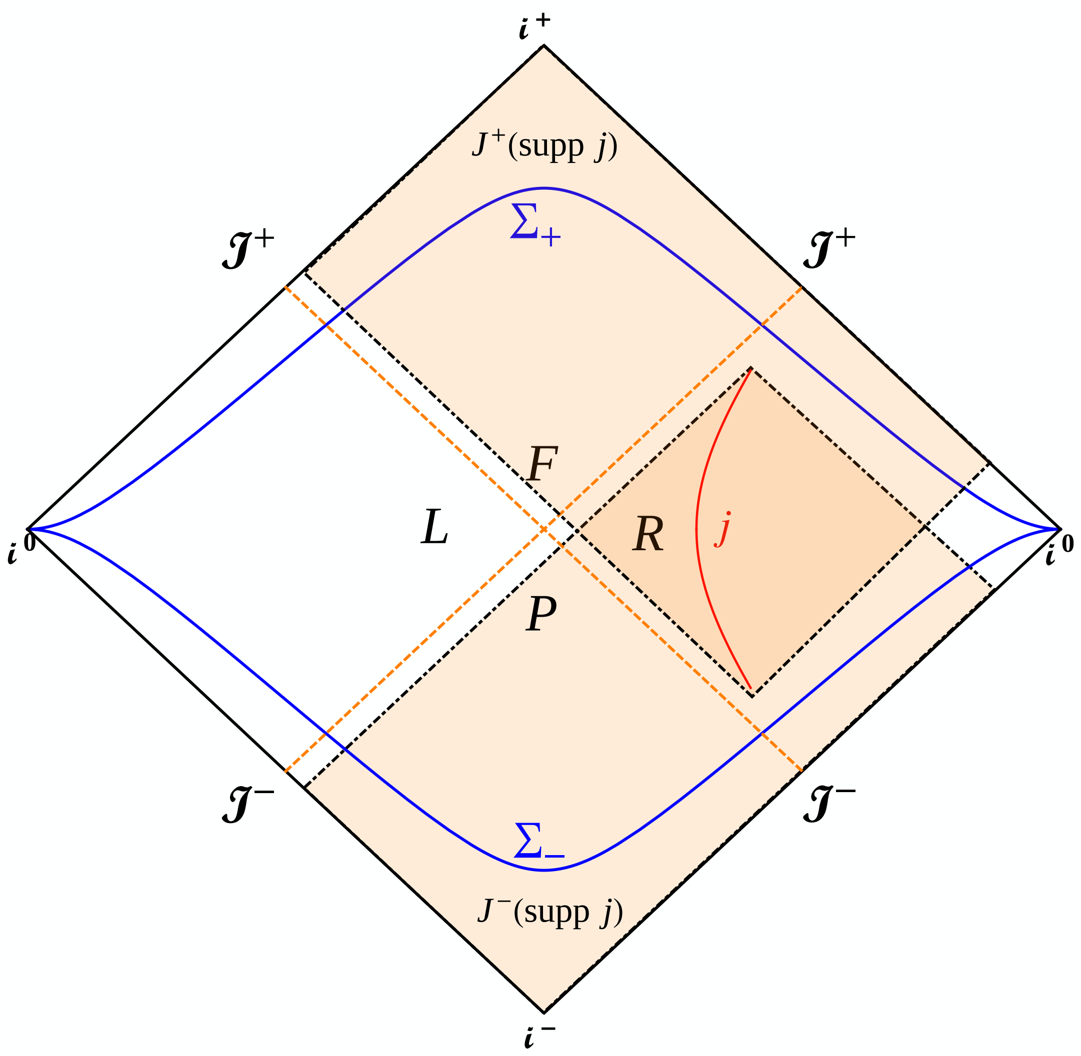
In the future (F) region

$$w^2_{\omega\mathbf{k}_\perp} = -\,i\frac{e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp + i\omega\zeta}}{\sqrt{32\pi^2 a}} e^{\pi\omega/2a} H^{(2)}_{i\omega/a}\left(k_\perp e^{a\eta}/a\right),$$

Classical radiation and zero-energy modes



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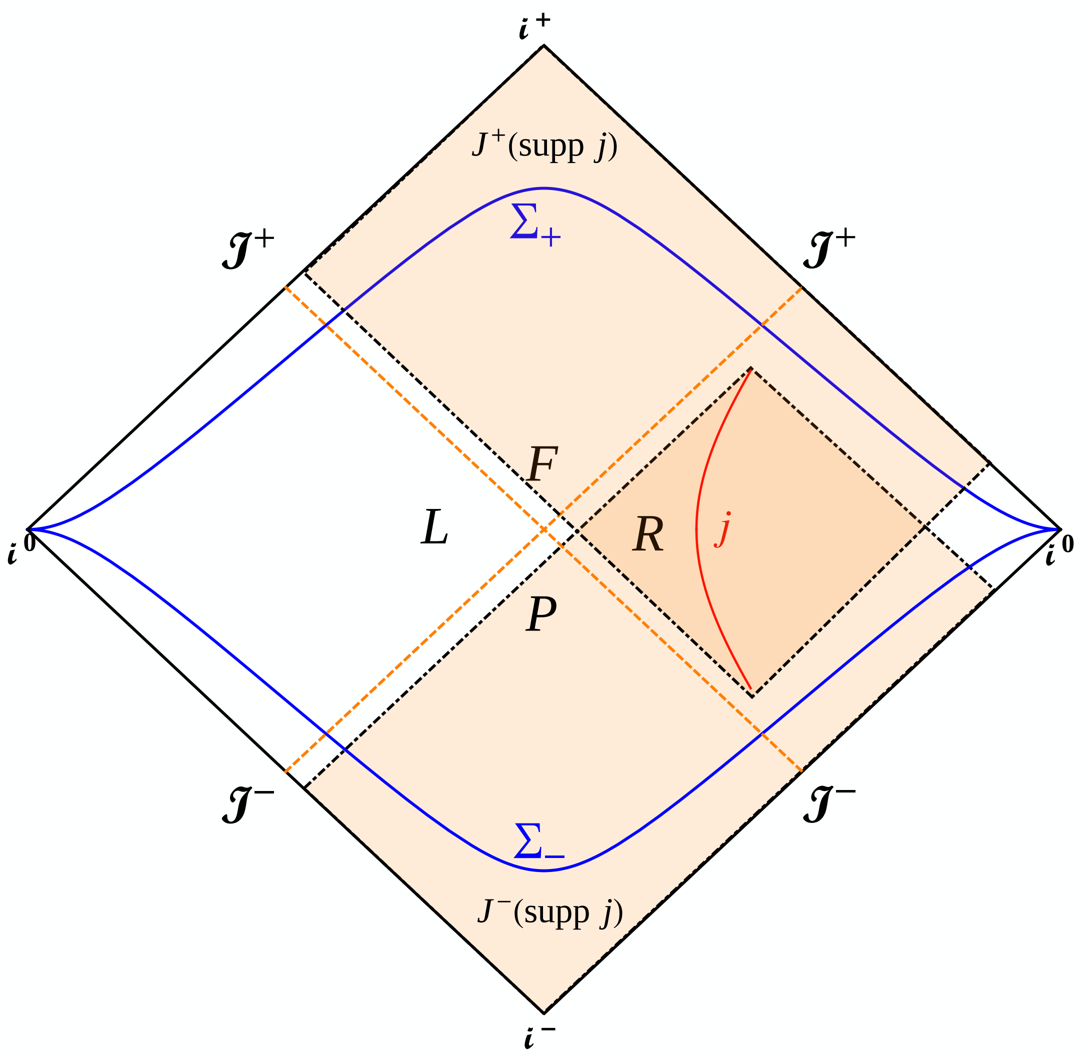
From which, we can cast Rj as

$$Rj = \frac{-q}{4\pi\rho_0(x)}$$

Classical radiation and zero-energy modes



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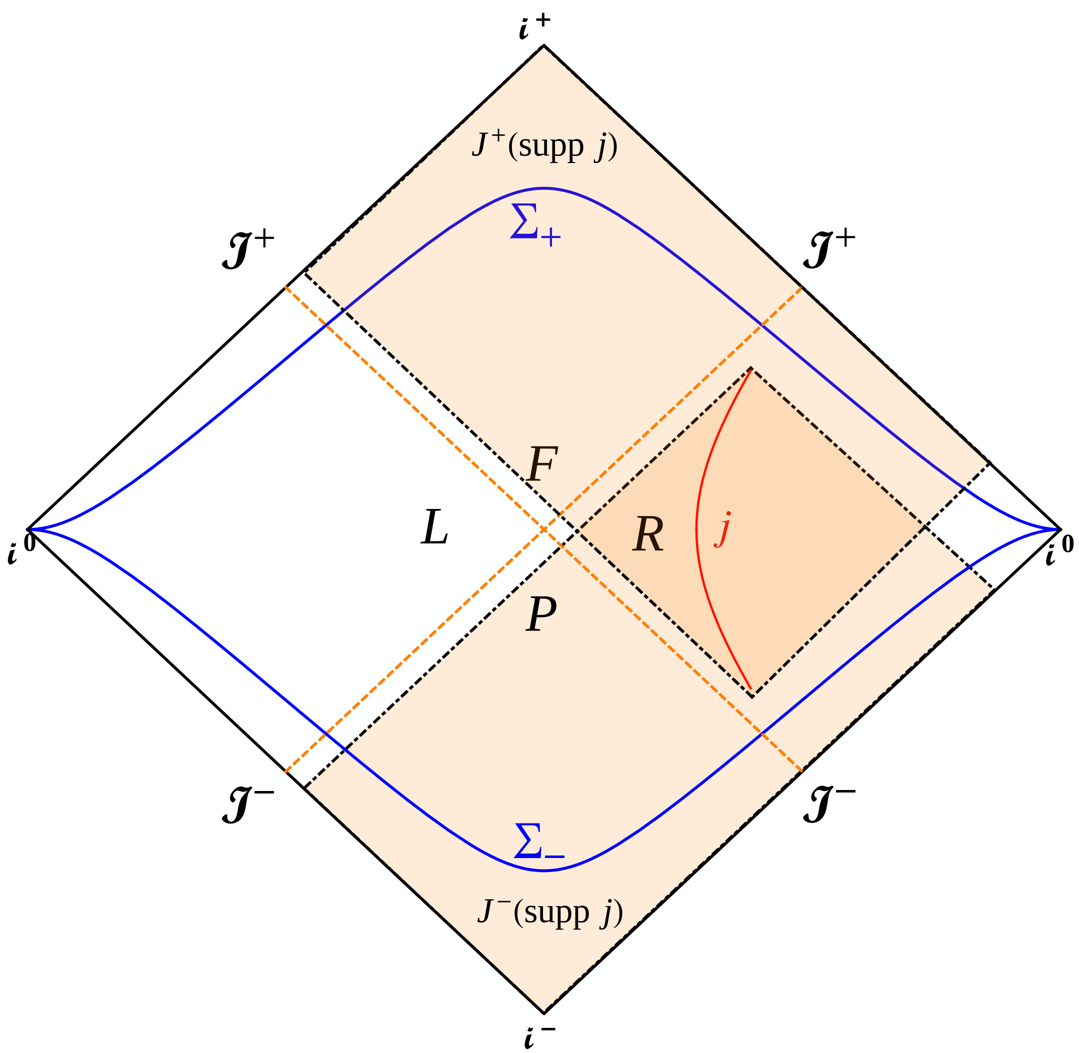
Classical radiation and zero-energy modes



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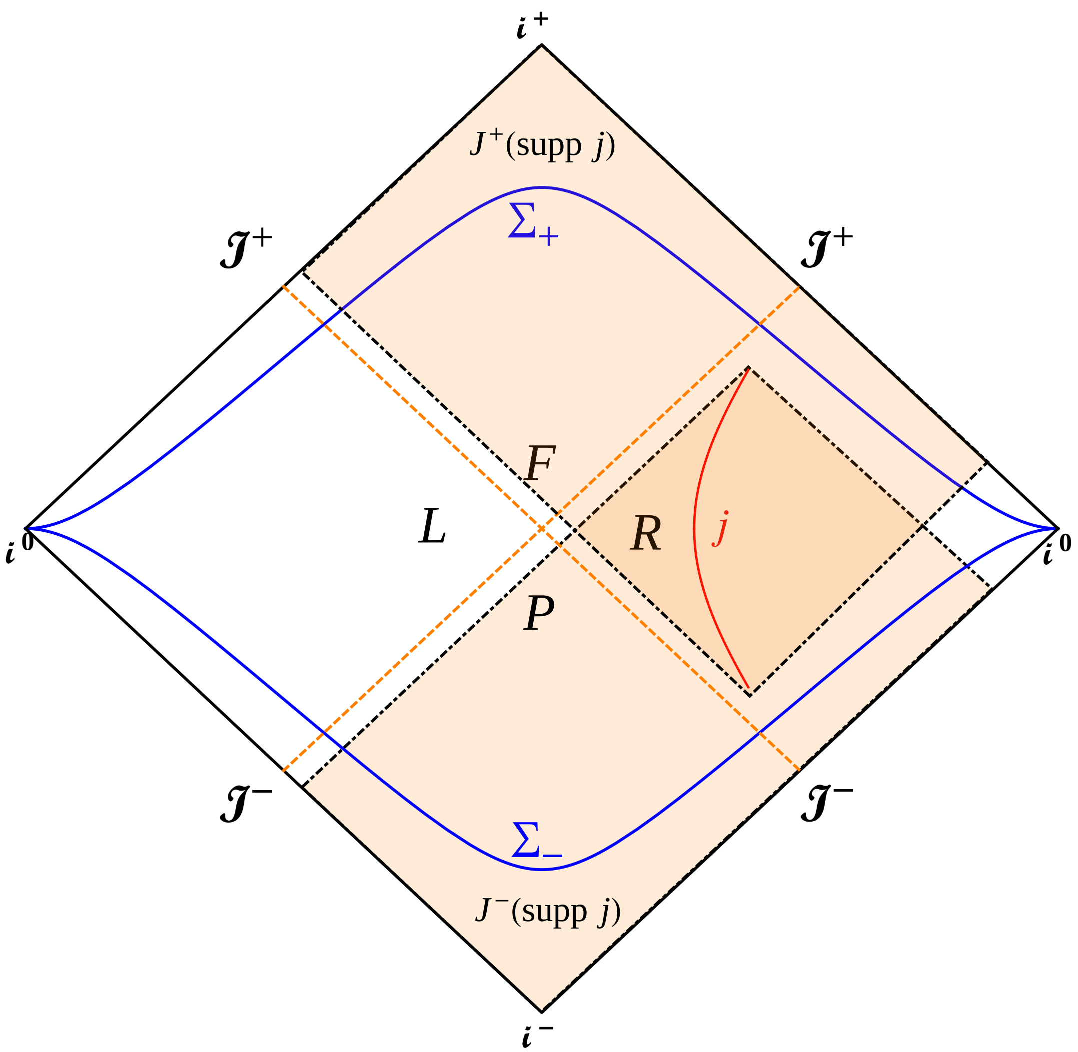
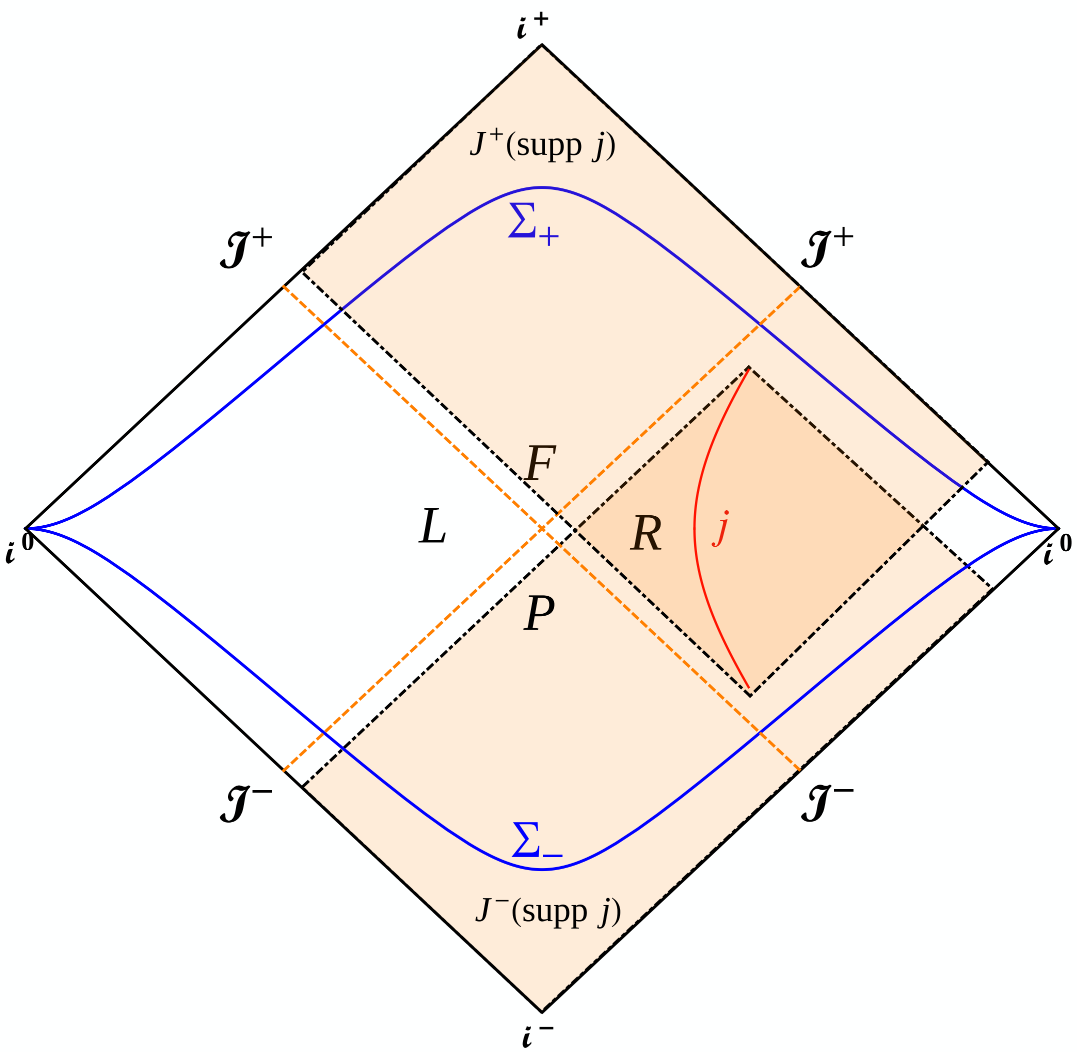
Retarded solution obtained by the usual Green function method of classical (scalar) electrodynamics

Classical radiation and zero-energy modes



Rj^+ (inertial) positive-frequency part of Rj

Classical radiation and zero-energy modes

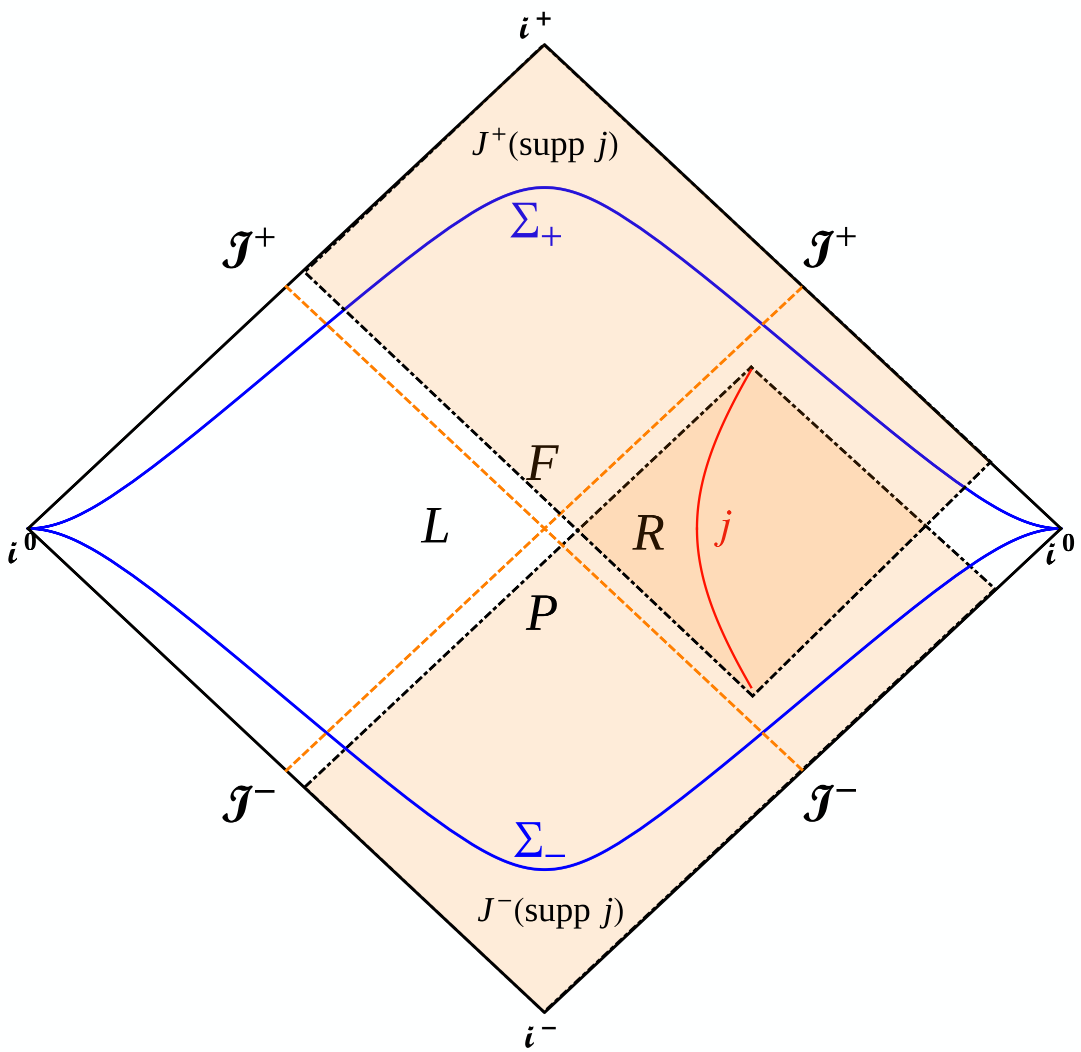


Rj^+ (inertial) positive-frequency part of Rj

From the classical Photon Number (on Σ_+)

$$N_M \equiv \langle Rj^+, Rj^+ \rangle_{\text{KG}}$$

Classical radiation and zero-energy modes



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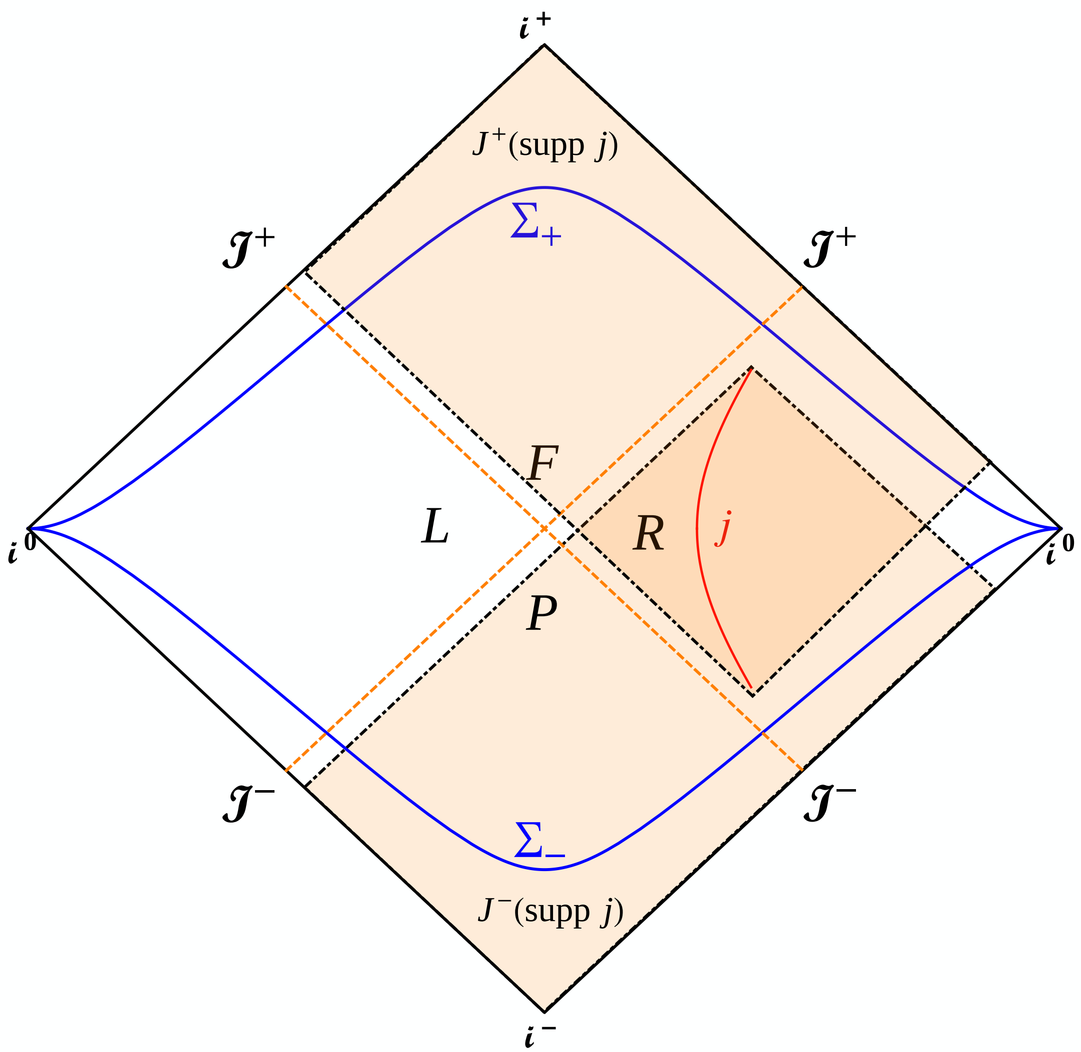

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yields

$$\frac{N_M}{T_{\mathrm{tot}}} = \frac{q^2 a}{4\pi^2}$$

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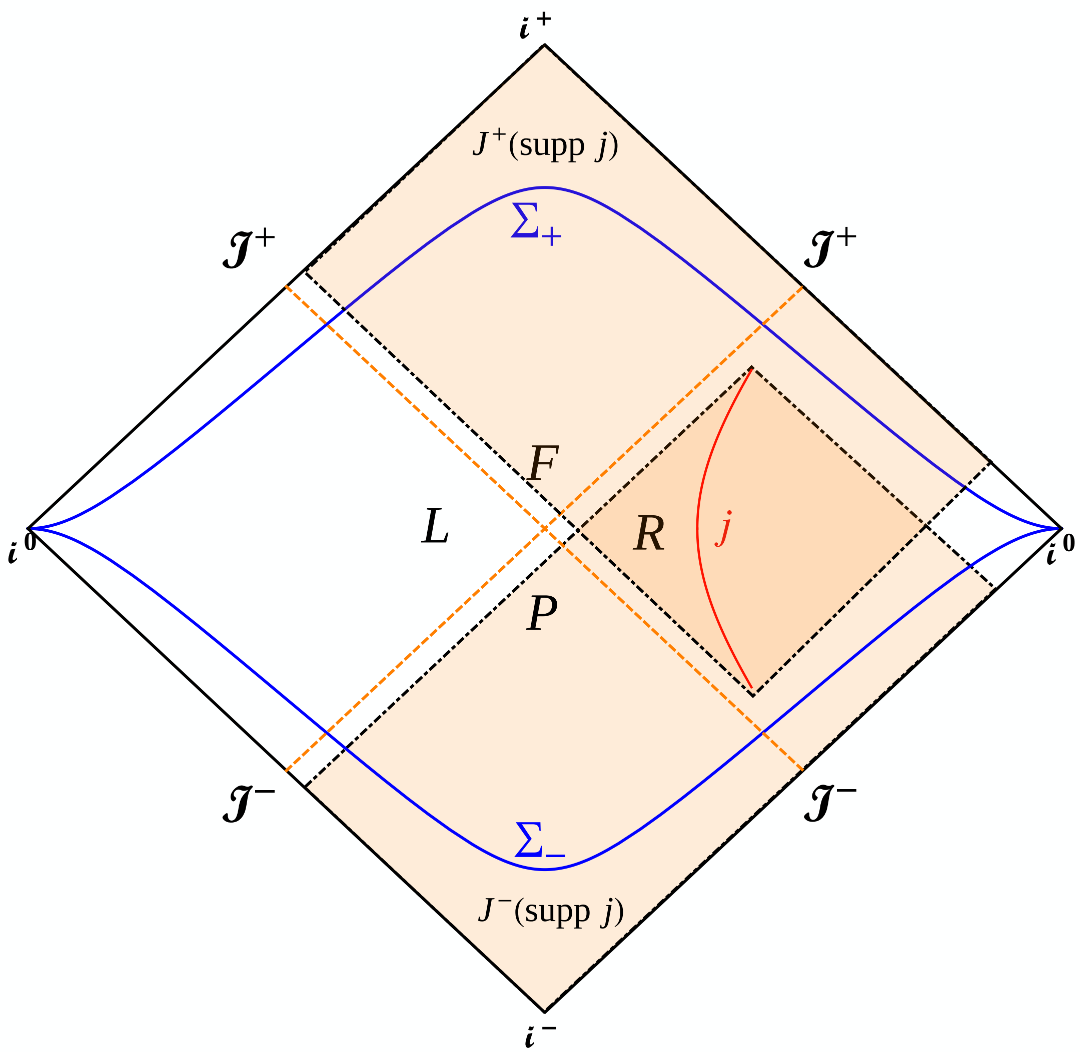

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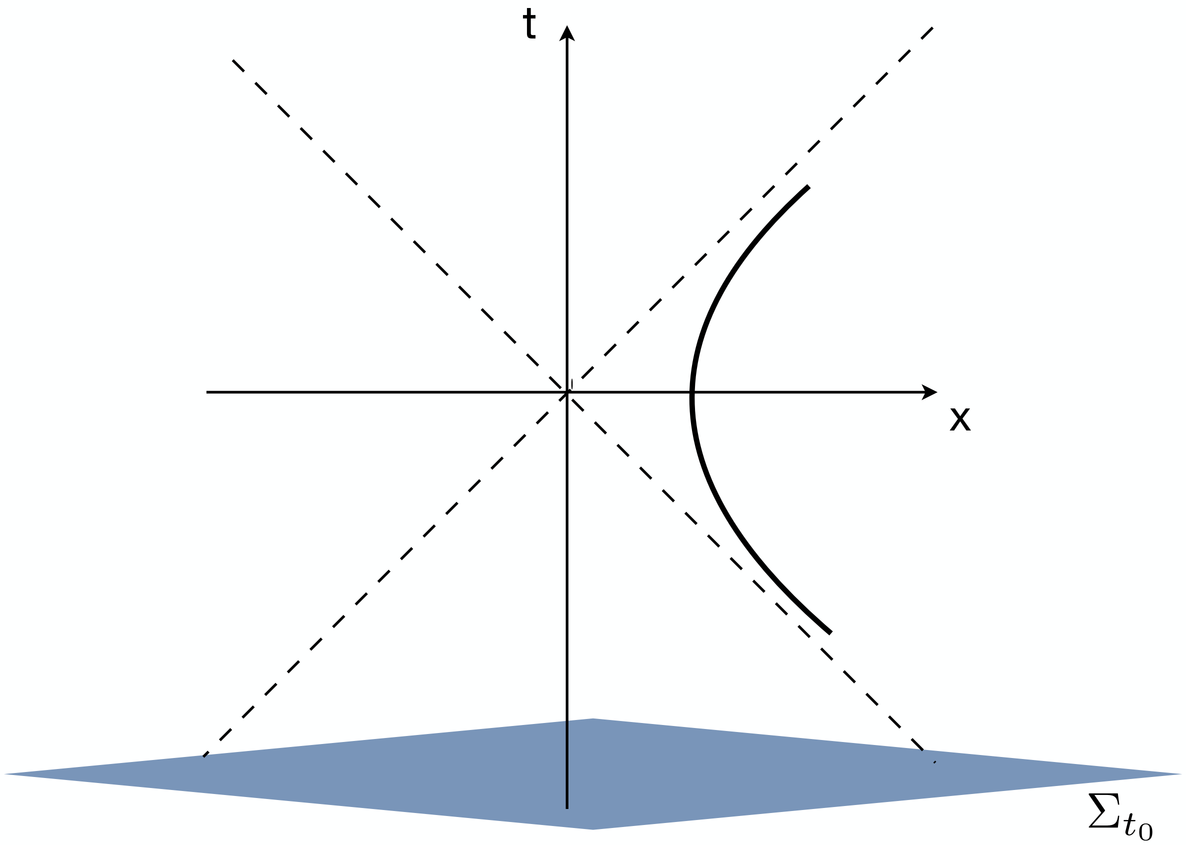
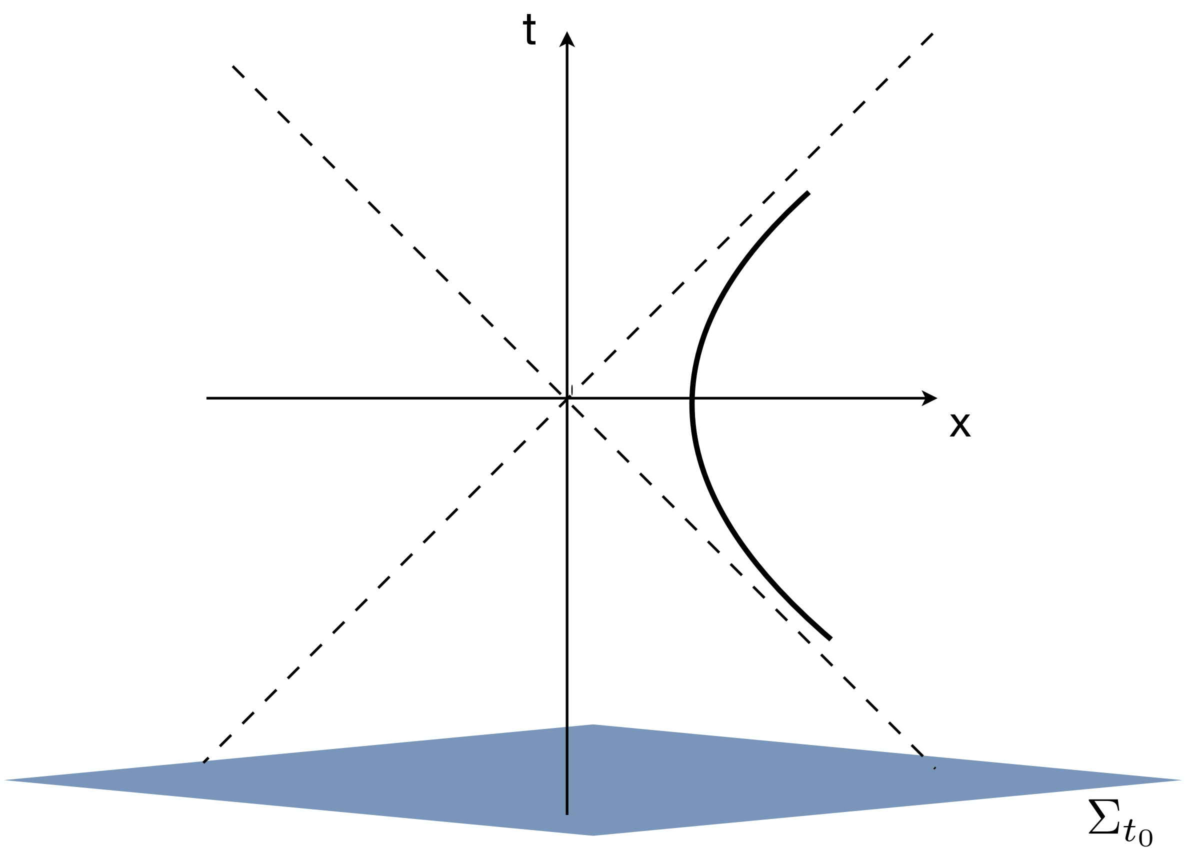
Exactly the result obtained using tree-level QFT
[Ren and Weinberg PRD 49, 6526,1994]

Quantum Radiation and zero-energy modes



$$\nabla^a \nabla_a \hat{\phi} = j$$

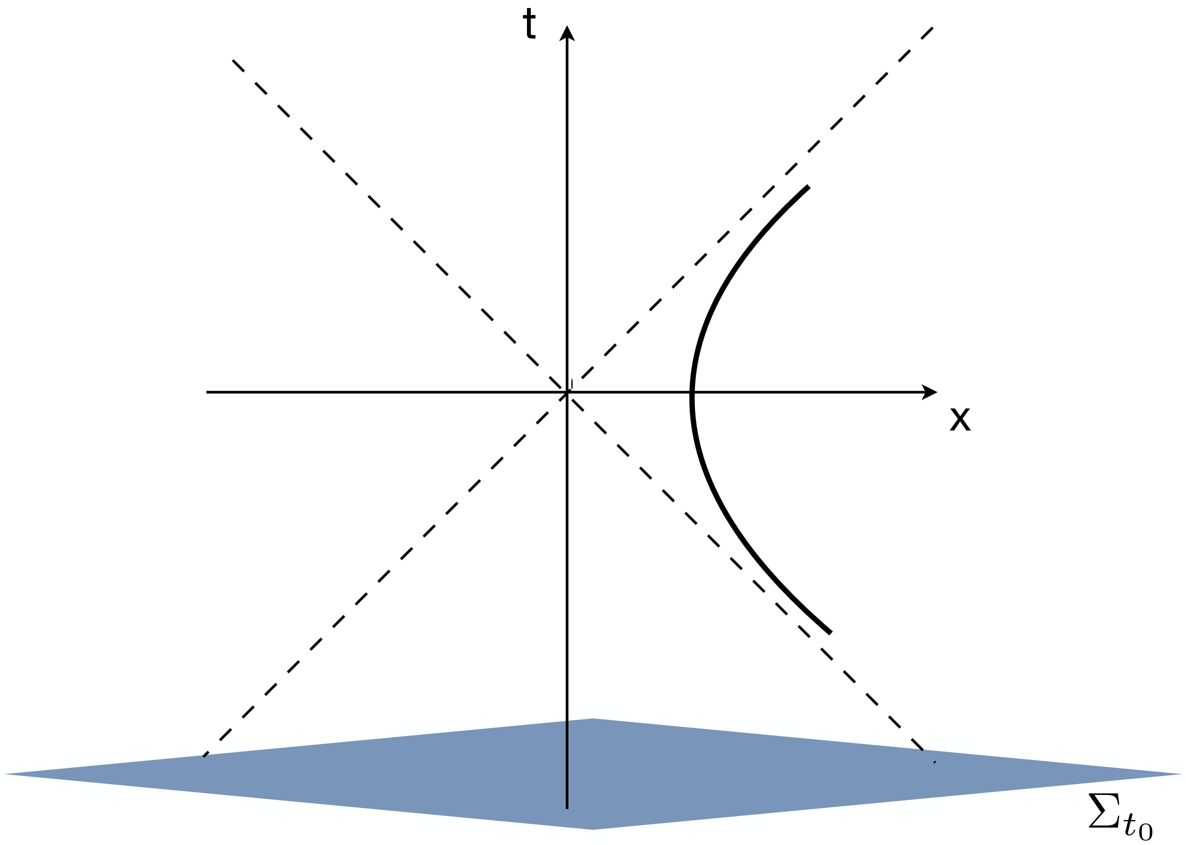
Quantum Radiation and zero-energy modes



$$\hat{\phi}(t, \mathbf{x}) = \phi_{\rm ret}(t, \mathbf{x})\hat{I} + \hat{\phi}_{\rm in}(t, \mathbf{x}),$$

$$\nabla^a \nabla_a \hat{\phi} = j$$

Quantum Radiation and zero-energy modes



$$\hat{\phi}(t,\mathbf{x}) = \phi_{\text{ret}}(t,\mathbf{x})\hat{I} + \hat{\phi}_{\text{in}}(t,\mathbf{x}),$$

↓ classical retarded solution

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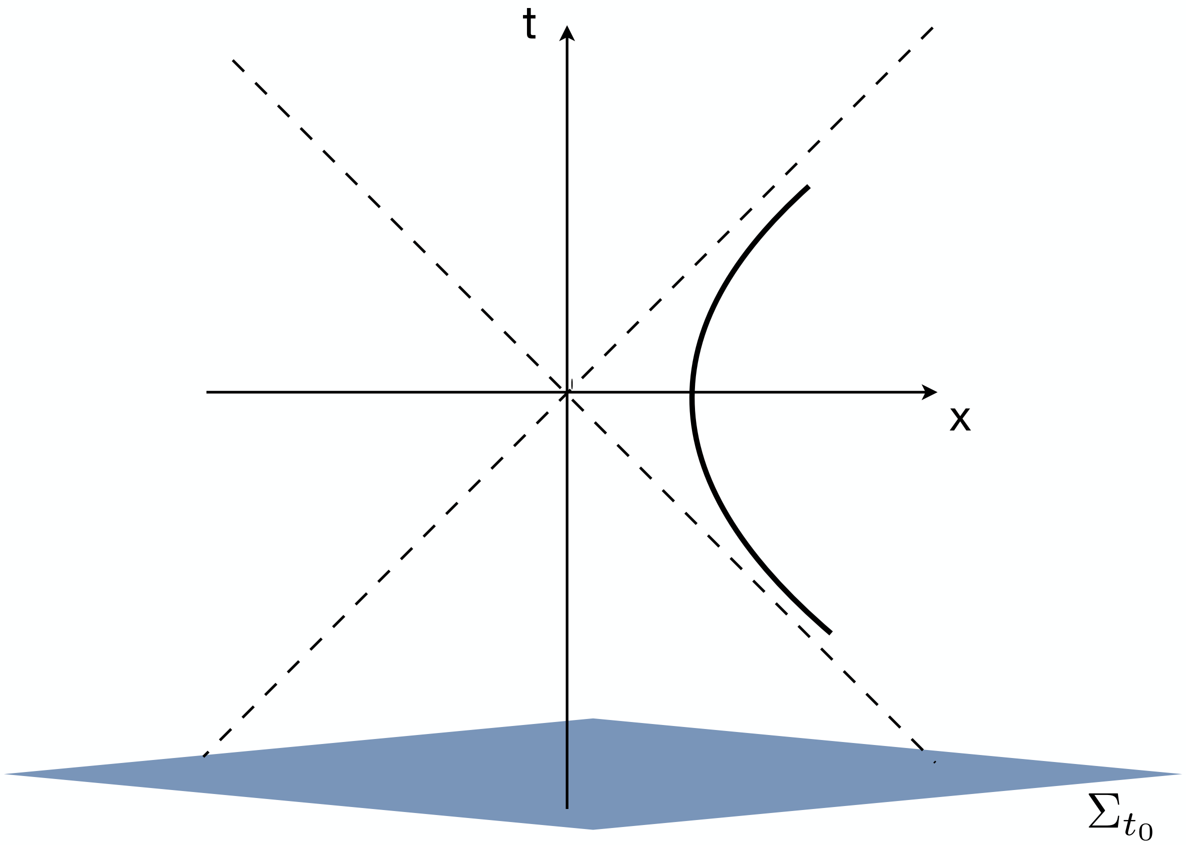
Quantum Radiation and zero-energy modes



free scalar field

$$\hat{\phi}(t,\mathbf{x}) = \phi_{\rm ret}(t,\mathbf{x})\hat{I} + \hat{\phi}_{\rm in}(t,\mathbf{x}),$$

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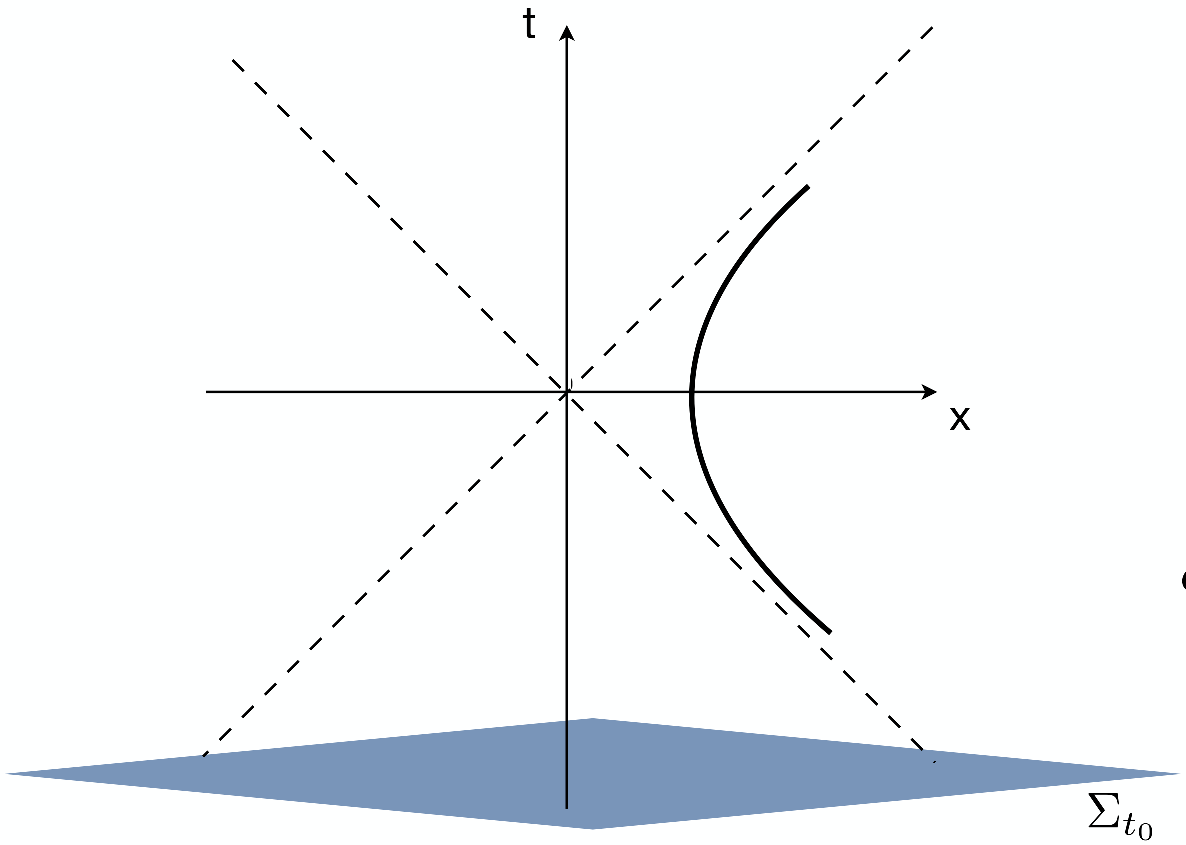
Quantum Radiation and zero-energy modes



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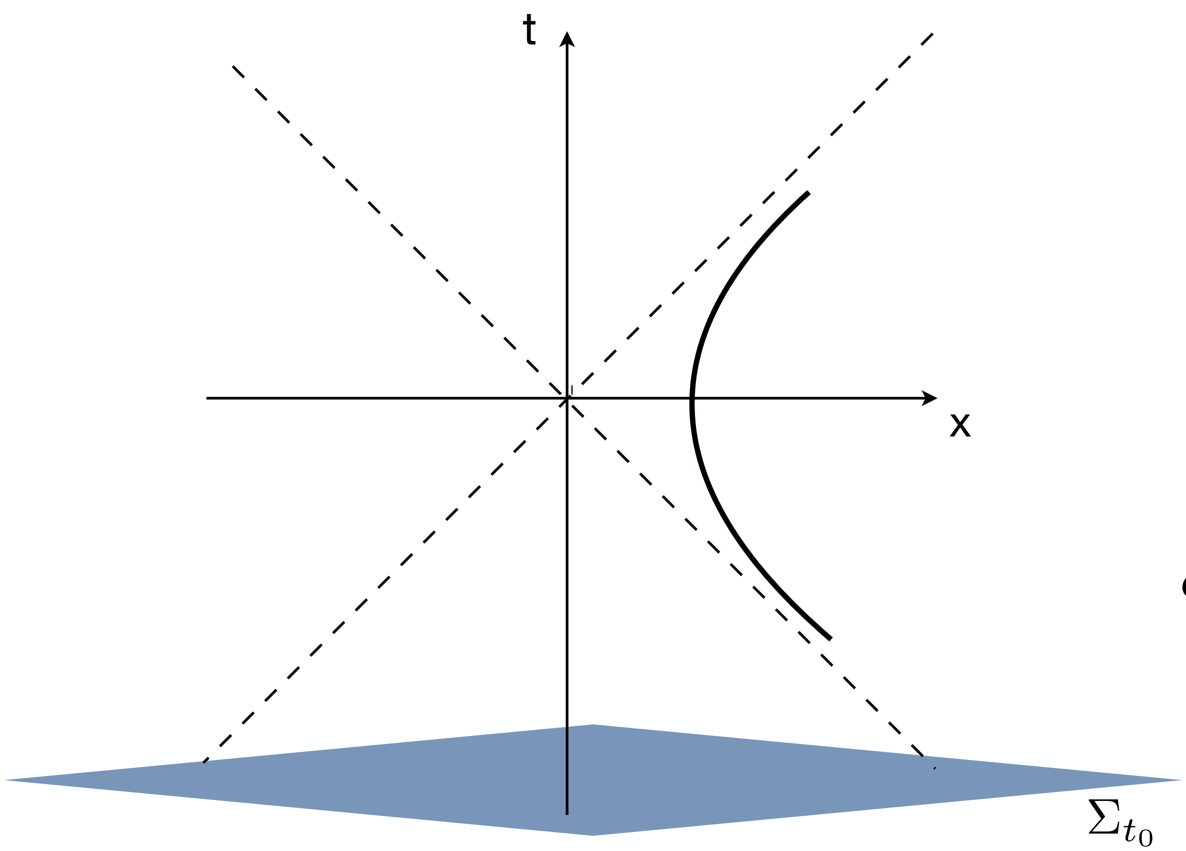
$$\hat{\phi}_{\rm in}(t, \mathbf{x}) \equiv \sum_j \left[u_j(t, \mathbf{x}) \hat{a}_{\rm in}(u_j^*) + u_j^*(t, \mathbf{x}) \hat{a}_{\rm in}^\dagger(u_j) \right]$$

Quantum Radiation and zero-energy modes



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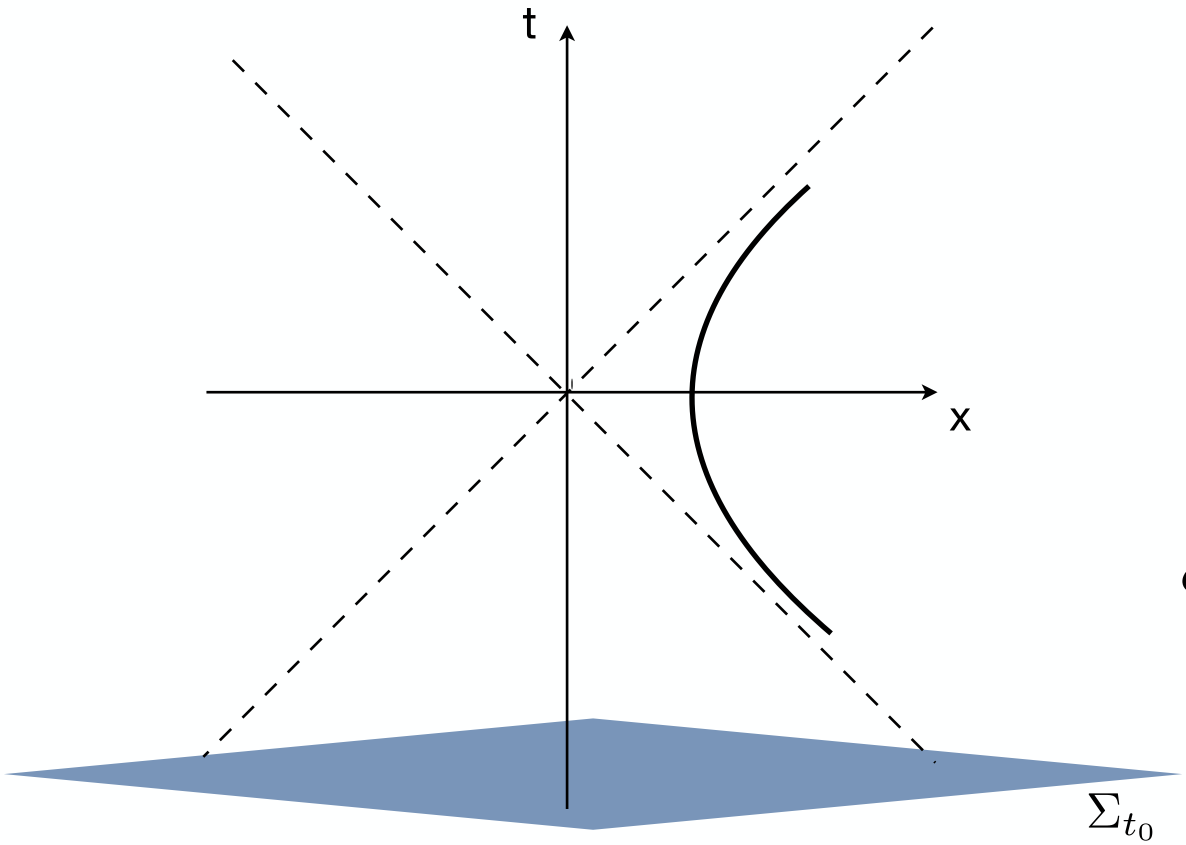
positive-frequency Minkowski mode

$$\nabla^a\nabla_a\hat{\phi} = j$$

Quantum Radiation and zero-energy modes



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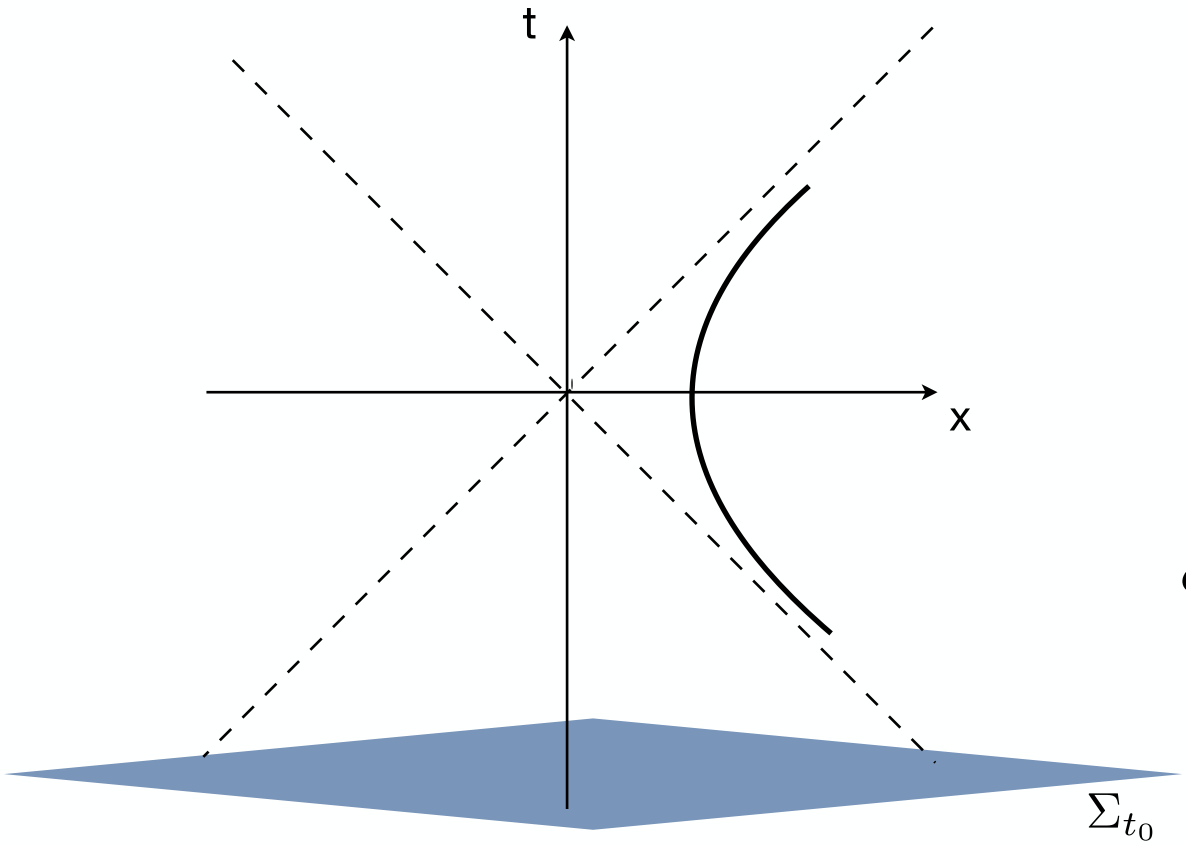
positive-frequency
Minkowski mode

$$\nabla^a\nabla_a\hat{\phi} = j$$

$$\hat{a}_{\rm in}(u_j^*)|0_{\rm in}^M\rangle = 0$$

Quantum Radiation and zero-energy modes



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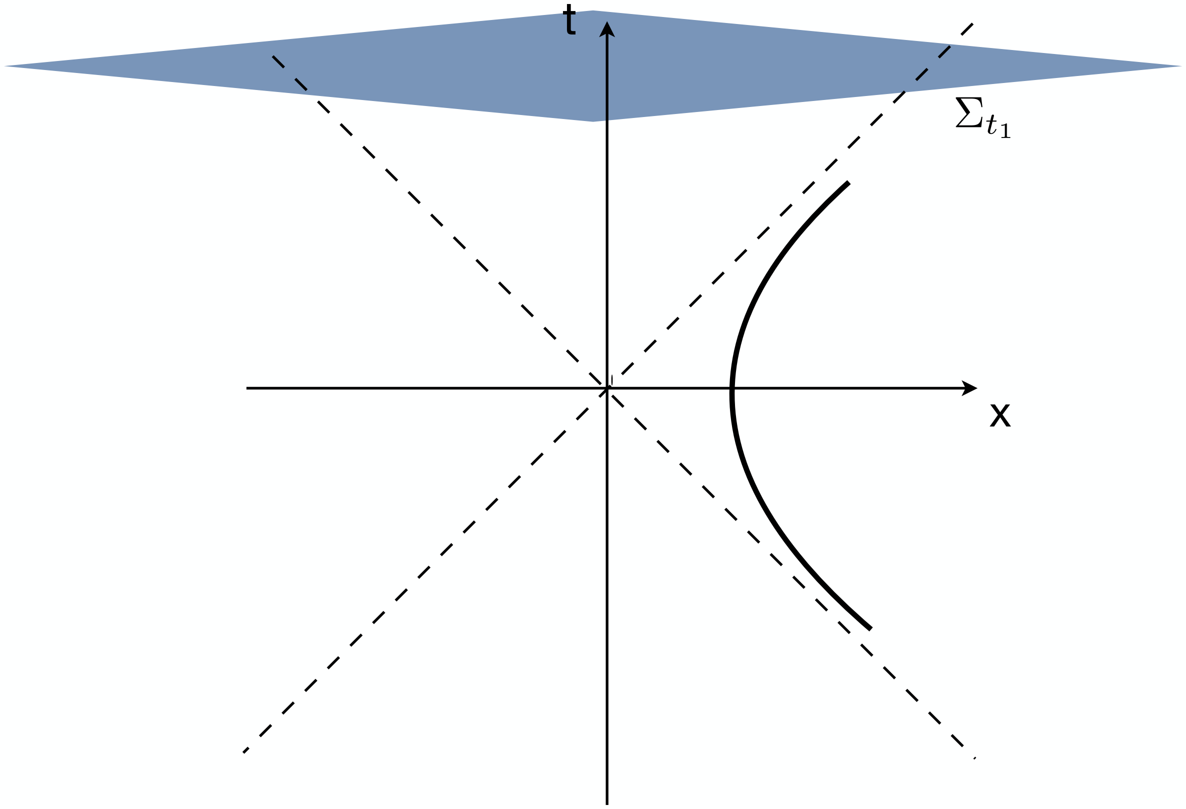
vacuum state for inertial observers in the asymptotic

Quantum Radiation and zero-energy modes



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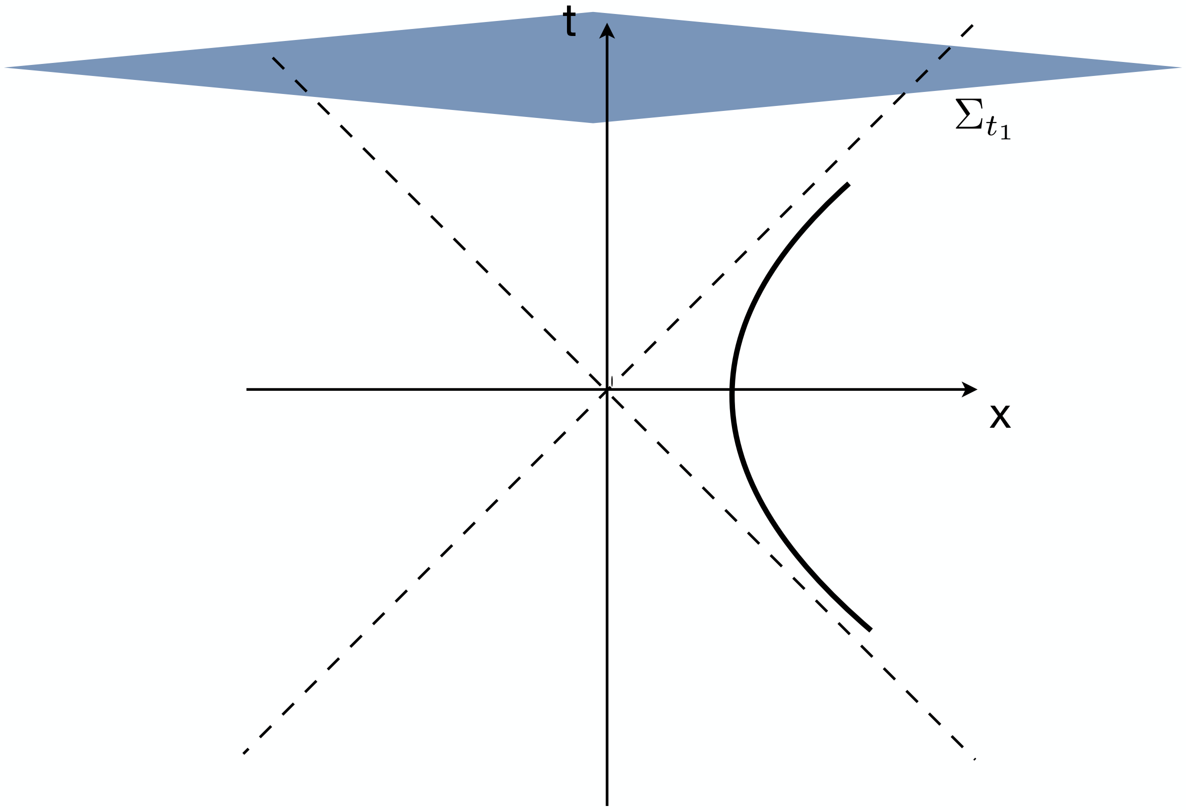
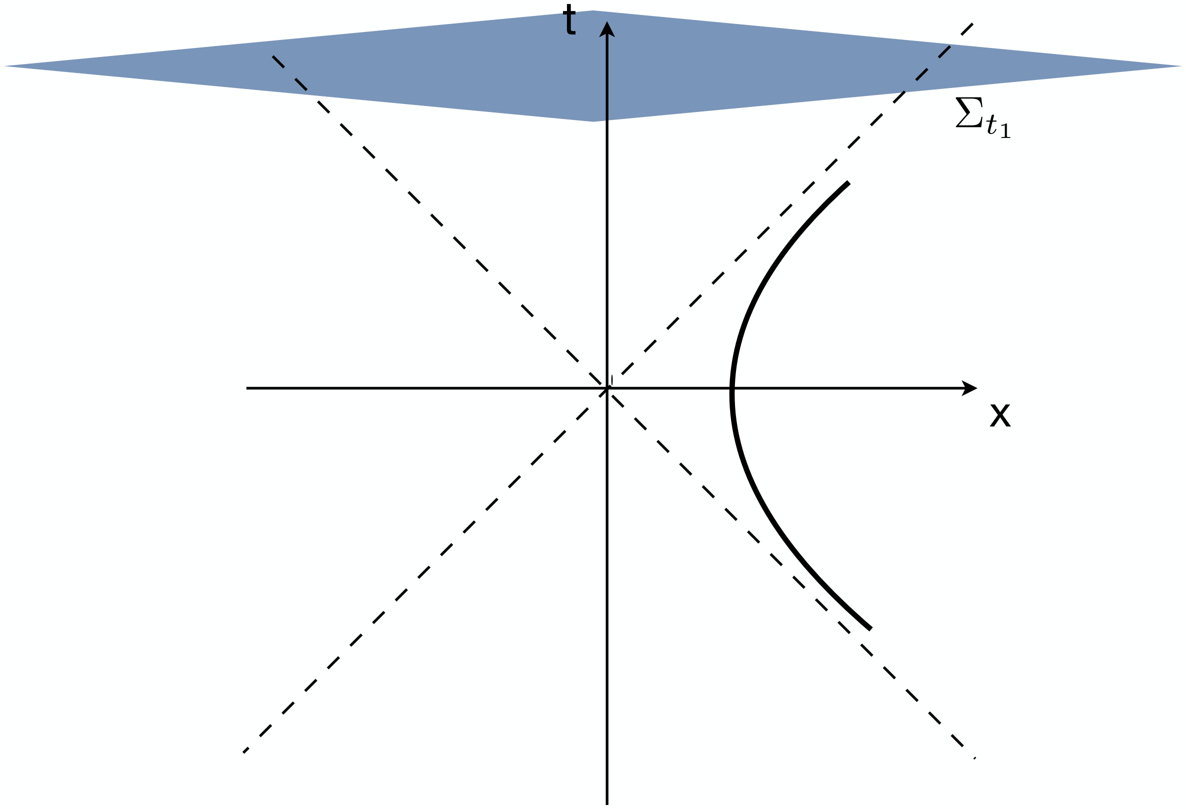
Quantum Radiation and zero-energy modes



$$\hat{\phi}(t,\mathbf{x}) = \phi_{\rm adv}(t,\mathbf{x})\hat{I} + \hat{\phi}_{\rm out}(t,\mathbf{x})$$

$$\nabla^a \nabla_a \hat{\phi} = j$$

Quantum Radiation and zero-energy modes



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classical advanced solution

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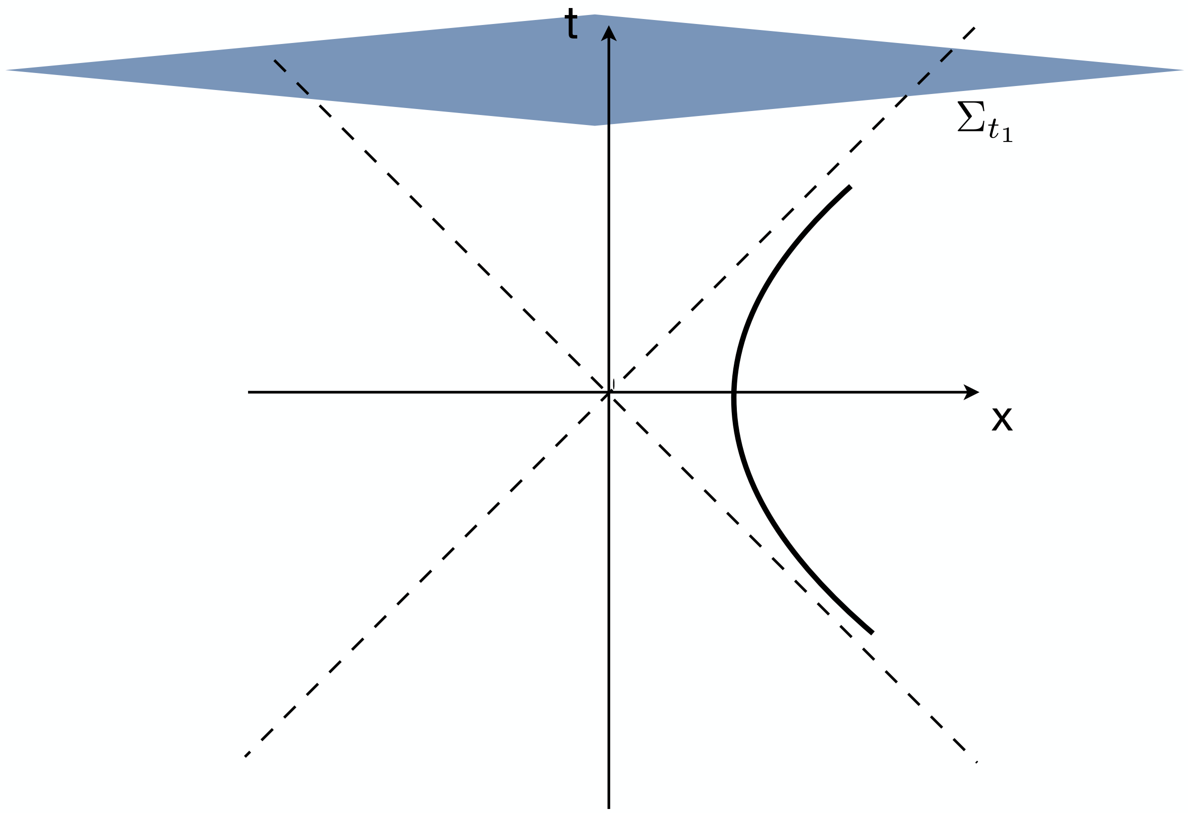
Quantum Radiation and zero-energy modes



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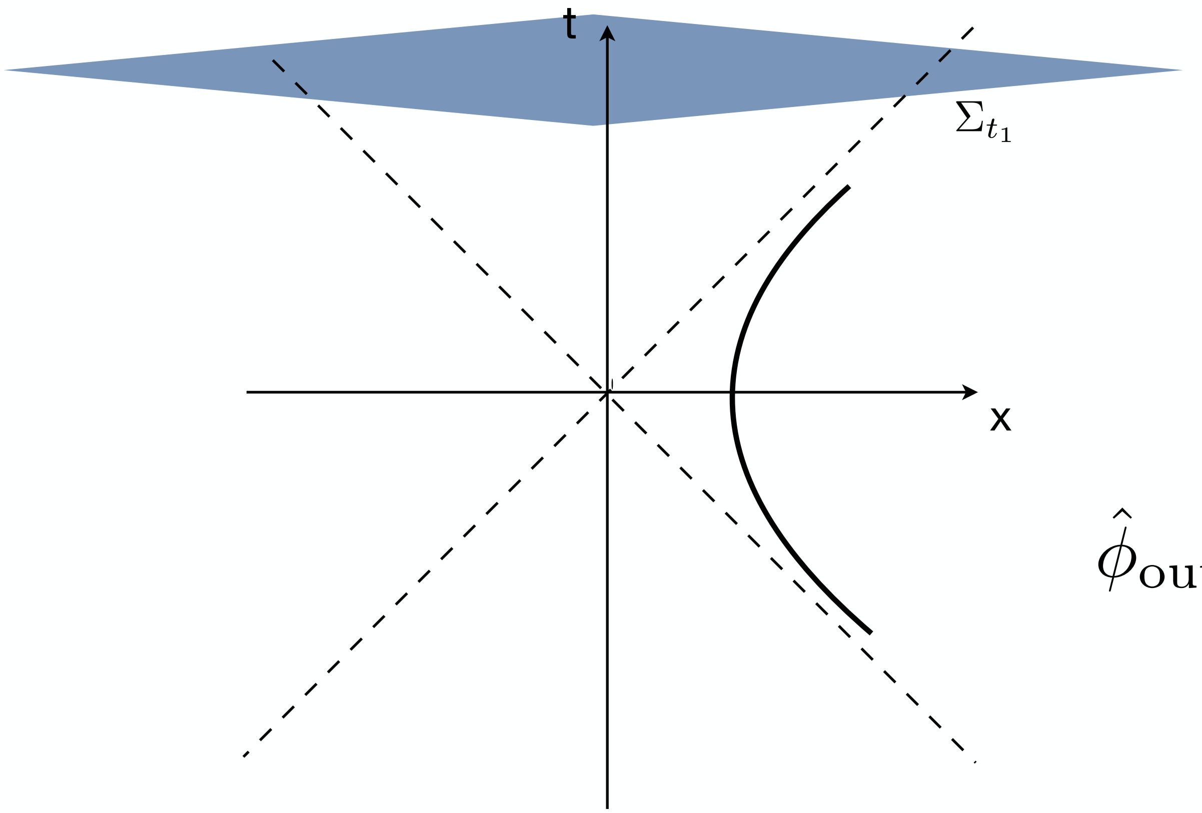

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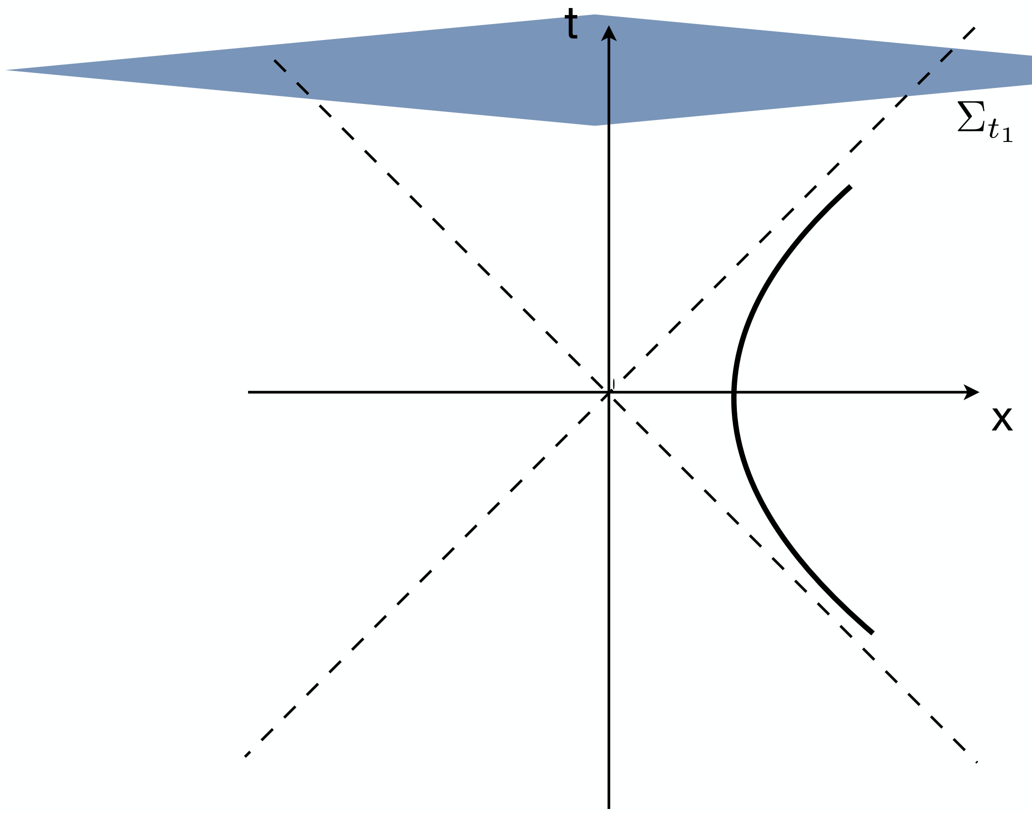
Quantum Radiation and zero-energy modes



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Minkowski mode

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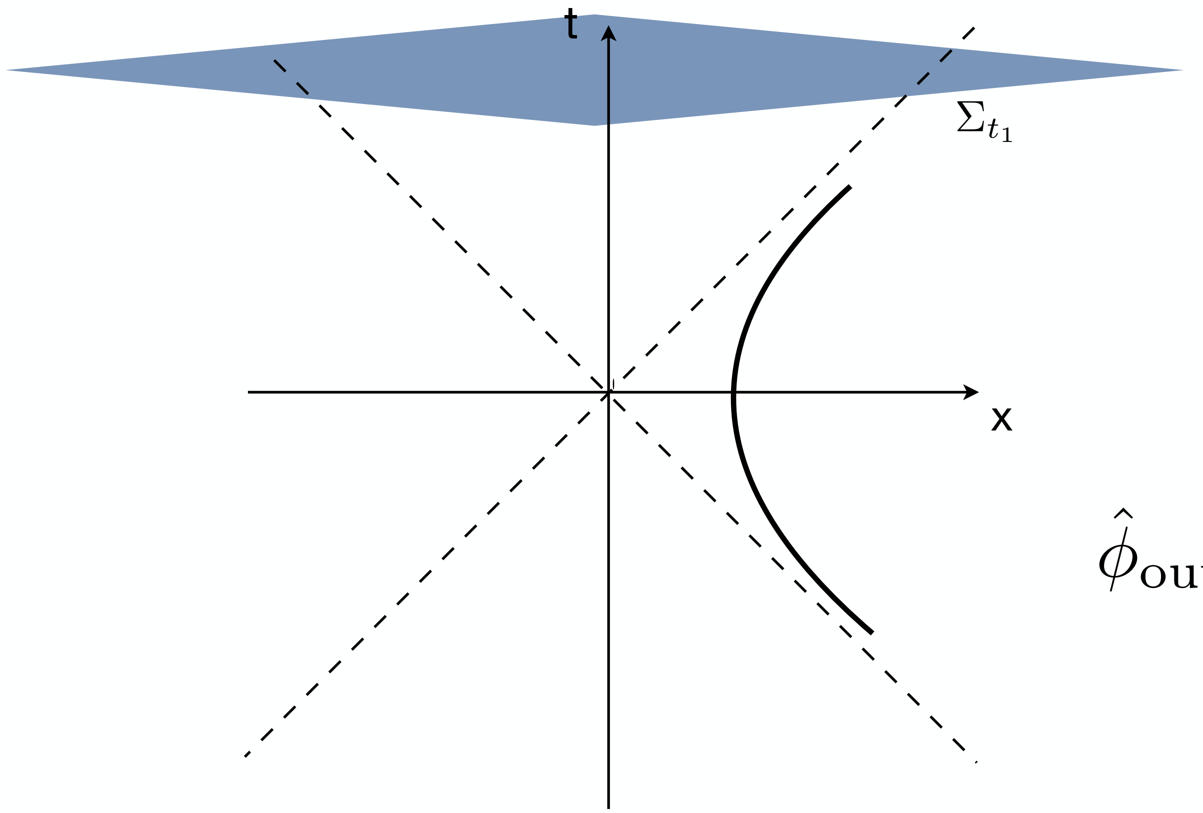
Quantum Radiation and zero-energy modes



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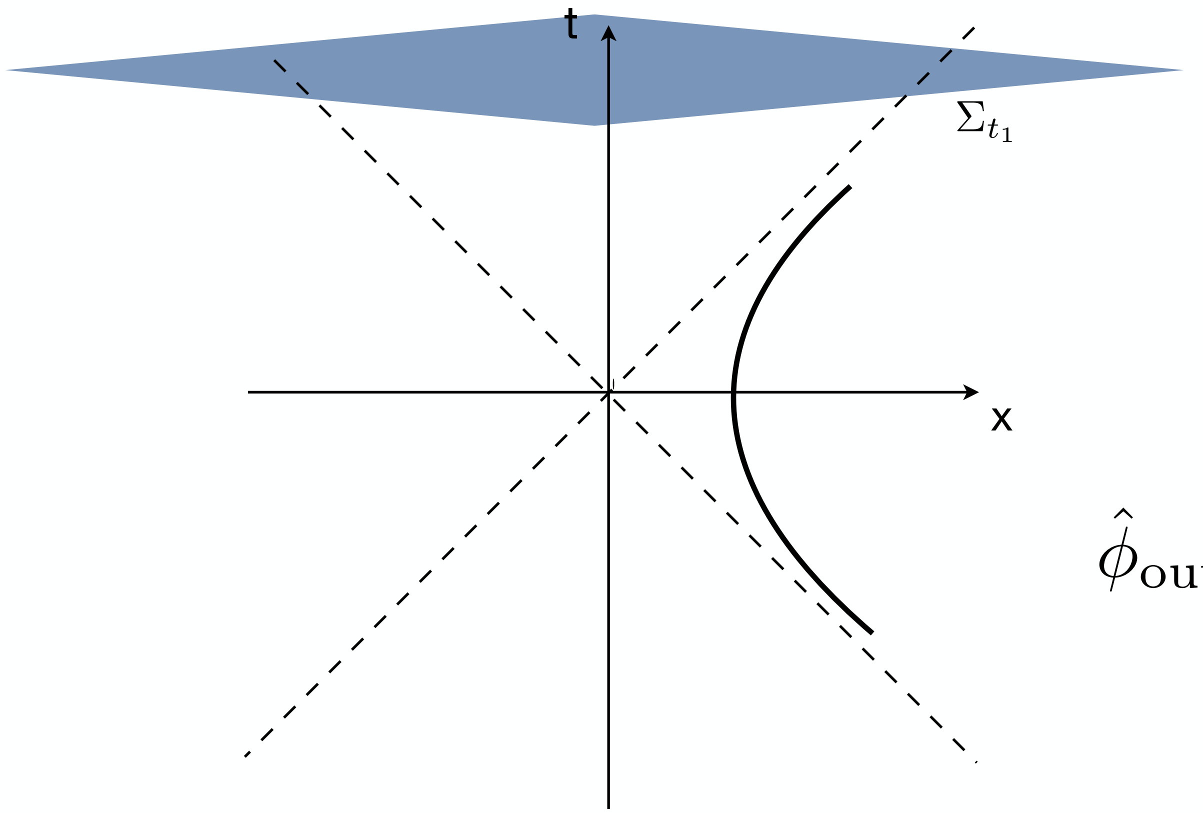
Quantum Radiation and zero-energy modes



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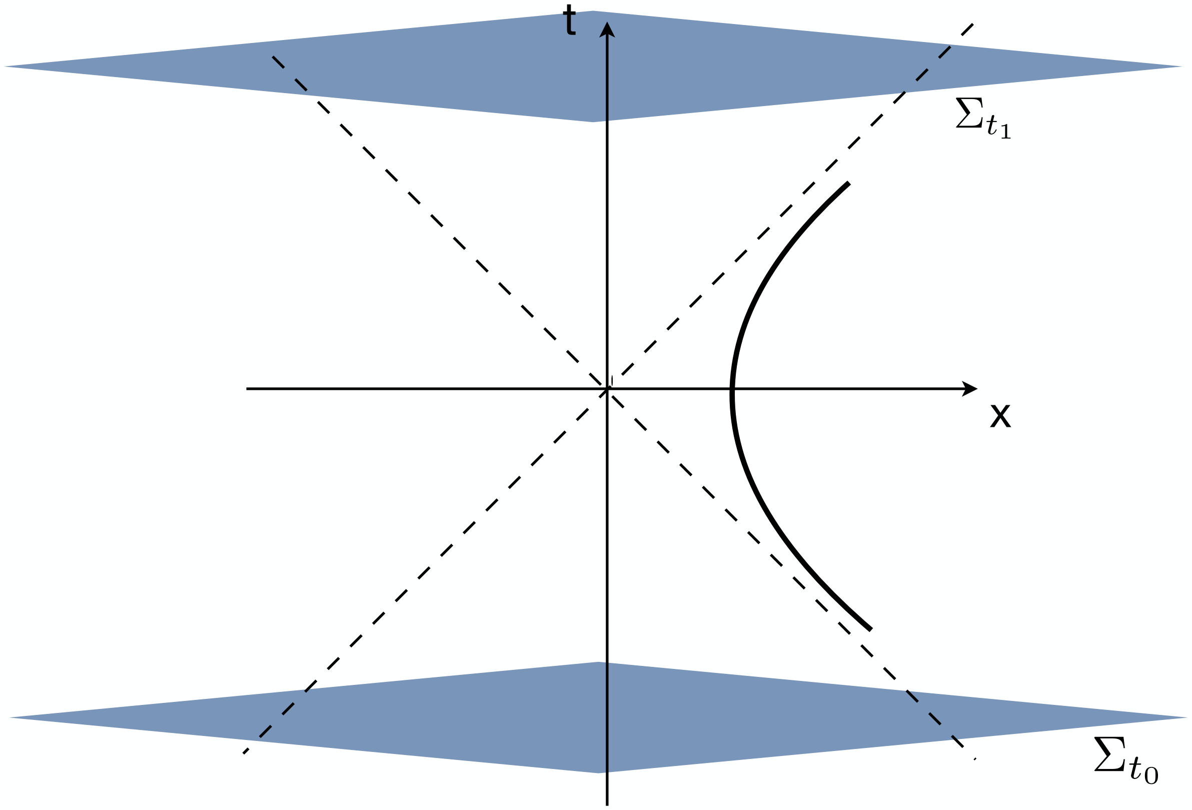
vacuum state for inertial observers in the asymptotic

Quantum Radiation and zero-energy modes

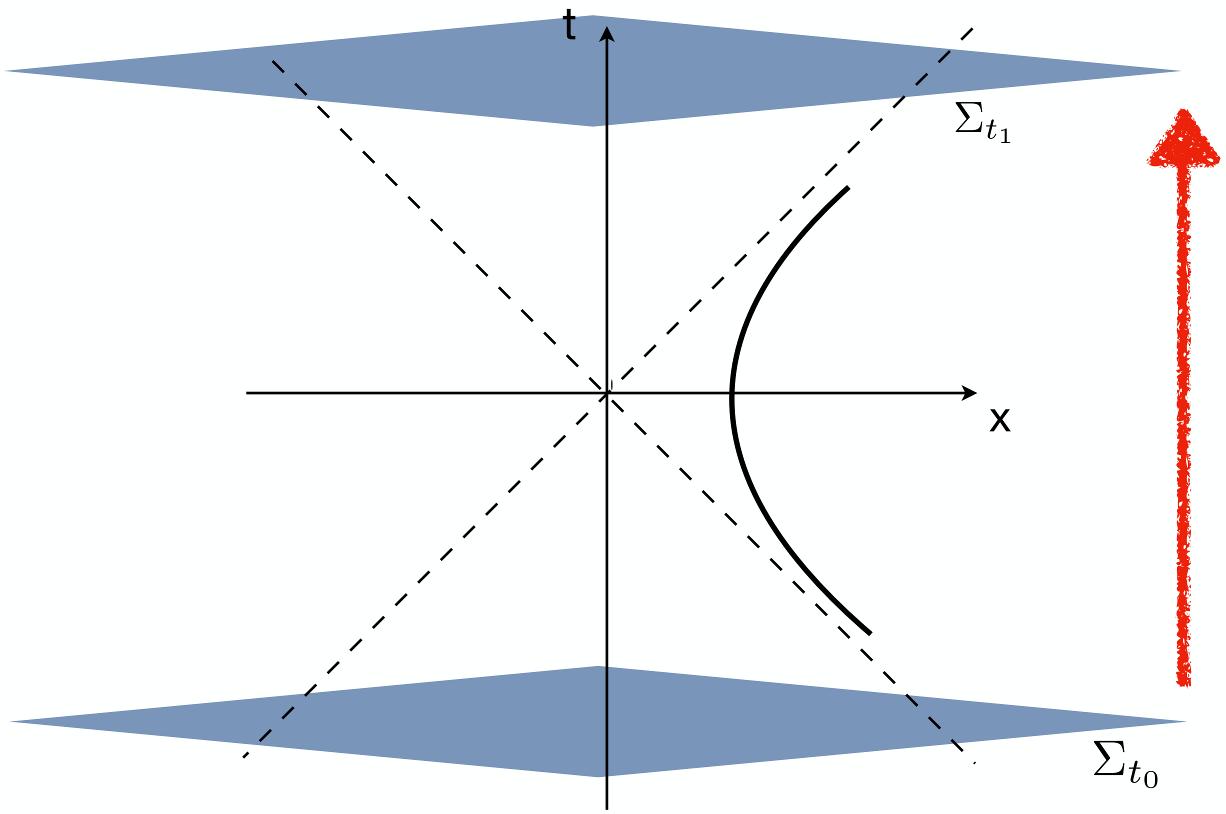
$$|0_{\rm in}^M\rangle = \hat{S}|0_{\rm out}^M\rangle \qquad \hat{\phi}_{\rm out}$$



$$|0_{\rm in}^M\rangle \qquad \hat{S}\hat{\phi}_{\rm out}\hat{S}^\dagger = \hat{\phi}_{\rm in}.$$

Quantum Radiation and zero-energy modes

$$|0^M_{\rm in}\rangle = \hat{S}|0^M_{\rm out}\rangle \qquad \hat{\phi}_{\rm out}$$



$$\hat{S} = T\exp\left(-i\int_{-\infty}^{\infty} dt H_I(t)\right)$$

S Matrix

$$|0^M_{\rm in}\rangle \qquad \hat{S}\hat{\phi}_{\rm out}\hat{S}^\dagger = \hat{\phi}_{\rm in}.$$

Quantum Radiation and zero-energy modes

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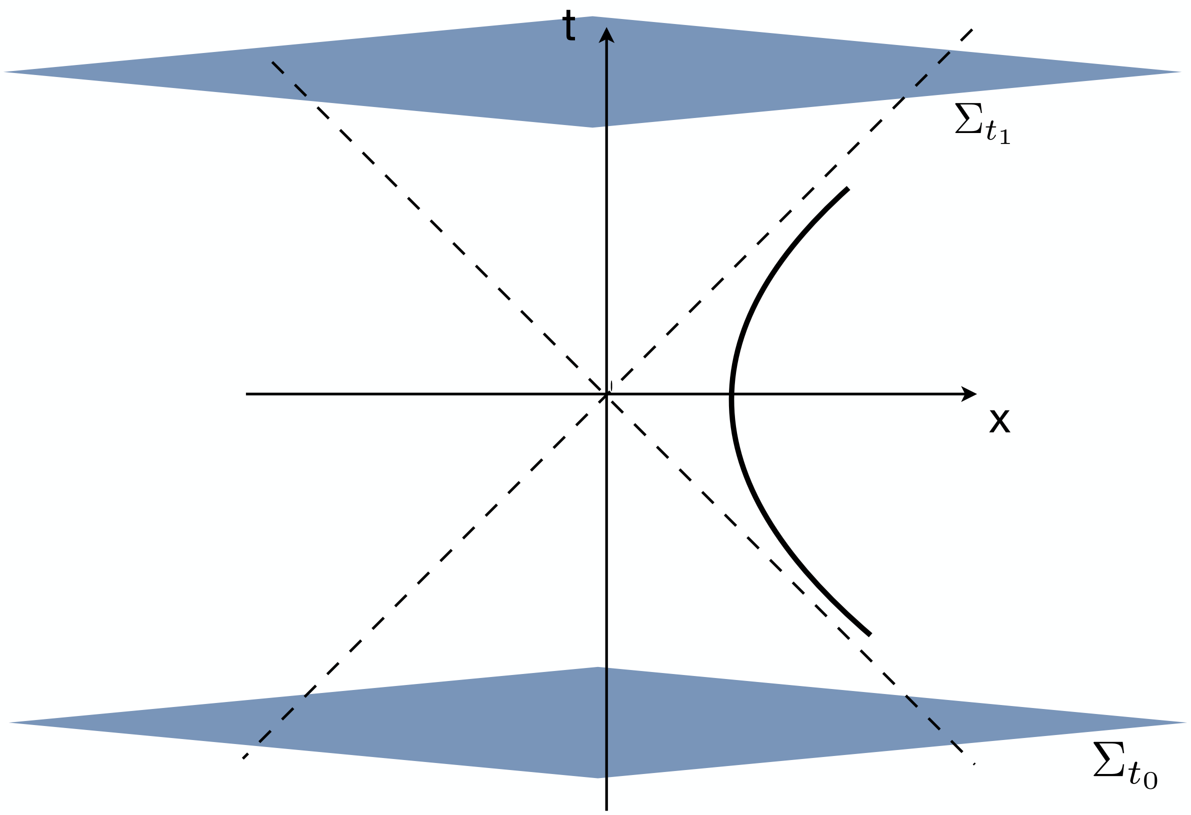

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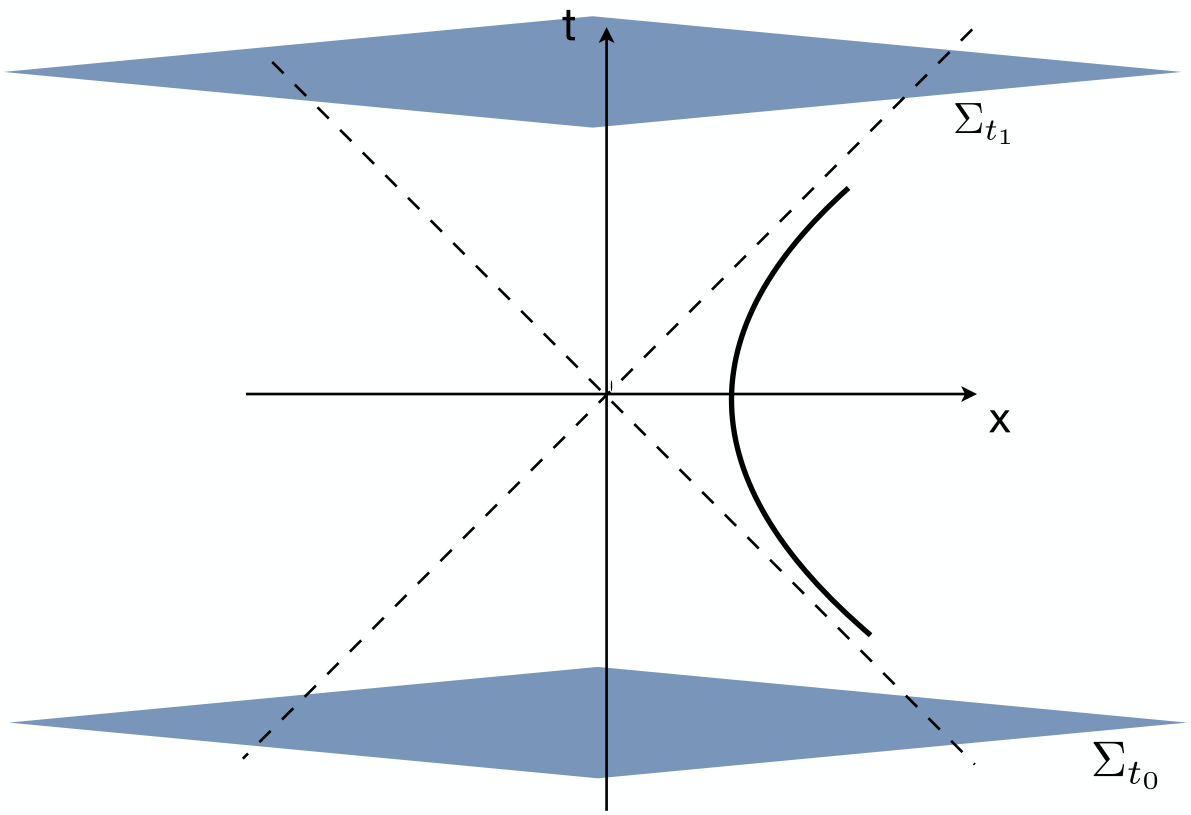
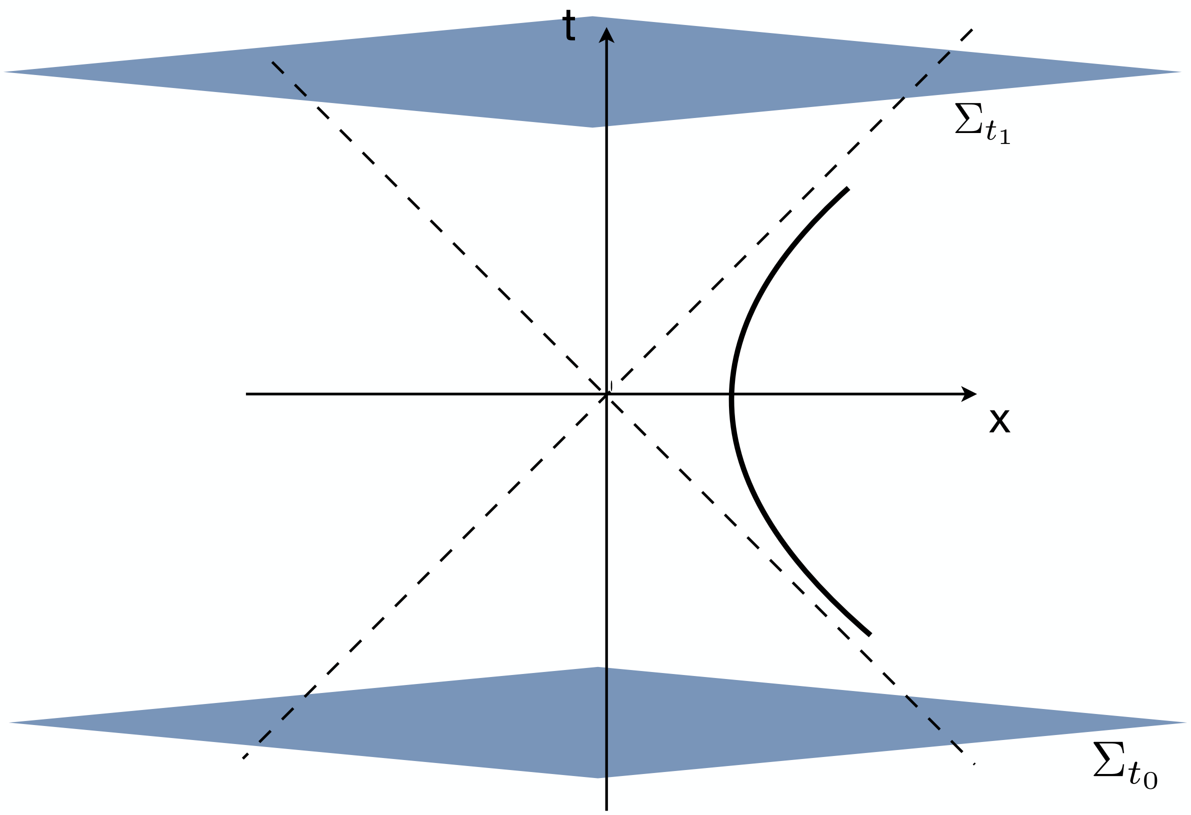


$$\hat{S} = \exp\left[-i\int d^4x\sqrt{-g}\;\hat{\phi}_{\rm out}(x)j(x)\right]$$

$$|0_{\rm in}^M\rangle \qquad \hat{S}\hat{\phi}_{\rm out}\hat{S}^\dagger = \hat{\phi}_{\rm in}.$$

Quantum Radiation and zero-energy modes

$$|0^M_{\rm in}\rangle = \hat{S}|0^M_{\rm out}\rangle \qquad \hat{\phi}_{\rm out}$$



$$\hat{S} = \exp\left[-i\int d^4x\sqrt{-g}\;\hat{\phi}_{\rm out}(x)j(x)\right]$$

By using Unruh modes:

$$|0^M_{\rm in}\rangle \qquad \hat{S}\hat{\phi}_{\rm out}\hat{S}^\dagger = \hat{\phi}_{\rm in}.$$

Quantum Radiation and zero-energy modes

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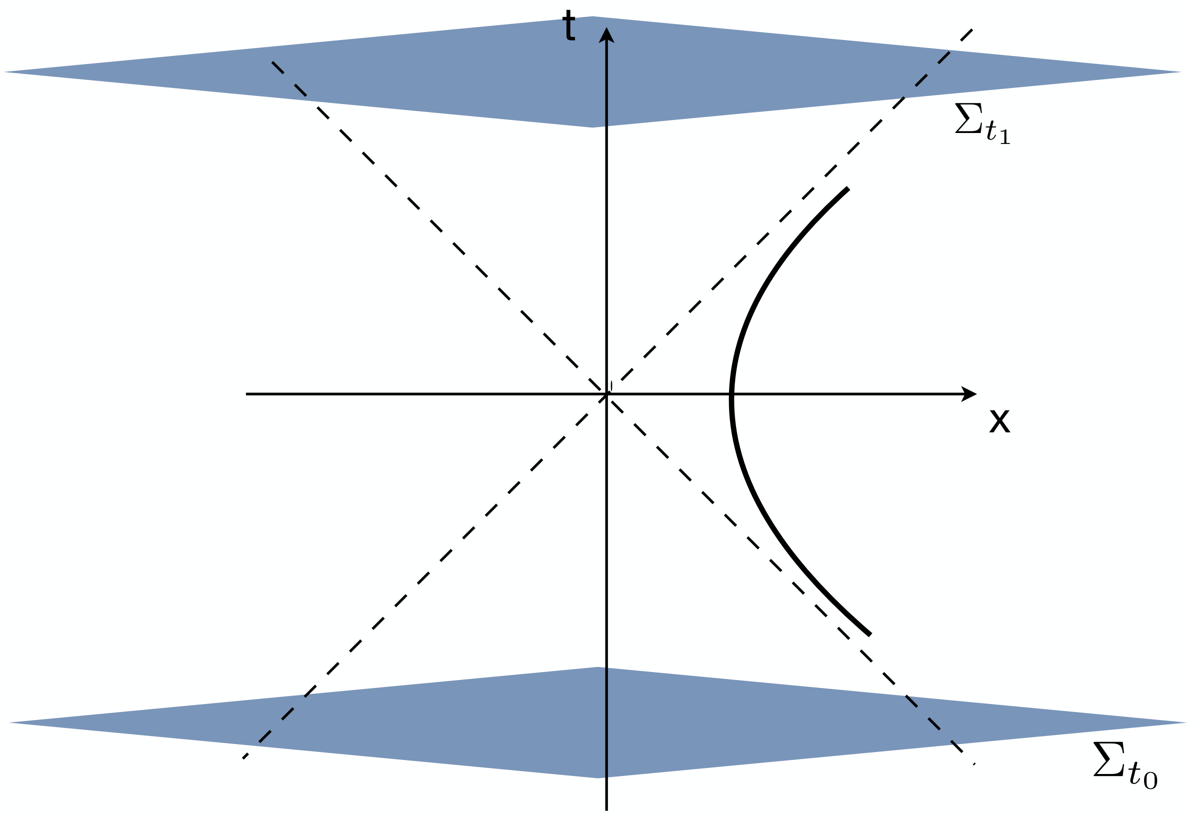

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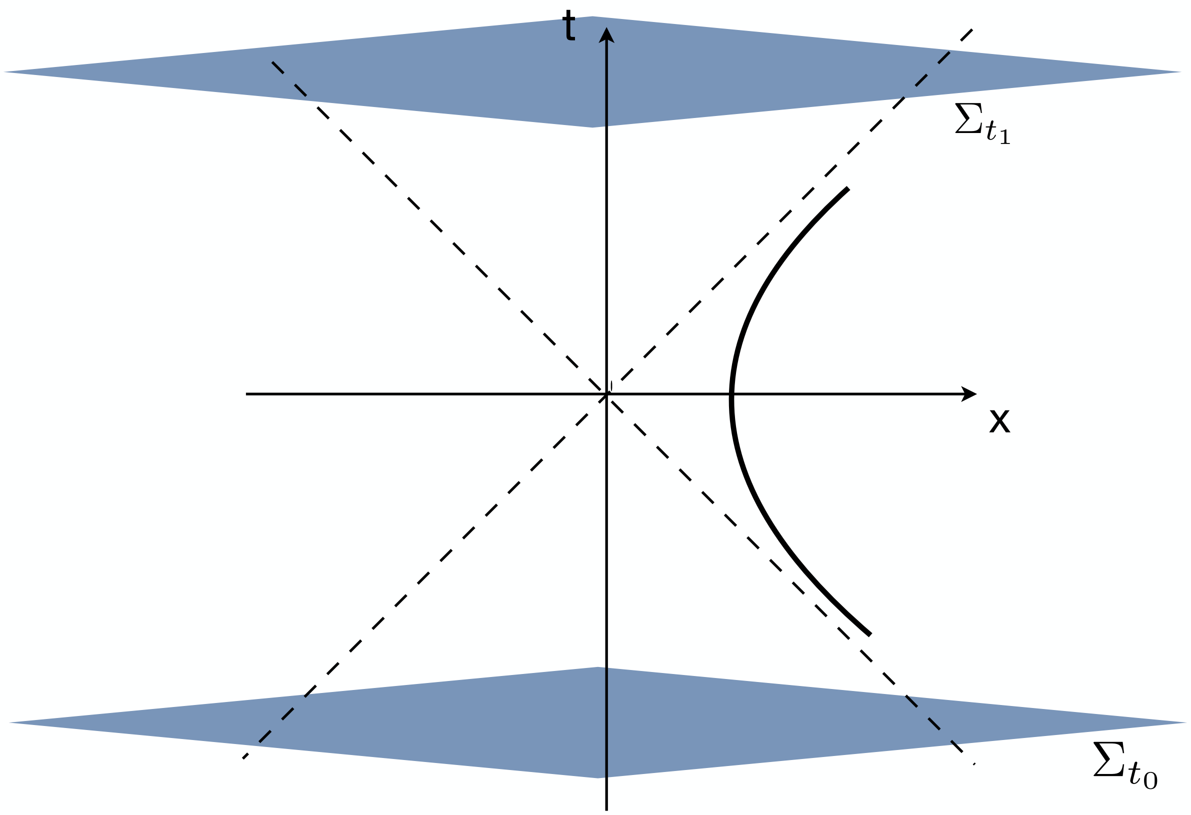
By using Unruh modes:

$$\hat{S} = \exp\left[\hat{a}_{\rm out}\left(KEj^*\right) - \hat{a}^\dagger_{\rm out}\left(KEj\right)\right]$$

Quantum Radiation and zero-energy modes

$|0_{\text{in}}^{M}\rangle = \hat{S}|0_{\text{out}}^{M}\rangle$ $\hat{\phi}_{\text{out}}$



$|0_{\text{in}}^{M}\rangle$ $\hat{S}\hat{\phi}_{\text{out}}\hat{S}^{\dagger} = \hat{\phi}_{\text{in}}.$

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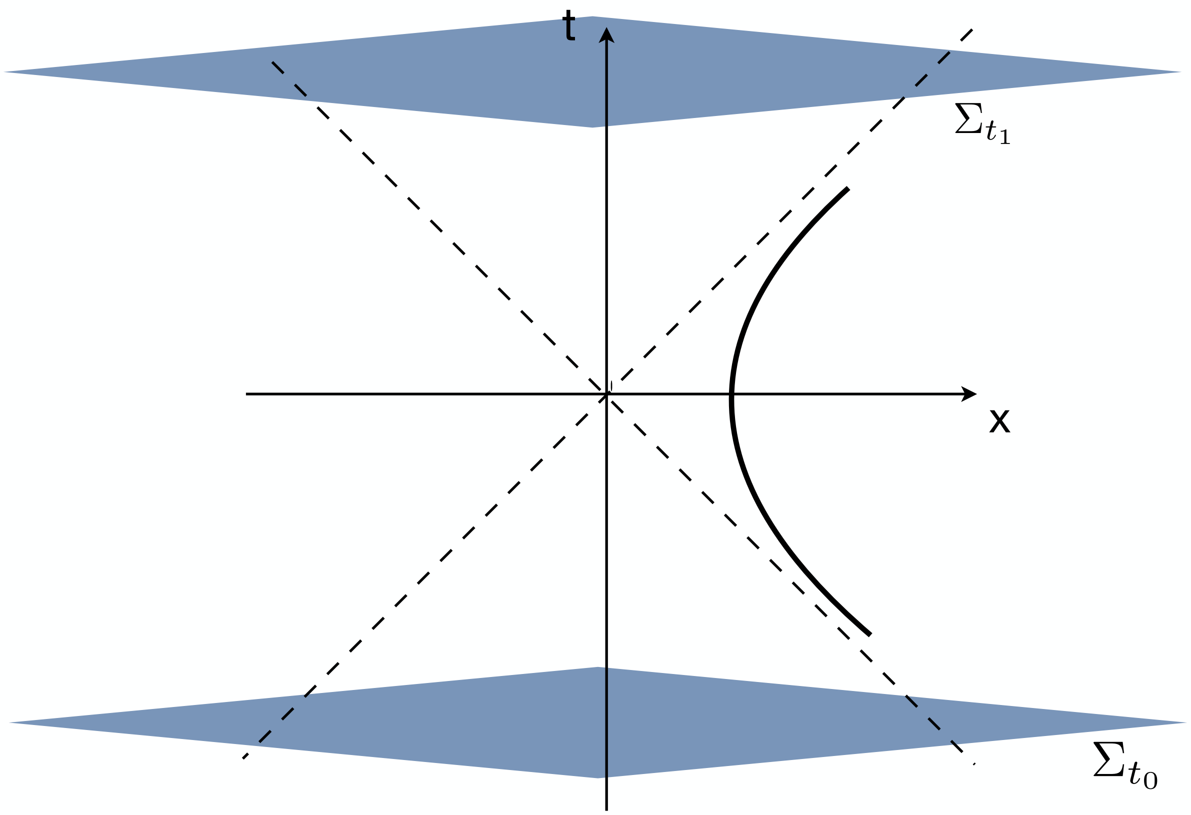
$$KEj \equiv \sum_{\sigma}\int_0^{\infty} d\omega \int d^2\mathbf{k}_{\perp}\langle w^{\sigma}_{\omega\mathbf{k}_{\perp}}, Ej\rangle_{\text{KG}}\, w^{\sigma}_{\omega\mathbf{k}_{\perp}}$$

Quantum Radiation and zero-energy modes

$$|0^M_{\rm in}\rangle = \hat{S}|0^M_{\rm out}\rangle \qquad \hat{\phi}_{\rm out}$$



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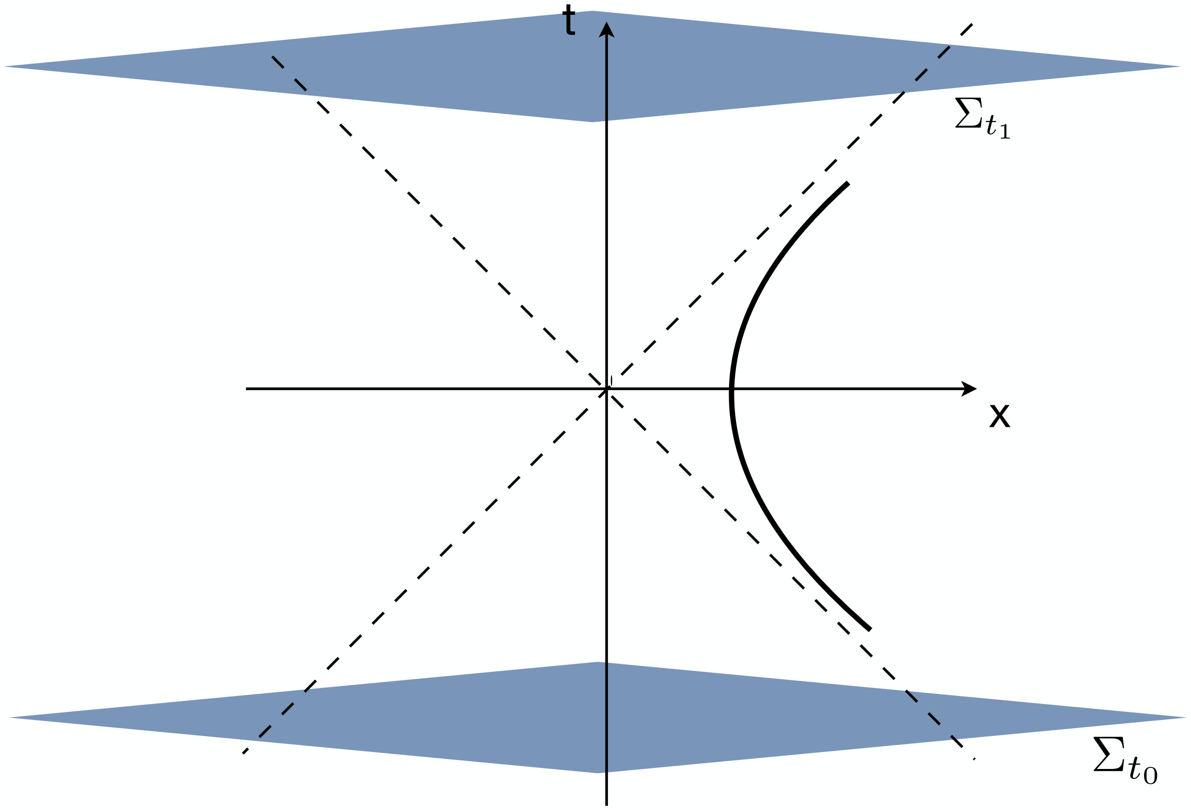
$$|0^M_{\rm in}\rangle \qquad \hat{S}\hat{\phi}_{\rm out}\hat{S}^\dagger = \hat{\phi}_{\rm in}.$$

$$|0^M_{\rm in}\rangle = e^{-\|KEj\|^2/2}e^{-\hat{a}^\dagger_{\rm out}(KEj)}|0^M_{\rm out}\rangle,$$

Quantum Radiation and zero-energy modes

$$|0_{\rm in}^M\rangle = \hat{S}|0_{\rm out}^M\rangle \qquad \hat{\phi}_{\rm out}$$



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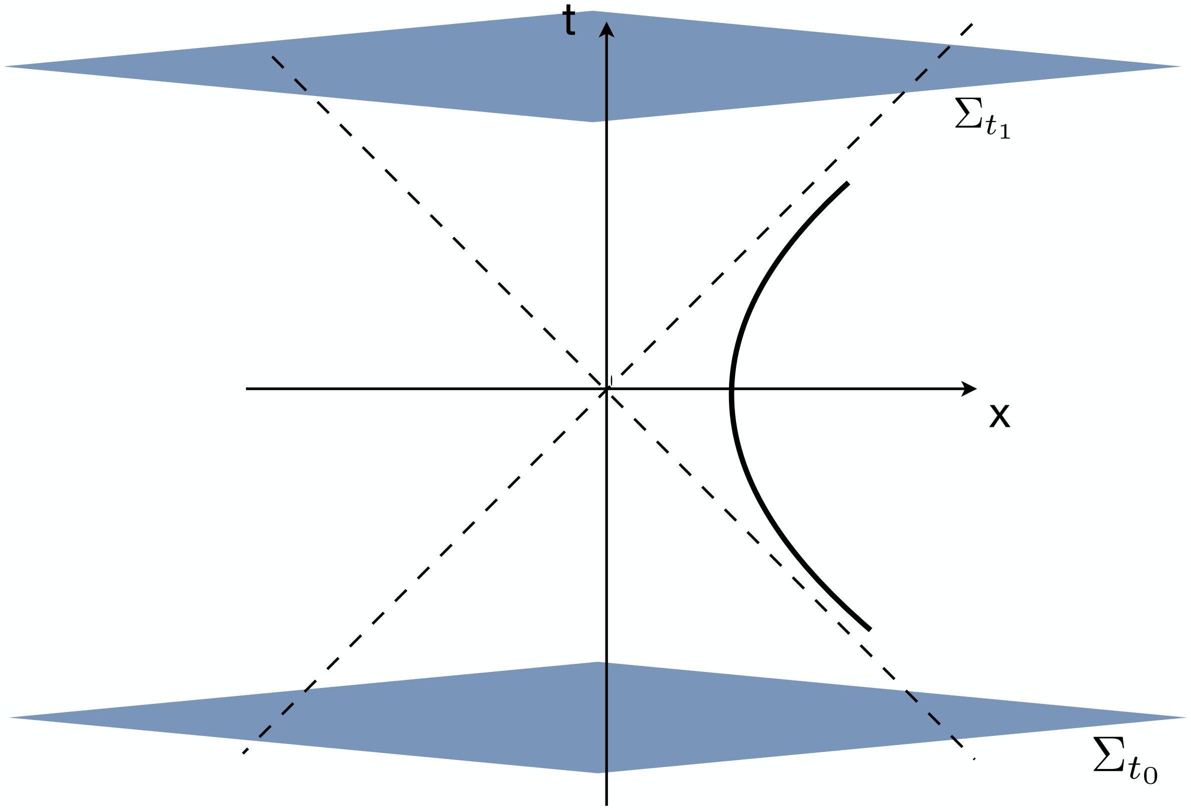
Quantum Radiation and zero-energy modes

$$|0^M_{in}\rangle = \hat{S}|0^M_{out}\rangle \qquad \hat{\phi}_{out}$$

$$|0^M_{in}\rangle = \exp\left[-\frac{T_{tot}q^2 a}{(4\pi^2}\right]\bigotimes_{\mathbf{k}_\perp}\exp\left[\frac{iqK_0(k_\perp/a)}{\sqrt{2\pi^2 a}}\hat{a}^\dagger_{out}\left(w^2_{0\mathbf{k}_\perp}\right)\right]|0^M_{out}\rangle,$$

coherent state



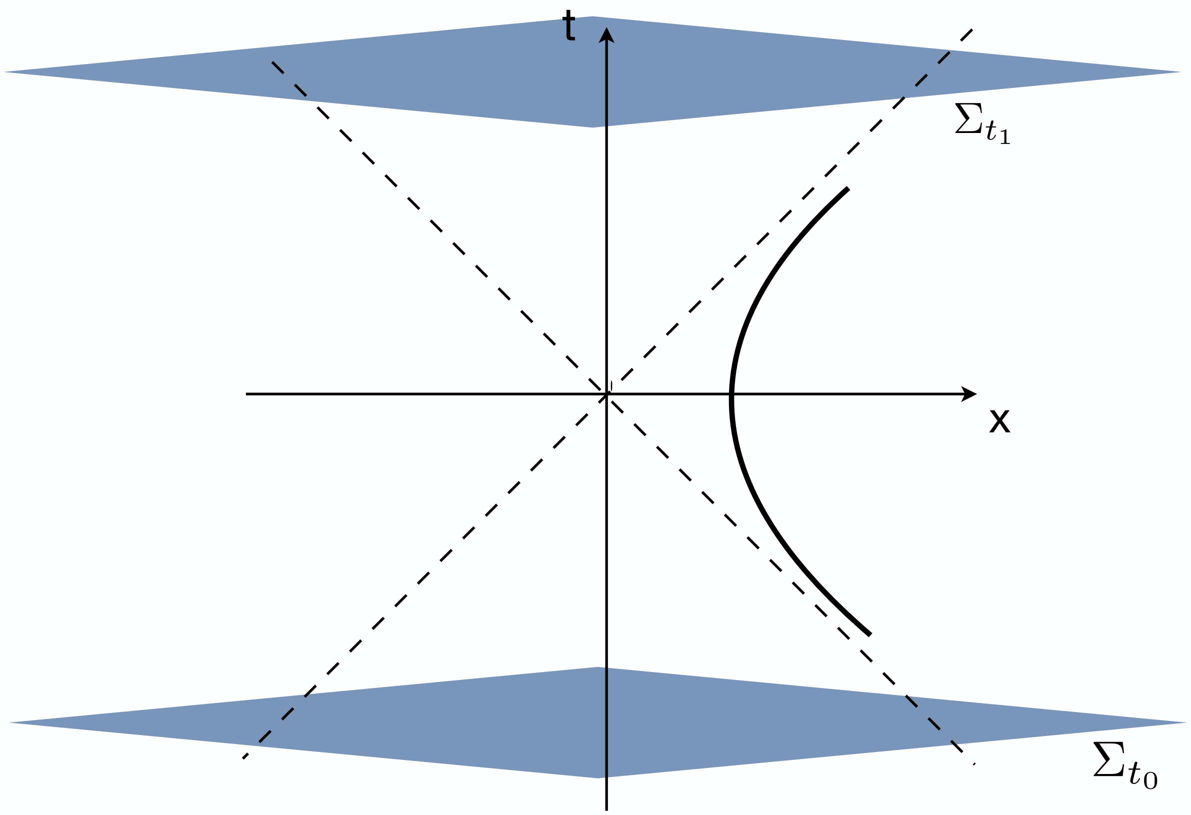
$$|0^M_{in}\rangle \qquad \hat{S}\hat{\phi}_{out}\hat{S}^\dagger = \hat{\phi}_{in}.$$

Quantum Radiation and zero-energy modes

$$|0^M_{\rm in}\rangle = \exp\left[-\frac{T_{\rm tot}q^2 a}{(4\pi^2}\right]\bigotimes_{\mathbf{k}_\perp}\exp\left[\frac{iqK_0(k_\perp/a)}{\sqrt{2\pi^2 a}}\hat{a}^\dagger_{\rm out}\left(w^2_{0\mathbf{k}_\perp}\right)\right]|0^M_{\rm out}\rangle,$$

coherent state

$$|0^M_{\rm in}\rangle = \hat{S}|0^M_{\rm out}\rangle \qquad \hat{\phi}_{\rm out}$$

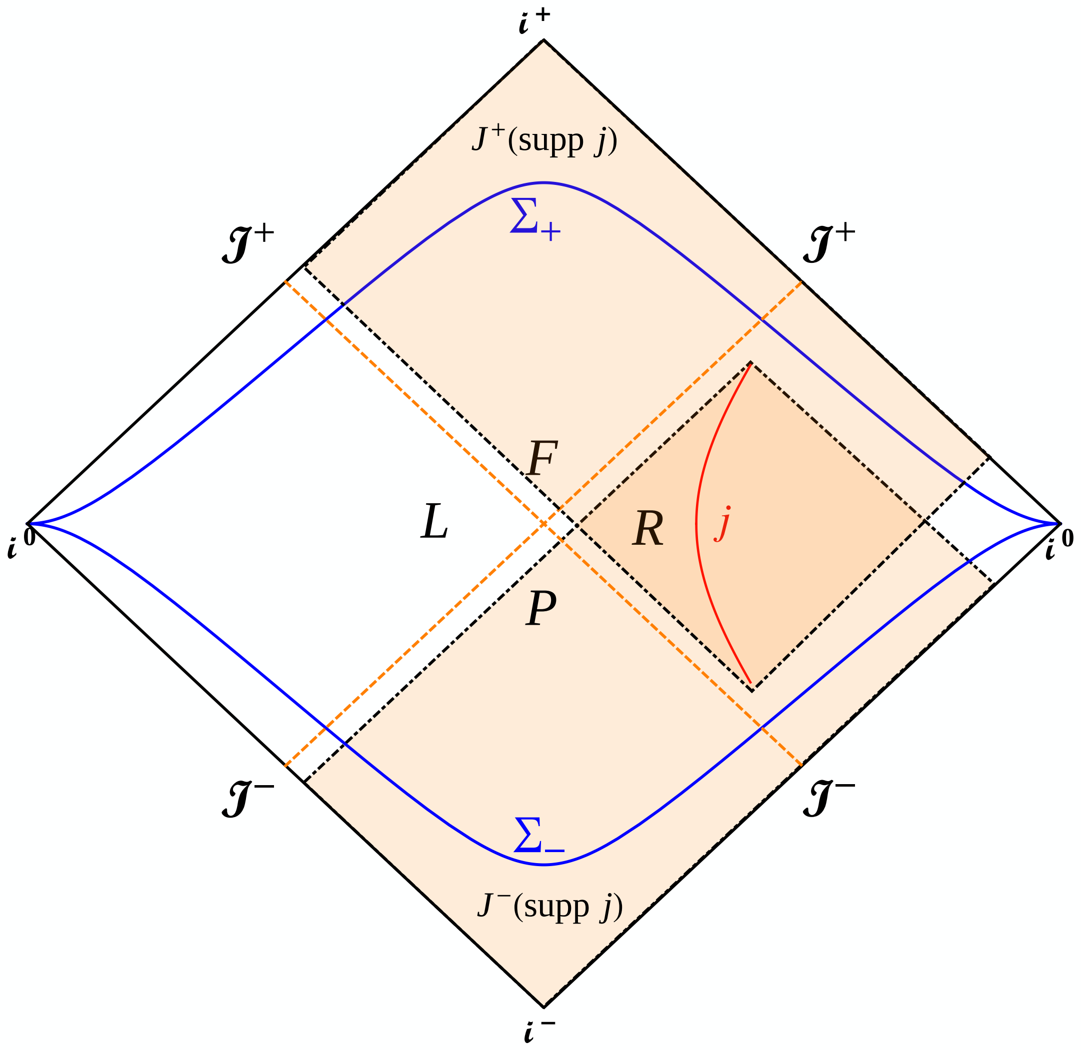


$$\langle 0^M_{\rm in}|\hat{\phi}_{\rm out}(x)|0^M_{\rm in}\rangle = Rj$$

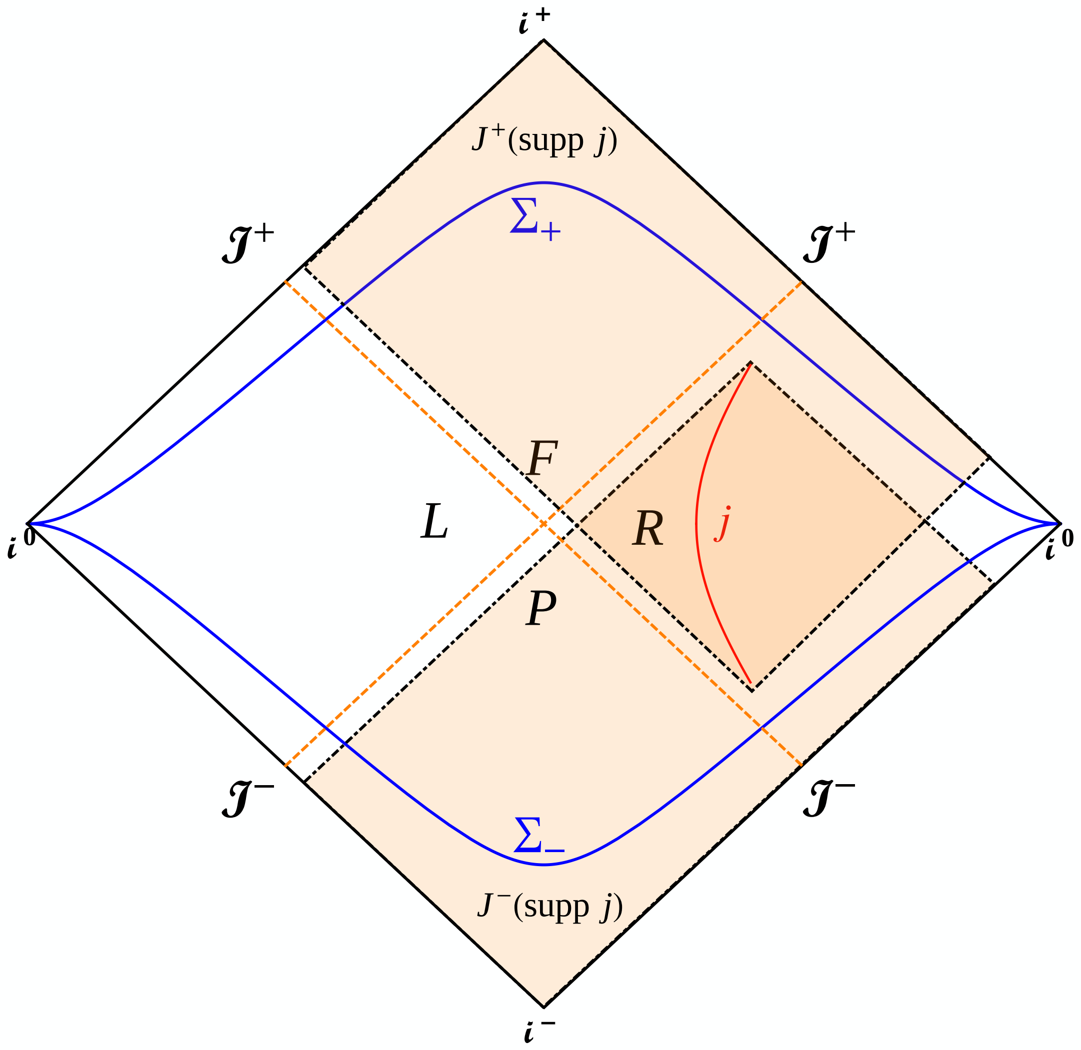
agrees with the classical solution!
(only if the state is the Minkowski vacuum)

$$|0^M_{\rm in}\rangle \qquad \hat{S}\hat{\phi}_{\rm out}\hat{S}^\dagger = \hat{\phi}_{\rm in}.$$

Quantum Radiation and bremsstrahlung



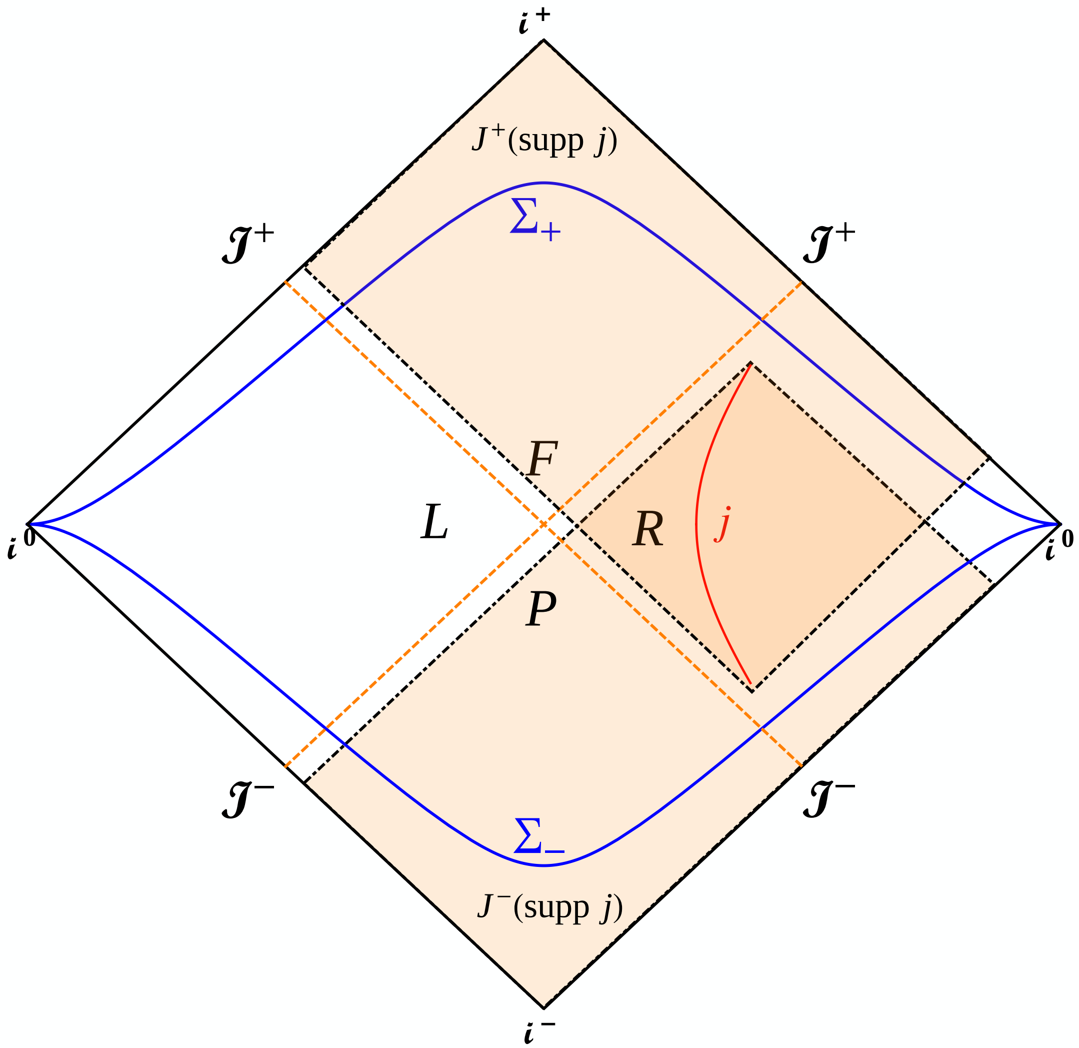
Quantum Radiation and bremsstrahlung



$$|0_{\text{in}}^M\rangle = \exp\left[-\frac{T_{\text{tot}} q^2 a}{(4\pi^2}\right] \bigotimes_{\mathbf{k}_\perp} \exp\left[\frac{iqK_0(k_\perp/a)}{\sqrt{2\pi^2 a}} \hat{a}_{\text{out}}^\dagger\left(w_{0\mathbf{k}_\perp}^2\right)\right] |0_{\text{out}}^M\rangle,$$

coherent state

Quantum Radiation and bremsstrahlung



$$|0_{\rm in}^M\rangle = \exp\left[-\frac{T_{\rm tot}q^2 a}{(4\pi^2}\right]\bigotimes_{\mathbf{k}_\perp}\exp\left[\frac{iqK_0(k_\perp/a)}{\sqrt{2\pi^2 a}}\hat{a}^\dagger_{\rm out}\left(w^2_{0\mathbf{k}_\perp}\right)\right]|0^M_{\rm out}\rangle,$$

coherent state

Classical Particle Number

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Exactly the classical result

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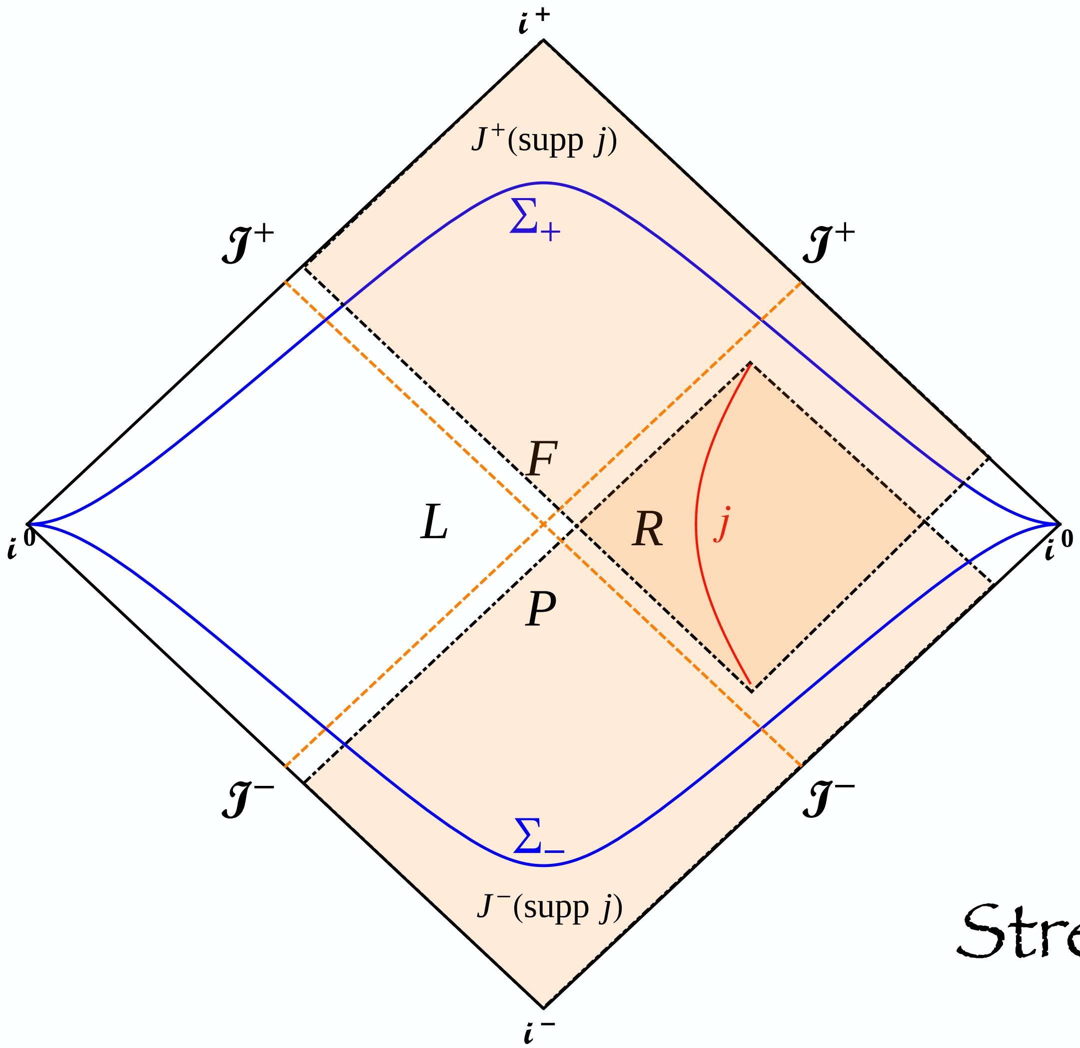

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Stress-energy tensor

$$\langle 0^M_{\text{in}}| : \hat{T}^{\text{out}}_{ab} : |0^M_{\text{in}}\rangle \equiv \nabla_a Rj \nabla_b Rj - \frac{1}{2}\eta_{ab}\nabla^c Rj \nabla_c Rj,$$

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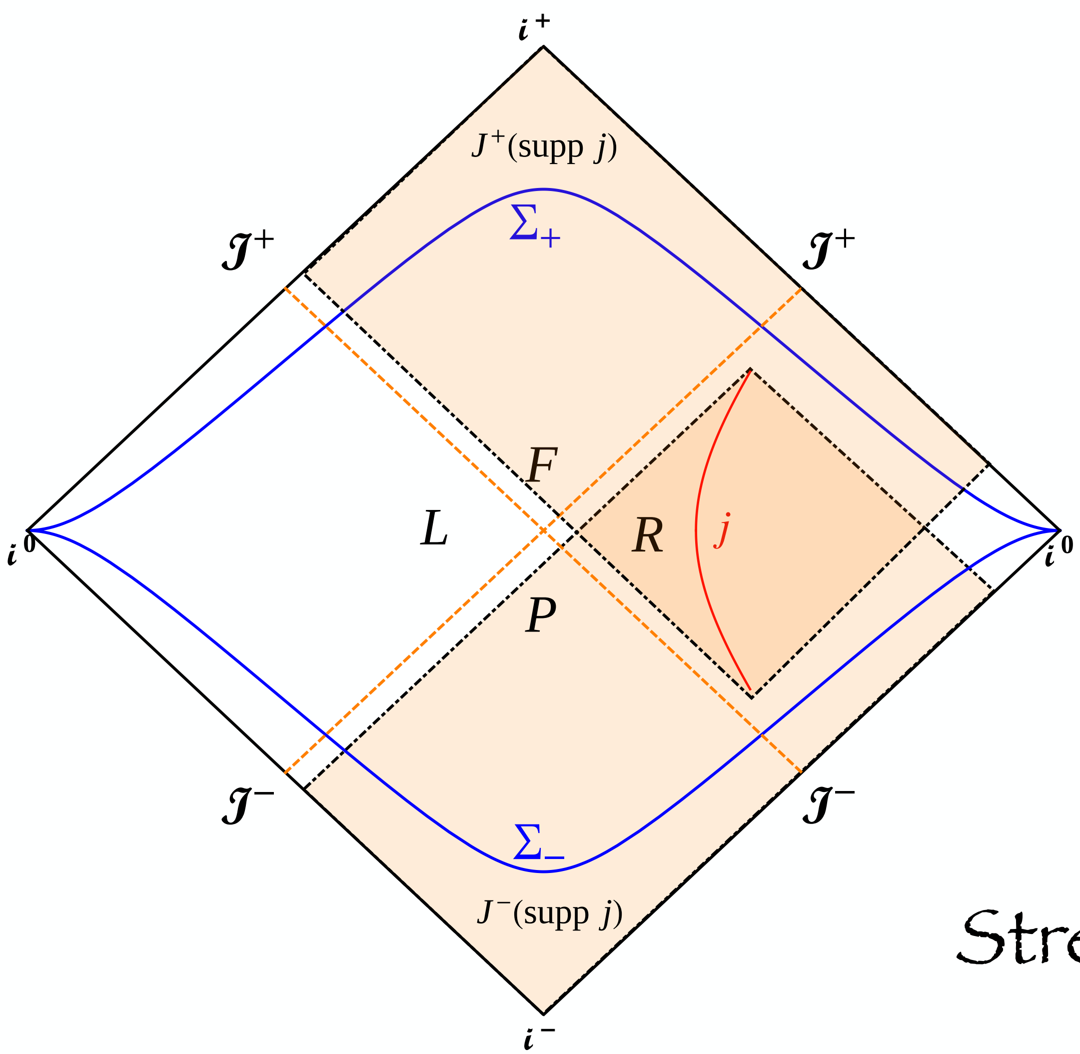

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(scalar) Larmor formula

$$\int dS^b\langle 0^M_{\rm in}| : \hat{T}^{\rm out}_{ab} : |0^M_{\rm in}\rangle(\partial_t)^a = \frac{q^2a^2}{12\pi}$$

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➡ We We believe that our results put to rest any doubts questioning the relationship between the Unruh effect and the classical Larmor radiation.