The Particle and Energy Cost of Vacuum Entanglement

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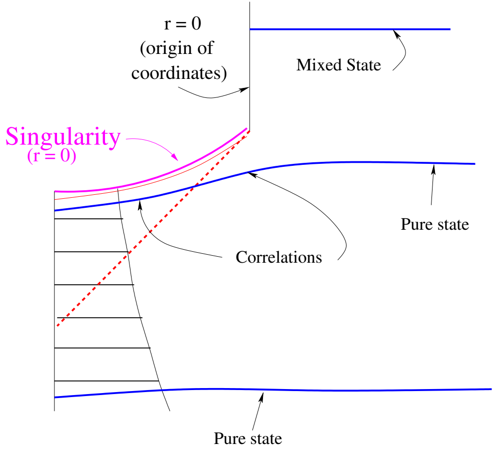
(Apparent) Information Loss in Black Hole Evaporation



In a spacetime in which a black hole forms, there will be entanglement between the state of quantum field observables inside and outside of the back hole.

Apparent Information Loss (cont.)

In particular, the Hawking particles emitted by the black hole are entangled with "particles" inside the black hole. After the black hole evaporates, the final state should be mixed. I have not understood the discomfort many physicists have with this conclusion. I particularly do not understand why ideas that require drastic modifications of local physics in a low curvature regime (firewalls, fuzzballs) have been embraced as alternatives.

However, one idea that does not require modification in a low curvature regime is that all the "information" comes out in a "final burst" at the end of the evaporation process. This idea is usually dismissed because it is assumed that one would need to emit as many particles in the burst as there are Hawking particles, but the energy of each particle should be Planckian, so one does not have nearly enough energy to carry away the information.

Possible Information Restoration Via Vacuum Entanglement

However, a few years ago, Hotta, Schutzhold, and Unruh showed that this is not necessarily the case: In a $1+1$ dimensional mirror model of Hawking radiation, they argued that the information is restored via entanglement of Hawking particles with late time vacuum fluctuations. This opens the possibility of having a way of restoring information with little or no energy cost.

Goal of this work: Estimate the energy cost of such vacuum entanglement.

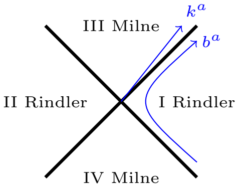
How to Define "Particles"

In this talk, I will restrict consideration to a free KG scalar field.

In any globally hyperbolic spacetime with a (global) time translation symmetry, can define a "one particle Hilbert space" as the positive frequency solutions to the KG equation with finite KG norm. One can then define a Fock space associated with this one-particle Hilbert space and express the quantum scalar field operator in terms of annihilation and creation operators on this Fock space.

In a general globally hyperbolic spacetime, one can construct a representation of the quantum field by a similar construction, choosing any definition of "positive frequency solutions" such that (i) the KG product is positive definite on the positive frequency solutions, (ii) the positive frequency solutions are KG orthogonal to the negative frequency solutions ($\equiv$ complex conjugate of positive frequency solutions) and (iii) the positive and negative frequency solutions are suitably "complete."

$1+1$ Dimensional Minkowski Spacetime



It is well known that one can use the boost Killing field b^a to define "Rindler quantization" in regions I and II. Similarly, for a massless (scale invariant) field, can use the dilation conformal Killing field k^a to define "Milne quantization" in regions III and IV. The boost and dilation Killing fields coincide (up to scale) on each horizon. This means that a left moving Rindler particle in wedge I becomes a Milne particle in wedge III! For convenience, normalize b^a and k^a so they have surface gravity κ on the horizon.

Positive Frequencies

Key fact that underlies the calculation of the Hawking and Unruh effects: Let $f_{I\omega}$ be a wave packet in region I that is positive frequency with respect to b^a, with frequency peaked sharply about ω. Let $\bar{f}_{II\omega}$ be the wave packet in region II obtained by wedge reflection of $f_{I\omega}$. Then

$$F_{1\omega} = f_{I\omega} + e^{-\pi\omega/\kappa}\bar{f}_{II\omega}$$

and

$$F_{2\omega} = f_{II\omega} + e^{-\pi\omega/\kappa}\bar{f}_{I\omega}$$

are purely positive frequency with respect to inertial time.

Minkowski Vacuum in Terms of Rindler and Milne Particles

This relationship between Rindler and Minkowski notions of positive frequency implies that the Minkowski vacuum $|0\rangle_M$ is given in terms of Rindler particles by

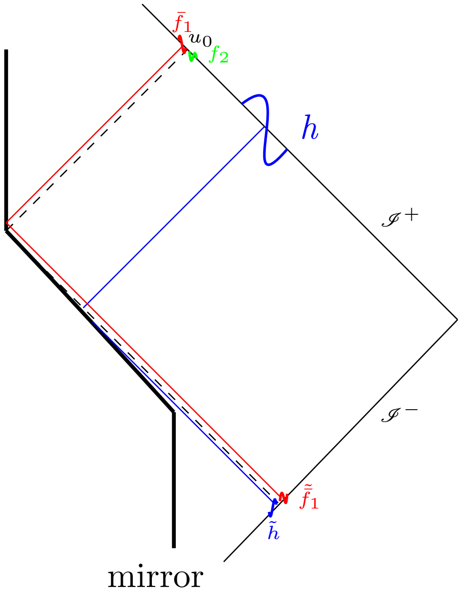
$$|0\rangle_M = \Pi_i \left(\sum_n e^{-n\pi\omega_i/\kappa} |n\rangle_{iI} |n\rangle_{iII} \right)$$

Similarly, the Minkowski vacuum $|0\rangle_M$ is given in terms of Milne particles by

$$|0\rangle_M = \Pi_i \left(\sum_n e^{-n\pi\omega_i/\kappa} |n\rangle_{iIII} |n\rangle_{iIV} \right)$$

When restricted to *any* of the 4 wedges, $|0\rangle_M$ is a thermal state at temperature $T = \kappa/2\pi$.

Moving Mirror Spacetime



Moving Mirror Spacetime (cont.)

Start with a static mirror and with the quantum field in its static ground state. During intermediate times, have the mirror move (in terms of null coordinates $v = t + x$, $u = t - x$) according to

$$v = -\frac{1}{\kappa} e^{-\kappa u} .$$

Finally, after a sufficiently long period of motion of this form, have the mirror move inertially again (shown "at rest" in the diagram, but it is allowed be moving relative to its original rest frame).

Two Important Observations

- The state of the field to the future of event at which the mirror becomes inertial again is identical to the static vacuum for that inertial mirror motion.
- A purely positive frequency "Hawking wave packet" h with (inertial) frequency peaked sharply about ω will propagate backwards in time to a Milne wave packet $\tilde{h}$ of frequency ω (blue). If we add $\exp(-\pi\omega/\kappa)$ times its reflection, $\bar{\tilde{\tilde{f}}}_1$ (red) about $v = 0$, we get a purely positive frequency solution with respect to inertial time.

Particle Creation

Nontrivial particle creation (with respect to given notions of "in" and "out" particles) starting from $|0\rangle_{\text{in}}$ occurs if an initially positive frequency solution picks up a negative frequency part under evolution. Define positive frequency for "in" states using the usual notion of inertial time translations. However, define "positive frequency" for "out" states using (i) inertial time translations for wave packets emerging at "early times" and Milne/Rindler time translations for wave packets emerging at times near the retarded time at which the mirror becomes inertial again. This is consistent if there is a large "time separation" between these time eras, as I will assume.

Particle Creation (cont.)

The initial positive frequency wave packet $\tilde{h} + \exp(-\pi\omega/\kappa)\tilde{\bar{f}}_1$ evolves to $h + \exp(-\pi\omega/\kappa)\bar{f}_1$, which gives its decomposition into positive and negative frequency parts. It follows that the "out" state is:

$$\Psi = \left(\sum_n e^{-n\pi\omega/\kappa} |n\rangle_h |n\rangle_{f_1} \right) \otimes \Psi'$$

where Ψ' describes the state of the system with respect to the modes that are orthogonal to both h and f_1 in our one-particle "out" Hilbert space. This shows that (i) the Hawking radiation is thermal at temperature $T = \kappa/2\pi$, and (ii) the Milne radiation is thermal at temperature $T = \kappa/2\pi$, as it must be, since the field is in the (inertial) vacuum state after the mirror becomes inertial again. The Hawking particles are entangled with Milne particles. All of the "information" about the Hawking radiation is stored in vacuum fluctuations!

The Particle Cost

At first sight, it may appear that information has been restored at no particle or energy cost, since the information is stored in the vacuum state, which has no particles or energy. However, this is not the case because $|n\rangle_{f_1}$ is entangled with $|n\rangle_h$ rather than with its reflection $|n\rangle_{f_2}$. This means that the Rindler mode f_2 cannot be in a thermal state (or would have to entangle in some improper way with some other mode, etc.). We can see more explicitly what goes wrong by considering the inertial positive frequency mode

$$F_1 = \frac{f_1 + e^{-\pi\omega/\kappa}\bar{f}_2}{\sqrt{1 - e^{-2\pi\omega/\kappa}}}$$

The annihilation operator for F_1 is given in terms of the annihilation and creation operators for f_1 and f_2 by

$$a(F_1) = \frac{a(f_1) - e^{-\pi\omega/\kappa}a^\dagger(f_2)}{\sqrt{1 - e^{-2\pi\omega/\kappa}}}$$

The Particle Cost (cont.)

Thus, the expected number of inertial particles in the mode F_1 is

$$\begin{aligned} \langle\Psi|a^\dagger(F_1)a(F_1)|\Psi\rangle &= \frac{1}{1-e^{-2\pi\omega/\kappa}}\left[\langle\Psi|a^\dagger(f_1)a(f_1)\right. \\ -e^{-\pi\omega/\kappa}a^\dagger(f_1)a^\dagger(f_2) &- e^{-\pi\omega/\kappa}a(f_1)a(f_2)+ \\ &+ \left. e^{-2\pi\omega/\kappa}a(f_2)a^\dagger(f_2)|\Psi\rangle\right] \end{aligned}$$

The terms $a^\dagger(f_1)a^\dagger(f_2)$ and $a(f_1)a(f_2)$ make vanishing contribution on account of the form of Ψ. Thus, we obtain,

$$\langle\Psi|N(F_1)|\Psi\rangle \geq \langle\Psi|N(f_1)|\Psi\rangle = \langle\Psi|N(h)|\Psi\rangle$$

Thus, there must be at least as many "real" (inertial) particles emitted modes of the form F_1 as there are Hawking particles!

The Energy cost

The mode F_1 is not an eigenstate of inertial energy so it is not obvious how to calculate a rigorous lower bound on the total energy E associated with "late time" emission. Nevertheless, it seems clear that we must have

$$\langle E \rangle \gtrsim \sum_i \langle N(F_{i1}) \rangle e(F_{i1})$$

where $e(F_{i1})$ is the classical energy of the mode F_{i1} corresponding to the Hawking mode h_i. Since we already know that there are at least as many F_1-particles as Hawking particles, the key issue is how large $e(F_{i1})$ is.

The Energy cost (cont.)

The propagation of h back into the past produces an enormous blueshift. The energy of $\tilde{h}$ and $\tilde{f}_1$ are enormous. If the mirror is brought to rest at late times, then the energies of f_1 and F_1 will be correspondingly enormous, and so will the energy cost of the vacuum entanglement.

However, suppose that instead of bringing the mirror back to rest, we let it "glide" at the end of the process, i.e., we (smoothly) turn off the acceleration without decreasing the velocity. Then there will be a larger redshift in the forward propagation of $\tilde{f}_1$ to f_1 then there is a blueshift in the backward propagation of h to $\tilde{h}$. Therefore $e(F_{i1})$ can be made much smaller than the energy of the Hawking particles. For a "gliding" mirror, it should be possible to recover the information in the Hawking particles via vacuum entanglement at negligible energy cost.

$3 + 1$ Dimensional Black Hole Evaporation

In higher dimensions, Milne quantization of scale invariant free fields with respect to the dilation conformal Killing field can be done in the future and past light cones of a point. The Minkowski vacuum takes the same form as in $1 + 1$ dimensions in terms of Milne particles, and is a thermal state of Milne particles at $T = \kappa/2\pi$ when restricted to either the future or past light cone.

There is no explicit model of an evaporating black hole spacetime where particle creation calculations can be done to determine what the Hawking particles are entangled with. Nevertheless, it is interesting to consider the possibility that the Hawking particles are entangled with vacuum fluctuations in the future light cone of the evaporation event:

$$\Psi = \left(\sum_n e^{-n\pi\omega/\kappa} |n\rangle_h |n\rangle_{f_1} \right) \otimes \Psi'$$

where f_1 denotes the Milne particle mode entangled with the Hawking mode h.

$3 + 1$ Dimensional Black Hole Evaporation (cont.)

The analysis is essentially unchanged from the $1 + 1$ dimensional case except that there is no analog of the "gliding mirror": The modes f_1 must emerge from the order of a Planck length of the vertex of the light cone of the evaporation event, so $e(f_1)$ and $e(F_1)$ must have energy of the order of the Planck energy. Thus, the energy cost of vacuum entanglement is of the same order as in the traditional "final burst scenario" to avoid information loss.

Conclusions

Entanglement with vacuum fluctuations provides an interesting possibility to avoid information loss in black hole evaporation. However, such vacuum entanglement necessarily requires a large amount of particle creation, and the energy cost in $3+1$ dimensions appears to be comparable to traditional "final burst scenarios."