Neural Networks
MLP ANNs

Advanced Institute for Artificial Intelligence

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Topics

- Multi-Layer Perceptron
- Training
- Backpropagation network
- Input space partition
- Classification

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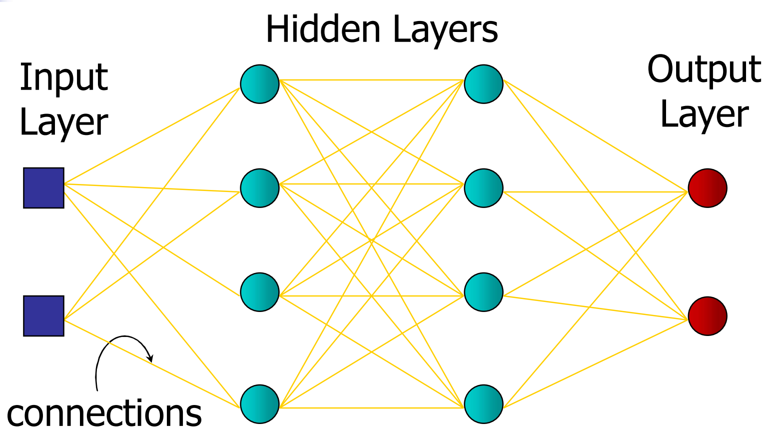
A bit of history

- (1982) Hopfield showed that ANNs could be seen as dynamic systems
- (1986) Hinton, Rumelhart e Williams, proposed an algorithm to train multi-layer perceptron (MLP) networks
  - *Parallel Distributed Processing*
  - Bryson e Ho (1959), Werbos (1974), Parker (1985) and Le Cun (1985)

Multi-Layer Perceptron (MLP)



Multi-Layer Perceptron (MLP)

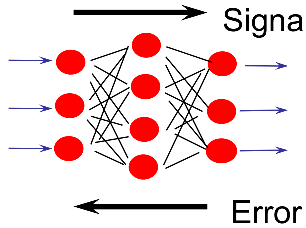
- Most used ANN model
  - One or more hidden layers
- Increased functionality
  - One hidden layer: any Boolean or continue function
  - Two hidden layers: any function
- Trained by the Backpropagation algorithm

Backpropagation

- Train by reducing errors made by the MLP network
  - Supervised
  - Error correction
    - Output layer
    - Hidden layers
      - Proportional to the error made by the nodes in the next layer

Backpropagation

- Training follows two directions
  - Forward
  - Backward



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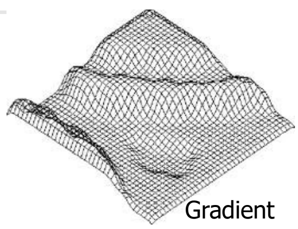
Backpropagation

- Training
  - Supervised
  - Adjust two weights: $\Delta w_{ij} = \eta x_i \delta_j$

$$\delta_j = \begin{cases} f'(net) error_j & \text{if } j \text{ is the output layer} \\ f'(net) \sum w_{jk} \delta_k & \text{if } j \text{ is a hidden layer} \end{cases}$$



Gradient function

$$error_j = \frac{1}{2} \sum_{q=1}^{c} (y_q - f(net_q)) \qquad net = \sum_{i=0}^{m} x_i w_i$$

If f (net) is a sigmoidal function, $f'(net) = f(net)(1 - f(net))$

Training is not guaranteed to converge

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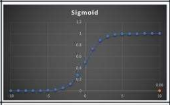
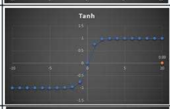
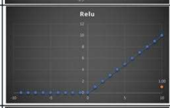
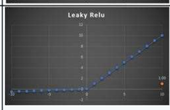
Activation functions

Name	Plot	Equation	Derivative
Sigmoid		$f(x) = \sigma(x) = \frac{1}{1+e^{-x}}$	$f'(x) = f(x)(1 - f(x))$
Tanh		$f(x) = \tanh(x) = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$	$f'(x) = 1 - f(x)^2$
Rectified Linear Unit (relu)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Leaky Rectified Linear Unit (Leaky relu)		$f(x) = \begin{cases} 0.01x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0.01 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$

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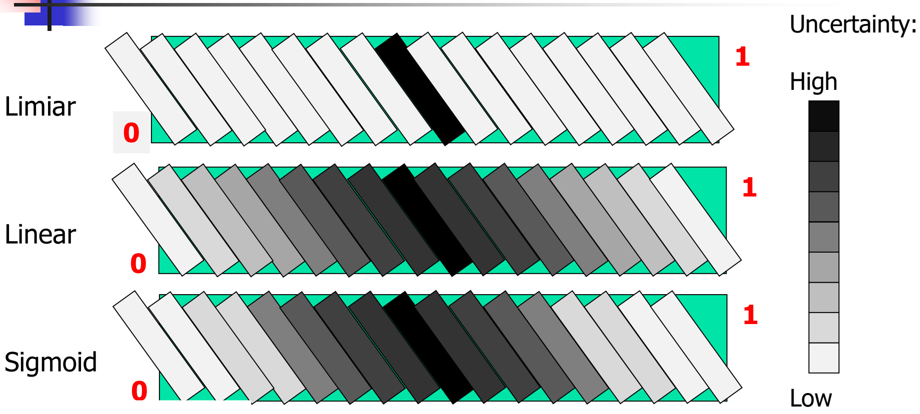
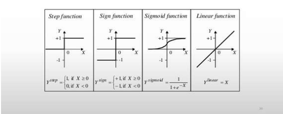
Activation functions and borders





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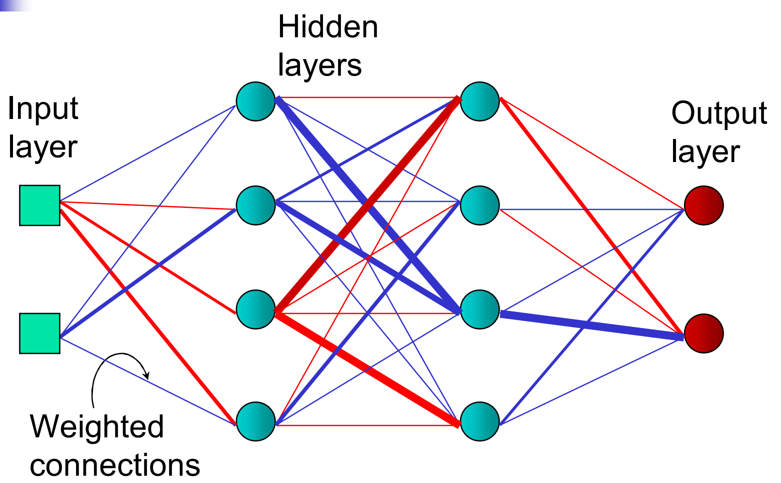
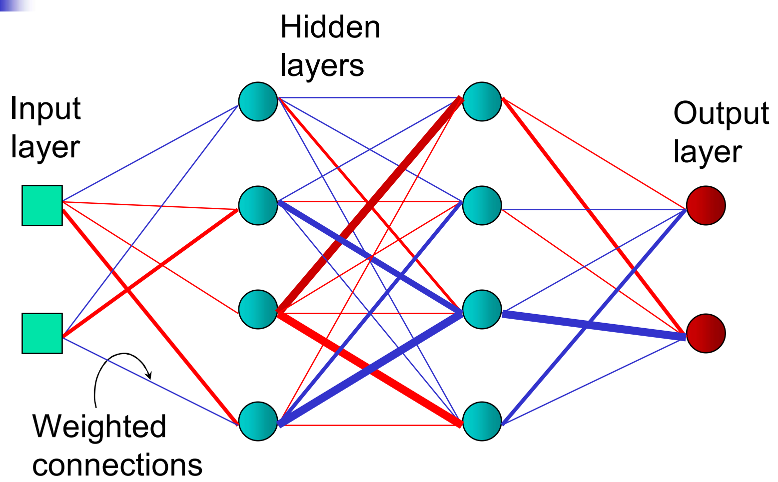
Training

start all the weights with random values
repeat
for *each training pattern (X, d)* ***do***
for *each layer k from 1 to n* ***do***
for *each node j from 1 to* m_k ***do***
calculate the output y_{jk}
if layer = n
then calculate error
if error $> \varepsilon$
then for *each layer k from n to 1* ***do***
for *each node j from 1 to* m_k ***do***
update weight values
until *error* $\leq \varepsilon$

Training updating weights



Training updating weights



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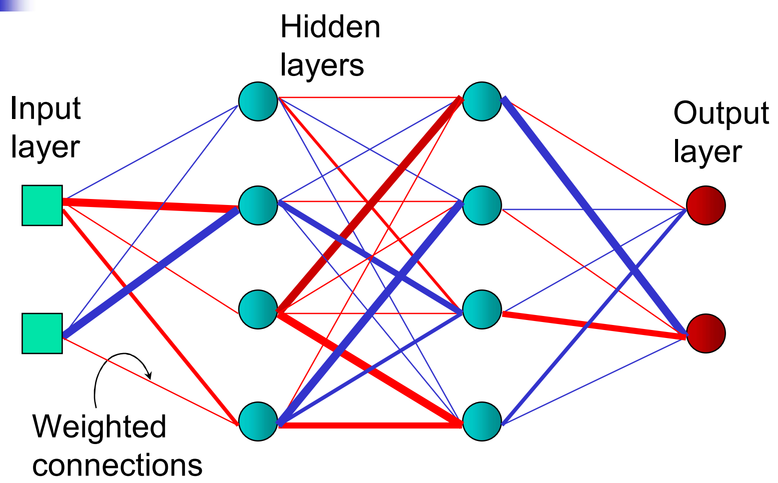
Training updating weights



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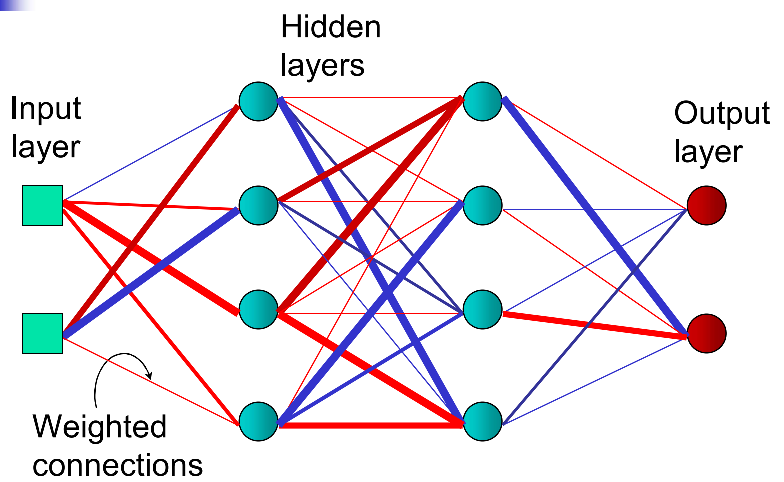
Training updating weights



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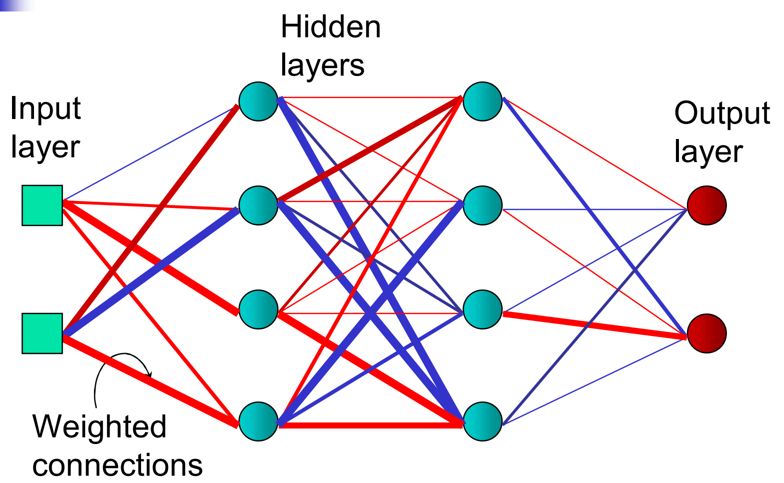
Training updating weights



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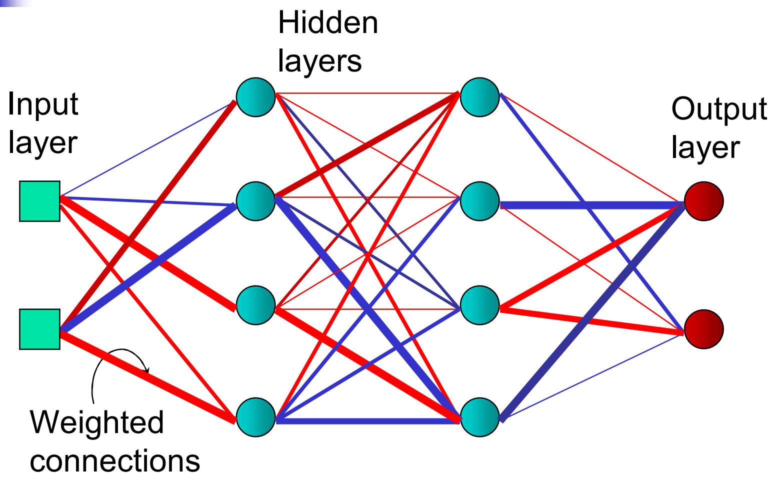
Training updating weights



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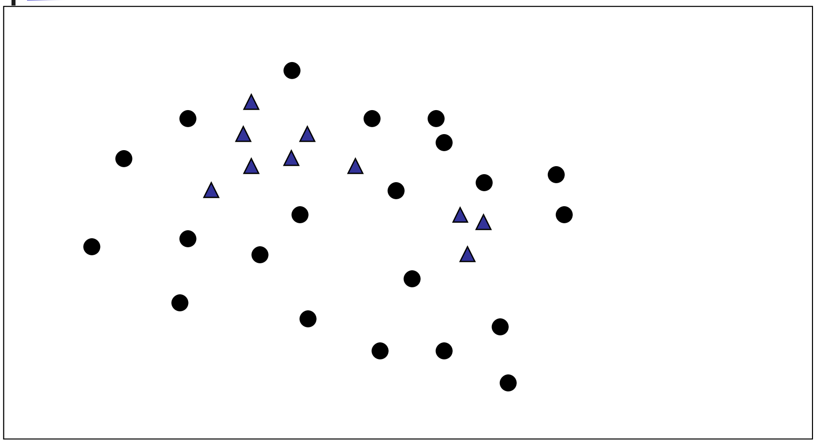
Training changing borders



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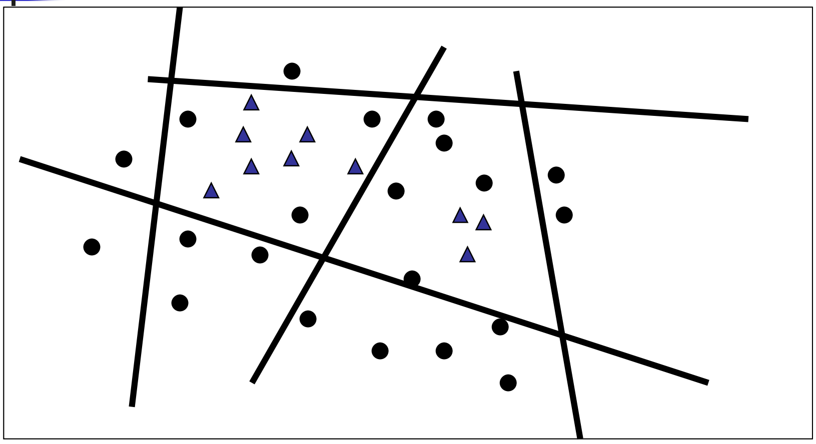
Training changing borders



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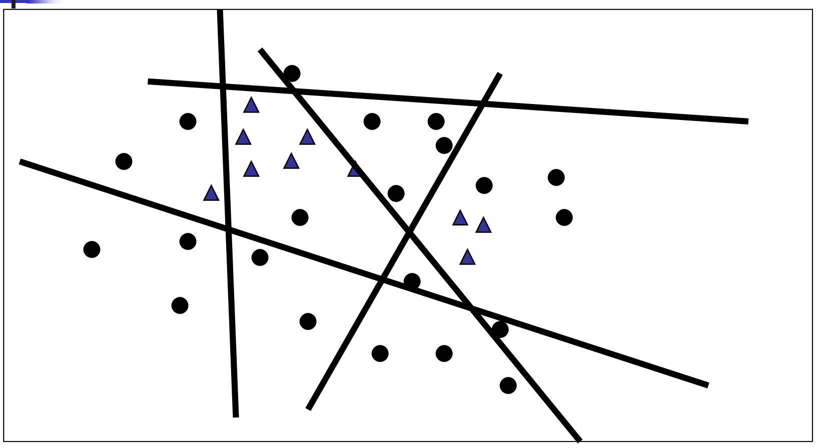
Training changing borders



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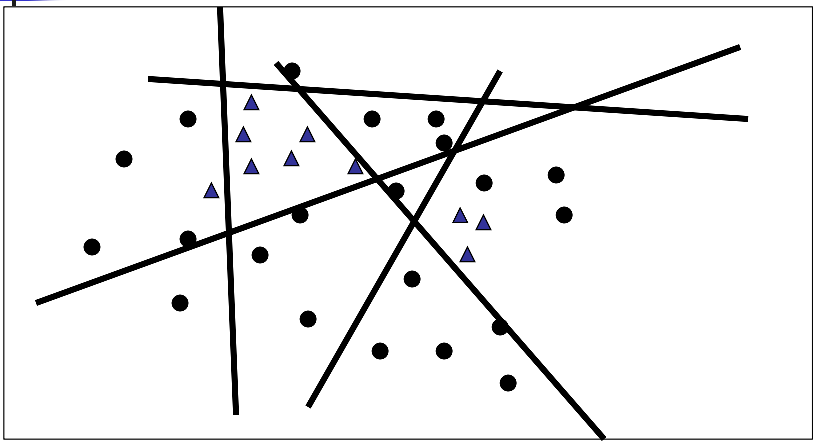
Training changing borders



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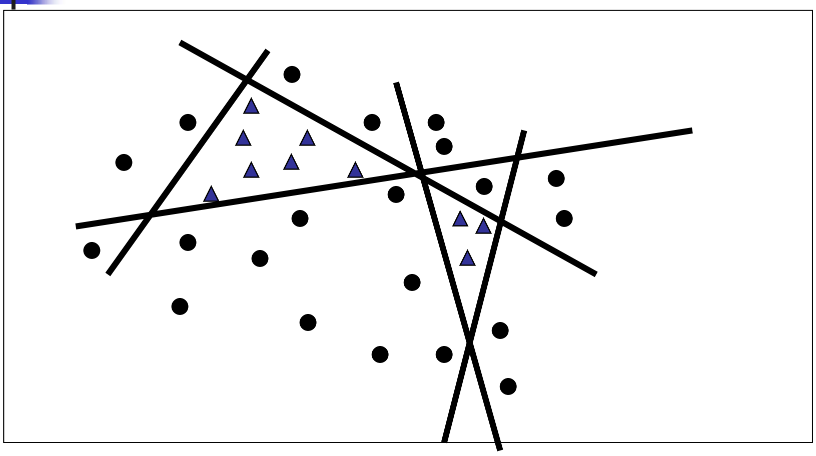
Training changing borders



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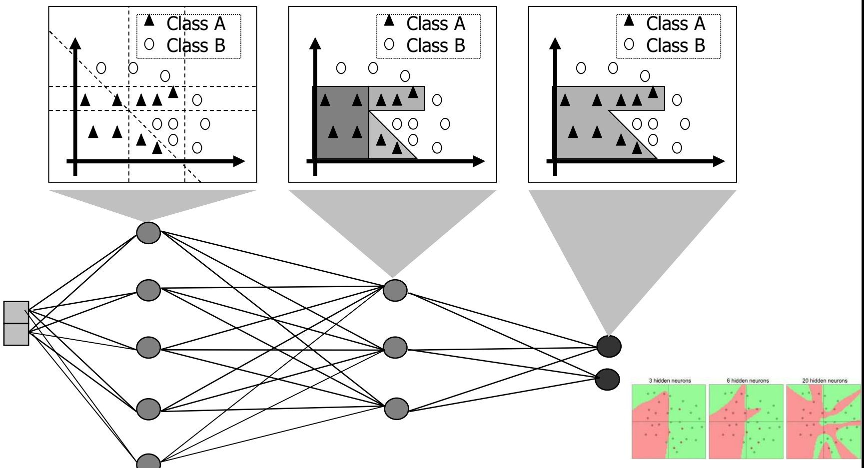
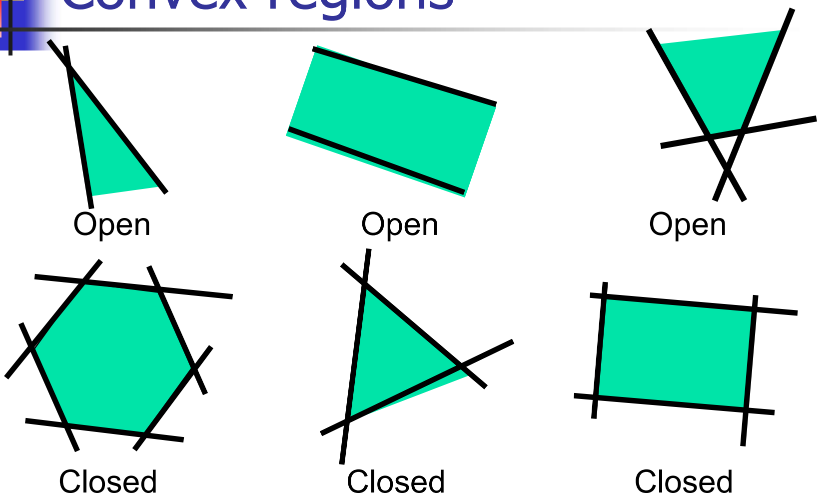
Test

for *each test pattern* ***do***
 for *each layer k from 1 to n* ***do***
  for *each node j from 1 to* m_k ***do***
   calculate the output y_{jk}
   compare Y and D
   classify the test pattern X as belonging to the class whose desired output is closer to the produced output

MLPs as classifiers



Convex regions



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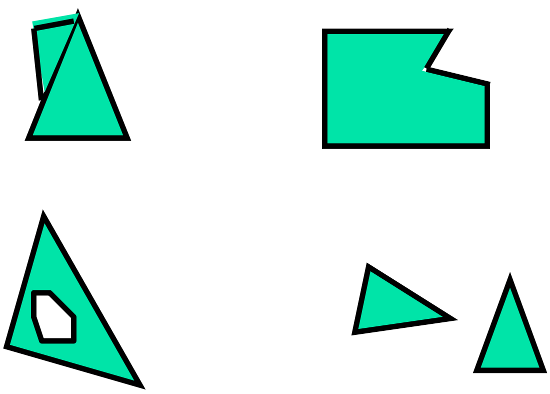
Combinations of convex regions



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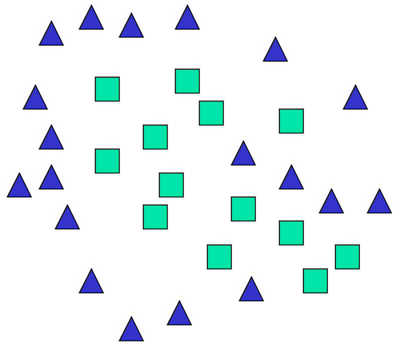
Combinations of convex regions

- Find decision boundaries that separate the data below:



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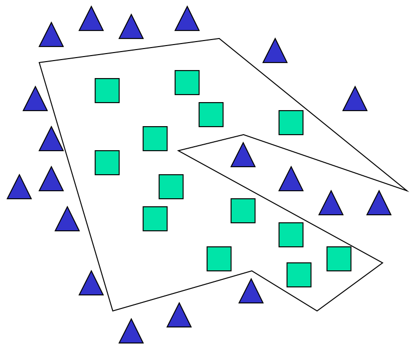
Combinations of convex regions

- Find decision boundaries that separate the data below:



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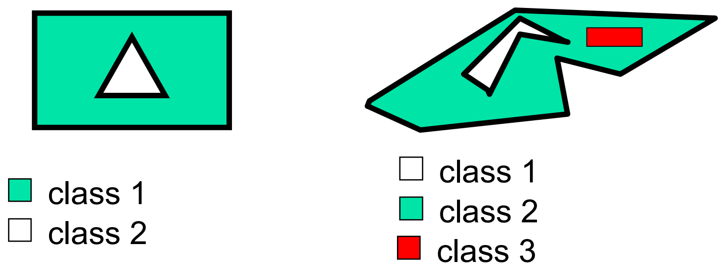
Quiz 1

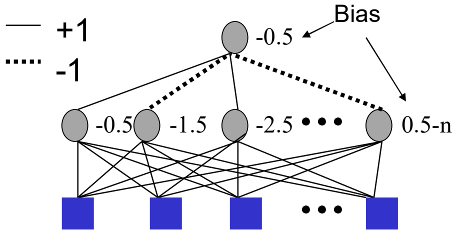
- How many layers and at least how many neurons in each layer have the networks that divide the input space of the shapes below:



Exercise

- Given the ANN below whose input is a n-bit binary vector and output is a binary value:
  - Indicate the function implemented:
  - Explain what each neuron does

The activation function uses step functin with threshold

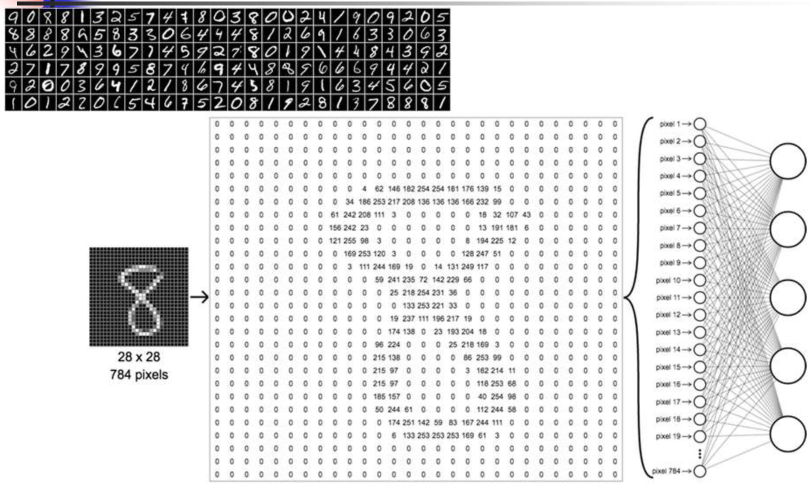


Exercise

- Parity
  - One of the limitations of the Perceptron raised by Minsky and Papert
- Difficult problem
  - More similar patterns require different responses
  - Uses n intermediate units to detect parity in binary vectors with n elements

Example

Example



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Other training algorithms

- Backpropagation momentum
- Resilient propagation (Rprop)
- Quickprop
- Newton
- Levenberg Marquardt
- Super Self-Adjusting Backpropagation (superSAB)
- Conjugate gradient algorithms
- ...

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Backpropagation momentum

- Adds a momentum term to the weight update equation
  - If the last and current weight update go in the same direction, current update is larger
    - Direction: increase or decrease weight
  - Specifies the amount of the old weight change to be added to the current change
  - Increase chance to escape from local minima

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Quiz 2

- What are limitations of MLP?

  A) Only works with up to 2 hidden layers

  B) Can learn any function using 2 hidden layers

  C) Can use any nonlinear activation function

  D) Can be used only for classification tasks

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Next:
Other Neural Networks

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